Comment on 'Salesforce compensation with Inventory Considerations'

Fabrizio Rodríguez
University of Montevideo

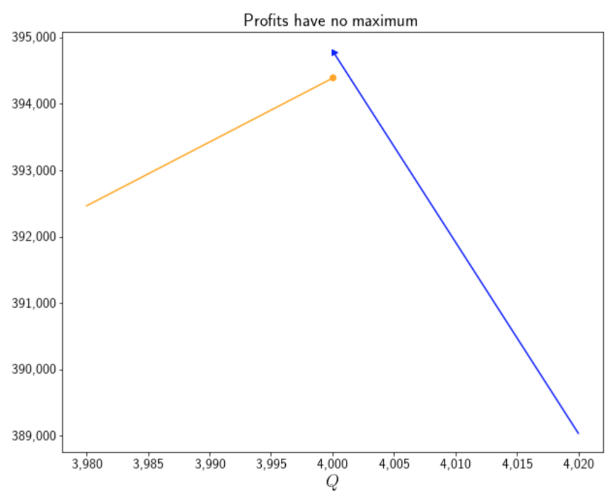
Online Appendix

This Online Appendix addresses four key issues in the following order: first, it presents an example demonstrating the non-existence of equilibrium under the original setting; second, it outlines the three possibilities for stocking; third, it explores the discretization solution to address the equilibrium existence problem; and finally, it discusses the validity of the main results in Dai and Jerath's original 2013 paper.

1. Example

Let $p = (p_L, p_M, p_H) = (0.01, 0.96, 0.03)$, $q = (q_L, q_M, q_H) = (0.5, 0.49, 0.01)$, which satisfy all of the assumptions in the paper. Additionally, let $L = 3000$, $M = 4000$, $H = 5000$, $\psi = 800$, $r = 400$, $c = 300$, which satisfies $c < C$ from (2). In this situation the firm's expected profits if it stocks $Q = H$ are 107,700; if it stocks $Q = M$ profits are 394,384; whereas if it stocks $Q = M + \Delta$ are 394,800$-$288Δ, which is larger than 394,384 if Δ is small.

Graphically:



2. Stocking possibilities

There are three posibilies of stocking: $Q = M$, $Q = H$ or $Q^H_M \in (M, H)$.

In the first one, $Q = M$. Here if demand is not L, it cannot be known if demand was H or M, so it is optimal to pay $S_{HM} = \frac{\psi}{q_L - p_L}$ and $S_L = 0$. The firm's expected profit is

$$\pi_M = (p_H + p_M)rM + p_L rL - \frac{\psi}{1 - \frac{q_H + q_M}{p_H + p_M}} - cM.$$

In the second one, $Q = H$, the demand is fully revealed ex post, in which case it is optimal to pay $S_H = \frac{\psi}{p_H - q_H}$ and $S_M = S_L = 0$ from (**??**). Furthermore, the firm's expected profit is

$$\pi_H = p_H r H + p_M r M + p_L r L - \frac{p_H \psi}{p_H - q_H} - cH.$$

In the last one, $Q_M^H \in (M, H)$, the demand will also be fully revealed and the firm optimally only pays a positive wage when demand is H. The firm's expected profit is:

$$\pi_{Q_M^H} = p_H r Q_M^H + p_M r M + p_L r L - \frac{p_H \psi}{p_H - q_H} - c Q_M^H.$$

I will now show that stocking $M + \Delta$ (for a small value of Δ) dominates stocking M. The reason is that because stocking that extra bit allows the principal to distinguish between M and H, and that lowers the compensation cost. This point can be seen by the following equation:

$$\pi_{Q_M^H} - \pi_M = (p_H r - c)\Delta + \frac{(p_H q_M - p_M q_H)\psi}{(p_H - q_H)(q_L - p_L)}.$$

This is the disparity in compensation costs attributed to understanding whether H has occurred as opposed to being uncertain about whether M or H occurred. From (1), $p_H q_M - p_M q_H > 0$, $p_H > q_H$, and $q_L > p_L$. Moreover, if $c \leq \bar{c}$, profits difference are positive for any $\Delta > 0$. However, if $c > \bar{c}$, profits difference are positive for any $\Delta < \frac{\psi}{(c - p_H r)} \frac{(p_H q_M - p_M q_H)}{(q_L - p_L)(p_H - q_H)} < H - M$.

In addition, when the stocking cost c surpasses the threshold $\bar{c}$, it becomes better to stock $M + \Delta$ instead of H: $M + \Delta$ yields the same demand information as H, but conveys a lower cost.

3. Discussion

To address the non-existence problem one potential solution is to discretize the decision variable Q to avoid the issue highlighted by the example when $c > p_H r$. In that case, for a fixed set of parameters, the optimal levels of inventory will be either $M + \Delta$ where $\Delta < \frac{\psi}{(c - p_H r)} \left(\frac{p_H q_M - p_M q_H}{(q_L - p_L)(p_H - q_H)} \right)$, or H as follows:

$$Q^* = \begin{cases} H & \text{if } c < \bar{c}, \\ M + \Delta & \text{if } c \geq \bar{c}. \end{cases}$$

In this situation, the reward for achieving the highest possible sales outcome is $S^* = \frac{\psi}{p_H - q_H}$ (it is cheaper to always pay the salary when demand is H) and the expected payment to the salesperson is $\mathbb{E}(S^*) = \frac{\psi}{1 - \frac{q_H}{p_H}}$. Therefore, the firm's expected profit is

$$\pi^* = \begin{cases} r(p_H H + p_M M + p_L L) - cH - \frac{\psi}{1 - q_H / p_H} & \text{if } c < \bar{c}, \\ r(p_H (M + \Delta) + p_M M + p_L L) - c(M + \Delta) - \frac{\psi}{1 - q_H / p_H} & \text{if } c \geq \bar{c}. \end{cases}$$

Taking into account this discretization, the constrained inventory does not distort the signal acquired by the company from the salesperson.

The only scenario in which the discussion arises between stocking M or stocking H is when Δ is small enough that the discrete unit does not allow stocking $M+\Delta$ to be beneficial than stocking M. In this situation, the choice comes down to stocking M or stocking H, depending on which benefit is greater. This disregards the consideration of informational value, thus leading to the discussion between stocking M or stocking H as follows:

$$Q^* = \begin{cases} H & \text{if } c < \widehat{c}, \\ M & \text{if } c \geq \widehat{c}. \end{cases}$$

Therefore, the firm's expected profit is

$$\pi^* = \begin{cases} r(p_H H + p_M M + p_L L) - cH - \frac{\psi}{1 - q_H/p_H} & \text{if } c < \widehat{c}, \\ \pi_M = (p_H + p_M)rM + p_L rL - \frac{\psi}{1 - \frac{q_H + q_M}{p_H + p_M}} - cM & \text{if } c \geq \widehat{c}. \end{cases}$$

The economic rationale behind this concept lies in the consideration of whether the acquisition of additional information holds enough value to justify its production costs. Even when the informational benefits are clear, there exists a tradeoff between the cost of producing an additional unit of information and its perceived value. If the production cost exceeds the informational worth of one unit (or any discrete "step" of stocking), it may be more advantageous to stock M rather than stocking H.

4. Main results: validity

It is worth emphasizing that, in the first case of variable discretization, optimal compensation should consistently align with high effort regardless of the circumstances. The consideration of operational factors does not alter the framework of the optimal compensation plan. Such a principle carries significant implications for the conclusions and propositions outlined in the paper. These are the three main conclusions of the paper:

1. Relative to when stocks are not considered, it may be optimal for the firm to pay a higher bonus even though limited inventory constrains sales.
2. As inventory becomes more expensive, thereby forcing the firm to lower its inventory, the firm may nevertheless pay the agent a higher bonus.
3. If there is a lower probability that the agent's effort exertion leads to high demand, rather than lowering inventory due to the lower sales potential, the firm may increase inventory.

The validity of the three conclusions is compromised. Relative to when inventories are not considered, the payment is the same; and as inventory becomes more expensive the agent receives the same bonus. On the other hand, if $p_H r$ decreases, it may change from being $c < \bar{c}$ to being $c \geq \bar{c}$, and thus change from stocking H to stocking $M + \Delta$.

In the second discretization scenario, which is plausible and preserves the essence of the original paper, the firm cannot stock a level within the interval $(M, M+\Delta)$, where $\Delta < \frac{\psi}{(c - p_H r)} \left(\frac{p_H q_M - p_M q_H}{(q_L - p_L)(p_H - q_H)} \right) \equiv \Delta^*$. One could interpret this constraint as follows: the firm can choose to stock a quantity belonging to an ordered set of real numbers. This set includes L as the first element, followed by M, and concludes with H. Importantly, the set does not contain any elements within the interval $(M, M+\Delta)$, where $\Delta < \Delta^*$. A similar, albeit inappropriate, solution would be to assume that the firm only operates at three levels: L, M, or H. In this context, it is noteworthy that this constitutes a technical flaw, as the results can be recovered without making such a strong assumption.