Performance Pay, Internal Control, and Employee Misconduct—Online Appendix

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Assumption to ensure equilibrium efforts strictly below one in the main model. The cost parameter γ is assumed to be sufficiently small to guarantee $e < 1$. Technically, this assumption is equivalent to the condition

$$\max_{\mu \in [\underline{\mu}, \bar{\mu}]} \left\{ \frac{\pi}{2},\, \eta f,\, \frac{1}{2}[\mu\pi + (1-\mu)\eta(\delta + f)] \right\} < \frac{1}{\gamma}. \tag{A1}$$

The first two arguments of the maximum operator refer to the possible effort levels without internal control (cf. the proof of Lemma 1 below) as well as to the effort under non-manipulation with internal control (cf. the proof of Lemma 3 below). The third argument refers to the equilibrium effort under manipulation with internal control (cf. the proof of Lemma 4 below).

Proof of Lemma 1. As the principal prevents manipulation, her problem reads:

$$\max_{e, w_0, w_1} e(\pi - w_1) - (1-e)w_0 \tag{A2}$$

$$e = \gamma(w_1 - w_0) \tag{A3}$$

$$w_0 + e(w_1 - w_0) - \frac{e^2}{2\gamma} \geq 0 \tag{A4}$$

$$w_0, w_1 \geq 0 \tag{A5}$$

$$w_1 - w_0 \leq \eta f \tag{A6}$$

Replacing e using (A3) yields the simplified problem:

$$\max_{w_0, w_1} \gamma(w_1 - w_0)(\pi - w_1) - (1 - \gamma(w_1 - w_0))w_0 \tag{A7}$$

$$w_0 + \gamma(w_1 - w_0)^2 - \frac{\gamma^2(w_1 - w_0)^2}{2\gamma} \geq 0 \tag{A8}$$

$$w_0, w_1 \geq 0 \tag{A9}$$

$$w_1 - w_0 \leq \eta f \tag{A10}$$

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Clearly, the participation constraint (A8) is satisfied for all $w_1, w_0 \geq 0$ and we can thus neglect it.

First, suppose the non-manipulation constraint (A10) is not binding. Then, $w_0 = 0$ is optimal. The problem becomes $\max_{w_1} \gamma w_1(\pi - w_1)$, which implies $w_1 = \frac{\pi}{2}$. Thus, (A10) is not binding if and only if $\frac{\pi}{2} \leq \eta f$. In this case, the principal's profit is $\frac{\gamma \pi^2}{4}$ and effort is $e = \gamma \frac{\pi}{2}$.

Otherwise, if $\frac{\pi}{2} > \eta f$, the non-manipulation constraint (A10) is binding. Substituting $w_1 - w_0 = \eta f$ in the objective function, we see that $w_0 = 0$ is again optimal. We thus obtain $w_1 = \eta f$, $\gamma \eta f(\pi - \eta f)$ for the principal's profit and $e = \gamma \eta f$ for the agent's effort. □

Proof of Lemma 2. We suppose the principal wants to induce manipulation. Her problem thus is:

$$\max_{e, w_0, w_1} e(\pi - w_1) + (1 - e)(\pi - \eta\delta - w_1) \tag{A11}$$

$$e = \gamma \eta f \tag{A12}$$

$$(1 - e)(w_1 - \eta f) + e w_1 - \frac{e^2}{2\gamma} \geq 0 \tag{A13}$$

$$w_0, w_1 \geq 0 \tag{A14}$$

$$w_1 - w_0 > \eta f \tag{A15}$$

It is optimal to set $w_0 = 0$, as this relaxes (A15). Using (A12) to substitute e yields:

$$\max_{w_1} \gamma \eta f(\pi - w_1) + (1 - \gamma \eta f)(\pi - \eta\delta - w_1) \tag{A16}$$

$$(1 - \gamma \eta f)(w_1 - \eta f) + \gamma \eta f w_1 - \frac{\gamma^2 (\eta f)^2}{2\gamma} \geq 0 \tag{A17}$$

$$w_1 > \eta f \tag{A18}$$

Setting $w_1 = \eta f + \varepsilon =: \hat{F}$, where $\varepsilon \searrow 0$, satisfies the participation constraint (A17) and is thus optimal. We thus obtain $w_1^M = \hat{F}$, $e^M = \gamma \eta f$, and

$$\pi^M = \gamma \eta f(\pi - \hat{F}) + (1 - \gamma \eta f)(\pi - \eta\delta - \hat{F}) = \gamma \eta f(\pi - \eta f) + (1 - \gamma \eta f)(\pi - \eta\delta - \eta f) - \varepsilon \tag{A19}$$

Letting ε approach zero, we obtain the results of Lemma 2. □

Proof of Proposition 1. (a) Suppose that $\frac{\pi}{2} \leq \eta\delta$, which is equivalent to $\frac{\pi - \eta\delta}{\eta} \leq \frac{\pi}{2\eta}$. We distinguish three cases regarding the size of f:

(i) We first consider the case $f < \frac{\pi - \eta\delta}{\eta}$, which implies that $S := \pi - \eta\delta - \eta f > 0$. Note that, because case (a) applies, we also have $f < \frac{\pi}{2\eta}$. Comparing π^{NM} and π^M as given by (5) and (6), respectively, we see that the principal prefers to induce manipulation.

(ii) Next, we consider the case $\frac{\pi - \eta\delta}{\eta} \leq f < \frac{\pi}{2\eta}$. In this case, $S \leq 0$. Thus, comparing (5) and (6), we obtain that the principal prefers to prevent manipulation.

(iii) Finally, we consider $\frac{\pi}{2\eta} \leq f$. In this case, if the principal prevents manipulation, she receives $\pi^{NM} = \gamma \frac{\pi^2}{4}$, which exceeds π^M because $S < 0$ and $\gamma \frac{\pi^2}{4} \geq \gamma \eta f(\pi - \eta f)$. The latter inequality holds because it is equivalent to $\left(\frac{\pi}{2} - \eta f\right)^2 \geq 0$. Thus, the principal prefers to prevent manipulation.

Regarding the implemented effort, Lemma 2 and equation (4) imply that $e = \gamma\eta f < \gamma\frac{\pi}{2}$ for $f < \frac{\pi}{2\eta}$. For $f \geq \frac{\pi}{2\eta}$, we obtain $e = \gamma\frac{\pi}{2}$ by equation (4).

(b) Now suppose that $\frac{\pi}{2} > \eta\delta$, which is equivalent to $\frac{\pi-\eta\delta}{\eta} > \frac{\pi}{2\eta}$. Again, we distinguish three cases.

(i) We first consider the case $f < \frac{\pi}{2\eta}$. Because case (b) applies, we also have $f < \frac{\pi-\eta\delta}{\eta}$, which implies $S > 0$. Comparing π^{NM} and π^{M} as given by (5) and (6), respectively, we see that the principal prefers to induce manipulation.

(ii) Now consider $\frac{\pi}{2\eta} \leq f < \frac{\pi-\eta\delta}{\eta}$. By Lemmas 1 and 2, the principal prefers to induce manipulation if and only if:

$$\gamma\eta f(\pi - \eta f) + (1 - \gamma\eta f)(\pi - \eta\delta - \eta f) > \gamma\frac{\pi^2}{4} \tag{A20}$$

$$\Leftrightarrow \quad (1 - \gamma\eta f)(\pi - \eta\delta - \eta f) > \gamma\left(\frac{\pi}{2} - \eta f\right)^2. \tag{A21}$$

Because $\frac{\pi}{2\eta} \leq f < \frac{\pi-\eta\delta}{\eta}$, the left-hand side of (A21) is strictly positive. Thus, inequality (A21) holds for $f = \frac{\pi}{2\eta}$, because in this case the right-hand side is zero. As the left-hand side is decreasing in f and approaches zero as f approaches $\frac{\pi-\eta\delta}{\eta}$, whereas the right-hand side is increasing in f starting at $f = \frac{\pi}{2\eta}$, the inequality continues to hold for all $f < \hat{f}$, where $\hat{f} \in \left(\frac{\pi}{2\eta}, \frac{\pi-\eta\delta}{\eta}\right)$ is given by:

$$(1 - \gamma\eta\hat{f})(\pi - \eta\delta - \eta\hat{f}) = \gamma\left(\frac{\pi}{2} - \eta\hat{f}\right)^2 \;\Leftrightarrow\; \hat{f} = \frac{4\pi - 4\delta\eta - \pi^2\gamma}{4\eta\,(1 - \gamma\delta\eta)}. \tag{A22}$$

(iii) Finally, we consider the case $\frac{\pi-\eta\delta}{\eta} \leq f$. Following the same argumentation as in (a), case (iii), the principal prefers to prevent manipulation.

For all $f < \hat{f}$, the principal induces manipulation. Thus, effort is $e = \gamma\eta f$. This is below the benchmark effort for $f < \frac{\pi}{2\eta}$, identical to the benchmark for $f = \frac{\pi}{2\eta}$, and exceeds the benchmark effort for all f such that $\frac{\pi}{2\eta} < f < \hat{f}$. Finally, for $\hat{f} \leq f$, the principal prevents manipulation and effort is equal to the benchmark effort. □

Proof of Lemma 3. We suppose the principal has implemented internal control and wants to prevent manipulation. The corresponding contracting problem is:

$$\max_{e,w_{11},w_{10},w_{1\emptyset},w_0} e[\pi - \mu w_{11} - (1-\mu)w_{1\emptyset}] - (1-e)w_0 \tag{A23}$$

$$\text{s.t.} \quad e = \gamma[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0] \tag{A24}$$

$$e[\mu w_{11} + (1-\mu)w_{1\emptyset}] + (1-e)w_0 - \frac{e^2}{2\gamma} \geq 0 \tag{A25}$$

$$\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f) \leq w_0 \tag{A26}$$

$$w_{11}, w_{10}, w_{1\emptyset}, w_0 \geq 0 \tag{A27}$$

Rewriting the objective function and the participation constraint (A25) gives:

$$\max_{e,w_{11},w_{10},w_{1\emptyset},w_0} e\pi - w_0 - e[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0] \tag{A28}$$

$$e = \gamma[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0] \tag{A29}$$

$$w_0 + e[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0] - \frac{e^2}{2\gamma} \geq 0 \tag{A30}$$

$$\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f) \leq w_0 \tag{A31}$$

$$w_{11}, w_{10}, w_{1\emptyset}, w_0 \geq 0 \tag{A32}$$

As w_{10} appears only in the non-manipulation constraint (A31), which is relaxed as w_{10} decreases, the principal cannot do better than setting $w_{10} = 0$. Substituting e shows that the participation constraint holds and the problem can thus be simplified to:

$$\max_{w_{11},w_{1\emptyset},w_0} \pi\gamma[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0] - w_0 - \gamma[\mu w_{11} + (1-\mu)w_{1\emptyset} - w_0]^2 \tag{A33}$$

$$(1-\mu)(w_{1\emptyset} - \eta f) \leq w_0 \tag{A34}$$

$$w_{11}, w_{1\emptyset}, w_0 \geq 0 \tag{A35}$$

For any given value of $\mu w_{11} + (1-\mu)w_{1\emptyset}$, the principal cannot do better than setting $w_{1\emptyset} = 0$ to relax the non-manipulation constraint. Then, setting $w_0 = 0$ is also optimal. We obtain:

$$\max_{w_{11}\geq 0} \pi\gamma\mu w_{11} - \gamma(\mu w_{11})^2, \tag{A36}$$

which yields $w_{11} = \frac{\pi}{2\mu}$. This contract yields the benchmark effort $\gamma\pi/2$ and benchmark profit $\gamma\pi^2/4$ for the principal.

Note that the non-manipulation constraint (A31) is not binding. Furthermore, the optimal contract is not unique. Maintaining $w_0 = w_{10} = 0$, the principal could also raise $w_{1\emptyset}$ above zero (so that the non-manipulation constraint remains satisfied) and reduce w_{11}, keeping $\mu w_{11} + (1-\mu)w_{1\emptyset} = \pi/2$ constant, which means that effort and wage costs remain unaffected. □

Proof of Lemma 4. We suppose the principal wants to induce manipulation under internal control and thus solves the following problem:

$$\max_{e,w_{11},w_{10},w_{1\emptyset},w_0} e[\pi - \mu w_{11} - (1-\mu)w_{1\emptyset}] + (1-e)[(1-\mu)(\pi - \eta\delta - w_{1\emptyset}) - \mu w_{10}] \tag{A37}$$

$$e = \gamma[\mu(w_{11} - w_{10}) + (1-\mu)\eta f] \tag{A38}$$

$$e[\mu w_{11} + (1-\mu)w_{1\emptyset}] + (1-e)[\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f)] - \frac{e^2}{2\gamma} \geq 0 \tag{A39}$$

$$\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f) > w_0 \tag{A40}$$

$$w_{11}, w_{10}, w_{1\emptyset}, w_0 \geq 0 \tag{A41}$$

We relax the problem by dropping the constraint $w_{1\emptyset} \geq 0$. We will see at the end that there are solutions to the relaxed problem that satisfy $w_{1\emptyset} \geq 0$. As w_0 appears only in the manipulation constraint (A40), $w_0 = 0$ is optimal. Rewriting the objective function yields:

$$\begin{aligned} & e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - e[\mu w_{11} + (1-\mu)w_{1\emptyset}] - (1-e)[(1-\mu)w_{1\emptyset} + \mu w_{10}] \\ &= e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - [(1-\mu)w_{1\emptyset} + \mu w_{10}] - e\mu(w_{11} - w_{10}) \end{aligned} \tag{A42}$$

Rewriting the participation constraint (A39) in a similar way, the problem becomes:

$$\max_{e, w_{11}, w_{10}, w_{1\emptyset}} e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - [\mu w_{10} + (1-\mu)w_{1\emptyset}] - e\mu(w_{11} - w_{10}) \tag{A43}$$

$$e = \gamma[\mu(w_{11} - w_{10}) + (1-\mu)\eta f] \tag{A44}$$

$$\mu w_{10} + (1-\mu)w_{1\emptyset} + e\mu(w_{11} - w_{10}) - (1-e)(1-\mu)\eta f - \frac{e^2}{2\gamma} \geq 0 \tag{A45}$$

$$\mu w_{10} + (1-\mu)w_{1\emptyset} > (1-\mu)\eta f \tag{A46}$$

$$w_{11}, w_{10} \geq 0 \tag{A47}$$

Note that the incentive constraint (A44) and the manipulation constraint (A46) jointly imply the participation constraint (A45): Substituting $\mu w_{10} + (1-\mu)w_{1\emptyset} = (1-\mu)\eta f$ in (A45) yields

$$e(\mu(w_{11} - w_{10}) + (1-\mu)\eta f) - \frac{e^2}{2\gamma} \geq 0, \tag{A48}$$

which holds because of (A44). We can thus neglect (A45).

Now, $w_{1\emptyset}$ appears only in the objective function and in (A46). It follows that (A46) needs to be binding at the optimal contract in the sense that $\mu w_{10} + (1-\mu)w_{1\emptyset} = (1-\mu)\eta f + \varepsilon$ with $\varepsilon \searrow 0$, because otherwise the principal benefits from decreasing $w_{1\emptyset}$. We can use this equation to replace $w_{1\emptyset}$ in the objective function, and the problem simplifies to:

$$\max_{e, w_{11}, w_{10}} e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - e\mu(w_{11} - w_{10}) - (1-\mu)\eta f - \varepsilon \tag{A49}$$

$$e = \gamma[\mu(w_{11} - w_{10}) + (1-\mu)\eta f] \tag{A50}$$

$$w_{11}, w_{10} \geq 0 \tag{A51}$$

Note that the principal can implement any effort e by choosing $w_{11} - w_{10}$ appropriately. We can thus use (A50) to replace $w_{11} - w_{10}$ in the objective function and obtain:

$$\begin{aligned} & \max_e \ e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - e\left(\frac{e}{\gamma} - (1-\mu)\eta f\right) - (1-\mu)\eta f - \varepsilon \\ &= \max_e \ e\left[\pi - (1-\mu)S\right] + (1-\mu)S - \frac{e^2}{\gamma} - \varepsilon \end{aligned} \tag{A52}$$

Hence, the optimal effort is $e^{MI} = \frac{\gamma}{2}[\pi - (1-\mu)S]$. Note that $e^{MI} > 0$ because $\pi \geq S$ and $\mu < 1$.

Moreover, $e^{MI} < 1$ by assumption (A1).

For the principal's profit we obtain:

$$\Pi^{MI}(\mu) = \frac{\gamma}{4}[\pi - (1-\mu)S]^2 + (1-\mu)S - \varepsilon \tag{A53}$$

To summarize, any contract that satisfies the following conditions solves the relaxed problem:

$$\frac{\gamma}{2}[\pi - (1-\mu)S] = \gamma[\mu(w_{11} - w_{10}) + (1-\mu)\eta f] \tag{A54}$$

$$\mu w_{10} + (1-\mu)w_{1\emptyset} = (1-\mu)\eta f + \varepsilon \tag{A55}$$

$$w_0 = 0,\ w_{11}, w_{10} \geq 0 \tag{A56}$$

where (A54) follows from the incentive constraint. The set of such contracts always includes a contract that satisfies the constraint $w_{1\emptyset} \geq 0$, e.g., $w_{1\emptyset} = 0$, $w_{10} = \frac{1-\mu}{\mu}\eta f + \frac{\varepsilon}{\mu}$, $w_{11} = \frac{1}{2\mu}[\pi - (1-\mu)S] + \frac{\varepsilon}{\mu}$. Letting ε approach zero yields the results presented in the lemma. □

Proof of Proposition 2. If $S = 0$, the principal's profit under manipulation is identical to the benchmark profit, which will also be achieved under manipulation prevention. Hence, the principal prefers to prevent manipulation according to the tie-breaking rules. Now, suppose $S \neq 0$. From comparing (12) with the benchmark profit, we obtain that manipulation occurs if and only if:

$$\begin{aligned} &\frac{\gamma}{4}(-2\pi(1-\mu)S + (1-\mu)^2 S^2) + (1-\mu)S > 0 \\ \Leftrightarrow\ &(1-\mu)S\left(1 - \frac{\gamma}{2}\pi\right) + \frac{\gamma}{4}(1-\mu)^2 S^2 > 0 \end{aligned} \tag{A57}$$

By assumption (A1), $\frac{\gamma}{2}\pi < 1$. Thus, condition (A57) holds for all $S > 0$.

For $S < 0$, condition (A57) is equivalent to:

$$\left(1 - \frac{\gamma}{2}\pi\right) + \frac{\gamma}{4}(1-\mu)S < 0 \tag{A58}$$

We show that this is never true. By assumption (A1),

$$\frac{1}{2}[\mu\pi + (1-\mu)\eta(\delta + f)] < \frac{1}{\gamma} \ \Leftrightarrow\ \frac{\pi - \frac{2}{\gamma}}{1-\mu} < S. \tag{A59}$$

Thus,

$$\left(1 - \frac{\gamma}{2}\pi\right) + \frac{\gamma}{4}(1-\mu)S > \left(1 - \frac{\gamma}{2}\pi\right) + \frac{\gamma}{4}(1-\mu)\frac{\pi - \frac{2}{\gamma}}{1-\mu} = \frac{1}{2} - \frac{\gamma}{4}\pi > 0,$$

where the last inequality follows from $\frac{\gamma}{2}\pi < 1$.

Overall, given the implementation of internal control, the principal induces manipulation if and only if $S > 0$. □

Proof of Proposition 3. Regarding case (a) of the proposition, suppose that $S > 0$ so that the principal induces manipulation when she implements internal control. We obtain:

$$\frac{d\Pi^{MI}}{d\mu} = \frac{\gamma}{2}(\pi - (1-\mu)S)S - S < 0 \;\Leftrightarrow\; \frac{\gamma}{2}(\pi - (1-\mu)S) < 1 \tag{A60}$$

The latter inequality holds, because its left-hand side is equal to e^{MI}, which is strictly below 1. Hence, as her profit is strictly decreasing in μ, the principal chooses $\mu = \underline{\mu}$.

To answer the question when the principal prefers internal control, we first consider case (b), where $S \leq 0$. From Propositions 1 and 2, both with and without internal control, the principal prevents manipulation. With internal control, by Lemma 3, the benchmark effort and profit is attained. Without internal control, by Lemma 1, the benchmark is also achieved except when $\frac{\pi}{2} \leq \eta\delta$ and $f \in \left[\frac{\pi-\eta\delta}{\eta}, \frac{\pi}{2\eta}\right)$. In this case, the non-manipulation constraint (NM) is binding under the optimal manipulation-proof contract and effort as well at the principal's profit are lower than in the benchmark. Thus, the principal strictly prefers to implement internal control, and internal control leads to a strictly higher effort.

Now consider case (c), where $S > 0$ and the principal prevents manipulation without internal control, i.e., $\frac{\pi}{2} > \eta\delta$ and $f \in \left[\hat{f}, \frac{\pi-\eta\delta}{\eta}\right)$. By Proposition 1, $f \geq \hat{f}$ implies $f > \frac{\pi}{2\eta}$. Thus, by Lemma 1, without internal control, the principal induces the benchmark effort and receives the benchmark profit. With internal control, by Proposition 2 and Lemma 4, the principal induces manipulation and effort $e^{MI}(\mu)$ and receives profit $\Pi^{MI}(\mu)$. The principal thus strictly prefers internal control if and only if $\Pi^{MI}(\mu) > \gamma\frac{\pi^2}{4}$, which is equivalent to:

$$\begin{aligned} &\frac{\gamma}{4}(\pi - (1-\mu)S)^2 + (1-\mu)S > \gamma\frac{\pi^2}{4} \\ \Leftrightarrow\; &\frac{\gamma}{4}(-2\pi(1-\mu)S + (1-\mu)^2S^2) + (1-\mu)S > 0 \\ \Leftrightarrow\; &(1-\mu)S\left(1 - \frac{\gamma}{2}\pi\right) + \frac{\gamma}{4}(1-\mu)^2S^2 > 0 \end{aligned} \tag{A61}$$

By assumption (A1), $\frac{\gamma}{2}\pi < 1$. Thus, condition (A61) holds for all $S > 0$. Moreover, for all $S > 0$, $e^{MI}(\mu)$ is strictly lower than the benchmark effort. □

Proof of Corollary 1. The corollary comprises the two cases (i) $\frac{\pi}{2} \leq \eta\delta$ and $f < \frac{\pi-\eta\delta}{\eta}$, and (ii) $\frac{\pi}{2} > \eta\delta$ and $f < \hat{f}$.

By Lemma 2, without internal control, the principal receives profit Π^M. With internal control, by Proposition 2 and Lemma 4, the principal induces manipulation and receives profit $\Pi^{MI}(\mu)$. From the proof of Proposition 3 (a), we know that $\Pi^{MI}(\mu)$ is strictly decreasing in μ. Furthermore, $\lim_{\mu\to 1}\Pi^{MI}(\mu) = \gamma\frac{\pi^2}{4}$, i.e., as μ goes to 1, $\Pi^{MI}(\mu)$ approaches the benchmark profit. Thus, $\Pi^{MI}(\mu) > \gamma\frac{\pi^2}{4}$ for all $\mu < 1$.

First, note that it is possible that $\Pi^M \leq \gamma\frac{\pi^2}{4}$, so that the principal prefers internal control for all $\mu < 1$. For example, in case (i) above, if $f \to 0$ and $\delta \to \frac{\pi}{\eta}$, profit Π^M will approach zero. However, $\Pi^M > \gamma\frac{\pi^2}{4}$ is also possible. For example, such a situation occurs in case (ii) above if $f \in \left(\frac{\pi}{2\eta}, \hat{f}\right)$, because then, without internal control, the principal would earn the benchmark profit if she prevents manipulation, which is however strictly dominated by inducing manipulation.

Suppose that $\Pi^M > \gamma\frac{\pi^2}{4}$ and consider $\lim_{\mu\to 0}[\Pi^{MI}(\mu) - \Pi^M]$. If the latter is strictly positive, there is a $\hat{\mu} \in (0,1)$ such that $\Pi^{MI}(\hat{\mu}) = \Pi^M$ and $\Pi^{MI}(\mu) > \Pi^M$ for all $\mu < \hat{\mu}$. We obtain

$$\begin{aligned}
\lim_{\mu\to 0}[\Pi^{MI}(\mu) - \Pi^M] &= \frac{\gamma}{4}(\pi - S)^2 + S - \gamma\eta f(\pi - \eta f) - (1 - \gamma\eta f)(\pi - \eta\delta - \eta f) \\
&= \frac{\gamma}{4}(\pi - S)^2 + S - \gamma\eta f(\pi - \eta f) - (1 - \gamma\eta f)S \\
&= \frac{\gamma}{4}(\pi - S)^2 - \gamma\eta f(\pi - \eta f - S) \\
&= \frac{\gamma}{4}(\pi - (\pi - \eta\delta - \eta f))^2 - \gamma\eta f(\pi - \eta f - (\pi - \eta\delta - \eta f)) \\
&= \frac{\gamma}{4}(\eta\delta + \eta f)^2 - \gamma\eta f \cdot \eta\delta \\
&= \eta^2 \cdot \frac{\gamma}{4}(\delta - f)^2,
\end{aligned} \tag{A62}$$

which is strictly positive as $\delta \neq f$. □

Proof of Proposition 4. In analogy to assumption (A1), we require a condition to guarantee that the agent's effort choice is strictly below 1 in our modified setting:

$$\max_{\mu\in[\underline{\mu},\bar{\mu}]} \left\{ \frac{\mu\hat{\pi} + (1-\mu)\pi}{2}, \eta f, \frac{1}{2}[\mu\hat{\pi} + (1-\mu)\eta(\delta + f)] \right\} < \frac{1}{\gamma}. \tag{A63}$$

The third argument of the maximum operator refers to the equilibrium effort under internal control and inducing manipulation, and guarantees that $\hat{e}^{MI} < 1$, whereas the remaining equilibrium efforts are captured by the first and the second argument of the maximum operator (cf. below).

Without internal control, the results of Lemmas 1 and 2, and Proposition 1 remain unchanged, as success is always unverified and the associated payoff is again denoted by π.

Let E_π denote the principal's expected payoff given success under internal control:

$$E_\pi := \mu\hat{\pi} + (1-\mu)\,\pi.$$

Suppose, the principal has implemented internal control and wants to prevent manipulation. In that case, she maximizes

$$\begin{aligned}
&e\,[\mu\,(\hat{\pi} - w_{11}) + (1-\mu)\,(\pi - w_{1\emptyset})] - (1-e)\,w_0 \\
&= e\,[E_\pi - \mu w_{11} - (1-\mu)\,w_{1\emptyset}] - (1-e)\,w_0
\end{aligned}$$

subject to the same set of constraints as in Lemma 3. Hence, in strict analogy to the proof of this lemma, the principal optimally induces effort $\gamma E_\pi/2$ and earns $\gamma(E_\pi)^2/4$.

Now suppose, the principal has implemented internal control and wants to induce manipulation.

She maximizes

$$\begin{aligned}
&e\left[\mu\left(\hat{\pi}-w_{11}\right)+(1-\mu)\left(\pi-w_{1\emptyset}\right)\right]+(1-e)\left[(1-\mu)\left(\pi-\eta\delta-w_{1\emptyset}\right)-\mu w_{10}\right] \\
&=e\left[E_{\pi}-\mu w_{11}-(1-\mu)\,w_{1\emptyset}\right]+(1-e)\left[(1-\mu)\left(\pi-\eta\delta-w_{1\emptyset}\right)-\mu w_{10}\right]
\end{aligned}$$

subject to to the same set of constraints as in Lemma 4. Again, in strict analogy, the principal optimally induces effort $\hat{e}^{MI}=\frac{\gamma}{2}\left[E_{\pi}-(1-\mu)\,S\right]$ and earns (for $\varepsilon\searrow 0$)

$$\hat{\Pi}^{MI}(\mu)=\frac{\gamma}{4}\left[E_{\pi}-(1-\mu)\,S\right]^{2}+(1-\mu)\,S.$$

(a) First, we can show that Proposition 2 still holds. Under internal control, the principal will prefer manipulation to non-manipulation if and only if

$$\begin{aligned}
\hat{\Pi}^{MI}(\mu)>\frac{\gamma\left(E_{\pi}\right)^{2}}{4}&\Leftrightarrow \\
\frac{\gamma}{4}\left[E_{\pi}-(1-\mu)\,S\right]^{2}+(1-\mu)\,S>\frac{\gamma\left(E_{\pi}\right)^{2}}{4}&\Leftrightarrow \\
(1-\mu)\,S\left(1-\frac{\gamma}{2}E_{\pi}\right)+\frac{\gamma}{4}(1-\mu)^{2}S^{2}>0&,
\end{aligned}$$

which is not satisfied for $S=0$. However, the condition holds for $S>0$ because of assumption (A63). It remains to show that the condition does not hold for $S<0$. For $S<0$, the condition can be rewritten as

$$1-\frac{\gamma}{2}E_{\pi}+\frac{\gamma}{4}(1-\mu)\,S<0.$$

Because, according to (A63), $\frac{1}{2}\left[\mu\hat{\pi}+(1-\mu)\,\eta\,(\delta+f)\right]<\frac{1}{\gamma}\Leftrightarrow(1-\mu)\,S>E_{\pi}-\frac{2}{\gamma}$ holds, we obtain

$$1-\frac{\gamma}{2}E_{\pi}+\frac{\gamma}{4}(1-\mu)\,S>1-\frac{\gamma}{2}E_{\pi}+\frac{\gamma}{4}\left(E_{\pi}-\frac{2}{\gamma}\right)=\frac{1}{2}\left(1-\frac{\gamma}{2}E_{\pi}\right)>0,$$

where the last inequality follows from (A63). Hence, $1-\frac{\gamma}{2}E_{\pi}+\frac{\gamma}{4}(1-\mu)\,S<0$ is not satisfied for $S<0$.

Concerning the shape of the profit function $\hat{\Pi}^{MI}(\mu)$, we obtain the following results:

$$\begin{aligned}
\frac{d\hat{\Pi}^{MI}(\mu)}{d\mu}&=\frac{\gamma}{2}[E_{\pi}-(1-\mu)S](\hat{\pi}-\pi+S)-S=\hat{e}^{MI}(\hat{\pi}-\pi)+(\hat{e}^{MI}-1)S, \\
\frac{d^{2}\hat{\Pi}^{MI}(\mu)}{d\mu^{2}}&=\frac{\gamma}{2}(\hat{\pi}-\pi+S)^{2}>0.
\end{aligned}$$

Because $\hat{\pi}-\pi>0$, $\hat{e}^{MI}>0$, and $(\hat{e}^{MI}-1)<0$ (due to (A63)), the profit function $\hat{\Pi}^{MI}(\mu)$ is strictly increasing in μ for $S\leq 0$. Thus, for $S\leq 0$, the principal prefers $\mu=\bar{\mu}$. For $S>0$, $\hat{\Pi}^{MI}(\mu)$ can be increasing, decreasing, or non-monotonic for $\mu\in[\underline{\mu},\bar{\mu}]$. However, as $\hat{\Pi}^{MI}(\mu)$ is a strictly convex function, the problem $\max_{\mu\in[\underline{\mu},\bar{\mu}]}\hat{\Pi}^{MI}(\mu)$ must have a corner solution. The principal strictly prefers $\mu = \bar{\mu}$ if and only if

$$\hat{\Pi}^{MI}(\bar{\mu}) > \hat{\Pi}^{MI}(\underline{\mu})$$
$$\Leftrightarrow \frac{\gamma}{4}\left([\pi + \bar{\mu}(\hat{\pi} - \pi) - (1-\bar{\mu})S]^2 - [\pi + \underline{\mu}(\hat{\pi} - \pi) - (1-\underline{\mu})S]^2\right) > (\bar{\mu} - \underline{\mu})S.$$

As the left-hand side of this condition is strictly increasing in $\hat{\pi}$ whereas the right-hand side is independent of $\hat{\pi}$, the principal will adopt $\mu = \bar{\mu}$ if and only if $\hat{\pi}$ exceeds a certain threshold.

(b) Suppose, $S \leq 0$. Irrespective of whether the principal has chosen internal control or not, she prefers to prevent manipulation. With internal control, she earns $\gamma(E_\pi)^2/4$. Without internal control, she earns $\gamma\pi^2/4 < \gamma(E_\pi)^2/4$ or an even lower profit. The comparison between the results of Lemma 1 and the optimal effort under internal control, $\gamma E_\pi/2$, shows that the optimal effort under internal control is strictly larger.

(c) Suppose, $S > 0$ and manipulation prevention is optimal without internal control. By Proposition 1 and Lemma 1, the principal induces the benchmark effort and receives the benchmark profit. For $S > 0$, the principal prefers manipulation under internal control, and receives $\hat{\Pi}^{MI}(\mu)$. The comparison of the two profits shows that

$$\hat{\Pi}^{MI}(\mu) > \gamma\frac{\pi^2}{4} \quad \Leftrightarrow \quad \frac{\gamma}{4}\left[(E_\pi)^2 - \pi^2\right] + (1-\mu)S\left(1 - \frac{\gamma}{2}E_\pi\right) + \frac{\gamma}{4}(1-\mu)^2 S^2 > 0$$

is true according to (A63). In addition,

$$\hat{e}^{MI} = \frac{\gamma}{2}\left[E_\pi - (1-\mu)S\right] > \frac{\gamma}{2}\pi \quad \Leftrightarrow \quad \mu\hat{\pi} + (1-\mu)\pi > \pi + (1-\mu)(\pi - \eta\delta - \eta f)$$
$$\Leftrightarrow \hat{\pi} > \frac{\pi - (1-\mu)\eta(\delta + f)}{\mu}$$

will be true if $\hat{\pi}$ is sufficiently large.

□

Proof of Proposition 5. The following technical assumption ensures equilibrium efforts strictly below 1:

$$\max_{\mu \in [\underline{\mu}, \bar{\mu}]} \left\{\frac{\pi}{2}, \eta f, \frac{1}{2}\left[\pi - (1-\mu)S + \mu\tilde{f}\right]\right\} < \frac{1}{\gamma}. \tag{A64}$$

In analogy to the previous sections, the third argument of the maximum operator refers to the equilibrium effort under internal control and inducing manipulation, whereas the remaining equilibrium efforts are captured by the first and the second argument of the maximum operator (cf. below).

(a) First, we consider the consequences of an additional loss from internally detected manipulation, $\tilde{f}$, for the solution when the principal has chosen internal control and wants to prevent manipulation. The principal's new optimization problem only differs in one respect from the one considered in Lemma 3. The non-manipulation constraint is now given by

$$\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\left(w_{1\emptyset}-\eta f\right)\leq w_{0}.$$

As the non-manipulation constraint was already slack in the previous problem of Lemma 3 and is now all the more slack due to $\tilde{f}>0$, the solution to the optimization problem with $\tilde{f}$ is the same as the one for the problem considered in Lemma 3. The principal induces the benchmark effort $\frac{\gamma}{2}\pi$ and receives the benchmark payoff $\frac{\gamma}{4}\pi^{2}$.

Second, we have to analyze the implications of $\tilde{f}$ for the principal's optimization problem when she has chosen internal control and wants to induce manipulation. Contrary to the optimization problem considered in Lemma 4, the new problem now reads as follows:

$$\max_{e,w_{11},w_{10},w_{1\emptyset},w_{0}} e\left[\pi-\mu w_{11}-(1-\mu)\,w_{1\emptyset}\right]+(1-e)\left[(1-\mu)\left(\pi-\eta\delta-w_{1\emptyset}\right)-\mu w_{10}\right]$$

$$e=\gamma\left[\mu\left(w_{11}-w_{10}+\tilde{f}\right)+(1-\mu)\,\eta f\right]$$

$$e\left[\mu w_{11}+(1-\mu)\,w_{1\emptyset}\right]+(1-e)\left[\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\left(w_{1\emptyset}-\eta f\right)\right]\geq\frac{e^{2}}{2\gamma}$$

$$\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\left(w_{1\emptyset}-\eta f\right)>w_{0}$$

$$w_{11},w_{10},w_{1\emptyset},w_{0}\geq 0.$$

We can proceed in the same way as in the solution to the problem in Lemma 4. Setting $w_{0}=0$ is optimal. For the moment, we drop the constraint $w_{1\emptyset}\geq 0$ and check it later. The relaxed problem can be rewritten as follows:

$$\max_{e,w_{11},w_{10},w_{1\emptyset}} e\pi+(1-e)(1-\mu)(\pi-\eta\delta)-\left[\mu w_{10}+(1-\mu)\,w_{1\emptyset}\right]-e\mu\left[w_{11}-w_{10}\right]$$

$$e=\gamma\left[\mu\left(w_{11}-w_{10}+\tilde{f}\right)+(1-\mu)\,\eta f\right]$$

$$\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\,w_{1\emptyset}+e\mu\left[w_{11}-\left(w_{10}-\tilde{f}\right)\right]-(1-e)(1-\mu)\,\eta f\geq\frac{e^{2}}{2\gamma}$$

$$\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\,w_{1\emptyset}>(1-\mu)\,\eta f$$

$$w_{11},w_{10}\geq 0.$$

Substituting $\mu\left(w_{10}-\tilde{f}\right)+(1-\mu)\,w_{1\emptyset}=(1-\mu)\,\eta f$ in the participation constraint leads to

$$(1-\mu)\,\eta f+e\mu\left[w_{11}-\left(w_{10}-\tilde{f}\right)\right]-(1-e)(1-\mu)\,\eta f\geq\frac{e^{2}}{2\gamma}\Leftrightarrow$$

$$e\left[\mu\left(w_{11}-w_{10}+\tilde{f}\right)+(1-\mu)\,\eta f\right]\geq\frac{e^{2}}{2\gamma}.$$

Together with the incentive constraint, this inequality shows that the participation constraint is slack under the optimal contract and, thus, can be neglected. The remaining relaxed program is given by:

$$\max_{e,w_{11},w_{10},w_{1\emptyset}} e\pi + (1-e)(1-\mu)(\pi - \eta\delta) - [\mu w_{10} + (1-\mu) w_{1\emptyset}] - e\mu [w_{11} - w_{10}]$$

$$e = \gamma \left[\mu (w_{11} - w_{10}) + \mu\tilde{f} + (1-\mu)\eta f\right]$$

$$\mu w_{10} + (1-\mu) w_{1\emptyset} > (1-\mu)\eta f + \mu\tilde{f}$$

$$w_{11}, w_{10} \geq 0.$$

As $w_{1\emptyset}$ only appears in the objective function and the manipulation constraint, it is optimal for the principal to set $\mu w_{10} + (1-\mu) w_{1\emptyset} = (1-\mu)\eta f + \mu\tilde{f} + \varepsilon$ with $\varepsilon \searrow 0$. By substituting for $\mu w_{10} + (1-\mu) w_{1\emptyset}$ in the objective function, the latter can be rewritten as

$$\max_{e,w_{11},w_{10}} e\pi - e[\mu (w_{11} - w_{10}) + (1-\mu)(\pi - \eta\delta)] + (1-\mu) S - \mu\tilde{f} - \varepsilon.$$

The incentive constraint can be used to replace $\mu(w_{11} - w_{10})$ in the objective function, so that the remaining relaxed program becomes

$$\max_{e\geq 0} e\pi - e\left[(1-\mu) S - \mu\tilde{f}\right] - \frac{e^2}{\gamma} + (1-\mu) S - \mu\tilde{f} - \varepsilon.$$

As this function is strictly concave, optimal effort is given by the first-order condition, yielding $\tilde{e}^{MI}(\mu) = \frac{\gamma}{2}\left[\pi - (1-\mu) S + \mu\tilde{f}\right]$. The corresponding optimal profit is

$$\tilde{\Pi}^{MI}(\mu) = \frac{\gamma}{4}\left[\pi - (1-\mu) S + \mu\tilde{f}\right]^2 + (1-\mu) S - \mu\tilde{f} - \varepsilon.$$

Overall, an optimal contract for implementing this solution to the relaxed program has to satisfy

$$\frac{\gamma}{2}\left[\pi - (1-\mu) S + \mu\tilde{f}\right] = \gamma\left[\mu (w_{11} - w_{10}) + \mu\tilde{f} + (1-\mu)\eta f\right],$$

$$\mu w_{10} + (1-\mu) w_{1\emptyset} = (1-\mu)\eta f + \mu\tilde{f} + \varepsilon,$$

$$w_{11}, w_{10} \geq 0, \quad \text{and} \quad w_0 = 0.$$

A contract that solves both the original and the relaxed program is the following one:

$$w_0 = w_{1\emptyset} = 0, \quad w_{10} = \frac{(1-\mu)\eta f + \varepsilon}{\mu} + \tilde{f}, \quad \text{and} \quad w_{11} = \frac{1}{2\mu}\left[\pi - (1-\mu) S + \mu\tilde{f}\right] + \frac{\varepsilon}{\mu}.$$

A comparison of the optimal profits with and without manipulation under internal control shows that the principal will induce manipulation if and only if (recall that $\varepsilon \searrow 0$)

$$\tilde{\Pi}^{MI} = \frac{\gamma}{4}\left[\pi - (1-\mu)\, S + \mu \tilde{f}\right]^2 + (1-\mu)\, S - \mu \tilde{f} > \frac{\gamma}{4}\pi^2 \Leftrightarrow$$
$$\left(\mu \tilde{f} - (1-\mu)\, S\right)\left(\frac{\gamma}{2}\left[\pi + \frac{1}{2}\left(\mu \tilde{f} - (1-\mu)\, S\right)\right] - 1\right) > 0.$$

This inequality will be satisfied if either

$$\mu \tilde{f} - (1-\mu)\, S > 0 \quad \text{and} \quad \frac{\gamma}{2}\left[\pi + \frac{1}{2}\left(\mu \tilde{f} - (1-\mu)\, S\right)\right] > 1, \tag{A65}$$

or if

$$\mu \tilde{f} - (1-\mu)\, S < 0 \quad \text{and} \quad \frac{\gamma}{2}\left[\pi + \frac{1}{2}\left(\mu \tilde{f} - (1-\mu)\, S\right)\right] < 1. \tag{A66}$$

From assumption (A64), we know that

$$\frac{\gamma}{2}\left[\pi + \left(\mu \tilde{f} - (1-\mu)\, S\right)\right] < 1,$$

which contradicts (A65). Rewriting (A66) leads to

$$\tilde{f} < \frac{1-\mu}{\mu} S \quad \text{and} \quad \tilde{f} < \frac{2}{\mu}\left(\frac{2}{\gamma} - \pi\right) + \frac{1-\mu}{\mu} S.$$

As we also know from (A64) that $\frac{\pi}{2} < \frac{1}{\gamma}$, which is equivalent to $\frac{2}{\gamma} > \pi$, the principal will prefer manipulation to non-manipulation under internal control if and only if

$$\tilde{f} < \frac{1-\mu}{\mu} S \;\Leftrightarrow\; S > \frac{\mu}{1-\mu}\tilde{f}.$$

If the principal induces manipulation under internal control, she will strictly prefer $\mu = \underline{\mu}$, as

$$\frac{d\tilde{\Pi}^{MI}(\mu)}{d\mu} = \frac{\gamma}{2}\left[\pi - (1-\mu)\, S + \mu \tilde{f}\right]\left(S + \tilde{f}\right) - \left(S + \tilde{f}\right) < 0 \Leftrightarrow$$
$$\frac{\gamma}{2}\left[\pi - (1-\mu)\, S + \mu \tilde{f}\right] < 1$$

is true because of (A64).

(b) For $S \leq 0$, the claim of Proposition 3(b) remains unchanged, as under internal control the principal still prefers non-manipulation and implements the benchmark solution.

(c) Suppose, $S > 0$ and manipulation prevention is optimal without internal control, i.e., $\frac{\pi}{2} > \eta\delta$ and $f \in \left[\hat{f}, \frac{\pi - \eta\delta}{\eta}\right)$. From the proof of Proposition 3(c), we know that the principal implements the benchmark solution when there is no internal control. For the case of internal control, we have to differentiate between two cases: If $S \leq \frac{\underline{\mu}}{1-\underline{\mu}}\tilde{f}$, the principal prevents manipulation and also implements the benchmark solution with internal control; otherwise, she induces manipulation and earns a strictly higher payoff, as we know from the proof of result (a). Overall, the principal weakly prefers to implement internal control. For those parameter constellations for which the principal strictly prefers internal control, i.e., for $S > \frac{\underline{\mu}}{1-\underline{\mu}}\tilde{f}$, she induces manipulation and a strictly lower effort compared to the situation without internal control, as

$$\tilde{e}^{MI} = \frac{\gamma}{2}\left[\pi - \left(1-\underline{\mu}\right)S + \underline{\mu}\tilde{f}\right] < \frac{\gamma}{2}\pi \quad \Leftrightarrow \quad S > \frac{\underline{\mu}}{1-\underline{\mu}}\tilde{f}$$

is true.

(d) Suppose, $0 < S \leq \frac{\underline{\mu}}{1-\underline{\mu}}\tilde{f} \Leftrightarrow \frac{\pi-\eta\delta}{\eta} - \frac{\underline{\mu}}{(1-\underline{\mu})\eta}\tilde{f} \leq f < \frac{\pi-\eta\delta}{\eta}$ and, without internal control, the principal prefers manipulation. By Lemma 2, the principal then receives the profit Π^M. With internal control, the principal prevents manipulation (by result (a) of the current proposition) and implements the benchmark solution with profit $\frac{\gamma}{4}\pi^2$. The principal will prefer internal control if and only if

$$\begin{gathered}
\frac{\gamma}{4}\pi^2 > \gamma\eta f\left(\pi - \eta f\right) + \left(1-\gamma\eta f\right)S = \Pi^M \Leftrightarrow \\
\frac{\gamma}{4}\pi^2 > \gamma\eta f\left(\pi - \eta f\right) + \left(1-\gamma\eta f\right)\left(\pi - \eta f - \eta\delta\right) \Leftrightarrow \\
\frac{\gamma}{4}\pi^2 > \pi - \eta f - \left(1-\gamma\eta f\right)\eta\delta \Leftrightarrow \\
-\pi^2 + \frac{4}{\gamma}\pi - \frac{4}{\gamma}\left[\eta f + \left(1-\gamma\eta f\right)\eta\delta\right] < 0.
\end{gathered}$$

Concerning π, the left-hand side describes a concave parabola with the possible zeros

$$\pi = \frac{2}{\gamma}\left(1 \pm \sqrt{(\gamma\delta\eta - 1)(f\gamma\eta - 1)}\right).$$

From (A64), we know that $f\gamma\eta < 1$. Thus, for the case $\gamma\delta\eta > 1$ considered in the proposition, the expression under the square root will be negative and feasible zeros do not exist. Consequently, the principal prefers internal control.

As the principal implements the benchmark effort $\frac{\gamma}{2}\pi$ under internal control and the effort $e^M = \gamma\eta f$ without internal control, effort under internal control is strictly higher if and only if $f < \frac{\pi}{2\eta}$. □

Proof of Corollary 2. In situations without internal control, we have to differentiate between (a) $\frac{\pi}{2} \leq \eta\delta$ and (b) $\frac{\pi}{2} > \eta\delta$. The left panel of Figure 1 on case (a) immediately shows that introducing vicarious liability leads to less manipulation in a situation in which external control of the principal is already high ($\frac{\pi}{2} \leq \eta\delta$). In case (b) on initially low external control of the principal, there will be manipulation if and only if $f < \hat{f}$ with

$$\hat{f} := \frac{4\pi - 4\delta\eta - \pi^2\gamma}{4\eta\left(1-\gamma\delta\eta\right)}$$

according to the proof of Proposition 1. As

$$\frac{\partial \hat{f}}{\partial \delta} = -\frac{(2-\pi\gamma)^2}{4(1-\gamma\eta\delta)^2} < 0,$$

the introduction of vicarious liability also reduces manipulation in case (b). Recall that the implemented efforts do not depend on δ in either case.

In situations with internal control, according to Proposition 2, the introduction of vicarious liability also decreases the parameter space for which the principal induces manipulation, as S decreases with δ. In addition, $e^{MI}(\mu)$ increases with δ.

The problem described by Proposition 3(c) will be mitigated if, in case (b), $\frac{\pi}{2} > \eta\delta$, the interval $\left[\hat{f}, \frac{\pi-\eta\delta}{\eta}\right)$ shrinks with an increase in δ. The derivative

$$\frac{\partial}{\partial \delta}\left(\frac{\pi-\eta\delta}{\eta} - \hat{f}\right) = -\frac{\gamma}{(1-\gamma\eta\delta)^2}\left(\frac{\pi}{2}-\eta\delta\right)\left[\gamma\left(\frac{1}{\gamma}-\frac{\pi}{2}\right) + (1-\gamma\eta\delta)\right]$$

is negative, because $\frac{\pi}{2} > \eta\delta$ in the given situation, $\frac{1}{\gamma} > \frac{\pi}{2}$ according to (A1), and $1 > \gamma\eta\delta$ as—in the given situation of Proposition 1(b)—$\frac{\pi}{2} > \eta\delta$ together with $f > \frac{\pi}{2\eta}$ imply that $f > \delta$ with $1 > \gamma\eta f$ according to (A1). □

Proof of Lemma 5. We assume that assumption (A1) holds to guarantee equilibrium efforts strictly below one.

(a) Suppose, the principal implements internal control and wants to prevent manipulation. As one of the previous solutions for the optimal contract, $w_{1\emptyset} = w_{10} = 0$ and $w_{11} = \frac{\pi}{2\mu}$ (compare Lemma 3), is still feasible under the new constraint $w_{11} \geq w_{1\emptyset} \geq w_{10}$, it also solves the principal's new problem.

(b) Now, suppose that the principal wants to maximize profits under internal control and manipulation, including the new restriction $w_{11} \geq w_{1\emptyset} \geq w_{10}$. Thus, the optimization problem for this modified setting reads as follows:

$$\max_{e,w_{11},w_{10},w_{1\emptyset},w_0} e\left[\pi - \mu w_{11} - (1-\mu) w_{1\emptyset}\right] + (1-e)\left[(1-\mu)(\pi - \eta\delta - w_{1\emptyset}) - \mu w_{10}\right]$$

$$e = \gamma\left[\mu(w_{11} - w_{10}) + (1-\mu)\eta f\right]$$

$$e\left[\mu w_{11} + (1-\mu) w_{1\emptyset}\right] + (1-e)\left[\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f)\right] \geq \frac{e^2}{2\gamma}$$

$$\mu w_{10} + (1-\mu)(w_{1\emptyset} - \eta f) \geq w_0 + \varepsilon \qquad (\text{with } \varepsilon \searrow 0)$$

$$w_{11} \geq w_{1\emptyset} \geq w_{10}$$

$$w_{11}, w_{10}, w_{1\emptyset}, w_0 \geq 0.$$

As in the original problem without the additional restriction, setting $w_0 = 0$ is optimal to relax the manipulation constraint. We can rewrite the problem as follows:

$$\max_{e,w_{11},w_{10},w_{1\emptyset}} e\pi + (1-e)(1-\mu)(\pi-\eta\delta) - [\mu w_{10} + (1-\mu) w_{1\emptyset}] - e\mu [w_{11} - w_{10}]$$

$$e = \gamma [\mu (w_{11} - w_{10}) + (1-\mu)\eta f]$$

$$\mu w_{10} + (1-\mu) w_{1\emptyset} - (1-\mu)\eta f + e [\mu (w_{11} - w_{10}) + (1-\mu)\eta f] \geq \frac{e^2}{2\gamma}$$

$$\mu w_{10} + (1-\mu) w_{1\emptyset} \geq (1-\mu)\eta f + \varepsilon$$

$$w_{11} \geq w_{1\emptyset} \geq w_{10}$$

$$w_{11}, w_{10}, w_{1\emptyset} \geq 0.$$

Combining the incentive constraint with the participation constraint as in the proof of Lemma 4 immediately shows that the participation constraint is always slack, so that we can neglect it. Replacing e by the incentive constraint in the principal's objective function yields

$$\max_{w_{11},w_{10},w_{1\emptyset}} \gamma [\mu (w_{11} - w_{10}) + (1-\mu)\eta f] [\pi - (1-\mu)(\pi-\eta\delta) - \mu (w_{11} - w_{10})]$$
$$+ (1-\mu)(\pi-\eta\delta) - [\mu w_{10} + (1-\mu) w_{1\emptyset}]$$

$$w_{11}, w_{10}, w_{1\emptyset} \geq 0$$

$$\mu w_{10} + (1-\mu) w_{1\emptyset} \geq (1-\mu)\eta f + \varepsilon$$

$$w_{11} \geq w_{1\emptyset} \geq w_{10}.$$

The corresponding Lagrangian to our problem is, hence, given by

$$\begin{aligned}\mathcal{L}(w_{11}, w_{10}, w_{1\emptyset}) = {} & \gamma [\mu (w_{11} - w_{10}) + (1-\mu)\eta f] [\pi - (1-\mu)(\pi-\eta\delta) - \mu (w_{11} - w_{10})] \\ & + (1-\mu)(\pi-\eta\delta) - [\mu w_{10} + (1-\mu) w_{1\emptyset}] \\ & + \lambda_1 w_{11} + \lambda_2 w_{10} + \lambda_3 w_{1\emptyset} + \lambda_4 [\mu w_{10} + (1-\mu) w_{1\emptyset} - (1-\mu)\eta f - \varepsilon] \\ & + \lambda_5 [w_{11} - w_{1\emptyset}] + \lambda_6 [w_{1\emptyset} - w_{10}]\end{aligned}$$

with $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \geq 0$ and $\varepsilon \searrow 0$. We obtain the following optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial w_{11}} = \gamma\mu [\pi - (1-\mu)(S + 2\eta f) - 2\mu (w_{11} - w_{10})] + \lambda_1 + \lambda_5 = 0 \tag{A67}$$

$$\frac{\partial \mathcal{L}}{\partial w_{10}} = -\gamma\mu [\pi - (1-\mu)(S + 2\eta f) - 2\mu (w_{11} - w_{10})] - \mu + \lambda_2 + \lambda_4\mu - \lambda_6 = 0 \tag{A68}$$

$$\frac{\partial \mathcal{L}}{\partial w_{1\emptyset}} = \lambda_3 + (\lambda_4 - 1)(1-\mu) - \lambda_5 + \lambda_6 = 0 \tag{A69}$$

$$w_{11}, w_{10}, w_{1\emptyset} \geq 0$$

$$\mu w_{10} + (1-\mu)\, w_{1\emptyset} \geq (1-\mu)\, \eta f + \varepsilon \tag{A70}$$

$$w_{11} \geq w_{1\emptyset} \geq w_{10} \tag{A71}$$

$$\lambda_1 \cdot w_{11} = \lambda_2 \cdot w_{10} = \lambda_3 \cdot w_{1\emptyset} = 0$$

$$\lambda_4 \cdot [\mu w_{10} + (1-\mu)\, w_{1\emptyset} - (1-\mu)\, \eta f - \varepsilon] = 0$$

$$\lambda_5 \cdot [w_{11} - w_{1\emptyset}] = 0 \tag{A72}$$

$$\lambda_6 \cdot [w_{1\emptyset} - w_{10}] = 0. \tag{A73}$$

Combining (A67) and (A68) yields

$$\lambda_1 + \lambda_2 + (\lambda_4 - 1)\, \mu + \lambda_5 - \lambda_6 = 0. \tag{A74}$$

This equation and (A69) together lead to

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1,$$

so that at least one of the first four constraints must be binding. Suppose, the first constraint is binding, i.e., $w_{11} = 0$. However, according to (A71), this result would imply $w_{1\emptyset} = w_{10} = 0$, so that (A70) is violated. Thus, we must have $w_{11} > 0$ and $\lambda_1 = 0$. Similarly, $w_{1\emptyset} = 0$ together with (A71) implies $w_{10} = 0$, but this result would again violate (A70). Consequently, $w_{1\emptyset} > 0$ and $\lambda_3 = 0$ under the optimal contract.

To sum up,

$$\lambda_2 + \lambda_4 = 1,$$

and we have to differentiate between three remaining cases:

(i) The second constraint of the optimization problem is binding whereas the fourth is not, so that $w_{10} = 0$ and $\mu w_{10} + (1-\mu)\, w_{1\emptyset} > (1-\mu)\, \eta f + \varepsilon$ and $\lambda_2 = 1$ and $\lambda_4 = 0$.

(ii) The fourth constraint is binding whereas the second is not, so that $\mu w_{10} + (1-\mu)\, w_{1\emptyset} = (1-\mu)\, \eta f + \varepsilon$ and $w_{10} > 0$ and $\lambda_2 = 0$ and $\lambda_4 = 1$.

(iii) The second and the fourth constraint are both binding, so that $w_{10} = 0$ and $\mu w_{10} + (1-\mu)\, w_{1\emptyset} = (1-\mu)\, \eta f + \varepsilon$ and $\lambda_2, \lambda_4 > 0$.

Consider *case (i)*. Because of $\lambda_1 = 0$ and $\lambda_2 = 1$, equation (A74) becomes $\lambda_6 = 1 - \mu + \lambda_5$. Thus, λ_6 is strictly positive, so that $w_{1\emptyset} - w_{10} = w_{1\emptyset} = 0$ due to (A73), because $w_{10} = 0$. However, $w_{10} = w_{1\emptyset} = 0$ contradicts the manipulation constraint (A70). Consequently, case (i) does not yield a solution to our optimization problem.

Now, consider *case (ii)*. Because of $\lambda_1 = \lambda_2 = 0$ and $\lambda_4 = 1$, equation (A74) becomes $\lambda_5 = \lambda_6$. *First*, suppose that $\lambda_5, \lambda_6 > 0$, i.e., the new restriction $w_{11} \geq w_{1\emptyset} \geq w_{10}$ becomes binding for all three wages in the optimum. In that case, (A72) and (A73) imply $w_{11} = w_{1\emptyset} = w_{10}$. From the binding manipulation constraint, we, thus, obtain $w_{11} = w_{1\emptyset} = w_{10} = (1-\mu)\,\eta f + \varepsilon$ with $\varepsilon \searrow 0$. In addition, for (A67) to be satisfied under $\lambda_1 = 0$ and $\lambda_5 > 0$ and $w_{11} = w_{10}$, i.e.,

$$\gamma\mu\left[\pi - (1-\mu)\,(S + 2\eta f)\right] + \lambda_5 = 0,$$

we must have that

$$\gamma\mu\left[\pi - (1-\mu)\,(S + 2\eta f)\right] < 0 \Leftrightarrow \frac{1}{2\,(1-\mu)}\left[\pi - (1-\mu)\,S\right] < \eta f.$$

Second, suppose that $\lambda_5 = \lambda_6 = 0$. Together with $\lambda_1 = 0$, equation (A67) becomes

$$\gamma\mu\left[\pi - (1-\mu)\,(S + 2\eta f) - 2\mu\,(w_{11} - w_{10})\right] = 0 \Leftrightarrow \pi - (1-\mu)\,(S + 2\eta f) = 2\mu\,(w_{11} - w_{10})\,.$$

As the new restriction (A71) must hold, this equation implies that we must have

$$\pi - (1-\mu)\,(S + 2\eta f) \geq 0 \Leftrightarrow \frac{1}{2\,(1-\mu)}\left[\pi - (1-\mu)\,S\right] \geq \eta f.$$

In addition, as

$$\pi - (1-\mu)\,(S + 2\eta f) = 2\mu\,(w_{11} - w_{10}) \Leftrightarrow \frac{1}{2}\left[\pi - (1-\mu)\,S\right] = \mu\,(w_{11} - w_{10}) + (1-\mu)\,\eta f$$

and the manipulation constraint (A70) is binding, this solution replicates the previous solution in the proof of Lemma 4.

Now consider *case (iii)*. Because of $w_{10} = 0$ and $\mu w_{10} + (1-\mu)\,w_{1\emptyset} = (1-\mu)\,\eta f + \varepsilon$, we immediately obtain the optimal wage $w_{1\emptyset} = \eta f + \frac{\varepsilon}{1-\mu}$. As $w_{1\emptyset} > 0 = w_{10}$, (A73) implies $\lambda_6 = 0$. Due to $\lambda_3 = \lambda_6 = 0$, (A69) simplifies to

$$(\lambda_4 - 1)\,(1-\mu) - \lambda_5 = 0.$$

Together with $\lambda_2 + \lambda_4 = 1 \Leftrightarrow \lambda_4 = 1 - \lambda_2$, this equation leads to $-\lambda_2\,(1-\mu) - \lambda_5 = 0$, which can only be satisfied if $\lambda_2 = \lambda_5 = 0$. For $\lambda_1 = \lambda_5 = 0 = w_{10}$, condition (A67) boils down to

$$\gamma\mu\left[\pi - (1-\mu)\,(S + 2\eta f) - 2\mu w_{11}\right] = 0 \Leftrightarrow \pi - (1-\mu)\,(S + 2\eta f) = 2\mu w_{11},$$

which—because of $w_{11} \geq w_{1\emptyset} \geq w_{10} = 0$—requires that

$$\pi - (1-\mu)\,(S + 2\eta f) \geq 0 \Leftrightarrow \frac{1}{2\,(1-\mu)}\left[\pi - (1-\mu)\,S\right] \geq \eta f.$$

Overall, this candidate solution collapses to a specific case of the previous solution in the proof of Lemma 4.

Finally, observe that, if

$$\frac{1}{2}[\pi - (1-\mu)S] \geq (1-\mu)\eta f,$$

the contract $w_0 = 0$, $w_{10} = w_{1\emptyset} = (1-\mu)\eta f$ and $w_{11} = \frac{1}{\mu}\left[\frac{1}{2}[\pi - (1-\mu)S] - (1-\mu)\eta f\right] + (1-\mu)\eta f$ is optimal in the absence of the new wage restriction (compare Lemma 4) as well as satisfies the new wage restrictions. Hence, in this case, the new wage restriction is not binding and the principal induces $e^{MI}(\mu)$ and earns $\Pi^{MI}(\mu)$ as specified in Lemma 4.

Otherwise, if $\frac{1}{2}[\pi - (1-\mu)S] < (1-\mu)\eta f$, by the above analysis, the new wage restriction is binding for all wages and the optimal wages are $w_0 = 0$ and $w_{11} = w_{10} = w_{1\emptyset} = (1-\mu)\eta f$. Given these optimal wages, the agent exerts effort $\check{e}^{MI} = \gamma(1-\mu)\eta f$, and the principal receives the profit $\check{\Pi}^{MI}(\mu) = (1-\mu)(\gamma\eta f\,[\pi - (1-\mu)\,(S+\eta f)] + S)$. We obtain

$$\check{e}^{MI}(\mu) > e^{MI}(\mu) \Leftrightarrow \gamma\,(1-\mu)\,\eta f > \frac{\gamma}{2}\,[\pi - (1-\mu)\,S] \Leftrightarrow$$
$$(1-\mu)\,\eta f > \frac{1}{2}\,[\pi - (1-\mu)\,S]\,,$$

which is satisfied. In addition, because of the binding wage restriction, we must have $\check{\Pi}^{MI}(\mu) < \Pi^{MI}(\mu)$. □

The case where condition (15) is only violated for a subset of $[\underline{\mu}, \bar{\mu}]$. Suppose that there exists a $\check{\mu} \in (\underline{\mu}, \bar{\mu})$ such that condition (15) is violated for all $\mu < \check{\mu}$ but holds for all $\mu \geq \check{\mu}$. This means that, if the principal picks a $\mu \geq \check{\mu}$, she can still earn the profit $\Pi^{MI}(\mu)$. In other words, implementing a sufficiently effective internal control system is now even more helpful to alleviate the effects of the wage restriction. However, as the profit $\Pi^{MI}(\mu)$ is strictly decreasing in μ, the principal may prefer $\mu = \check{\mu}$, but will never choose an effectiveness strictly above $\check{\mu}$. As $\Pi^{MI}(\check{\mu}) = \check{\Pi}^{MI}(\check{\mu})$ and the profit function $\check{\Pi}^{MI}(\mu)$ may attain its maximum at a lower value of μ, the principal may prefer an effectiveness strictly below $\check{\mu}$ as the associated profit $\check{\Pi}^{MI}(\mu)$ can exceed $\Pi^{MI}(\check{\mu})$. Overall, compared to the case analyzed in Proposition 6, the wage restriction leads to a weakly lower effectiveness and a weakly higher effort.

Proof of Proposition 6. (a) Suppose, the principal has implemented internal control. If she wants to prevent manipulation, the choice of μ will be irrelevant, as she always obtains the benchmark payoff.

If she wants to induce manipulation, she maximizes the function $\check{\Pi}^{MI}(\mu)$, which describes a concave parabola with a maximum at

$$\mu = 1 - \frac{S + \gamma\pi\eta f}{2\gamma\eta f\,(S+\eta f)} =: \tilde{\mu}.$$

Obviously, $\tilde{\mu} < 1$. However, it is not clear, whether $\tilde{\mu}$ is positive or not. In addition, $\check{\Pi}^{MI}(0) = \gamma\eta^2 f\delta + S > 0$ and $\check{\Pi}^{MI}(1) = 0$. We have to differentiate between two case. If $\tilde{\mu} \leq 0$, then $d\check{\Pi}^{MI}(\mu)/d\mu < 0$ for all $\mu \in [\underline{\mu}, \bar{\mu}]$, so that $\mu^* = \underline{\mu}$. If $\tilde{\mu} > 0$, then $\mu^* = \tilde{\mu}$ given that $\tilde{\mu} \in [\underline{\mu}, \bar{\mu}]$;

otherwise, $\mu^* = \underline{\mu}$ if $\underline{\mu} > \tilde{\mu}$, and $\mu^* = \bar{\mu}$ if $\bar{\mu} < \tilde{\mu}$. Result (a) summarizes these findings.

In the next step, we analyze, under which parameter constellations the principal will prefer manipulation to non-manipulation if she has implemented internal control. Given internal control, she will prefer to induce manipulation if and only if the respective profit $\check{\Pi}^{MI}(\mu^*)$ exceeds the benchmark profit, which would be realized under non-manipulation:

$$\check{\Pi}^{MI}(\mu^*) = (1-\mu^*)(\gamma\eta f[\pi - (1-\mu^*)(S+\eta f)] + S) > \gamma\frac{\pi^2}{4} \Leftrightarrow$$
$$[1-\gamma(1-\mu^*)\eta f](1-\mu^*) S > \frac{\gamma}{4}[\pi - 2(1-\mu^*)\eta f]^2 .$$

As the term in square brackets at the left-hand side of the inequality is strictly positive due to $\check{e}^{MI}(\mu^*) = \gamma(1-\mu^*)\eta f < 1$ (see assumption (A1)), we obtain

$$S > \frac{\gamma}{4}\frac{[\pi - 2(1-\mu^*)\eta f]^2}{(1-\mu^*)[1-\gamma(1-\mu^*)\eta f]}. \tag{A75}$$

We can immediately see that the right-hand side of (A75) is always positive. In addition, we can show that there exist feasible parameter constellations for which (A75) holds. By replacing S, inequality (A75) can be rewritten as

$$\pi - \eta\delta - \eta f > \frac{\gamma}{4}\frac{[\pi - 2(1-\mu^*)\eta f]^2}{(1-\mu^*)[1-\gamma(1-\mu^*)\eta f]} \Leftrightarrow$$
$$\frac{4(1-\mu^*)(\pi[1+f\gamma\mu^*\eta] - \eta f) - \pi^2\gamma}{4(1-\mu^*)[1-(1-\mu^*)\gamma\eta f]} > \eta\delta.$$

Consider, for example, $\gamma, \delta \to 0$. As the denominator is strictly positive, the condition simplifies to

$$\pi > f\eta.$$

In addition, as $\gamma \to 0$, assumption (A1) is clearly satisfied. Finally, condition (15) has to be violated, which can be rewritten as $f > \frac{\mu^*}{(1-\mu^*)\eta}\pi + \delta$. Due to $\delta \to 0$, both inequalities together yield

$$\frac{\mu^*}{1-\mu^*}\pi < \eta f < \pi,$$

which can be satisfied for $\mu^* < \frac{1}{2}$. In the derivation of μ^* above, we have seen that the principal chooses $\mu^* = \underline{\mu}$ for $\gamma, \delta \to 0$.

(b) Suppose, $S > 0$ and preventing manipulation is optimal without internal control, i.e., we have $\frac{\pi}{2} > \eta\delta$ and $f \in \left[\hat{f}, \frac{\pi-\eta\delta}{\eta}\right)$. In that case, without internal control, the principal implements the benchmark solution, yielding profit $\gamma\frac{\pi^2}{4}$. If for $\frac{\pi}{2} > \eta\delta$ and $f \in \left[\hat{f}, \frac{\pi-\eta\delta}{\eta}\right)$ the principal prefers manipulation to non-manipulation under internal control—that is, condition (19) is satisfied—, the manipulation profit $\check{\Pi}^{MI}(\mu^*)$ will exceed the benchmark profit $\gamma\frac{\pi^2}{4}$, which implies that the principal prefers to implement internal control in order to induce manipulation.[1]

□

[1] There exist feasible parameter values that satisfy the conditions for result (b) to hold, for example, $\delta = 0$, $\eta = \gamma = 0.1$, and $\pi = 1.8$.