

# Internet Appendix to “Dark Knights: The Rise in Firm Intervention by CDS Investors”

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## A. General model with an active CDS buyer

We present a model that builds on Bolton and Oehmke (2011), with a few extensions. We allow the firm to issue both equity and debt to finance investment, as opposed to using only debt. We also introduce an optimal trade-off between debt and equity, a continuously distributed profitability shock, and a continuous investment variable. This version of the model can also be viewed as a static version of the dynamic model in Danis and Gamba (2018). The static nature of the model allows us to derive results in closed form, at least for some special cases.

All agents in the model are risk-neutral. We model a single firm, owned by an entrepreneur who makes investment and financing decisions to maximize her expected payoff at the beginning of the period. The main driver of the model is the firm’s profit shock, a continuous-state random variable. The probability of the end-of-period shock,  $z$ , is determined by the cumulative distribution function  $\Gamma(z)$ , where  $z > 0$ .

We denote by  $k$  the capital stock for the period. To finance the investment, the firm issues debt alongside a possible equity injection. The debt contract is an unsecured zero-coupon bond with face value  $b$  paid at the end of the period. Both  $k$  and  $b$  are non-negative. Debt financing is cheaper than equity financing for two reasons. First, we assume that there are proportional equity issuance costs, captured by the parameter  $\lambda$ . Second, we assume that debt payments reduce the corporate income tax base. The corporate tax rate is  $\tau$ . Since the debt is a zero-coupon bond, we assume for simplicity that the whole face value is deductible. Because this overstates the tax benefits of debt compared to the real world, we compensate by parameterizing the tax rate accordingly.

The firm’s profit shock determines the asset value at the end of the period:

$$a(z, k) = zk^\alpha, \tag{IA.1}$$

where,  $\alpha \in ]0, 1[$  is the return-to-scale parameter.

After the realization of  $z$ , the owner decides whether to pay  $b$  in full, to renegotiate the debt by paying an amount  $b_r$ , or to file for bankruptcy and liquidate the firm. The owner cannot commit not to default on the debt in the future. If the debt is renegotiated, we assume that renegotiation may fail for exogenous reasons with probability  $\gamma \in [0, 1[$ , in which case the firm is liquidated, the debt holder receives  $(1 - \xi)a(z, k)$ , and the owner receives nothing. The parameter  $\xi$  represents bankruptcy costs, with  $\xi \in ]0, 1[$ .

We assume a competitive market for insuring against credit risk. In particular, the debt holder can purchase a CDS from a dealer (protection seller) at the time the debt contract is issued. The lender (protection buyer) chooses the fraction  $h$  of the debt exposure covered

by the CDS contract. The dollar amount, or notional amount, of insured debt is therefore  $hb$ . After observing  $h$ , the protection seller sets the CDS spread (the insurance premium) accordingly. The CDS spread is endogenously determined and the protection seller has rational expectations: he understands that selling CDS protection to the debt holder may change both the probability of default and the debt payoff in default and adjusts the CDS spread accordingly.

The debt holder chooses the hedge ratio,  $h$ , to maximize his expected payoff. Because we assume that the credit risk market is competitive, the CDS spread is fair (and the transaction has zero-NPV for the protection seller). In the first part of this section,  $h$  will be an arbitrary hedge ratio. We discuss later how the optimal hedge ratio is determined.<sup>1</sup>

The sequence of events is as follows. The firm owner chooses a capital level  $k$  and a face value of debt  $b$ . The debt holder observes the outcome of these decisions and chooses a hedge ratio  $h$ . At the end of the model, nature chooses a productivity shock  $z$ , and the owner decides between repaying the debt, renegotiating with the debt holder, or liquidating the firm. The timeline of events is the same as in Figure 2 in the paper.

We next describe the payoffs to equity and debt as a function of the owner's default decision. The payoff to equity at the end of the period is  $a - b$  if the debt is repaid,  $a - b_r$  if the debt is successfully renegotiated, and  $\max\{(1 - \xi)a - b, 0\}$  if the firm is liquidated. The corresponding payoff to debt is  $b$  when it is repaid in full, and  $b_r$  in case of successful renegotiation. In the case of liquidation, if the liquidation payoff is less than the face value of debt, or  $(1 - \xi)a \leq b$ , the payoff to debt is  $hb + (1 - h)(1 - \xi)a$ . The term  $hb$  represents the payment from the protection seller, while the term  $(1 - h)(1 - \xi)a$ , which can be written as  $(1 - \xi)a - h(1 - \xi)a$ , is the liquidation value of the firm, net of the payment to the protection seller.

We derive the optimal default decision for the two cases  $(1 - \xi)a > b$  and  $(1 - \xi)a \leq b$  separately. When the owner's liquidation payoff is positive, or  $(1 - \xi)a > b$ , the threat of debt renegotiation is not credible because the debt holder can recover the full face value of debt in liquidation. The owner herself prefers repayment to liquidation, because the payoff under debt repayment,  $a - b$ , is higher than the liquidation payoff,  $(1 - \xi)a - b$ . Therefore, debt is always repaid if  $(1 - \xi)a > b$ .

On the other hand, if the liquidation payoff to equity is zero,  $(1 - \xi)a \leq b$ , then debt renegotiation leads to the following Nash bargaining problem:

$$b_r(z, k, b, h) = \arg \max_p [a - p]^{1-q} \times [p - hb - (1 - h)(1 - \xi)a]^q, \quad (\text{IA.2})$$

where  $q \in [0, 1]$  is the exogenous bargaining power of the debt holder. The bargaining problem has two constraints,  $a - p \geq 0$  and  $p \geq hb + (1 - h)(1 - \xi)a$ . The first constraint states that the owner's payoff after successful renegotiation,  $a - p$ , must be at least as large as her outside option. Similarly, the second constraint makes sure that the debt holder's renegotiation payoff  $p$  is not below his outside option. The Nash bargaining problem, together

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<sup>1</sup>There are no speculators in the CDS market in our model, and therefore no so-called naked CDS positions. The only agent who buys CDS protection in equilibrium is the lender. However, speculators would not change much in this framework. By definition, they do not own the debt of the underlying firm, so they cannot participate in any debt restructuring. Therefore, they would not have any effect on the probability of default or on the recovery rate in default.

with the constraints, is important for the results of the model. If the hedge ratio  $h$  is sufficiently high, then the outside option of the debt holder  $hb + (1 - h)(1 - \xi)a$  is so large that there is no  $p$  that can satisfy the two constraints. The result is that renegotiation is not feasible. And even if renegotiation is feasible, a higher hedge ratio increases the payoff to the debt holder, because the  $p$  that solves the bargaining game is increasing in the debt holder's outside option.

The debt holder's optimal hedging policy, the firm's optimal default policy, and the outcome of the bargaining game is summarized in the following proposition:

PROPOSITION 1: *Given the choices  $k$  and  $b > 0$ :*

1. *The optimal hedge ratio is  $h^* \in [0, 1]$ .*
2. *The optimal default policy depends on  $a$ : it is optimal for the owner to repay the debt for  $a \geq a_P$ , to attempt renegotiation for  $a_R \leq a < a_P$ , and to liquidate the firm for  $a < a_R$ , where*

$$a_R = \frac{hb}{1 - (1 - h)(1 - \xi)}, \quad a_P = \frac{b[1 - h(1 - \gamma)(1 - q)]}{1 - (1 - \gamma)(1 - q)[\xi + h(1 - \xi)]},$$

and  $a_R < b < a_P$ .

3. *The introduction of a CDS market is equivalent to an increase in the hedge ratio from  $h = 0$  to  $h^* \in [0, 1]$ . This decreases  $a_P$  and increases  $a_R$ . The first effect leads to more likely repayment of debt and less likely renegotiation. The second effect increases the probability of liquidation and reduces the probability of renegotiation.*
4. *If renegotiation occurs in equilibrium, the renegotiated debt payment can be written as*

$$b_r = hb + (1 - h)(1 - \xi)a + q[a - hb - (1 - h)(1 - \xi)a]. \quad (\text{IA.3})$$

*Since  $b_r$  is increasing in the hedge ratio  $h$ , the introduction of a CDS market increases the payoff to the debt holder in renegotiation.*

The proof is in Appendix B. Figure IA.12 shows the optimal default decision. Low asset values lead to liquidation, intermediate asset values trigger renegotiation, and high values of  $a$  lead to debt repayment.

Proposition 1 and Figure IA.12 also summarize nicely the positive and negative effects of CDS contracts. On the one hand, if the hedge ratio increases from  $h = 0$  to  $h = h^*$ , the threshold  $a_P$  decreases, which means that repayment is more likely to occur, while renegotiation is less likely. This is good for the debt holder, which he anticipates when the debt is issued, and this decreases credit spreads. There is a second positive effect of introducing a CDS market for the lender. This effect operates through an increase in  $b_r$ , the payoff to debt holder in a future renegotiation. On the other hand, if the hedge ratio  $h$  increases, the threshold  $a_R$  increases, which makes liquidation more likely and renegotiation less likely, reducing the expected payoff to the debt holder. Anticipating this outcome, he adjusts credit spreads upwards when the debt is issued.

The model encompasses also the case without a CDS market. More details about this case are in Appendix C. If the debt holder has no CDS protection (i.e.,  $h = 0$ ), the debt is renegotiated if  $a < a_P$ , and it is repaid if  $a \geq a_P$ .

We now turn to the owner's ex ante decision. The owner maximizes the cum-dividend value of equity (i.e., of the firm), defined as

$$\begin{aligned}
V(k^*, b^*, h(k^*, b^*)) &= \max_{k, b} V(k, b, h(k, b)) \\
&= \max_{k, b} \left\{ [m(k, b) - k](1 + \lambda \mathbf{1}_{\{m(k, b) < k\}}) \right. \\
&\quad \left. + \frac{1 - \tau}{1 + r} \left[ \int_{z_R}^{z_P} (1 - \gamma)(a(z, k) - b_r) d\Gamma(z) + \int_{z_P}^{\infty} (a(z, k) - b) d\Gamma(z) \right] \right\}, \tag{IA.4}
\end{aligned}$$

where  $m(k, b)$  denotes the equilibrium price of debt, which we will derive later on. The dividend  $m(k, b) - k$  can have either sign; if it is negative, it is the amount of injected equity capital. In this case, the firm incurs a transaction cost  $\lambda$  per unit of equity capital. The parameter  $\tau$  is the corporate income tax rate. The limits of the integrals are  $z_R = a_R/k^\alpha$  and  $z_P = a_P/k^\alpha$ . While  $a_P$  and  $a_R$  are functions of  $(b, h)$ , and  $b_r$  is a function of  $(a(z, k), b, h)$ , in what follows we suppress these dependencies for notational convenience.

The debt holder maximizes his expected payoff, denoted by  $M$ , by choosing the hedge ratio:

$$m(k, b) = \max_h \frac{1}{1 + r} M(k, b, h). \tag{IA.5}$$

The solution of the above program,  $h = h(k, b)$ , is the state-contingent optimal hedge ratio that is considered in the owner's program in (IA.4).

To find  $M$ , the expected payoff to the debt holder, we first derive the fair price of the CDS contract. The credit event that triggers the CDS payment is bankruptcy/liquidation. An out-of-court debt restructuring does not trigger a CDS payment, in line with the Standard North American Contract (SNAC) of the International Swaps and Derivatives Association (e.g., Bolton and Oehmke (2011)). The price of credit protection for a given hedge ratio  $h \in [0, 1]$  is the expectation of the net compensation from the protection seller:

$$C(k, b, h) = \int_0^{z_R} [hb - h(1 - \xi)a(z, k)] d\Gamma(z) + \int_{z_R}^{z_P} \gamma [hb - h(1 - \xi)a(z, k)] d\Gamma(z). \tag{IA.6}$$

The first integral is the expected payoff in a deliberate liquidation, while the second integral is the expected payoff in a failed renegotiation.

The expected payoff to the debt holder, including the payment from the protection seller in case of a credit event, but excluding the insurance premium  $C(k, b, h)$ , is

$$\begin{aligned}
\psi(k, b, h) &= \int_0^{z_R} [hb + (1 - h)(1 - \xi)a(z, k)] d\Gamma(z) \\
&\quad + \int_{z_R}^{z_P} [(1 - \gamma)b_r + \gamma [hb + (1 - h)(1 - \xi)a(z, k)]] d\Gamma(z) + \int_{z_P}^{\infty} b d\Gamma(z). \tag{IA.7}
\end{aligned}$$

The expected value of the debt for a given hedge ratio,  $h$ , net of the cost of the CDS is

$$M(k, b, h) = \psi(k, b, h) - C(k, b, h).$$

After simplifying, the price of debt and the optimal  $h$  are found by solving the program

$$m(k, b) = \max_h \frac{1}{1+r} \left[ \int_0^{z_R} (1-\xi)a(z, k) d\Gamma(z) + \int_{z_R}^{z_P} [(1-\gamma)b_r + \gamma(1-\xi)a(z, k)] d\Gamma(z) + \int_{z_P}^{\infty} b d\Gamma(z) \right]. \quad (\text{IA.8})$$

### A.1. The effect of CDSs on firm value

Since the full model cannot be solved in closed-form, we solve it numerically. We assume that  $z$  follows a uniform distribution,  $z \sim U[0, Z]$ , and use the parameter values in Table IA.2. We define several variables that allow us to illustrate how the introduction of a CDS market affects various firm outcomes. These variables are the total firm value,  $V(k^*, b^*, h(k^*, b^*))$ , the dividend,  $m(k^*, b^*) - k^*$ , the optimal hedge ratio,  $h^*$ , the credit spread, the optimal face value of debt,  $b^*$ , the optimal capital stock,  $k^*$ , and quasi-market leverage,  $b^*/V(k^*, b^*, h(k^*, b^*))$ .

Figure IA.13 presents a sensitivity analysis of the above metrics with respect to  $q$ , the debt holder's bargaining power. Most importantly, it compares firm value with a CDS market to firm value in an economy without CDSs. The plot for firm value shows nicely that there are both positive and negative value effects, and that the net value effect depends on parameter values. For low values of the lender's bargaining power, with-CDS firm value exceeds the no-CDS firm value. For high values of bargaining power, the opposite is true.

To understand where these opposing effects on firm value come from, it is important to understand the constrained situation of this firm. Most importantly, there is a commitment problem between the owner and the lender. The owner cannot commit to repay the debt in the future. In some states of nature, she renegotiates even though the firm would have enough profits to repay the debt. Other frictions include taxes, equity issuance costs, bankruptcy costs, and renegotiation failure. As a result of all these frictions, the firm cannot invest as much in capital as it would want to in a first-best world with no frictions.

The reason why CDSs create value is that they alleviate the commitment problem between the owner and the lender. By hedging with CDS contracts, the debt holder improves his bargaining position in any future renegotiation. This has two positive effects: First, it increases the lender's payoff in renegotiation. Second, it reduces the owner's incentive to renegotiate, especially in the states of nature where the asset value would be high enough to repay the debt. In other words, it reduces the probability of strategic default. This is why the effect of CDSs on firm value is particularly large for low values of  $q$ . If the lender's bargaining power is low, then he is particularly vulnerable to strategic default, because the owner can extract larger rents from him. As a consequence, a low  $q$  should create a strong incentive to hedge with CDS contracts. This is exactly what is shown in the subplot for the hedge ratio.

The lower risk of strategic default ex post leads to lower credit spreads ex ante. This is shown in the plot for the credit spread. As the cost of debt financing decreases, the owner issues more debt, which can be seen in the plot for the book value of debt. Also, the owner substitutes costly equity financing with cheaper debt financing. In the plot for quasi-market leverage, one can see that at low values of  $q$ , CDSs increase leverage. The reduction in the cost of debt is an important source of value creation. It exists even for a fixed amount of

capital  $k$ . Additionally, these new funds are used to invest in capital, which is shown in the plot for capital stock. So the owner can move closer to the first-best level of capital stock. Overall, the net effect of CDSs on firm value, or value effect, can be decomposed into a financial effect, which holds even for constant capital, and a real effect, which allows for an endogenous increase in investment.

For intermediate to high values of the lender's bargaining power, the net effect of CDSs on firm value is negative. The downside of allowing the debt holder to hedge with CDS contracts is that renegotiation is sometimes infeasible. Since renegotiation is less costly than bankruptcy, in some states of nature it would have been better to renegotiate debt, but instead, the firm has to file for bankruptcy. The underlying problem here is that the lender chooses the hedge ratio  $h$  to maximize the value of his own claim, not total firm value. As a result, the lender over-insures relative to the hedge ratio that would maximize firm value. For high values of  $q$ , this negative effect of CDS contracts dominates the positive effects, which creates the negative net effect on firm value.

In the limit, as  $q$  moves closer to 1, the debt holder has all the bargaining power. The owner cannot extract any rents from the lender, which alleviates the agency conflict between the two parties. As a result, the cases with and without CDSs are identical, making CDSs *redundant*.

While the magnitude of the effect of CDSs on firm value is relatively small (less than 1% in Figure IA.13), the purpose of this analysis is to examine the determinants of the sign of the effect. An extension to a dynamic model and a detailed calibration is outside of the scope of this paper.

Figure IA.14 shows that the amount of value created is positive for high values of the liquidation cost  $\xi$ , and negative for low values of  $\xi$ . The intuition is that a higher liquidation cost reduces the outside option of the debt holder in bargaining, which allows the owner to extract higher rents. The debt holder is aware of this, so he purchases more CDS contracts to improve his outside option. CDS contracts become redundant in the limit where  $\xi$  approaches zero.

Figure IA.15 depicts comparative statics with respect to  $\gamma$ , the probability of renegotiation failure. For low values of  $\gamma$ , the value effect is negative, and for high values of  $\gamma$ , it is positive. The probability of renegotiation failure has both positive and negative consequences for the debt holder. On the one hand, it reduces the incentive of the owner to renegotiate. This decreases the likelihood of (attempted) debt renegotiation and increases the probability of debt repayment. Since CDSs are another way to reduce the probability of renegotiation, a higher  $\gamma$  reduces the need to buy CDS contracts. On the other hand, conditional on an attempted debt renegotiation, it increases the chance of failure. The lender's payoff in a successful renegotiation,  $b_r$ , is increasing in  $h$ . Therefore, buying CDSs can offset the loss in expected payoff. These two effects approximately offset each other, which is why the plot for the hedge ratio is almost constant. For low values of  $\gamma$  one effect dominates, and for high values of  $\gamma$  the other dominates.

In addition to revealing the parameter values under which the value effect of CDS introduction is positive, our comparative statics exercises also have important implications for empirical tests. Our results imply that the theory in Bolton and Oehmke (2011) or its many extensions cannot be easily tested using a reduced-form regression.

For example, regressing firm value (or Tobin's Q, or investment) on a proxy for one of

the three exogenous parameters  $q$ ,  $\xi$ , or  $\gamma$ , interacted with a CDS dummy, cannot be used to test the theory. This is important because similar regression specifications are often used in the empirical literature mentioned in the introduction. Figures IA.13–IA.15 show that the net effect on firm value, i.e., the difference between the diamonds and the crosses in the first subplot of each figure, is not a monotonic function of the exogenous parameter. Additionally, we have found in unreported tests that the shape of the relationship between the value effect and a particular exogenous parameter changes with different values for the *other* exogenous parameters. Making things even more difficult for econometricians, the shape of the relationship between the value effect and a parameter changes with different distributional assumptions for profitability  $z$ . We show this in unreported tests by assuming a log-normal distribution for  $z$ .

## B. Proof of Proposition 1

From Equation (IA.2), renegotiation is feasible if

$$hb + (1 - h)(1 - \xi)a \leq a. \quad (\text{IA.9})$$

If renegotiation is feasible, the solution of (IA.2) is

$$b_r(z, k, b, h) = hb + (1 - h)(1 - \xi)a + q[a - hb - (1 - h)(1 - \xi)a]. \quad (\text{IA.10})$$

Debt repayment is preferred to renegotiation when  $a - b \geq (1 - \gamma)(a - b_r)$ , or

$$a[1 - (1 - \gamma)(1 - q)s(h)] \geq b[1 - (1 - \gamma)(1 - q)h], \quad (\text{IA.11})$$

in which  $s(h) = \xi + h(1 - \xi)$ . Renegotiation feasibility depends on the sign of  $s(h)$ , whose only zero is

$$h_0 = -\frac{\xi}{1 - \xi} < 0.$$

When  $h > h_0$  (i.e.,  $s(h) > 0$ ), renegotiation is feasible if  $a \geq a_R$ , where  $a_R$  is defined as

$$a_R = \frac{hb}{1 - (1 - h)(1 - \xi)}. \quad (\text{IA.12})$$

Because the numerator on the right-hand side of (IA.12) is negative when  $h \in ]h_0, 0]$ , then renegotiation is feasible for all  $a > 0$ . Otherwise, for  $h > 0$ , renegotiation is feasible for  $a \geq a_R > 0$ . When  $h = h_0$  (i.e.,  $s(h) = 0$ ), then Equation (IA.9) is always satisfied, and so renegotiation is feasible for all  $a > 0$ . Finally, if  $h < h_0$  (i.e.,  $s(h) < 0$ ), renegotiation is feasible if  $a \leq a_R$ , and in this case,  $a_R > 0$ . Therefore, renegotiation is feasible for  $a \in ]0, a_R]$  when  $h < h_0$ .

The choice between debt repayment and renegotiation depends on the sign of  $1 - (1 - \gamma)(1 - q)s(h)$ , and the only zero of this function is

$$h_1 = \frac{1 - \xi(1 - q)(1 - \gamma)}{(1 - q)(1 - \xi)(1 - \gamma)}, \quad (\text{IA.13})$$

with  $h_1 > h_0$ , and  $h_1 > 1$ . When  $h < h_1$  (i.e.,  $1 - (1 - \gamma)(1 - q)s(h) > 0$ ), repayment is optimal for  $a \geq a_P$ , where  $a_P$  is defined as

$$a_P = \frac{b[1 - h(1 - \gamma)(1 - q)]}{1 - (1 - \gamma)(1 - q)s(h)}. \quad (\text{IA.14})$$

Because the numerator in (IA.14) is negative for  $1/[(1 - \gamma)(1 - q)] \leq h$ , and  $1/[(1 - \gamma)(1 - q)] < h_1$  (this is equivalent to  $(1 - \gamma)(1 - q) < 1$ , which is consistent with our assumptions), then for  $h \in [1/[(1 - \gamma)(1 - q)], h_1[$ ,  $a_P \leq 0$ , and repayment is optimal for all  $a > 0$ . Otherwise, for  $h \leq 1/[(1 - \gamma)(1 - q)]$ , repayment is optimal for  $a \geq a_P$ .

If  $h = h_1$  (i.e.,  $1 - (1 - \gamma)(1 - q)s(h) = 0$ ), the left-hand side of (IA.11) vanishes, and in the right-hand side, from  $1 - (1 - \gamma)(1 - q)h_1$ , after rearranging, we have  $-\xi[\gamma + q(1 - \gamma)]/(1 - \xi)$ , which is negative. Because the left-hand side is zero and the right-hand side is negative, inequality (IA.11) holds true and repayment is preferred to renegotiation for all  $a > 0$ .

When  $h > h_1$  (i.e.,  $1 - (1 - \gamma)(1 - q)s(h) < 0$ ), repayment is optimal for  $a \leq a_P$ . However, for  $h > h_1$ , we have that  $[1 - h(1 - q)] < 0$  in (IA.14). Therefore,  $a_P > 0$ , and repayment is optimal for  $a \in ]0, a_P]$ .

When renegotiation is not feasible, the equity payoff is  $a - b$  under debt repayment and zero under liquidation. Therefore, if  $a \geq b$ , the owner prefers repayment, and if  $a < b$ , she prefers liquidation.

To prove Point 2 of Proposition 1, we need to study the relation between  $b/(1 - \xi)$ ,  $a_P$ ,  $a_R$ , and  $b$ . We will do this in the following Lemmas.

### *The relation between $b/(1 - \xi)$ , $a_P$ , $a_R$ , and $b$*

Lemma B.1 (Compare  $a_P$  and  $b$ ): *If  $1 - (1 - \gamma)(1 - q)s(h) > 0$  (i.e.  $h < h_1$ ), and  $h > 1$ , then  $a_P < b$ .*

*If  $h < 1$  then  $b < a_P$ .*

*If  $1 - (1 - \gamma)(1 - q)s(h) < 0$  (i.e.  $h > h_1$ ), then  $b < a_P$ .*

This can be shown by rearranging the inequality

$$\frac{b[1 - h(1 - \gamma)(1 - q)]}{1 - (1 - \gamma)(1 - q)s(h)} < b$$

and using the different conditions on  $h$ .

Lemma B.2 (Compare  $a_R$  and  $b$ ): *If  $s(h) > 0$  (i.e.  $h > h_0$ ), and  $h < 1$ , then  $a_R < b$ .*

*If  $s(h) > 0$  (i.e.  $h > h_0$ ), and  $h > 1$ , then  $a_R > b$ .*

*If  $s(h) < 0$  (i.e.  $h < h_0$ ), and  $h < 1$ , then  $a_R > b$ .*

To show this, assume  $s(h) > 0$  and  $h < 1$ . Then, from  $h/s(h) < 1$ , after rearranging using the different conditions, it follows that the inequality holds. The other two cases are analogous.

Lemma B.3 (Compare  $a_P$  and  $b/(1 - \xi)$ ): *If  $1 - (1 - \gamma)(1 - q)s(h) > 0$  (i.e.  $h < h_1$ ), then  $a_P < b/(1 - \xi)$ .*

*If  $1 - (1 - \gamma)(1 - q)s(h) < 0$  (i.e.  $h > h_1$ ), then  $b/(1 - \xi) < a_P$ .*

This can be shown by rearranging the inequality

$$\frac{b[1 - h(1 - \gamma)(1 - q)]}{1 - (1 - \gamma)(1 - q)s(h)} < \frac{b}{1 - \xi}$$

and using the different conditions on  $h$ .

Lemma B.4 (Compare  $a_R$  and  $b/(1 - \xi)$ ): *If  $s(h) > 0$  (i.e.  $h > h_0$ ), then  $a_R < b/(1 - \xi)$ . If  $s(h) < 0$  (i.e.  $h < h_0$ ), then  $a_R > b/(1 - \xi)$ .*

To show this, assume  $s(h) > 0$  and  $h < 1$ . Then, from  $h/s(h) < 1/(1 - \xi)$ , after rearranging using the conditions, it follows that the inequality holds. The other two cases are analogous.

Lemma B.5 (Compare  $a_P$  and  $a_R$ ):  *$a_P \geq a_R$  if and only if*

$$\frac{1 - h(1 - \gamma)(1 - q)}{1 - (1 - \gamma)(1 - q)s(h)} \geq \frac{h}{s(h)}.$$

This is proved as follows. Since the denominator can be either positive or negative, we have the following four subcases:

1. Assume  $s(h) > 0$  and  $1 - (1 - \gamma)(1 - q)s(h) > 0$ , which is the same as  $h > h_0$  and  $h < h_1$ . In this subcase, it can be shown that
  - If  $h_0 < h < 1$ , then  $a_P > a_R$ .
  - If  $1 < h < h_1$ , then  $a_P < a_R$ .

This follows from rearranging the inequality above using the conditions, and using the fact that  $h_0 < 1 < h_1$ .

2. Assume  $s(h) > 0$  and  $1 - (1 - \gamma)(1 - q)s(h) < 0$ , which is the same as  $h > h_0$  and  $h > h_1$ . Because  $h_0 < h_1$ , these assumptions are equivalent to  $h > h_1$ . In this case, it can be shown that the inequality above always holds strictly, because  $h_1 > 1$ . Therefore,  $a_P > a_R$  for all  $h > h_1$ .
3. Assume  $s(h) < 0$  and  $1 - (1 - \gamma)(1 - q)s(h) > 0$ , which is the same as  $h < h_0$  and  $h < h_1$ . Because  $h_0 < h_1$ , these assumptions reduce to  $h < h_0$ . In this case, it can be shown that the inequality above never holds. Therefore,  $a_P < a_R$  for all  $h < h_0$ .
4. Assume  $s(h) < 0$  and  $1 - (1 - \gamma)(1 - q)s(h) < 0$ , which is equivalent to  $h < h_0$  and  $h > h_1$ . This is impossible, so this subcase can be dropped.

Lemma B.6 (Compare 0 and  $a_R$ ): *If  $h > 0$  and  $h > h_0$  then  $a_R > 0$ .*

*If  $h_0 < h < 0$  then  $a_R < 0$ .*

*If  $h < 0$  and  $h < h_0$  then  $a_R > 0$ .*

This follows from the definition

$$a_R = \frac{hb}{1 - (1 - h)(1 - \xi)}.$$

Lemma B.7 (Compare 0 and  $a_P$ ): *If  $h < 1/[(1 - \gamma)(1 - q)]$  and  $h < h_1$  then  $0 < a_P$ . If  $1/[(1 - \gamma)(1 - q)] < h < h_1$  then  $0 > a_P$ .*

If  $h > 1/[(1-\gamma)(1-q)]$  and  $h > h_1$  then  $0 < a_P$ .

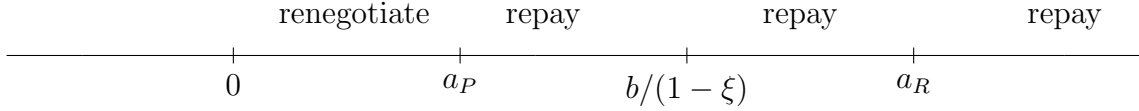
This follows from the definition

$$a_P = \frac{b[1 - h(1-\gamma)(1-q)]}{1 - (1-\gamma)(1-q)s(h)}.$$

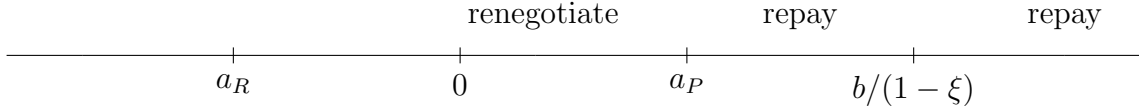
### The optimal default policy

The above Lemmas enable us to derive the optimal default policy. Because  $h_0 < 0 < 1 < 1/[(1-\gamma)(1-q)] < h_1$ , there are six regions for  $h$ . We will describe the default policy in these regions, from lowest to highest  $h$ , and in each of the six cases, for the different  $a$ . Initially we will consider only the interior of these intervals. We will deal with the boundaries (i.e.,  $h = h_0$ ,  $h = 0$ , etc.) later on.

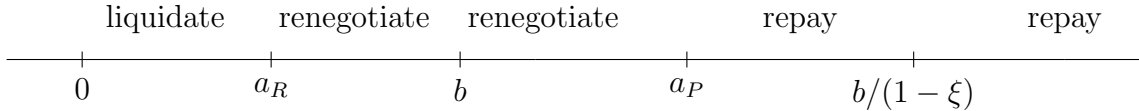
1. If  $h < h_0$ , because in Lemmas 3, 4, and 7 we have determined that  $0 < a_P < b/(1-\xi) < a_R$ , then the following diagram summarizes the optimal actions:



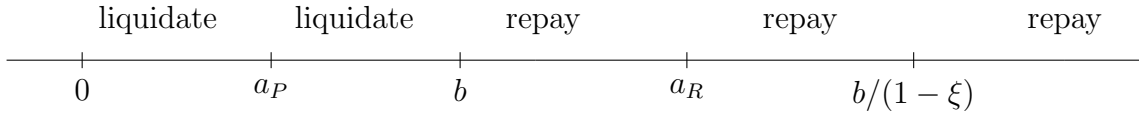
2. If  $h_0 < h < 0$ , because  $a_R < 0 < a_P < b/(1-\xi)$  from Lemmas 3, 6, and 7, we can derive the following diagram:



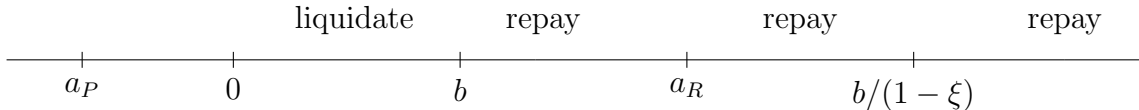
3. If  $0 < h < 1$ , we know from Lemmas 1, 2, 3, and 6 that  $0 < a_R < b < a_P < b/(1-\xi)$ .



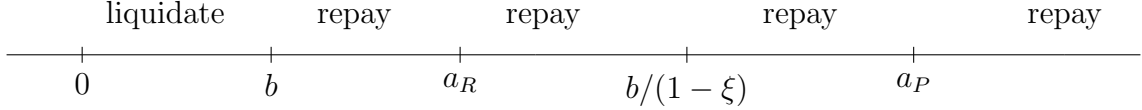
4. If  $1 < h < 1/[(1-\gamma)(1-q)]$ , we have determined in Lemmas 1, 2, 4, and 7 that  $0 < a_P < b < a_R < b/(1-\xi)$ .



5. If  $1/[(1-\gamma)(1-q)] < h < h_1$ , we know from Lemmas 2, 4, and 7 that  $a_P < 0 < b < a_R < b/(1-\xi)$ . Then the optimal actions are



6. Finally, if  $h_1 < h$ , because from Lemmas 2, 3, and 4 we have  $0 < b < a_R < b/(1-\xi) < a_P$ , then we can derive the diagram



The six diagrams above can be summarized into three main cases. This is done in Figure IA.1.

Finally, we analyze the optimal decision at the boundaries  $h = 0$  and  $h = 1$ .<sup>2</sup> At  $h = 0$ , from Lemmas 1 and 2 we have  $0 = a_R < b < a_P$ . From this, a graph similar to the above Cases 1 or 2 can be derived, whereby renegotiation is optimal below  $a_P$ , and repayment is optimal above  $a_P$ . At  $h = 1$ , from the previous section we know that  $0 < b = a_P = a_R$ . From this, a graph similar to Cases 4, 5, or 6 can be derived, whereby liquidation is optimal below  $b$ , and repayment is optimal above  $b$ , and renegotiation never occurs.

*Proof that the equilibrium hedge ratio is in  $[0, 1]$*

Here, we will prove Point 1 of Proposition 1. The proof consists of two parts. In the first part, we show that  $h < 0$  is not optimal. Then we show that  $h > 1$  never occurs.

For the first part, assume that  $h < 0$ , which corresponds to Case (a). Note that  $\partial a_P / \partial h < 0$ , for any  $h$ . This is because

$$\frac{\partial b [1 - h(1 - \gamma)(1 - q)]}{\partial h 1 - (1 - \gamma)(1 - q)s(h)} < 0$$

can be shown to be equivalent to  $(1 - \gamma)(1 - q) < 1$ , which is always true, given our assumptions. Also, we know that the payoff to the debt holder is higher for  $a > a_P$  than for  $a < a_P$ . It follows that the debt holder can always increase his expected payoff by increasing the hedge ratio  $h$ . Therefore,  $h < 0$  cannot be optimal.

For the second part, assume that  $h > 1$ , which corresponds to Case (c). As Figure IA.1 shows, the debt holder receives the liquidation payoff if  $a < b$ , and the repayment payoff if  $a > b$ . Neither of the two payoffs, nor the threshold  $b$  separating them, depends on  $h$ . Therefore, the debt holder cannot be made better off (or worse off) by increasing his hedge ratio above the point  $h = 1$ .

### C. The model without a CDS market

The solution of the model without a CDS market is very similar to the solution of the model with a CDS market. Let's derive the solution following the same logic as before. The case  $(1 - \xi)a > b$ , is simple, as it does not depend on  $h$ , with repayment being always optimal.

If  $(1 - \xi)a \leq b$ , renegotiation is feasible if  $(1 - \xi)a \leq a$ , so renegotiation is feasible for all  $a \leq b/(1 - \xi)$ . If renegotiation is feasible, the solution can be written as

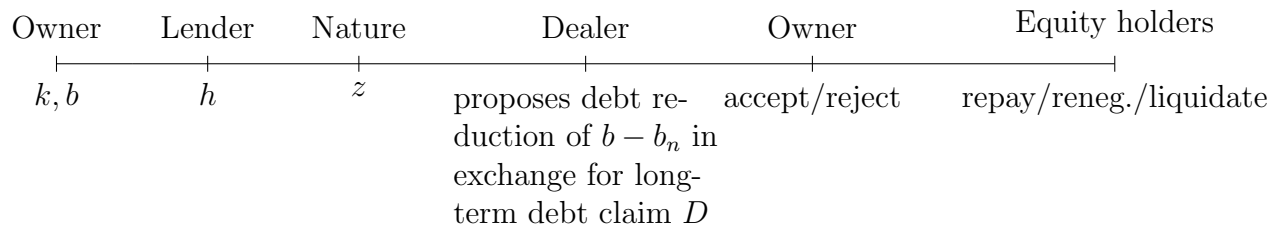
$$b_r = (1 - \xi)a + q[a - (1 - \xi)a]. \tag{IA.15}$$

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<sup>2</sup>The other boundaries,  $h_0$ ,  $1/[(1 - \gamma)(1 - q)]$ , and  $h_1$  are not relevant, as they are interior points of intervals in which the same action is optimal.



**Figure IA.2.** Timeline of events with a debt injection



Repayment is preferred to renegotiation when  $a - b \geq (1 - \gamma)(a - b_r)$ , or,

$$a[1 - (1 - \gamma)(1 - q)\xi] \geq b. \quad (\text{IA.16})$$

We can define  $a_P = b/[1 - (1 - q)(1 - \gamma)\xi]$ , where  $a_P \geq 0$  under our parameter assumptions. Then, repayment is optimal for  $a \geq a_P$  and renegotiation is optimal for  $a \in [0, a_P[$ .

It is easy to show that  $a_P < b/(1 - \xi)$ . Therefore, the final solution is that renegotiation is optimal for  $a \in [0, a_P[$ , and repayment is optimal for  $a \geq a_P$ .

### *D. Debt vs. equity injection*

In the baseline model, we assume the CDS seller recapitalizes the firm ex post by injecting equity capital. Our results remain unchanged if the capital injection is done by buying long-term debt issued by the firm.

The debt contract used for this transaction has a longer maturity relative to the debt owed at  $t = 1$ , which for this reason will be referred to as short-term debt thereafter. The dealer offers to inject an amount  $b - b_n$  in exchange for a long-term debt claim with expected payoff  $D$ .<sup>3</sup> Because the proceeds of debt issuance would be used to repay the existing debt, after the recapitalization the face value of short-term debt is reduced to  $b_n$ .

We write down the terminal payoffs for each player under each possible outcome in Table IA.1. Using this table, we can derive the firm's optimal default policy. We assume that the equity holder and the dealer bargain as a single party, so their aggregate payoff in renegotiation is  $a - b_r - D + D = a - b_r$ . For the same reasons illustrated in the main model, their aggregate outside option is 0. The short-term lender must receive his outside option as well, or  $b_r \geq hb$ . Hence, renegotiation is feasible if  $hb \leq a$ . These are the same inequalities as in the paper, in the case of an equity injection. It follows that the optimal default decision is the same as in Lemma 1 in the paper.

Next, we find the owner's optimal decision on whether to accept the protection seller's refinancing offer. In equilibrium, the proposed debt injection,  $b - b_n$ , and the expected long-

<sup>3</sup>While not strictly necessary for the derivations below, we can derive the expected payoff  $D$  by making an explicit assumption on the face value and maturity,  $T > 1$ , of debt and on the conditional distribution of a future state variable,  $\Gamma(z'|z)$  at  $T$ .

	Owner	Dealer	Short-term lender
Debt repayment	$a - b_n - D$	$D$	$b_n$
Debt renegotiation	$a - b_r - D$	$D$	$b_r$
Liquidation	0	$-hb$	$hb$

**Table IA.1 Terminal payoffs for each player.** The symbol  $a$  denotes the asset value of the firm;  $b$  is the face value of short-term debt;  $b_n$  is the new face value of short-term debt after a possible injection of long-term debt;  $b_r$  is the face value of short-term debt that is agreed on in a debt renegotiation;  $h$  is the short-term lender's CDS hedge ratio;  $D$  is the expected payoff of long-term debt after a possible debt injection by the CDS dealer.

term debt payoff,  $D$ , make the owner indifferent between acceptance and rejection. Figure 4 shows the possible scenarios and for each of them. For a given value of  $b - b_n$ , we determine  $D$  by equating the owner's payoff under acceptance to his payoff under rejection.

We have a new version of Lemma 2 in the main next.

Lemma D.1: *If  $b_n \leq hb$ , then the protection seller's expected debt payoff is*

$$D = \begin{cases} hb - b_n & \text{if } a \geq hb, \\ a - b_n & \text{if } b_n \leq a < hb \\ 0 & \text{if } a < b_n. \end{cases}$$

*If  $b_n > hb$ , then*

$$D = \begin{cases} 0 & \text{if } a \geq b_n, \\ 0 & \text{if } hb \leq a < b_n, \\ 0 & \text{if } a < hb. \end{cases}$$

**Proof.** We consider two possible cases:  $b_n \leq hb$  and  $b_n > hb$ . As for the first case,

- if  $a \geq hb$ , the owner's payoff if the offer is accepted is  $a - b_n - D$ , corresponding to debt repayment, and if the offer is rejected it is  $a - b_r(b) = a - hb$ , corresponding to renegotiation. Equating the two payoffs we have  $D = hb - b_n$ . Because we are in the case  $hb \geq b_n$ , then  $D \geq 0$ , with strict inequality if  $hb > b_n$ .
- if  $b_n \leq a < hb$ , the payoff if the offer is accepted is  $a - b_n - D$  from debt repayment, and 0 from liquidation if the offer is rejected. From the equality of the two payoffs we find  $D = a - b_n$ ;
- if  $a < b_n$ , the payoff from liquidation is 0 regardless if the offer is accepted or rejected. Hence, it does not matter what amount the dealer injects, and there is a large number of possible equilibria. We assume that the dealer does not inject any capital, or  $D = 0$ .

As for the case  $b_n > hb$ ,

- if  $a \geq b_n$ , the owner's payoff if the offer is accepted is  $a - hb - D$ , deriving from renegotiation of the debt  $b_n$ , and if the offer is rejected it is  $a - b_r(b) = a - hb$ , corresponding to renegotiation of the debt  $b$ . Equating the two payoffs we have  $D = 0$ ;

- if  $hb \leq a < b_n$ , the payoff if the offer is accepted is  $a - hb - D$ , from renegotiation of  $b_n$ , and if the offer is rejected it is  $a - b_r(b)$ , from renegotiation of  $b$ . Hence,  $D = 0$ ;
- if  $a < hb$ , the payoff is 0 from liquidation regardless if the offer is accepted or rejected. Hence, it does not matter what amount the dealer injects, and there is a large number of possible equilibria. We assume that the dealer does not inject any capital, or  $D = 0$ .

□

Next, we solve the dealer's program of how much debt to inject, which determines  $b_n$ . It is easy to show, based on Lemma D.1, that Lemma 3 in the paper remains unchanged. The proof under the assumption of debt refinancing differs from the baseline one as follows.

**Proof.** If  $a > hb$ ,

- If  $b_n \leq hb$ , because there would be repayment of  $b_n$ , his payoff is  $\max_{b_n} \{-(b - b_n) + D\}$ . Replacing  $D = hb - b_n$  from Lemma D.1, this simplifies to  $\max_{b_n} \{-b + hb\}$ , which is independent of  $b_n$  and negative.
- If  $b_n > hb$ , because there would be renegotiation, his payoff is  $\max_{b_n} \{-(b - b_n) + D\}$ . Based on Lemma D.1, the optimal  $D$  is 0 in this case. Hence, the payoff simplifies to  $\max_{b_n} \{-(b - b_n)\}$ , which is maximized at  $b_n = b$ . This implies that no recapitalization takes place, and the dealer's payoff is zero, which is higher than in the case considered above.

Therefore, if  $a > hb$ , the dealer does not inject capital to reduce the debt.

If  $a < hb$ :

- for the case  $b_n \leq hb$ , there are two possible sub-cases to consider.
  - If  $b_n \leq a < hb$ , followed by repayment, the dealer gets  $\max_{b_n} \{-(b - b_n) + D\}$ . After replacing  $D = a - b_n$  from Lemma D.1, this simplifies to  $\max_{b_n} (a - b)$ , which is independent of  $b_n$  and negative, because  $a < hb$  implies  $a < b$ .
  - If  $a < b_n < hb$ , the dealer gets  $\max_{b_n} \{-(b - b_n) - hb\}$ , because this would be a case of liquidation and the CDS dealer does not inject any capital.

Using the same argument as in the proof of Lemma 3, the dealer optimally chooses  $b_n$  such that  $b_n \leq a < hb$  rather than  $b_n > a$ . Hence,  $b_n \leq a$ .

- If  $b_n > hb$ , the payoff is  $\max_{b_n} \{-(b - b_n) - hb\}$  because this would be a case of liquidation. The maximum payoff is  $-hb$ , which is attained at  $b_n = b$ .

At this point, Figure 5 can be used to draw the same conclusions as in Lemma 3. As Lemma 3 is unchanged, Lemma 4 and the equity program are the same as in the paper.

## *E. General bargaining power between debt and equity*

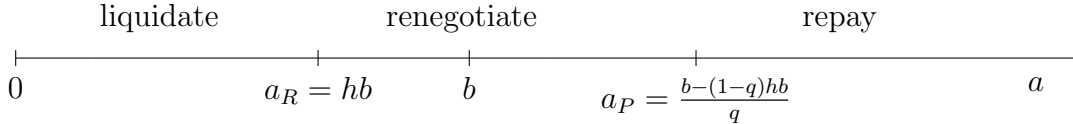
In this extension, we relax the assumption that the equity holders have all the bargaining power vis-a-vis the debt holders. Mathematically, we remove the assumption  $q = 0$  and allow  $0 \leq q \leq 1$ , where  $q$  measures the bargaining power of debt holders.

This extension also allows us to think about the role of commitment problems between equity and debt. We can think of the case  $q = 1$  as a case without any commitment problems, and  $q = 0$  as a case with a severe commitment problem, where the equity holder reneges on debt payments in many states of the world and extracts large concessions from the lenders. As we have shown in the paper, the introduction of a CDS market (with only an active CDS buyer) “allocates commitment” to the lender, by allowing him to commit to being a tough negotiator in a future debt renegotiation. We have also shown that allowing the lender to commit always improves firm value for  $q = 0$ . However, it will turn out that this is not always true for  $q \geq 0$ . We show that for some parameter values, it is possible that the introduction of a CDS market allocates too much commitment to the lender, which results in a negative net effect on firm value. We then show that two-sided activism, by both the CDS buyer and seller, is nevertheless able to increase firm value to a level higher than in a no-CDS world.

In fact, both investment and firm value reach first-best, just like in the case  $q = 0$ . Together with the previous result, this means that going from a world with only an active CDS buyer to a world with both an active CDS buyer and seller not only increases firm value, but it increases firm value even more than in the case  $q = 0$ .

The following is based on the general model, as presented in Section D.1, by assuming  $\xi = 1$ ,  $\gamma = 0$ , and  $r = 0$ . We develop this extension by just covering the parts that are changed by assuming  $q \in [0, 1]$ . We start with the model with an active CDS buyer and then move to the case where we add an active CDS dealer.

We denote by  $a = zk^\alpha$  the asset value at the debt maturity. Renegotiation is feasible when  $a \geq hb$ . When feasible, renegotiation gives  $b_r = qa + (1 - q)hb$ . The optimal default decision is then:



The debt optimization program, after netting out the fair price of the CDS protection, is

$$m(k, b) = \max_h \int_{z_R}^{z_P} b_r d\Gamma(z) + \int_{z_P}^Z b d\Gamma(z),$$

where  $z_R = a_R/k^\alpha$  and  $z_P = a_P/k^\alpha$ . Solving the first order condition gives  $h^* = 1 - q$ .

The equity maximization program is

$$V(k, b, h)|_{h=h^*} = -k + m(k, b) + (1 - \tau) \left[ \int_{z_R}^{z_P} (a - b_r) d\Gamma(z) + \int_{z_P}^Z (a - b) d\Gamma(z) \right].$$

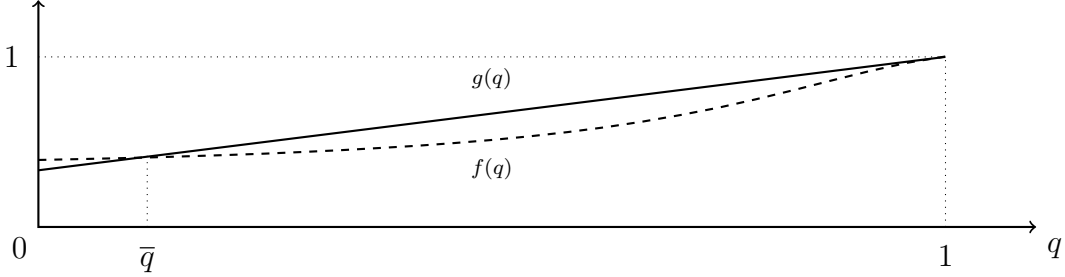
Solving the first order condition for optimal debt we find

$$b^*(k) = \frac{\tau}{1 + \tau - q[1 + (1 - q)(1 - \tau)]} Zk^\alpha,$$

and replacing  $b^*(k)$  in the equity value expression, from the first order condition for optimal

**Figure IA.3.** Comparison of optimal capital

The figure plots the expressions  $f(q) = \frac{1-q[1+(1-\tau)(1-q)](1-\tau)}{(1-q)^2 + \tau[1+q(1-q)]}$  and  $g(q) = 1 - \tau(1 - q)$ . The value of  $\bar{q}$  is such that  $f(q) < g(q)$  for  $q < \bar{q}$ .



capital stock we have

$$k^* = \left( \alpha \frac{Z}{2} \frac{1 - q[1 + (1 - \tau)(1 - q)](1 - \tau)}{(1 - q)^2 + \tau[1 + q(1 - q)]} \right)^{\frac{1}{1-\alpha}}.$$

When considering the case without CDS market ( $h \equiv 0$ ),

$$m_0(k, b) = \int_0^{\frac{b}{qk^\alpha}} qa \, d\Gamma(z) + \int_{\frac{b}{qk^\alpha}}^Z b \, d\Gamma(z),$$

$$V(k, b, h)|_{h=0} = -k + m_0(k, b) + (1 - \tau) \left[ \int_0^{\frac{b}{qk^\alpha}} (1 - q)ad\Gamma(z) + \int_{\frac{b}{qk^\alpha}}^Z (a - b)d\Gamma(z) \right].$$

from which we find  $b_0^*(k) = qk^\alpha Z$ , and the corresponding optimal investment is

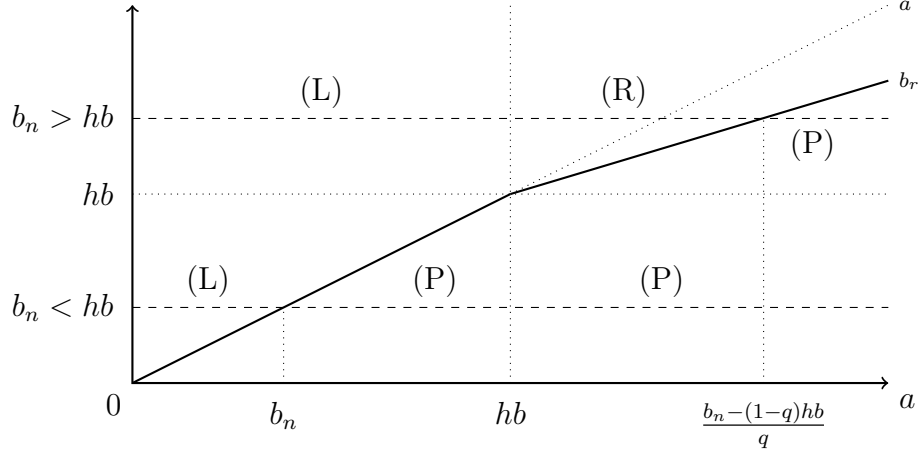
$$k_0^* = \left( \alpha \frac{Z}{2} [1 - \tau(1 - q)] \right)^{\frac{1}{1-\alpha}},$$

which is increasing in  $q$  and achieves first best at  $q = 1$ .

To compare the case with CDS to the case without CDS, we can just focus on the parts of the expressions of  $k^*$  and  $k_0^*$  that are different, which depend only on  $q$  and  $\tau$  (and which is equivalent to holding fixed  $\alpha$  and  $Z$ ). As such a comparison is not algebraically possible, so we resort to a numerical calculation, in which the only free parameter is  $\tau$ , which we set at 0.1. The result is in Figure IA.3. In particular, there are  $q > \bar{q}$  for which investment is lower with CDSs. Overall, with higher bargaining power of the creditors, CDSs have a negative effect on investment. Hence, the implications of the model may reverse if CDSs make renegotiation too hard. In other words, CDSs destroy value when they misallocate commitment.

**Figure IA.4.** Optimal default decision

We plot the optimal decision between payment of  $b_n$  (P), renegotiation to  $b_r$  (R), and liquidation (L), depending on the position of  $b_n$  relative to  $hb$ , contingent on  $a$ . We discuss separately two cases:  $b_n < hb$  (bottom horizontal dashed line) and  $b_n > hb$  (upper horizontal dashed line). In the figure, point  $a = (b_n - (1 - q)hb)/q$  is defined by condition  $b_r = b_n$ , which separates the asset values for which renegotiation is optimal, from those for which it is optimal to repay because paying  $b_n$  is cheaper for the owner than  $b_r$ .



We now extend the model with an active CDS dealer. Lemma 1 regarding the optimal default/repayment/renegotiation decision becomes

**Lemma E.1:** *When  $b_n \leq hb$ , if  $a < b_n$  the optimal decision is to liquidate, and if  $a \geq b_n$  to repay  $b_n$ . When  $b_n > hb$ , if  $a < hb$  the optimal decision is to liquidate, if  $hb \leq a < \frac{b_n - (1-q)hb}{q}$ , to renegotiate the debt to  $b_r$ , and if  $a \geq \frac{b_n - (1-q)hb}{q}$  to repay  $b_n$ .*

**Proof.** We first consider the case  $b_n < hb$ , which in Figure IA.4 is represented by the bottom dashed line. There are three possible scenarios of  $a$  to consider:

- if  $a < b_n$ , renegotiation is not feasible (as  $a < b_n < hb$ ) and because  $a < b_n$  the firm is liquidated;
- if  $b_n \leq a < hb$  renegotiation is not feasible (as  $a < hb$ ) and because  $a > b_n$ , the debt is repaid;
- if  $a > hb$ , renegotiation is feasible, and since  $b_n < b_r$ , the owners choose debt repayment over renegotiation.

We next consider the case  $b_n > hb$ , which is the upper dashed line in Figure IA.4. Also in this case there are three scenarios to consider for  $a$ :

- if  $a < hb$ , renegotiation is not feasible and because  $a < hb < b_n$  the firm is liquidated;

- if  $hb \leq a < \frac{b_n - (1-q)hb}{q}$ , renegotiation is feasible (as  $hb < a$ ) and because  $b_r < b_n$ , it is optimal to renegotiate;
- if  $a \geq \frac{b_n - (1-q)hb}{q}$ , renegotiation is feasible, and because  $b_r > b_n$ , the amount  $b_n$  is repaid.

□

The new version of Lemma 2, which establishes the share of ownership that the CDS dealer receives by injecting equity capital in the firm, is

Lemma E.2: *If  $b_n \leq hb$ , then the protection seller's equity stake is*

$$\theta = \begin{cases} 1 & \text{if } a < b_n, \\ 1 & \text{if } b_n \leq a < hb, \\ 1 - \frac{a-b_r}{a-b_n} & \text{if } hb \leq a < \frac{b-(1-q)hb}{q}, \\ 1 - \frac{a-b}{a-b_n} & \text{if } a \geq \frac{b-(1-q)hb}{q}, \end{cases}$$

If  $b_n > hb$ , then

$$\theta = \begin{cases} 1 & \text{if } a < hb, \\ 0 & \text{if } hb \leq a < \frac{b_n - (1-q)hb}{q}, \\ 1 - \frac{a-b_r}{a-b_n} & \text{if } \frac{b_n - (1-q)hb}{q} \leq a < \frac{b-(1-q)hb}{q}, \\ 1 - \frac{a-b}{a-b_n} & \text{if } a > \frac{b-(1-q)hb}{q}. \end{cases}$$

**Proof.** For the discussion of the different cases, it is useful to note that

$$\frac{b_n - (1-q)hb}{q} \leq \frac{b - (1-q)hb}{q}, \quad (\text{IA.17})$$

as we conjecture that  $b_n \leq b$ .<sup>4</sup>

We first consider the case  $b_n \leq hb$ , for which, based on Figure IA.4, there are three possible scenarios for  $a$ :

- if  $a < b_n$ , and the refinancing offer is accepted, there is liquidation; if the offer is rejected, there is also liquidation. As the owner's payoff is zero either way, any  $\theta$  would make the two payoffs equal and the owner indifferent. We choose  $\theta = 1$ ;
- if  $b_n < a < hb$ , and the offer is accepted, there is repayment and the owner's payoff is  $(1 - \theta)(a - b_n)$ . If the offer is rejected, there is liquidation and the payoff is zero. Equating the two payoffs to find the break-even equity stake gives  $\theta = 1$ ;
- if  $a \geq hb$ , and the refinancing offer is accepted, there is repayment of  $b_n$  and the owner's payoff is  $(1 - \theta)(a - b_n)$ . If instead the offer is rejected, there are two possible scenarios. The first is  $hb \leq a < \frac{b-(1-q)hb}{q}$ , for which there is renegotiation of the debt to  $b_r$  and payoff  $a - b_r$ . The indifference  $\theta$  is found by equating the two payoffs, that is  $\theta = 1 - \frac{a-b_r}{a-b_n}$ . The second scenario is  $a > \frac{b-(1-q)hb}{q}$ , for which  $b$  is repaid and the owner's payoff is  $a - b$ . By equating the two payoffs, we find  $\theta = 1 - \frac{a-b}{a-b_n}$ .

Next, we consider the case  $b_n > hb$ , for which, based on Figure IA.4, there are three possible scenarios for  $a$ :

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<sup>4</sup>The next lemma will confirm this conjecture.

- if  $a < hb$ , and the refinancing offer is accepted, there is liquidation; if the offer is rejected, there is also liquidation. Then any  $\theta$  is possible and we set  $\theta = 1$ ;
- if  $hb < a < \frac{b_n - (1-q)hb}{q}$ , and the offer is accepted, there is renegotiation and the owner's payoff is  $(1 - \theta)(a - b_r)$ . If the offer is rejected, there are two possible cases. The first is  $a < \frac{b - (1-q)hb}{q}$ , for which there is renegotiation of  $b$  down to  $b_r$ , with payoff  $a - b_r$ . Equating the payoffs gives  $\theta = 0$ . The second would be  $a \geq \frac{b - (1-q)hb}{q}$ , but this cannot happen, as we noted in equation (IA.17);
- if  $a \geq \frac{b_n - (1-q)hb}{q}$ , and the refinancing offer is accepted,  $b_n$  is repaid and the owner's payoff is  $(1 - \theta)(a - b_n)$ . If the offer is rejected, there are two cases to consider. The first is  $a < \frac{b - (1-q)hb}{q}$  for which there is renegotiation of  $b$  to  $b_r$ , and the payoff is  $a - b_r$ . The indifference equity stake is  $\theta = 1 - \frac{a - b_r}{a - b_n}$ . The second is  $a \geq \frac{b - (1-q)hb}{q}$ , for which there is repayment of  $b$ , and the owner's payoff is  $a - b$ . Then, the indifference equity stake is  $\theta = 1 - \frac{a - b}{a - b_n}$ .

□

The new version of Lemma 3, which determines the optimal CDS dealer's decision to refinance, is similar to the old one. For the same reasons discussed in the baseline model, we will focus only on the equilibrium scenario of  $h > \frac{1}{2}$ .

Lemma E.3: *Assuming  $h > \frac{1}{2}$ , the new amount of debt is*

$$b_n = \begin{cases} b & \text{if } a < b - hb, \text{ followed by liquidation,} \\ a & \text{if } b - hb \leq a < hb, \text{ with repayment of } a, \\ b & \text{if } a \geq hb, \text{ with renegotiation if } a < a_P \text{ or repayment if } a \geq a_P. \end{cases}$$

**Proof.** The CDS dealer must choose the amount to inject in the firm, which is equivalent to choosing  $b_n$ . Based on Figure IA.4, we first consider  $a \leq hb$ . We compare two alternative refinancing options:  $b_n \leq a$  and  $a < b_n \leq hb$ .

- if  $b_n \leq a$ , then  $\theta = 1$ , and follow-on decision is repayment of  $b_n$ . The dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) = \max_{b_n} -(b - b_n) + (a - b_n) = a - b,$$

which is independent of  $b_n$  and negative;

- if  $a < b_n \leq hb$ , then there is liquidation and  $\theta = 1$ . The dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) - hb = \max_{b_n} -(b - b_n) - hb.$$

The optimal choice between these two options is the first, using the same argument of the proof of Lemma 3 in the main text. That is, the best between the two options is  $b_n \leq a$  with payoff for the dealer of  $a - b < 0$ .

There is a third option,  $b_n > hb$ , for which  $\theta = 1$  and is followed by liquidation. The dealer's payoff is  $\max_{b_n} -(b - b_n) - hb$ , which is maximized for  $b_n = b$  (this is a feasible choice, as we are in the case  $b_n > hb$  and the maximum is  $-hb$ ).

The choice between  $b_n = a$  and  $b_n = b$  depends on the relation between  $a - b$  and  $-hb$ : if  $-hb > a - b$ ,  $b_n = b$ ; if  $-hb \leq a - b$ ,  $b_n = a$ . As the final choice depends on the relation between  $hb$  and  $b - hb$ , using the same arguments as in the proof of Lemma 3 in the main text, we can focus on the case  $hb > b - hb$ , which is equivalent to  $h > 1/2$ .

We now consider the case  $a \geq hb$ . If

- $b_n < hb$ , then  $b_n$  is repaid. There are two scenarios for  $a$ . The first is  $hb \leq a < \frac{b-(1-q)hb}{q}$ , for which  $\theta = 1 - \frac{a-b_r}{a-b_n}$ , and the dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) = \max_{b_n} -(b - b_n) + (a - b_n) - (a - b_r) = b_r - b,$$

which is negative and independent of  $b_n$ . The second is  $a > \frac{b-(1-q)hb}{q}$ , for which  $\theta = 1 - \frac{a-b}{a-b_n}$ , and the payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) = \max_{b_n} -(b - b_n) + (a - b_n) - (a - b) = 0,$$

which is independent of  $b_n$ ;

- if  $b_n > hb$ , there are three scenarios for  $a$ . The first is  $hb \geq a < \frac{b_n-(1-q)hb}{q}$ , for which  $\theta = 0$  and renegotiation to  $b_r$ . The dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_r) = \max_{b_n} -(b - b_n),$$

which is maximized at  $b_n = b$  and the optimal value is zero.

The second is  $\frac{b_n-(1-q)hb}{q} \leq a < \frac{b-(1-q)hb}{q}$ , for which  $\theta = 1 - \frac{a-b_r}{a-b_n}$ , and the dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) = \max_{b_n} -(b - b_n) + (a - b_n) - (a - b_r) = b_r - b,$$

which is negative and independent of  $b_n$ .

The third scenario is  $a \geq \frac{b-(1-q)hb}{q}$ , for which  $\theta = 1 - \frac{a-b}{a-b_n}$  and there is repayment of  $b_n$ . The dealer's payoff is

$$\max_{b_n} -(b - b_n) + \theta(a - b_n) = \max_{b_n} -(b - b_n) + (a - b_n) - (a - b) = 0,$$

which is independent of  $b_n$ .

Overall, the dealer's payoff for  $a \geq hb$  is maximized by not recapitalizing, or  $b_n = b$ . □

The new version of Lemma 4 is

Lemma E.4: *The debt holder's program, net of the fair CDS premium, is (for the case  $h > \frac{1}{2}$ )*

$$m(k, b) = \max_h \int_{z_L}^{z_R} a \, d\Gamma(z) + \int_{z_R}^{z_P} b_r \, d\Gamma(z) + \int_{z_P}^Z b \, d\Gamma(z)$$

where  $z_L = (b - hb)/k^\alpha$ .

The optimal hedge ratio found from the first order condition is  $h^* = 1$ , confirming the conjecture that  $h^* > \frac{1}{2}$ .

The owner's program becomes

$$V(k, b, h)|_{h=1} = -k + m(k, b) + (1 - \tau) \left[ \int_{z_R}^{z_P} (a - b_r) d\Gamma(z) + \int_{z_P}^Z (a - b) d\Gamma(z) \right].$$

From this, the optimal debt is

$$b^*(k) = Zk^\alpha,$$

and, after replacing  $b^*(k)$  in the owner's program, the optimal capital stock is

$$k^* = \left( \frac{\alpha Z}{2} \right)^{\frac{1}{1-\alpha}}.$$

Therefore, the CDS dealer's activism delivers first-best investment for any distribution of bargaining power  $q \in [0, 1]$ . More importantly, if we compare this outcome to the one with only CDS buyer's activism for a  $q > \bar{q}$ , the dealer's intervention is even more beneficial to the CDS market, as it allows to achieve first best also if CDS would decrease investment.

## *F. The model with equity injections and taking into account dealer's payoffs*

In this section, we take the model from the paper where the dealer is allowed to inject equity into the firm, but we modify one of the assumptions. In the final stage of the game, when the owners of the firm (i.e., the original owner and the dealer, who becomes a new owner) decide whether and how to default on the firm's debt, we now take into account that the dealer would incur a cash outflow in a hypothetical liquidation.

To be clear, we write down the aggregate payoffs of all owners in a debt repayment, debt renegotiation, and liquidation. The aggregate payoffs under debt repayment and renegotiation are  $a - b_n$  and  $a - b_r$ , respectively, which are the same as in the paper. The aggregate payoff under liquidation is  $-hb$ , which is the main change relative to the model in the body of the paper. This payoff is because the old owner receives 0 from her shares, and the new owner (dealer) receives 0 from his shares but pays  $hb$  on his CDS position. It is not obvious whether, empirically, the CDS dealer can convince the other shareholders in a debt renegotiation with the lender to take into account his payoffs from a derivative that has nothing to do with the firm's shares. However, we will allow for this alternative assumption to show that our results are robust to it.

We will now solve the model with this new assumption using analogous methods as in the paper. Using backwards induction, we will first solve the owners' default decision, then the original owner's decision whether to accept an equity injection, then the dealer's decision how much equity to inject, then we derive the expected payoff of the lender, followed by the lender's decision on the hedge ratio.

To derive the optimal default decision, we need to check under which conditions debt renegotiation is feasible. Two conditions need to hold:  $a - b_r \geq -hb$  and  $b_r \geq hb$ . Combining

these two inequalities, we see that debt renegotiation is always feasible. This is different from the result in the paper.

Under the assumption that the lender has no bargaining power, we have that  $b_r = hb$ . We assume that  $h^* \leq 1$ . Equity holders prefer repayment to renegotiation if  $a - b_n \geq a - b_r$ , or  $b_n \leq hb$ . Combining these results we have the following:

*Lemma F.1: If  $b_n \leq hb$ , the firm repays the debt for all values of  $a$ . If  $b_n > hb$ , the firm renegotiates the debt for all values of  $a$ .*

Note that this lemma implies that there will be some states of the world where the owners pay the lender an amount that is higher than the value of the firm's assets. Consider the case  $b_n > hb$ , which is followed by renegotiation, where the lender has to receive  $b_r = hb$ . In the states  $a < hb$  the assets of the firm are insufficient to pay  $hb$ . In these states, the dealer provides the remaining funds  $hb - a$ . This is optimal for the dealer because spending  $hb - a$  is better for him than spending  $hb$  if he would allow the firm to be liquidated.

We now derive the original owner's choice whether to accept the dealer's equity injection. We consider two possible cases:  $b_n \leq hb$  and  $b_n > hb$ . As for the first case,

- if  $a \geq hb$ , the owner's payoff if the offer is accepted is  $(1 - \theta)(a - b_n)$ , corresponding to debt repayment, and if the offer is rejected it is  $a - b_r(b) = a - hb$ , corresponding to renegotiation. From equating the two payoffs we have

$$\theta = 1 - \frac{a - hb}{a - b_n}.$$

- if  $a < hb$ , the payoff is 0 from liquidation if the offer is rejected. If the offer is accepted, it is followed by repayment, with payoff  $(1 - \theta)(a - b_n)$ . Equating the payoffs yields  $\theta = 1$ .

As for the case  $b_n > hb$ ,

- if  $a \geq hb$ , the payoff if the offer is accepted is  $(1 - \theta)(a - b_r(b_n))$ , from renegotiation of  $b_n$ , and if the offer is rejected it is  $a - b_r(b)$ , from renegotiation of  $b$ . Hence,  $\theta = 0$ ;
- if  $a < hb$ , the payoff is 0 from liquidation if the offer is rejected. If it is accepted, it is followed by renegotiation, with payoff  $(1 - \theta)(a - b_r)$ . Equating the payoffs yields  $\theta = 1$ .

We summarize these results as follows.

*Lemma F.2: If  $b_n \leq hb$ , then the protection seller's equity stake is*

$$\theta = \begin{cases} 1 - \frac{a - hb}{a - b_n} & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

*If  $b_n > hb$ , then*

$$\theta = \begin{cases} 0 & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

Note that this lemma is identical to the corresponding result in the paper.

We now derive the dealer's optimal amount of debt injection, which is determined by the new face value of debt  $b_n$ .

Motivated by Lemma F.2, and the expression of the optimal  $\theta$ , we consider two possible cases:  $a > hb$  and  $a < hb$ . As for the first case:

- If the dealer injects a lot of capital, i.e., chooses a sufficiently low  $b_n$  such that  $b_n \leq hb$ , this would eventually lead to repayment of  $b_n$  (see Lemma F.1), so his payoff is

$$\max_{b_n} \{-(b - b_n) + \theta(a - b_n)\}.$$

After substituting for  $\theta = 1 - \frac{a-hb}{a-b_n}$  from Lemma F.2, this simplifies to  $\max_{b_n} (-b + hb)$ , which is independent of  $b_n$  and negative.

- If the dealer injects little capital, i.e., chooses a sufficiently high  $b_n$  such that  $b_n > hb$ , this eventually leads to renegotiation (see Lemma F.1), so his payoff is

$$\max_{b_n} \{-(b - b_n) + \theta(a - b_r(b_n))\}.$$

Based on Lemma F.2, the optimal  $\theta$  is 0 in this case. Hence, the payoff simplifies to  $\max_{b_n} (-(b - b_n))$ , which is maximized at  $b_n = b$ . This implies that no recapitalization takes place, and the dealer's payoff is zero, which is higher than in the case considered above.

Therefore, if  $a > hb$ , it is optimal for the dealer not to reduce the debt.

As for the second case,  $a < hb$ :

- Suppose the dealer chooses a sufficiently low  $b_n$  such that  $b_n \leq hb$ . This would be a case of repayment, so he gets

$$\max_{b_n} \{-(b - b_n) + \theta(a - b_n)\}.$$

After substituting for  $\theta = 1$  from Lemma F.2, this simplifies to  $\max_{b_n} (a - b)$ , which is independent of  $b_n$ . Because  $a < hb$ , then  $a < b$ , and therefore the payoff is negative.

- If the dealer chooses a sufficiently high  $b_n$  such that  $b_n > hb$ , then this would be a case of renegotiation with payoff

$$\max_{b_n} \{-(b - b_n) + \theta(a - b_r)\}.$$

Since  $\theta = 1$  (from Lemma F.2), the payoff simplifies to  $-(b - b_n) + (a - hb)$ , which is negative. The maximum payoff is  $a - hb$ , which is attained at  $b_n = b$ .

**Lemma F.3:** *The optimal new face value of debt is  $b_n = b$  for all values of  $a$ .*

This implies that no capital is injected by the dealer at this stage of the game, which is different from the result in the paper. However, we have seen in Lemma F.1 that the dealer will spend some money, but that happens at the default stage, not at this stage.

We can now derive the lender's ex ante expected payoff. We include both his payoffs from bonds and from the CDS contract. Also, we subtract the upfront CDS premium. When calculating the CDS premium, we recognize the possible future losses of the CDS seller: In some future states of the world, when  $a < hb$ , the debt will be renegotiated but the assets of the firm will be insufficient to pay  $hb$  to the protection buyer. In these states, the protection seller will have to pay the remaining  $hb - a$  out of his pocket. These future losses of the protection seller are priced into the upfront insurance premium.

The lender's expected payoff is:

Lemma F.4: *The expected payoff to the debt holder is*

$$M(k, b, h) = \underbrace{\int_0^Z hbd\Gamma(z)}_{\text{renegotiation}} - \underbrace{\int_0^{z_R} (hb - a)d\Gamma(z)}_{\text{CDS premium}},$$

where  $z_R = hb/k^\alpha$  is the threshold from Lemma F.3. The value of debt can be simplified to

$$M(k, b, h) = \int_0^{z_R} ad\Gamma(z) + \int_{z_R}^Z hbd\Gamma(z). \quad (\text{IA.18})$$

This expression is similar to the corresponding debt payoff in the paper but slightly different. The difference is the lower bound of the first integral is 0 here but  $z_L$  in the paper. In other words, for arbitrary values of  $h \leq 1$ , the probability of liquidation is reduced to zero. However, it should be noted that even in the paper, the equilibrium hedge ratio will be  $h^* = 1$ , which reduces the lower bound to  $z_L = 0$ , which implies that the probability of liquidation in equilibrium is zero. This suggests that the final results might be similar to the model in the paper, which we show conclusively in the next step.

One can show that the optimal hedge ratio is  $h^* = 1$ . It is sufficient to show that  $\partial M/\partial h$  is positive for all  $h \in [0, 1]$ , because of our conjecture that  $h \leq 1$ . The derivative is  $b(Zk^\alpha - hb)/Zk^\alpha$ . This is trivially positive for  $h = 0$ . At  $h = 1$ , it is positive if and only if  $b \leq Zk^\alpha$ . We will make the assumption  $b \leq Zk^\alpha$  at this point and verify that it is true below. Under this assumption, the derivative  $\partial M/\partial h$  is positive for all  $h \in [0, 1]$ .

The remaining part of the proof is identical to the main paper. We briefly summarize it here for completeness.

The ex-ante firm value is

$$V(k, b, h)|_{h=1} = m(k, b) - k + (1 - \tau) \int_{z_R}^Z (a(z, k) - b)d\Gamma(z).$$

After substituting the market value of debt  $m = \max_h M(k, b, h)$ , firm value becomes

$$V(k, b, h)|_{h=1} = -k + \int_0^Z zk^\alpha d\Gamma(z) - \tau \int_{z_R}^Z (zk^\alpha - b)d\Gamma(z). \quad (\text{IA.19})$$

For an arbitrary level of capital  $k$ , the optimal amount of debt can be found by solving the

first-order condition with respect to  $b$ , which yields

$$b^*(k) = Zk^\alpha. \quad (\text{IA.20})$$

This is strictly higher than the optimal amount of debt in the active CDS buyer model. Using the previous result, one can solve the first-order condition with respect to  $k$  to find the optimal level of capital:

$$k^* = \left( \frac{\alpha Z}{2} \right)^{\frac{1}{1-\alpha}}. \quad (\text{IA.21})$$

This concludes the solution of this alternative version of the model. The solution shows that the main results of the paper, especially the first-best optimality of capital  $k^*$ , are robust to alternative assumptions about the firm's default decision.

## *G. Multiple CDS dealers*

We assume that there are two CDS dealers instead of one. In Section G.1, we take the existing model in the paper and allow for two dealers, with no other changes. Both dealers are allowed to inject equity into the firm.

In Section G.2 we also allow for two dealers, but we additionally relax the assumption that the dealers make a take-it-or-leave-it offer to the initial owner of the firm. Instead, we allow the owner to move first and to set the price at which she sells equity to the two dealers.

In both of these model extensions, we show that the likelihood of liquidation is higher than in the model in the paper with only a single protection seller.

### *G.1. Existing model with two dealers*

The extensive form of the game is as follows:

1. Owner chooses  $k$  and  $b$ .
2. Lender chooses the two hedge ratios  $h_1$  and  $h_2$ :
  - The dealers make zero profits on average.
3. Nature chooses  $z$ .
4. The two dealers simultaneously propose how much equity they want to inject.
  - Denote by  $x_i$  the dollar amount injected by Dealer  $i$ ,  $i = 1, 2$ . The total amount injected is  $x = x_1 + x_2$ , and the new face value of debt is  $b_n = b - x$ .
5. Owner accepts or rejects the offers. We focus on the equilibrium where she either accepts or rejects both offers.
  - Denote the fraction of ownership Dealer  $i$  acquires by  $\theta_i$ ,  $i = 1, 2$ .
6. Equity holders decide whether to repay the debt, to renegotiate, or to liquidate.

We assume that the dealers have a disincentive to holding large positions and therefore they charge an insurance premium that increases in the dollar amount of the exposure. We do not model this disincentive explicitly, as it would be outside of the scope of the paper. But one can think of various micro-foundations for such an assumption. For example, the dealer might be subject to value-at-risk constraints that limit the size of the position he can take in a single underlying. The result of this assumption is that it is not optimal to buy only from one protection seller. It is important to note that this is a relatively weak assumption in the sense that we do not require that the protection buyer splits his orders equally across both dealers. We only rule out extreme asymmetry where  $h_1 = 0$  and  $h_2 > 0$ , or vice versa.

We solve the model by backwards induction, starting with the firm's default decision. The solution is the same as in the paper. If renegotiation is feasible, the equity holders prefer repayment to renegotiation if  $a - b_n \geq a - b_r$ , or equivalently, if  $b_n \leq hb$ . If renegotiation is infeasible, the equity holders prefer repayment to liquidation if  $a - b_n \geq 0$ , or  $b_n \leq a$ .

### G.1.1. Optimal default decision

The optimal default decision of the equity holders depends on  $b_n$  and  $hb$  and is described in the following lemma.

Lemma G.1: *We can distinguish between two cases. If  $b_n \leq hb$ , the optimal decision is to liquidate if  $a < b_n$  and to repay the debt if  $a \geq b_n$ . If  $b_n > hb$ , the optimal decision is to liquidate if  $a < hb$  and to renegotiate if  $a \geq hb$ .*

### G.1.2. Optimal equity stake

Next, we find the owner's optimal decision on whether to accept the protection sellers' offer. In equilibrium, the protection sellers will propose an equity stake  $\theta = \theta_1 + \theta_2$  for themselves that makes the owner indifferent between accepting and rejecting. As Lemma G.1 shows, the default decision and the resulting payoffs depend on the region in which  $a$  lies. For each case and each region, we determine  $\theta = \theta_1 + \theta_2$  by equating the owner's payoff under acceptance to his payoff under rejection.

Because the owner's decision only depends on  $\theta$  and not on  $\theta_1$  and  $\theta_2$ , the solution is identical to the one in the paper. We reproduce it in the following lemma:

Lemma G.2: *If  $b_n \leq hb$ , then the protection seller's equity stake is*

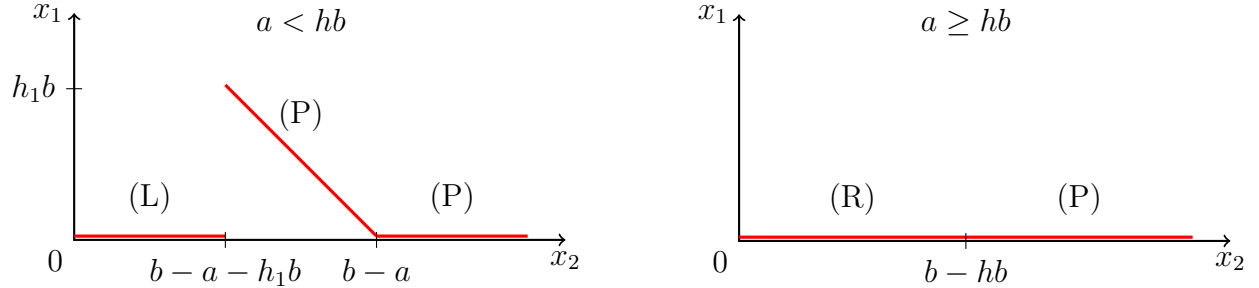
$$\theta = \begin{cases} 1 - \frac{a-hb}{a-b_n} & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

*If  $b_n > hb$ , then*

$$\theta = \begin{cases} 0 & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

**Figure IA.5.** Best response function

This figure shows the best response function  $x_1(x_2)$ , and the related optimal outcome (L is liquidation, R is renegotiation, and P repayment) for low (left panel) vs. high (right panel) values of  $a$ .



### G.1.3. Optimal equity injection amount

Next, we solve the dealers' choice of how much equity to inject, which determines the new face value of debt  $b_n$ . Denote by  $x_i$  the dollar amount injected by Dealer  $i$ ,  $i = 1, 2$ . The total amount injected is  $x = x_1 + x_2$ , which is by definition equal to  $x = b - b_n$ . Hence, the new debt value is  $b_n = b - x = b - x_1 - x_2$ , and if the firm does not default, the dealers will receive shares in proportion to their capital injection (e.g., Dealer 1 receives  $x_1/x$  of  $\theta$ ).

The protection sellers maximize the algebraic sum of the expected value of their (negative) CDS payoff in liquidation, of their (positive) payoff as a future equity holder, and their (negative) payoff from injecting equity capital into the firm, respectively. We do not include their cash inflow from the upfront insurance premium because that is sunk at this point.

Lemma G.3: *Dealers 1's best response function is as follows:*

- For  $a \geq hb$  (high  $a$ ),  $x_1 = 0$  for all  $x_2$ ;
- For  $a < hb$  (low  $a$ ),

$$x_1 = \begin{cases} 0 & \text{if } 0 \leq x_2 < b - a - h_1b \\ b - a - x_2 & \text{if } b - a - h_1b \leq x_2 < b - a \\ 0 & \text{if } x_2 \geq b - a. \end{cases}$$

**Proof:** See Section G.3.

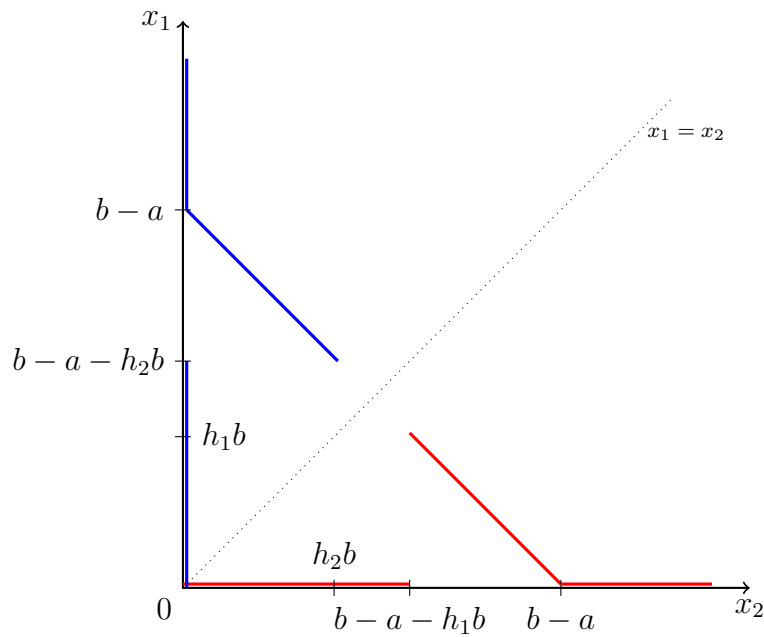
Dealer 1's best response function is illustrated in Figure IA.5.

### G.1.4. Find Nash equilibria

One can see from the best response function in Figure IA.5 that if  $a \geq hb$ , there is only a single pure-strategy Nash equilibrium,  $(x_1, x_2) = (0, 0)$ . Neither of the dealers injects any equity into the firm. This is not surprising, since for high asset values the firm is not in financial distress.

**Figure IA.6.** Equilibrium

This figure shows the two best response functions in the case  $a < hb$ , with  $h_1 = h_2$ . (With  $h_1 \neq h_2$  the two sloped lines would have different ranges.) It shows that, for a non-zero (i.e.,  $x_1 \neq 0 \neq x_2$ ) equilibrium to exist it is necessary that  $h_1$  and  $h_2$  are large enough, so that the two sloped lines meet.



More interestingly, for low asset values where  $a < hb$ , there is always at least one pure-strategy Nash equilibrium,  $(x_1, x_2) = (0, 0)$ . In addition, however, there can be other pure-strategy Nash equilibria. This can be seen in Figure IA.6. If the downward-sloping sections of the two best response functions overlap, there can be a continuum of additional equilibria.

When do these equilibria exist? Consider Dealer 2. On the one hand,  $b - a - h_1b$  is the *lowest* possible capital injection that she can make for Dealer 1's best response injection to be positive (in other words, if  $x_2 < b - a - h_1b$ ,  $x_1 = 0$  for sure, and there cannot be an equilibrium alternative to  $(0, 0)$ ). On the other hand,  $h_2b$  is the *highest* possible capital injection that Dealer 2 can ever make. Hence, an equilibrium is possible if  $h_2b \geq b - a - h_1b$ , or reshuffling it,

$$a \geq b - (h_1 + h_2)b = b - hb. \quad (\text{IA.22})$$

An analogous argument can be used by taking Dealer 1's perspective, then getting to exactly the same condition (IA.22). To conclude, an equilibrium is possible only if the asset value is sufficiently high,  $a \geq b - hb$ .

To summarize, there are two possible types of equilibria: one in which the firm is liquidated, and one in which the firm is bailed out, but only if  $a \geq b - hb$  (with no bailout equilibrium if  $a < b - hb$ ).

Lemma G.4: *The game between the dealers has the following pure-strategy Nash equilibria. Given  $(h_1, h_2)$ , and  $h = h_1 + h_2$ :*

1. *For high asset values where  $a \geq hb$ , there is only a single pure-strategy Nash equilibrium,  $(x_1, x_2) = (0, 0)$ , which leads to renegotiation.*
2. *For low asset values where  $a < hb$ , there is always at least one pure-strategy Nash equilibrium,  $(x_1, x_2) = (0, 0)$ , which leads to liquidation.*
3. *For  $a \geq b - hb$ , there exists a continuum of pure-strategy Nash equilibria in addition to the equilibrium  $(x_1, x_2) = (0, 0)$ . This type of equilibrium involves an equity injection by both dealers and leads to debt repayment. If  $a < b - hb$  there is no equilibrium alternative to  $(x_1, x_2) = (0, 0)$ .*

Note that up to now, we have not imposed any assumption on symmetry. If we impose symmetry in both the hedge ratios ( $h_1 = h_2$ ) and the injections ( $x_1 = x_2$ ), then we can find a symmetric pure-strategy Nash equilibrium by solving the equation  $x_1(x_2) = x_2$ , which produces  $x_1 = (b - a)/2$ . By symmetry, we have  $x_2 = (b - a)/2$ . Therefore, there is a second *symmetric* pure-strategy Nash equilibrium,  $(x_1, x_2) = (\frac{b-a}{2}, \frac{b-a}{2})$ . This equilibrium implies  $x = b - a$ ,  $b_n = a$ , and debt repayment.

There are a couple of interesting observations at this point. First, the second set of (bailout) Nash equilibria is very similar to the equilibrium in the paper. Lemma 3 in the paper shows that equity injection only takes place if  $a \geq b - hb$ , and that in this case the debt will be reduced to  $b_n = a$ . Both these features are identical to the equilibrium found here.

Second, the downward-sloping part of the best response function in Figure IA.6 has an interesting economic interpretation: Given Dealer 2's injection  $x_2$ , Dealer  $x_1$  optimally injects just enough equity to avoid liquidation. This is another similarity to the equilibrium in the paper.

### G.1.5. Optimal hedge ratio

To find the optimal hedge ratio, we maximize the creditor's expected payoff at  $t = 0$ . In the case the no-bailout equilibrium is played,  $(x_1, x_2) = (0, 0)$ , the creditor's value is as in the case with no CDS dealer intervention in the paper. It is easy to show that also the solution is the same,  $h^* = 1$ , and any combination of positive  $(h_1, h_2)$ , such that  $h_1 + h_2 = 1$ , works.

To find the optimal hedge ratio in the bailout equilibrium, we assume that the hedge ratio is such that  $b - hb < hb$ , or  $h > 1/2$ , to simplify the analysis. As in the paper, we also assume that  $h \leq 1$ .

The following happens in the bailout equilibrium: If asset values are high, or  $a \geq hb$ , there is no equity injection and debt is renegotiated. If asset values are low, or  $a < b - hb$ , there is also no injection and the firm is liquidated. If asset values fall in the intermediate range  $b - hb \leq a < hb$ , there is an injection of  $x = b - a$ , debt is reduced to  $b_n = a$ , and the remaining debt is repaid.

The creditor maximizes her value with respect to  $(h_1, h_2)$ , with additional constraints  $h_1 > 0$ ,  $h_2 > 0$ . It is easy to show that although the optimization is with respect to  $(h_1, h_2)$ , the creditor only cares about their sum,  $h = h_1 + h_2$ .

The equilibrium described here corresponds exactly to the equilibrium in Lemma 3 of the paper. Therefore, the expected payoff to the debt holder is also the same as in the paper (Lemma 4). It follows that the optimal hedge ratio is  $h^* = 1$ , as in the paper. Figure IA.7 illustrates this equilibrium at the optimal hedge ratio.

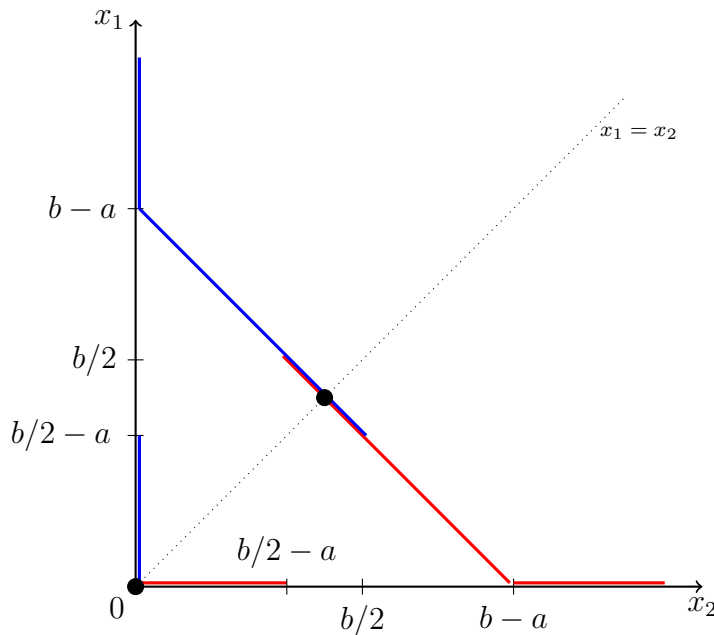
Formally, the creditor's expected payoff is

$$\begin{aligned}
M(k, b, h_1, h_2) &= \int_0^{\frac{b-(h_1+h_2)b}{k^\alpha}} (h_1 + h_2)bd\Gamma(z) + \int_{\frac{b-(h_1+h_2)b}{k^\alpha}}^{\frac{(h_1+h_2)b}{k^\alpha}} [(x_1 + x_2) + a]d\Gamma(z) \\
&+ \int_{\frac{(h_1+h_2)b}{k^\alpha}}^Z (h_1 + h_2)bd\Gamma(z) - \left[ \int_0^{\frac{b-(h_1+h_2)b}{k^\alpha}} (h_1 + h_2)bd\Gamma(z) + \int_{\frac{b-(h_1+h_2)b}{k^\alpha}}^{\frac{(h_1+h_2)b}{k^\alpha}} (x_1 + x_2)d\Gamma(z) \right] \\
&= \int_{\frac{b-(h_1+h_2)b}{k^\alpha}}^{\frac{(h_1+h_2)b}{k^\alpha}} ad\Gamma(z) + \int_{\frac{(h_1+h_2)b}{k^\alpha}}^Z (h_1 + h_2)bd\Gamma(z).
\end{aligned}$$

The solution is  $h^* = 1$ , and any combination of positive  $(h_1, h_2)$ , such that  $h_1 + h_2 = 1$ , works. Under this equilibrium the outcome is exactly the same as in the model in the paper.

The following proposition summarizes the solution of the two-dealer model:

**Figure IA.7. Nash equilibria at  $h^* = 1$ .** The red lines represent the best response function of Dealer 1. The blue lines illustrate the best response of Dealer 2. The two black dots illustrate the symmetric pure-strategy Nash equilibria of the game between the two dealers. The first equilibrium is  $(x_1, x_2) = (0, 0)$ , which leads to liquidation. The second equilibrium is  $(x_1, x_2) = (\frac{b-a}{2}, \frac{b-a}{2})$ , which leads to debt repayment.



**PROPOSITION 2:** *The creditor's optimal hedge ratio is  $h^* = 1$ . At the optimal hedge ratio, the following pure strategy Nash equilibria exist:*

1. *An equilibrium where neither of the dealers inject equity,  $(x_1, x_2) = (0, 0)$ , and the firm is liquidated.*
2. *A type of equilibrium in which both dealers inject equity and the remaining debt is repaid in full.*

*In the single-dealer model, the liquidation equilibrium does not exist. In this sense, liquidation is more likely to occur in the two-dealer model than in the single-dealer model.*

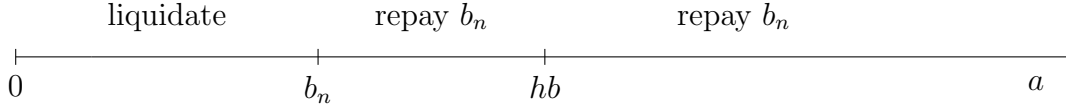
Next, we show that even in a game where the original firm owner has some bargaining power vis-a-vis the protection sellers, we will have a very similar solution, with two types of equilibria, one with bailout one with liquidation.

## G.2. Model with two dealers where the owner sets the stock price

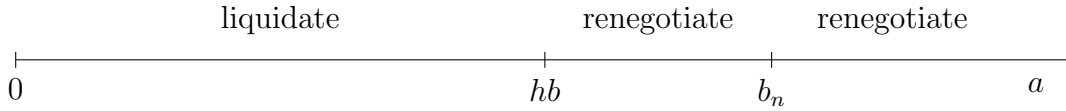
We relax the assumption that the protection sellers make a take-it-or-leave-it offer to the owner of the firm. Instead, the owner moves first and offers a price at which she is willing

**Figure IA.8.** Optimal default decision

If  $b_n \leq hb$ :



If  $b_n > hb$ :



to sell shares to the dealers. The extensive form of the game is as follows:

1. Owner chooses  $k$  and  $b$ .
2. Lender chooses  $h_1$  and  $h_2$ . The dealers make zero profits on average.
3. Nature chooses  $z$ .
4. Firm offers to sell equity by quoting a price per share  $p$ .
5. The dealers simultaneously choose how many shares they want to buy at the price.
6. Equity holders decide whether to repay the debt, to renegotiate, or to liquidate.

We assume for simplicity that the hedge ratio  $h \in [0, 1]$ . Denote the utility of the Dealer  $i$  by  $U_i(x_1; x_2, p)$ , where  $x_i \geq 0$  is the number of shares purchased by Dealer  $i$ ,  $i = 1, 2$ , and  $p > 0$  is the price per share quoted by the firm. For simplicity, the total number of shares is 1. Dealer 1 solves  $\max_{x_1} U_1(x_1; x_2, p)$  taking  $x_2$  and  $p$  as given. Dealer 2 solves  $\max_{x_2} U_2(x_2; x_1, p)$  taking  $x_1$  and  $p$  as given. The new face value of debt, after an equity injection, is defined as  $b_n = b - px$ , where  $x = x_1 + x_2$ . As before, the new face value of debt cannot be negative,  $b_n \geq 0$ .

We solve the game by backwards induction, starting with the equity holders' default decision. Nothing has changed at the default stage compared to the paper, so the optimal default decision is the same as before. Figure IA.8 summarizes the equity holders' optimal default policy.

We can write down Dealer 1's utility function depending on whether the debt is repaid,

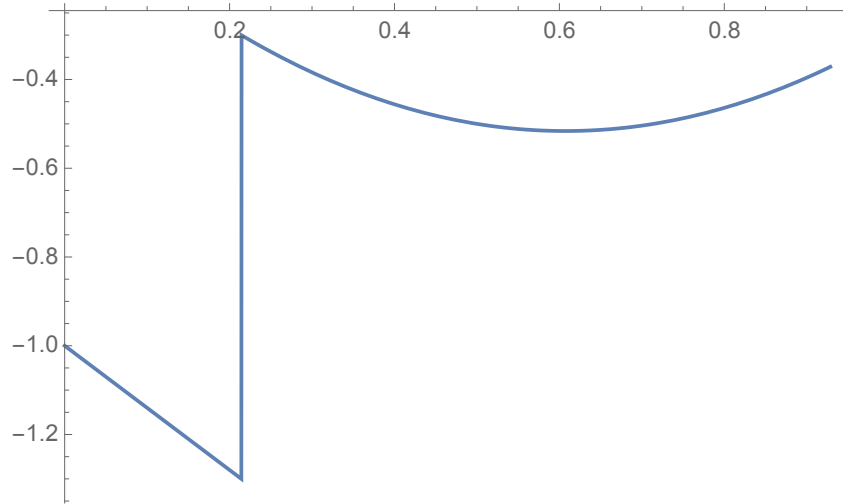
the debt is renegotiated, or the firm is liquidated:

$$U_1(x_1; x_2, p) = \begin{cases} -px_1 + x_1[a - b_n(x_1)] & \text{if } a \geq b_n \text{ and } b_n \leq hb \text{ (repayment),} \\ -px_1 + x_1[a - b_r(b_n(x_1))] & \text{if } a \geq hb \text{ and } b_n > hb \text{ (renegotiation),} \\ -px_1 - h_1b & \text{else (liquidation).} \end{cases} \quad (\text{IA.23})$$

The utility of Dealer 2 can be defined analogously.

We proceed with deriving the dealers' optimal equity demand  $x_1$  and  $x_2$ . We focus on the case where  $a < hb$ , which is the more interesting case, as this is where the asset value is so low that the firm is financially distressed. We derive Dealer 1's optimal demand  $x_1$  taking  $x_2$  as given. Then we derive Dealer 2's analogous demand, taking  $x_1$  as given, which then allows us to find the Nash equilibria of the game.

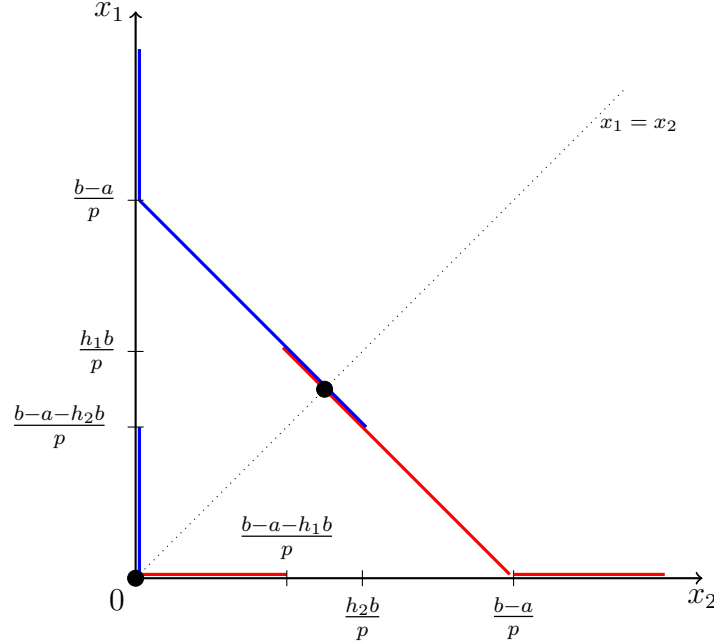
Dealer 1's utility function has a discontinuity at the point  $x_1$  where  $b_n(x_1) = a$ . This is the injection amount where the equity holders are indifferent between liquidation and repayment. A slightly higher injection by Dealer 1 avoids liquidation and leads to debt repayment. This saves liquidation costs, creating a discontinuous increase in his payoff. Because of the discontinuity, we cannot use first-order conditions to find the optimal demand  $x_1$ . However, looking at the utility function  $U_1(x_1)$  in Figure IA.9 reveals that there are only three candidates for the global optimum: (1)  $x_1 = 0$ , (2)  $x_1$  that solves  $b_n(x_1) = a$ , which is  $x_1 = \frac{b-a-px_2}{p}$ , and (3)  $x_1 = b/p - x_2$ , which is the maximum possible  $x_1$  so that the constraint  $b_n \geq 0$  is not violated.



**Figure IA.9.** Dealer 1's utility  $U_1(x_1; x_2, p)$  as a function of his demand  $x_1$ . The parameter values are  $a = 1, b = 2, x_2 = 0.5, p = 1.4$ , and  $h_1 = 0.5$ .

We conjecture that the owner will choose a sufficiently high price in equilibrium that rules out the third candidate,  $x_1 = b/p - x_2$ . This happens if the dealer's utility at the third candidate does not exceed his utility at the first and second candidates. Formally, we need that  $U_1(x_1 = 0) \geq U_1(x_1 = b/p - x_2)$  and  $U_1(x_1 = \frac{b-a-px_2}{p}) \geq U_1(x_1 = b/p - x_2)$ . These inequalities can be simplified to  $p \geq \frac{ab}{b-h_1b+ax_2-x_2}$  and  $p \geq \frac{b}{1+x_2}$ . These conditions are also necessary if we want to make sure that the Nash equilibrium we find is symmetric. Finally,

**Figure IA.10. Nash equilibria of extensive form 2.** The red lines represent the best response function of Dealer 1. The blue lines illustrate the best response of Dealer 2. The two black dots illustrate the symmetric pure-strategy Nash equilibria of the game between the two dealers. The first equilibrium is  $(x_1, x_2) = (0, 0)$ , which leads to liquidation. The second equilibrium is  $(x_1, x_2) = (\frac{b-a}{2p}, \frac{b-a}{2p})$ , which leads to debt repayment.



there are analogous conditions from Dealer 2's utility function, which we omit for brevity.

Under symmetry, we can write the conditions as  $p \geq \frac{ab}{b-hb/2+ax/2-x/2}$  and  $p \geq \frac{b}{1+x/2}$ . We assume that the conditions hold and examine their implications for the equilibrium price later. Therefore, we only need to compare the first two candidates for the global optimum in  $x_1$ .

By comparing the utility at  $x_1 = 0$  to the payoff at  $x_1 = \frac{b-a-px_2}{p}$ , we find the optimal demand to be

$$x_1(x_2) = \begin{cases} 0 & \text{if } x_2 < \frac{b-a-h_1b}{p}, \\ \frac{b-a-px_2}{p} & \text{if } x_2 \geq \frac{b-a-h_1b}{p}, \end{cases} \quad (\text{IA.24})$$

subject to  $x_1(x_2) \geq 0$ . This is the best response function of Dealer 1. Dealer 2's best response function,  $x_2(x_1)$ , can be found analogously, which we omit for brevity. By solving for the fixed points of the best response functions, we find the Nash equilibria of the game. Figure IA.10 illustrates.

One can see in Figure IA.10 that there are two symmetric pure-strategy Nash equilibria: A liquidation equilibrium,  $(x_1, x_2) = (0, 0)$  and a bailout equilibrium,  $(x_1 > 0, x_2 > 0)$ .

The first equilibrium always exists. When does the second equilibrium exist? We can write the conditions for existence as follows. The sloped section of the best response function of Dealer 1,  $x_1(x_2) = (b-a-px_2)/p$ , at its highest point, must extend to above the dotted

line, which is the line where  $x_1 = x_2$ . Mathematically, we need that

$$x_1(x_2)|_{x_2=(b-a-h_1b)/p} \geq x_2,$$

and if we substitute for  $x_1(x_2) = (b - a - px_2)/p$ , this becomes

$$h_1b/p \geq x_2. \tag{IA.25}$$

Analogously, we can derive a condition for the other dealer, which is

$$h_2b/p \geq x_1.$$

Let's assume that the two conditions hold. If we impose symmetry in both the hedge ratios ( $h_1 = h_2$ ) and the injections ( $x_1 = x_2$ ), then we can find the equilibrium by solving the equation  $x_1(x_2) = x_2$ , which produces  $x_1 = (b - a)/(2p)$ . By symmetry, we have  $x_2 = (b - a)/(2p)$ . Therefore, the bailout equilibrium is  $(x_1, x_2) = (\frac{b-a}{2p}, \frac{b-a}{2p})$ .

When does this equilibrium exist? If we substitute  $(x_1, x_2) = (\frac{b-a}{2p}, \frac{b-a}{2p})$  into the condition for existence in Equation (IA.25) and impose  $h_1 = h_2 = h/2$ , we get  $a \geq b - hb$ .

This means that for an arbitrary hedge ratio  $h \in [0, 1]$  the bailout equilibrium exists only if the asset value ends up being sufficiently high. We will determine the optimal hedge ratio  $h^*$  later.

### G.2.1. Equilibrium stock price $p$

Next, we find the equilibrium price  $p$  at which the owner sells shares to the protection sellers. If the dealers play the equilibrium  $(x_1, x_2) = (0, 0)$ , i.e., if they do not inject any equity, then the price  $p$  does not affect the owner's payoff (because it is zero), and the owner can choose any price subject to  $p \geq \frac{ab}{b-hb/2+ax/2-x/2}$ .<sup>5</sup> If we plug in the equilibrium demand function  $x(p) = 0$ , the constraint on the price becomes  $p \geq 2a$ .

If the dealers play the bailout equilibrium,  $(x_1, x_2) = (\frac{b-a}{2p}, \frac{b-a}{2p})$ , then we can write the owner's problem as

$$\max_p px(p) + (1 - x(p))[a - b_n(x(p))], \tag{IA.26}$$

subject to  $p \geq \frac{b}{1+x(p)/2}$ , where  $x(p)$  is the aggregate demand function for shares,  $x(p) = x_1(p) + x_2(p) = \frac{b-a}{p}$ . Substituting into the objective function, we get

$$\max_p (b - a), \tag{IA.27}$$

The objective function is independent of the price  $p$ . There are two reasons why the objective function simplifies so much. First, the term in the square brackets,  $a - b_n$ , is zero. This is because the dealers choose their demand for shares exactly so that the owner will be indifferent between repayment and liquidation. Since the owner's liquidation payoff is zero,

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<sup>5</sup>The relevant constraint is  $p \geq \frac{ab}{b-hb/2+ax/2-x/2}$ , not  $p \geq \frac{b}{1+x/2}$ , because the first constraint makes sure that  $x_1 = 0$  is not dominated by  $x_1 = b/p - x_2$  for Dealer 1, and analogously for Dealer 2.

his repayment payoff  $a - b_n$  is also zero at the equilibrium demand. Second, the term  $px(p) = p^{\frac{b-a}{p}}$  simplifies to  $b - a$ .

The price is only subject to the constraint  $p \geq \frac{b}{1+x(p)/2}$ . If we plug in the equilibrium demand function  $x(p) = \frac{b-a}{p}$ , the constraint on the price becomes  $p \geq \frac{b+a}{2}$ .

To conclude, we have a very similar situation as in the solution of extensive form 1. There is always a symmetric pure-strategy Nash equilibrium that leads to liquidation. At an arbitrary hedge ratio  $h \in [0, 1]$ , a second symmetric pure-strategy Nash equilibrium might exist, at least for sufficiently high asset values.

### G.2.2. Optimal hedge ratio $h$

If the liquidation equilibrium  $(x_1, x_2) = (0, 0)$  is played, there is never an equity injection. Therefore, the outcome is very similar to the model in the paper where the protection seller cannot intervene in the underlying firm. It follows that the optimal hedge ratio is also the same, which means  $h^* = 1$ .

If the dealers play the bailout equilibrium  $(x_1, x_2) = (\frac{b-a}{2p}, \frac{b-a}{2p})$ , there is liquidation for low asset values  $a < b - hb$  and repayment for high asset values  $a \geq b - hb$ . We conjecture, without proof, that in the states  $a \geq hb$ , there is not equity injection, because these are the good states of the world where the firm is not financially distressed. In the states where an injection takes place,  $b - hb \leq a < hb$ , the injection amount is  $px(p) = p^{\frac{b-a}{p}} = b - a$ , and the new face value of debt is  $b_n = b - (b - a) = a$ . Note that this is the same as in the model in the paper, which implies that the bondholder's objective function with respect to  $h$  is the same, so the optimal hedge ratio must also be the same,  $h^* = 1$ .

### G.2.3. Conclusion

To summarize, the solution of the model is virtually identical to the previous model (Section G.1) where the protection sellers made a take-it-or-leave-it offer to the firm's owner.

At the optimal hedge ratio  $h^* = 1$ , two types of pure strategy Nash equilibria exist. First, an equilibrium where neither of the dealers inject equity,  $(x_1, x_2) = (0, 0)$ , and the firm is liquidated. Second, a type of equilibrium in which both dealers inject equity and the remaining debt is repaid in full.

In the single-dealer model, the liquidation equilibrium does not exist. In this sense, liquidation is more likely to occur in the two-dealer model than in the single-dealer model.

This shows that our result in Section G.1 does not depend on the assumption that protection sellers made a take-it-or-leave-it offer to the firm's owner.

## G.3. Derivation of optimal equity injection in Section G.1

Motivated by Lemma G.2 and the optimal  $\theta$ , we consider two possible cases:  $a \geq hb$  and  $a < hb$ . As for the first case,  $a \geq hb$ :

- If Dealer 1 injects a lot of capital, i.e., chooses a sufficiently high  $x_1$  such that  $b_n \leq hb$ , or  $x_1 \geq b - x_2 - hb$ , this would eventually lead to repayment of  $b_n$  (see Lemma G.1),

so his payoff is

$$\max_{x_1} \{-x_1 + \theta_1(x_1)(a - b_n(x_1))\}.$$

After substituting for  $\theta_1 = \frac{x_1}{x} \theta = \frac{x_1}{x} (1 - \frac{a-hb}{a-b_n})$  from Lemma G.2, this becomes

$$\max_{x_1} \left\{ \frac{x_1}{x_1 + x_2} (hb - b) \right\},$$

which is decreasing in  $x_1$  because  $h \leq 1$ . So Dealer 1 chooses the smallest  $x_1$  that satisfies the constraints  $x_1 \geq b - x_2 - hb$  and  $x_1 \geq 0$ . If  $b - x_2 > hb$ , the first constraint is binding, or  $x_1 = b - x_2 - hb > 0$ . Substituting into the objective function, this leads to repayment with payoff  $-(b - x_2 - hb) < 0$ .

If  $b - x_2 < hb$ , then  $x_1 \geq 0$  is binding, which implies  $x_1 = 0$  and  $b_n = b - x_2$ . The outcome is repayment, with payoff 0.

- Suppose Dealer 1 injects little capital, i.e., chooses a sufficiently low  $x_1$  such that  $b_n > hb$ , or  $x_1 < b - x_2 - hb$ . The injection  $x_1$  needs to satisfy the constraints  $x_1 < b - x_2 - hb$  and  $x_1 \geq 0$ . If  $b - x_2 > hb$ , there is a feasible  $x_1$ , which leads to renegotiation (see Lemma G.1), so his payoff is

$$\max_{x_1} \{-x_1 + \theta_1(a - b_r(b_n))\}.$$

Based on Lemma G.2,  $\theta = 0$  and the payoff simplifies to  $\max_{x_1} (-x_1)$ , which is maximized at  $x_1 = 0$ , or  $b_n = b - x_2$ . This implies that Dealer 1's injection is zero and his payoff is zero.

If  $b - x_2 < hb$ , the feasible set for  $x_1$  is empty and no choice can be made.

To conclude the case  $a \geq hb$ : If  $b - x_2 > hb$ ,  $x_1 = 0$ , the optimal payoff for the dealer is zero, and the outcome is renegotiation. If  $b - x_2 < hb$ , we have  $x_1 = 0$ , repayment, and the optimal payoff 0.

As for the case  $a < hb$ , we distinguish between  $b - x_2 > hb$  and  $b - x_2 < hb$ . We start with  $b - x_2 > hb$ .

- Suppose Dealer 1 chooses a sufficiently high  $x_1$  such that  $b_n \leq a < hb$ , or  $b - x_1 - x_2 \leq a < hb$ , because this would be a case of repayment, he gets

$$\max_{x_1} \{-x_1 + \theta_1(a - b_n(x_1))\}.$$

After substituting for  $\theta = 1$  (i.e.,  $\theta_1 = \frac{x_1}{x_1 + x_2}$ ) from Lemma G.2, this becomes

$$\max_{x_1} \left\{ \frac{x_1}{x_1 + x_2} (a - b) \right\},$$

which is decreasing in  $x_1$ . Therefore, Dealer 1 chooses the smallest possible injection that satisfies  $b - x_1 - x_2 \leq a < hb$ , which is  $x_1 = b - x_2 - a > 0$ . The maximum payoff is  $-(b - x_2 - a) < 0$ .

- If he chooses  $x_1$  such that  $a < b_n \leq hb$ , that is  $a < b - x_1 - x_2 \leq hb$ , then he gets

$$\max_{x_1} \{-x_1 - h_1b\},$$

where  $h_1b$  denotes the notional amount of protection sold by Dealer 1, because this would be a case of liquidation. Dealer 1 chooses the smallest value of  $x_1$  that satisfies the constraint  $x_1 \geq b - x_2 - hb$ , which is  $x_1 = b - x_2 - hb > 0$ . The maximum payoff is  $-(b - x_2 - hb) - h_1b < 0$ .

- If the dealer chooses a sufficiently low  $x_1$  such that  $a < hb < b_n$ , or  $a < hb < b - x_1 - x_2$ , then this would be a case of liquidation with payoff

$$\max_{x_1} \{-x_1 - h_1b\}.$$

The maximum payoff is  $-h_1b$ , which is attained at  $x_1 = 0$ .

To conclude the case  $a < hb$  and  $b - x_2 > hb$ , we compare the payoff  $-(b - x_2 - a)$  to the payoff  $-h_1b$ . Dealer 1, if  $h_1b < b - x_2 - a$ , injects  $x_1 = 0$  and the outcome is liquidation; if  $h_1b > b - x_2 - a$ , he injects  $x_1 = b - x_2 - a$  and the outcome is repayment.

As for the case  $a < hb$  and  $b - x_2 < hb$ , we analyze the case  $a < b - x_2$  and  $a > b - x_2$  separately. In the first case:

- If Dealer 1 chooses a sufficiently high  $x_1$  such that  $b_n \leq a < b - x_2 < hb$ , that is  $b - x_1 - x_2 \leq a < b - x_2 < hb$ , because this would be a case of repayment, he gets

$$\max_{x_1} \{-x_1 + \theta_1(a - b_n(x_1))\}.$$

After substituting for  $\theta = 1$  (i.e.,  $\theta_1 = \frac{x_1}{x_1 + x_2}$ ) from Lemma G.2, this becomes

$$\max_{x_1} \left\{ \frac{x_1}{x_1 + x_2} (a - b) \right\},$$

which is decreasing in  $x_1$ . Therefore, Dealer 1 chooses the smallest possible injection that satisfies  $b - x_1 - x_2 \leq a$ , which is  $x_1 = b - x_2 - a > 0$ . The maximum payoff is  $-(b - x_2 - a) < 0$ .

- If he chooses  $x_1$  such that  $a < b_n \leq b - x_2 < hb$ , that is  $a < b - x_1 - x_2 \leq b - x_2 < hb$ , then he gets

$$\max_{x_1} \{-x_1 - h_1b\},$$

because this is a case of liquidation. Dealer 1 chooses the smallest  $x_1$  that satisfies the constraint  $x_1 \geq 0$ , which is  $x_1 = 0$ . The maximum payoff is  $-h_1b < 0$ .

- The dealer cannot choose an  $x_1$  such that  $a < b - x_2 < b_n < hb$ , or  $a < b - x_2 < b - x_1 - x_2 < hb$ , because there is no  $x_1$  that simultaneously satisfies  $x_1 \geq 0$  and  $b_n > b - x_2$ .
- Similarly, the dealer cannot choose an  $x_1$  such that  $a < b - x_2 < hb < b_n$ , that is  $a < b - x_2 < hb < b - x_1 - x_2$ , as this case is infeasible.

To conclude the case  $a < hb$ ,  $b - x_2 < hb$ , and  $a < b - x_2$ , we compare the payoff  $-(b - x_2 - a)$  to the payoff  $-h_1b$ . If  $h_1b < b - x_2 - a$ , Dealer 1 injects  $x_1 = 0$  and the outcome is liquidation. If  $h_1b > b - x_2 - a$ , Dealer 1 injects  $x_1 = b - x_2 - a$  and the outcome is repayment.

As for the case  $a < hb$ ,  $b - x_2 < hb$ ,  $a > b - x_2$ :

- If Dealer 1 chooses a value of  $x_1$  such that  $b_n < b - x_2 < a < hb$ , or  $b - x_1 - x_2 < b - x_2 < a < hb$ , this leads to repayment, then he gets

$$\max_{x_1} \{-x_1 + \theta_1(a - b_n(x_1))\}.$$

In this case,  $\theta = 1$ , which implies

$$\max_{x_1} \left\{ \frac{x_1}{x_1 + x_2} (a - b) \right\}.$$

This is decreasing in  $x_1$ , which implies  $x_1 = 0$ , or  $b_n = b - x_2$ . The maximum payoff is 0.

- All remaining cases, in which  $b - x_2 < b_n$ , there is no  $x_1$  that satisfies the constraints  $x_1 \geq 0$  and  $b_n > b - x_2$ .

To conclude the case  $a < hb$ ,  $b - x_2 < hb$ ,  $a > b - x_2$ , there is only one solution, where  $x_1 = 0$ , or  $b_n = b - x_2$ . The maximum payoff is 0, which leads to repayment.

These results are illustrated in Figure IA.11 and summarized in the following lemma.

Lemma G.5: *For a given injection by Dealer 2,  $x_2$ , Dealer 1's optimal injection is:*

*If  $b - x_2 - h_1b \leq b - x_2 < hb$ :*

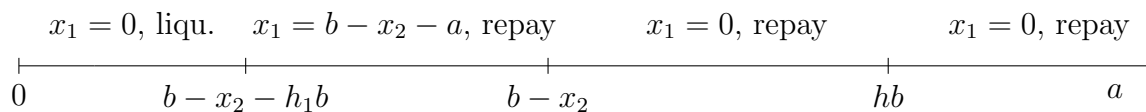
$$x_1 = \begin{cases} 0 & \text{if } a < b - x_2 - h_1b, \text{ followed by liquidation,} \\ b - a - x_2 & \text{if } b - x_2 - h_1b \leq a < b - x_2, \text{ followed by repayment,} \\ 0 & \text{if } b - x_2 \leq a < hb, \text{ followed by repayment,} \\ 0 & \text{if } a \geq hb, \text{ followed by repayment.} \end{cases}$$

*If  $hb \leq b - x_2 - h_1b \leq b - x_2$ :*

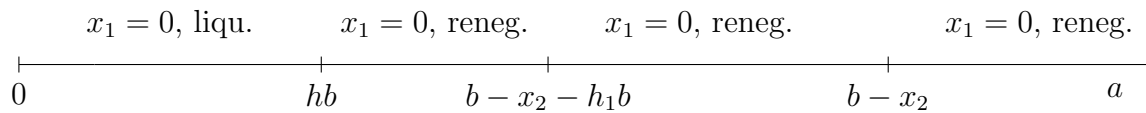
$$x_1 = \begin{cases} 0 & \text{if } a < hb, \text{ followed by liquidation,} \\ 0 & \text{if } hb \leq a \leq b - x_2 - h_1b, \text{ followed by renegotiation,} \\ 0 & \text{if } b - x_2 - h_1b \leq a \leq b - x_2, \text{ followed by renegotiation,} \\ 0 & \text{if } a > b - x_2, \text{ followed by renegotiation.} \end{cases}$$

**Figure IA.11.** Optimal intervention and optimal default decision – extended model, proportional

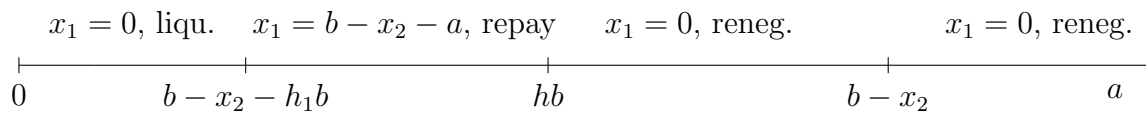
If  $b - x_2 - h_1b \leq b - x_2 < hb$ :



If  $hb \leq b - x_2 - h_1b \leq b - x_2$ :



If  $b - x_2 - h_1b < hb < b - x_2$ :



If  $b - x_2 - h_1b < hb < b - x_2$ :

$$x_1 = \begin{cases} 0 & \text{if } a < b - x_2 - h_1b, \text{ followed by liquidation,} \\ b - a - x_2 & \text{if } b - x_2 - h_1b \leq a < hb, \text{ followed by repayment,} \\ 0 & \text{if } hb \leq a < b - x_2, \text{ followed by renegotiation,} \\ 0 & \text{if } a \geq b - x_2, \text{ followed by renegotiation.} \end{cases}$$

Analogously, we can derive Dealer 2's optimal injection by taking Dealer 1's injection as given. One needs to swap  $x_1$  and  $x_2$ , as well as  $h_1$  and  $h_2$  in the expressions above.

From Lemma G.5, we derive Dealer 1's best response,  $x_1(x_2)$ , contingent on  $a$ . Figure IA.5 illustrates it.

**Proof:** We first consider the case  $a \geq hb$ , for which the best response is  $x_1 = 0$ .

Next, we consider the case  $a < hb$ . From the first case in Lemma G.5, conditions  $b - x_2 - h_1b \leq b - x_2 < hb$  are equivalent to  $x_2 > b - hb$ . It is possible, depending on  $a$ , that either  $b - hb > b - a - h_1b$  or  $b - hb < b - a - h_1b$ , that is equivalent to either  $a > hb - h_1b$  or  $a < hb - h_1b$ , respectively. In the first case, the best response is

$$x_1 = \begin{cases} b - a - x_2 & \text{if } b - hb \leq x_2 < b - a \\ 0 & \text{if } x_2 \geq b - a. \end{cases} \quad (\text{IA.28})$$

In the second case, it is

$$x_1 = \begin{cases} 0 & \text{if } b - hb < x_2 < b - a - h_1b \\ b - a - x_2 & \text{if } b - a - h_1b \leq x_2 < b - a \\ 0 & \text{if } x_2 \geq b - a. \end{cases} \quad (\text{IA.29})$$

In the second case of Lemma G.5, conditions  $hb \leq b - x_2 - h_1b \leq b - x_2$  are equivalent to  $x_2 \leq b - hb - h_1b$ , and in this case the best response is

$$x_1 = 0 \quad \text{if} \quad 0 \leq x_2 \leq b - hb - h_1b. \quad (\text{IA.30})$$

Finally, the third case in Lemma G.5,  $b - x_2 - h_1b < hb < b - x_2$  sets two conditions:  $x_2 > b - hb - h_1b$ , and  $x_2 < b - hb$ . As before, depending on  $a$ , it can be  $b - hb > b - a - h_1b$  or vice versa. In the first case, corresponding to  $a > hb - h_1b$ ,

$$x_1 = \begin{cases} 0 & \text{if } b - hb - h_1b < x_2 < b - a - h_1b \\ b - a - x_2 & \text{if } b - a - h_1b < x_2 < b - hb. \end{cases} \quad (\text{IA.31})$$

In the second case, which is equivalent to  $a < hb - h_1b$ , the best response is

$$x_1 = 0 \quad \text{if} \quad b - hb - h_1b < x_2 < b - hb. \quad (\text{IA.32})$$

For the case  $a > hb - h_1b$ , the conclusion is derived by putting together the expression of  $x_1$

in (IA.28) and (IA.31), together with the expression in (IA.30):

$$x_1 = \begin{cases} 0 & \text{if } 0 \leq x_2 < b - a - h_1 b \\ b - a - x_2 & \text{if } b - a - h_1 b \leq x_2 < b - a \\ 0 & \text{if } x_2 \geq b - a. \end{cases}$$

For the case  $a < hb - h_1 b$ , we put together the expression of  $x_1$  in (IA.29), (IA.32), and (IA.30):

$$x_1 = \begin{cases} 0 & \text{if } 0 \leq x_2 < b - a - h_1 b \\ b - a - x_2 & \text{if } b - a - h_1 b \leq x_2 < b - a \\ 0 & \text{if } x_2 \geq b - a. \end{cases}$$

The best response functions for  $a > hb - h_1 b$  and  $a < hb - h_1 b$  are in fact the same. So, we simplify the analysis and consider just two cases:  $a < hb$  and  $a \geq hb$ .

## *H. Relation to manufactured defaults*

While we do not directly model manufactured defaults, our framework indirectly has something to say about another type of CDS intervention: manufactured defaults. In narrowly tailored credit events (NTCEs), as they are formally called, typically the protection buyer offers cheap financing to a financially distressed firm. The catch is that the CDS buyer asks the firm to make a payment on existing debt contracts with a slight delay. This triggers a credit event in the CDS market, creating a payoff to the protection buyer.

Our model implies that a manufactured default such as in the case of Codere or Hovnanian cannot be an equilibrium outcome with rational expectations and complete information. The reason is that under these two assumptions, the CDS dealer, when selling CDS to the protection buyer, anticipates future actions of the buyer and prices them into the CDS spread. Using the simple model in this section, we can illustrate how this works by looking at Equation (4), which shows how the protection seller chooses the appropriate CDS spread  $C$  by incorporating the hedge ratio  $h$  of the protection buyer. A higher hedge ratio increases the likelihood that the protection buyer pushes the firm into liquidation in the future as well as the payout to the protection buyer in a liquidation. Both of these things increase the expected loss of the protection seller. To protect himself, he charges a higher insurance premium  $C$ .

The same logic applies to NTCEs: If the protection seller anticipates the buyer to manufacture a default in the future, then he will charge a higher CDS spread today.

So why do we observe a few of these cases in real life? It could be that investors were surprised by the possibility of a manufactured default, but have now learned about it and have updated their expectations going forward. Also, the assumptions of our model are sometimes violated. There are two ways this can happen. One possibility is that market participants are not fully rational. A few investors make a mistake sometimes and fail to anticipate that their counterparty might engage in opportunistic actions.

The other possibility is that the protection seller has incomplete information about who the protection buyer is. Since the CDS market is opaque, this is very possible. With

incomplete information, manufactured defaults could arise endogenously. We do not solve such a model explicitly, for reasons explained below. Briefly, a parsimonious model where manufacturing default arises in equilibrium might look like this: The dealer knows that his counterparty can be one of two types. With probability  $p$ , the protection buyer has expertise in manufacturing defaults (e.g., a distressed debt hedge fund, like Blackstone’s GSO). With probability  $1 - p$ , the buyer does not have that expertise (e.g., mutual fund, insurance company, or pension fund). If  $p$  is sufficiently small, the friction created by asymmetric information is so little that the market does not collapse. The protection seller knows that a higher  $p$  increases his future losses, so he sets a CDS spread that is increasing with  $p$ . With a certain probability, the protection buyer will randomly turn out to be of the sophisticated type and will manufacture a default. It is in this sense that manufactured default can arise even in a model with rational expectations.

To be clear, we do not claim that manufactured defaults increase firm value or are welfare-enhancing in any other way. This is in contrast to the rest of the paper, where we argue that some forms of CDS intervention can be welfare-enhancing. Our views are consistent with a recent reform of the CDS market’s definition of a credit event, introduced by the ISDA in 2019<sup>6</sup> and fully implemented in 2020.<sup>7</sup> In the so-called NTCE Supplement, the ISDA acknowledges that manufactured defaults can create problems in the CDS market and it requires that for a credit event to be triggered, one has to prove that there was a deterioration in the creditworthiness of the underlying firm.

We also contribute to a debate in the legal literature about whether manufactured defaults should be banned by law. In a recent paper, Fletcher (2019, p. 1094) writes that a manufactured default is the “most feasible” type of CDS intervention. We argue that this is not true: Only if the protection seller makes a mistake, or if he does not know the identity of the protection buyer, can a CDS buyer profit from such a trading strategy. Additionally, once this trading strategy becomes more common, it will be even easier for protection sellers to anticipate it. Therefore, we argue that the more interesting cases of CDS intervention are not related to manufactured defaults. More broadly, our analysis adds to the legal literature by incorporating important ex ante effects, such as anticipating future opportunistic behavior and setting the upfront CDS spread accordingly.

While we agree with Fletcher (2019) that manufactured defaults are unlikely to increase welfare, it is not obvious that additional legislation is necessary. The recent changes in the CDS market introduced by the ISDA might be a sufficient solution to the problem. Only if manufactured defaults continue to emerge in the following years can one conclusively say that the ISDA’s solution was insufficient.

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<sup>6</sup><https://www.isda.org/a/KDqME/Final-NTCE-Supplement.pdf>

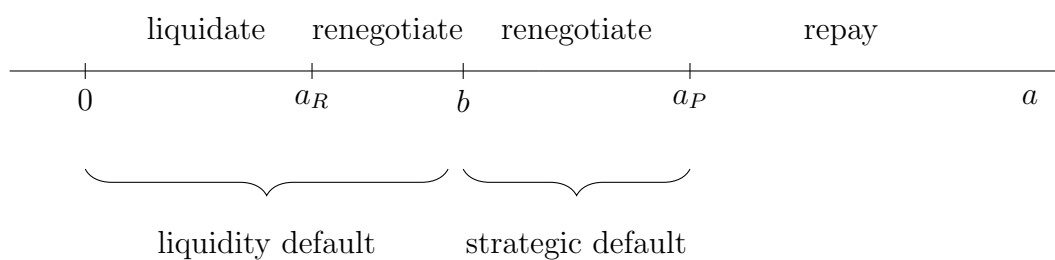
<sup>7</sup><https://www.isda.org/protocol/isda-2019-ntce-protocol/>

Symbol	Economic interpretation	Value
$r$	Risk-free rate	0.05
$\tau$	Corporate income tax rate	0.02
$\alpha$	Return to scale	0.5
$Z$	Upper bound of uniform distribution for profitability shock	2
$\lambda$	Equity issuance costs	0.1
$\xi$	Proportional liquidation costs	0.6
$\gamma$	Probability of renegotiation failure	0.7
$q$	Bargaining power of the debt holder	0.1

**Table IA.2 Base case parameter values.** This table provides the base case parameters used in the numerical solutions of the model. The corporate tax rate  $\tau$  is lower than the real-world corporate tax rate, because we assume that the whole face value of debt is tax deductible. Without correcting  $\tau$  downwards, the model would overstate the tax benefits of debt. One can show that our model is equivalent to one where the discount rate for equity is the discount rate for debt plus a parameter  $i$ , with  $\tau = i/(1 + r + i)$ . The value of  $\tau = 0.02$ , together with  $r = 0.05$ , implies that  $i = 0.021$ , i.e., the discount rate of equity is 2.1 percentage points higher than the risk-free interest rate.

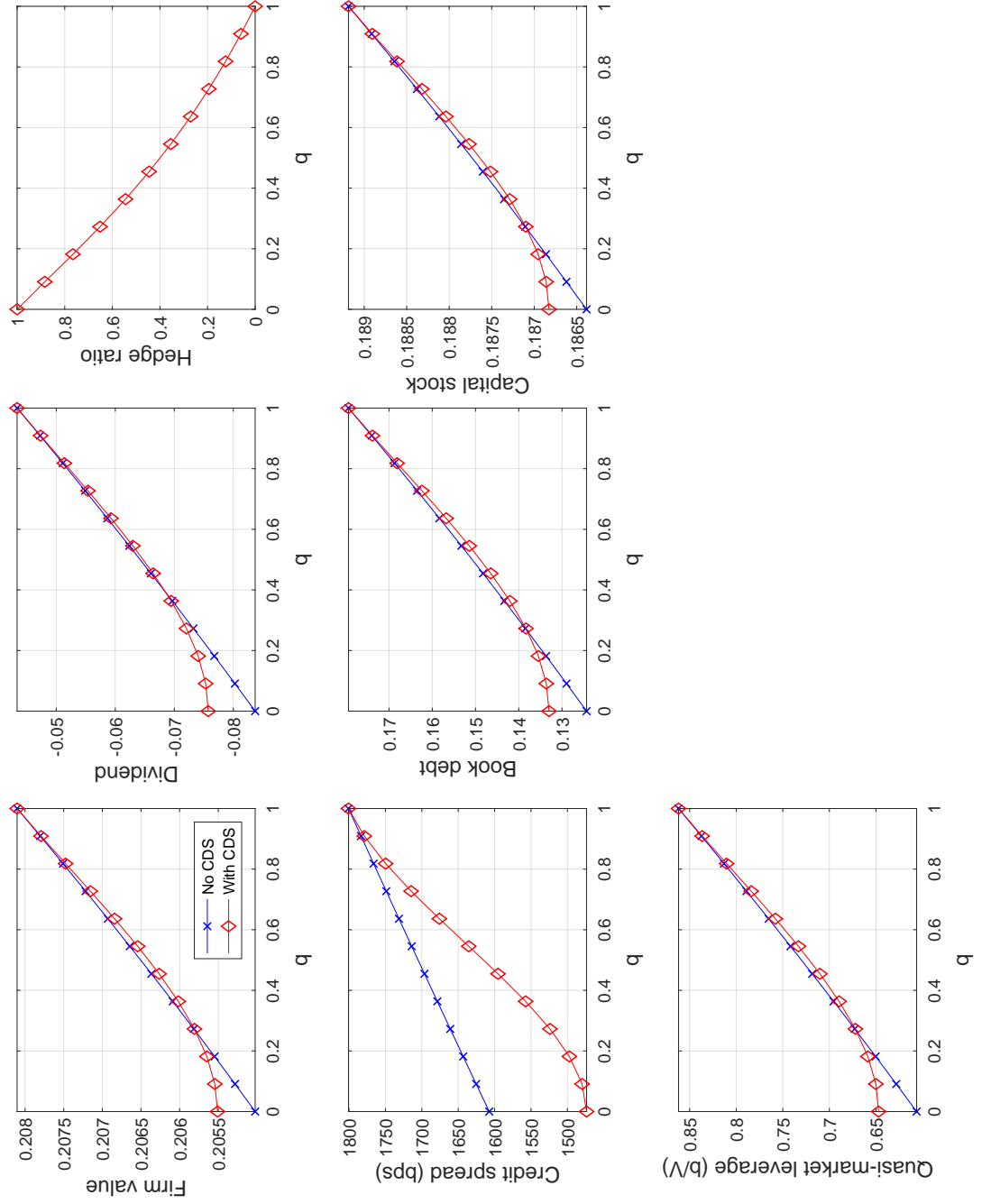
**Figure IA.12.** Optimal default decision

The figure presents the owner's optimal default decision, as a function of the asset value  $a$ . "Liquidity default" denotes the region where the firm would default even in a world where debt cannot be renegotiated. "Strategic default" denotes the region where the firm would not default if debt could not be renegotiated.



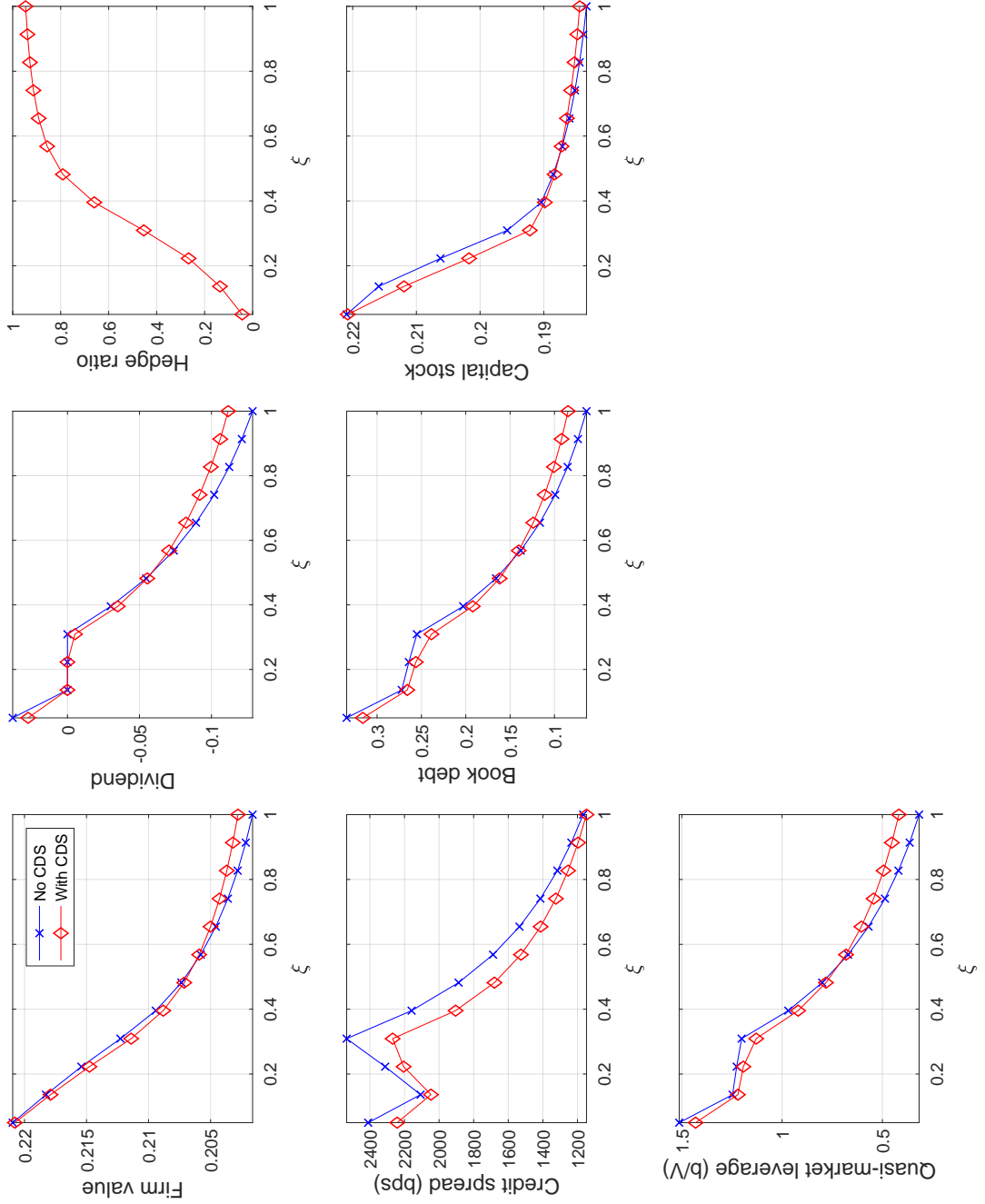
**Figure IA.13. Sensitivity to different levels of debt bargaining power  $q$ .**

This figure is based on the parameters in Table IA.2. The variables shown are the total firm value,  $V(k^*, b^*, h(k^*, b^*))$ , the dividend,  $m(k^*, b^*) - k^*$ , the optimal hedge ratio,  $h^*$ , the credit spread in basis points, the optimal face value of debt,  $b^*$ , the optimal capital stock,  $k^*$ , and quasi-market leverage,  $b^*/V(k^*, b^*, h(k^*, b^*))$ . The with-CDS model is shown with diamonds, while the no-CDS model is represented by crosses.



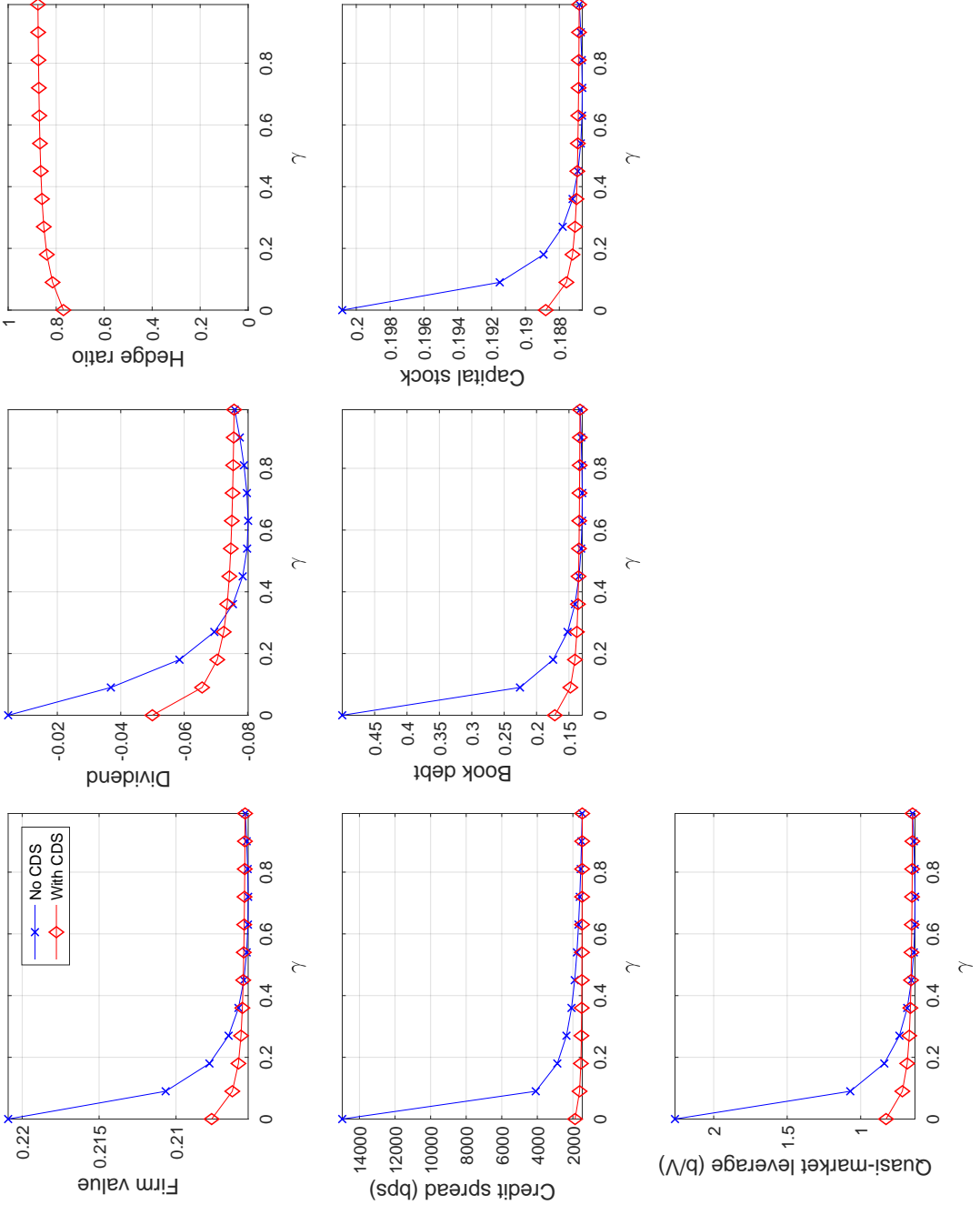
**Figure IA.14. Sensitivity to different levels of liquidation cost  $\xi$ .**

This figure is based on the parameters in Table IA.2. The variables shown are the total firm value,  $V(k^*, b^*, h(k^*, b^*))$ , the dividend,  $m(k^*, b^*) - k^*$ , the optimal hedge ratio,  $h^*$ , the credit spread in basis points, the optimal face value of debt,  $b^*$ , the optimal capital stock,  $k^*$ , and quasi-market leverage,  $b^*/V(k^*, b^*, h(k^*, b^*))$ . The with-CDS model is shown with diamonds, while the no-CDS model is represented by crosses.



**Figure IA.15. Sensitivity to different levels of renegotiation cost  $\gamma$ .**

This figure is based on the parameters in Table IA.2. The variables shown are the total firm value,  $V(k^*, b^*, h(k^*, b^*))$ , the dividend,  $m(k^*, b^*) - k^*$ , the optimal hedge ratio,  $h^*$ , the credit spread in basis points, the optimal face value of debt,  $b^*$ , the optimal capital stock,  $k^*$ , and quasi-market leverage,  $b^*/V(k^*, b^*, h(k^*, b^*))$ . The with-CDS model is shown with diamonds, while the no-CDS model is represented by crosses.



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