Internet Appendix

Countercyclical Risks, Consumption and Portfolio Choice: Theory and Evidence

A Supplementary Data

A.1 Panel Study of Income Dynamics

The PSID, the longest longitudinal household survey, started in 1968, was an annual survey from 1968 to 1997 and became a biennial survey after 1997. The PSID provides extremely rich information on households' socioeconomic characteristics, labor market experiences, income, wealth, health status, and family structure. Total family labor income includes the labor income of the head of the household and the labor income of the spouse. Labor income is the sum of wages and salaries, bonuses, overtime, tips, commissions, professional practice or trade, market gardening, additional job income, and miscellaneous labor income. Riskless assets comprise cash (checking and savings accounts, money market funds, certificates of deposits, savings bonds, and Treasury bills), plus bonds and life insurance (bonds, bond funds, cash value in a life insurance, valuable collection for investment purposes, and rights in a trust or estate). Risky financial assets are defined as the amount reported in the PSID survey question asking for the combined value of the shares of stock in publicly held corporations, mutual funds, and investment trusts.

A.2 Survey of Consumer Finances Data

The SCF has been conducted by the Federal Reserve Board every three years to provide detailed information on the finances of US households. The survey deliberately oversamples relatively wealthy households to produce more accurate statistics; in my analysis, I then use the sampling weights provided by the SCF to obtain unbiased statistics for the US population. The SCF also handles survey nonrespondents by using weighting adjustments. These weights are used to calculate the values reported in the tables and graphs. I use data from the 1989 to 2013 wave. Variables are constructed using the codebook and macrovariable definitions from the Federal Reserve Board website.

Wealth is made up of checking accounts, savings accounts, certificates of deposit, saving bonds, money market accounts, cash/call money accounts, trusts, life insurance, thrift plans, individual retirement accounts (IRAs), future pensions, total directly held mutual funds, stocks, bonds, savings bonds, other managed assets, and other financial assets. Household income refers to the household's cash income, before taxes, for the full calendar year preceding the survey. The components of income are the sum of wages and salaries, unemployment insurance, worker's compensation, Social Security income, other pension income, annuities, and other disability or retirement programs. Wealth invested in risky assets is the sum of directly held stock, stock mutual funds, and amounts of stock in retirement accounts. Stock market participants are those who have the full value of stocks greater than zero. The risky assets share is constructed as the ratio of wealth invested in risky assets, which are defined above.

B Numerical Solution

At each time period t, depending on different states, households control their consumption $\{C_{it}^*\}_{t=1}^T$ and allocation to the stock $\{\alpha_{it}^*\}_{t=1}^T$ to maximize the value function. More specifically, households always know the current aggregate state $s(t)$ and then decide their consumption and investment in stock. However, households are uncertain of the future aggregate state $s(t+1)$. Because of the unit-root process assumption for the labor income process, the state space can be reduced to two variables by standardizing the entire problem by the permanent component of labor income, $e^{v_{it}}$, which is denoted by P_{it} for simplicity.

Let $x_{it} = \frac{X_{it}}{P_{it}}$ and $c_{it} = \frac{C_{it}}{P_{it}}$ be the normalized cash on hand and consumption. The normalized value function is

(1)

$$v_{it}(x_{it}, s(t)) = \max_{\{\alpha_{it}\}_{t=1}^T, \{c_{it}\}_{t=1}^T} \left\{ \begin{array}{c} (1-\beta)c_{it}^{1-\frac{1}{\psi}} + \beta(E_t((\frac{P_{i,t+1}}{P_{it}})^{1-\gamma} p_{t+1} V_{it+1}(x_{i,t+1}, s(t+1))^{1-\gamma} \\ +b(\frac{P_{i,t+1}}{P_{it}})^{1-\gamma}(1-p_{t+1})x_{i,t+1}^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}} \end{array} \right\}^{\frac{1}{1-1/\psi}},$$

subject to

$$x_{i,t+1} = \left\{ \begin{array}{c} (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^p \frac{P_{it}}{P_{i,t+1}} + e^{\varepsilon_{i,t+1}} \text{ for } t \leq K \\ (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^p \frac{P_{it}}{P_{i,t+1}} + \lambda \text{ for } t > K \ . \end{array} \right. \quad (2)$$

For the extended model, where the participation decision is endogenized, I introduce the stock market participation status (E_{it}, a zero-one variable indicating whether or not the entry cost has been paid), and the corresponding optimization problem can be written as

(3)

$$V_{it}(x_{it}, s(t), E_{it}) = \max \left\{ \begin{array}{c} (1-\beta)c_{it}^{1-\frac{1}{\psi}} + \beta(E_t((\frac{P_{i,t+1}}{P_{it}})^{1-\gamma} p_{t+1} V_{it+1}(x_{i,t+1}, s(t+1), E_{i,t+1})^{1-\gamma} \\ +b(\frac{P_{i,t+1}}{P_{it}})^{1-\gamma}(1-p_{t+1})x_{i,t+1}^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}} \end{array} \right\}^{\frac{1}{1-1/\psi}},$$

subject to

$$x_{i,t+1} = \left\{ \begin{array}{c} (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^{p}\frac{P_{it}}{P_{i,t+1}} - I_E^i F^0 - I_S^i F^1 + e^{\varepsilon_{i,t+1}} \text{ for } t \leq K \\ (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^{p}\frac{P_{it}}{P_{i,t+1}} - I_E^i F^0 - I_S^i F^1 + \lambda \text{ for } t > K \end{array} \right. , \quad (4)$$

where I define the dummy variable I_E^i as equal to one in the period in which the entry cost is paid, and zero otherwise, and the dummy variable I_S^i as equal to one if the household has a positive holding of stocks, and zero otherwise.

The model does not have an analytical solution but can be numerically solved with backward induction. The policy functions and value functions are functions of the state variables: time t, a business-cycle indicator $s(t)$, and cash on hand relative to the permanent labor income, which is continuous and thus needs to be discretized appropriately. In the last period, the policy functions are determined by the bequest motive, and the value function corresponds to the bequest function. I use a grid search to optimize the value function. I compute the value associated with each level of consumption and the share of wealth invested in stock. Then I choose the level of consumption and the share of wealth invested in stock achieving the maximum value, which are saved as the policy rules for the previous period. For every time t prior to T, and for each point in the state space, this procedure is iterated backward.

To approximate the distributions of innovations to the permanent labor income shocks, I use numerical integrations. My density function for permanent income shocks can be rewritten as a sum of the Hermite polynomials with the Gaussian kernel so that I can use the Gaussian quadrature points with some adjusted weights to approximate numerical integrations. For points that do not lie on the state space grid, I evaluate the value function using a cubic spine interpolation. I use cubic spline interpolation for value function evaluation of the chosen grids. As for the transition matrix between expansion and recession, I assume that the probability of the current state staying the same in the next period is 0.75, and the probability of the current state changing to the other state in the next period is 0.25. During a recession, there is a small probability (3%) of losing 55% of the stock returns.

After the optimal policy rules are derived, I start simulating the life-cycle profile for each household in the SCF data from 1989 to 2013. Following the NBER dating methodology specified in the main paper, I have three recessions from the 1989 SCF to the 2013 SCF: 1992, 2001, and 2010. To make the results comparable, I use the 1989 to 2013 waves for the US Financial Accounts as well. All households face the same annual stock returns from CRSP and choose the income distribution based on the business-cycle status. Once households die at age 100, they are dropped from the simulation. New 20-year-old households enter the labor market every year with an initial level of wealth.

C Sensitivity Analysis

I perform sensitivity analysis with respect to the importance of the correlations between permanent income shocks and stock returns. The empirical evidence is mixed on the magnitude of this correlation. In Figure G1, I explore the implications of a positive correlation with countercyclical earnings risk and check whether the results are sensitive to these values.

Huggett and Kaplan (2016) argue that the correlation should be quantitatively small, whereas Bonaparte, Korniotis, and Kumar (2014) find that this correlation can vary from -0.6 to 0.6 for different households. Campbell et al. (2001) find a positive correlation only for specific population groups. Heaton and Lucas (2000) also report a positive correlation for entrepreneurs, while Davis and Willen (2000) obtain estimates of between 0.1 and 0.3. The benchmark correlation is 0.20. Figure G1 illustrates the effects of a correlation from 0.0 to 0.3. Although a larger correlation mildly discourages households from holding risky assets, I find that the effects of countercyclical earnings risk on portfolio choice stay strong for young households, independent of correlation. This finding indicates that moderate risky asset holdings are not driven by a positive correlation between permanent income shocks and stock returns.

D Model Extension with a Fixed Cost: Model Results

I present results with the calibrated preference parameters and bequest motive: the discount factor (β) is equal to 0.98, the coefficient of relative risk aversion (γ) is set to 5.6, the EIS (ψ) is 0.5, and the bequest motive (b) is 2. To facilitate a comparison with Fagereng et al. (2017), I set both the entry cost of participation (F^0) and the per-period cost of participation (F^1) to 2%.

Figure G4 plots the optimal risky share of financial wealth conditional on participating in the stock market. To facilitate a comparison with the benchmark, I plot the extended model as well as benchmark 2. Similar to benchmark 2, the optimal risky asset shares decrease with financial wealth and age. Introducing a fixed cost lowers the share of wealth in stocks across all ages compared with benchmark 2.

Figure G5 shows the wealth participation threshold. The wealth threshold of participation decreases with age until retirement. In the beginning of the working period, households accumulate very little wealth, and the fixed cost prevents those households from participating in the market. Later on, households accumulate more financial wealth, which makes stock market participation more appealing, and they are willing to pay for the fixed cost and enter the market. Therefore, the wealth participation threshold drops and remains low until retirement. When households retire, their human wealth drops significantly, and households start reducing their optimal risky asset shares. So participation in the stock market becomes less attractive, and the wealth participation threshold increases.

Figure G6 reports the life-cycle profiles. Participation in the stock market increases steadily until retirement. During retirement, households start slowly exiting the market. Moreover, households rebalance their risky asset holdings before retirement. Introducing a fixed cost lowers the conditional risky asset shares and wealth accumulation compared with benchmark 2 at all ages. Households allocate more financial wealth to riskless assets; therefore, the returns on financial wealth are lower, and accumulated wealth decreases.

E Preference Heterogeneity

Table G3 shows the mean consumption-to-wealth ratio for different values of the preference parameters under three models. I allow the risk aversion coefficient to vary from 1 to 5 and the EIS to vary from 0.2 to 0.8, which are both relatively standard choices in the existing empirical literature. First, with the lognormal model, the results are consistent with Gomes and Michaelides (2005): the consumption-to-wealth ratio is a decreasing function of both γ and ψ at every stage of the life cycle.

Then, I compare the mean consumption-to-wealth ratio between the lognormal model and benchmark 1. I find that overall, the consumption-to-wealth ratio drops in the early adult years (ages 20 to 35) and in middle age (ages 36 to 65). During the early adult years, wealth accumulation is mostly driven by the precautionary savings motive. Therefore, the optimal consumption-to-wealth ratio is significantly more affected by prudence than by the EIS. During the middle age, savings are now determined by the preference for low-frequency consumption smoothing. A riskier labor income process increases households' total risk exposure and discourages households from consuming more. During the retirement period, labor income risk by assumption disappears; therefore, the results are not quantitatively affected. Adding rare disasters in the stock market further lowers the consumption-to-wealth ratio, and this effect is consistent at every stage of the life cycle. Although labor income risk disappears during retirement, households suffer from a stock market crash as long as they participate in the market.

The optimal consumption-to-wealth ratio is driven by the trade-off between the expected return on invested wealth and the discount rate, combined with the household's sensitivity to these incentives. The less risk-averse households invest more aggressively, and therefore those households earn more from the stock market. Thus, since for a given $\psi < 1$ the income effect dominates the substitution effect, they have higher consumption-to-wealth ratios. The same intuition explains why the consumption-to-wealth ratio is a decreasing function of the EIS for a given γ. Riskier labor income and rare disaster amplify these effects. However, I find the pattern is weaker for the highest γ because the expected return on invested wealth is close to the discount rate and the consumption-to-wealth ratio is almost independent of ψ. For the lowest values of γ, I also find a weaker pattern. This result is because of the borrowing constraint, which does not allow the consumption-to-wealth ratio to exceed 100%.

From the results in Table G3, I can conclude that the consumption-to-wealth ratio decreases more significantly in benchmark 1 and benchmark 2 with γ and ψ, compared with the lognormal model.

F Approximation of Continuous Distributions

I need to approximate the density functions for stock returns and labor income. Since the innovations to stock returns and the transitory shocks to labor income follow a normal distribution, their density functions can be easily approximated by the Gaussian quadrature method. Innovations to stock returns and permanent labor income are not normally distributed; therefore, their expectations cannot be approximated by the Gaussian quadrature method. I follow the method of Zoia (2009) and Faliva et al. (2016), which enables me to tailor the shape of any given distribution and link the marginal density to the joint density by using orthogonal polynomials. They focus on between-squares dependence, and I extend their method to the linear correlation. Next, I will discuss how to generate grids and weights for the joint density in two cases: independent marginal distributions and dependent marginal distributions.

F.1 Independent Marginal Distributions

Assume that z_1 and z_2 are two independent distributions with $D(\mu_1, \sigma_1, \alpha_1, \beta_1)$ and $D(\mu_2, \sigma_2, \alpha_2, \beta_2)$. Then their joint density can be written as

$$\psi(z_1, z_2) = \phi(z_1)\phi(z_2). \tag{1}$$

From Zoia (2009), I know that

$$\phi(z_1) = f(z_1)g(z_1, \alpha_1, \beta_1), \tag{2}$$

$$\phi(z_2) = f(z_2)g(z_2, \alpha_2, \beta_2), \tag{3}$$

where $f(\cdot)$ is the Gaussian kernel,

$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{z-\mu}{\sigma}^2)}, \tag{4}$$

and $g(\cdot)$ is an adjustment term for excess kurtosis β and skewness α via the Hermite polynomials:

$$g(z,\alpha,\beta) = 1 + \frac{\alpha}{6}H_3(\frac{z-\mu}{\sigma}) + \frac{\beta}{24}H_4(\frac{z-\mu}{\sigma}) \tag{5}$$

$$H_3 = z^3 - 3z \tag{6}$$

$$H_4 = z^4 - 6z^2 + 3. \tag{7}$$

So, for any $V = v(z_1, z_2)$,

$$E(V) = \int\int v(z_1,z_2)\psi(z_1,z_2)dz_1dz_2 \tag{8}$$

$$= \int\int v(z_1,z_2)\phi(z_1)\phi(z_2)dz_1dz_2 \tag{9}$$

$$= \int \phi(z_1)\int v(z_1,z_2)\phi(z_2)dz_2dz_1 \tag{10}$$

$$= \int f(z_1)g(z_1,\alpha_1,\beta_1)\int v(z_1,z_2)f(z_2)g(z_2,\alpha_2,\beta_2)dz_2dz_1 \tag{11}$$

(12)

$$\overline{\overline{\sqrt{2}x_1=\frac{z_1-\mu_1}{\sigma_1},\sqrt{2}x_2=\frac{z_2-\mu_2}{\sigma_2}}} \int \frac{1}{\sqrt{2\pi\sigma_1^2}}e^{-x_1^2}[1+\frac{\alpha}{6}H_3(\sqrt{2}x_1)+\frac{\beta}{24}H_4(\sqrt{2}x_1)]$$

$$\int v(\sqrt{2}\sigma_1x_1+\mu_1,\sqrt{2}\sigma_2x_2+\mu_2)\frac{1}{\sqrt{2\pi\sigma_2^2}}e^{-x_2^2}[1+\frac{\alpha}{6}H_3(\sqrt{2}x_2)+\frac{\beta}{24}H_4(\sqrt{2}x_2)] \tag{13}$$

$$\sqrt{2\sigma_1^2}\sqrt{2\sigma_2^2}dx_1dx_2 \tag{14}$$

$$= \int e^{-x_1^2}[1+\frac{\alpha}{6}H_3(\sqrt{2}x_1)+\frac{\beta}{24}H_4(\sqrt{2}x_1)] \tag{15}$$

$$\int v(\sqrt{2}\sigma_1x_1+\mu_1,\sqrt{2}\sigma_2x_2+\mu_2)e^{-x_2^2}[1+\frac{\alpha}{6}H_3(\sqrt{2}x_2)+\frac{\beta}{24}H_4(\sqrt{2}x_2)]dx_1dx_2. \tag{16}$$

Hence, the grids and the associated weights for each marginal distribution $z_i, i \in 1, 2$ are

$$n_{i,j} = \sqrt{2}\sigma_i m_j + \mu_i \tag{17}$$

$$w_{i,j}^* = \frac{1}{\sqrt{\pi}}[1+\frac{\alpha}{6}H_3(\sqrt{2}m_j)+\frac{\beta}{24}H_4(\sqrt{2}m_j)]w_j, \tag{18}$$

where m_j and w_j are the Gauss-Hermite polynomial grids and weights.

F.2 Dependent Marginal Distributions

Faliva et al. (2016) show that if $f(\cdot)$ is a Gaussian kernel, the first two monic orthogonal polynomials of a system $p_n(x)$ associated with $f(\cdot)$ are given as

$$p_1(x) = x \tag{19}$$

$$p_2(x) = x^2 - 1 \tag{20}$$

and have the following two important properties:

$$\int p_1(x)p_2(x)f(x)dx = 0 \tag{21}$$

$$\int x^j p_m(x)f(x)dx = \begin{cases} = 0 & j < m \\ = 1 & j = m = 1. \\ = \frac{1}{2} & j = m = 2 \end{cases} \tag{22}$$

Theorem A.0.2 Consider a pair of normal distributions x_1 and x_2 with marginal distributions:

$$\phi(x_1) = f(x_1)g(x_1, \alpha_1, \beta_1), \tag{23}$$

$$\phi(x_2) = f(x_2)g(x_2, \alpha_2, \beta_2), \tag{24}$$

and assume the existence of a correlation ρ and a between-squares correlation γ that are

$$\rho = E(x_1 x_2) - E(x_1)E(x_2) \tag{25}$$

$$\gamma = E(x_1^2 x_2^2) - E(x_1^2)E(x_2^2). \tag{26}$$

Consider the orthogonal polynomial function

$$\tau = 1 + \rho\frac{p_1(x_1)p_1(x_2)}{g(x_1, \alpha_1, \beta_1)g(x_2, \alpha_2, \beta_2)} + \frac{1}{4}\gamma\frac{p_2(x_1)p_2(x_2)}{g(x_1, \alpha_1, \beta_1)g(x_2, \alpha_2, \beta_2)} \tag{27}$$

linking two marginal distributions through τ, and the joint density function can be written as

$$\psi(x_1, x_2) = \phi(x_1)\phi(x_2)\tau(x_1, x_2). \tag{28}$$

Proof. The product-moment $\mu_{i,j}$ of the function $\psi(x_1, x_2)$ is given by

$$\mu_{i,j} = \int\int x_1^j x_2^i \psi(x_1, x_2) dx_1 dx_2 \tag{29}$$

and can be written as

$$\begin{aligned}
\mu_{i,j} &= \int\int x_1^i x_2^j \phi(x_1)\phi(x_2)\tau(x_1, x_2) && (30)\\
&= \int x_1^i g(x_1, \alpha_1, \beta_1) f(x_1) dx_1 \int x_2^j g(x_2, \alpha_2, \beta_2) f(x_2) dx_2 && (31)\\
&\quad + \rho \int x_1^i p_1(x_1) f(x_1) dx_1 \int x_2^j p_1(x_2) f(x_2) dx_2 && (32)\\
&\quad + \frac{1}{4}\gamma \int x_1^i p_2(x_1) f(x_1) dx_1 \int x_2^j p_2(x_2) f(x_2) dx2. && (33)
\end{aligned}$$

When i and j both equal zero, $\mu_{i,j}$ is

$$\begin{aligned}
\mu_{0,0} &= \int g(x_1, \alpha_1, \beta_1) f(x_1) dx_1 \int g(x_2, \alpha_2, \beta_2) f(x_2) dx_2 && (34)\\
&= \int \phi(x_1) dx_1 \int \phi(x_2) dx_2 && (35)\\
&= 1. && (36)
\end{aligned}$$

When i or j equal zero, $\mu_{i,j}$ are reduced to marginal moments

$$\begin{aligned}
\mu_{i,0} &= \int x_1^i g(x_1, \alpha_1, \beta_1) f(x_1) dx_1 && (37)\\
&= \int x_1^i \phi(x_1) dx_1 && (38)\\
\mu_{0,j} &= \int x_2^j g(x_2, \alpha_2, \beta_2) f(x_2) dx_2 && (39)\\
&= \int x_2^j \phi(x_2) dx_2. && (40)
\end{aligned}$$

For $1 \le i, j \le 2$, $\mu_{i,j}$ can be rewritten as

$$\mu_{1,1} = E(x_1 x_2) = E(x1)E(x_2) + \rho \tag{41}$$

$$\mu_{2,1} = E(x_1^2 x_2) \tag{42}$$

$$\mu_{1,2} = E(x_1 x_2^2) \tag{43}$$

$$\mu_{2,2} = E(x_1^2 x_2^2) = E(x1^2)E(x_2^2) + \gamma, \tag{44}$$

which completes the proof. ■

So, for any $V = v(z_1, z_2)$,

$$E(V) = \int\int v(z_1, z_2)\psi(z_1, z_2)dz_1dz_2 \tag{45}$$

$$= \int\int v(z_1, z_2)\phi(x_1)\phi(x_2)\tau(x_1, x_2)dz_1dz_2 \tag{46}$$

$$= \int\int v(z_1, z_2)f(z_1)g(x_1, \alpha_1, \beta_1)f(z_2)g(x_2, \alpha_2, \beta_2)dz_1dz_2 \tag{47}$$

$$+ \int\int v(z_1, z_2)\rho f(z_1)p_1(z_1)f(z_2)p_1(z_2)dz_1dz_2 \tag{48}$$

$$+ \frac{1}{4}\int\int v(z_1, z_2)\gamma f(z_1)p_2(z_1)f(z_2)p_2(z_2)dz_1dz_2 \tag{49}$$

(50)

$$\overset{\sqrt{2}x_1=\frac{z_1-\mu_1}{\sigma_1}, \sqrt{2}x_2=\frac{z_2-\mu_2}{\sigma_2}}{=\!=\!=\!=} \int\int \frac{1}{\sqrt{\pi}}e^{-x_1^2}[1 + \frac{\alpha}{6}H_3(\sqrt{2}x_1) + \frac{\beta}{24}H_4(\sqrt{2}x_1)]$$

$$v(\sqrt{2}\sigma_1 x_1 + \mu_1, \sqrt{2}\sigma_2 x_2 + \mu_2)\frac{1}{\sqrt{\pi}}e^{-x_2^2}[1 + \frac{\alpha}{6}H_3(\sqrt{2}x_2) + \frac{\beta}{24}H_4(\sqrt{2}x_2)]dx_1dx_2 \tag{51}$$

$$+ \rho\int\int \frac{1}{\sqrt{\pi}}e^{-x_1^2}\sqrt{2}x_1 v(\sqrt{2}\sigma_1 x_1 + \mu_1, \sqrt{2}\sigma_2 x_2 + \mu_2)\frac{1}{\sqrt{\pi}}e^{-x_2^2}\sqrt{2}x_2 dx_1dx_2 \tag{52}$$

$$+ \frac{1}{4}\gamma\int\int \frac{1}{\sqrt{\pi}}e^{-x_1^2}(2x_1 - 1)v(\sqrt{2}\sigma_1 x_1 + \mu_1, \sqrt{2}\sigma_2 x_2 + \mu_2)\frac{1}{\sqrt{\pi}}e^{-x_2^2}(zx_2 - 1)dx_1dx_2. \tag{53}$$

From the above equations, I decompose $E(V)$ into three parts: the first term as the integration in the independent case and two adjustment terms for the correlation and between-squares correlation. So the grids and weights for joint density $\psi(z_1, z_2)$ are generated separately for the three terms.

The grids and the associated weights for each marginal distribution $z_i, i \in 1, 2$ in the first term are

$$n_{i,j} = \sqrt{2}\sigma_i m_j + \mu_i \tag{54}$$

$$w^*_{i,j} = \frac{1}{\sqrt{\pi}}[1 + \frac{\alpha}{6}H_3(\sqrt{2}m_j) + \frac{\beta}{24}H_4(\sqrt{2}m_j)]w_j. \tag{55}$$

The grids and the associated adjustments for each marginal distribution $z_i, i \in 1, 2$ in the second term are

$$n_{i,j} = \sqrt{2}\sigma_i m_j + \mu_i \tag{56}$$

$$w^{**}_{i,j} = \frac{1}{\sqrt{\pi}}\sqrt{2}m_j w_j. \tag{57}$$

The grids and the associated adjustments for each marginal distribution $z_i, i \in 1, 2$ in the third term are

$$n_{i,j} = \sqrt{2}\sigma_i m_j + \mu_i \tag{58}$$

$$w^{**}_{i,j} = \frac{1}{2\sqrt{\pi}}(2m_j^2 - 1)w_j, \tag{59}$$

where m_j and w_j are the Gauss-Hermite polynomial grids and weights.

F.3 Simulation of Correlated Mixture Normal Distribution

Consider a pair of distributions z_1 and z_2 with $D(\mu_1, \sigma_1, \alpha_1, \beta_1)$ and $D(\mu_2, \sigma_2, \alpha_2, \beta_2)$, which can be generated by the following two normal distributions, respectively:

$$z_1 = \begin{cases} z_{1,1} \sim N_1(\mu_1, \sigma_1) & w.p \quad p_1 \\ z_{1,2} \sim N_2(\mu_2, \sigma_2) & w.p \quad 1 - p_1 \end{cases} \tag{60}$$

$$z_2 = \begin{cases} z_{2,1} \sim N_3(\mu_3, \sigma_3) & w.p \quad p_2 \\ z_{2,2} \sim N_4(\mu_4, \sigma_4) & w.p \quad 1 - p_2 \end{cases} \tag{61}$$

and if the covariance between each pair of normal distributions is

$$\begin{pmatrix} \sigma_1^2 & 0 & \rho^*\sigma_1\sigma_3 & \rho^*\sigma_1\sigma_4 \\ 0 & \sigma_2^2 & \rho^*\sigma_2\sigma_3 & \rho^*\sigma_2\sigma_4 \\ \rho^*\sigma_1\sigma_3 & \rho^*\sigma_2\sigma_3 & \sigma_3^2 & 0 \\ \rho^*\sigma_1\sigma_4 & \rho^*\sigma_2\sigma_4 & 0 & \sigma_4^2 \end{pmatrix} \tag{62}$$

then there exists a correlation ρ between z_1 and z_2:

$$\rho = \frac{\rho^*}{\sqrt{p_1(\mu_1^2+\sigma_1^2)+(1-p_1)(\mu_2^2+\sigma_2^2)-(p_1\mu_1+(1-p_1)\mu_2)^2}} \tag{63}$$

$$\frac{[p_1p_2\sigma_1\sigma_3+p_1(1-p_2)\sigma_1\sigma_4+(1-p_1)p_2\sigma_2\sigma_3+(1-p_1)(1-p_2)\sigma_2\sigma_4]}{\sqrt{p_2(\mu_3^2+\sigma_3^2)+(1-p_2)(\mu_4^2+\sigma_4^2)-(p_2\mu_3+(1-p_2)\mu_4)^2}}. \tag{64}$$

Proof. Let

$$1_A = \begin{cases} 1 & x \in A \quad w.p. \quad p_1 \\ 0 & x \notin A \quad w.p. \quad 1-p_1 \end{cases} \tag{65}$$

$$1_B \begin{cases} 1 & x \in B \quad w.p. \quad p_2 \\ 0 & x \notin B \quad w.p. \quad 1-p_2, \end{cases} \tag{66}$$

where 1_A and 1_B are random variables, 1_A is independent of N_1, N_2, and 1_B is independent of N_3, N_4. Then $z_1 = 1_AN_1 + (1-1_A)N_2$, $z_2 = 1_BN_3 + (1-1_B)N_4$, and the covariance between z_1 and z_2 can be given as

$$cov(z_1, z_2) = cov(1_AN_1 + (1-1_A)N_2, 1_BN_3 + (1-1_B)N_4) \tag{67}$$

$$= cov(1_AN_1, 1_BN_3) + cov(1_AN_1, (1-1_B)N_4) \tag{68}$$

$$+ cov((1-1_A)N_2, N_3) + cov((1-1_A)N_2, (1-1_B)N_4). \tag{69}$$

I calculate $cov(z_1, z_2)$ part by part:

$$cov(1_A N_1, 1_B N_3) = E(1_A N_1, 1_B N_3) - E(1_A N_1)E(1_B N_3), \tag{70}$$

$$= E(1_A)E(1_B)E(N_1 N_3) - E(1_A)E(N_1)E(1_B)E(N_3), \tag{71}$$

$$= p_1 p_2(\rho^* \sigma_1 \sigma_3 + \mu_1 \mu_3) - p_1 p_2 \mu_1 \mu_3, \tag{72}$$

$$= p_1 p_2 \rho^* \sigma_1 \sigma_3, \tag{73}$$

$$cov(1_A N_1, (1 - 1_B)N_4) = E(1_A N_1, (1 - 1_B)N_4) - E(1_A N_1)E((1 - 1_B)N_4), \tag{74}$$

$$= E(1_A)E(1 - 1_B)E(N_1 N_4) - E(1_A)E(N_1)E(1 - 1_B)E(N_4), \tag{75}$$

$$= p_1(1 - p_2)(\rho^* \sigma_1 \sigma_4 + \mu_1 \mu_4) - p_1(1 - p_2)\mu_1 \mu_4, \tag{76}$$

$$= p_1(1 - p_2)\rho^* \sigma_1 \sigma_4, \tag{77}$$

$$cov((1 - 1_A)N_2, 1_B N_3) = E((1 - 1_A)N_2, 1_B N_3) - E((1 - 1_A)N_2)E(1_B N_3), \tag{78}$$

$$= E(1 - 1_A)E(1_B)E(N_2 N_3) - E(1 - 1_A)E(N_2)E(1_B)E(N_3), \tag{79}$$

$$= (1 - p_1)p_2(\rho^* \sigma_2 \sigma_3 + \mu_2 \mu_3) - (1 - p_1)p_2 \mu_2 \mu_3, \tag{80}$$

$$= (1 - p_1)p_2 \rho^* \sigma_2 \sigma_3, \tag{81}$$

$$cov((1 - 1_A)N_2, (1 - 1_B)N_4) = E((1 - 1_A)N_2, (1 - 1_B)N_4) - E((1 - 1_A)N_2)E((1 - 1_B)N_4), \tag{82}$$

$$= E(1 - 1_A)E(1 - 1_B)E(N_2 N_4) - E(1 - 1_A)E(N_2)E(1 - 1_B)E(N_4), \tag{83}$$

$$= (1 - p_1)(-p_2)(\rho^* \sigma_2 \sigma_4 + \mu_2 \mu_4) - (1 - p_1)(1 - p_2)\mu_2 \mu_4, \tag{84}$$

$$= (1 - p_1)(1 - p_2)\rho^* \sigma_2 \sigma_4. \tag{85}$$

So $cov(z_1, z_2)$ can be rewritten as

$$cov(z_1, z_2) = \rho^*[p_1 p_2 \sigma_1 \sigma_3 + p_1(1 - p_2)\sigma_1 \sigma_4 + (1 - p_1)p_2 \sigma_2 \sigma_3 + (1 - p_1)(1 - p_2)\sigma_2 \sigma_4], \tag{86}$$

$$(87)\qquad var(z_1) = E(z_1^2) - E(z_1)^2$$

$$(88)\qquad E(z_1) = E(1_A N_1 + (1-1_A)N_2),$$

$$(89)\qquad = E(1_A)E(N_1) + E(1-1_A)E(N_2),$$

$$(90)\qquad = p_1\mu_1 + (1-p_1)\mu_2,$$

$$(91)\qquad E(z_1^2) = E(1_A^2 N_1^2 + (1-1_A)^2 N_2^2 + 21_A(1-1_A)N_1N_2),$$

$$(92)\qquad = E(1_A^2)E(N_1^2) + E((1-1_A)^2)E(N_2^2) + 2E(1_A(1-1_A))E(N_1)E(N_2),$$

$$(93)\qquad = p_1(\mu_1^2 + \sigma_1^2) + (1-p_1)(\mu_2^2 + \sigma_2^2).$$

So $var(z_1)$ can be rewritten as

$$(94)\qquad var(z_1) = p_1(\mu_1^2 + \sigma_1^2) + (1-p_1)(\mu_2^2 + \sigma_2^2) - (p_1\mu_1 + (1-p_1)\mu_2)^2.$$

Similarly, $var(z_2) = p_2(\mu_3^2 + \sigma_3^2) + (1-p_2)(\mu_4^2 + \sigma_4^2) - (p_2\mu_3 + (1-p_2)\mu_4)^2$. Hence, the correlation between z_1 and z_2 can be derived as

$$(95)\qquad \rho = \frac{cov(z_1, z_2)}{\sqrt{var(z_1)var(z_2)}}$$

$$(96)\qquad \frac{\rho^*}{\sqrt{p_1(\mu_1^2 + \sigma_1^2) + (1-p_1)(\mu_2^2 + \sigma_2^2) - (p_1\mu_1 + (1-p_1)\mu_2)^2}}$$

$$(97)\qquad \frac{[p_1p_2\sigma_1\sigma_3 + p_1(1-p_2)\sigma_1\sigma_4 + (1-p_1)p_2\sigma_2\sigma_3 + (1-p_1)(1-p_2)\sigma_2\sigma_4]}{\sqrt{p_2(\mu_3^2 + \sigma_3^2) + (1-p_2)(\mu_4^2 + \sigma_4^2) - (p_2\mu_3 + (1-p_2)\mu_4)^2}}.$$

■

F.4 Continuous Distributions Approximation Experiments

I now provide experimentation with the orthogonal polynomials approximation method in Zoia (2009) and Faliva, Poti, and Zoia (2016). To test the accuracy of the approximation method, I use two different methods. The first method is based on simulation. I simulate based on the discretization for a given number of grid points and then perform a Monte Carlo analysis to investigate how close the estimated parameters are to the actual parameters used to generate the discrete approximation. I generate 100,000 simulation paths and report the means, variance, skewness, and kurtosis of each variable and the distance between the estimations and true values. The second method uses the nodes and weights used in the numerical solution to compute the first four moments of the variables. These values should be close to the simulations.

I test the orthogonal polynomials approximation method for three different situations: (1) a variable distributed normally, (2) a variable distributed as a mixture of a normal distribution with negative skewness and excess kurtosis, and (3) two correlated variables.

Experiment 1: Assume that a variable follows a normal distribution $N(0, 0.1)$. I report the first four moments of this variable and change the number of grid points (N) to check whether the accuracy can be improved by increasing the number of grid points. The first four moments and the average distance by simulation are as follows:

N	Mean	Variance	Skewness	Kurtosis	Avg. distance
5	-3.3043×10^{-5}	0.0101	-0.0085	3.0044	9.2008×10^{-5}
10	2.0576×10^{-6}	0.0100	-1.1483×10^{-4}	2.9963	1.4082×10^{-5}
15	-3.4842×10^{-6}	0.0100	-0.0027	3.0018	1.0594×10^{-5}
20	2.4127×10^{-17}	0.0100	-1.1044×10^{-15}	3.0000	8.1986×10^{-6}

The first four moments computed using the numerical integration method are as follows:

N	Mean	Variance	Skewness	Kurtosis
5	2.0817×10^{-17}	0.0100	3.5128×10^{-16}	3.0000
10	-3.1113×10^{-17}	0.0100	3.0709×10^{-16}	3.0000
15	-5.5311×10^{-17}	0.0100	2.1441×10^{-16}	3.0000
20	-1.3772×10^{-17}	0.0100	-9.8642×10^{-17}	3.0000

From these two tables, I find that the orthogonal polynomials approximation method can produce accurate first four moments for the normal variable with only five grid points. Increasing the number of grid points does not improve the accuracy very much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation. Now, an interesting question is whether this orthogonal polynomials approximation method also can be applied to the nonnormal variables. Answering this question leads to experiment 2.

Experiment 2: Assume that a variable follows a mixture of normal distributions with a mean of 0, a standard deviation of 0.1, a skewness of −0.5, and a kurtosis of 5. I report the first four moments of this variable and change the number of grid points (N) to check whether the accuracy can be improved by increasing the number of grid points. The first four moments and the average distance by simulation are as follows:

N	Mean	Variance	Skewness	Kurtosis	Avg. distance
5	2.9796×10^{-4}	0.0100	−0.4944	4.9972	3.9133×10^{-5}
10	-9.0775×10^{-5}	0.0100	−0.4995	5.0047	2.2194×10^{-5}
15	-4.6554×10^{-5}	0.0100	−0.4996	4.9988	1.5714×10^{-6}
20	-2.4788×10^{-5}	0.0100	−0.5001	5.0001	8.1986×10^{-7}

The first four moments computed using the numerical integration method are as follows:

N	Mean	Variance	Skewness	Kurtosis
5	3.4694×10^{-17}	0.0100	−0.5000	5.0000
10	-4.8843×10^{-17}	0.0100	−0.5000	5.0000
15	-2.6057×10^{-17}	0.0100	−0.5000	5.0000
20	-2.3259×10^{-17}	0.0100	−0.5000	5.0000

From these two tables, I find that the orthogonal polynomials approximation method can produce accurate first four moments for the variable with nonzero skewness and excess kurtosis with only five grid points. Increasing the number of grid points does not improve the accuracy very much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation.

Experiment 3: Assume that there are two correlated variables with correlation 0.15: one (v_1) follows a normal distribution $N(0, 0.1)$, and the other one (v_2) follows a mixture of normal distributions with a mean of 0, a standard deviation of 0.1, a skewness of −0.5, and a kurtosis of 5. I report the correlation and the first four moments of each variable and change the number of grid points (N) to check whether the accuracy can be improved by increasing the number of grid points. For each N, I report the correlation, the first four moments (v_1 in the first row and v_2 in the second row), and the average distance by simulation:

N	Correlation	Mean	Variance	Skewness	Kurtosis	Avg. distance
5	0.1674	-1.3310×10^{-4}	0.0100	-0.0024	2.9907	9.3236×10^{-5}
		-2.7394×10^{-5}	0.0100	-0.4981	4.9951	2.7826×10^{-5}
10	0.1662	5.1730×10^{-5}	0.0100	-0.0036	3.0016	1.5511×10^{-5}
		-1.8385×10^{-5}	0.0100	-0.5015	5.0025	1.3567×10^{-5}
15	0.1571	1.3257×10^{-5}	0.0100	6.4496×10^{-4}	2.9979	4.8294×10^{-6}
		1.6052×10^{-5}	0.0100	-0.5005	5.0019	6.3069×10^{-6}
20	0.1533	5.9415×10^{-6}	0.0100	-3.0275×10^{-4}	3.0015	2.2885×10^{-6}
		-1.7116×10^{-6}	0.0100	-0.4998	4.9994	3.1394×10^{-6}

The correlation and the first four moments (v_1 in the first row and v_2 in the second row) computed using the numerical integration method are as follows:

N	Correlation	Mean	Variance	Skewness	Kurtosis
5	0.1500	2.7322×10^{-17}	0.0100	1.0192×10^{-16}	3.0000
		3.8164×10^{-17}	0.0100	-0.5000	5.0000
10	0.1500	-2.7566×10^{-17}	0.0100	3.2543×10^{-16}	3.0000
		-4.8843×10^{-17}	0.0100	-0.5000	5.0000
15	0.1500	-6.1494×10^{-17}	0.0100	3.7788×10^{-16}	3.0000
		-2.0095×10^{-17}	0.0100	-0.5000	5.0000
20	0.1500	2.4127×10^{-17}	0.0100	1.1044×10^{-16}	3.0000
		1.7961×10^{-17}	0.0100	-0.5000	5.0000

From these two tables, I find that the orthogonal polynomials approximation method can produce an accurate correlation and the first four moments for the correlated processes with only five grid points. Increasing the number of grid points does not improve the accuracy very much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation.

G Tables and Figures

Table G1

Baseline Results (Benchmark 1)

This table presents the preference parameters of the model with skewed permanent shocks but without a rare event (benchmark 1) calibrated to the Survey of Consumer Finances (SCF) (1989-2016), and it compares the data with the model for different age groups. Both stockholders and nonstockholders have Epstein-Zin preferences. I calibrate the discount factor (β) to match the average ratio of financial wealth to labor income during households' working time, the strength of the bequest motive (b) to match the average ratio of financial wealth to labor income during the retirement phase, and the coefficient of relative risk aversion (γ) to match the optimal risky asset shares. The EIS (ψ) is set at 0.5. For nonstockholders, the coefficient of relative risk aversion (γ) is the same as that of stockholders; the discount factor (β) matches the average ratio of financial wealth to labor income during households' working time; and the strength of the bequest motive (b) matches the average ratio of financial wealth to labor income during the retirement phase.

Benchmark 1			Stockholders			Nonstockholders	
			Data	Model		Data	Model
Discount factor (β)	(mean W/Y work phase)	0.98			0.92		
	20 – 34		1.529	1.433		0.232	0.061
	35 – 44		1.911	3.342		0.944	0.792
	45 – 54		2.572	5.739		1.489	1.481
	55 – 64		5.110	7.394		2.314	2.184
Strength of bequest motive (b)	(mean W/Y retirement)	2.0			1.5		
	65 – 74		8.691	9.102		2.857	3.011
	75 – 64		9.248	10.281		3.132	3.783
Coefficient of relative risk aversion (γ)	(mean α)	5.6			5.6		
	20 – 34		0.442	0.543			
	35 – 44		0.501	0.463			
	45 – 54		0.453	0.418			
	55 – 64		0.432	0.381			
	65 – 74		0.510	0.496			
	75 – 100		0.493	0.485			
EIS (ψ)		0.5			0.17		

Table G2

Baseline Results (Benchmark 2)

This table presents the preference parameters of the model with skewed permanent shocks and rare event (benchmark 2) calibrated to the Survey of Consumer Finances (SCF) (1989-2016), and it compares the data with the model for different age groups. Both stockholders and nonstockholders have Epstein-Zin preferences. I calibrate the discount factor (β) to match the average ratio of financial wealth to labor income during households' working time, the strength of the bequest motive (b) to match the average ratio of financial wealth to labor income during the retirement phase, and the coefficient of the relative risk aversion (γ) to match the optimal risky asset shares. The EIS (ψ) is set at 0.5. For nonstockholders, the coefficient of relative risk aversion (γ) is the same as that for stockholders; the discount factor (β) matches the average ratio of financial wealth to labor income during households' working time; and the strength of the bequest motive (b) matches the average ratio of financial wealth to labor income during the retirement phase.

Benchmark 2			Stockholders			Nonstockholders	
			Data	Model		Data	Model
Discount factor (β)	(mean W/Y work phase)	0.98			0.90		
	20 – 34		1.529	1.384		0.232	0.058
	35 – 44		1.911	3.192		0.944	0.673
	45 – 54		2.572	4.831		1.489	1.328
	55 – 64		5.110	6.743		2.314	2.090
Strength of bequest motive (b)	(mean W/Y retirement)	2.0			1.5		
	65 – 74		8.691	8.826		2.857	2.993
	75 – 64		9.248	9.502		3.132	3.618
Coefficient of relative risk aversion (γ)	(mean α)	5.2			5.2		
	20 – 34		0.442	0.462			
	35 – 44		0.501	0.510			
	45 – 54		0.453	0.463			
	55 – 64		0.432	0.425			
	65 – 74		0.510	0.464			
	75 – 100		0.493	0.407			
EIS (ψ)		0.5			0.17		

Table G3

Average Consumption-Wealth Ratio (C/X)

This table shows the average consumption-wealth ratio for different values of risk aversion (γ) and the EIS (ψ) for different age groups. Panel A shows the average consumption-wealth ratio implied by the lognormal earnings model; Panel B shows the average consumption-wealth ratio implied by benchmark 1, and Panel C shows the average consumption-wealth ratio implied by benchmark 2.

	$20-35$			$36-65$			$66-100$		
γ	$\psi=0.8$	$\psi=0.5$	$\psi=0.2$	$\psi=0.8$	$\psi=0.5$	$\psi=0.2$	$\psi=0.8$	$\psi=0.5$	$\psi=0.2$
				Panel A. Lognormal Earnings Model					
1	98%	99%	99%	98%	99%	99%	100%	100%	100%
1.2	89%	93%	94%	47%	89%	95%	95%	100%	100%
2	76%	86%	90%	18%	35%	67%	25%	71%	97%
4	67%	69%	75%	14%	18%	27%	24%	32%	59%
5	58%	62%	66%	13%	17%	22%	23%	29%	51%
				Panel B. Benchmark 1					
1	91%	94%	98%	96%	96%	98%	99%	100%	100%
1.2	84%	89%	92%	46%	77%	85%	95%	100%	100%
2	72%	82%	86%	14%	28%	59%	24%	71%	97%
4	61%	63%	72%	10%	15%	24%	24%	30%	59%
5	50%	57%	62%	8%	11%	10%	23%	29%	50%
				Panel C. Benchmark 2					
1	89%	90%	91%	91%	92%	94%	84%	90%	92%
1.2	75%	84%	87%	43%	74%	84%	77%	83%	91%
2	71%	81%	84%	12%	23%	49%	13%	49%	84%
4	59%	62%	66%	7%	14%	17%	12%	23%	53%
5	46%	54%	62%	6%	7%	9%	9%	19%	41%

Table G4

Regressions on Skewness of Labor Income Shocks (PSID)

This table presents the results for the 1999-2017 sample with financial assets. It shows how risky asset shares (α) responds to the skewness of earnings shocks (l_{skew}) conditional on variance of earnings shocks (l_{var}), inertia, and a vector of the variables that may cause common movements. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Explanatory variable	Stockholders	Nonstockholders
Δl_{skew}	-0.009^{***}	
	(0.001)	
Δl_{var}	-0.097^{**}	
	(0.051)	

Table G5
Regressions on Changes in Skewness of Labor Income Shocks by Age Groups (PSID)

This table presents the results for the 1999-2017 sample with financial assets for different age groups. Panel A shows how changes in risky assets shares ($\Delta\alpha$) respond to changes in the skewness of earnings shocks (Δl_{skew}) conditional on changes in the variance of earnings shocks (Δl_{var}) and a vector of the variables that may cause common movements. Panel B shows how changes in the skewness of consumption growth (Δc_{skew}) respond to changes in the skewness of earnings shocks (Δl_{skew}) conditional on changes in the variance of earnings shocks (Δl_{var}), and a vector of the variables that may cause common movements. Panel C reports the results with dependent variable, changes in consumption growth (Δc) conditional on changes in the variance of earnings shocks (Δl_{var}) and a vector of the variables that may cause common movements. Financial assets are defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Total wealth is defined as the sum of financial wealth, home equity, and equity in private business. Regressions control for preference shifters and life-cycle controls (not reported). Preference shifters include changes in household characteristics. Life-cycle controls are related to the life cycle, background, and financial situation of the household. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Explanatory variable	Stockholders				Nonstockholders			
Age	20 − 29	30 − 39	40 − 49	50 − 65	20 − 29	30 − 39	40 − 49	50 − 65
Panel A. Dependent variable: Changes in risky assets shares ($\Delta\alpha$)								
Δl_{skew}	−0.031***	−0.024***	−0.022***	−0.009				
	(0.007)	(0.006)	(0.006)	(0.007)				
Δl_{var}	−0.172***	−0.152**	−0.083**	−0.071				
	(0.053)	(0.058)	(0.042)	(0.064)				
Panel B. Dependent variable: Changes in skewness of consumption growth (Δc_{skew})								
Δl_{skew}	0.087***	0.094***	0.078***	0.067***	0.013	0.011	0.011	0.007
	(0.004)	(0.004)	(0.004)	(0.003)	(0.013)	(0.011)	(0.013)	(0.015)
Δl_{var}	0.171	0.171	0.169	0.134	0.102	0.098	0.098	0.095
	(0.373)	(0.339)	(0.343)	(0.346)	(0.339)	(0.344)	(0.371)	(0.370)
Panel C. Dependent variable: Consumption growth(Δc)								
Δl_{skew}	−0.042***	−0.029***	−0.031***	−0.020***	−0.014***	−0.014***	−0.010*	−0.009
	(0.006)	(0.006)	(0.004)	(0.004)	(0.003)	(0.004)	(0.006)	(0.006)
Δl_{var}	−0.168	−0.161	−0.159	−0.142	−0.014	−0.016	−0.013	−0.011
	(0.167)	(0.149)	(0.183)	(0.180)	(0.033)	(0.025)	(0.028)	(0.041)

Table G6
Regressions on Changes in Skewness of Labor Income Shocks with Borrowing Constraints (PSID)

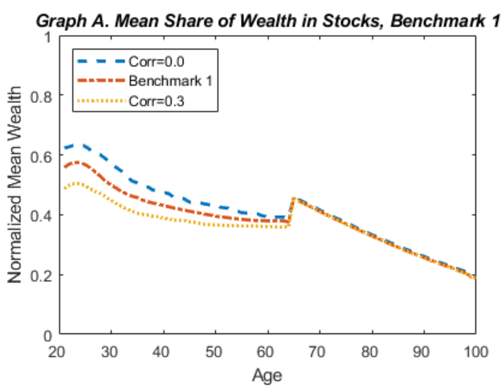
This table presents the results with borrowing constraints for the 1999-2017 sample with financial assets. Panel A shows how changes in risky asset shares ($\Delta\alpha$) respond to changes in the skewness of earnings shocks (Δl_{skew}) conditional on changes in the variance of earnings shocks (Δl_{var}) and a vector of the variables that may cause common movements. Panel B shows how changes in the skewness of consumption growth (Δc_{skew}) respond to changes in the skewness of earnings shocks (Δl_{skew}) conditional on changes in the variance of earnings shocks (Δl_{var}) and a vector of the variables that may cause common movements. Panel C reports the results with dependent variable, changes in consumption growth (Δc) conditional on changes in the variance of earnings shocks (Δl_{var}), and a vector of the variables that may cause common movements. Financial assets are defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Regressions control for preference shifters and life-cycle controls (not reported). Preference shifters include changes in household characteristics. Life-cycle controls are related to the life cycle, background, and financial situation of the household. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Explanatory variable	Stockholders	Nonstockholders
	Panel A. Dependent variable: Changes in risky assets shares ($\Delta\alpha$)	
Δl_{skew}	-0.008^{**}	
	(0.004)	
Δl_{var}	-0.069^{**}	
	(0.035)	
Borrowing Constrained Dummy	-0.023^{**}	
	0.011	
	Panel B. Dependent variable: Changes in skewness of consumption growth (Δc_{skew})	
Δl_{skew}	0.063^{***}	0.008
	(0.004)	(0.009)
Δl_{var}	0.143	0.081
	(0.197)	(0.311)
Borrowing Constrained Dummy	0.051	0.036^{*}
	(0.034)	(0.021)
	Panel C. Dependent variable: Consumption growth (Δc)	
Δl_{skew}	-0.021^{***}	-0.007^{**}
	(0.005)	(0.006)
Δl_{var}	-0.173	-0.035
	(0.150)	(0.049)
Borrowing Constrained Dummy	-0.058^{*}	-0.048^{**}
	(0.034)	(0.022)

Figure G1

Mean Share of Wealth in Stocks

This figure compares the mean share of wealth in stocks across different correlations between permanent earnings shocks and stock returns innovation during recessions. Graph A plots the model with skewed permanent shocks (benchmark 1), and Graph B plots the model with skewed permanent shocks and rare events in the stock market (benchmark 2).



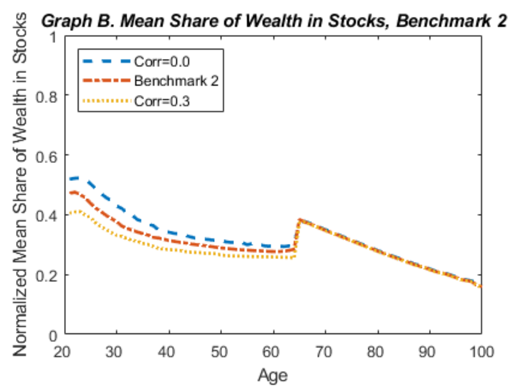


Figure G2

Policy Functions during Recessions

This figure compares the policy functions across different models during recessions. Graphs A, B, and C plot policy functions for the share of wealth in stocks during recessions and compare different models. Graphs D, E, and F plot policy functions for consumption during recessions and compare different models.

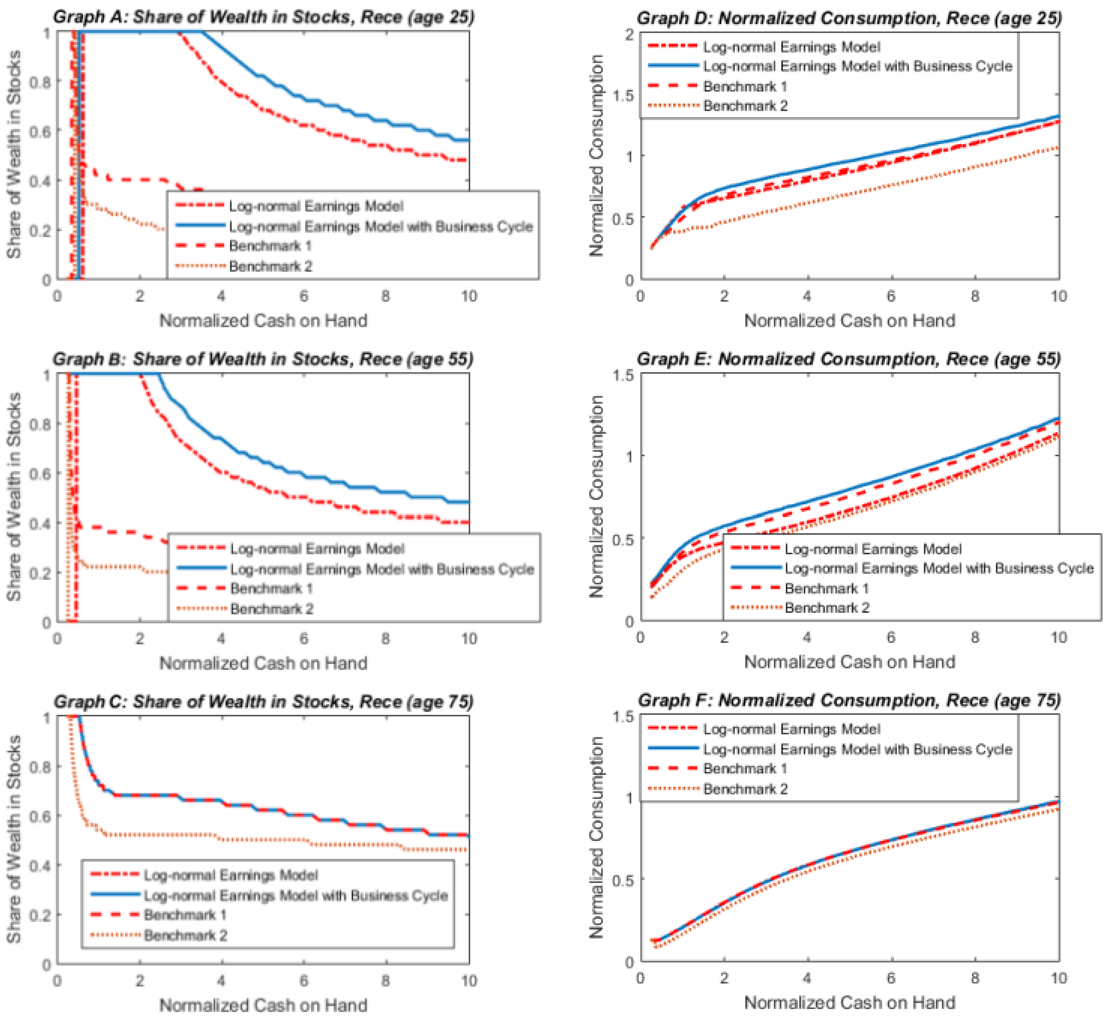


Figure G3

Business-Cycle Variation in Policy Functions (Benchmark 2)

This figure presents policy functions during booms and recessions for benchmark 2. Graphs A, B, and C show the policy functions for the share of wealth in stocks and compare the current state being a boom and the current state being a recession. Graphs D, E, and F show the policy functions for consumption and compare the current state being a boom and the current state being a recessions.

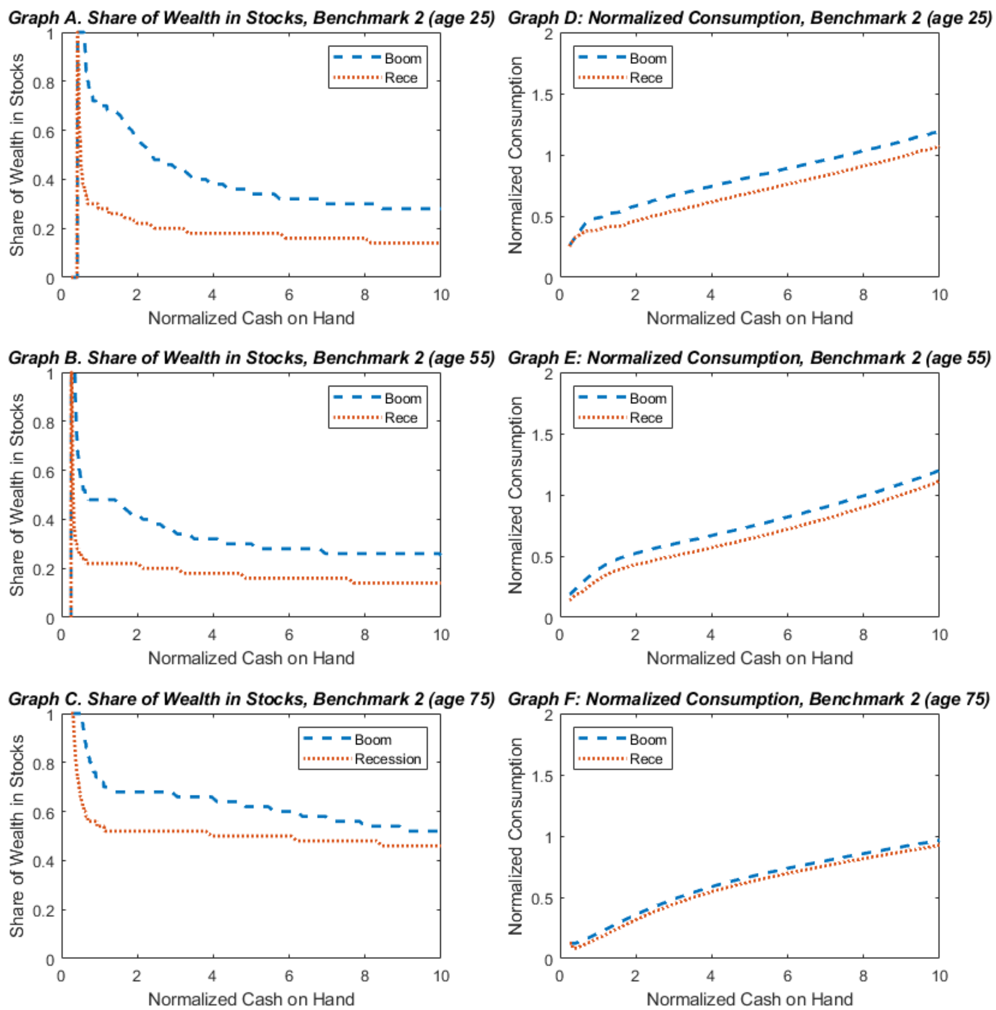


Figure G4
Conditional Risky Asset Shares

This figure presents the policy functions of the extended model. To make the comparison clear, I also plot the model with skewed permanent shocks and rare events in the stock market (benchmark 2).

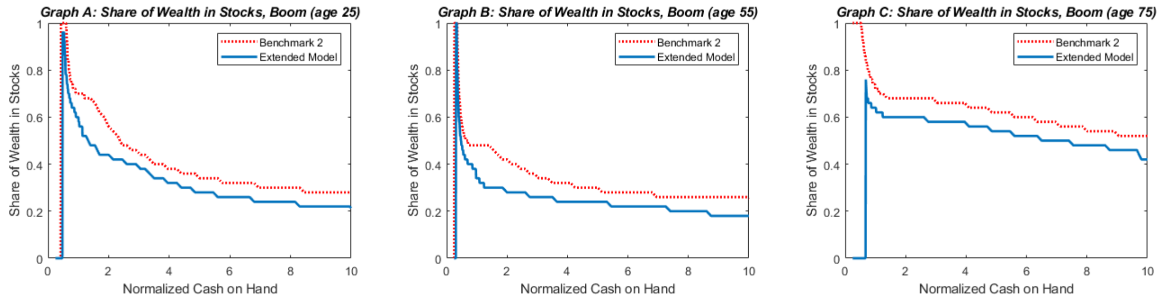


Figure G5
Conditional Risky Asset Shares

This figure presents the policy functions of the extended model. To make the comparison clear, I also plot the model with skewed permanent shocks and rare events in the stock market (benchmark 2).

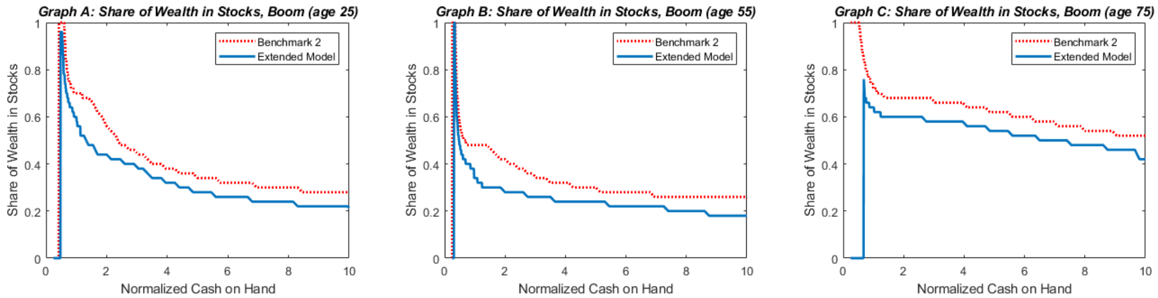


Figure G6
Wealth Threshold of Participation

This figure presents the wealth threshold of participation as a function of age for the extended model.

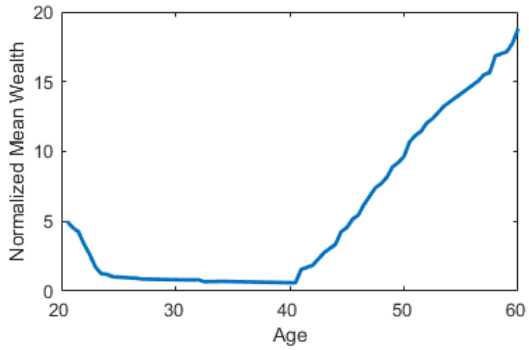


Figure G7
Life-Cycle Profile of the Extended Model

This figure presents the life-cycle profile of the extended model. To make the comparison clear, I also plot the model with skewed permanent shocks and rare events in the stock market (benchmark 2).