Online Appendix for “A Direct Utility Model for Access Costs and Economies of Scope”

Dong Soo Kim, Sanghak Lee, Taegyu Hur, Jaehwan Kim, and Greg M. Allenby

1 Details of the marginal time cost ξ_j

ξ_j is the marginal time cost for product j:

$$\xi_j = \frac{\partial Q(\boldsymbol{x})}{\partial x_j} = \gamma_j + \delta_j \rho_j^{\theta_{n_j}-1}, \tag{A1}$$

where

$$\rho_j = \begin{cases} 1, & \text{if } x_{j'} = 0 \text{ for all } j' \text{ such that } m_{j'n_j} = 1, \\ \dfrac{\delta_j x_j}{\left\{\sum_{j'=1}^{J} m_{j'n_j} \left(\delta_{j'} x_{j'}\right)^{\theta_{n_j}}\right\}^{\frac{1}{\theta_{n_j}}}}, & \text{otherwise,} \end{cases} \tag{A2}$$

and n_j indicates the group in which product j is assigned, i.e., $n_j = k$ such that $m_{jk} = 1$.

2 Proof for the global concavity

This appendix provides the proof for the global concavity of the utility maximization in the proposed model. The proposed model is equivalent to the following utility maximization:

$$\begin{aligned} \max_{\boldsymbol{x}} \quad & U^*(\boldsymbol{x}) = \sum_{j=1}^{J} \psi_j \log(x_j+1) + \log(E-P) + \log(C-Q), \\ & P = \sum_{j=1}^{J} p_j x_j, \quad Q = \sum_{j=1}^{J} \gamma_j x_j + \sum_{k=1}^{K} \left\{ \sum_{j=1}^{J} m_{jk} (\delta_j x_j)^{\theta_k} \right\}^{\frac{1}{\theta_k}}, \\ \text{s.t.} \quad & E-P>0, \quad C-Q>0. \end{aligned} \tag{A3}$$

For brevity, time (t) and individual (h) subscripts are suppressed. The utility function, U^*, is globally concave over the feasible set if $\theta_k \geq 1$ for all $k = 1, \ldots, K$. Proof is as follows: The (i,j)-th element of the Hessian of this utility function is

$$\mathbf{H}_{ij} = \frac{\partial^2 U^*}{\partial x_j \partial x_i} = -I_{i=j} \times \frac{\psi_i}{(x_i+1)^2} - \frac{p_i p_j}{(E-P)^2} - \frac{\xi_i \xi_j}{(C-Q)^2} - \frac{\omega_{ij}}{C-Q}, \tag{A4}$$

where $\xi_i = \partial Q / \partial x_i$, $\omega_{ij} = \partial \xi_i / \partial x_j$, and I is an indicator function. If the Hessian is negative-semidefinite over the feasible set, the utility function is globally concave, i.e., if $\boldsymbol{\alpha}'\mathbf{H}\boldsymbol{\alpha} \leq 0$ for any arbitrary vector $\boldsymbol{\alpha}$ over the feasible set, the utility function is globally concave. From Equation (A4),

$$\begin{aligned} \boldsymbol{\alpha}'\mathbf{H}\boldsymbol{\alpha} = & -\sum_{i=1}^{J} \frac{\alpha_i^2 \psi_i}{(x_i+1)^2} - \frac{1}{(E-P)^2} \left(\sum_{i=1}^{J} \alpha_i p_i \right)^2 - \frac{1}{(C-Q)^2} \left(\sum_{i=1}^{J} \alpha_i \xi_i \right)^2 \\ & - \frac{1}{C-Q} \left\{ \sum_{j=1}^{J} \left(\alpha_j \sum_{i=1}^{J} \alpha_i \omega_{ij} \right) \right\}. \end{aligned} \tag{A5}$$

Since $\psi_i > 0$, the first three terms in Equation (A5) are always nonpositive. Thus, if the fourth term is always nonpositive, the Hessian is negative-semidefinite and the utility function is globally concave over the feasible set. ω_{ij} is given by

$$\omega_{ij} = (\theta_{n_i} - 1) \left(I_{i=j} \cdot \delta_i^{\theta_{n_i}} x_i^{\theta_{n_i}-2} \cdot v_{n_i}^{\frac{1}{\theta_{n_i}}-1} - I_{n_i=n_j} \cdot \delta_i^{\theta_{n_i}} x_i^{\theta_{n_i}-1} \cdot \delta_j^{\theta_{n_i}} x_j^{\theta_{n_i}-1} \cdot v_{n_i}^{\frac{1}{\theta_{n_i}}-2} \right), \tag{A6}$$

where $v_{n_i} = \sum_{j'=1}^{J} m_{j'n_i} (\delta_{j'} x_{j't})^{\theta_{n_i}}$. Since ω_{ij} has a non-zero value only if i and j are assigned into the same group, the fourth term can be written as a summation with respect to k:

$$
\begin{aligned}
& -\frac{1}{C-Q}\left\{\sum_{j=1}^{J}\left(\alpha_j \sum_{i=1}^{J} \alpha_i \omega_{ij}\right)\right\} \qquad \text{(A7)} \\
& = -\frac{1}{C-Q}\left[\sum_{k=1}^{K}\left(\theta_k-1\right) v_k^{\frac{1}{\theta_k}-2}\left\{v_k \sum_{i=1}^{J} m_{ik} \alpha_i^2 \delta_i^{\theta_k} x_i^{\theta_k-2}-\left(\sum_{i=1}^{J} m_{ik} \alpha_i \delta_i^{\theta_k} x_i^{\theta_k-1}\right)^2\right\}\right].
\end{aligned}
$$

The brace term is always nonnegative since

$$
\begin{aligned}
& v_k \sum_{i=1}^{J} m_{ik} \alpha_i^2 \delta_i^{\theta_k} x_i^{\theta_k-2}-\left(\sum_{i=1}^{J} m_{ik} \alpha_i \delta_i^{\theta_k} x_i^{\theta_k-1}\right)^2 \qquad \text{(A8)} \\
& =\left(\sum_{i=1}^{J} m_{ik} \delta_i^{\theta_k} x_i^{\theta_k}\right)\left(\sum_{i=1}^{J} m_{ik} \alpha_i^2 \delta_i^{\theta_k} x_i^{\theta_k-2}\right)-\left(\sum_{i=1}^{J} m_{ik} \alpha_i \delta_i^{\theta_k} x_i^{\theta_k-1}\right)^2 \\
& =\sum_{i=1}^{J} m_{ik} \alpha_i^2 \delta_i^{2\theta_k} x_i^{2\theta_k-2}+\sum_{i=1}^{J-1} \sum_{j=i+1}^{J} m_{ik} m_{jk} \alpha_j^2 \delta_i^{\theta_k} x_i^{\theta_k} \delta_j^{\theta_k} x_j^{\theta_k-2}+\sum_{i=1}^{J-1} \sum_{j=i+1}^{J} m_{ik} m_{jk} \alpha_i^2 \delta_j^{\theta_k} x_j^{\theta_k} \delta_i^{\theta_k} x_i^{\theta_k-2} \\
& \quad -\sum_{i=1}^{J} m_{ik} \alpha_i^2 \delta_i^{2\theta_k} x_i^{2\theta_k-2}-2 \sum_{i=1}^{J-1} \sum_{j=i+1}^{J} m_{ik} m_{jk} \alpha_i \alpha_j \delta_i^{\theta_k} x_i^{\theta_k-1} \delta_j^{\theta_k} x_j^{\theta_k-1} \\
& =\sum_{i=1}^{J-1} \sum_{j=i+1}^{J} m_{ik} m_{jk} \delta_i^{\theta_k} \delta_j^{\theta_k} x_i^{\theta_k-2} x_j^{\theta_k-2}\left(\alpha_j^2 x_i^2-2 \alpha_i \alpha_j x_i x_j+\alpha_i^2 x_j^2\right) \\
& =\sum_{i=1}^{J-1} \sum_{j=i+1}^{J} m_{ik} m_{jk} \delta_i^{\theta_k} \delta_j^{\theta_k} x_i^{\theta_k-2} x_j^{\theta_k-2}\left(\alpha_j x_i-\alpha_i x_j\right)^2 \geq 0.
\end{aligned}
$$

Therefore, if $\theta_k \geq 1$ for all $k=1, \ldots, K$, Equation (A5) is nonpositive, the Hessian is negative-semidefinite, and the utility function is globally concave over the feasible set.

3 Estimation procedure

This appendix shows estimation procedure for the proposed model with indivisible demand. This estimation procedure is based on a Bayesian error augmentation method proposed by **?**. The full model specification with indivisibility can be written as follows:

$$\max_{\boldsymbol{x}_{ht}} \quad U_h^*\left(\boldsymbol{x}_{ht}\right) = \sum_{j=1}^{J} \psi_{hjt} \log\left(x_{hjt}+1\right) + \log\left(E_h - \sum_{j=1}^{J} p_{hjt} x_{hjt}\right) + \log\left(C_h - Q_h\left(\boldsymbol{x}_{ht}\right)\right),$$
$$\log\left(\psi_{hjt}\right) = \boldsymbol{a}'_{hjt}\boldsymbol{\beta}_h + \epsilon_{hjt}, \quad \epsilon_{hjt} \sim EV\left(0,\sigma_h\right), \tag{A9}$$
$$Q_h\left(\boldsymbol{x}_{ht}\right) = \sum_{j=1}^{J} \gamma_j x_{hjt} + \sum_{k=1}^{K}\left\{\sum_{j=1}^{J} m_{hjk}\left(\delta_j x_{hjt}\right)^{\theta_{hk}}\right\}^{\frac{1}{\theta_{hk}}},$$
$$\text{s.t.} \quad x_{hjt} \in \{0,1,2,\ldots\}, \quad E_h - \sum_{j=1}^{J} p_{hjt} x_{hjt} > 0, \quad C_h - Q_h\left(\boldsymbol{x}_{ht}\right) > 0.$$

An individual's grouping of products (m_{hjk}) and heterogeneity of individual parameters are modeled as in Section 3.4 in the main text of the paper:

$$n_{hj} \sim \text{multinomial}_K\left(\phi_{j1},\ldots,\phi_{jK}\right), \quad \text{for } j=1,\ldots,J, \tag{A10}$$

where n_{hj} indicates the group in which product j is assigned, i.e., $n_{hj}=k$ such that $m_{hjk}=1$, and

$$\boldsymbol{\eta}_h = \left[\ \boldsymbol{\beta}'_h \ \ \log\sigma_h \ \ \log E_h \ \ \log C_h \ \ \log\left(\theta_{h1}-1\right) \ \cdots \ \log\left(\theta_{hK}-1\right)\ \right]' \sim N\left(\bar{\boldsymbol{\eta}}, \mathbf{V}\right). \tag{A11}$$

We henceforth use a tilde ($\tilde{}$) mark for indicating the log- or logarithmically transformed variables, i.e., $\tilde{\sigma}_h \equiv \log\sigma_h$, $\tilde{E}_h \equiv \log E_h$, $\tilde{C}_h \equiv \log C_h$, and $\tilde{\theta}_{hk} \equiv \log\left(\theta_{hk}-1\right)$, and define $\tilde{\boldsymbol{\theta}}_h$ as a vector of $\tilde{\theta}_{hk}$, i.e., $\tilde{\boldsymbol{\theta}}'_h \equiv \left[\tilde{\theta}_{h1} \ \cdots \ \tilde{\theta}_{hK}\right]$, for brevity.

Since U_h^* is globally concave over the feasible set (see Online Appendix 2 for the proof), we can extend **?**'s (**?**) method for a one-constraint model to a two-constraint model in Equation (A9). The optimality condition with indivisibility is

$$U_h^*\left(\boldsymbol{x}_{ht}\right) \geq \max\left\{U_h^*\left(x_{h1t}+\Delta_1,\ldots,x_{hJt}+\Delta_J\right) \middle| \left(x_{h1t}+\Delta_1,\ldots,x_{hJt}+\Delta_J\right) \in F_{ht}\right\},$$
$$\Delta_j \in \{-1,0,1\} \ \text{ for } j=1,\ldots,J, \tag{A12}$$

where F_{ht} is the feasible set of the utility maximization for individual h at observation t. This condition implies that a point where the utility value is greater than (or equal to) the value at all the adjacent points is the optimum.

The likelihood for individual h at observation t with indivisibility, ℓ_{ht}^{Int}, is

$$\ell_{ht}^{\text{Int}} = \int_{\epsilon_{hJt}} \cdots \int_{\epsilon_{h1t}} I\left(U_h^*\left(\boldsymbol{x}_{ht} \mid \epsilon_{h1t}, \ldots, \epsilon_{hJt}\right) \geq \bar{U}_h^*\left(\boldsymbol{x}_{ht} \mid \epsilon_{h1t}, \ldots, \epsilon_{hJt}\right)\right) dp\left(\epsilon_{h1t}\right) \cdots dp\left(\epsilon_{hJt}\right), \tag{A13}$$

where $\bar{U}_h^*\left(\boldsymbol{x}_{ht}\right) \equiv \max\left\{U_h^*\left(x_{h1t}+\Delta_1, \ldots, x_{hJt}+\Delta_J\right) \mid \left(x_{h1t}+\Delta_1, \ldots, x_{hJt}+\Delta_J\right) \in F_{ht}\right\}$ in Equation (A12), i.e., the maximum attainable utility at the adjacent points to $\boldsymbol{x}_{ht}$, and $I(\cdot)$ is an indicator function. The region of integration in Equation (A13) is irregular because utility functions for outside goods are assumed to be log-linear. We employ a Bayesian error augmentation according to **?**'s (**?**) method. Rather than directly evaluate the likelihood in Equation (A13), we first draw the errors, $\epsilon_{h1t}, \ldots, \epsilon_{hJt}$, given $\boldsymbol{\eta}_h$ and $\{n_{hj}\}$. We then draw the model parameters given the errors. The likelihood function becomes an indicator function because the errors are given. Details of this estimation procedure are below.

We estimate (i) the individual parameters, $\boldsymbol{\eta}_h$, (ii) the augmented assignment variables, $\{n_{hj}\}$, (iii) the augmented baseline utility, $\psi_{hjt}^* \equiv \log\left(\psi_{hjt}\right)$, (iv) the hyperparameters for heterogeneity on the individual parameters, $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$, and (v) the assignment probabilities, $\{\phi_{jk}\}$. The estimation procedure is as follows:

(Step 1) Set initial values for the individual parameters, $\boldsymbol{\eta}_h$, and the assignment variables, $\{n_{hj}\}_{j=1,\ldots,J}$ for each individual.

(Step 2) Set initial values for $\left\{\psi_{hjt}^*\right\}_{\substack{j=1,\ldots,J\\t=1,\ldots,T}}$ for each individual by solving the Kuhn-Tucker conditions in Equation (20) in the main text with given $\boldsymbol{\eta}_h$ and $\{n_{hj}\}_{j=1,\ldots,J}$. For a zero demand, the first order condition with respect to ψ_{hjt}^* is an inequality and we set any value satisfying the inequality for ψ_{hjt}^*.

(Step 3) For each product, draw ψ_{hjt}^* given $\{\psi_{hit}^*\}_{i\neq j}$, $\boldsymbol{\eta}_h$, and $\{n_{hj}\}_{j=1,\ldots,J}$ through the following posterior truncated extreme value distribution:[1]

$$\psi_{hjt}^* \mid \{\psi_{hit}^*\}_{i\neq j}, \boldsymbol{\eta}_h, \{n_{hj}\}_{j=1,\ldots,J}; \boldsymbol{x}_{ht} \sim EV\left(\boldsymbol{a}_{hjt}'\boldsymbol{\beta}_h, \sigma_h\right) \times I\left(lb_{hjt} \leq \psi_{hjt}^* \leq ub_{hjt}\right), \tag{A14}$$

where lb_{hjt} and ub_{hjt} are the lower and upper truncation points for ψ_{hjt}^* calculated from the optimality condition in Equation (A12). Substituting the utility

[1]This step is identical to draw the errors, $\epsilon_{h1t}, \ldots, \epsilon_{hJt}$. We directly draw the baseline utility values instead of the errors for computational efficiency.

function in Equation (A9) into the optimality condition and rearranging terms, we have

$$\psi_{hjt}\log\left(\frac{x_{hjt}+1}{x_{hjt}+\Delta_j+1}\right)\geq\psi^{**}_{hjt}\left(\Delta_j,\{\Delta_i\}_{i\neq j}\right)\equiv\sum_{i\neq j}\psi_{hit}\log\left(\frac{x_{hit}+\Delta_i+1}{x_{hit}+1}\right)+\log\left(\frac{E_h-\sum_{i=1}^{J}p_{hit}\left(x_{hit}+\Delta_i\right)}{E_h-\sum_{i=1}^{J}p_{hit}x_{hit}}\right)+\log\left(\frac{C_h-Q_h\left(\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\right)}{C_h-Q_h\left(\boldsymbol{x}_{ht}\right)}\right),\tag{A15}$$

for any $\Delta_i\in\{-1,0,1\}$ satisfying $\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\in F_{ht}$ for all $i\neq j$, where $\boldsymbol{\Delta}=[\Delta_1\ \cdots\ \Delta_J]'$. If $\Delta_j=1$, we have

$$\psi_{hjt}\leq\psi^{**}_{hjt}\left(1,\{\Delta_i\}_{i\neq j}\right)\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}+2}\right),\tag{A16}$$

and if $\Delta_j=-1$, we have

$$\psi_{hjt}\geq\psi^{**}_{hjt}\left(-1,\{\Delta_i\}_{i\neq j}\right)\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}}\right).\tag{A17}$$

From Equations (A16) and (A17), we finally have

$$\begin{aligned}ub_{hjt}&=\log\left(\min\left[\psi^{**}_{hjt}\left(1,\{\Delta_i\}_{i\neq j}\right)\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}+2}\right)\right]_{\{\Delta_i\}_{i\neq j},\,\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\in F_{ht}\,\mid\,\Delta_j=1}\right),\\ lb_{hjt}&=\log\left(\max\left[\psi^{**}_{hjt}\left(-1,\{\Delta_i\}_{i\neq j}\right)\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}}\right)\right]_{\{\Delta_i\}_{i\neq j},\,\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\in F_{ht}\,\mid\,\Delta_j=-1}\right).\end{aligned}\tag{A18}$$

If any $\boldsymbol{\Delta}$ with $\Delta_j=1$ does not satisfy $\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\in F_{ht}$, $ub_{hjt}=\infty$. If any $\boldsymbol{\Delta}$ with $\Delta_j=-1$ does not satisfy $\boldsymbol{x}_{ht}+\boldsymbol{\Delta}\in F_{ht}$, $lb_{hjt}=-\infty$.

(Step 4) Draw $\boldsymbol{\beta}_h$ and $\tilde{\sigma}_h$ given $\left\{\psi^{*}_{hjt}\right\}_{\substack{j=1,\dots,J\\t=1,\dots,T}}$, $\tilde{E}_h$, $\tilde{C}_h$, $\tilde{\boldsymbol{\theta}}_h$, $\bar{\boldsymbol{\eta}}$, and $\mathbf{V}$ using a Random-Walk Metropolis-Hastings algorithm for the following linear equation:

$$\psi^{*}_{hjt}=\boldsymbol{a}'_{hjt}\boldsymbol{\beta}_h+\epsilon_{hjt},\quad\epsilon_{hjt}\sim EV\left(0,\sigma_h\right).\tag{A19}$$

(Step 4.1) Draw $\boldsymbol{\beta}_h^{\mathrm{NEW}}=\boldsymbol{\beta}_h^{\mathrm{OLD}}+\boldsymbol{v}$, $\boldsymbol{v}\sim N\left(\mathbf{0},d^2\mathbf{I}\right)$. d is a scaling factor set a priori.

(Step 4.2) With probability $\min\left[1, \frac{\ell_\psi\left(\boldsymbol{\beta}_h^{\text{NEW}}, \tilde{\sigma}_h\right)\pi_{\boldsymbol{\beta}|\text{else}}\left(\boldsymbol{\beta}_h^{\text{NEW}} \mid \text{else}\right)}{\ell_\psi\left(\boldsymbol{\beta}_h^{\text{OLD}}, \tilde{\sigma}_h\right)\pi_{\boldsymbol{\beta}|\text{else}}\left(\boldsymbol{\beta}_h^{\text{OLD}} \mid \text{else}\right)}\right]$, take $\boldsymbol{\beta}_h^{\text{NEW}}$, else take $\boldsymbol{\beta}_h^{\text{OLD}}$, as the new draw of $\boldsymbol{\beta}_h$. $\ell_\psi(\cdot)$, the likelihood function, is derived from Equation (A19) as follows:

$$\ell_\psi = \prod_{t=1}^{T}\prod_{j=1}^{J}\frac{1}{\sigma_h}\exp\left(-\epsilon_{hjt}^{*} - e^{-\epsilon_{hjt}^{*}}\right), \tag{A20}$$

where $\epsilon_{hjt}^{*} = \frac{\psi_{hjt}^{*} - \boldsymbol{a}_{hjt}'\boldsymbol{\beta}_h}{\sigma_h}$. $\pi_{\boldsymbol{\beta}|\text{else}}(\cdot)$ indicating the prior distribution of $\boldsymbol{\beta}_h$ is the pdf of a conditional normal distribution of $\boldsymbol{\beta}_h$ given $\tilde{\sigma}_h$, $\tilde{E}_h$, $\tilde{C}_h$, and $\tilde{\boldsymbol{\theta}}_h$ with mean $\bar{\boldsymbol{\eta}}$ and variance $\mathbf{V}$.

(Step 4.3) Draw $\tilde{\sigma}_h^{\text{NEW}} = \tilde{\sigma}_h^{\text{OLD}} + v$, $v \sim N(0, d^2)$. d is a scaling factor set a priori.

(Step 4.4) With probability $\min\left[1, \frac{\ell_\psi\left(\boldsymbol{\beta}_h, \tilde{\sigma}_h^{\text{NEW}}\right)\pi_{\tilde{\sigma}|\text{else}}\left(\tilde{\sigma}_h^{\text{NEW}} \mid \text{else}\right)}{\ell_\psi\left(\boldsymbol{\beta}_h, \tilde{\sigma}_h^{\text{OLD}}\right)\pi_{\tilde{\sigma}|\text{else}}\left(\tilde{\sigma}_h^{\text{OLD}} \mid \text{else}\right)}\right]$, take $\tilde{\sigma}_h^{\text{NEW}}$, else take $\tilde{\sigma}_h^{\text{OLD}}$, as the new draw of $\tilde{\sigma}_h$. $\ell_\psi(\cdot)$ is the same as Equation (A20). $\pi_{\tilde{\sigma}|\text{else}}(\cdot)$ indicating the prior distribution of $\tilde{\sigma}_h$ is the pdf of a conditional normal distribution of $\tilde{\sigma}_h$ given $\boldsymbol{\beta}_h$, $\tilde{E}_h$, $\tilde{C}_h$, and $\tilde{\boldsymbol{\theta}}_h$ with mean $\bar{\boldsymbol{\eta}}$ and variance $\mathbf{V}$.

(Step 5) Draw $\tilde{E}_h$, $\tilde{C}_h$, and $\tilde{\boldsymbol{\theta}}_h$ given $\left\{\psi_{hjt}^{*}\right\}_{\substack{j=1,\dots,J,\\ t=1,\dots,T}}$, $\{n_{hj}\}_{j=1,\dots,J}$, $\boldsymbol{\beta}_h$, $\tilde{\sigma}_h$, $\bar{\boldsymbol{\eta}}$, and $\mathbf{V}$ using a Random-Walk Metropolis-Hastings algorithm:

(Step 5.1) Draw $\tilde{E}_h^{\text{NEW}} = \tilde{E}_h^{\text{OLD}} + v$, $v \sim N(0, d^2)$. d is a scaling factor set a priori.

(Step 5.2) With probability $\min\left[1, \frac{\ell_h\left(\tilde{E}_h^{\text{NEW}}, \text{else}\right)\pi_{\tilde{E}|\text{else}}\left(\tilde{E}_h^{\text{NEW}} \mid \text{else}\right)}{\ell_h\left(\tilde{E}_h^{\text{OLD}}, \text{else}\right)\pi_{\tilde{E}|\text{else}}\left(\tilde{E}_h^{\text{OLD}} \mid \text{else}\right)}\right]$, take $\tilde{E}_h^{\text{NEW}}$, else take $\tilde{E}_h^{\text{OLD}}$, as the new draw of $\tilde{E}_h$. $\ell_h(\cdot)$, the likelihood function, is a product of an indicator function:

$$\ell_h = \prod_{t=1}^{T}\ell_{ht}\left(\boldsymbol{x}_{ht} \mid \left\{\psi_{hjt}^{*}\right\}, \{n_{hj}\}, \tilde{E}_h, \tilde{C}_h, \tilde{\boldsymbol{\theta}}_h\right), \tag{A21}$$

where $\ell_{ht}(\cdot)$ is an indicator function that equals 1, if the optimality condition in Equation (A12) is satisfied given $\left\{\psi_{hjt}^{*}\right\}$, $\{n_{hj}\}$, $\tilde{E}_h$, $\tilde{C}_h$, and $\tilde{\boldsymbol{\theta}}_h$, and otherwise, 0. $\pi_{\tilde{E}|\text{else}}(\cdot)$ indicating the prior distribution of $\tilde{E}_h$ is the pdf of a conditional normal distribution of $\tilde{E}_h$ given $\boldsymbol{\beta}_h$, $\tilde{\sigma}_h$, $\tilde{C}_h$, and $\tilde{\boldsymbol{\theta}}_h$ with mean $\bar{\boldsymbol{\eta}}$ and variance $\mathbf{V}$.

(Step 5.3) Draw $\tilde{C}_h^{\text{NEW}} = \tilde{C}_h^{\text{OLD}} + v$, $v \sim N(0, d^2)$. d is a scaling factor set a priori.

(Step 5.4) With probability $\min\left[1, \dfrac{\ell_h\left(\tilde{C}_h^{\text{NEW}}, \text{else}\right)\pi_{\tilde{C}|\text{else}}\left(\tilde{C}_h^{\text{NEW}} \mid \text{else}\right)}{\ell_h\left(\tilde{C}_h^{\text{OLD}}, \text{else}\right)\pi_{\tilde{C}|\text{else}}\left(\tilde{C}_h^{\text{OLD}} \mid \text{else}\right)}\right]$, take $\tilde{C}_h^{\text{NEW}}$, else take $\tilde{C}_h^{\text{OLD}}$, as the new draw of $\tilde{C}_h$. $\ell_h(\cdot)$ is the same as Equation (A21). $\pi_{\tilde{C}|\text{else}}(\cdot)$ indicating the prior distribution of $\tilde{C}_h$ is the pdf of a conditional normal distribution of $\tilde{C}_h$ given $\boldsymbol{\beta}_h$, $\tilde{\sigma}_h$, $\tilde{E}_h$, and $\tilde{\boldsymbol{\theta}}_h$ with mean $\bar{\boldsymbol{\eta}}$ and variance $\mathbf{V}$.

(Step 5.5) Draw $\tilde{\boldsymbol{\theta}}_h^{\text{NEW}} = \tilde{\boldsymbol{\theta}}_h^{\text{OLD}} + \boldsymbol{v}$, $\boldsymbol{v} \sim N(\mathbf{0}, d^2\mathbf{I})$. d is a scaling factor set a priori.

(Step 5.6) With probability $\min\left[1, \dfrac{\ell_h\left(\tilde{\boldsymbol{\theta}}_h^{\text{NEW}}, \text{else}\right)\pi_{\tilde{\boldsymbol{\theta}}|\text{else}}\left(\tilde{\boldsymbol{\theta}}_h^{\text{NEW}} \mid \text{else}\right)}{\ell_h\left(\tilde{\boldsymbol{\theta}}_h^{\text{OLD}}, \text{else}\right)\pi_{\tilde{\boldsymbol{\theta}}|\text{else}}\left(\tilde{\boldsymbol{\theta}}_h^{\text{OLD}} \mid \text{else}\right)}\right]$, take $\tilde{\boldsymbol{\theta}}_h^{\text{NEW}}$, else take $\tilde{\boldsymbol{\theta}}_h^{\text{OLD}}$, as the new draw of $\tilde{\boldsymbol{\theta}}_h$. $\ell_h(\cdot)$ is the same as Equation (A21). $\pi_{\tilde{\boldsymbol{\theta}}|\text{else}}(\cdot)$ indicating the prior distribution of $\tilde{\boldsymbol{\theta}}_h$ is the pdf of a conditional normal distribution of $\tilde{\boldsymbol{\theta}}_h$ given $\boldsymbol{\beta}_h$, $\tilde{\sigma}_h$, $\tilde{E}_h$, and $\tilde{C}_h$ with mean $\bar{\boldsymbol{\eta}}$ and variance $\mathbf{V}$.

(Step 6) Draw n_{hj} given $\{n_{hi}\}_{i\neq j}$, $\boldsymbol{\eta}_h$, and $\{\phi_{jk}\}_{k=1,\ldots,K}$ through an Independent Metropolis-Hastings algorithm for each product. We use the prior multinomial distribution, Equation (A8), as the proposal distribution and, therefore, acceptance of a candidate draw is determined by the likelihood ratio:

(Step 6.1) Draw $n_{hj}^{\text{NEW}} \sim \text{multinomial}_K(\phi_{j1}, \ldots, \phi_{jK})$.

(Step 6.2) With probability $\min\left[1, \dfrac{\ell_h\left(\boldsymbol{\eta}_h, n_{hj}^{\text{NEW}}, \{n_{hi}\}_{i\neq j}\right)}{\ell_h\left(\boldsymbol{\eta}_h, n_{hj}^{\text{OLD}}, \{n_{hi}\}_{i\neq j}\right)}\right]$, take n_{hj}^{NEW}, else take n_{hj}^{OLD}, as the new draw. The likelihood function, $\ell_h(\cdot)$, is the same as Equation (A21).

(Step 7) Repeat (Step 3) through (Step 6) for each individual.

(Step 8) Draw $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ given $\{\boldsymbol{\eta}_h\}_{h=1,\ldots,H}$ through the following posterior distributions for a Bayesian multivariate regression:

$$\mathbf{V} \mid \{\boldsymbol{\eta}_h\}_{h=1,\ldots,H} \sim \text{IW}(\nu_0 + H, \mathbf{V}_0 + \mathbf{S}), \tag{A22}$$
$$\bar{\boldsymbol{\eta}} \mid \{\boldsymbol{\eta}_h\}_{h=1,\ldots,H}, \mathbf{V} \sim N\left(\frac{\mathbf{Y}'\mathbf{1}_H + A\bar{\boldsymbol{\eta}}_0}{H+A}, \frac{\mathbf{V}}{H+A}\right),$$

where $\mathbf{Y}$ is the matrix where $\boldsymbol{\eta}_h'$ is stacked up, i.e., $\mathbf{Y} = [\boldsymbol{\eta}_1 \cdots \boldsymbol{\eta}_H]'$, $\mathbf{1}_H$ is a H by 1 vector of one, and $\mathbf{S} = \left(\mathbf{Y} - \frac{\mathbf{1}_H \mathbf{1}_H' \mathbf{Y}}{H}\right)' \left(\mathbf{Y} - \frac{\mathbf{1}_H \mathbf{1}_H' \mathbf{Y}}{H}\right)$. ν_0, $\mathbf{V}_0$, A, and $\bar{\boldsymbol{\eta}}_0$ are priors. We use a typically used diffuse prior for the Normal distribution in Equation (A22), i.e., $A = 0.01$ and $\bar{\boldsymbol{\eta}}_0 = \mathbf{0}$. For the IW distribution, we note that $\tilde{\sigma}_h$, $\tilde{E}_h$, $\tilde{C}_h$, and $\tilde{\theta}_h$ in $\boldsymbol{\eta}_h$ are log- or logarithmically transformed elements and a typically used diffuse prior will result in very thick tails and admit extremely big and small values of the transformed variables on their original scale (**?**). We thus use a tighter prior for the IW distribution, as suggested by **?**: $\nu_0 =$ (the number of parameters in $\boldsymbol{\eta}_h$) + 15 and $\mathbf{V}_0 = \nu_0 \tilde{\mathbf{I}}$, where $\tilde{\mathbf{I}}$ is a diagonal matrix with 0.5 for the log- or logarithmically transformed elements and 1.0 for the other elements.

(Step 9) For each product, draw $\{\phi_{jk}\}_{k=1,\ldots,K}$ given $\{n_{hj}\}_{h=1,\ldots,H}$ through the following posterior Dirichlet distribution:

$$\phi_{j1}, \ldots, \phi_{jK} \mid \{n_{hj}\}_{h=1,\ldots,H} \sim \text{Dirichlet}\left(\alpha_{j1} + \sum_{h=1}^{H} m_{hj1}, \ldots, \alpha_{jK} + \sum_{h=1}^{H} m_{hjK}\right), \tag{A23}$$

where α_{jk} is a prior. We set $\alpha_{j1} = \cdots = \alpha_{jK} = 3$.

(Step 10) Repeat (Step 3) through (Step 9) at each iteration of the MCMC.

When an order restriction is imposed for $\phi_{j1}, \ldots, \phi_{jK}$, an ordered Dirichlet distribution substitutes for the Dirichlet distribution in Equation (A23). We employ a simple accept/reject method to draw the ϕ parameters from the ordered Dirichlet distribution: (*i*) continuously draw again a candidate of the parameters from the original Dirichlet distribution until the order restriction is satisfied and (*ii*) take the candidate satisfying the restriction as the new draw.

4 Data patterns with heterogeneity

We extend the simulation that allows for heterogeneity of the parameters. We generate data of 1,000 individuals each with 1,000 observations for five inside goods ($J = 5$) for each of the three models (Satiation, Complementarity, and Scope models) used for the simulation in Section 4.1 in the main text of the paper. Details of the model specifications are as follows:

Satiation model (data pattern simulation).

$$\max_{x_{h1t},\ldots,x_{hJt}} \quad U_h\left(x_{h1t},\ldots,x_{hJt},z_{ht}\right) = \sum_{j=1}^{J} \frac{\psi_{hj} e^{\epsilon_{hjt}}}{\kappa_{hj}} \log\left(\kappa_{hj} x_{hjt} + 1\right) + \log\left(z_{ht}\right), \tag{A24}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} p_{hjt} x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0.$$

Complementarity model (data pattern simulation).

$$\begin{aligned}
\max_{x_{h1t},\ldots,x_{hJt}} \quad U_h\left(x_{h1t},\ldots,x_{hJt},z_{ht}\right) = & \sum_{j=1}^{J} \psi_{C,hj} e^{\epsilon_{hjt}} \log\left(x_{hjt}+1\right) \\
& + \sum_{i=1}^{J-1} \sum_{j=i+1}^{J} \mu_{hij} \psi_{C,hi} e^{\epsilon_{hit}} \log\left(x_{hit}+1\right) \psi_{C,hj} e^{\epsilon_{hjt}} \log\left(x_{hjt}+1\right) \\
& + \log\left(z_{ht}\right),
\end{aligned} \tag{A25}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} p_{hjt} x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0.$$

Scope model (data pattern simulation).

$$\max_{x_{h1t},\ldots,x_{hJt}} \quad U_h\left(x_{h1t},\ldots,x_{hJt},z_{ht},s_{ht}\right) = \sum_{j=1}^{J} \psi_{S,hj} e^{\epsilon_{hjt}} \log\left(x_{hjt}+1\right) + \log\left(z_{ht}\right) + \log\left(s_{ht}\right), \tag{A26}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{j=1}^{J} p_{hjt} x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0, \\
& \sum_{j=1}^{J} x_{hjt} + \left(\sum_{j=1}^{J} x_{hjt}^{\theta_h}\right)^{\frac{1}{\theta_h}} + s_{ht} = C_h, \quad s_{ht} > 0.
\end{aligned}$$

For a fair comparison, the same individual along with the same price and error for each product and individual is used for data generation across the models. Table A1 displays the parameter setting used for data generation.

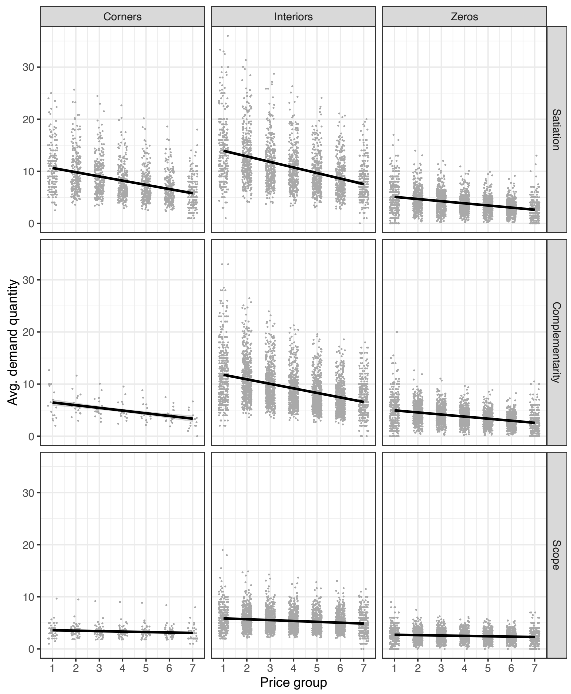
Table A1: Parameter settings for data generation

Parameters/ data	Satiation model	Complementarity model	Scope model
Utility (ψ_{hj}, $\psi_{C,hj}$, $\psi_{S,hj}$)	$\log \psi_{hj} \sim N(-2, 1)$ for each j	$\psi_{C,hj} = 0.8 \times \psi_{hj}$	$\psi_{S,hj} = 2.5 \times \psi_{hj}$
Satiation (κ_{hj})	$\log \kappa_{hj} \sim N(0, 4)$ for each j	fixed to 1 for all h and j	fixed to 1 for all h and j
Complementarity (μ_{hij})	–	$\log \mu_{hij} \sim N(\bar{\mu}_{ij}, 4)$ for each (i, j) *	–
Scope (θ_h)	–	–	$\log(\theta_h - 1) \sim N(0, 4)$
Monetary budget (E_h)	The same values generated from $\log E_h \sim N(\log 500, 0.25)$ are used across the models.		
Time budget (C_h)	–	–	$\log C_h \sim N(\log 11, 0.25)$
Errors (ϵ_{hjt})	The same values generated from $\epsilon_{hjt} \sim EV(0, 1)$ for all h, j and t are used across the models.		
Prices (p_{hjt})	The same values generated from $p_{hjt} \sim Unif(35, 70)$ for all h, j and t are used across the models.		

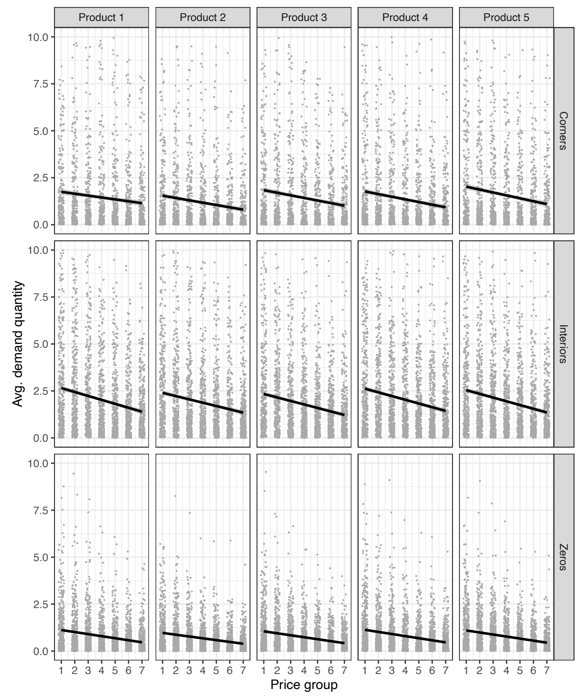
* $\bar{\mu}_{ij} = -2$ for $(i, j) = (1, 2)$ and $(3, 4)$; and -10000 for the other pairs.

We classify the individuals into three different types by the number of interior solutions, corner solutions and zero demand. "Interiors" denotes the individuals who are generated to mostly choose interior solutions over the observations, "Corners" indicates the individuals who are generated to mostly choose corner solutions over the observations, and "Zeros" represents the individuals who are generated to make no-purchase observations significantly. The price is randomly sampled from the range ($35, $70) and seven price groups are constructed by $5 intervals. Figure A1 displays individuals' primary and secondary demand quantities averaged over the observations for each price group across different individual types and different models. The black solid line in each plot is the trend line of the demand quantities. Figure A1(a) displays primary demand quantities and the flat trend lines across all individual types are observed from the Scope model (plots in the third row of Figure A1(a)), whereas downward slopes are observed from the conventional models (plots in the first and second rows of Figure A1(a)). Figures A1(b), (c) and (d) display secondary demand quantities under different models and show the same patterns as in the primary demand quantities: flat trend lines are observed from the Scope model across all types and products (Figure A1(d)), whereas downward trend lines are observed from the conventional models (Figures A1(b) and (c)). This implies that the effect of price discounts on primary and secondary demand is very weak in the Scope model, consistent with our findings in Section 4.1 in the main text of the paper. We also investigate the trend lines at individual-level (Figures A2 through A7) and find that the flat slopes of the demand curves for the Scope model do not emanate from data aggregation.

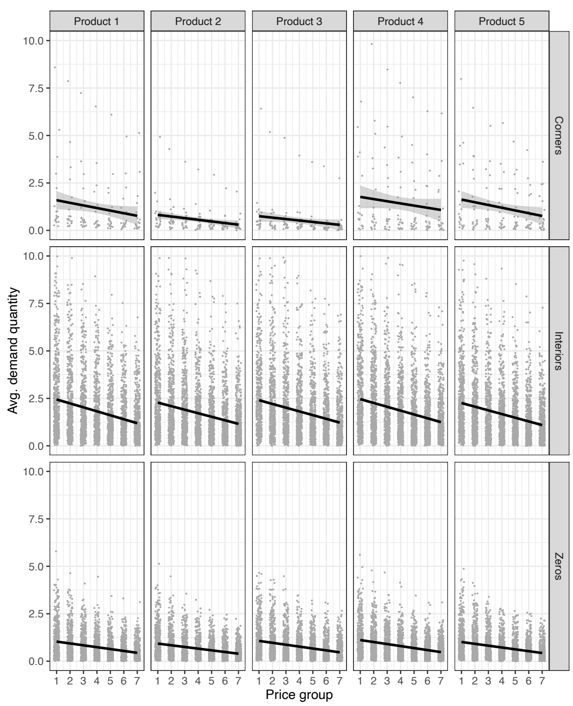
Figure A1: Primary and secondary demand quantities under different models



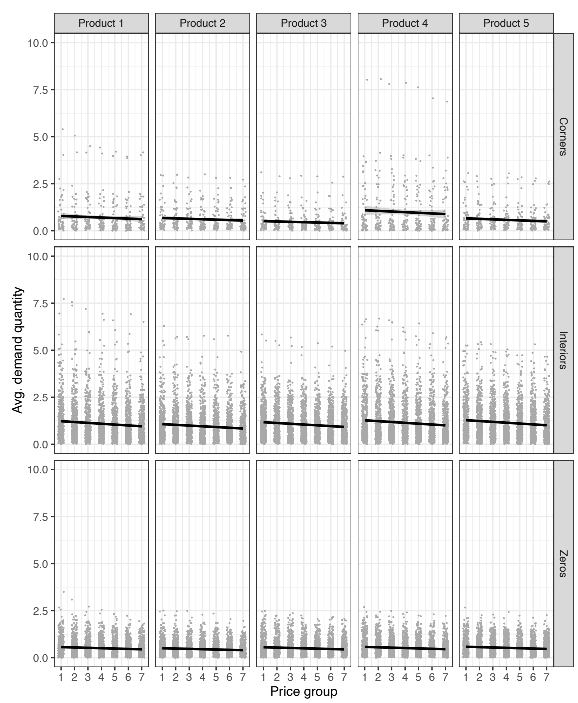
(a) Primary demand



(b) Secondary demand: Satiation model



(c) Secondary demand: Complementarity model



(d) Secondary demand: Scope model

Figure A2: Primary demand curves at individual-level: Satiation model

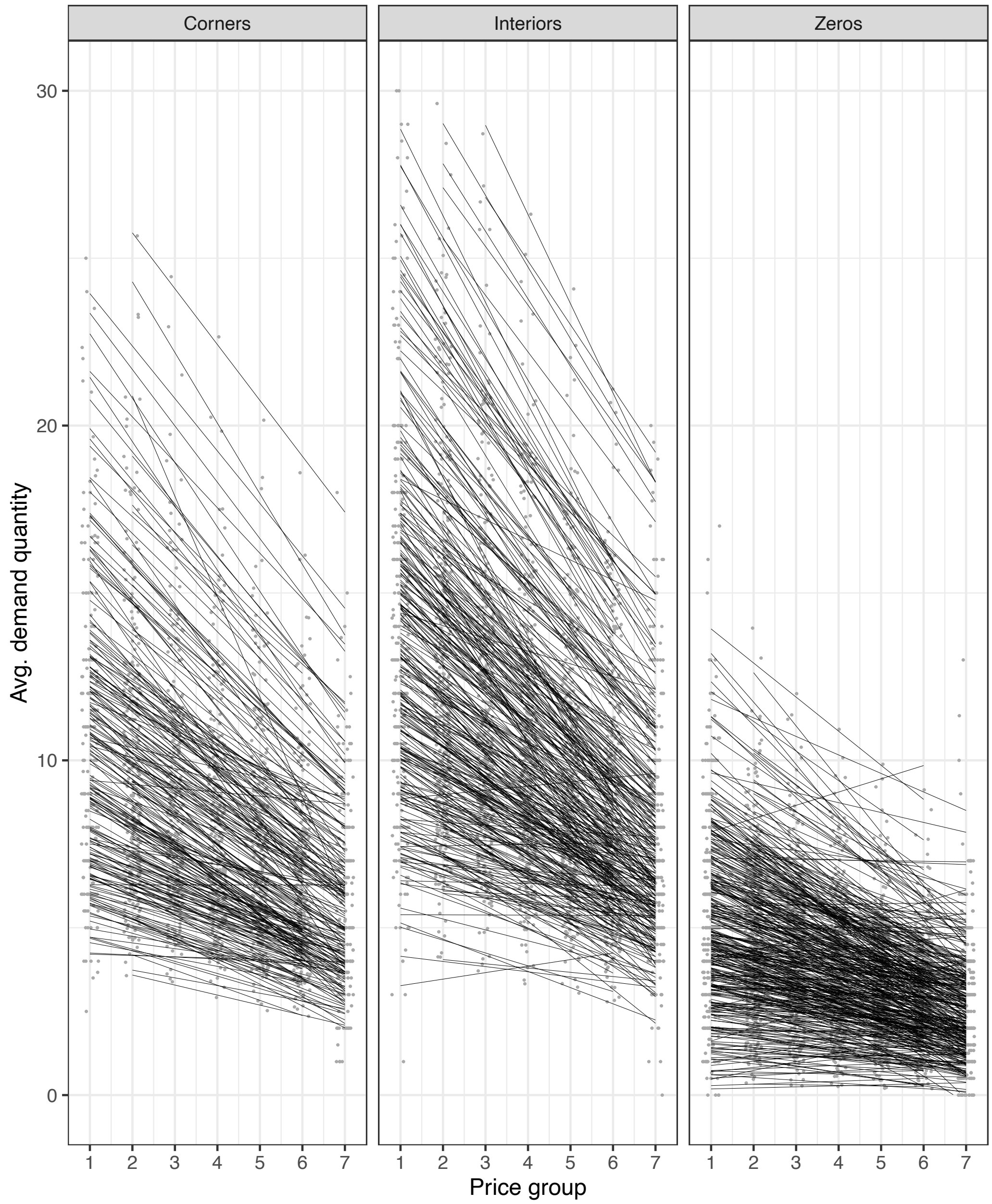


Figure A3: Primary demand curves at individual-level: Complementarity model

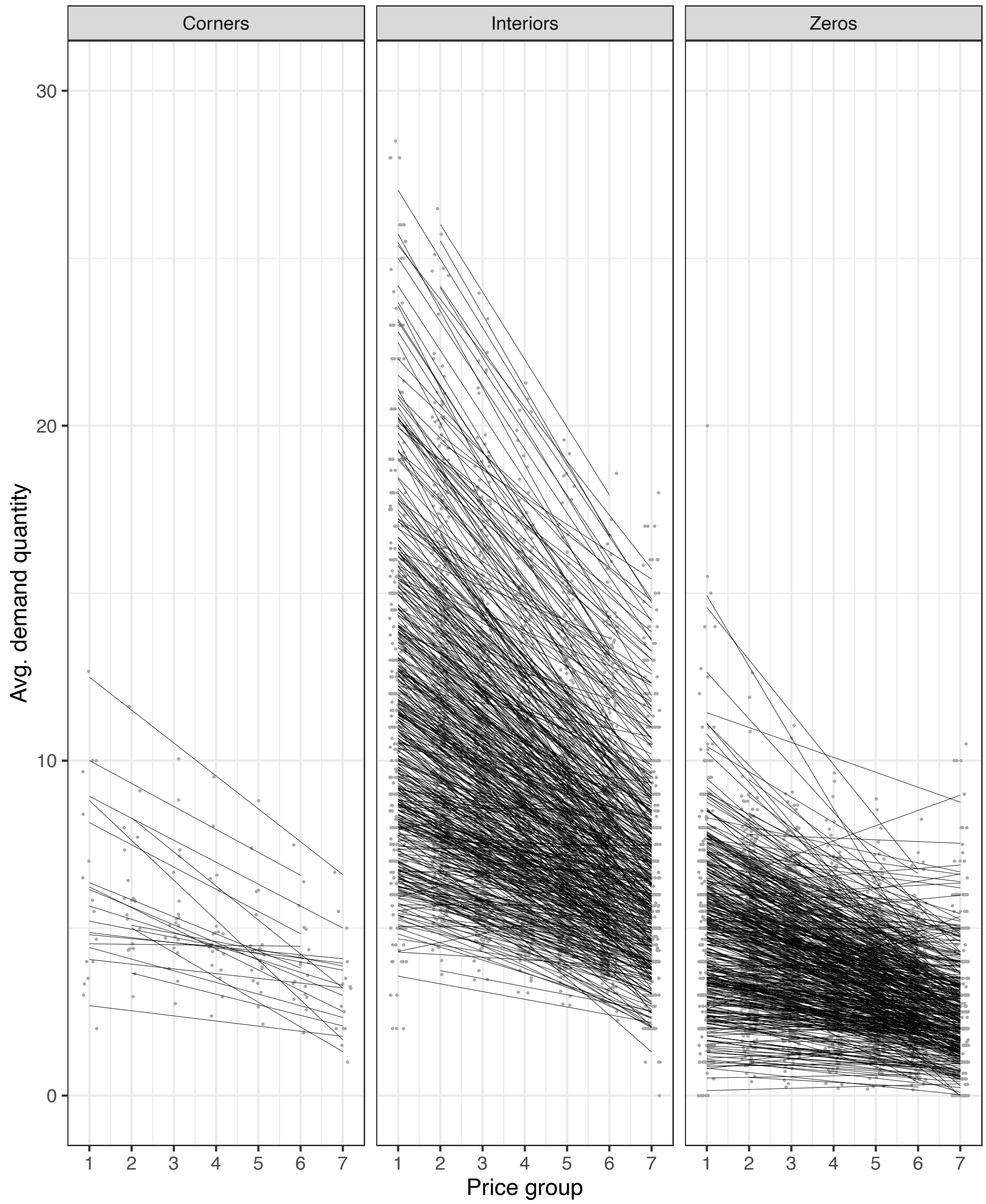


Figure A4: Primary demand curves at individual-level: Scope model

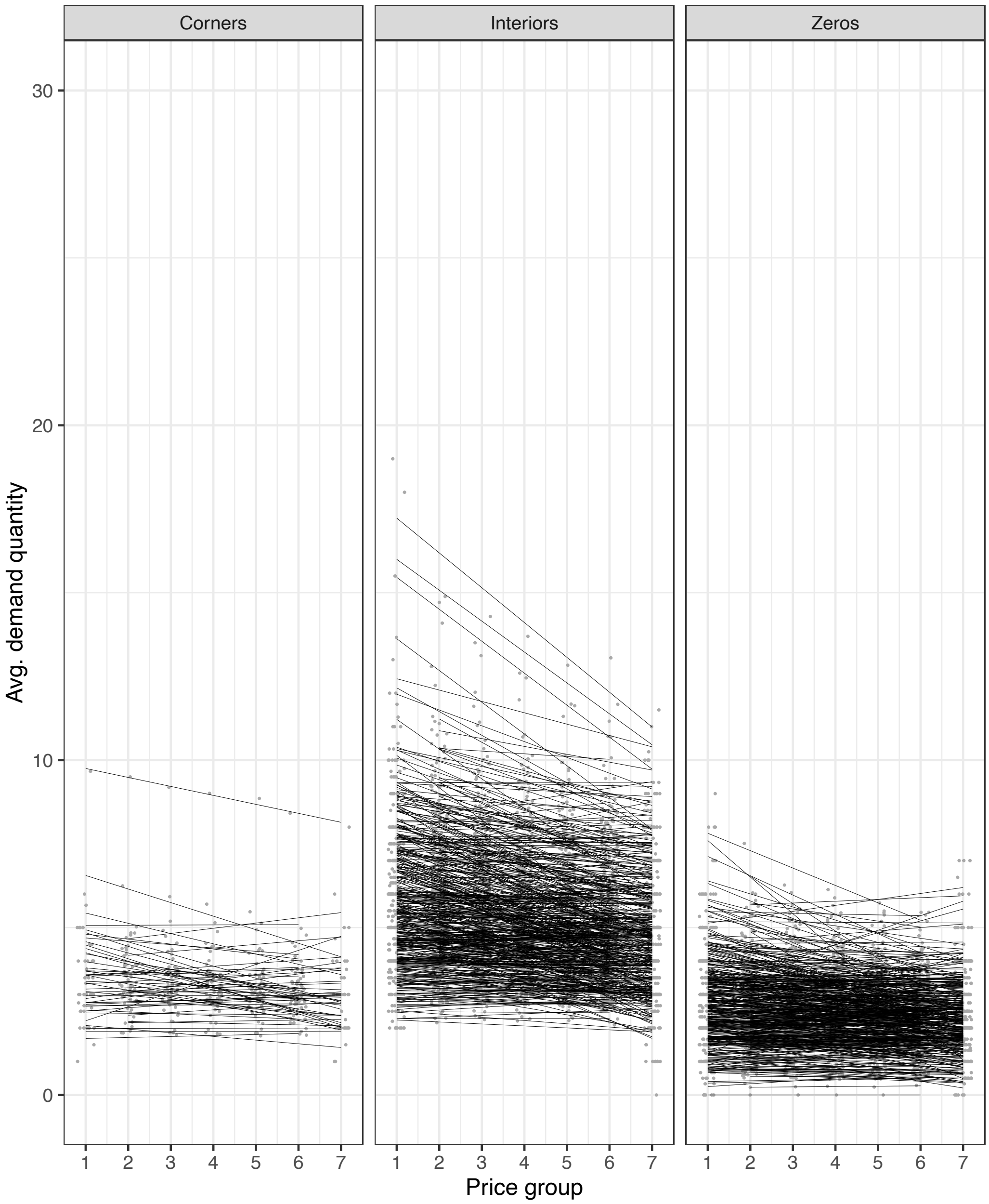


Figure A5: Secondary demand curves at individual-level: Satiation model

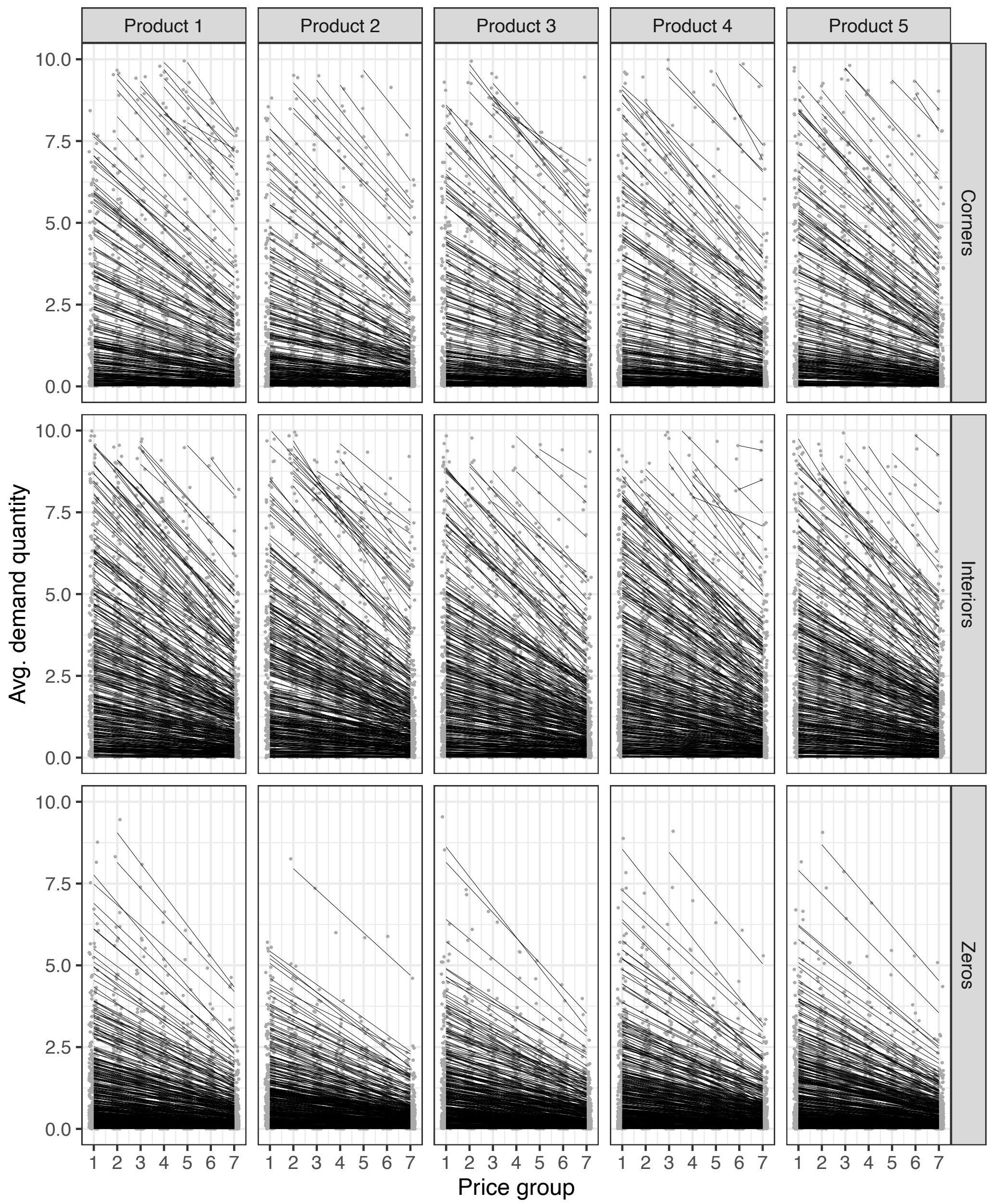


Figure A6: Secondary demand curves at individual-level: Complementarity model

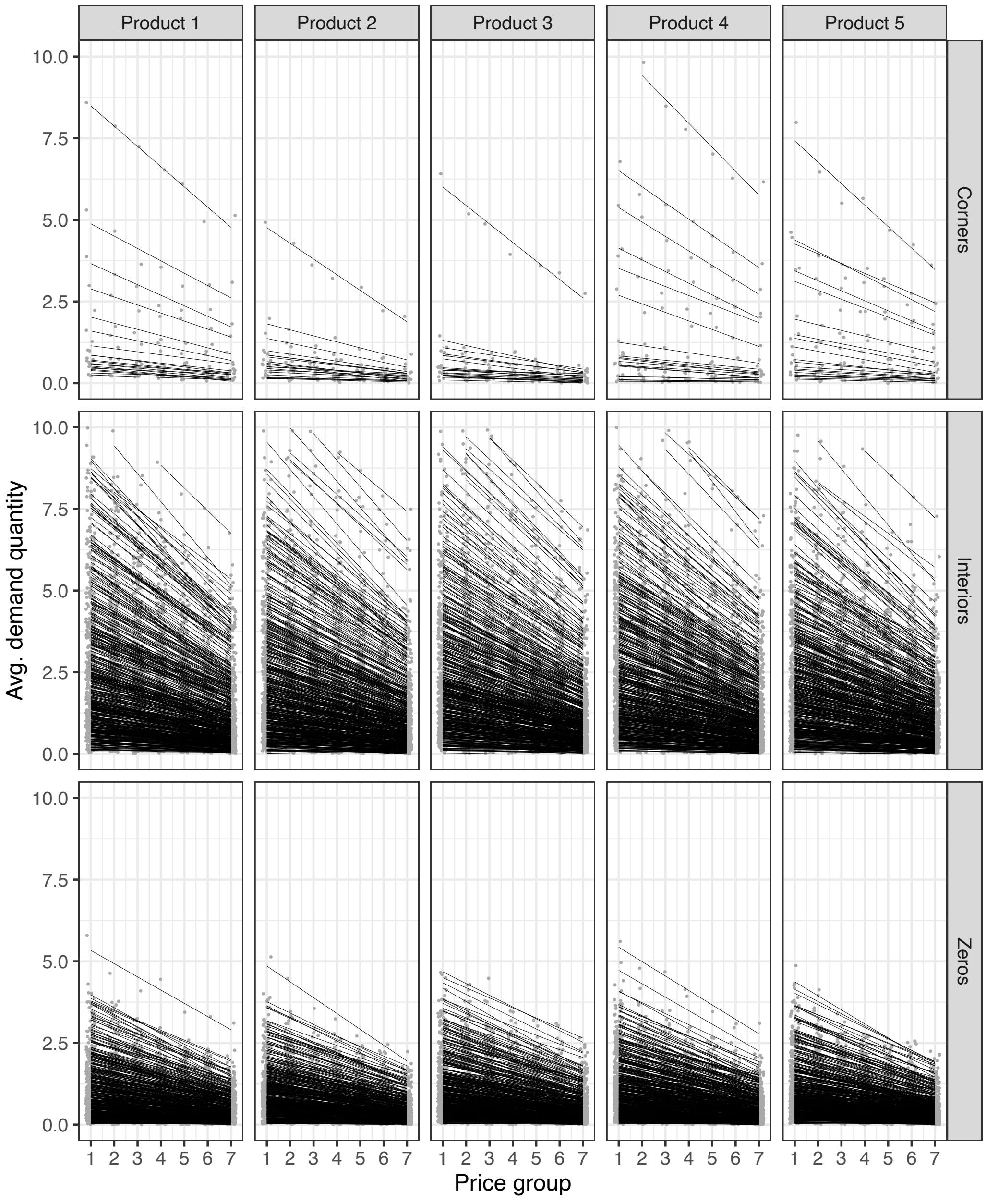
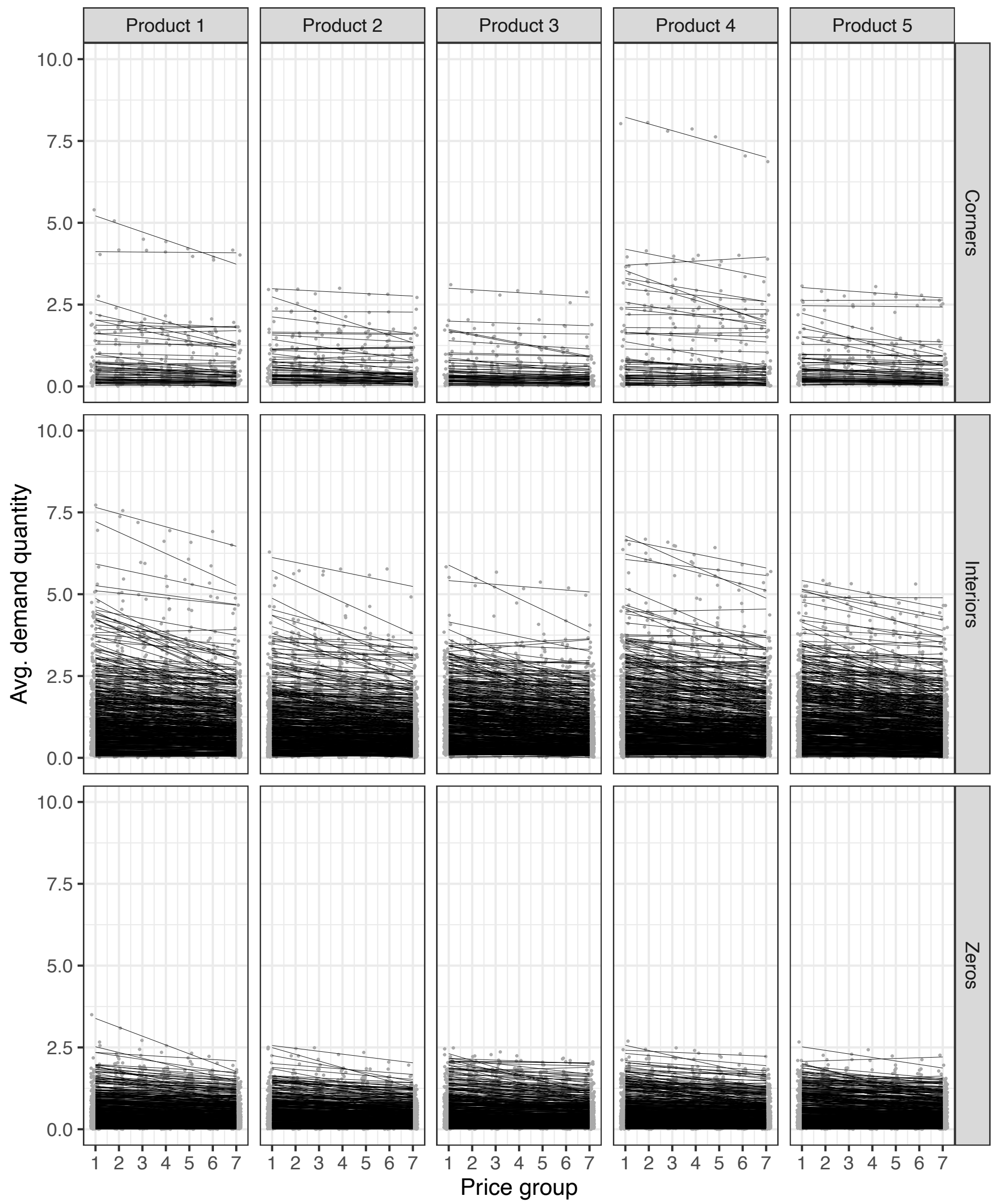


Figure A7: Secondary demand curves at individual-level: Scope model



5 Details of testing recovery of the DGP

This appendix provides the details of the data generation and the result of the true value recovery in Section 4.2 in the main text. For all data-generating processes, the following parameters are fixed to avoid generating too-extreme observations where there are not enough variations to identify the parameters:

- The heterogeneity mean of the log of the error variance ($\log \sigma$) is fixed to $\log(0.5)$.
- The heterogeneity mean of the log of the monetary budget ($\log E$) is fixed to $\log(6)$.
- The heterogeneity mean of the log of the time allotment ($\log C$) is fixed to $\log(3)$.
- The heterogeneity variances of the preference parameters ($\bar{\psi}$) are fixed to 1.
- The heterogeneity variances of the other (log- or logarithmic-transformed) parameters are fixed to 0.5.

For each dataset, we first draw the heterogeneity means of the parameters that are not fixed, then draw 100 individuals' parameters given the heterogeneity distribution, and lastly draw 30 observations given each individual's parameters. The heterogeneity means of those not-fixed parameters are drawn from uniform distributions as follows:

For the cases where the Scope model is the true data-generating process,

- The heterogeneity means of the preference parameters ($\bar{\psi}$) are drawn from $Unif\,(0, 0.5)$.
- The heterogeneity mean of the logarithmic transformed scope parameter ($\log(\theta - 1)$) is drawn from $Unif\,(0.5, 2.5)$.
- The assignment probabilities for group 1 are drawn from $Unif\,(0.15, 0.85)$.

For the cases where the Satiation model is the true data-generating process,

- The heterogeneity means of the preference parameters ($\bar{\psi}$) are drawn from $Unif\,(0.5, 1.0)$.
- The heterogeneity mean of the log of the common satiation parameter ($\log \kappa$) is drawn from $Unif\,(0.5, 2.5)$.

For the cases where the Complementarity model is the true data-generating process,

- The heterogeneity means of the preference parameters ($\bar{\psi}$) are drawn from $Unif\,(-0.5, 0)$.
- The heterogeneity mean of the log of the common complementarity parameter ($\log \mu$) is drawn from $Unif\,(-0.5, 0.5)$.

In addition, each individual's monetary budget and time allotment parameters are bounded as follows: $E_h \in (\$150, \$1500)$ and $C_h \in (5, 50)$. We also discard individuals whose generated demand quantities are zeros for all products in more than 90% of observations.

Table A2: Counts of cases where the true value is included in the 95% credible interval

	True: Scope ($K = 2$)			True: Satiation			True: Complementarity		
Parameters	**Est'd: Scope2**	Est'd: Satia.	Est'd: Compl.	Est'd: Scope2	**Est'd: Satia.**	Est'd: Compl.	Est'd: Scope2	Est'd: Satia.	**Est'd: Compl.**
$\bar{\psi}_1$	**10**	3	0	2	**10**	0	0	0	**10**
$\bar{\psi}_2$	**10**	3	0	1	**10**	1	0	0	**10**
$\bar{\psi}_3$	**10**	4	0	2	**10**	0	0	0	**10**
$\bar{\psi}_4$	**10**	4	0	2	**10**	1	0	0	**10**
$\bar{\psi}_5$	**10**	5	0	3	**10**	0	0	0	**10**
$\log \sigma$	**10**	1	4	3	**10**	5	0	0	**10**
$\log \kappa$	**10**	–	–	–	**10**	–	–	–	–
$\log \mu$	**10**	–	–	–	–	–	–	–	**10**
$\log E$	**10**	10	6	10	**10**	10	10	10	**10**
$\log C$	**10**	–	–	–	–	–	–	–	–
$\log(\theta - 1)$	**8**	–	–	–	–	–	–	–	–
ϕ_{11}	**8**	–	–	–	–	–	–	–	–
ϕ_{21}	**9**	–	–	–	–	–	–	–	–
ϕ_{31}	**10**	–	–	–	–	–	–	–	–
ϕ_{41}	**10**	–	–	–	–	–	–	–	–
ϕ_{51}	**10**	–	–	–	–	–	–	–	–

* $\{\phi_{j2}\}$ are omitted for brevity: the true value of ϕ_{j2} is recovered if and only if that of ϕ_{j1} is recovered.

Table A2 reports the number of cases where the true value is included in the 95% credible interval of the posterior distribution of each parameter. Mostly, the true values of the parameters are well recovered in the true models (columns in boldface), but not in the misspecified models. We find only the exceptions where the scope effect ($\log(\theta - 1)$) is overestimated or one of the assignment probabilities (ϕ_{jk}) is incorrectly estimated, implying that the model is able to, at least, clearly detect the scope effect if it exists. Considering the complexity of the models and the small size of the datasets, we conclude that the proposed model can be empirically identified from the conventional models.

6 Likelihood approximation

This appendix provides a method for evaluating the likelihood value given the individual parameters. Some of equations in Online Appendix 3 are also used here. The likelihood for individual h at observation t with the indivisibility is given by an integration over irregular regions in Equation (A13). This integration is hard to be numerically computed through a simple Monte Carlo simulation over the entire space of errors when the dimension of errors, J, is large. We rather employ an importance sampling method to approximate the likelihood value.

From the optimality condition in Equation (A12), we have the following inequality if $\Delta_i = 0$ for all $i \neq j$:

$$\psi_{hjt} \log\left(\frac{x_{hjt}+1}{x_{hjt}+\Delta_j+1}\right) \geq \log\left(\frac{E_h - \sum_{i=1}^{J} p_{hit}x_{hit} - p_{hjt}\Delta_j}{E_h - \sum_{i=1}^{J} p_{hit}x_{hit}}\right) + \log\left(\frac{C_h - Q_h\left(\boldsymbol{x}_{ht}+\boldsymbol{\Delta}_j\right)}{C_h - Q_h\left(\boldsymbol{x}_{ht}\right)}\right), \tag{A27}$$

for any $\Delta_j \in \{-1, 0, 1\}$ satisfying $\boldsymbol{x}_{ht} + \boldsymbol{\Delta}_j \in F_{ht}$, where $\boldsymbol{\Delta}_j$ is a vector where the j-th element is equal to Δ_j and the others are equal to zero. From this equation, we have

$$\begin{aligned} ub^*_{hjt} &= \log\left[\left\{\log\left(\frac{E_h - \sum_{i=1}^{J} p_{hit}x_{hit} - p_{hjt}}{E_h - \sum_{i=1}^{J} p_{hit}x_{hit}}\right) + \log\left(\frac{C_h - Q_h\left(\boldsymbol{x}_{ht}+\boldsymbol{\Delta}_j^+\right)}{C_h - Q_h\left(\boldsymbol{x}_{ht}\right)}\right)\right\}\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}+2}\right)\right], \\ lb^*_{hjt} &= \log\left[\left\{\log\left(\frac{E_h - \sum_{i=1}^{J} p_{hit}x_{hit} - p_{hjt}}{E_h - \sum_{i=1}^{J} p_{hit}x_{hit}}\right) + \log\left(\frac{C_h - Q_h\left(\boldsymbol{x}_{ht}+\boldsymbol{\Delta}_j^-\right)}{C_h - Q_h\left(\boldsymbol{x}_{ht}\right)}\right)\right\}\Big/\log\left(\frac{x_{hjt}+1}{x_{hjt}}\right)\right], \end{aligned} \tag{A28}$$

where $\boldsymbol{\Delta}_j^+$ is $\boldsymbol{\Delta}_j$ when $\Delta_j = 1$ and $\boldsymbol{\Delta}_j^-$ is $\boldsymbol{\Delta}_j$ when $\Delta_j = -1$. If $\boldsymbol{\Delta}_j^+$ does not satisfy $\boldsymbol{x}_{ht} + \boldsymbol{\Delta}_j^+ \in F_{ht}$, then $ub^*_{hjt} = \infty$. If $\boldsymbol{\Delta}_j^-$ does not satisfy $\boldsymbol{x}_{ht} + \boldsymbol{\Delta}_j^- \in F_{ht}$, then $lb^*_{hjt} = -\infty$. Because $ub^*_{hjt} \geq ub_{hjt}$ and $lb^*_{hjt} \leq lb_{hjt}$, where ub_{hjt} and lb_{hjt} are the upper and lower bounds defined in Equation (A18), the optimality condition is not satisfied (i.e., the indicator function in Equation (A13) is zero) at any points outside of the following rectangular space:

$$lb^*_{hjt} - \boldsymbol{a}'_{hjt}\boldsymbol{\beta}_h \leq \epsilon_{hjt} \leq ub^*_{hjt} - \boldsymbol{a}'_{hjt}\boldsymbol{\beta}_h, \quad \text{for } j = 1, \ldots, J. \tag{A29}$$

Thus, we can compute the approximate likelihood (denoted $\tilde{\ell}^{\text{Int}}$) by the use of an importance sampling method through the following steps:

(Step 1) For each product, draw R errors, $\epsilon_{hjt}^{(1)}, \ldots, \epsilon_{hjt}^{(R)}$:

$$\epsilon_{hjt}^{(r)} \sim EV\left(0, \sigma_h\right) \times I\left(lb_{hjt}^{*} - \boldsymbol{a}_{hjt}^{\prime}\boldsymbol{\beta}_h \leq \epsilon_{hjt} \leq ub_{hjt}^{*} - \boldsymbol{a}_{hjt}^{\prime}\boldsymbol{\beta}_h\right). \tag{A30}$$

(Step 2) For the r-th set of errors, $\left\{\epsilon_{h1t}^{(r)}, \ldots, \epsilon_{hJt}^{(r)}\right\}$, compute the indicator function in Equation (A13) and let the value be M_r.

(Step 3) Calculate the approximate likelihood for individual h at observation t, $\tilde{\ell}_{ht}^{\text{Int}}$:

$$\tilde{\ell}_{ht}^{\text{Int}} = \frac{1}{R}\sum_{r=1}^{R} M_r \times \prod_{j=1}^{J}\left\{\exp\left(-e^{-\frac{ub_{hjt}^{*} - \boldsymbol{a}_{hjt}^{\prime}\boldsymbol{\beta}_h}{\sigma_h}}\right) - \exp\left(-e^{-\frac{lb_{hjt}^{*} - \boldsymbol{a}_{hjt}^{\prime}\boldsymbol{\beta}_h}{\sigma_h}}\right)\right\}. \tag{A31}$$

(Step 4) Repeat (Step 1) through (Step 3) for each individual and each observation.

(Step 5) Compute the approximate likelihood, $\tilde{\ell}^{\text{Int}}$:

$$\tilde{\ell}^{\text{Int}} = \prod_{h=1}^{H}\prod_{t=1}^{T}\tilde{\ell}_{ht}^{\text{Int}}. \tag{A32}$$

To ensure that $\sum_{r=1}^{R} I_r > 0$, R must be large enough.

7 Model-free evidence for the Theme Park data

This appendix provides individual-level model-free evidence from the Theme Park data.

Figure A8: Primary demand curves at individual-level: Theme Park data

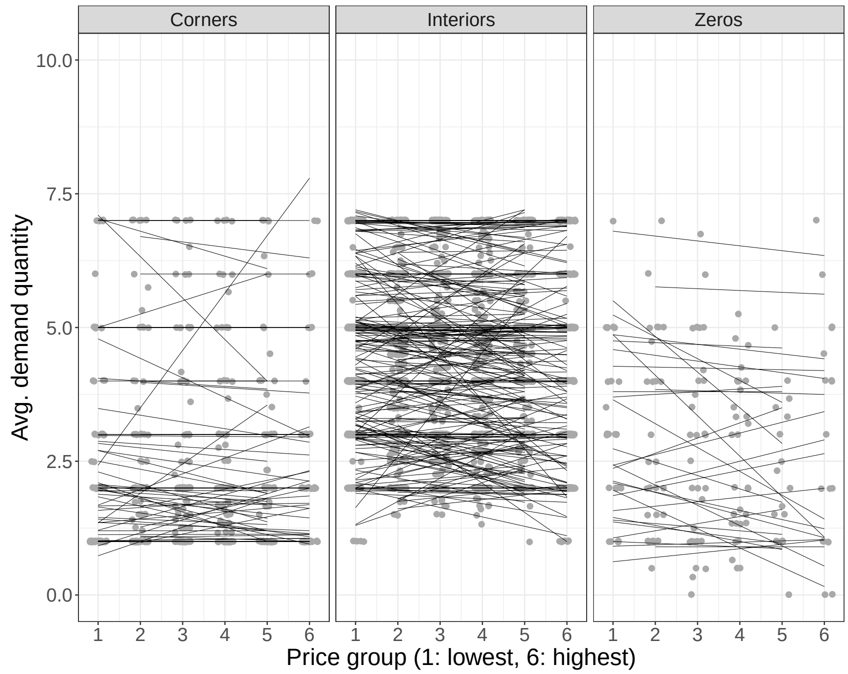
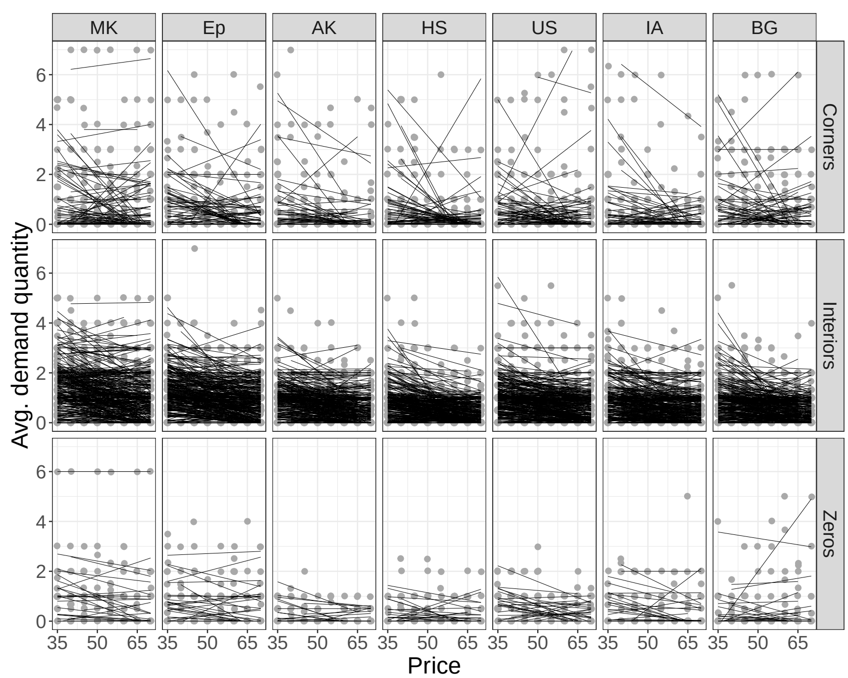


Figure A9: Secondary demand curves at individual-level: Theme Park data



8 Validation of the empirical strategy with no information on the unit time costs (γ and δ)

This appendix validates the empirical strategy assuming γ and δ to be ones as used for the Theme Park data. We first explain that (*i*) γ is analytically unidentifiable from δ and C due to the scale-invariant property of the time constraint and (*ii*) it is empirically hard to identify δ from C given γ. We then present a simulation study that shows the robustness of assuming δ to be one in demand prediction.

If γ and δ are assumed to be the same across inside goods and θ is assumed to be identical across the groups, the inequality version of the time constraint given γ, δ, θ, and C becomes

$$\gamma \sum_j x_j + \delta \sum_k \left\{ \sum_j m_{jk} x_j^{\theta} \right\}^{\frac{1}{\theta}} \leq C. \tag{A33}$$

This inequality is scale-invariant, i.e., homogeneous of degree 1 with respect to (γ, δ, C), and thus identical to the following inequality:

$$\sum_j x_j + \frac{\delta}{\gamma} \sum_k \left\{ \sum_j m_{jk} x_j^{\theta} \right\}^{\frac{1}{\theta}} \leq \frac{C}{\gamma}, \tag{A34}$$

implying that fixing γ to an arbitrary number (e.g., one) does not affect the feasible set to be navigated in estimation. Therefore, γ cannot be separately identified from δ and C.

Note that the access cost part in the constraint above, i.e., $\delta \sum_k \left\{ \sum_j m_{jk} x_j^{\theta} \right\}^{\frac{1}{\theta}}$, is proportional to δ. Therefore, it is empirically hard to identify having a large time allotment (C) but large access costs incurred by a large δ from having a small time allotment (C) but small access costs incurred by a small δ although γ is fixed. To resolve the identification problem, we assume δ to be one in addition to fixing γ to an arbitrary number.

We conduct a simulation study to test the robustness of assuming δ to be one in demand prediction. Fixing γ to one without loss of generality as shown above, we generate synthetic datasets of the Scope model with four different values of δ $(= 2, 3, \frac{1}{2}, \frac{1}{3})$ and fit the model to the synthetic data with fixing δ to one. The setting of the data generation is the same as used in the main simulation study in Section 4.2: five inside goods ($J = 5$), two groups ($K = 2$), 100 individuals, and 30 observations per individual. True values of the model parameters except for δ and the prices used in the data generation are almost the same as in the simulation study in Section 4.2. Details of the setting and true values are available upon request. Those 30 observations per individual serve as the in-sample data for estimating the model parameters.

If the assumption of δ is robust, the estimates of the model parameters with the arbitrary value of δ (= 1) are a good approximation that makes the same feasible set and demand prediction as the true model parameters with the true value of δ. We thus generate extra 30 observations per individual from the same true values of the model parameters but with different price scenarios as the out-of-sample data. We then investigate the in-sample and out-of-sample prediction errors for the "WRONG" case where the estimates with fixing $\delta = 1$ are used for prediction, and for the "TRUE" case where the true model parameters with the true value of δ are used for prediction.

Table A3: Prediction errors: Robustness test for assuming $\delta = 1$

	In-sample prediction errors				Out-of-sample prediction errors			
	Case "WRONG": from the estimates with fixing $\delta = 1$		Case "TRUE": from the true values with true δ		Case "WRONG": from the estimates with fixing $\delta = 1$		Case "TRUE": from the true values with true δ	
Value of δ	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
$\delta = 2$	0.6619	1.7238	0.6707	1.7222	0.6985	1.8696	0.6721	1.7553
$\delta = 3$	0.5648	1.2869	0.5695	1.2685	0.5871	1.3493	0.5711	1.2537
$\delta = \frac{1}{2}$	0.8953	3.0115	0.9033	2.9990	0.9500	3.4201	0.9167	3.2023
$\delta = \frac{1}{3}$	0.9335	3.2848	0.9417	3.2658	0.9924	3.7542	0.9564	3.5016

Table A3 reports the in-sample and out-of-sample prediction errors for the two cases. For each of the in-sample and the out-of-sample, we compute the mean absolute error (MAE) and the mean squared error (MSE). We do not find any significant difference in the prediction errors between the two cases, implying that assuming $\delta = 1$ does not affect the overall demand prediction performance.

Table A4: Deviation from the "TRUE" case: Robustness test for assuming $\delta = 1$

	In-sample prediction errors			Out-of-sample prediction errors		
Value of δ	MAD	% of D>1	MAD (agg.)	MAD	% of D>1	MAD (agg.)
$\delta = 2$	0.2322	2.86 %	0.0317	0.2358	3.00 %	0.0331
$\delta = 3$	0.1934	1.73 %	0.0375	0.1949	1.73 %	0.0398
$\delta = \frac{1}{2}$	0.2618	4.15 %	0.0349	0.2599	4.15 %	0.0356
$\delta = \frac{1}{3}$	0.2680	4.32 %	0.0324	0.2682	4.41 %	0.0319

Table A4 reports the deviation of the demand prediction in the "WRONG" case from that in the "TRUE" case. If the assumption of δ is robust, the deviation should be small.

The mean absolute deviation (MAD) values displayed in the second (for the in-sample data) and the fifth (for the out-of-sample data) columns are not large in that each absolute deviation should be at least one if deviated due to the integer demand. The third (for the in-sample data) and the sixth (for the out-of-sample data) columns show that the proportion of the absolute deviations larger than one (% of D>1) is small, implying that the most deviations are a one unit difference in the demand quantity, which is the smallest size of a deviation. In addition, relying on the price scenarios being identical across all individuals in each dataset, we investigate the deviation at the aggregate level. The fourth (for the in-sample data) and the seventh (for the out-of-sample data) columns report the MAD of the aggregate-level demand prediction in the "WRONG" case from that in the "TRUE" case (MAD (agg.)). The aggregate-level MAD is close to zero, supporting the robustness of the assumption of δ in the overall demand counterfactual.

9 Posterior estimates

This appendix provides the posterior estimates of all parameters for all models.

Table A5: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Standard model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(E)$
MK	-1.23	1.05	0.76	0.73	0.70	0.67	0.67	0.55	-0.01	-0.04	-0.05	-0.00	0.05	-0.66
(SD)	(0.05)	(0.07)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.03)
Ep	-1.26	0.81	1.08	0.77	0.75	0.74	0.68	0.67	-0.05	-0.03	-0.05	-0.01	0.02	-0.72
(SD)	(0.05)	(0.07)	(0.08)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.02)
AK	-1.51	0.77	0.82	1.06	0.76	0.70	0.70	0.68	-0.07	-0.06	-0.02	0.02	-0.07	-0.66
(SD)	(0.06)	(0.06)	(0.07)	(0.08)	(0.02)	(0.03)	(0.03)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
HS	-1.66	0.75	0.81	0.81	1.08	0.80	0.72	0.67	-0.08	-0.04	-0.02	-0.00	-0.08	-0.66
(SD)	(0.05)	(0.07)	(0.07)	(0.07)	(0.08)	(0.02)	(0.03)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.03)
US	-1.43	0.74	0.83	0.78	0.89	1.16	0.78	0.76	-0.06	-0.02	-0.04	-0.03	0.04	-0.73
(SD)	(0.05)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.02)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.02)
IA	-1.59	0.76	0.78	0.79	0.82	0.93	1.22	0.69	-0.03	-0.01	-0.02	0.01	-0.00	-0.69
(SD)	(0.05)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.09)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.03)
BG	-1.63	0.66	0.82	0.82	0.82	0.95	0.89	1.38	-0.04	-0.02	0.00	0.00	0.08	-0.72
(SD)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.10)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.02)
Speed	0.05	-0.00	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	0.09	0.05	-0.02	0.00	0.08	-0.01
(SD)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
Mobile	0.02	-0.01	-0.01	-0.02	-0.01	-0.01	-0.00	-0.01	0.00	0.09	-0.04	-0.02	0.09	-0.02
(SD)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.01)	(0.05)	(0.05)	(0.06)	(0.05)
Early	0.01	-0.02	-0.02	-0.00	-0.01	-0.01	-0.01	0.00	-0.00	-0.00	0.09	0.16	-0.01	0.01
(SD)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.00)	(0.01)	(0.05)	(0.06)	(0.05)
Late	0.02	-0.00	-0.00	0.01	-0.00	-0.01	0.00	-0.00	0.00	-0.00	0.02	0.10	-0.04	0.00
(SD)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.00)	(0.00)	(0.01)	(0.06)	(0.05)
$\log(\sigma)$	-1.34	0.03	0.01	-0.04	-0.04	0.02	-0.00	0.05	0.01	0.02	-0.00	-0.01	0.30	-0.29
(SD)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.05)
$\log(E)$	6.08	-0.49	-0.54	-0.49	-0.49	-0.56	-0.55	-0.61	-0.00	-0.00	0.00	0.00	-0.11	0.52
(SD)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A6: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Satiation model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(\kappa)$	$\log(E)$
MK	1.27	5.56	0.89	0.88	0.87	0.84	0.84	0.75	-0.12	-0.09	-0.02	-0.03	-0.32	0.84	-0.15
(SD)	(0.13)	(0.50)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.05)
Ep	1.22	5.15	6.06	0.90	0.89	0.87	0.83	0.82	-0.14	-0.09	-0.02	-0.02	-0.31	0.85	-0.19
(SD)	(0.14)	(0.50)	(0.56)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.05)
AK	0.79	5.19	5.54	6.25	0.90	0.86	0.85	0.84	-0.15	-0.09	-0.01	-0.01	-0.32	0.86	-0.17
(SD)	(0.14)	(0.48)	(0.51)	(0.51)	(0.01)	(0.02)	(0.02)	(0.02)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.05)
HS	0.50	5.01	5.31	5.46	5.90	0.91	0.87	0.84	-0.14	-0.09	0.00	-0.02	-0.33	0.84	-0.20
(SD)	(0.14)	(0.46)	(0.50)	(0.48)	(0.50)	(0.01)	(0.02)	(0.02)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.04)
US	0.89	4.75	5.14	5.16	5.31	5.75	0.90	0.88	-0.13	-0.07	-0.01	-0.03	-0.26	0.83	-0.25
(SD)	(0.14)	(0.48)	(0.52)	(0.49)	(0.48)	(0.52)	(0.01)	(0.01)	(0.05)	(0.06)	(0.05)	(0.05)	(0.06)	(0.02)	(0.05)
IA	0.61	4.94	5.11	5.30	5.28	5.39	6.25	0.83	-0.11	-0.07	-0.01	-0.01	-0.29	0.82	-0.22
(SD)	(0.14)	(0.53)	(0.54)	(0.53)	(0.52)	(0.54)	(0.61)	(0.02)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)	(0.02)	(0.05)
BG	0.50	4.33	4.93	5.12	4.95	5.11	5.06	5.91	-0.11	-0.07	0.00	-0.02	-0.20	0.78	-0.31
(SD)	(0.13)	(0.46)	(0.51)	(0.48)	(0.46)	(0.49)	(0.51)	(0.51)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)	(0.03)	(0.05)
Speed	0.10	-0.10	-0.13	-0.13	-0.13	-0.11	-0.10	-0.10	0.13	0.08	-0.02	-0.02	0.10	-0.12	-0.04
(SD)	(0.02)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.01)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)
Mobile	0.04	-0.07	-0.08	-0.08	-0.08	-0.06	-0.06	-0.06	0.01	0.12	-0.05	-0.02	0.09	-0.10	-0.04
(SD)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.01)	(0.01)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
Early	0.02	-0.02	-0.01	-0.01	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.14	0.20	-0.02	0.02	0.03
(SD)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.06)
Late	0.02	-0.02	-0.02	-0.01	-0.02	-0.02	-0.01	-0.02	0.00	0.00	0.03	0.14	-0.04	-0.01	0.00
(SD)	(0.02)	(0.05)	(0.06)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)	(0.06)	(0.06)
$\log(\sigma)$	-1.02	-0.41	-0.41	-0.44	-0.43	-0.34	-0.39	-0.26	0.02	0.02	0.00	-0.01	0.30	-0.39	-0.29
(SD)	(0.03)	(0.07)	(0.07)	(0.08)	(0.08)	(0.07)	(0.08)	(0.07)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.05)	(0.05)
$\log(\kappa)$	3.21	5.32	5.64	5.77	5.49	5.33	5.55	5.12	-0.12	-0.09	0.02	-0.01	-0.57	7.25	0.10
(SD)	(0.15)	(0.52)	(0.56)	(0.54)	(0.51)	(0.53)	(0.55)	(0.52)	(0.05)	(0.06)	(0.06)	(0.06)	(0.08)	(0.59)	(0.05)
$\log(E)$	5.97	-0.25	-0.33	-0.30	-0.34	-0.42	-0.39	-0.53	-0.01	-0.01	0.01	0.00	-0.11	0.20	0.51
(SD)	(0.03)	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.10)	(0.09)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.09)	(0.04)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A7: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Complementarity model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(\mu)$	$\log(E)$
MK	-1.48	0.94	0.52	0.44	0.39	0.31	0.33	0.16	0.10	-0.02	-0.06	-0.05	0.22	-0.48	-0.35
(SD)	(0.05)	(0.07)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.04)	(0.05)	(0.06)	(0.05)	(0.04)	(0.04)
Ep	-1.52	0.48	0.90	0.53	0.48	0.47	0.31	0.41	0.03	-0.01	-0.05	-0.05	0.19	-0.47	-0.49
(SD)	(0.05)	(0.07)	(0.08)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.06)	(0.04)
AK	-1.86	0.38	0.44	0.77	0.50	0.37	0.34	0.42	-0.01	-0.06	-0.01	-0.01	0.07	-0.35	-0.46
(SD)	(0.04)	(0.06)	(0.06)	(0.07)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.04)
HS	-2.05	0.32	0.38	0.37	0.72	0.58	0.39	0.41	0.00	-0.02	-0.02	-0.04	0.11	-0.38	-0.48
(SD)	(0.05)	(0.06)	(0.06)	(0.05)	(0.07)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.04)
US	-1.76	0.28	0.41	0.30	0.46	0.87	0.55	0.58	0.02	0.01	-0.05	-0.08	0.24	-0.47	-0.54
(SD)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.07)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.06)	(0.05)	(0.05)	(0.04)
IA	-1.98	0.31	0.29	0.29	0.32	0.50	0.95	0.42	0.06	0.03	-0.03	-0.01	0.15	-0.36	-0.49
(SD)	(0.05)	(0.05)	(0.07)	(0.06)	(0.06)	(0.06)	(0.08)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)
BG	-2.02	0.18	0.44	0.42	0.40	0.61	0.46	1.30	0.01	0.00	-0.01	-0.04	0.24	-0.46	-0.53
(SD)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.11)	(0.05)	(0.06)	(0.05)	(0.06)	(0.05)	(0.04)	(0.04)
Speed	0.07	0.03	0.01	0.00	0.00	0.01	0.02	0.00	0.11	0.05	-0.02	0.00	0.07	-0.10	-0.04
(SD)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
Mobile	0.02	-0.01	0.00	-0.02	-0.01	0.00	0.01	0.00	0.01	0.10	-0.05	-0.03	0.07	-0.08	-0.01
(SD)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.00)	(0.01)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
Early	0.02	-0.02	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.11	0.19	-0.04	0.06	-0.02
(SD)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.06)
Late	0.02	-0.02	-0.02	0.00	-0.01	-0.02	0.00	-0.02	0.00	0.00	0.02	0.12	-0.06	0.08	-0.01
(SD)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)	(0.05)	(0.05)
$\log(\sigma)$	-1.06	0.11	0.09	0.03	0.05	0.12	0.08	0.15	0.01	0.01	-0.01	-0.01	0.28	-0.63	-0.16
(SD)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)	(0.05)
$\log(\mu)$	0.45	-1.10	-1.04	-0.73	-0.76	-1.03	-0.83	-1.23	-0.08	-0.06	0.05	0.07	-0.78	5.51	0.27
(SD)	(0.12)	(0.14)	(0.17)	(0.15)	(0.13)	(0.14)	(0.13)	(0.17)	(0.04)	(0.04)	(0.05)	(0.04)	(0.07)	(0.47)	(0.04)
$\log(E)$	5.65	-0.17	-0.23	-0.20	-0.20	-0.25	-0.23	-0.30	-0.01	0.00	0.00	0.00	-0.04	0.31	0.25
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)	(0.02)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A8: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Linear Time model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(E)$	$\log(C)$
MK	0.09	1.73	0.88	0.86	0.85	0.84	0.84	0.77	0.00	-0.01	-0.03	0.00	-0.15	-0.35	-0.65
(SD)	(0.06)	(0.12)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.03)
Ep	0.05	1.52	1.72	0.88	0.88	0.87	0.84	0.83	-0.02	-0.01	-0.03	-0.00	-0.18	-0.38	-0.69
(SD)	(0.06)	(0.11)	(0.11)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
AK	-0.17	1.50	1.53	1.73	0.88	0.86	0.86	0.84	-0.03	-0.02	-0.03	-0.00	-0.24	-0.35	-0.68
(SD)	(0.06)	(0.10)	(0.10)	(0.11)	(0.01)	(0.01)	(0.01)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
HS	-0.31	1.51	1.55	1.56	1.80	0.90	0.87	0.84	-0.03	-0.02	-0.02	-0.00	-0.23	-0.40	-0.65
(SD)	(0.06)	(0.11)	(0.11)	(0.11)	(0.12)	(0.01)	(0.01)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
US	-0.10	1.50	1.55	1.53	1.64	1.83	0.89	0.87	-0.01	0.00	-0.03	-0.01	-0.16	-0.42	-0.69
(SD)	(0.06)	(0.11)	(0.11)	(0.11)	(0.11)	(0.12)	(0.01)	(0.01)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
IA	-0.23	1.49	1.49	1.53	1.57	1.63	1.82	0.84	-0.01	-0.00	-0.03	0.00	-0.20	-0.40	-0.69
(SD)	(0.06)	(0.11)	(0.10)	(0.10)	(0.11)	(0.11)	(0.12)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)
BG	-0.26	1.39	1.48	1.51	1.54	1.62	1.55	1.87	-0.00	-0.00	-0.02	-0.02	-0.10	-0.47	-0.71
(SD)	(0.06)	(0.11)	(0.11)	(0.11)	(0.11)	(0.12)	(0.11)	(0.13)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.02)
Speed	0.06	0.00	-0.01	-0.01	-0.01	-0.01	-0.00	-0.00	0.09	0.05	-0.01	0.01	0.14	-0.06	-0.01
(SD)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
Mobile	0.03	-0.00	-0.01	-0.01	-0.01	0.00	-0.00	-0.00	0.00	0.08	-0.02	-0.01	0.08	-0.03	-0.01
(SD)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.01)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
Early	0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.00	-0.00	0.08	0.11	0.00	0.01	0.02
(SD)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.00)	(0.01)	(0.05)	(0.06)	(0.05)	(0.05)
Late	0.00	0.00	-0.00	-0.00	-0.00	-0.01	0.00	-0.01	0.00	-0.00	0.01	0.09	-0.02	-0.01	0.01
(SD)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.00)	(0.00)	(0.00)	(0.01)	(0.06)	(0.05)	(0.05)
$\log(\sigma)$	-1.88	-0.22	-0.27	-0.35	-0.35	-0.24	-0.29	-0.15	0.05	0.03	0.00	-0.01	1.25	-0.29	-0.03
(SD)	(0.05)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.02)	(0.02)	(0.02)	(0.02)	(0.10)	(0.06)	(0.05)
$\log(E)$	6.85	-0.47	-0.51	-0.48	-0.55	-0.58	-0.55	-0.65	-0.02	-0.01	0.00	-0.00	-0.33	1.05	0.36
(SD)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.02)	(0.02)	(0.02)	(0.02)	(0.07)	(0.11)	(0.06)
$\log(C)$	2.61	-0.57	-0.61	-0.59	-0.58	-0.63	-0.62	-0.65	-0.00	-0.00	0.00	0.00	-0.02	0.24	0.44
(SD)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.04)	(0.04)	(0.03)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A9: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope1 model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
MK	0.50	2.25	0.77	0.74	0.75	0.68	0.69	0.49	-0.02	0.00	0.00	0.00	-0.14	-0.10	-0.41	0.09
(SD)	(0.07)	(0.15)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.06)	(0.05)	(0.08)	(0.04)	(0.06)
Ep	0.43	1.72	2.18	0.80	0.81	0.77	0.72	0.66	-0.05	0.01	-0.03	-0.02	-0.18	-0.16	-0.50	0.12
(SD)	(0.06)	(0.12)	(0.15)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.05)	(0.05)	(0.07)	(0.06)	(0.06)	(0.08)	(0.04)	(0.05)
AK	0.07	1.62	1.72	2.10	0.81	0.73	0.73	0.66	-0.07	0.00	-0.03	-0.02	-0.22	-0.16	-0.50	0.19
(SD)	(0.07)	(0.11)	(0.13)	(0.14)	(0.02)	(0.02)	(0.03)	(0.03)	(0.05)	(0.05)	(0.07)	(0.06)	(0.06)	(0.08)	(0.04)	(0.06)
HS	-0.13	1.65	1.75	1.71	2.13	0.83	0.79	0.68	-0.04	0.00	-0.02	-0.03	-0.21	-0.23	-0.48	0.14
(SD)	(0.07)	(0.12)	(0.13)	(0.12)	(0.13)	(0.01)	(0.02)	(0.03)	(0.05)	(0.05)	(0.07)	(0.06)	(0.05)	(0.08)	(0.04)	(0.06)
US	0.17	1.56	1.74	1.61	1.85	2.33	0.82	0.75	-0.04	0.02	-0.03	-0.06	-0.10	-0.25	-0.51	0.12
(SD)	(0.06)	(0.13)	(0.13)	(0.12)	(0.13)	(0.15)	(0.02)	(0.02)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.07)	(0.03)	(0.05)
IA	-0.04	1.59	1.65	1.63	1.76	1.94	2.37	0.69	-0.02	0.01	-0.04	-0.04	-0.16	-0.24	-0.50	0.12
(SD)	(0.06)	(0.13)	(0.13)	(0.13)	(0.13)	(0.14)	(0.14)	(0.03)	(0.05)	(0.05)	(0.06)	(0.07)	(0.05)	(0.07)	(0.04)	(0.06)
BG	-0.14	1.14	1.49	1.47	1.53	1.77	1.62	2.36	0.00	0.04	-0.03	-0.05	0.02	-0.33	-0.52	0.08
(SD)	(0.06)	(0.11)	(0.12)	(0.13)	(0.12)	(0.13)	(0.13)	(0.17)	(0.06)	(0.05)	(0.07)	(0.06)	(0.05)	(0.08)	(0.03)	(0.05)
Speed	0.08	-0.01	-0.03	-0.03	-0.02	-0.02	-0.01	0.00	0.12	0.06	0.00	0.00	0.13	-0.06	0.01	-0.11
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.06)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.06)
Mobile	0.05	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.01	0.11	-0.05	-0.03	0.08	-0.03	0.00	-0.02
(SD)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.01)	(0.01)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.06)
Early	0.02	0.00	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	0.00	-0.01	0.12	0.17	0.03	0.03	-0.01	-0.01
(SD)	(0.02)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.01)	(0.01)	(0.01)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
Late	0.00	0.00	-0.01	-0.01	-0.02	-0.04	-0.02	-0.03	0.00	0.00	0.02	0.13	0.00	0.01	0.00	0.00
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)	(0.06)	(0.05)	(0.06)
$\log(\sigma)$	-1.31	-0.21	-0.25	-0.31	-0.30	-0.15	-0.23	0.03	0.05	0.03	0.01	0.00	0.93	-0.20	-0.05	-0.23
(SD)	(0.04)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)	(0.08)	(0.07)	(0.02)	(0.02)	(0.02)	(0.02)	(0.08)	(0.06)	(0.05)	(0.05)
$\log(E)$	6.71	-0.14	-0.23	-0.22	-0.31	-0.35	-0.35	-0.47	-0.02	-0.01	0.01	0.00	-0.18	0.88	0.15	-0.13
(SD)	(0.09)	(0.12)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.02)	(0.02)	(0.02)	(0.02)	(0.06)	(0.11)	(0.09)	(0.08)
$\log(C)$	2.20	-0.34	-0.41	-0.40	-0.39	-0.43	-0.42	-0.44	0.00	0.00	0.00	0.00	-0.03	0.07	0.30	-0.12
(SD)	(0.02)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.05)	(0.02)	(0.05)
$\log(\theta-1)$	2.53	0.65	0.84	1.29	0.96	0.86	0.85	0.60	-0.19	-0.04	-0.01	0.00	-1.06	-0.56	-0.30	22.62
(SD)	(0.22)	(0.46)	(0.39)	(0.42)	(0.41)	(0.40)	(0.43)	(0.41)	(0.11)	(0.09)	(0.10)	(0.10)	(0.27)	(0.38)	(0.14)	(2.34)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A10: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope2 model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
MK	0.48	2.25	0.77	0.74	0.74	0.70	0.70	0.58	-0.01	-0.03	0.01	0.04	-0.21	-0.07	-0.45	0.02
(SD)	(0.06)	(0.15)	(0.02)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.04)	(0.05)
Ep	0.44	1.73	2.24	0.80	0.79	0.77	0.71	0.70	-0.04	-0.05	-0.02	0.01	-0.24	-0.13	-0.51	0.10
(SD)	(0.06)	(0.13)	(0.16)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.04)	(0.05)
AK	0.07	1.67	1.78	2.24	0.79	0.72	0.74	0.71	-0.08	-0.06	-0.02	0.01	-0.32	-0.10	-0.50	0.15
(SD)	(0.06)	(0.13)	(0.13)	(0.15)	(0.02)	(0.02)	(0.03)	(0.03)	(0.06)	(0.05)	(0.06)	(0.05)	(0.05)	(0.06)	(0.03)	(0.05)
HS	-0.16	1.63	1.74	1.72	2.13	0.84	0.77	0.71	-0.04	-0.05	0.00	0.02	-0.27	-0.21	-0.47	0.09
(SD)	(0.06)	(0.12)	(0.14)	(0.13)	(0.14)	(0.01)	(0.02)	(0.03)	(0.06)	(0.05)	(0.06)	(0.05)	(0.04)	(0.06)	(0.03)	(0.05)
US	0.18	1.58	1.73	1.63	1.83	2.26	0.81	0.77	-0.02	-0.02	-0.02	-0.02	-0.17	-0.22	-0.52	0.09
(SD)	(0.07)	(0.13)	(0.15)	(0.13)	(0.14)	(0.16)	(0.02)	(0.03)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.06)	(0.04)	(0.05)
IA	-0.01	1.60	1.62	1.70	1.72	1.86	2.31	0.71	-0.02	-0.01	-0.02	0.00	-0.23	-0.20	-0.50	0.09
(SD)	(0.07)	(0.13)	(0.14)	(0.14)	(0.13)	(0.15)	(0.16)	(0.02)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.04)	(0.05)
BG	-0.06	1.35	1.60	1.63	1.59	1.79	1.65	2.37	0.00	-0.01	-0.01	-0.02	-0.06	-0.27	-0.56	0.10
(SD)	(0.07)	(0.13)	(0.14)	(0.14)	(0.14)	(0.15)	(0.14)	(0.18)	(0.05)	(0.06)	(0.06)	(0.05)	(0.05)	(0.06)	(0.03)	(0.05)
Speed	0.09	-0.01	-0.02	-0.04	-0.02	-0.01	-0.01	0.00	0.13	0.06	0.00	-0.02	0.16	-0.07	0.01	-0.10
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.05)
Mobile	0.05	-0.02	-0.03	-0.03	-0.02	-0.01	0.00	-0.01	0.01	0.11	-0.03	-0.02	0.12	-0.04	0.02	-0.05
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.06)
Early	0.02	0.01	-0.01	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.12	0.18	0.02	0.02	0.01	0.02
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.06)	(0.07)
Late	0.00	0.02	0.01	0.01	0.01	-0.01	0.00	-0.01	0.00	0.00	0.02	0.13	-0.03	0.01	0.03	0.03
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.07)	(0.06)	(0.05)	(0.06)
$\log(\sigma)$	-1.29	-0.31	-0.35	-0.46	-0.38	-0.25	-0.33	-0.09	0.06	0.04	0.00	-0.01	0.92	-0.21	-0.04	-0.19
(SD)	(0.05)	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)	(0.09)	(0.07)	(0.02)	(0.02)	(0.02)	(0.03)	(0.08)	(0.07)	(0.05)	(0.05)
$\log(E)$	6.69	-0.10	-0.19	-0.14	-0.29	-0.31	-0.29	-0.40	-0.02	-0.01	0.01	0.00	-0.20	0.91	0.18	-0.11
(SD)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.10)	(0.02)	(0.02)	(0.02)	(0.02)	(0.07)	(0.13)	(0.08)	(0.12)
$\log(C)$	2.20	-0.37	-0.42	-0.41	-0.38	-0.43	-0.42	-0.47	0.00	0.00	0.00	0.01	-0.02	0.09	0.30	-0.11
(SD)	(0.02)	(0.04)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.04)	(0.02)	(0.06)
$\log(\theta-1)$	2.67	0.15	0.69	1.09	0.64	0.63	0.67	0.70	-0.16	-0.08	0.03	0.06	-0.84	-0.52	-0.29	22.26
(SD)	(0.24)	(0.38)	(0.38)	(0.38)	(0.34)	(0.36)	(0.40)	(0.39)	(0.09)	(0.09)	(0.12)	(0.10)	(0.25)	(0.55)	(0.15)	(2.26)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A11: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope3 model: Theme Park data

Variables	Mean	MK	Ep	AK	HS	US	IA	BG	Speed	Mobile	Early	Late	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
MK	0.42	2.24	0.77	0.74	0.74	0.68	0.69	0.51	0.01	-0.03	-0.03	0.01	-0.20	-0.07	-0.42	0.01
(SD)	(0.08)	(0.16)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)	(0.05)	(0.06)	(0.06)	(0.06)	(0.03)	(0.07)
Ep	0.38	1.75	2.29	0.79	0.78	0.75	0.68	0.64	-0.02	-0.04	-0.04	-0.01	-0.22	-0.15	-0.49	0.10
(SD)	(0.08)	(0.15)	(0.16)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.06)	(0.05)	(0.05)	(0.06)	(0.05)	(0.07)	(0.04)	(0.08)
AK	-0.01	1.64	1.79	2.21	0.79	0.70	0.70	0.66	-0.03	-0.06	-0.05	-0.02	-0.27	-0.11	-0.48	0.12
(SD)	(0.08)	(0.13)	(0.15)	(0.16)	(0.02)	(0.03)	(0.02)	(0.03)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.04)	(0.08)
HS	-0.23	1.62	1.74	1.72	2.15	0.83	0.74	0.68	-0.01	-0.05	-0.02	0.01	-0.19	-0.26	-0.46	0.07
(SD)	(0.09)	(0.13)	(0.14)	(0.15)	(0.14)	(0.01)	(0.02)	(0.03)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.04)	(0.08)
US	0.14	1.54	1.70	1.56	1.82	2.26	0.78	0.77	-0.01	0.00	-0.02	-0.02	-0.11	-0.26	-0.51	0.07
(SD)	(0.08)	(0.13)	(0.14)	(0.14)	(0.13)	(0.15)	(0.02)	(0.02)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.06)	(0.03)	(0.07)
IA	-0.11	1.53	1.53	1.54	1.63	1.76	2.22	0.67	0.01	-0.01	-0.03	-0.01	-0.18	-0.22	-0.48	0.09
(SD)	(0.08)	(0.13)	(0.14)	(0.13)	(0.12)	(0.13)	(0.15)	(0.03)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.03)	(0.08)
BG	-0.15	1.20	1.50	1.52	1.56	1.80	1.55	2.44	0.03	0.01	0.00	-0.01	0.01	-0.31	-0.52	0.10
(SD)	(0.08)	(0.13)	(0.13)	(0.13)	(0.11)	(0.13)	(0.12)	(0.14)	(0.06)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.04)	(0.07)
Speed	0.09	0.01	-0.01	-0.02	-0.01	0.00	0.00	0.02	0.13	0.06	0.00	0.01	0.13	-0.05	-0.04	-0.11
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.05)	(0.06)	(0.05)	(0.06)	(0.06)	(0.04)	(0.05)
Mobile	0.05	-0.01	-0.02	-0.03	-0.02	0.00	-0.01	0.00	0.01	0.11	-0.04	-0.03	0.09	-0.01	0.00	-0.03
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.05)	(0.05)	(0.07)
Early	0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.12	0.16	0.03	0.01	-0.01	-0.02
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)	(0.05)	(0.06)	(0.06)	(0.05)	(0.07)
Late	0.01	0.00	0.00	-0.01	0.01	-0.01	-0.01	-0.01	0.00	0.00	0.02	0.13	0.01	-0.02	0.02	0.02
(SD)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.07)	(0.07)	(0.05)	(0.06)
$\log(\sigma)$	-1.28	-0.28	-0.32	-0.39	-0.27	-0.15	-0.25	0.01	0.04	0.03	0.01	0.00	0.91	-0.23	-0.05	-0.19
(SD)	(0.05)	(0.09)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)	(0.07)	(0.02)	(0.02)	(0.02)	(0.02)	(0.06)	(0.06)	(0.05)	(0.06)
$\log(E)$	6.68	-0.10	-0.22	-0.16	-0.37	-0.38	-0.32	-0.47	-0.02	0.00	0.00	-0.01	-0.21	0.94	0.19	-0.03
(SD)	(0.08)	(0.09)	(0.10)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.02)	(0.02)	(0.02)	(0.02)	(0.06)	(0.10)	(0.07)	(0.09)
$\log(C)$	2.22	-0.35	-0.41	-0.40	-0.37	-0.42	-0.39	-0.45	-0.01	0.00	0.00	0.00	-0.02	0.10	0.30	-0.17
(SD)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.04)	(0.02)	(0.07)
$\log(\theta-1)$	2.80	0.09	0.75	0.89	0.50	0.54	0.64	0.73	-0.19	-0.05	-0.04	0.03	-0.88	-0.14	-0.44	22.79
(SD)	(0.26)	(0.51)	(0.62)	(0.59)	(0.58)	(0.53)	(0.57)	(0.56)	(0.09)	(0.11)	(0.11)	(0.10)	(0.27)	(0.40)	(0.19)	(2.17)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A12: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope3 model: Theme Park data

Company	Group 1	Group 2	Group 3
Disney	0.86	0.09	0.05
	(0.02)	(0.02)	(0.02)
Universal	0.94	0.02	0.04
	(0.02)	(0.01)	(0.01)
Busch	0.89	0.04	0.07
	(0.03)	(0.01)	(0.02)

Table A13: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Standard model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$
SupR	-3.13	2.26	0.72	0.61	0.66	0.67	0.07	-0.54
(SD)	(0.29)	(0.57)	(0.08)	(0.10)	(0.09)	(0.09)	(0.15)	(0.11)
UnsR	-3.34	1.58	2.08	0.61	0.65	0.65	0.09	-0.53
(SD)	(0.25)	(0.51)	(0.54)	(0.11)	(0.08)	(0.10)	(0.14)	(0.12)
SupPy	-2.77	1.30	1.26	2.00	0.68	0.65	0.24	-0.66
(SD)	(0.24)	(0.43)	(0.44)	(0.44)	(0.08)	(0.10)	(0.15)	(0.09)
UnsPy	-2.90	1.43	1.36	1.39	2.06	0.67	0.13	-0.61
(SD)	(0.27)	(0.42)	(0.42)	(0.37)	(0.46)	(0.09)	(0.14)	(0.09)
Case	-2.78	1.50	1.41	1.37	1.43	2.18	0.16	-0.64
(SD)	(0.24)	(0.48)	(0.48)	(0.40)	(0.42)	(0.48)	(0.15)	(0.09)
$\log(\sigma)$	-0.37	0.07	0.09	0.21	0.12	0.15	0.39	-0.40
(SD)	(0.10)	(0.15)	(0.14)	(0.14)	(0.14)	(0.15)	(0.07)	(0.11)
$\log(E)$	4.56	-0.98	-0.93	-1.11	-1.05	-1.13	-0.30	1.40
(SD)	(0.19)	(0.37)	(0.39)	(0.34)	(0.31)	(0.34)	(0.11)	(0.32)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A14: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Satiation model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(\kappa)$	$\log(E)$
SupR	-1.50	3.46	0.77	0.64	0.68	0.70	-0.24	0.48	-0.02
(SD)	(0.33)	(0.88)	(0.07)	(0.12)	(0.10)	(0.09)	(0.14)	(0.14)	(0.18)
UnsR	-1.71	2.48	2.99	0.67	0.69	0.68	-0.23	0.50	-0.01
(SD)	(0.32)	(0.70)	(0.77)	(0.11)	(0.08)	(0.10)	(0.14)	(0.13)	(0.17)
SupPy	-1.04	1.89	1.85	2.49	0.70	0.66	-0.10	0.53	-0.10
(SD)	(0.32)	(0.64)	(0.63)	(0.63)	(0.08)	(0.10)	(0.15)	(0.13)	(0.17)
UnsPy	-1.27	2.22	2.06	1.93	2.96	0.70	-0.22	0.55	-0.02
(SD)	(0.36)	(0.73)	(0.63)	(0.59)	(0.69)	(0.08)	(0.15)	(0.13)	(0.18)
Case	-0.88	2.39	2.13	1.90	2.19	3.27	-0.14	0.53	-0.14
(SD)	(0.34)	(0.74)	(0.65)	(0.63)	(0.67)	(0.82)	(0.14)	(0.12)	(0.16)
$\log(\sigma)$	-0.11	-0.32	-0.28	-0.11	-0.27	-0.17	0.51	-0.25	-0.43
(SD)	(0.12)	(0.19)	(0.19)	(0.17)	(0.19)	(0.18)	(0.13)	(0.13)	(0.13)
$\log(\kappa)$	2.60	1.97	1.85	1.79	2.05	2.06	-0.37	4.54	0.41
(SD)	(0.39)	(0.94)	(0.81)	(0.73)	(0.82)	(0.83)	(0.22)	(1.40)	(0.16)
$\log(E)$	4.43	-0.03	-0.02	-0.18	-0.04	-0.30	-0.35	1.05	1.32
(SD)	(0.22)	(0.37)	(0.34)	(0.31)	(0.36)	(0.35)	(0.15)	(0.55)	(0.30)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A15: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Complementarity model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(\mu)$	$\log(E)$
SupR	-2.90	1.42	0.34	-0.07	0.06	0.14	-0.04	-0.02	-0.09
(SD)	(0.22)	(0.39)	(0.17)	(0.16)	(0.19)	(0.17)	(0.14)	(0.16)	(0.13)
UnsR	-3.22	0.42	0.98	0.01	0.09	0.08	-0.07	0.03	-0.04
(SD)	(0.17)	(0.26)	(0.23)	(0.15)	(0.15)	(0.15)	(0.16)	(0.18)	(0.13)
SupPy	-2.45	-0.09	0.00	0.93	0.17	0.07	0.15	-0.14	-0.18
(SD)	(0.16)	(0.20)	(0.15)	(0.23)	(0.15)	(0.16)	(0.15)	(0.17)	(0.15)
UnsPy	-2.69	0.08	0.09	0.17	0.97	0.16	0.00	-0.05	-0.17
(SD)	(0.19)	(0.24)	(0.16)	(0.17)	(0.23)	(0.16)	(0.14)	(0.17)	(0.14)
Case	-2.42	0.18	0.09	0.07	0.17	1.18	0.10	-0.20	-0.24
(SD)	(0.19)	(0.23)	(0.17)	(0.17)	(0.18)	(0.28)	(0.13)	(0.16)	(0.12)
$\log(\sigma)$	-0.10	-0.03	-0.05	0.09	0.00	0.07	0.42	-0.48	-0.29
(SD)	(0.10)	(0.11)	(0.10)	(0.10)	(0.09)	(0.10)	(0.08)	(0.12)	(0.11)
$\log(\mu)$	0.86	-0.03	0.03	-0.21	-0.07	-0.33	-0.47	2.20	0.30
(SD)	(0.25)	(0.31)	(0.27)	(0.27)	(0.27)	(0.29)	(0.17)	(0.58)	(0.12)
$\log(E)$	3.56	-0.07	-0.03	-0.12	-0.11	-0.17	-0.12	0.29	0.42
(SD)	(0.11)	(0.10)	(0.08)	(0.10)	(0.10)	(0.10)	(0.05)	(0.13)	(0.07)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A16: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Linear Time model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$	$\log(C)$
SupR	-1.69	2.72	0.77	0.70	0.74	0.75	-0.07	-0.64	-0.59
(SD)	(0.30)	(0.66)	(0.06)	(0.07)	(0.07)	(0.06)	(0.15)	(0.10)	(0.10)
UnsR	-1.82	1.97	2.39	0.70	0.74	0.74	-0.04	-0.67	-0.60
(SD)	(0.29)	(0.54)	(0.56)	(0.08)	(0.07)	(0.07)	(0.13)	(0.08)	(0.09)
SupPy	-1.46	1.78	1.69	2.37	0.76	0.74	0.07	-0.71	-0.67
(SD)	(0.29)	(0.51)	(0.48)	(0.55)	(0.06)	(0.06)	(0.13)	(0.07)	(0.08)
UnsPy	-1.56	2.02	1.90	1.95	2.72	0.76	-0.02	-0.71	-0.65
(SD)	(0.29)	(0.60)	(0.53)	(0.53)	(0.65)	(0.06)	(0.14)	(0.07)	(0.09)
Case	-1.37	2.06	1.91	1.91	2.10	2.75	0.01	-0.72	-0.67
(SD)	(0.28)	(0.56)	(0.54)	(0.52)	(0.57)	(0.62)	(0.14)	(0.07)	(0.08)
$\log(\sigma)$	-0.85	-0.11	-0.06	0.10	-0.04	0.01	0.87	-0.19	-0.16
(SD)	(0.15)	(0.23)	(0.18)	(0.19)	(0.21)	(0.22)	(0.20)	(0.12)	(0.12)
$\log(E)$	4.56	-1.31	-1.29	-1.36	-1.45	-1.50	-0.22	1.52	0.70
(SD)	(0.22)	(0.43)	(0.39)	(0.39)	(0.43)	(0.44)	(0.15)	(0.37)	(0.07)
$\log(C)$	3.72	-1.25	-1.21	-1.33	-1.39	-1.43	-0.19	1.11	1.64
(SD)	(0.24)	(0.42)	(0.40)	(0.38)	(0.44)	(0.44)	(0.15)	(0.32)	(0.44)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A17: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope1 model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
SupR	-1.45	2.73	0.71	0.59	0.66	0.66	-0.16	-0.50	-0.40	-0.12
(SD)	(0.25)	(0.56)	(0.07)	(0.11)	(0.09)	(0.09)	(0.14)	(0.13)	(0.11)	(0.19)
UnsR	-1.57	1.69	2.02	0.55	0.64	0.63	-0.13	-0.51	-0.41	-0.06
(SD)	(0.24)	(0.47)	(0.50)	(0.12)	(0.09)	(0.09)	(0.14)	(0.12)	(0.12)	(0.21)
SupPy	-1.08	1.35	1.10	1.91	0.69	0.61	0.04	-0.62	-0.52	-0.21
(SD)	(0.26)	(0.43)	(0.40)	(0.44)	(0.08)	(0.10)	(0.16)	(0.08)	(0.10)	(0.18)
UnsPy	-1.26	1.73	1.44	1.54	2.55	0.67	-0.11	-0.58	-0.49	-0.20
(SD)	(0.27)	(0.44)	(0.38)	(0.42)	(0.50)	(0.09)	(0.16)	(0.10)	(0.10)	(0.18)
Case	-0.97	1.73	1.42	1.36	1.70	2.52	-0.07	-0.61	-0.50	-0.11
(SD)	(0.27)	(0.46)	(0.41)	(0.42)	(0.44)	(0.53)	(0.17)	(0.10)	(0.10)	(0.18)
$\log(\sigma)$	-0.55	-0.25	-0.17	0.05	-0.17	-0.10	0.91	-0.21	-0.17	-0.13
(SD)	(0.16)	(0.23)	(0.19)	(0.22)	(0.25)	(0.26)	(0.22)	(0.14)	(0.13)	(0.21)
$\log(E)$	4.37	-0.97	-0.86	-1.02	-1.09	-1.14	-0.23	1.38	0.59	0.15
(SD)	(0.21)	(0.37)	(0.31)	(0.28)	(0.31)	(0.33)	(0.16)	(0.27)	(0.09)	(0.20)
$\log(C)$	2.79	-0.61	-0.53	-0.65	-0.71	-0.72	-0.15	0.63	0.82	0.21
(SD)	(0.15)	(0.23)	(0.21)	(0.21)	(0.22)	(0.23)	(0.12)	(0.17)	(0.18)	(0.18)
$\log(\theta-1)$	1.85	-0.18	-0.09	-0.28	-0.31	-0.16	-0.13	0.18	0.19	0.94
(SD)	(0.31)	(0.32)	(0.30)	(0.27)	(0.30)	(0.30)	(0.20)	(0.23)	(0.18)	(0.36)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A18: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope2 model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
SupR	-1.41	2.53	0.68	0.59	0.65	0.65	-0.19	-0.52	-0.39	0.18
(SD)	(0.26)	(0.51)	(0.09)	(0.09)	(0.09)	(0.07)	(0.15)	(0.11)	(0.12)	(0.26)
UnsR	-1.58	1.48	1.83	0.55	0.59	0.59	-0.12	-0.51	-0.37	0.17
(SD)	(0.23)	(0.43)	(0.44)	(0.12)	(0.11)	(0.11)	(0.15)	(0.12)	(0.12)	(0.25)
SupPy	-1.06	1.25	1.00	1.78	0.66	0.60	-0.01	-0.60	-0.49	0.17
(SD)	(0.23)	(0.35)	(0.34)	(0.39)	(0.08)	(0.09)	(0.15)	(0.11)	(0.12)	(0.24)
UnsPy	-1.24	1.62	1.25	1.37	2.40	0.67	-0.20	-0.57	-0.44	0.17
(SD)	(0.25)	(0.43)	(0.41)	(0.34)	(0.47)	(0.08)	(0.14)	(0.10)	(0.11)	(0.26)
Case	-0.98	1.62	1.25	1.24	1.61	2.42	-0.12	-0.61	-0.47	0.22
(SD)	(0.26)	(0.41)	(0.39)	(0.36)	(0.42)	(0.51)	(0.15)	(0.09)	(0.10)	(0.26)
$\log(\sigma)$	-0.59	-0.29	-0.16	-0.01	-0.30	-0.18	0.93	-0.13	-0.16	-0.08
(SD)	(0.17)	(0.24)	(0.20)	(0.20)	(0.23)	(0.24)	(0.23)	(0.15)	(0.13)	(0.24)
$\log(E)$	4.38	-0.94	-0.79	-0.91	-1.00	-1.07	-0.14	1.27	0.53	-0.17
(SD)	(0.19)	(0.31)	(0.29)	(0.29)	(0.31)	(0.31)	(0.17)	(0.29)	(0.11)	(0.27)
$\log(C)$	2.93	-0.55	-0.45	-0.58	-0.61	-0.66	-0.14	0.54	0.79	-0.10
(SD)	(0.15)	(0.20)	(0.18)	(0.20)	(0.19)	(0.22)	(0.12)	(0.17)	(0.15)	(0.24)
$\log(\theta-1)$	4.72	0.33	0.26	0.25	0.30	0.38	-0.08	-0.23	-0.11	0.90
(SD)	(0.95)	(0.48)	(0.41)	(0.38)	(0.46)	(0.51)	(0.23)	(0.37)	(0.23)	(0.51)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A19: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope2 model: Machine Learning data

Course	Group 1	Group 2
SupR	0.65	0.35
	(0.18)	(0.18)
UnsR	0.63	0.37
	(0.19)	(0.19)
SupPy	0.62	0.38
	(0.17)	(0.17)
UnsPy	0.62	0.38
	(0.18)	(0.18)
Case	0.66	0.34
	(0.17)	(0.17)

Table A20: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope3 model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
SupR	-1.43	2.32	0.70	0.59	0.65	0.62	-0.16	-0.50	-0.38	0.10
(SD)	(0.25)	(0.50)	(0.08)	(0.11)	(0.10)	(0.10)	(0.16)	(0.12)	(0.13)	(0.18)
UnsR	-1.49	1.48	1.89	0.58	0.64	0.61	-0.13	-0.53	-0.37	0.09
(SD)	(0.23)	(0.40)	(0.43)	(0.10)	(0.09)	(0.10)	(0.14)	(0.11)	(0.12)	(0.18)
SupPy	-1.09	1.20	1.07	1.78	0.66	0.60	0.01	-0.58	-0.45	0.04
(SD)	(0.25)	(0.39)	(0.37)	(0.44)	(0.08)	(0.09)	(0.14)	(0.10)	(0.13)	(0.18)
UnsPy	-1.25	1.48	1.33	1.31	2.21	0.63	-0.14	-0.57	-0.43	0.08
(SD)	(0.24)	(0.43)	(0.40)	(0.39)	(0.47)	(0.10)	(0.16)	(0.11)	(0.13)	(0.18)
Case	-0.99	1.39	1.24	1.18	1.37	2.13	-0.07	-0.60	-0.45	0.11
(SD)	(0.22)	(0.42)	(0.40)	(0.38)	(0.43)	(0.50)	(0.14)	(0.10)	(0.12)	(0.18)
$\log(\sigma)$	-0.58	-0.23	-0.16	0.01	-0.19	-0.10	0.86	-0.14	-0.16	-0.21
(SD)	(0.16)	(0.24)	(0.19)	(0.18)	(0.23)	(0.20)	(0.20)	(0.13)	(0.13)	(0.18)
$\log(E)$	4.34	-0.81	-0.77	-0.82	-0.91	-0.93	-0.14	1.11	0.49	-0.02
(SD)	(0.21)	(0.33)	(0.30)	(0.31)	(0.34)	(0.33)	(0.14)	(0.31)	(0.11)	(0.20)
$\log(C)$	2.99	-0.52	-0.46	-0.54	-0.57	-0.59	-0.14	0.46	0.79	0.05
(SD)	(0.13)	(0.22)	(0.20)	(0.22)	(0.23)	(0.23)	(0.12)	(0.17)	(0.17)	(0.18)
$\log(\theta-1)$	5.62	0.12	0.09	0.03	0.09	0.12	-0.14	-0.01	0.03	0.53
(SD)	(0.26)	(0.22)	(0.19)	(0.18)	(0.20)	(0.20)	(0.13)	(0.16)	(0.13)	(0.18)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A21: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope3 model: Machine Learning data

Course	Group 1	Group 2	Group 3
SupR	0.39	0.28	0.33
	(0.21)	(0.16)	(0.19)
UnsR	0.39	0.28	0.33
	(0.21)	(0.17)	(0.20)
SupPY	0.36	0.30	0.33
	(0.14)	(0.13)	(0.14)
UnsPy	0.38	0.30	0.32
	(0.15)	(0.13)	(0.14)
Case	0.39	0.28	0.33
	(0.22)	(0.17)	(0.19)

Table A22: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope4 model: Machine Learning data

Variables	Mean	SupR	UnsR	SupPy	UnsPy	Case	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
SupR	-1.43	2.22	0.66	0.53	0.64	0.60	-0.13	-0.52	-0.37	0.07
(SD)	(0.28)	(0.51)	(0.09)	(0.13)	(0.08)	(0.10)	(0.14)	(0.12)	(0.13)	(0.18)
UnsR	-1.53	1.30	1.72	0.52	0.59	0.57	-0.07	-0.50	-0.36	0.05
(SD)	(0.22)	(0.37)	(0.38)	(0.11)	(0.09)	(0.12)	(0.14)	(0.12)	(0.12)	(0.16)
SupPy	-1.10	1.04	0.90	1.71	0.62	0.54	0.10	-0.57	-0.46	0.01
(SD)	(0.26)	(0.39)	(0.34)	(0.41)	(0.09)	(0.11)	(0.14)	(0.10)	(0.14)	(0.16)
UnsPy	-1.26	1.38	1.13	1.18	2.07	0.61	-0.08	-0.59	-0.45	0.05
(SD)	(0.26)	(0.38)	(0.35)	(0.37)	(0.48)	(0.10)	(0.13)	(0.09)	(0.11)	(0.17)
Case	-1.03	1.28	1.06	1.01	1.23	1.97	-0.04	-0.59	-0.44	0.08
(SD)	(0.25)	(0.41)	(0.37)	(0.34)	(0.38)	(0.45)	(0.13)	(0.11)	(0.11)	(0.16)
$\log(\sigma)$	-0.62	-0.19	-0.10	0.13	-0.11	-0.06	0.93	-0.17	-0.23	-0.23
(SD)	(0.18)	(0.21)	(0.18)	(0.19)	(0.19)	(0.18)	(0.25)	(0.12)	(0.13)	(0.18)
$\log(E)$	4.32	-0.83	-0.71	-0.82	-0.92	-0.91	-0.18	1.16	0.54	0.01
(SD)	(0.18)	(0.29)	(0.25)	(0.27)	(0.28)	(0.30)	(0.14)	(0.26)	(0.10)	(0.17)
$\log(C)$	3.06	-0.52	-0.45	-0.58	-0.62	-0.59	-0.21	0.55	0.89	0.05
(SD)	(0.16)	(0.24)	(0.19)	(0.24)	(0.22)	(0.21)	(0.13)	(0.17)	(0.19)	(0.17)
$\log(\theta-1)$	5.66	0.08	0.06	0.02	0.05	0.09	-0.16	0.01	0.04	0.54
(SD)	(0.21)	(0.21)	(0.16)	(0.17)	(0.19)	(0.17)	(0.13)	(0.14)	(0.12)	(0.15)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A23: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope4 model: Machine Learning data

Course	Group 1	Group 2	Group 3	Group 4
SupR	0.22	0.28	0.27	0.23
	(0.12)	(0.16)	(0.15)	(0.13)
UnsR	0.23	0.29	0.26	0.22
	(0.14)	(0.18)	(0.17)	(0.13)
SupPy	0.22	0.27	0.26	0.25
	(0.11)	(0.14)	(0.14)	(0.13)
UnsPy	0.21	0.28	0.27	0.24
	(0.12)	(0.19)	(0.17)	(0.14)
Case	0.22	0.28	0.28	0.23
	(0.13)	(0.16)	(0.16)	(0.14)

Table A24: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Standard model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$
CPn	-2.54	1.43	0.18	0.23	-0.08	-0.05	0.11	-0.43
(SD)	(0.17)	(0.37)	(0.18)	(0.16)	(0.20)	(0.19)	(0.15)	(0.12)
JPn	-3.09	0.28	1.51	0.22	0.21	0.22	0.05	-0.30
(SD)	(0.21)	(0.29)	(0.36)	(0.15)	(0.16)	(0.18)	(0.14)	(0.16)
CGt	-2.85	0.35	0.33	1.45	0.22	0.07	0.02	-0.31
(SD)	(0.19)	(0.28)	(0.26)	(0.34)	(0.17)	(0.16)	(0.14)	(0.14)
JGt	-3.62	-0.14	0.39	0.42	2.17	0.51	-0.09	-0.11
(SD)	(0.30)	(0.36)	(0.34)	(0.36)	(0.67)	(0.14)	(0.14)	(0.17)
Sax	-4.53	-0.09	0.39	0.14	1.11	2.04	-0.19	0.10
(SD)	(0.36)	(0.32)	(0.34)	(0.31)	(0.53)	(0.65)	(0.14)	(0.16)
$\log(\sigma)$	-0.36	0.07	0.04	0.02	-0.07	-0.15	0.30	-0.29
(SD)	(0.09)	(0.10)	(0.09)	(0.10)	(0.12)	(0.12)	(0.06)	(0.12)
$\log(E)$	4.22	-0.43	-0.31	-0.32	-0.15	0.10	-0.13	0.67
(SD)	(0.13)	(0.19)	(0.19)	(0.20)	(0.23)	(0.20)	(0.06)	(0.17)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A25: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Satiation model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(\kappa)$	$\log(E)$
CPn	-1.21	1.57	-0.06	-0.10	-0.31	-0.12	-0.04	0.00	-0.11
(SD)	(0.29)	(0.39)	(0.18)	(0.16)	(0.17)	(0.17)	(0.13)	(0.17)	(0.14)
JPn	-1.94	-0.10	2.17	0.06	-0.02	0.15	0.01	0.19	0.03
(SD)	(0.31)	(0.33)	(0.64)	(0.19)	(0.21)	(0.19)	(0.14)	(0.20)	(0.15)
CGt	-1.66	-0.14	0.13	1.67	0.00	0.00	-0.08	0.17	0.15
(SD)	(0.26)	(0.28)	(0.40)	(0.51)	(0.19)	(0.20)	(0.15)	(0.18)	(0.14)
JGt	-2.70	-0.65	-0.02	0.00	2.87	0.54	-0.12	-0.13	0.05
(SD)	(0.38)	(0.42)	(0.54)	(0.47)	(0.89)	(0.13)	(0.15)	(0.20)	(0.14)
Sax	-3.74	-0.25	0.38	0.00	1.50	2.72	-0.20	0.06	0.27
(SD)	(0.37)	(0.38)	(0.48)	(0.46)	(0.59)	(0.91)	(0.15)	(0.22)	(0.16)
$\log(\sigma)$	-0.08	-0.03	0.01	-0.06	-0.11	-0.19	0.33	0.03	-0.20
(SD)	(0.10)	(0.09)	(0.13)	(0.11)	(0.15)	(0.16)	(0.06)	(0.15)	(0.13)
$\log(\kappa)$	2.39	0.02	0.36	0.28	-0.28	0.15	0.02	1.52	0.52
(SD)	(0.32)	(0.27)	(0.41)	(0.31)	(0.44)	(0.48)	(0.10)	(0.50)	(0.13)
$\log(E)$	4.24	-0.13	0.03	0.17	0.06	0.40	-0.11	0.59	0.81
(SD)	(0.15)	(0.15)	(0.20)	(0.17)	(0.21)	(0.26)	(0.07)	(0.24)	(0.17)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A26: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Complementarity model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(\mu)$	$\log(E)$
CPn	-2.36	1.50	-0.17	-0.02	-0.30	-0.16	0.07	-0.13	-0.18
(SD)	(0.19)	(0.37)	(0.16)	(0.14)	(0.15)	(0.17)	(0.14)	(0.14)	(0.13)
JPn	-3.04	-0.29	1.86	-0.08	-0.06	0.12	0.04	0.17	-0.08
(SD)	(0.24)	(0.30)	(0.54)	(0.17)	(0.18)	(0.19)	(0.14)	(0.16)	(0.14)
CGt	-2.80	-0.03	-0.14	1.51	0.01	-0.06	-0.05	-0.14	-0.04
(SD)	(0.21)	(0.22)	(0.29)	(0.37)	(0.14)	(0.17)	(0.11)	(0.15)	(0.13)
JGt	-3.72	-0.60	-0.11	0.01	2.62	0.46	-0.06	0.00	0.05
(SD)	(0.27)	(0.34)	(0.39)	(0.29)	(0.68)	(0.16)	(0.13)	(0.17)	(0.13)
Sax	-4.81	-0.31	0.28	-0.12	1.22	2.56	-0.18	0.34	0.17
(SD)	(0.39)	(0.34)	(0.43)	(0.33)	(0.61)	(0.76)	(0.15)	(0.18)	(0.13)
$\log(\sigma)$	-0.12	0.05	0.03	-0.03	-0.06	-0.16	0.31	-0.17	-0.25
(SD)	(0.09)	(0.10)	(0.11)	(0.08)	(0.12)	(0.14)	(0.06)	(0.15)	(0.12)
$\log(\mu)$	0.76	-0.22	0.30	-0.22	0.00	0.71	-0.13	1.68	0.22
(SD)	(0.25)	(0.24)	(0.29)	(0.24)	(0.36)	(0.44)	(0.13)	(0.42)	(0.12)
$\log(E)$	3.48	-0.13	-0.06	-0.03	0.05	0.15	-0.08	0.16	0.33
(SD)	(0.08)	(0.10)	(0.11)	(0.10)	(0.13)	(0.13)	(0.04)	(0.10)	(0.06)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A27: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Linear Time model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$	$\log(C)$
CPn	-1.16	2.94	0.70	0.70	0.58	0.59	-0.03	-0.71	-0.69
(SD)	(0.25)	(0.59)	(0.09)	(0.08)	(0.10)	(0.11)	(0.14)	(0.07)	(0.07)
JPn	-1.53	2.15	3.19	0.68	0.65	0.66	-0.04	-0.66	-0.62
(SD)	(0.28)	(0.59)	(0.70)	(0.08)	(0.09)	(0.10)	(0.13)	(0.08)	(0.09)
CGt	-1.43	1.93	1.95	2.50	0.63	0.58	-0.06	-0.70	-0.63
(SD)	(0.25)	(0.52)	(0.56)	(0.55)	(0.10)	(0.10)	(0.15)	(0.08)	(0.09)
JGt	-1.88	1.68	1.95	1.66	2.76	0.71	0.01	-0.70	-0.52
(SD)	(0.27)	(0.49)	(0.57)	(0.49)	(0.59)	(0.08)	(0.13)	(0.07)	(0.11)
Sax	-2.50	1.66	1.94	1.51	1.94	2.65	-0.10	-0.63	-0.45
(SD)	(0.26)	(0.55)	(0.64)	(0.53)	(0.58)	(0.70)	(0.13)	(0.09)	(0.13)
$\log(\sigma)$	-1.03	-0.03	-0.05	-0.06	0.01	-0.10	0.41	-0.09	-0.04
(SD)	(0.11)	(0.16)	(0.15)	(0.16)	(0.14)	(0.14)	(0.09)	(0.13)	(0.13)
$\log(E)$	4.89	-1.63	-1.57	-1.50	-1.56	-1.39	-0.08	1.79	0.58
(SD)	(0.22)	(0.44)	(0.46)	(0.45)	(0.43)	(0.47)	(0.13)	(0.49)	(0.10)
$\log(C)$	3.31	-1.31	-1.23	-1.12	-0.97	-0.82	-0.03	0.86	1.23
(SD)	(0.18)	(0.34)	(0.35)	(0.33)	(0.31)	(0.33)	(0.09)	(0.27)	(0.25)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A28: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope1 model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
CPn	-0.85	2.21	0.48	0.46	0.17	0.27	0.05	-0.59	-0.49	-0.05
(SD)	(0.24)	(0.46)	(0.13)	(0.12)	(0.16)	(0.14)	(0.15)	(0.10)	(0.10)	(0.22)
JPn	-1.35	1.22	2.89	0.49	0.35	0.43	0.06	-0.47	-0.44	-0.15
(SD)	(0.26)	(0.47)	(0.66)	(0.14)	(0.15)	(0.14)	(0.15)	(0.12)	(0.13)	(0.20)
CGt	-1.26	0.98	1.19	2.03	0.30	0.28	0.01	-0.53	-0.41	-0.17
(SD)	(0.24)	(0.39)	(0.46)	(0.49)	(0.15)	(0.16)	(0.16)	(0.10)	(0.12)	(0.20)
JGt	-1.91	0.39	0.90	0.65	2.34	0.55	0.06	-0.42	-0.21	0.15
(SD)	(0.27)	(0.39)	(0.45)	(0.37)	(0.56)	(0.12)	(0.14)	(0.13)	(0.13)	(0.21)
Sax	-2.67	0.59	1.09	0.59	1.24	2.15	-0.05	-0.38	-0.11	0.10
(SD)	(0.32)	(0.37)	(0.47)	(0.38)	(0.44)	(0.49)	(0.16)	(0.14)	(0.14)	(0.22)
$\log(\sigma)$	-0.68	0.05	0.07	0.01	0.06	-0.05	0.41	-0.16	-0.16	0.04
(SD)	(0.11)	(0.15)	(0.18)	(0.15)	(0.14)	(0.16)	(0.08)	(0.15)	(0.15)	(0.17)
$\log(E)$	4.73	-1.07	-0.97	-0.94	-0.79	-0.70	-0.13	1.50	0.43	-0.03
(SD)	(0.24)	(0.32)	(0.35)	(0.32)	(0.34)	(0.33)	(0.13)	(0.42)	(0.11)	(0.21)
$\log(C)$	2.64	-0.58	-0.60	-0.47	-0.26	-0.13	-0.08	0.42	0.63	0.23
(SD)	(0.12)	(0.18)	(0.23)	(0.20)	(0.18)	(0.16)	(0.08)	(0.15)	(0.12)	(0.17)
$\log(\theta-1)$	0.56	-0.11	-0.30	-0.31	0.29	0.19	0.03	-0.05	0.22	1.55
(SD)	(0.31)	(0.41)	(0.43)	(0.38)	(0.44)	(0.44)	(0.14)	(0.34)	(0.18)	(0.60)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A29: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope2 model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
CPn	-0.80	2.26	0.50	0.49	0.25	0.35	-0.03	-0.58	-0.47	-0.24
(SD)	(0.23)	(0.41)	(0.12)	(0.12)	(0.13)	(0.14)	(0.15)	(0.09)	(0.10)	(0.26)
JPn	-1.24	1.26	2.71	0.51	0.45	0.48	-0.01	-0.51	-0.41	-0.28
(SD)	(0.27)	(0.41)	(0.61)	(0.12)	(0.13)	(0.12)	(0.16)	(0.12)	(0.12)	(0.27)
CGt	-1.11	1.07	1.20	2.05	0.40	0.36	-0.05	-0.55	-0.40	-0.26
(SD)	(0.23)	(0.35)	(0.40)	(0.45)	(0.13)	(0.16)	(0.15)	(0.11)	(0.11)	(0.27)
JGt	-1.72	0.62	1.24	0.95	2.73	0.58	0.07	-0.54	-0.24	-0.12
(SD)	(0.28)	(0.34)	(0.53)	(0.40)	(0.82)	(0.11)	(0.16)	(0.11)	(0.14)	(0.26)
Sax	-2.41	0.78	1.17	0.78	1.40	2.17	-0.08	-0.46	-0.12	-0.19
(SD)	(0.29)	(0.36)	(0.42)	(0.40)	(0.48)	(0.58)	(0.15)	(0.13)	(0.14)	(0.26)
$\log(\sigma)$	-0.75	-0.03	-0.01	-0.04	0.08	-0.08	0.41	-0.12	-0.11	0.05
(SD)	(0.12)	(0.16)	(0.18)	(0.15)	(0.18)	(0.14)	(0.10)	(0.15)	(0.13)	(0.18)
$\log(E)$	4.63	-1.02	-0.98	-0.94	-1.04	-0.79	-0.09	1.38	0.40	0.18
(SD)	(0.25)	(0.27)	(0.31)	(0.31)	(0.39)	(0.30)	(0.12)	(0.36)	(0.13)	(0.26)
$\log(C)$	2.68	-0.57	-0.54	-0.45	-0.32	-0.15	-0.06	0.38	0.63	0.23
(SD)	(0.12)	(0.18)	(0.20)	(0.17)	(0.22)	(0.17)	(0.07)	(0.15)	(0.13)	(0.21)
$\log(\theta-1)$	1.72	-0.43	-0.54	-0.43	-0.22	-0.32	0.05	0.24	0.21	1.18
(SD)	(0.56)	(0.50)	(0.58)	(0.47)	(0.46)	(0.47)	(0.13)	(0.36)	(0.21)	(0.64)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A30: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope2 model: Musical Instruments data

Course	Group 1	Group 2
CPn	0.48 (0.15)	0.52 (0.15)
JPn	0.51 (0.16)	0.49 (0.16)
CGt	0.47 (0.18)	0.53 (0.18)
JGt	0.50 (0.17)	0.50 (0.17)
Sax	0.52 (0.19)	0.48 (0.19)

Table A31: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope3 model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
CPn	-0.83	2.38	0.60	0.60	0.37	0.43	0.02	-0.62	-0.54	-0.05
(SD)	(0.24)	(0.65)	(0.09)	(0.09)	(0.13)	(0.14)	(0.14)	(0.09)	(0.09)	(0.33)
JPn	-1.27	1.69	3.29	0.61	0.54	0.55	0.10	-0.60	-0.50	-0.08
(SD)	(0.31)	(0.55)	(0.69)	(0.10)	(0.13)	(0.12)	(0.15)	(0.10)	(0.11)	(0.36)
CGt	-1.20	1.44	1.73	2.38	0.46	0.43	0.04	-0.63	-0.47	-0.06
(SD)	(0.25)	(0.50)	(0.55)	(0.53)	(0.14)	(0.13)	(0.15)	(0.10)	(0.12)	(0.34)
JGt	-1.80	0.97	1.65	1.22	2.87	0.60	0.16	-0.61	-0.34	-0.08
(SD)	(0.30)	(0.46)	(0.58)	(0.53)	(0.74)	(0.11)	(0.16)	(0.09)	(0.15)	(0.34)
Sax	-2.45	1.01	1.49	1.02	1.54	2.25	0.00	-0.51	-0.22	-0.15
(SD)	(0.24)	(0.50)	(0.53)	(0.47)	(0.55)	(0.59)	(0.15)	(0.12)	(0.16)	(0.33)
$\log(\sigma)$	-0.81	0.02	0.12	0.04	0.18	0.00	0.43	-0.19	-0.18	0.07
(SD)	(0.10)	(0.15)	(0.18)	(0.16)	(0.19)	(0.16)	(0.09)	(0.14)	(0.13)	(0.16)
$\log(E)$	4.75	-1.28	-1.45	-1.30	-1.37	-1.04	-0.17	1.77	0.49	0.03
(SD)	(0.25)	(0.43)	(0.44)	(0.44)	(0.44)	(0.41)	(0.13)	(0.48)	(0.13)	(0.35)
$\log(C)$	2.80	-0.72	-0.79	-0.64	-0.50	-0.29	-0.10	0.56	0.74	0.04
(SD)	(0.15)	(0.24)	(0.28)	(0.25)	(0.27)	(0.25)	(0.08)	(0.21)	(0.17)	(0.28)
$\log(\theta-1)$	2.13	-0.12	-0.18	-0.14	-0.14	-0.24	0.04	0.08	0.05	0.98
(SD)	(0.55)	(0.59)	(0.75)	(0.60)	(0.62)	(0.59)	(0.11)	(0.52)	(0.27)	(0.44)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A32: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope3 model: Musical Instruments data

Course	Group 1	Group 2	Group 3
CPn	0.31	0.36	0.33
	(0.13)	(0.14)	(0.15)
JPn	0.32	0.35	0.33
	(0.15)	(0.15)	(0.15)
CGt	0.35	0.34	0.31
	(0.14)	(0.15)	(0.13)
JGt	0.33	0.32	0.34
	(0.15)	(0.15)	(0.13)
Sax	0.33	0.33	0.34
	(0.15)	(0.16)	(0.15)

Table A33: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope4 model: Musical Instruments data

Variables	Mean	CPn	JPn	CGt	JGt	Sax	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
CPn	-0.96	2.44	0.59	0.56	0.40	0.44	0.07	-0.65	-0.55	-0.07
(SD)	(0.26)	(0.53)	(0.09)	(0.11)	(0.13)	(0.14)	(0.16)	(0.10)	(0.11)	(0.33)
JPn	-1.39	1.57	2.86	0.55	0.53	0.52	0.09	-0.60	-0.49	-0.10
(SD)	(0.27)	(0.46)	(0.56)	(0.10)	(0.12)	(0.11)	(0.17)	(0.10)	(0.11)	(0.34)
CGt	-1.29	1.20	1.29	1.87	0.45	0.39	0.03	-0.61	-0.44	-0.07
(SD)	(0.26)	(0.41)	(0.42)	(0.45)	(0.12)	(0.13)	(0.17)	(0.10)	(0.13)	(0.29)
JGt	-1.86	1.05	1.51	1.05	2.83	0.61	0.17	-0.62	-0.32	-0.12
(SD)	(0.28)	(0.45)	(0.49)	(0.40)	(0.64)	(0.11)	(0.16)	(0.09)	(0.12)	(0.31)
Sax	-2.59	1.07	1.35	0.82	1.59	2.39	0.01	-0.53	-0.18	-0.18
(SD)	(0.30)	(0.45)	(0.42)	(0.36)	(0.50)	(0.59)	(0.17)	(0.12)	(0.14)	(0.31)
$\log(\sigma)$	-0.86	0.08	0.10	0.02	0.19	0.00	0.45	-0.18	-0.17	0.04
(SD)	(0.11)	(0.17)	(0.20)	(0.16)	(0.18)	(0.18)	(0.09)	(0.16)	(0.16)	(0.18)
$\log(E)$	4.81	-1.36	-1.35	-1.12	-1.38	-1.09	-0.16	1.76	0.48	0.06
(SD)	(0.24)	(0.42)	(0.40)	(0.34)	(0.40)	(0.40)	(0.14)	(0.45)	(0.12)	(0.35)
$\log(C)$	2.84	-0.72	-0.70	-0.52	-0.45	-0.24	-0.10	0.54	0.71	-0.02
(SD)	(0.14)	(0.23)	(0.21)	(0.20)	(0.20)	(0.19)	(0.09)	(0.19)	(0.14)	(0.30)
$\log(\theta-1)$	2.34	-0.13	-0.21	-0.12	-0.22	-0.30	0.03	0.11	0.00	0.99
(SD)	(0.87)	(0.59)	(0.68)	(0.46)	(0.56)	(0.54)	(0.13)	(0.53)	(0.28)	(0.47)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A34: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope4 model: Musical Instruments data

Course	Group 1	Group 2	Group 3	Group 4
CPn	0.26	0.23	0.25	0.26
	(0.11)	(0.12)	(0.11)	(0.12)
JPn	0.23	0.25	0.28	0.24
	(0.10)	(0.12)	(0.13)	(0.11)
CGt	0.26	0.25	0.24	0.25
	(0.13)	(0.12)	(0.12)	(0.11)
JGt	0.26	0.25	0.24	0.25
	(0.15)	(0.13)	(0.11)	(0.12)
Sax	0.26	0.24	0.25	0.25
	(0.13)	(0.11)	(0.13)	(0.12)

Table A35: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Standard model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$
Brd	-3.41	1.84	0.67	0.60	0.50	0.57	-0.11	-0.51
(SD)	(0.21)	(0.35)	(0.09)	(0.09)	(0.12)	(0.11)	(0.12)	(0.12)
Dst	-3.07	1.38	2.29	0.61	0.58	0.59	-0.11	-0.61
(SD)	(0.23)	(0.35)	(0.46)	(0.07)	(0.09)	(0.10)	(0.12)	(0.08)
Pst	-2.79	1.23	1.40	2.25	0.69	0.60	-0.18	-0.56
(SD)	(0.22)	(0.34)	(0.37)	(0.47)	(0.08)	(0.10)	(0.12)	(0.10)
Meat	-2.82	1.09	1.42	1.67	2.59	0.71	-0.06	-0.62
(SD)	(0.23)	(0.37)	(0.42)	(0.47)	(0.62)	(0.07)	(0.13)	(0.09)
Fish	-3.62	1.27	1.44	1.46	1.87	2.64	-0.03	-0.59
(SD)	(0.21)	(0.38)	(0.41)	(0.45)	(0.49)	(0.56)	(0.13)	(0.10)
$\log(\sigma)$	-0.60	-0.08	-0.09	-0.14	-0.05	-0.03	0.30	-0.09
(SD)	(0.09)	(0.10)	(0.10)	(0.10)	(0.12)	(0.12)	(0.05)	(0.11)
$\log(E)$	4.65	-0.68	-0.91	-0.83	-0.98	-0.94	-0.05	0.94
(SD)	(0.15)	(0.24)	(0.26)	(0.29)	(0.32)	(0.28)	(0.06)	(0.21)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A36: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Satiation model: Cooking Data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(\kappa)$	$\log(E)$
Brd	-2.42	2.14	0.61	0.52	0.38	0.50	-0.27	-0.24	-0.38
(SD)	(0.27)	(0.64)	(0.12)	(0.13)	(0.15)	(0.13)	(0.11)	(0.15)	(0.16)
Dst	-1.96	1.42	2.43	0.46	0.38	0.43	-0.29	-0.29	-0.45
(SD)	(0.29)	(0.63)	(0.75)	(0.15)	(0.16)	(0.14)	(0.12)	(0.15)	(0.15)
Pst	-1.57	1.45	1.36	3.36	0.67	0.53	-0.27	0.00	-0.38
(SD)	(0.32)	(0.66)	(0.71)	(0.88)	(0.08)	(0.12)	(0.12)	(0.17)	(0.12)
Meat	-1.64	1.13	1.19	2.42	3.80	0.66	-0.11	0.00	-0.46
(SD)	(0.27)	(0.59)	(0.64)	(0.72)	(0.78)	(0.08)	(0.14)	(0.14)	(0.10)
Fish	-2.75	1.33	1.23	1.79	2.33	3.23	-0.19	-0.14	-0.43
(SD)	(0.32)	(0.57)	(0.59)	(0.67)	(0.62)	(0.69)	(0.13)	(0.15)	(0.12)
$\log(\sigma)$	-0.33	-0.23	-0.26	-0.28	-0.12	-0.19	0.34	0.09	0.02
(SD)	(0.08)	(0.11)	(0.13)	(0.14)	(0.16)	(0.14)	(0.06)	(0.13)	(0.14)
$\log(\kappa)$	2.44	-0.51	-0.66	-0.01	0.00	-0.37	0.07	1.99	0.57
(SD)	(0.25)	(0.37)	(0.43)	(0.42)	(0.40)	(0.41)	(0.12)	(0.64)	(0.13)
$\log(E)$	4.88	-0.72	-0.90	-0.86	-1.08	-0.94	0.02	1.00	1.45
(SD)	(0.30)	(0.45)	(0.51)	(0.44)	(0.40)	(0.44)	(0.10)	(0.47)	(0.47)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A37: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Complementarity model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(\mu)$	$\log(E)$
Brd	-3.15	1.06	0.29	0.13	-0.26	-0.02	-0.12	0.02	-0.04
(SD)	(0.18)	(0.26)	(0.13)	(0.15)	(0.16)	(0.16)	(0.14)	(0.14)	(0.12)
Dst	-2.68	0.35	1.36	0.07	-0.09	-0.08	0.00	-0.16	-0.19
(SD)	(0.18)	(0.18)	(0.28)	(0.15)	(0.16)	(0.14)	(0.16)	(0.16)	(0.12)
Pst	-2.28	0.17	0.10	1.64	0.25	-0.04	-0.17	0.00	-0.15
(SD)	(0.22)	(0.20)	(0.22)	(0.37)	(0.14)	(0.18)	(0.14)	(0.15)	(0.12)
Meat	-2.33	-0.36	-0.15	0.42	1.75	0.28	0.06	-0.25	-0.15
(SD)	(0.20)	(0.24)	(0.27)	(0.26)	(0.44)	(0.13)	(0.13)	(0.14)	(0.11)
Fish	-3.40	-0.02	-0.12	-0.06	0.47	1.54	0.01	-0.06	-0.06
(SD)	(0.21)	(0.22)	(0.22)	(0.29)	(0.28)	(0.41)	(0.13)	(0.15)	(0.14)
$\log(\sigma)$	-0.28	-0.07	0.00	-0.13	0.05	0.01	0.32	-0.28	-0.24
(SD)	(0.08)	(0.08)	(0.11)	(0.11)	(0.10)	(0.10)	(0.06)	(0.14)	(0.11)
$\log(\mu)$	0.97	0.02	-0.19	0.00	-0.33	-0.07	-0.16	0.95	0.33
(SD)	(0.19)	(0.14)	(0.19)	(0.20)	(0.20)	(0.18)	(0.08)	(0.19)	(0.12)
$\log(E)$	3.57	-0.02	-0.13	-0.12	-0.12	-0.04	-0.08	0.20	0.37
(SD)	(0.09)	(0.08)	(0.09)	(0.10)	(0.09)	(0.10)	(0.04)	(0.09)	(0.07)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A38: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Linear Time model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$	$\log(C)$
Brd	-1.74	2.35	0.77	0.78	0.76	0.74	-0.04	-0.72	-0.64
(SD)	(0.24)	(0.49)	(0.06)	(0.05)	(0.06)	(0.06)	(0.14)	(0.07)	(0.08)
Dst	-1.47	1.96	2.71	0.77	0.77	0.74	-0.07	-0.72	-0.61
(SD)	(0.22)	(0.46)	(0.53)	(0.05)	(0.06)	(0.06)	(0.14)	(0.07)	(0.09)
Pst	-1.38	2.14	2.27	3.18	0.82	0.77	-0.08	-0.75	-0.60
(SD)	(0.26)	(0.47)	(0.47)	(0.56)	(0.04)	(0.05)	(0.13)	(0.06)	(0.08)
Meat	-1.34	2.29	2.50	2.88	3.88	0.83	-0.02	-0.73	-0.67
(SD)	(0.27)	(0.50)	(0.52)	(0.54)	(0.67)	(0.04)	(0.13)	(0.07)	(0.08)
Fish	-1.90	2.08	2.25	2.53	3.01	3.36	-0.02	-0.72	-0.62
(SD)	(0.25)	(0.47)	(0.49)	(0.51)	(0.56)	(0.58)	(0.12)	(0.07)	(0.07)
$\log(\sigma)$	-1.22	-0.05	-0.08	-0.10	-0.02	-0.03	0.46	-0.13	0.01
(SD)	(0.09)	(0.15)	(0.16)	(0.16)	(0.17)	(0.16)	(0.09)	(0.13)	(0.13)
$\log(E)$	4.92	-1.40	-1.51	-1.70	-1.84	-1.68	-0.11	1.62	0.51
(SD)	(0.18)	(0.34)	(0.32)	(0.34)	(0.38)	(0.35)	(0.12)	(0.30)	(0.12)
$\log(C)$	3.68	-1.16	-1.18	-1.26	-1.55	-1.34	0.01	0.76	1.38
(SD)	(0.17)	(0.27)	(0.30)	(0.28)	(0.33)	(0.27)	(0.10)	(0.24)	(0.23)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A39: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope1 model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
Brd	-1.38	1.79	0.64	0.62	0.54	0.58	-0.08	-0.56	-0.44	0.13
(SD)	(0.19)	(0.38)	(0.08)	(0.10)	(0.11)	(0.10)	(0.13)	(0.10)	(0.11)	(0.15)
Dst	-1.07	1.27	2.20	0.60	0.57	0.55	-0.10	-0.61	-0.39	0.18
(SD)	(0.21)	(0.34)	(0.39)	(0.09)	(0.09)	(0.09)	(0.11)	(0.09)	(0.12)	(0.15)
Pst	-0.95	1.32	1.40	2.46	0.70	0.61	-0.17	-0.61	-0.38	0.04
(SD)	(0.20)	(0.38)	(0.36)	(0.45)	(0.07)	(0.09)	(0.13)	(0.09)	(0.11)	(0.16)
Meat	-0.89	1.25	1.45	1.87	2.90	0.72	-0.03	-0.61	-0.47	-0.02
(SD)	(0.24)	(0.41)	(0.38)	(0.44)	(0.55)	(0.08)	(0.13)	(0.08)	(0.10)	(0.15)
Fish	-1.53	1.28	1.36	1.57	2.04	2.71	0.02	-0.60	-0.45	0.13
(SD)	(0.24)	(0.40)	(0.36)	(0.42)	(0.51)	(0.57)	(0.13)	(0.08)	(0.11)	(0.15)
$\log(\sigma)$	-0.89	-0.09	-0.12	-0.21	-0.04	0.02	0.62	-0.10	-0.12	0.36
(SD)	(0.12)	(0.14)	(0.13)	(0.16)	(0.17)	(0.18)	(0.11)	(0.13)	(0.12)	(0.11)
$\log(E)$	4.70	-0.89	-1.06	-1.14	-1.23	-1.17	-0.09	1.39	0.38	-0.32
(SD)	(0.19)	(0.31)	(0.27)	(0.33)	(0.34)	(0.33)	(0.12)	(0.32)	(0.12)	(0.13)
$\log(C)$	2.82	-0.51	-0.51	-0.52	-0.70	-0.64	-0.08	0.39	0.75	0.11
(SD)	(0.09)	(0.19)	(0.20)	(0.21)	(0.22)	(0.23)	(0.09)	(0.17)	(0.16)	(0.14)
$\log(\theta-1)$	1.08	0.32	0.52	0.11	-0.07	0.43	0.54	-0.71	0.18	3.59
(SD)	(0.36)	(0.41)	(0.45)	(0.51)	(0.47)	(0.48)	(0.22)	(0.34)	(0.24)	(1.03)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A40: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope2 model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
Brd	-1.46	2.30	0.73	0.73	0.70	0.71	-0.01	-0.68	-0.57	0.11
(SD)	(0.23)	(0.49)	(0.07)	(0.06)	(0.07)	(0.07)	(0.13)	(0.08)	(0.10)	(0.17)
Dst	-1.15	1.83	2.71	0.70	0.70	0.68	-0.07	-0.66	-0.52	0.12
(SD)	(0.23)	(0.45)	(0.51)	(0.07)	(0.07)	(0.08)	(0.13)	(0.07)	(0.10)	(0.16)
Pst	-1.03	1.97	2.04	3.14	0.79	0.73	-0.11	-0.69	-0.51	0.01
(SD)	(0.23)	(0.46)	(0.46)	(0.61)	(0.06)	(0.07)	(0.14)	(0.07)	(0.10)	(0.16)
Meat	-0.95	2.09	2.27	2.74	3.85	0.80	0.01	-0.69	-0.59	0.00
(SD)	(0.26)	(0.53)	(0.54)	(0.60)	(0.73)	(0.05)	(0.13)	(0.07)	(0.08)	(0.17)
Fish	-1.60	2.15	2.22	2.54	3.11	3.87	0.08	-0.70	-0.57	0.14
(SD)	(0.30)	(0.55)	(0.52)	(0.59)	(0.67)	(0.74)	(0.13)	(0.08)	(0.09)	(0.16)
$\log(\sigma)$	-1.00	-0.01	-0.09	-0.15	0.02	0.12	0.65	-0.18	-0.15	0.41
(SD)	(0.12)	(0.16)	(0.18)	(0.20)	(0.21)	(0.21)	(0.12)	(0.13)	(0.12)	(0.14)
$\log(E)$	4.80	-1.35	-1.42	-1.59	-1.77	-1.79	-0.19	1.68	0.53	-0.31
(SD)	(0.22)	(0.33)	(0.32)	(0.34)	(0.39)	(0.40)	(0.14)	(0.29)	(0.10)	(0.15)
$\log(C)$	3.07	-0.93	-0.92	-0.96	-1.22	-1.19	-0.13	0.73	1.12	0.05
(SD)	(0.15)	(0.27)	(0.28)	(0.27)	(0.31)	(0.32)	(0.11)	(0.21)	(0.20)	(0.16)
$\log(\theta-1)$	1.52	0.31	0.37	0.01	-0.02	0.52	0.64	-0.77	0.11	3.70
(SD)	(0.41)	(0.50)	(0.50)	(0.54)	(0.66)	(0.61)	(0.25)	(0.40)	(0.32)	(1.05)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A41: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope2 model: Cooking data

Course	Group 1	Group 2
Brd	0.51	0.49
	(0.21)	(0.21)
Dst	0.50	0.50
	(0.22)	(0.22)
Pst	0.50	0.50
	(0.20)	(0.20)
Meat	0.48	0.52
	(0.14)	(0.14)
Fish	0.51	0.49
	(0.16)	(0.16)

Table A42: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope3 model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
Brd	-1.47	2.03	0.70	0.67	0.63	0.64	-0.01	-0.63	-0.51	0.25
(SD)	(0.21)	(0.47)	(0.07)	(0.09)	(0.10)	(0.09)	(0.14)	(0.09)	(0.11)	(0.23)
Dst	-1.16	1.55	2.38	0.65	0.64	0.63	-0.03	-0.65	-0.48	0.28
(SD)	(0.22)	(0.41)	(0.45)	(0.08)	(0.08)	(0.10)	(0.12)	(0.09)	(0.10)	(0.22)
Pst	-1.02	1.57	1.66	2.68	0.75	0.67	-0.10	-0.67	-0.46	0.21
(SD)	(0.23)	(0.45)	(0.46)	(0.59)	(0.06)	(0.08)	(0.12)	(0.07)	(0.10)	(0.23)
Meat	-0.96	1.66	1.85	2.27	3.40	0.78	0.04	-0.68	-0.54	0.24
(SD)	(0.23)	(0.52)	(0.54)	(0.63)	(0.77)	(0.06)	(0.12)	(0.07)	(0.09)	(0.24)
Fish	-1.61	1.66	1.76	1.98	2.59	3.24	0.06	-0.65	-0.49	0.33
(SD)	(0.26)	(0.49)	(0.51)	(0.55)	(0.66)	(0.64)	(0.11)	(0.08)	(0.10)	(0.20)
$\log(\sigma)$	-1.00	-0.01	-0.03	-0.13	0.06	0.09	0.64	-0.17	-0.13	0.36
(SD)	(0.13)	(0.17)	(0.16)	(0.17)	(0.18)	(0.16)	(0.14)	(0.13)	(0.14)	(0.15)
$\log(E)$	4.73	-1.08	-1.20	-1.32	-1.49	-1.40	-0.17	1.42	0.45	-0.44
(SD)	(0.17)	(0.33)	(0.31)	(0.33)	(0.38)	(0.35)	(0.14)	(0.30)	(0.11)	(0.18)
$\log(C)$	3.05	-0.71	-0.72	-0.74	-0.97	-0.85	-0.10	0.53	0.93	-0.07
(SD)	(0.14)	(0.24)	(0.23)	(0.25)	(0.29)	(0.28)	(0.11)	(0.17)	(0.17)	(0.25)
$\log(\theta-1)$	2.17	0.55	0.65	0.51	0.66	0.89	0.45	-0.80	-0.09	2.44
(SD)	(0.53)	(0.54)	(0.54)	(0.59)	(0.71)	(0.58)	(0.22)	(0.41)	(0.38)	(0.73)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A43: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope3 model: Cooking data

Course	Group 1	Group 2	Group 3
Brd	0.34	0.37	0.29
	(0.20)	(0.20)	(0.17)
Dst	0.29	0.39	0.32
	(0.19)	(0.21)	(0.18)
Pst	0.33	0.36	0.31
	(0.15)	(0.14)	(0.14)
Meat	0.33	0.36	0.32
	(0.17)	(0.17)	(0.17)
Fish	0.33	0.37	0.30
	(0.13)	(0.14)	(0.14)

Table A44: Posterior estimates of $\bar{\boldsymbol{\eta}}$ and $\mathbf{V}$ for Scope4 model: Cooking data

Variables	Mean	Brd	Dst	Pst	Meat	Fish	$\log(\sigma)$	$\log(E)$	$\log(C)$	$\log(\theta-1)$
Brd	-1.60	2.63	0.76	0.75	0.70	0.72	-0.03	-0.73	-0.61	0.46
(SD)	(0.21)	(0.56)	(0.07)	(0.06)	(0.08)	(0.08)	(0.14)	(0.08)	(0.09)	(0.17)
Dst	-1.35	2.18	3.07	0.73	0.71	0.69	-0.06	-0.73	-0.58	0.49
(SD)	(0.22)	(0.53)	(0.56)	(0.07)	(0.07)	(0.08)	(0.15)	(0.06)	(0.09)	(0.16)
Pst	-1.20	2.23	2.34	3.35	0.79	0.73	-0.11	-0.75	-0.58	0.44
(SD)	(0.26)	(0.53)	(0.53)	(0.61)	(0.05)	(0.06)	(0.14)	(0.06)	(0.09)	(0.18)
Meat	-1.14	2.25	2.47	2.87	3.90	0.81	0.02	-0.75	-0.62	0.48
(SD)	(0.26)	(0.60)	(0.58)	(0.61)	(0.69)	(0.05)	(0.14)	(0.06)	(0.08)	(0.17)
Fish	-1.81	2.26	2.36	2.58	3.07	3.71	0.07	-0.74	-0.62	0.52
(SD)	(0.27)	(0.60)	(0.57)	(0.58)	(0.64)	(0.76)	(0.13)	(0.07)	(0.09)	(0.16)
$\log(\sigma)$	-1.04	-0.04	-0.08	-0.16	0.02	0.11	0.63	-0.11	-0.13	0.31
(SD)	(0.10)	(0.19)	(0.21)	(0.21)	(0.21)	(0.20)	(0.12)	(0.14)	(0.12)	(0.15)
$\log(E)$	4.93	-1.72	-1.86	-1.99	-2.14	-2.08	-0.13	2.09	0.61	-0.61
(SD)	(0.23)	(0.43)	(0.40)	(0.45)	(0.48)	(0.53)	(0.16)	(0.43)	(0.09)	(0.13)
$\log(C)$	3.19	-1.11	-1.12	-1.17	-1.36	-1.32	-0.12	0.98	1.21	-0.38
(SD)	(0.16)	(0.33)	(0.32)	(0.32)	(0.36)	(0.38)	(0.11)	(0.28)	(0.25)	(0.19)
$\log(\theta-1)$	2.47	1.23	1.39	1.33	1.53	1.63	0.40	-1.44	-0.69	2.61
(SD)	(0.48)	(0.59)	(0.62)	(0.70)	(0.69)	(0.66)	(0.20)	(0.51)	(0.42)	(0.77)

* Numbers in shaded cells are the correlation coefficients.
* Posterior standard deviations are given in parentheses ().

Table A45: Posterior means (standard deviations) of grouping probabilities (ϕ) for Scope4 model: Cooking data

Course	Group 1	Group 2	Group 3	Group 4
Brd	0.19	0.17	0.36	0.28
	(0.11)	(0.09)	(0.17)	(0.17)
Dst	0.16	0.14	0.41	0.28
	(0.15)	(0.10)	(0.26)	(0.24)
Pst	0.18	0.17	0.38	0.27
	(0.13)	(0.09)	(0.21)	(0.20)
Meat	0.18	0.16	0.40	0.26
	(0.10)	(0.09)	(0.20)	(0.16)
Fish	0.20	0.20	0.36	0.24
	(0.11)	(0.10)	(0.17)	(0.15)

10 Open online course: the other categories

The datasets consist of 49 respondents in the Musical Instruments survey and 59 respondents in the Cooking survey. The survey design for both categories is the same as that for the Machine Learning category (see Section 5.2.1 in the main text of the paper for details). Table A46 summarizes attributes and levels used in the choice tasks.

Table A46: Attributes and levels: the other open online course datasets

Attribute	Level 1	Level 2	Level 3	Level 4	Level 5
Musical Instruments course					
Topics	Classical Piano	Jazz Piano	Classical Guitar	Jazz Guitar	Saxophone
Price	Continuous variable ranging from \$2 to \$8				
Homework Time	Continuous variable ranging from 0.5 hrs to 2 hrs				
Cooking course					
Topics	Bread	Desserts	Pasta	Meat	Fish
Price	Continuous variable ranging from \$2 to \$8				
Homework Time	Continuous variable ranging from 0.5 hrs to 2 hrs				

The average primary and secondary demand curves are provided in Online Appendix 11. We do not observe any strong downward-sloping demand as in the Machine Learning data: the weak income and substitution effects suggest existence of an additional nonlinear time constraint to describe the data patterns.

We fit the same proposed and benchmark models as in Section 5.1.3 in the main text of the paper to the data. The values of γ (i.e., lecture time) are given as one per credit hour and those of δ (i.e., homework time) are observed in the data. Tables A47 and A48 summarize in-sample and predictive performance of the models. The LMD values indicate that the Scope model with $K = 3$ groups (henceforth, the Scope3 model) shows the best in-sample fit to the Musical Instruments data and the Scope model with $K = 2$ groups (henceforth, the Scope2 model) provides the best in-sample fit to the Cooking data. We find that large improvements in the measures come from the additional time constraint (Standard/Satiation/Complementarity vs. others) and the scope effects lead to further

Table A47: In-sample and predictive fit: Musical Instruments data

Models	K	In-sample			Predictive	
		LMD	MAE	MSE	MAE	MSE
Standard		-2243.43	0.4806	0.8070	0.4635	0.8291
Satiation		-2056.98	0.4251	0.6669	0.4292	0.7178
Complementarity		-1931.36	0.5135	0.8962	0.4997	0.9410
Linear Time		-1557.36	0.3311	0.4747	0.3996	0.6152
	1	-1502.95	0.3355	0.4821	0.3898	0.6026
Scope	2	-1479.32	0.3347	0.4826	0.3902	0.6047
	3	**-1452.27**	**0.3312**	**0.4786**	**0.3856**	**0.5942**
	4	-1482.92	0.3322	0.4802	0.3953	0.6109

Table A48: In-sample and predictive fit: Cooking data

Models	K	In-sample			Predictive	
		LMD	MAE	MSE	MAE	MSE
Standard		-2917.69	0.5219	0.8359	0.5035	0.8043
Satiation		-2585.56	0.4457	0.6831	0.4423	0.6670
Complementarity		-2581.54	0.5607	0.9576	0.5487	0.9603
Linear Time		-2082.99	0.3773	0.5445	0.3898	0.5393
	1	-1937.78	0.3732	0.5342	0.3897	0.5402
Scope	**2**	**-1873.23**	**0.3724**	**0.5373**	**0.3931**	**0.5482**
	3	-1901.49	0.3739	0.5394	0.3924	0.5469
	4	-1880.17	0.3742	0.5395	0.3939	0.5514

improvements in fit (Linear Time vs. Scope). This suggests that the consumer demand for the open online courses of different categories is constrained not only by the monetary budget but also the amount of time available and that the time cost is affected by the economies of scope. This is consistent with the general notion that the time saving is expected when taking multiple courses that require common knowledge or expertise.

The details of the posterior estimates are provided in Online Appendix 9. The estimates show the patterns similar to those for the Machine Learning data. So, we omit detailed interpretation of the estimates.

11 Model-free evidence for the open online courses

This appendix provides model-free evidence from the open online course datasets.

Figure A10: Average demand quantities versus price: Machine Learning data

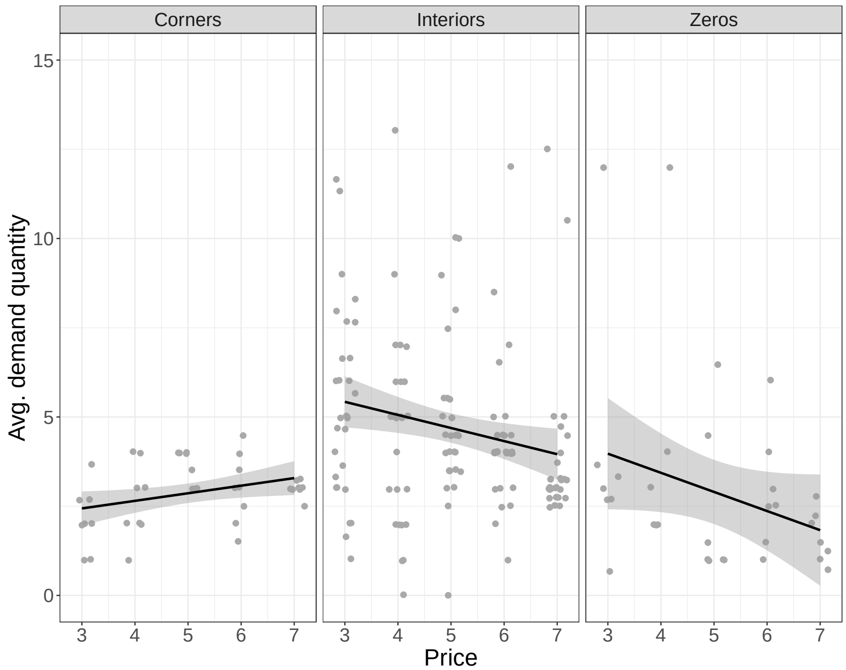


Figure A11: Primary demand curves at individual-level: Machine Learning data

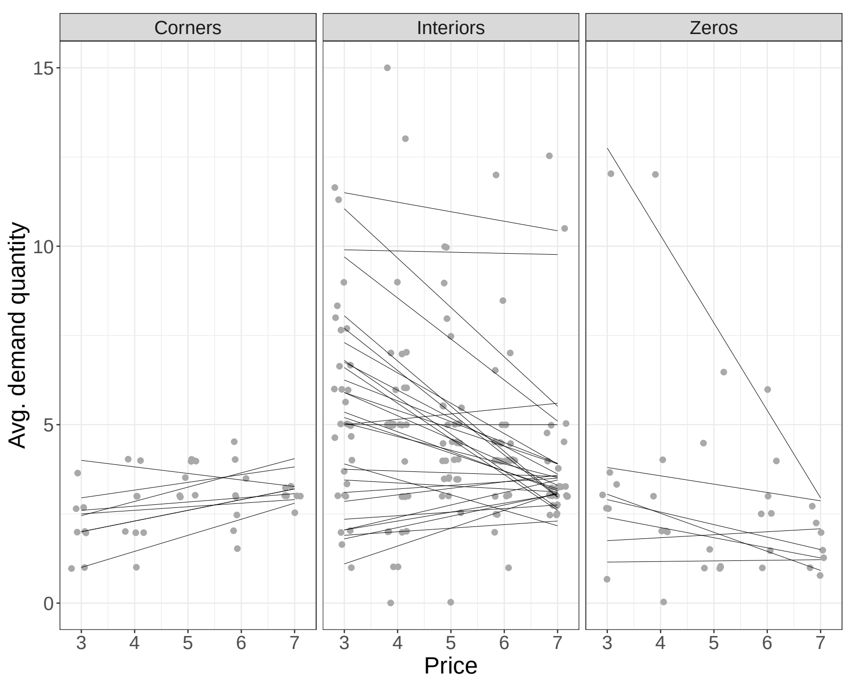


Figure A12: Secondary demand quantities versus price: Machine Learning data

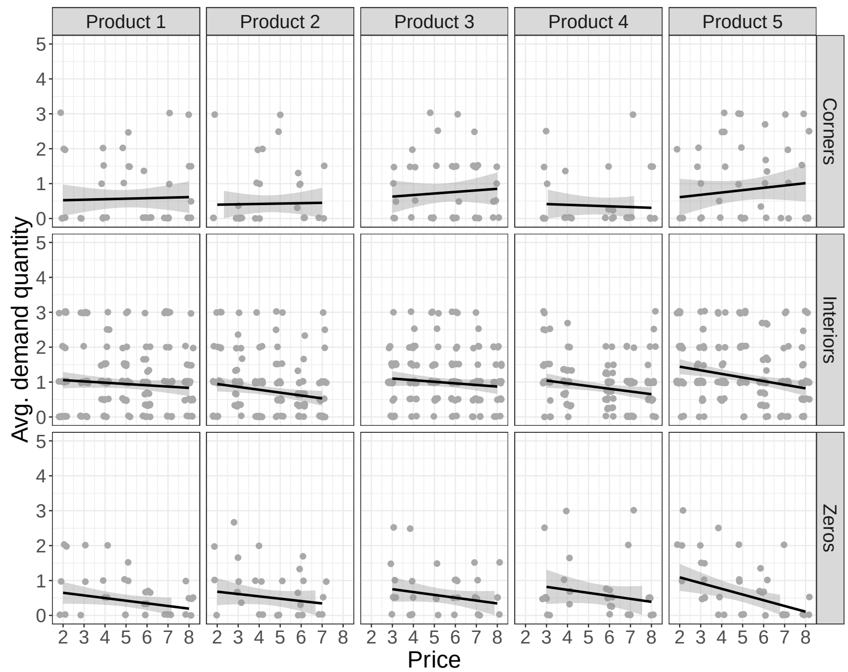


Figure A13: Secondary demand curves at individual-level: Machine Learning data

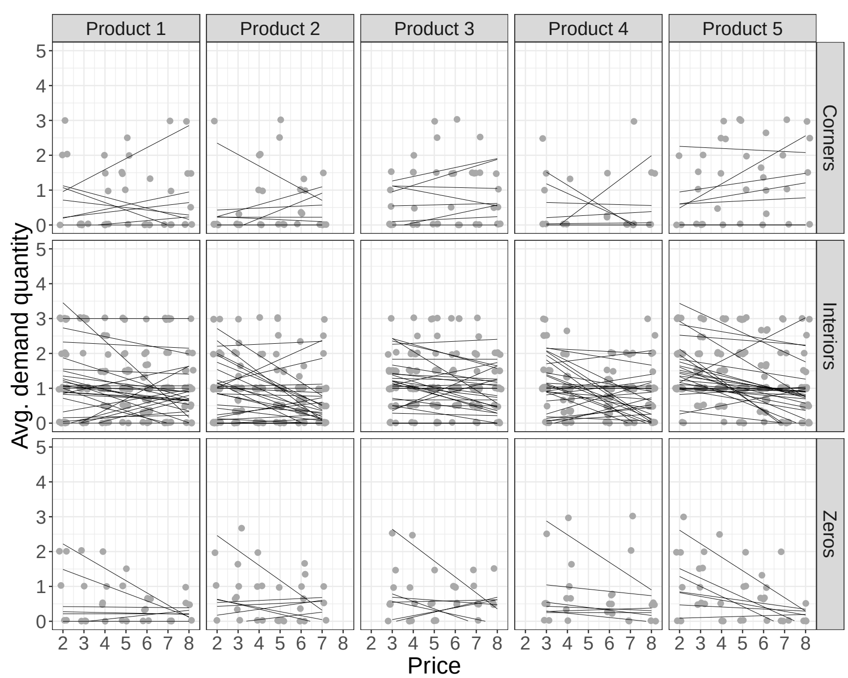


Figure A14: Average demand quantities versus price: Musical Instruments data

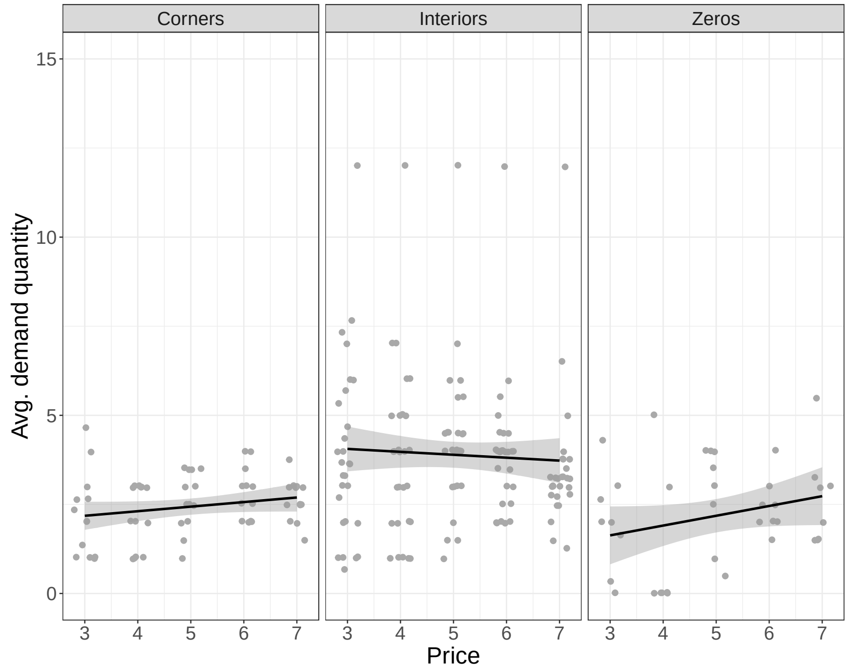


Figure A15: Primary demand curves at individual-level: Musical Instruments data

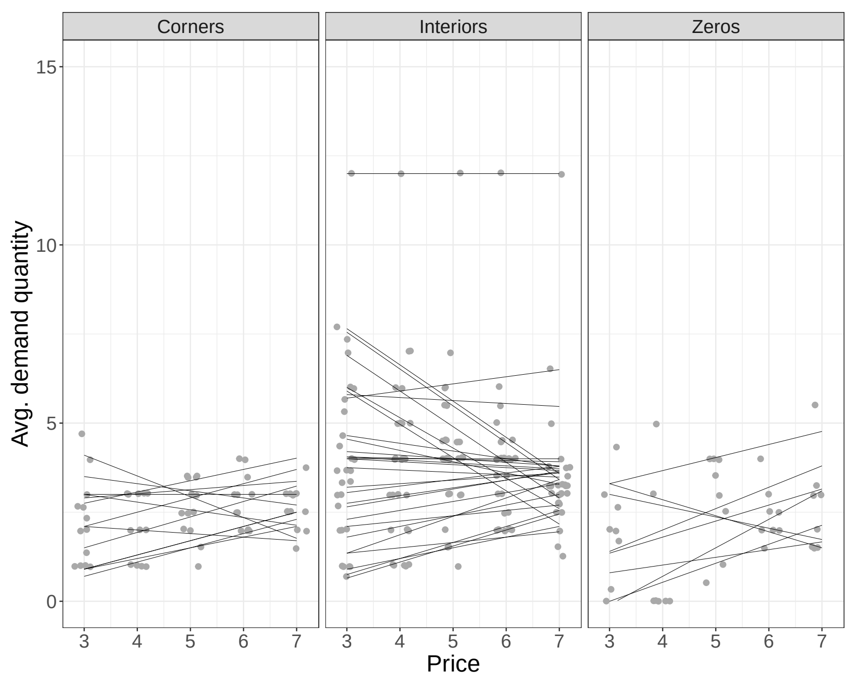


Figure A16: Secondary demand quantities versus price: Musical Instruments data

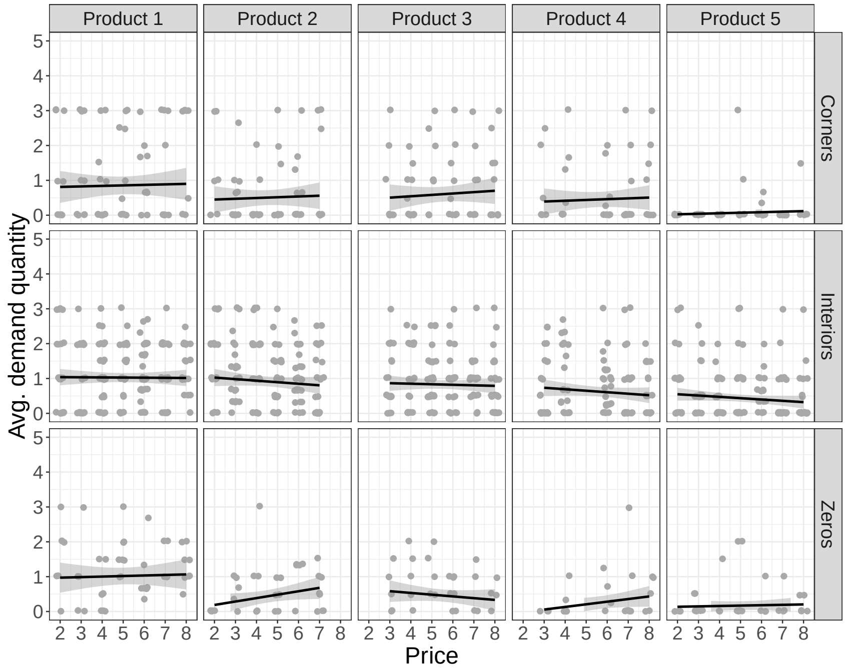


Figure A17: Secondary demand curves at individual-level: Musical Instruments data

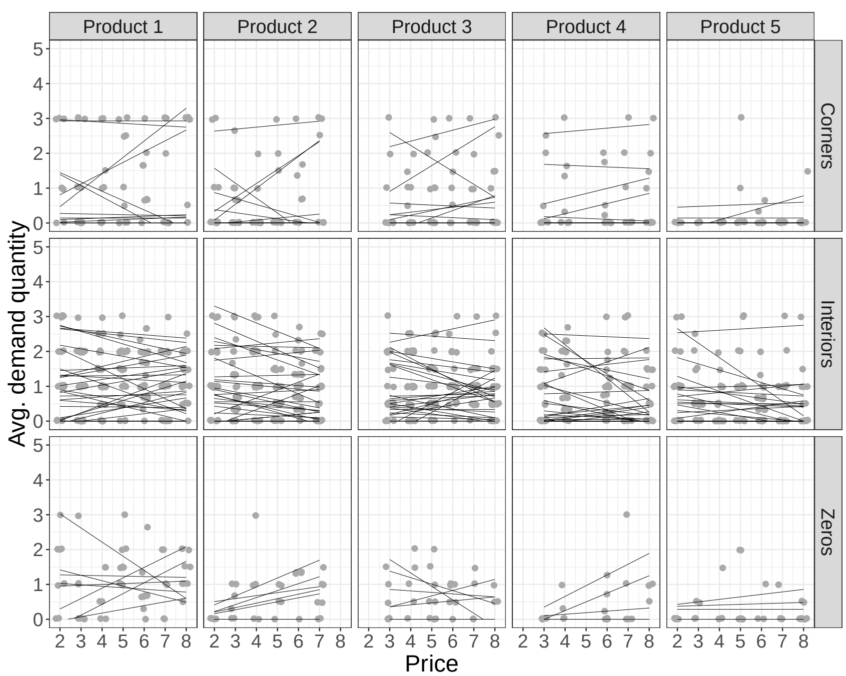


Figure A18: Average demand quantities versus price: Cooking data

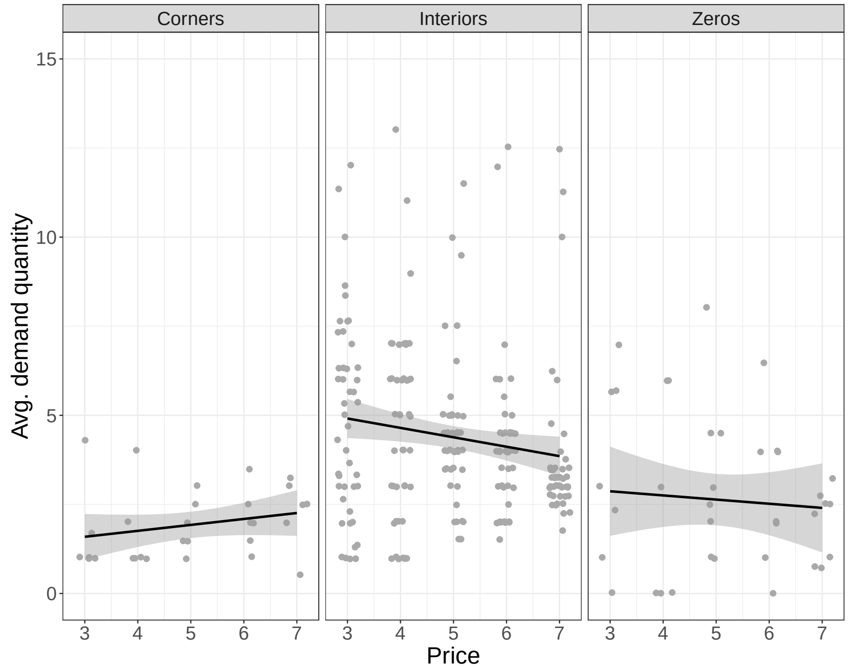


Figure A19: Primary demand curves at individual-level: Cooking data

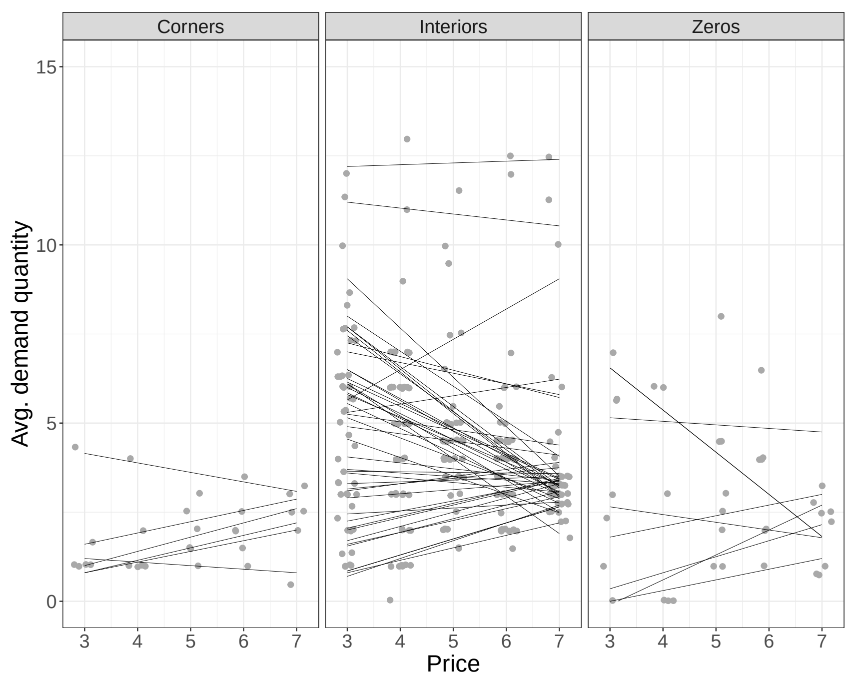


Figure A20: Secondary demand quantities versus price: Cooking data

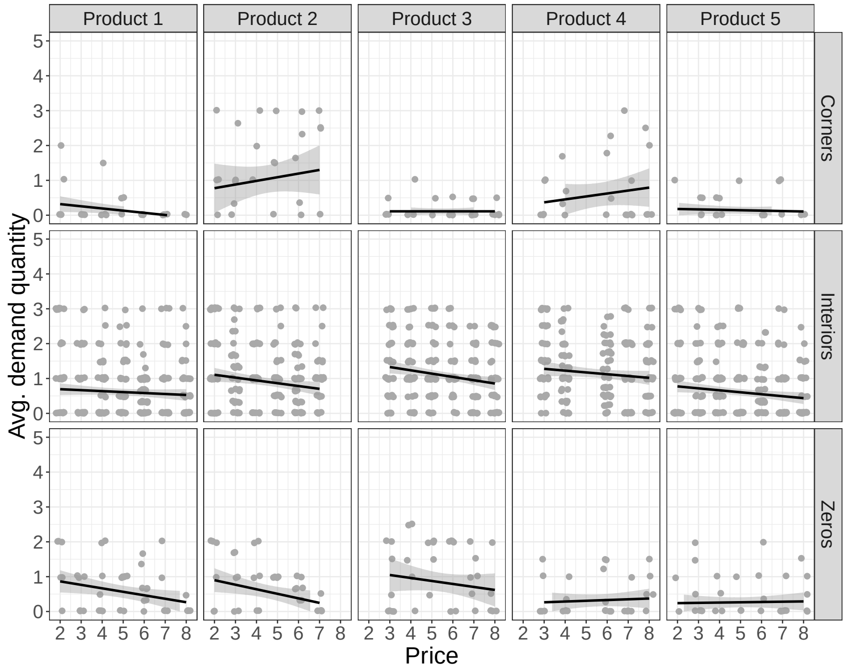
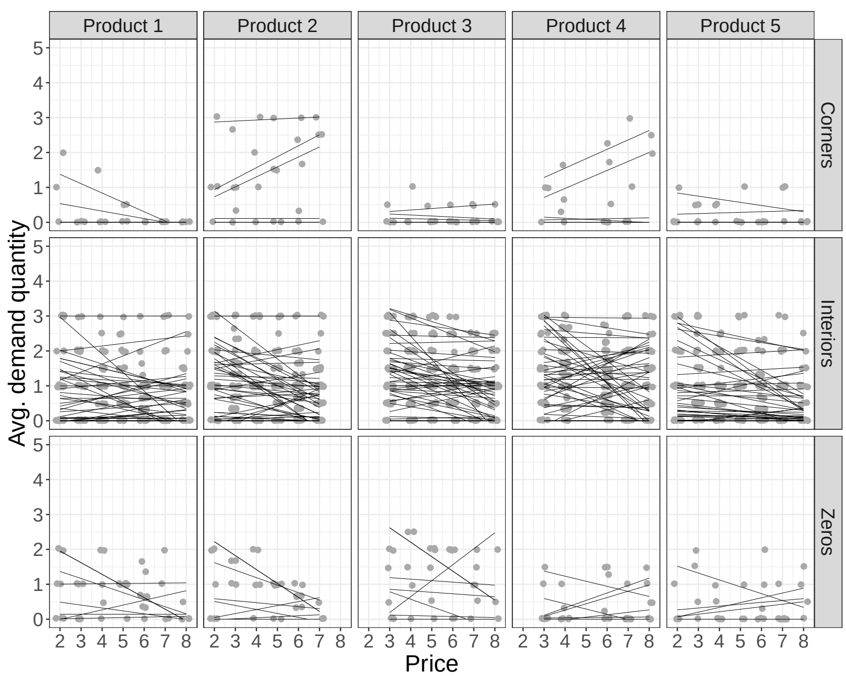


Figure A21: Secondary demand curves at individual-level: Cooking data



12 Computation of expected demand

This appendix provides the detailed computation of the expected demand ($\{\hat{x}_j\}$) in Section 6.1 in the main text of the paper. If no promotion is offered, the expected demand is given by solving the utility maximization problem in the main text for the each model. When a free one-day entrance ticket for park i is offered, the expected demand is given by solving the following utility maximization problems:

for the **Standard model**,

$$\begin{aligned}
\max_{\boldsymbol{x}_{ht}} \quad & U\left(\boldsymbol{x}_{ht}, z_{ht}\right) = \sum\nolimits_j \psi_{hjt} \log\left(x_{hjt}+1\right) + \log\left(z_{ht}\right), \qquad \text{(A35)}\\
\text{s.t.} \quad & p_{hit}\left(x_{hit}-1\right) + \sum\nolimits_{j\neq i} p_{hjt}x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0,\\
& x_{hit} \in \{1,2,\ldots\} \text{ and } x_{hjt} \in \{0,1,2,\ldots\}, \text{ for } j \neq i,
\end{aligned}$$

for the **Satiation model**,

$$\begin{aligned}
\max_{\boldsymbol{x}_{ht}} \quad & U\left(\boldsymbol{x}_{ht}, z_{ht}\right) = \sum\nolimits_j \frac{\psi_{hjt}}{\kappa_h} \log\left(\kappa_h x_{hjt}+1\right) + \log\left(z_{ht}\right), \qquad \text{(A36)}\\
\text{s.t.} \quad & p_{hit}\left(x_{hit}-1\right) + \sum\nolimits_{j\neq i} p_{hjt}x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0,\\
& x_{hit} \in \{1,2,\ldots\} \text{ and } x_{hjt} \in \{0,1,2,\ldots\}, \text{ for } j \neq i,
\end{aligned}$$

for the **Complementarity model**,

$$\begin{aligned}
\max_{\boldsymbol{x}_{h}} \quad & U_h\left(\boldsymbol{x}_{ht}, z_{ht}\right) = \sum\nolimits_j \psi_{hjt} \log\left(x_{hjt}+1\right) \qquad \text{(A37)}\\
& \qquad + \mu_h \sum\nolimits_j \sum\nolimits_{j'>j} \psi_{hjt} \log\left(x_{hjt}+1\right) \psi_{hj't} \log\left(x_{hj't}+1\right) + \log\left(z_{ht}\right),\\
\text{s.t.} \quad & p_{hit}\left(x_{hit}-1\right) + \sum\nolimits_{j\neq i} p_{hjt}x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0,\\
& x_{hit} \in \{1,2,\ldots\} \text{ and } x_{hjt} \in \{0,1,2,\ldots\}, \text{ for } j \neq i,
\end{aligned}$$

for the **Scope model**,

$$\begin{aligned}
\max_{\boldsymbol{x}_{ht}} \quad & U\left(\boldsymbol{x}_{ht}, z_{ht}, s_{ht}\right) = \sum\nolimits_j \psi_{hjt} \log\left(x_{hjt}+1\right) + \log\left(z_{ht}\right) + \log\left(s_{ht}\right), \qquad \text{(A38)}\\
\text{s.t.} \quad & p_{hit}\left(x_{hit}-1\right) + \sum\nolimits_{j\neq i} p_{hjt}x_{hjt} + z_{ht} = E_h, \quad z_{ht} > 0,\\
& \sum\nolimits_j x_{hjt} + \sum\nolimits_{k=1}^{K} \left(\sum\nolimits_j m_{hjk} x_{hjt}^{\theta_h}\right)^{\frac{1}{\theta_h}} + s_{ht} = C_h, \quad s_{ht} > 0,\\
& x_{hit} \in \{1,2,\ldots\} \text{ and } x_{hjt} \in \{0,1,2,\ldots\}, \text{ for } j \neq i.
\end{aligned}$$