Online Appendix
"The Impact of Input and Output Farm Subsidies on Farmer Welfare, Income Disparity, and Consumer Surplus"

Appendix A: Proofs of Lemmas and Propositions

Recall from (1) that $H = \int_0^1 \mu(r) q_r dr$. Since $Q = H \cdot \varepsilon$ and $E[\varepsilon] = 1$, $E[Q] = H$. In this proof, for ease of exposition, we use $E[\tilde{Q}(\cdot)]$ and $\tilde{H}(\cdot)$ interchangably and replace $\tilde{H}(\cdot)$ with $E[\tilde{Q}(\cdot)]$ whenever necessary.

***Proof of Proposition 1*:** Since $\tilde{q}_r(\delta)$, $\mu(r)\tilde{q}_r(\delta)$ and $\pi_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$ share the same monotonicity with respect to δ, it is enough to examine the monotonicity of $\tilde{q}_r(\delta)$. According to Table 2, differentiating $\tilde{q}_r(\delta)$ with respect to δ yields

$$\frac{d\tilde{q}_r(\delta)}{d\delta} = -\frac{1}{\beta}\left(\frac{bM_1(1+\sigma^2)\mu(r)}{\beta + M_2 \cdot b(1+\sigma^2)} - 1\right) = -\frac{1}{\beta}\frac{b(1+\sigma^2)(M_1\mu(r) - M_2) - \beta}{\beta + M_2 \cdot b(1+\sigma^2)}. \tag{22}$$

Recall that $M_1 = \int_0^1 \mu(r)dr$ and $M_2 = \int_0^1 (\mu(r))^2 dr$. Define

$$\tau := b(1+\sigma^2)(M_1\mu_h - M_2).$$

Since $\mu(r)$ increases in r, $M_1\mu_l - M_2 < 0$ and $M_1\mu_h - M_2 > 0$, where $\mu(0) = \mu_l$ and $\mu(1) = \mu_h$. Therefore, when $\beta \geq \tau = b(1+\sigma^2)(M_1\mu_h - M_2)$, $\tilde{q}_r(\delta)$ increases in δ for all $r \in [0,1]$. When $\beta < \tau = b(1+\sigma^2)(M_1\mu_h - M_2)$, there exists a $\bar{r} \in (0,1)$ such that $\beta = b(1+\sigma^2)(M_1\mu(\bar{r}) - M_2)$. Hence, when $\beta < \tau$, $\tilde{q}_r(\delta)$ increases in δ if and only if $r < \bar{r}$.

Suppose $r_1 > r_2$. Since $\mu(r)$ increases in r,

$$\tilde{q}_{r_1}(\delta) - \tilde{q}_{r_2}(\delta) = \frac{(a\beta + (\alpha - \delta)bM_1(1+\sigma^2))(\mu(r_1) - \mu(r_2))}{\beta(\beta + M_2 \cdot b(1+\sigma^2))} > 0.$$

In consequence, $\tilde{q}_{r_1}(\delta) > \tilde{q}_{r_2}(\delta)$, $\mu(r_1)\tilde{q}_{r_1}(\delta) > \mu(r_2)\tilde{q}_{r_2}(\delta)$ and $\tilde{\pi}_{r_1}(\delta) > \tilde{\pi}_{r_2}(\delta)$.

Next, by noting that $\pi_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$, differentiating $\tilde{\pi}_r(\delta)$ with respect to δ and r yields

$$\frac{\partial^2 \tilde{\pi}_r(\delta)}{\partial\delta\partial r} = \beta\frac{\partial\tilde{q}_r(\delta)}{\partial r}\frac{\partial\tilde{q}_r(\delta)}{\partial\delta} + \beta\tilde{q}_r(\delta)\frac{\partial^2\tilde{q}_r(\delta)}{\partial\delta\partial r}.$$

According to Table 2, it can be easily verified that $\tilde{q}_r(\delta)$ increases in r. Recall that we have shown that when $\beta < \tau$ and $r_1 > \bar{r}$, $\tilde{q}_r(\delta)$ decreases in δ. Furthermore, since $\mu(r)$ increases in r, by (22), we have $\frac{\partial^2\tilde{q}_r(\delta)}{\partial\delta\partial r} < 0$. Therefore, we can conclude that when $\beta < \tau$ and $r_1 > \bar{r}$, $\frac{\partial^2\tilde{\pi}_r(\delta)}{\partial\delta\partial r} < 0$. Thus, when $\beta < \tau$, this together with that $\tilde{q}_r(\delta)$ increases in δ only if $r < \bar{r}$ imply that if $r_1 > r_2$, $\tilde{\pi}_{r_1}(\delta) - \tilde{\pi}_{r_2}(\delta)$ decreases in δ.

According to (5),

$$\tilde{G}(\delta) = 1 - 2\frac{\int_0^1\int_0^x \tilde{\pi}_r(\delta)drdx}{\tilde{\Pi}(\delta)}, \tag{23}$$

where $\tilde{\Pi}(\delta) = \int_0^1 \tilde{\pi}_r(\delta)dr$. Differentiating $\tilde{G}(\delta)$ with respect to δ yields

$$\frac{d\tilde{G}(\delta)}{d\delta} = -2\frac{\tilde{\Pi}(\delta)\int_0^1\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\delta}drdx - \int_0^1\int_0^x \tilde{\pi}_r(\delta)drdx\frac{d\tilde{\Pi}(\delta)}{d\delta}}{\tilde{\Pi}^2(\delta)} = -2\frac{\int_0^1\int_0^x \tilde{\pi}_r(\delta)dr\left(\frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\delta}dr}{\int_0^x \tilde{\pi}_r(\delta)dr} - \frac{\frac{d\tilde{\Pi}(\delta)}{d\delta}}{\tilde{\Pi}(\delta)}\right)dx}{\tilde{\Pi}(\delta)}.$$

Let $f(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\delta}dr}{\int_0^x \tilde{\pi}_r(\delta)dr}$. It is easy to see that $f(1) = \frac{\frac{d\tilde{\Pi}(\delta)}{d\delta}}{\tilde{\Pi}(\delta)}$. By noting that $\tilde{\pi}_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$, differentiating $f(x)$ with respect to x yields

$$\frac{df(x)}{dx} = \frac{\frac{d\tilde{\pi}_x(\delta)}{d\delta}\int_0^x \tilde{\pi}_r(\delta)dr - \tilde{\pi}_x(\delta)\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\delta}dr}{(\int_0^x \tilde{\pi}_r(\delta)dr)^2} = \frac{\beta^2\left(\int_0^x \tilde{q}_x^2(\delta)\tilde{q}_r^2(\delta)\left(\frac{\frac{d\tilde{q}_x(\delta)}{d\delta}}{\tilde{q}_x(\delta)} - \frac{\frac{d\tilde{q}_r(\delta)}{d\delta}}{\tilde{q}_r(\delta)}\right)dr\right)}{2(\int_0^x \tilde{\pi}_r(\delta)dr)^2}.$$

From (22), by noting that $\mu(r)$ increases in r, we can show that

$$\frac{d}{dr}\left(\frac{\frac{d\tilde{q}_r(\delta)}{d\delta}}{\tilde{q}_r(\delta)}\right) = \frac{\tilde{q}_r(\delta)\frac{\partial^2\tilde{q}_r(\delta)}{\partial\delta\partial r} - \frac{d\tilde{q}_r(\delta)}{d\delta}\frac{d\tilde{q}_r(\delta)}{dr}}{\tilde{q}_r^2(\delta)} = -\frac{a}{\beta(\beta + M_2 b(1+\sigma^2))\tilde{q}_r^2(\delta)}\frac{d\mu(r)}{dr} < 0.$$

This implies that $\frac{\frac{d\tilde{q}_x(\delta)}{d\delta}}{\tilde{q}_x(\delta)} - \frac{\frac{d\tilde{q}_r(\delta)}{d\delta}}{\tilde{q}_r(\delta)} < 0$ for $r \in (0, x)$. Hence, $\frac{df(x)}{dx} < 0$ and $f(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\delta}dr}{\int_0^x \tilde{\pi}_r(\delta)dr} > f(1) = \frac{\frac{d\tilde{\Pi}(\delta)}{d\delta}}{\tilde{\Pi}(\delta)}$ for $x \in (0,1)$. Therefore, $\frac{d\tilde{G}(\delta)}{d\delta} < 0$.

Finally, differentiating $\tilde{q}_r(\delta)$ with respect to σ yields

$$\frac{d\tilde{q}_r(\delta)}{d\sigma} = -\frac{2\sigma b\mu(r)(aM_2 - M_1(\alpha-\delta))}{(\beta + M_2\cdot b(1+\sigma^2))^2} = -\frac{2\sigma b\mu(r)E[\tilde{Q}(\delta)]}{\beta + M_2\cdot b(1+\sigma^2)} < 0. \tag{24}$$

This also implies that $\mu(r)\tilde{q}_r(\delta)$ and $\tilde{\pi}_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$ are decreasing in σ.

Suppose $r_1 > r_2$. Recall from the above that $\mu(r_1)\tilde{q}_{r_1}(\delta) - \mu(r_2)\tilde{q}_{r_2}(\delta) > 0$. Since $\tilde{\pi}_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$, we can show that

$$\frac{d(\tilde{\pi}_{r_1}(\delta) - \tilde{\pi}_{r_2}(\delta))}{d\sigma} = \beta\tilde{q}_{r_1}(\delta)\frac{d\tilde{q}_{r_1}(\delta)}{d\sigma} - \beta\tilde{q}_{r_2}(\delta)\frac{d\tilde{q}_{r_2}(\delta)}{d\sigma} = -\frac{2\beta\sigma bE[\tilde{Q}(\delta)](\mu(r_1)\tilde{q}_{r_1}(\delta) - \mu(r_2)\tilde{q}_{r_2}(\delta))}{\beta + M_2\cdot b(1+\sigma^2)} < 0.$$

From (23), differentiating $\tilde{G}(\delta)$ with respect to σ yields

$$\frac{d\tilde{G}(\delta)}{d\sigma} = -2\frac{\tilde{\Pi}(\delta)\int_0^1\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\sigma}drdx - \int_0^1\int_0^x \tilde{\pi}_r(\delta)drdx\frac{d\tilde{\Pi}(\delta)}{d\sigma}}{\tilde{\Pi}^2(\delta)} = -2\frac{\int_0^1\int_0^x \tilde{\pi}_r(\delta)dr\left(\frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\delta)dr} - \frac{\frac{d\tilde{\Pi}(\delta)}{d\sigma}}{\tilde{\Pi}(\delta)}\right)dx}{\tilde{\Pi}(\delta)}.$$

Let $g(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\delta)dr}$. Obviously, $g(1) = \frac{\frac{d\tilde{\Pi}(\delta)}{d\sigma}}{\tilde{\Pi}(\delta)}$. Differentiating $g(x)$ with respect to x yields

$$\frac{dg(x)}{dx} = \frac{\frac{d\tilde{\pi}_x(\delta)}{d\sigma}\int_0^x \tilde{\pi}_r(\delta)dr - \tilde{\pi}_x(\delta)\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\sigma}dr}{(\int_0^x \tilde{\pi}_r(\delta)dr)^2} = \frac{\beta^2\left(\int_0^x \tilde{q}_x^2(\delta)\tilde{q}_r^2(\delta)\left(\frac{\frac{d\tilde{q}_x(\delta)}{d\sigma}}{\tilde{q}_x(\delta)} - \frac{\frac{d\tilde{q}_r(\delta)}{d\sigma}}{\tilde{q}_r(\delta)}\right)dr\right)}{2(\int_0^x \tilde{\pi}_r(\delta)dr)^2}.$$

From (24), since $\mu(r)$ increases in r, we can show that

$$\frac{d}{dr}\left(\frac{\frac{d\tilde{q}_r(\delta)}{d\sigma}}{\tilde{q}_r(\delta)}\right) = \frac{\tilde{q}_r(\delta)\frac{\partial^2\tilde{q}_r(\delta)}{\partial\sigma\partial r} - \frac{d\tilde{q}_r(\delta)}{d\sigma}\frac{d\tilde{q}_r(\delta)}{dr}}{\tilde{q}_r^2(\delta)} = \frac{2\sigma bE[\tilde{Q}(\delta)](\alpha-\delta)}{\beta(\beta+M_2b(1+\sigma^2))\tilde{q}_r^2(\delta)}\frac{d\mu(r)}{dr} > 0.$$

Hence, $\frac{\frac{d\tilde{q}_x(\delta)}{d\sigma}}{\tilde{q}_x(\delta)} - \frac{\frac{d\tilde{q}_r(\delta)}{d\sigma}}{\tilde{q}_r(\delta)} > 0$ for $r \in (0,x)$. This further implies that $\frac{dg(x)}{dx} > 0$ and $g(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\delta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\delta)dr} < f(1) = \frac{\frac{d\tilde{\Pi}(\delta)}{d\sigma}}{\tilde{\Pi}(\delta)}$ for $x \in (0,1)$. Therefore, $\frac{d\tilde{G}(\delta)}{d\sigma} > 0$.

***Proof of Proposition 2*:** To facilitate our analysis, we denote $\triangle = \beta + b(1+\sigma^2)(M_2 - M_1^2)$. According to Table 2, the total planting quantity is as follows:

$$\tilde{I}(\delta) = \int_0^1 \tilde{q}_r(\delta)dr = \frac{(a\beta + (\alpha-\delta)bM_1(1+\sigma^2))M_1}{\beta(\beta+M_2\cdot b(1+\sigma^2))} - \frac{\alpha-\delta}{\beta}. \tag{25}$$

Takeing the first derivative of $\tilde{I}(\delta)$ with respect to δ yields

$$\frac{d\tilde{I}(\delta)}{d\delta} = \frac{\beta + (M_2 - M_1^2)\cdot b(1+\sigma^2)}{\beta(\beta+M_2\cdot b(1+\sigma^2))}.$$

According to the Jensen's inequality, $M_1^2 = (\int_0^1 \mu(r)dr)^2 < M_2 = \int_0^1(\mu(r))^2dr$. Therefore, $\tilde{I}(\delta)$ increases in δ. From Table 2, by noting that $\tilde{Q}(\delta) = E[\tilde{Q}(\delta)]\varepsilon$, $\tilde{U}(\delta) = \frac{b}{2}E[(\tilde{Q}(\delta))^2] = \frac{b(1+\sigma^2)}{2}(E[(\tilde{Q}(\delta)])^2$ and $\tilde{\Pi}(\delta) = \int_0^1 \tilde{\pi}_r(\delta)dr$, differentiating $\tilde{\Pi}(\delta) + \tilde{U}(\delta)$ with respect to δ yields

$$\begin{aligned}\frac{d(\tilde{\Pi}(\delta)+\tilde{U}(\delta))}{d\delta} &= \int_0^1 \beta\tilde{q}_r(\delta)\frac{d\tilde{q}_r(\delta)}{d\delta}dr + b(1+\sigma^2)E[\tilde{Q}(\delta)]\frac{dE[\tilde{Q}(\delta)]}{d\delta} \\ &= -\frac{bM_1(1+\sigma^2)\int_0^1 \mu(r)\tilde{q}_r(\delta)dr}{\beta+M_2\cdot b(1+\sigma^2)} + \int_0^1 \tilde{q}_r(\delta)dr + \frac{b(1+\sigma^2)M_1E[\tilde{Q}(\delta)]}{\beta+M_2\cdot b(1+\sigma^2)} \\ &= \int_0^1 \tilde{q}_r(\delta)dr > 0.\end{aligned}$$

Recall from Proposition 1 that $\tilde{G}(\delta)$ decreases in δ. Therefore, the net social welfare $\tilde{W}(\delta)$ increases in δ. Combining this observation with the fact that $\delta\cdot\tilde{I}(\delta)$ increases in δ, we can conclude that the budget constraint (10) will be binding in equilibrium. From (25), the binding budget constraint can be rewritten as

$$\triangle\delta^2 + (a\beta M_1 - \triangle\alpha)\delta - \beta(\beta+M_2\cdot b(1+\sigma^2))\Omega = 0.$$

Solving the above equation yields

$$\delta^* = \frac{-(a\beta M_1 - \triangle\alpha) + \sqrt{(a\beta M_1-\triangle\alpha)^2 + 4\triangle\beta(\beta+M_2\cdot b(1+\sigma^2))\Omega}}{2\triangle}.$$

According to Proposition 1, $\tilde{q}_r(\delta)$ decreases in σ. Therefore, $\tilde{I}(\delta) = \int_0^1 \tilde{q}_r(\delta)dr$ is also decreasing in σ. Because δ^* satisfies the binding constraint $\delta^*\cdot\tilde{I}(\delta^*) = \Omega$, by using the implicit function theorem, we can show that

$$\frac{d\delta^*}{d\sigma} = -\frac{\delta^* \frac{d\tilde{I}(\delta)}{d\sigma}\big|_{\delta=\delta^*}}{\tilde{I}(\delta^*) + \delta^* \frac{d\tilde{I}(\delta)}{d\delta}\big|_{\delta=\delta^*}} > 0.$$

***Proof of Proposition 3*:** According to Table 3, it is easy to show that $\tilde{q}_r(\theta)$, $\mu(r)\tilde{q}_r(\theta)$, and $\tilde{\pi}_r(\theta)$ are increasing in θ. Furthermore, for $r_1 > r_2$, through the direct comparison, we can easily show that

$$\tilde{q}_{r_1}(\theta) - \tilde{q}_{r_2}(\theta) = \frac{((a+\theta)\beta + \alpha b M_1(1+\sigma^2))(\mu(r_1) - \mu(r_2))}{\beta(\beta + M_2 \cdot b(1+\sigma^2))} > 0. \tag{26}$$

This further implies that $\mu(r_1)\tilde{q}_{r_1}(\theta) > \mu(r_2)\tilde{q}_{r_2}(\theta)$ and $\tilde{\pi}_{r_1}(\theta) = \frac{\beta}{2}\left(\tilde{q}_{r_1}(\theta)\right)^2 > \frac{\beta}{2}\left(\tilde{q}_{r_2}(\theta)\right)^2 = \tilde{\pi}_{r_2}(\theta)$.

Furthermore, from (26), it can be easily verified that $\tilde{q}_{r_1}(\theta) - \tilde{q}_{r_1}(\theta)$ increases in θ. Because $\tilde{\pi}_{r_1}(\theta) - \tilde{\pi}_{r_2}(\theta) = \frac{\beta(\tilde{q}_{r_1}(\theta)-\tilde{q}_{r_2}(\theta))(\tilde{q}_{r_1}(\theta)+\tilde{q}_{r_2}(\theta))}{2}$ and $\tilde{q}_r(\theta)$ increases in θ for all $r \in (0,1)$, $\tilde{\pi}_{r_1}(\theta) - \tilde{\pi}_{r_2}(\theta)$ is also increasing in θ.

From (5), the Gini coefficient under the output subsidy scheme is as follows:

$$\tilde{G}(\theta) = 1 - 2\frac{\int_0^1 \int_0^x \tilde{\pi}_r(\theta) dr dx}{\tilde{\Pi}(\theta)}, \tag{27}$$

where $\tilde{\Pi}(\theta) = \int_0^1 \tilde{\pi}_r(\theta) dr$. Differentiating $\tilde{G}(\theta)$ with respect to θ yields

$$\frac{d\tilde{G}(\theta)}{d\theta} = -2\frac{\tilde{\Pi}(\theta)\int_0^1 \int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\theta} dr dx - \int_0^1 \int_0^x \tilde{\pi}_r(\theta) dr dx \frac{d\tilde{\Pi}(\theta)}{d\theta}}{\tilde{\Pi}^2(\theta)} = -2\frac{\int_0^1 \int_0^x \tilde{\pi}_r(\theta) dr \left(\frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\theta} dr}{\int_0^x \tilde{\pi}_r(\theta) dr} - \frac{\frac{d\tilde{\Pi}(\theta)}{d\theta}}{\tilde{\Pi}(\theta)}\right) dx}{\tilde{\Pi}(\theta)}.$$

Denote $\bar{f}(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\theta} dr}{\int_0^x \tilde{\pi}_r(\theta) dr}$. Then, $\bar{f}(1) = \frac{\frac{d\tilde{\Pi}(\theta)}{d\theta}}{\tilde{\Pi}(\theta)}$. Differentiating $\bar{f}(x)$ with respect to x yields

$$\frac{d\bar{f}(x)}{dx} = \frac{\frac{d\tilde{\pi}_x(\theta)}{d\theta}\int_0^x \tilde{\pi}_r(\theta) dr - \tilde{\pi}_x(\theta)\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\theta} dr}{(\int_0^x \tilde{\pi}_r(\theta) dr)^2} = \frac{\beta^2 \left(\int_0^x \tilde{q}_x^2(\theta)\tilde{q}_r^2(\theta)\left(\frac{\frac{d\tilde{q}_x(\theta)}{d\theta}}{\tilde{q}_x(\theta)} - \frac{\frac{d\tilde{q}_r(\theta)}{d\theta}}{\tilde{q}_r(\theta)}\right) dr\right)}{2(\int_0^x \tilde{\pi}_r(\theta) dr)^2}.$$

According to Table 3, by noting that $\mu(r)$ increases in r, we can show that

$$\frac{d}{dr}\left(\frac{\frac{d\tilde{q}_r(\theta)}{d\theta}}{\tilde{q}_r(\theta)}\right) = \frac{\tilde{q}_r(\theta)\frac{\partial^2 \tilde{q}_r(\theta)}{\partial\theta\partial r} - \frac{d\tilde{q}_r(\theta)}{d\theta}\frac{d\tilde{q}_r(\theta)}{dr}}{\tilde{q}_r^2(\theta)} = -\frac{\alpha}{\beta(\beta + M_2 b(1+\sigma^2))\tilde{q}_r^2(\theta)}\frac{d\mu(r)}{dr} < 0.$$

This implies that $\frac{\frac{d\tilde{q}_x(\theta)}{d\theta}}{\tilde{q}_x(\theta)} - \frac{\frac{d\tilde{q}_r(\theta)}{d\theta}}{\tilde{q}_r(\theta)} < 0$ for $r \in (0,x)$. Hence, $\frac{d\bar{f}(x)}{dx} < 0$ and $\bar{f}(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\theta} dr}{\int_0^x \tilde{\pi}_r(\theta) dr} > \bar{f}(1) = \frac{\frac{d\tilde{\Pi}(\theta)}{d\theta}}{\tilde{\Pi}(\theta)}$ for $x \in (0,1)$. Therefore, $\frac{d\tilde{G}(\theta)}{d\theta} < 0$.

According to Table 3, differentiating $\tilde{q}_r(\theta)$ with respect to σ yields

$$\frac{d\tilde{q}_r(\theta)}{d\sigma} = -\frac{2\sigma b\mu(r)((a+\theta)M_2 - M_1\alpha)}{(\beta + M_2 \cdot b(1+\sigma^2))^2} = -\frac{2\sigma b\mu(r)E[\tilde{Q}(\theta)]}{\beta + M_2 \cdot b(1+\sigma^2)} < 0.$$

This also implies that $\mu(\theta)\tilde{q}_r(\theta)$ and $\tilde{\pi}_r(\theta) = \frac{\beta(\tilde{q}_r(\theta))^2}{2}$ are decreasing in σ. Furthermore, it can be easily verified that

$$\frac{d(\tilde{q}_{r_1}(\theta) - \tilde{q}_{r_2}(\theta))}{d\sigma} = -\frac{2\sigma b(\mu(r_1) - \mu(r_2))E[\tilde{Q}(\theta)]}{\beta + M_2 \cdot b(1+\sigma^2)} < 0.$$

Because $\tilde{\pi}_{r_1}(\theta) - \tilde{\pi}_{r_2}(\theta) = \frac{\beta(\tilde{q}_{r_1}(\theta) - \tilde{q}_{r_2}(\theta))(\tilde{q}_{r_1}(\theta) + \tilde{q}_{r_2}(\theta))}{2}$ and $\tilde{q}_r(\theta)$ decreases in σ for all $r \in (0,1)$, $\tilde{\pi}_{r_1}(\theta) - \tilde{\pi}_{r_2}(\theta)$ is also decreasing in σ.

Finally, from (27), differentiating $\tilde{G}(\theta)$ with respect to σ yields

$$\frac{d\tilde{G}(\theta)}{d\sigma} = -2\frac{\tilde{\Pi}(\theta)\int_0^1\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\sigma}drdx - \int_0^1\int_0^x \tilde{\pi}_r(\theta)drdx\frac{d\tilde{\Pi}(\theta)}{d\sigma}}{\tilde{\Pi}^2(\theta)} = -2\frac{\int_0^1\int_0^x \tilde{\pi}_r(\theta)dr\left(\frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\theta)dr} - \frac{\frac{d\tilde{\Pi}(\theta)}{d\sigma}}{\tilde{\Pi}(\theta)}\right)dx}{\tilde{\Pi}(\theta)}.$$

Let $\bar{g}(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\theta)dr}$. Obviously, $\bar{g}(1) = \frac{\frac{d\tilde{\Pi}(\theta)}{d\sigma}}{\tilde{\Pi}(\theta)}$. Differentiating $\bar{g}(x)$ with respect to x yields

$$\frac{d\bar{g}(x)}{dx} = \frac{\frac{d\tilde{\pi}_x(\theta)}{d\sigma}\int_0^x \tilde{\pi}_r(\theta)dr - \tilde{\pi}_x(\theta)\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\sigma}dr}{(\int_0^x \tilde{\pi}_r(\theta)dr)^2} = \frac{\beta^2\left(\int_0^x \tilde{q}_x^2(\theta)\tilde{q}_r^2(\theta)\left(\frac{\frac{d\tilde{q}_x(\theta)}{d\sigma}}{\tilde{q}_x(\theta)} - \frac{\frac{d\tilde{q}_r(\theta)}{d\sigma}}{\tilde{q}_r(\theta)}\right)dr\right)}{2(\int_0^x \tilde{\pi}_r(\theta)dr)^2}.$$

From (24), since $\mu(r)$ increases in r, we can show that

$$\frac{d}{dr}\left(\frac{\frac{d\tilde{q}_r(\theta)}{d\sigma}}{\tilde{q}_r(\theta)}\right) = \frac{\tilde{q}_r(\theta)\frac{\partial^2\tilde{q}_r(\theta)}{\partial\sigma\partial r} - \frac{d\tilde{q}_r(\theta)}{d\sigma}\frac{d\tilde{q}_r(\theta)}{dr}}{\tilde{q}_r^2(\theta)} = \frac{2\sigma bE[\tilde{Q}(\theta)]\alpha}{\beta(\beta + M_2 b(1+\sigma^2))\tilde{q}_r^2(\theta)}\frac{d\mu(r)}{dr} > 0.$$

Hence, $\frac{\frac{d\tilde{q}_x(\theta)}{d\sigma}}{\tilde{q}_x(\theta)} - \frac{\frac{d\tilde{q}_r(\theta)}{d\sigma}}{\tilde{q}_r(\theta)} > 0$ for $r \in (0,x)$. This further implies that $\frac{d\bar{g}(x)}{dx} > 0$ and $\bar{g}(x) = \frac{\int_0^x \frac{d\tilde{\pi}_r(\theta)}{d\sigma}dr}{\int_0^x \tilde{\pi}_r(\theta)dr} < f(1) = \frac{\frac{d\tilde{\Pi}(\theta)}{d\sigma}}{\tilde{\Pi}(\theta)}$ for $x \in (0,1)$. Therefore, $\frac{d\tilde{G}(\theta)}{d\sigma} > 0$.

***Proof of Proposition 4*:** According to Table 3, it can be easily verified that $E[\tilde{Q}(\theta)]$ increases in θ. Based on Proposition 3, we can easily show that $\tilde{W}(\theta)$ increases in θ. Therefore, the budget constraint (15) will be binding in equilibrium. By using $E[\tilde{Q}(\theta)] = \frac{(a+\theta)M_2 - M_1\alpha}{\beta + M_2 b(1+\sigma^2)}$, the binding budget constraint can be rewritten as

$$M_2\theta^2 + (aM_2 - M_1\alpha)\theta - (\beta + M_2 b(1+\sigma^2))\Omega = 0.$$

Solving the above equation yields

$$\theta^* = \frac{-(aM_2 - M_1\alpha) + \sqrt{(aM_2 - M_1\alpha)^2 + 4M_2(\beta + M_2 b(1+\sigma^2))\Omega}}{2M_2}.$$

Next, we are going to show that θ^* increases in σ. According to Table 3, we can easily show that $E[\tilde{Q}(\theta)]$ decreases in σ and increases in θ. Recall that θ^* satisfies the binding constraint $\theta E[\tilde{Q}(\theta)] - \Omega = 0$. Therefore, by using the implicit function theorem, we can obtain that

$$\frac{d\theta^*}{d\sigma} = -\frac{\theta^*\frac{dE[\tilde{Q}(\theta)]}{d\sigma}\Big|_{\theta=\theta^*}}{E[\tilde{Q}(\theta^*)] + \theta^*\frac{dE[\tilde{Q}(\theta)]}{d\theta}\Big|_{\theta=\theta^*}} > 0.$$

***Proof of Proposition 5*:** To facilitate our analysis, here we use the superscripts o and i to indicate the equilibrium outcomes under the input and output subsidy schemes, respectively. For example, $\tilde{q}_r^o(\cdot)$ and $\tilde{q}_r^i(\cdot)$ represent the planting quantity of farmer r under the input and output subsidy schemes, respectively.

Recall that $M_1 = \int_0^1 \mu(r)dr$ and $M_2 = \int_0^1 (\mu(r))^2 dr$. By noting that $\mu(1) = \mu_h$, $\mu(0) = \mu_l$, and $\mu(r)$ increases in r, we can easily show that $\mu_h > M_1$, $\mu_h M_1 > M_2$, and $\mu_l M_1 < M_2$. Furthermore, the Jensen's inequality reveals that $M_2 > M_1^2$. According to Tables 2 and 3, we have

$$\tilde{I}^o(\theta) = \int_0^1 \tilde{q}_r(\theta)dr = \frac{((a+\theta)\beta + \alpha b M_1(1+\sigma^2))M_1}{\beta(\beta + M_2 \cdot b(1+\sigma^2))} - \frac{\alpha}{\beta},$$

$$\tilde{I}^i(\delta) = \int_0^1 \tilde{q}_r(\delta)dr = \frac{(a\beta + (\alpha-\delta) b M_1(1+\sigma^2))M_1}{\beta(\beta + M_2 \cdot b(1+\sigma^2))} - \frac{\alpha-\delta}{\beta}.$$

Through the direct comparison, we can easily show that

$$\begin{aligned}
\tilde{q}_r^o(\theta^*) - \tilde{q}_r^i(\delta^*) &= \frac{\beta(\theta^*\mu(r) - \delta^*) + \delta^* b(1+\sigma^2)(M_1\mu(r) - M_2)}{\beta(\beta + M_2 \cdot b(1+\sigma^2))}, \qquad (28)\\
\tilde{I}^i(\mu_h\theta^*) - \tilde{I}^o(\theta^*) &= \frac{\beta\theta^*(\mu_h - M_1) + b(1+\sigma^2)\mu_h\theta^*(M_2 - M_1^2)}{\beta(\beta + M_2 \cdot b(1+\sigma^2))} > 0,\\
E[\tilde{Q}^i(\mu_l\theta^*)] - E[\tilde{Q}^o(\theta^*)] &= \frac{\theta^*(M_1\mu_l - M_2)}{\beta + M_2 \cdot b(1+\sigma^2)} < 0.
\end{aligned}$$

As $\mu(r)$ increases in r, it is straightforward to see that $\tilde{q}_r^o(\theta^*) - \tilde{q}_r^i(\delta^*)$ also increases in r. Because $\mu_h \tilde{I}^o(\theta^*) - E[\tilde{Q}^o(\theta^*)] = \int_0^1 (\mu_h - \mu(r))\tilde{q}_r^o(\theta^*)dr > 0$,

$$\mu_h\theta^*\tilde{I}^i(\mu_h\theta^*) > \mu_h\theta^*\tilde{I}^o(\theta^*) > \theta^* E[\tilde{Q}^o(\theta^*)] = \Omega.$$

Recall that $\delta\tilde{I}^i(\delta)$ increases in δ and $\delta^*\tilde{I}^i(\delta^*) = \Omega$. Therefore, $\mu_h\theta^* > \delta^*$. From (28), since $M_1\mu_h > M_2$, we have $\tilde{q}_1^o(\theta^*) - \tilde{q}_1^i(\delta^*) > 0$. Similarly, because $\mu_l\tilde{I}^i(\mu_l\theta^*) - E[\tilde{Q}^i(\mu_l\theta^*)] = \int_0^1 (\mu_l - \mu(r))\tilde{q}_r^i(\mu_l\theta^*)dr < 0$,

$$\mu_l\theta^*\tilde{I}^i(\mu_l\theta^*) < \theta^* E[\tilde{Q}^i(\mu_l\theta^*)] < \theta^* E[\tilde{Q}^o(\theta^*)] = \Omega.$$

By using the facts that $\delta\tilde{I}^i(\delta)$ increases in δ and $\delta^*\tilde{I}^i(\delta^*) = \Omega$, we have $\mu_l\theta^* < \delta^*$. From (28), because $M_1\mu_l < M_2$, we can conclude that $\tilde{q}_0^o(\theta^*) - \tilde{q}_0^i(\delta^*) < 0$. Because $\tilde{q}_r^o(\theta^*) - \tilde{q}_r^i(\delta^*)$ increases in r, there exists a $\hat{r}$ such that $\tilde{q}_r^o(\theta^*) > \tilde{q}_r^i(\delta^*)$ if and only if $r > \hat{r}$. This also implies that $\mu(r)\tilde{q}_r^o(\theta^*) > \mu(r)\tilde{q}_r^i(\delta^*)$ and $\tilde{\pi}_r^o(\theta^*) = \frac{\beta(\tilde{q}_r^o(\theta^*))^2}{2} > \frac{\beta(\tilde{q}_r^i(\delta^*))^2}{2} = \tilde{\pi}_r^i(\delta^*)$ if and only if $r > \hat{r}$.

Also, through the direct comparison, we can obtain that

$$\begin{aligned}
E[\tilde{Q}^o(\theta^*)] - E[\tilde{Q}^i(\delta^*)] &= \frac{\theta^* M_2 - \delta^* M_1}{\beta + M_2 \cdot b(1+\sigma^2)}, \qquad (29)\\
\tilde{I}^o(\theta^*) - \tilde{I}^i(\delta^*) &= \frac{\beta(\theta^* M_1 - \delta^*) - \delta^* b(\sigma^2+1)(M_2 - M_1^2)}{\beta(\beta + M_2 \cdot b(1+\sigma^2))}. \qquad (30)
\end{aligned}$$

According to Table 3, plugging $\theta = \frac{M_1\delta^*}{M_2}$ into $E[\tilde{Q}^o(\theta)]$ yields

$$E\left[\tilde{Q}^o\left(\frac{M_1\delta^*}{M_2}\right)\right] = \frac{aM_2 - (\alpha - \delta^*)M_1}{\beta + M_2 \cdot b(1+\sigma^2)} = E[\tilde{Q}^i(\delta^*)].$$

Because $M_2 > M_1^2$,

$$\frac{M_1}{M_2}E[\tilde{Q}^i(\delta^*)] - \tilde{I}^i(\delta^*) = \frac{(\alpha - \delta^*)(M_2 - M_1^2)}{\beta M_2} > 0, \quad (31)$$

$$\frac{M_1}{M_2}E[\tilde{Q}^o(\theta^*)] - \tilde{I}^o(\theta^*) = \frac{\alpha(M_2 - M_1^2)}{\beta M_2} > 0. \quad (32)$$

Therefore,

$$\frac{M_1\delta^*}{M_2}E\left[\tilde{Q}^o\left(\frac{M_1\delta^*}{M_2}\right)\right] = \frac{M_1\delta^*}{M_2}E[\tilde{Q}^i(\delta^*)] > \delta^*\tilde{I}^i(\delta^*) = \Omega.$$

Because $\theta E[\tilde{Q}^o(\theta)]$ increases in θ and $\theta^* E[\tilde{Q}^o(\theta^*)] = \Omega$, $\frac{M_1\delta^*}{M_2} > \theta^*$. From (29), we can obtain that $E[\tilde{Q}^o(\theta^*)] < E[\tilde{Q}^i(\delta^*)]$. Since $M_2 > M_1^2$, $\frac{\delta^*}{M_1} > \frac{M_1\delta^*}{M_2} > \theta^*$. From (30), we can conclude that $\tilde{I}^o(\theta^*) < \tilde{I}^i(\delta^*)$. As $\tilde{U}^j(\cdot) = \frac{bE[(\tilde{Q}^j(\cdot))^2]}{2} = \frac{b(1+\sigma^2)(E[\tilde{Q}^j(\cdot)])^2}{2}$, $j \in \{o, i\}$ and $E[\tilde{Q}^o(\theta^*)] < E[\tilde{Q}^i(\delta^*)]$, $\tilde{U}^o(\theta^*) < \tilde{U}^i(\delta^*)$.

We next consider the farmers' total income. From (28), by noting that $\tilde{\pi}_r^j(\cdot) = \frac{\beta(\tilde{q}_r^j(\cdot))^2}{2}$, $j \in \{o, i\}$, we have

$$\tilde{\pi}_r^o(\theta^*) - \tilde{\pi}_r^i(\delta^*) = \frac{\beta \cdot (\tilde{q}_r^o(\theta^*) - \tilde{q}_r^i(\delta^*))(\tilde{q}_r^o(\theta^*) + \tilde{q}_r^i(\delta^*))}{2} \quad (33)$$

$$= \frac{(\beta(\theta^*\mu(r) - \delta^*) + \delta^* b(1+\sigma^2)(M_1\mu(r) - M_2))(\tilde{q}_r^o(\theta^*) + \tilde{q}_r^i(\delta^*))}{2(\beta + M_2 \cdot b(1+\sigma^2))}. \quad (34)$$

By noting that $E[\tilde{Q}^j(\cdot)] = \int_0^1 \mu(r)\tilde{q}_r^j(\cdot)dr$ and $\tilde{I}^j(\cdot) = \int_0^1 \tilde{q}_r^j(\cdot)dr$, $j \in \{o, i\}$, from (31) and (32), we can show that

$$\int_0^1 (M_1\mu(r) - M_2))(\tilde{q}_r^o(\theta^*) + \tilde{q}_r^i(\delta^*))dr = M_1(E[\tilde{Q}^o(\theta^*)] + E[\tilde{Q}^i(\delta^*)]) - M_2(\tilde{I}^o(\theta^*) + \tilde{I}^i(\delta^*)) > 0. \quad (35)$$

We have shown that $E[\tilde{Q}^i(\delta^*)] > E[\tilde{Q}^o(\theta^*)]$ and $\tilde{I}^i(\delta^*) > \tilde{I}^o(\theta^*)$. Because $\theta^* E[\tilde{Q}^o(\theta^*)] = \delta^*\tilde{I}^i(\delta^*) = \Omega$,

$$\int_0^1 (\theta^*\mu(r) - \delta^*)(\tilde{q}_r^o(\theta^*) + \tilde{q}_r^i(\delta^*))dr = \theta^* E[\tilde{Q}^i(\delta^*)] - \delta^*\tilde{I}^o(\theta^*) = \frac{\Omega(E[\tilde{Q}^i(\delta^*)]\tilde{I}^i(\delta^*) - E[\tilde{Q}^o(\theta^*)]\tilde{I}^o(\theta^*))}{E[\tilde{Q}^o(\theta^*)]\tilde{I}^i(\delta^*)} > 0. \quad (36)$$

As $\tilde{\Pi}^j(\cdot) = \int_0^1 \tilde{\pi}_r^j(\cdot)dr$, $j \in \{o, i\}$, $\tilde{\Pi}(\theta^*) - \tilde{\Pi}(\delta^*) = \int_0^1 (\tilde{\pi}_r^o(\theta^*) - \tilde{\pi}_r^i(\delta^*))dr$. From (34), (35), and (36), we can conclude that $\tilde{\Pi}(\theta^*) > \tilde{\Pi}(\delta^*)$.

From (28), it is straightforward to see that $\tilde{q}_r^o(\theta^*) - \tilde{q}_r^i(\delta^*)$ increases in r. Because both $\tilde{q}_r^o(\theta^*)$ and $\tilde{q}_r^i(\delta^*)$ increase in r, from (33), we can know that $\tilde{\pi}_r^o(\theta^*) - \tilde{\pi}_r^i(\delta^*)$ also increases in r. Therefore, given $r_1 > r_2$, we have $\tilde{\pi}_{r_1}^o(\theta^*) - \tilde{\pi}_{r_1}^i(\delta^*) > \tilde{\pi}_{r_2}^o(\theta^*) - \tilde{\pi}_{r_2}^i(\delta^*)$. Equivalently, $\tilde{\pi}_{r_1}^o(\theta^*) - \tilde{\pi}_{r_2}^o(\theta^*) > \tilde{\pi}_{r_1}^i(\delta^*) - \tilde{\pi}_{r_2}^i(\delta^*)$. Finally, we consider the Gini coefficient. Through the direct comparison, we obtain that

$$\begin{aligned}\tilde{G}^o(\theta^*) - \tilde{G}^i(\delta^*) &= \frac{2\int_0^1\int_0^x \tilde{\pi}_r^i(\delta^*)drdx}{\tilde{\Pi}^i(\delta^*)} - \frac{2\int_0^1\int_0^x \tilde{\pi}_r^o(\theta^*)drdx}{\tilde{\Pi}^o(\theta^*)} = 2\int_0^1 \left(\frac{\int_0^x \tilde{\pi}_r^i(\delta^*)dr}{\tilde{\Pi}^i(\delta^*)} - \frac{\int_0^x \tilde{\pi}_r^o(\theta^*)dr}{\tilde{\Pi}^o(\theta^*)}\right)dx \\ &= 2\int_0^1 \frac{\int_0^x \tilde{\pi}_r^o(\theta^*)dr}{\tilde{\Pi}^i(\delta^*)}\left(\frac{\int_0^x \tilde{\pi}_r^i(\delta^*)dr}{\int_0^x \tilde{\pi}_r^o(\theta^*)dr} - \frac{\tilde{\Pi}^i(\delta^*)}{\tilde{\Pi}^o(\theta^*)}\right)dx.\end{aligned}$$

Denote $h(x) = \frac{\int_0^x \tilde{\pi}_r^i(\delta^*)dr}{\int_0^x \tilde{\pi}_r^o(\theta^*)dr}$. It is easy to see that $h(1) = \frac{\tilde{\Pi}^i(\delta^*)}{\tilde{\Pi}^o(\theta^*)}$. Differentiating $h(x)$ with respect to x yields

$$\frac{dh(x)}{dx} = \frac{\tilde{\pi}_x^i(\delta^*)\int_0^x \tilde{\pi}_r^o(\theta^*)dr - \tilde{\pi}_x^o(\theta^*)\int_0^x \tilde{\pi}_r^i(\delta^*)dr}{(\int_0^x \tilde{\pi}_r^o(\theta^*)dr)^2} = \frac{\beta^2(\tilde{q}_x^i(\delta^*))^2\left(\int_0^x (\tilde{q}_r^i(\delta^*))^2\left(\frac{(\tilde{q}_r^o(\theta^*))^2}{(\tilde{q}_r^i(\delta^*))^2} - \frac{(\tilde{q}_x^o(\theta^*))^2}{(\tilde{q}_x^i(\delta^*))^2}\right)dr\right)}{4(\int_0^x \tilde{\pi}_r^o(\theta^*)dr)^2}.$$

Denote $n(r) = \frac{\tilde{q}_r^o(\theta^*)}{\tilde{q}_r^i(\delta^*)}$. Differentiating $n(r)$ with respect to r yields

$$\frac{dn(r)}{dr} = \frac{\tilde{q}_r^i(\delta^*)\frac{d\tilde{q}_r^o(\theta^*)}{dr} - \tilde{q}_r^o(\theta^*)\frac{d\tilde{q}_r^i(\delta^*)}{dr}}{(\tilde{q}_r^i(\delta^*))^2} = \frac{a\delta^* - (\alpha - \delta^*)\theta^*}{(\tilde{q}_r^i(\delta^*))^2\beta(\beta + M_2 b(1+\sigma^2))}\frac{d\mu(r)}{dr}.$$

In equilibrium, the expected harvest quantity should be positive such that $E[\tilde{Q}^i(\delta^*)] > 0$. According to Table 2, $E[\tilde{Q}^i(\delta^*)] > 0$ implies that $a > \frac{(\alpha-\delta^*)M_1}{M_2}$. We have shown that $\frac{\delta^* M_1}{M_2} > \theta^*$. Therefore, $a\delta^* - (\alpha - \delta^*)\theta^* > (\alpha - \delta^*)\left(\frac{\delta^* M_1}{M_2} - \theta^*\right) > 0$. Hence, $n(r)$ increases in r. This implies that $\frac{(\tilde{q}_r^o(\theta^*))^2}{(\tilde{q}_r^i(\delta^*))^2} < \frac{(\tilde{q}_x^o(\theta^*))^2}{(\tilde{q}_x^i(\delta^*))^2}$ for $r \in (0,x)$. Hence, $\frac{dh(x)}{dx} < 0$ and $h(x) = \frac{\int_0^x \tilde{\pi}_r^i(\delta^*)dr}{\int_0^x \tilde{\pi}_r^o(\theta^*)dr} > h(1) = \frac{\tilde{\Pi}^i(\delta^*)}{\tilde{\Pi}^o(\theta^*)}$ for $x \in (0,1)$. Therefore, $\tilde{G}^o(\theta^*) > \tilde{G}^i(\delta^*)$.

Proof of Proposition 6: From (29), by noting that $\tilde{U}^j(\cdot) = \frac{b}{2}E[(\tilde{Q}^j(\cdot))^2] = \frac{b(1+\sigma^2)}{2}(E[\tilde{Q}^j(\cdot)])^2$, $j \in \{o,i\}$,

$$\begin{aligned}\tilde{U}^o(\theta^*) - \tilde{U}^i(\delta^*) &= \frac{b(1+\sigma^2)(E[\tilde{Q}^o(\theta^*)] - E[\tilde{Q}^i(\delta^*)])(E[\tilde{Q}^o(\theta^*)] + E[\tilde{Q}^i(\delta^*)])}{2} \\ &= \frac{b(1+\sigma^2)(M_2\theta^* - M_1\delta^*)(E[\tilde{Q}^o(\theta^*)] + E[\tilde{Q}^i(\delta^*)])}{2(\beta + M_2 \cdot b(1+\sigma^2))}.\end{aligned}$$

From (35) and (36), since $\theta^* E[\tilde{Q}^o(\theta^*)] = \delta^* \tilde{I}^i(\delta^*) = \Omega$, $\theta^* E[\tilde{Q}^o(\theta^*)] = \Omega$, and $\tilde{I}^o(\theta^*) < \tilde{I}^i(\delta^*)$, by the direct comparison, we have

$$\int_0^1 \frac{\delta^* b(1+\sigma^2)(M_1\mu(r) - M_2))(\tilde{q}_r^o(\theta^*) + \tilde{q}_r^i(\delta^*))dr}{2(\beta + M_2 \cdot b(1+\sigma^2))} + \tilde{U}^o(\theta^*) - \tilde{U}^i(\delta^*) = \frac{b(1+\sigma^2)M_2(\theta^* E[\tilde{Q}^i(\delta^*)] - \delta^* \tilde{I}^o(\theta^*))}{2(\beta + M_2 \cdot b(1+\sigma^2))} > 0.$$

By noting that $\tilde{\Pi}(\theta^*) - \tilde{\Pi}(\delta^*) = \int_0^1 (\tilde{\pi}_r^o(\theta^*) - \tilde{\pi}_r^i(\delta^*))dr$, from (34) and (36), we can conclude that $\tilde{\Pi}^o(\theta^*) + \tilde{U}^o(\theta^*) > \tilde{\Pi}^i(\delta^*) + \tilde{U}^i(\delta^*)$.

Note that both δ^* given in (11) and θ^* given in (16) are independent of λ. Thus, $\tilde{W}(\delta^*) > \tilde{W}(\theta^*)$ if and only if

$$\lambda > \hat{\lambda} := \frac{\tilde{\Pi}(\theta^*) + \tilde{U}(\theta^*) - \left(\tilde{\Pi}(\delta^*) + \tilde{U}(\delta^*)\right)}{\tilde{G}(\theta^*) - \tilde{G}(\delta^*)}. \tag{37}$$

***Proof of Proposition 7*:** By noting that $\pi_r(\delta,\theta) = \frac{\beta}{2}\left(\tilde{q}_r(\delta,\theta)\right)^2$, we have

$$\frac{\partial^2 \tilde{\pi}_r(\delta,\theta)}{\partial\delta\partial\theta} = \beta \frac{\partial \tilde{q}_r(\delta,\theta)}{\partial\delta}\frac{\partial \tilde{q}_r(\delta,\theta)}{\partial\theta}.$$

Recall that if we let $\acute{a} := a+\theta$ ($\acute{\alpha} := \alpha-\delta$), the equilibrium outcomes degenerate to their counterparts under the input (output) subsidy scheme. Therefore, according to Propositions 1 and 3, when $\beta \geq b(\mu_h M_1 - M_2)(1+\sigma^2)$, $\frac{\partial^2 \tilde{\pi}_r(\delta,\theta)}{\partial\delta\partial\theta} \geq 0$ for all $r \in [0,1]$. While when $\beta < b(\mu_h M_1 - M_2)(1+\sigma^2)$, $\frac{\partial^2 \tilde{\pi}_r(\delta,\theta)}{\partial\delta\partial\theta} > 0$ if and only if $r < \bar{r}$, where $\bar{r}$ is given in Proposition 1.

Furthermore, differentiating $\tilde{\Pi}(\delta,\theta) = \int_0^1 \tilde{\pi}_r(\delta,\theta)dr$ with respect to (δ,θ) yields

$$\frac{\partial^2 \tilde{\Pi}(\delta,\theta)}{\partial\delta\partial\theta} = \beta\int_0^1 \frac{\partial \tilde{q}_r(\delta,\theta)}{\partial\delta}\frac{\partial \tilde{q}_r(\delta,\theta)}{\partial\theta}dr = \frac{\beta M_1}{(\beta+M_2 b(1+\sigma^2))^2} > 0.$$

Suppose $r_1 > r_2$ and $r_1 > \bar{r}$. Then, we can show that

$$\begin{aligned}\frac{\partial^2(\tilde{\pi}_{r_1}(\delta,\theta) - \tilde{\pi}_{r_2}(\delta,\theta))}{\partial\delta\partial\theta} &= \beta\left(\frac{\partial \tilde{q}_{r_1}(\delta,\theta)}{\partial\delta}\frac{\partial \tilde{q}_{r_1}(\delta,\theta)}{\partial\theta} - \frac{\partial \tilde{q}_{r_2}(\delta,\theta)}{\partial\delta}\frac{\partial \tilde{q}_{r_2}(\delta,\theta)}{\partial\theta}\right) \\ &= \frac{(\mu(r_1)-\mu(r_2))(\beta + b(1+\sigma^2)(M_2 - (\mu(r_1)+\mu(r_2))M_1))}{(\beta+M_2 b(1+\sigma^2))^2}.\end{aligned}$$

Let $v(\beta) = \beta + b(1+\sigma^2)(M_2 - (\mu(r_1)+\mu(r_2))M_1)$. It is straightforward to see that $v(\beta)$ increases in β and when β is large enough, $v(\beta) > 0$. Also, note that when $\beta = b(1+\sigma^2)(M_1\mu(\bar{r}) - M_2)$, $v(\beta) = b(1+\sigma^2)(\mu(\bar{r}) - \mu(r_1) - \mu(r_2))M_1 < 0$. Therefore, we can find a $\breve{\beta}$ such that $v(\beta) > 0$ if and only if $\beta > \breve{\beta}$. This further implies that $\frac{\partial^2(\tilde{\pi}_{r_1}(\delta,\theta) - \tilde{\pi}_{r_2}(\delta,\theta))}{\partial\delta\partial\theta} > 0$ if and only if $\beta > \breve{\beta}$.

***Proof of Proposition 8*:** The first statement can be easily shown according to Proposition 3. We next consider the second statement. From (21), the government's optimization problem here becomes

$$\max\ \tilde{W}(\delta) = \max_{\delta}\{\lambda_f \cdot \tilde{\Pi}(\delta) + \lambda_c \cdot \tilde{U}(\delta) - \lambda_g \cdot \tilde{G}(\delta)\}, \tag{38}$$

$$s.t. \quad \delta \cdot \tilde{I}(\delta) \leq \Omega, \tag{39}$$

According to (25), $\tilde{I}(\delta)$ can be written as

$$\tilde{I}(\delta) = \frac{a\beta M_1 - (\alpha-\delta)(\beta + b(1+\sigma^2)(M_2 - M_1^2))}{\beta(\beta + M_2 \cdot b(1+\sigma^2))}.$$

In equilibrium, $\tilde{I}(\delta)$ should be always positive such that $a\beta M_1 > \alpha(\beta + b(1+\sigma^2)(M_2 - M_1^2))$. According to Table 2, it is straightforward to show that $\tilde{\pi}_r(\delta)$ is convex in δ. As $\tilde{\Pi}(\delta) = \int_0^1 \tilde{\pi}_r(\delta)dr$, $\tilde{\Pi}(\delta)$ is also convex in δ. By noting that $\tilde{\pi}_r(\delta) = \frac{\beta(\tilde{q}_r(\delta))^2}{2}$, we can show that

$$\begin{aligned}
\frac{d\tilde{\Pi}(\delta)}{d\delta}|_{\delta=0} &= \int_0^1 \beta\tilde{q}_r(0)\frac{d\tilde{q}_r(\delta)}{d\delta}|_{\delta=0}dr \\
&= -\frac{bM_1(1+\sigma^2)\int_0^1\mu(r)\tilde{q}_r(0)dr}{\beta+M_2\cdot b(1+\sigma^2)} + \int_0^1 \tilde{q}_r(0)dr \\
&= -\frac{bM_1(1+\sigma^2)(aM_2-\alpha M_1)}{(\beta+M_2\cdot b(1+\sigma^2))^2} + \frac{a\beta M_1-\alpha(\beta+b(1+\sigma^2)(M_2-M_1^2))}{\beta(\beta+M_2\cdot b(1+\sigma^2))} \\
&= \frac{a\beta^2M_1-\alpha(\beta^2+2\beta b(1+\sigma^2)(M_2-M_1^2)+b^2(1+\sigma^2)^2M_2(M_2-M_1^2))}{\beta(\beta+M_2\cdot b(1+\sigma^2)^2)}.
\end{aligned}$$

Let $h(\beta) = a\beta^2M_1-\alpha(\beta^2+2\beta b(1+\sigma^2)(M_2-M_1^2)+b^2(1+\sigma^2)^2M_2(M_2-M_1^2))$. Then, $\frac{dh(\beta)}{d\beta} = 2(a\beta M_1 - \alpha(\beta+b(1+\sigma^2)(M_2-M_1^2)) > 0$. It can be easily verified that when β is sufficiently large (small), $h(\beta)$ is positive (negative). Therefore, there exists a $\hat{\beta}$ such that $h(\beta) > 0$ if and only if $\beta > \hat{\beta}$. As $\tilde{\Pi}(\delta)$ is convex in δ, this further implies that when $\beta \geq \hat{\beta}$, $\frac{d\tilde{\Pi}(\delta)}{d\delta}|_{\delta=0} \geq 0$ and $\tilde{\Pi}(\delta)$ increases in δ for $\delta \geq 0$; while when $\beta < \hat{\beta}$, $\frac{d\tilde{\Pi}(\delta)}{d\delta}|_{\delta=0} < 0$ and $\tilde{\Pi}(\delta)$ is first decreasing and then increasing in δ. Note that the consumer surplus $\tilde{U}(\delta) = \frac{b}{2}E[(\tilde{Q}(\delta))^2] = \frac{b(1+\sigma^2)}{2}(E[(\tilde{Q}(\delta)])^2$. It can be easily shown that $\tilde{U}(\delta)$ increases in δ and recall that $\tilde{G}(\delta)$ decreases in δ according to Proposition 1. Thus, when $\beta \geq \hat{\beta}$, $\tilde{W}(\delta)$ increases in δ and the budget constraint (39) should be binding. When $\beta < \hat{\beta}$ and λ_f is sufficiently large, $\frac{d\tilde{W}(\delta)}{d\delta}$ will be negative if δ is small. In this case, without loss of generality, we assume that $\frac{d\tilde{W}(\delta)}{d\delta} < 0$ for $\delta \leq \hat{\delta}$. By noting that $\tilde{I}(\delta)$ increases in δ, if Ω is sufficiently small such that $\hat{\delta}\cdot\tilde{I}(\hat{\delta}) \geq \Omega$, $\delta \in [\hat{\delta}, \alpha]$ cannot satisfy the budget constraint (39). In this case, as $\tilde{W}(\delta)$ decreases in δ for $\delta \in [0, \hat{\delta}]$, $\delta^* = 0$.

Finally, it is worth noting that the proof of Proposition 5 only requires the binding constraints $\delta^*\cdot\tilde{I}(\delta^*) = \Omega$ and $\theta^*\cdot E[\tilde{Q}(\theta^*)] = \Omega$, and does not rely on the values of δ^* and θ^*. Therefore, by repeating the proof of Proposition 5, we can show that if δ^* satisfies the binding budget constraint (39), the results given in Proposition 5 continue to hold.

Appendix B: Multiplicative Demand Function

In the base model, we use the inverse linear demand function to capture the relationship between the market price and the total harvest quantity. Although this function is commonly used in the existing literature, one may wonder whether the main insights under our base model still hold under other types of demand functions. According to Petruzzi and Dada (1999), additive and multiplicative demand functions are two representative demand functions, and linear demand function is a special case of additive demand functions. Following Petruzzi and Dada (1999), we now consider the following multiplicative function:

$$\tilde{P} = m\cdot Q^{-b} = m\cdot\left(\int_0^1 \mu(r)q_r(\cdot)dr\right)^{-b}\cdot\varepsilon^{-b}, \tag{40}$$

where $m > 0$, $b \in (0, 1)$, and ε represents the common yield uncertainty with $E[\varepsilon] = 1$, $E[\varepsilon^{1-b}] = \sigma$, and $E[\varepsilon^2] = k\sigma$.

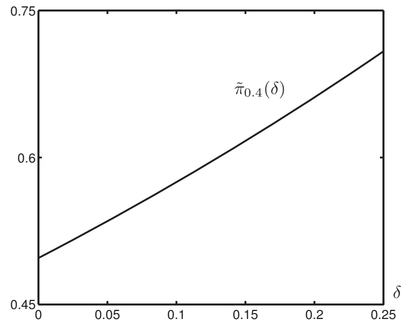
It it worth noting that now the price and the yield uncertainty satisfy the log-normal linear function. Furthermore, since $Q = H \cdot \varepsilon$, where $H = \int_0^1 \mu(r) q_r dr$, (40) can also be rewritten as $\tilde{P} = m \cdot (H)^{-b} \cdot \varepsilon^{-b}$. We assume that by learning from their past experience, farmers can form rational expectations about the expected total harvest quantity H.

By repeating the analysis under the base model, we can obtain the equilibrium planting quantity under each subsidy scheme. Then, by using the farmers' expectations, we can obtain one equation that contains the unknown variable H. However, since the equilibrium price and the total harvest quantity satisfy the power function given in (40), we cannot obtain the closed-form solutions regarding H. Therefore, we have to resort to extensive numerical studies to examine equilibrium outcomes. In these numerical studies, without any further notice, we let $\mu_l = 0.5$, $\mu_h = 0.8$, $b = 0.8$, $m = 11.67$, $t = 1$, $\alpha = 1.8$, $\sigma = 1$, $\beta = 0.15$, $k = 0.2$, $\Omega = 1$, and $\mu(r) = (\mu_h - \mu_l)r - \mu_l$.

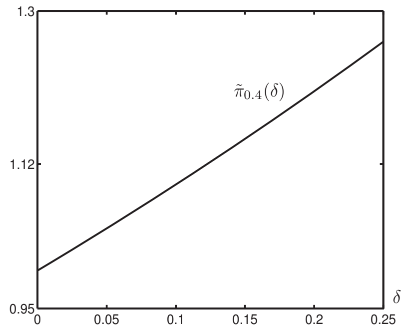
Figure 5 illustrates the equilibrium outcomes under the input subsidy scheme. These numerical illustrations show that as the input subsidy δ increases, the income of the farmer with a low yield rate (i.e., $r = 0.4$) $\tilde{\pi}_{0.4}(\delta)$ always increases, however, the income of the farmer with a high yield rate (i.e., $r = 1$) may decrease when the effort cost coefficient β is small. The Gini coefficient $\tilde{G}(\delta)$ and the income gap $\tilde{\pi}_1(\delta) - \tilde{\pi}_{0.4}(\delta)$ both decrease. These indicate that the major insights associated with Proposition 1 under a linear inverse demand function still hold here. Regarding the output subsidy scheme, Figure 6 shows that an increase in θ, the output subsidy level, makes all farmers better off. Furthermore, a higher θ will create a wider income gap but it can reduce the aggregate income inequality in terms of the Gini coefficient. Again, this indicates that the major insights associated with Proposition 3 also hold here. Last, Figure 7 depicts the comparison results with regard to the two subsidy schemes. It shows that relatively speaking, the output subsidy scheme benefits the farmer with a high yield rate; by contrast, the input subsidy scheme benefits the farmer with a low yield rate. The input subsidy scheme performs comparatively better with regard to reducing the aggregate income inequality and improving consumer surplus, while the output subsidy scheme performs comparatively better with regard to improving the total farmer income. Only when the government places great emphasis on reducing the aggregate income inequality in terms of the Gini coefficient (i.e., a high λ) should the input subsidy scheme be adopted. These comparison results are consistent with those stated in Propositions 5 and 6. In summary, we can conclude that the main results in our base model (i.e., under a linear inverse demand function) continue to hold under a multiplicative demand function.

References

Petruzzi, N. C., M. Dada. 1999. Pricing and the newsvendor problem: a review with extensions. *Operations Research* **47**(2): 183-194.



(a) $\beta = 0.08$



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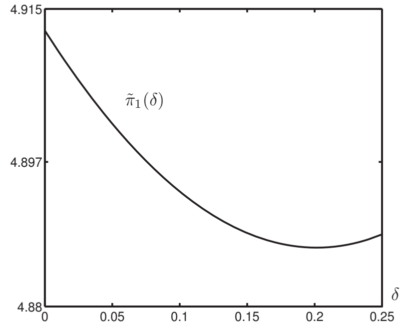

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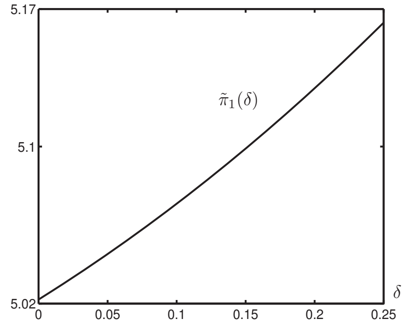

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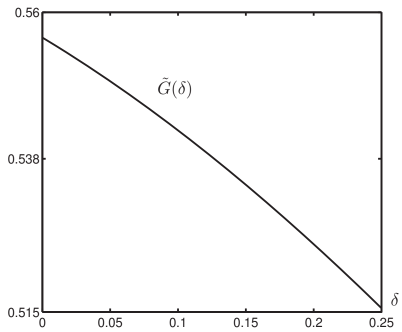

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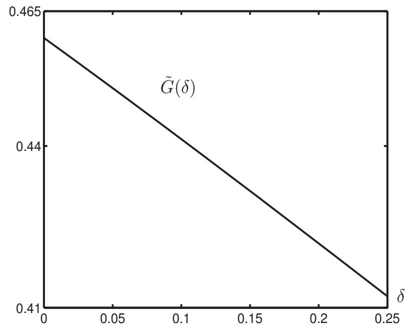

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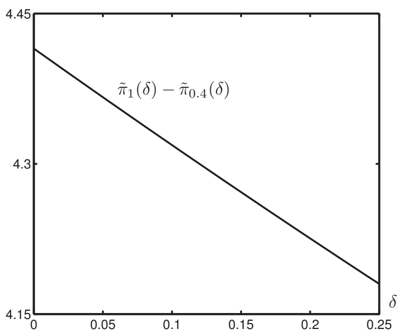

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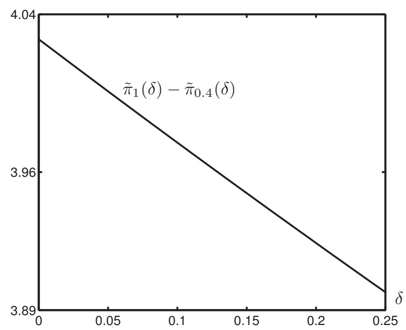
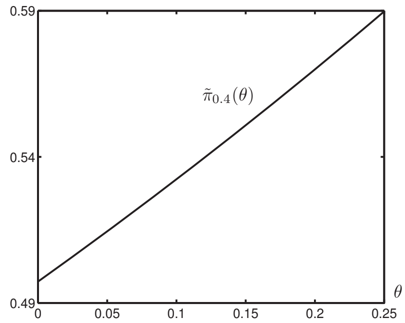


(h) $\beta = 0.15$

Figure 5: The effects of the input subsidy on the equilibrium outcomes



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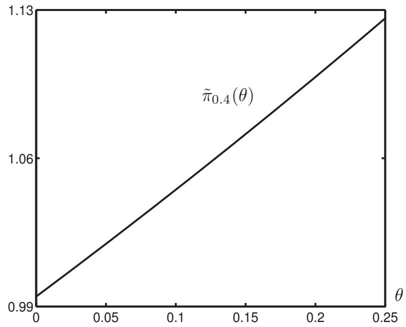

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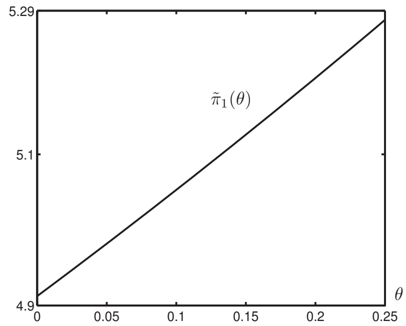

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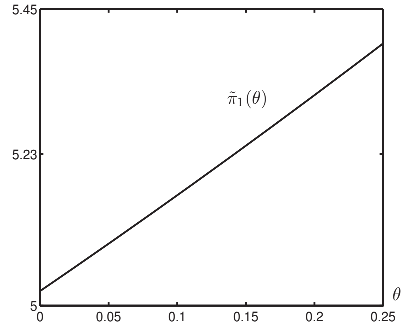

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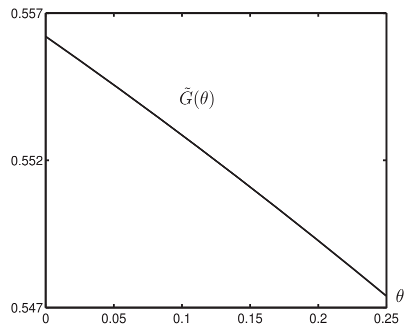

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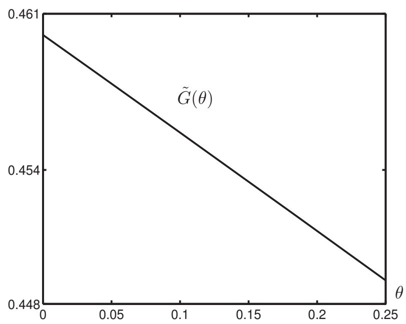

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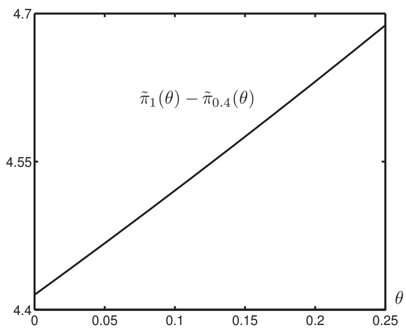

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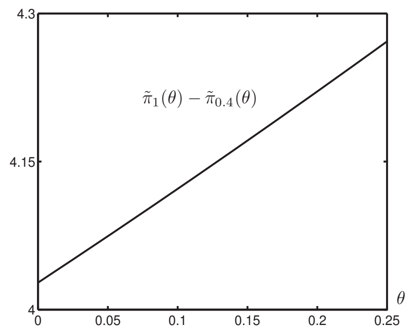

(h) $\beta = 0.15$

Figure 6: The effects of the output subsidy on the equilibrium outcomes

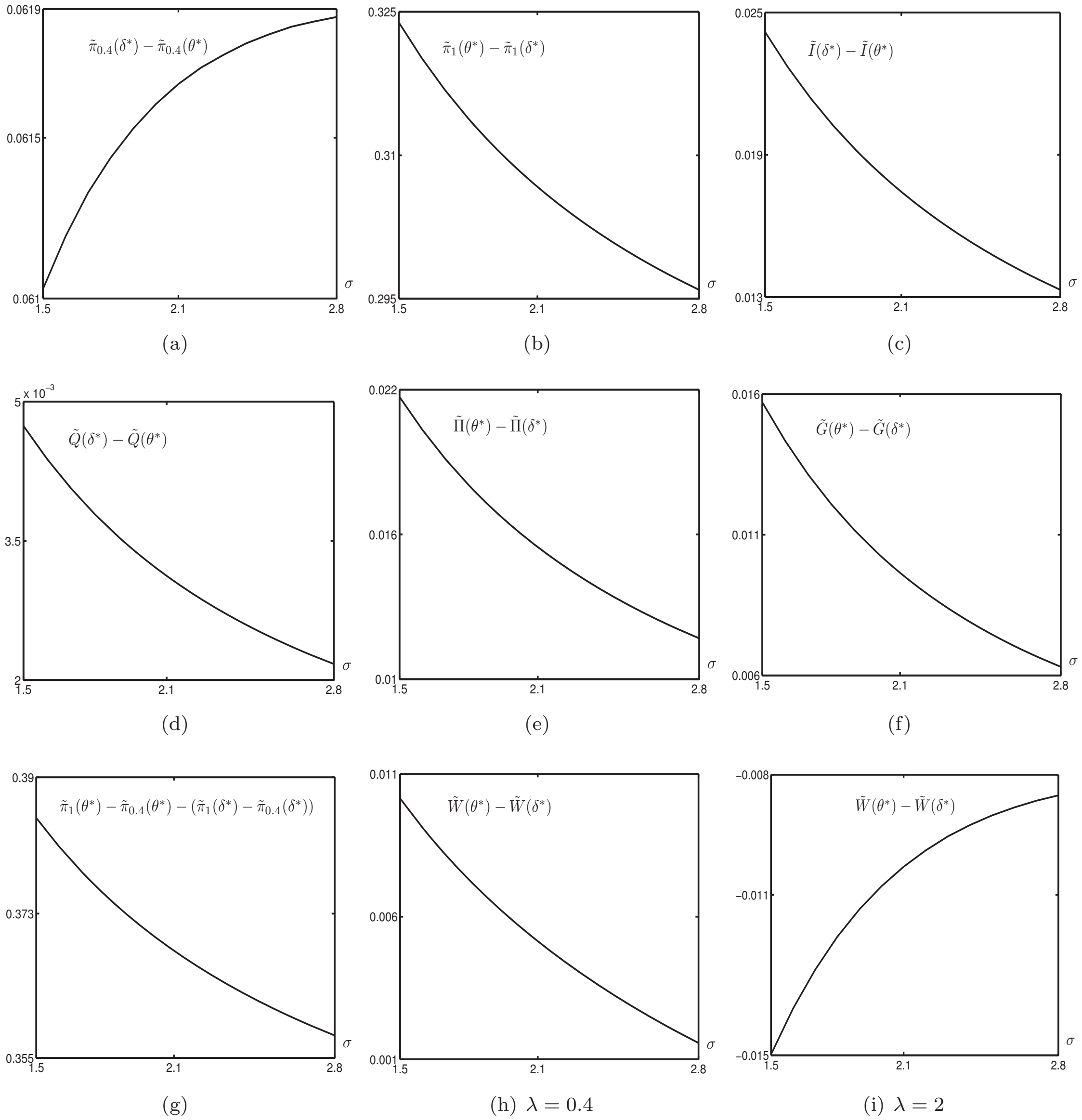

Figure 7: Comparison of the equilibrium outcomes under the multiplicative demand function: input vs. output subsidy