Online Appendix to "Impact of Physician Payment Scheme on Diagnostic Effort and Testing"

A1: Notations

$s \in \{\bar{s}, \underline{s}\}$	the patient's true state: severe or mild
p	the prior probability that the patient's state is severe
$f(\cdot)$, $F(\cdot)$, and μ	the pdf, cdf, and mean of the distribution of the patient priors
$e \in \{L, H\}$	the physician's effort level: low or high
c^e	the cost of effort when effort level is $e = H$
$\sigma \in \{\bar{\sigma}, \underline{\sigma}\}$	the signal (indicative or not indicative), which imperfectly reflects patient state
θ	the probability that the signal matches the true state
$t \in \{0, 1\}$	testing decision: do not test or test
c^t	the patient's cost share of the test
C^t	total cost of the test (payer and patient shares combined)
h	the penalty incurred to patient in the case of false negative diagnosis
δ	the physician's degree of patient-centeredness
b	the benefit incurred to the patient in the case of a true positive diagnosis
r^t, r^n	physician compensation, based on testing decision: with test or without test
Δr	the net payment differential: $r^t - r^n$
U_{patient}	the patient's utility
$\Pi_{\text{physician}}$	the physician's payoff
$U_{\text{physician}}$	the physician's utility
Π_{payer}	the payer's payoff
SW	social welfare
$\bar{c}^F, \bar{c}^S, \bar{c}^D$	thresholds on the cost of effort
$p_1^F, p_2^F, p_1^S, p_2^S$, p_1^D, p_2^D	thresholds on patient prior probability
r^+, r^-	parameters of the diagnosis-based payment scheme

Table A1 Notations

A2: Proofs of Technical Results

Proof of Lemma 1. The physician should order a test if $r^t + \delta(pb - c^t) > r^n - \delta ph$, which is equivalent to

$$p > \frac{\delta c^t - \Delta r}{\delta(b+h)},$$

and provide a diagnosis of mild otherwise.

We have $0 < \frac{\delta c^t - \Delta r}{\delta(b+h)} < 1$ if and only if

$$\delta(c^t - b - h) < \Delta r < \delta c^t. \tag{A1}$$

If $\Delta r \leq \delta(c^t - b - h)$, the physician never orders a test, regardless of p. If $\Delta r \geq \delta c^t$, the physician orders a test for all patients. *Q.E.D.*

Proof of Lemma 2. If the consultation generates a positive signal, the physician updates the patient's probability of suffering from the condition:

$$\Pr(s = \bar{s} | \sigma = \bar{\sigma}) = \frac{\theta p}{\theta p + (1-\theta)(1-p)}.$$

The physician should order a test if

$$r^t + \delta \cdot \frac{\theta p}{\theta p + (1-\theta)(1-p)} \cdot b - \delta c^t > r^n - \delta \cdot \frac{\theta p}{\theta p + (1-\theta)(1-p)} \cdot h,$$

which gives

$$p > \frac{(1-\theta)(\delta c^t - \Delta r)}{(1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t) + \Delta r]}.$$

We have $0 < \frac{(1-\theta)(\delta c^t - \Delta r)}{(1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t)+\Delta r]} < 1$ if and only if $\delta(c^t - b - h) < \Delta r < \delta c^t$, which coincides with (A1). If $\Delta r \le \delta(c^t - b - h)$, the physician never orders a test, regardless of p. If $\Delta r \ge \delta c^t$, the physician orders a test for all patients.

If the consultation generates a negative signal, the physician updates the patient's probability of suffering from the condition:

$$\Pr(s = \bar{s} | \sigma = \underline{\sigma}) = \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)}.$$

The physician should order a test if

$$r^t + \delta \cdot \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)} \cdot b - \delta c^t > r^n - \delta \cdot \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)} \cdot h,$$

which is equivalent to

$$p > \frac{\theta(\delta c^t - \Delta r)}{\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t) + \Delta r]}.$$

We have $0 < \frac{\theta(\delta c^t - \Delta r)}{\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t)+\Delta r]} < 1$ if and only if $\delta(c^t - b - h) < \Delta r < \delta c^t$, which coincides with (A1). If $\Delta r \le \delta(c^t - b - h)$, the physician never orders a test, regardless of p. If $\Delta r \ge \delta c^t$, the physician orders a test for all patients.

If $\delta(c^t - b - h) < \Delta r < \delta c^t$, the inequality

$$\frac{(1-\theta)(\delta c^t - \Delta r)}{(1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t) + \Delta r]} < \frac{\delta c^t - \Delta r}{\delta(b+h)} < \frac{\theta(\delta c^t - \Delta r)}{\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t) + \Delta r]}$$

can be obtained from simple algebra using the condition $\theta > 1/2$. *Q.E.D.*

Proof of Lemma 3. We consider the following cases:

(i) If $\Delta r \le \delta(c^t - b - h)$ or $\Delta r \ge \delta c^t$, based on Lemmas 1 and 2, the final decision is the same under low and high effort (regardless of the signal), so the physician has no incentive to exert costly high effort.

(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$, the physician's effort decision can be determined as follows, according to Lemmas 1 and 2:

(a) If $\frac{\theta(\delta c^t - \Delta r)}{\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t)+\Delta r]} < p < 1$, the physician orders the test regardless of the effort or the signal (if applicable). Thus, the physician chooses not to exert high effort.

(b) If $0 < p \le \frac{(1-\theta)(\delta c^t - \Delta r)}{(1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t)+\Delta r]}$, the physician provides a mild diagnosis regardless of the effort or the signal. Thus, the physician chooses not to exert high effort.

(c) If $\frac{(1-\theta)(\delta c^t - \Delta r)}{(1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t)+\Delta r]} < p \le \frac{\theta(\delta c^t - \Delta r)}{\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t)+\Delta r]}$, the physician follows the signal if exerting high effort. The condition for the physician to exert high effort is as follows:

i. If $\frac{(1-\theta)(\delta c^t-\Delta r)}{(1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r]} < p \le \frac{\delta c^t-\Delta r}{\delta(b+h)}$, the physician exerts high effort if and only if

$$[\theta p+(1-\theta)(1-p)]\cdot\left[r^t+\delta\cdot\frac{\theta p}{\theta p+(1-\theta)(1-p)}\cdot b-\delta c^t\right] + [(1-\theta)p+\theta(1-p)]\cdot\left[r^n-\delta\cdot\frac{(1-\theta)p}{(1-\theta)p+\theta(1-p)]}\cdot h\right]-c^e > r^n-\delta ph,$$

which gives

$$p > \frac{(1-\theta)(\delta c^t-\Delta r)+c^e}{(1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r]} = p_1^F.$$

We can easily verify that

$$p_1^F > \frac{(1-\theta)(\delta c^t-\Delta r)}{(1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r]}.$$

Moreover, we have

$$p_1^F \le \frac{\delta c^t-\Delta r}{\delta(b+h)},$$

if and only if

$$c^e \le (2\theta-1)(\delta c^t-\Delta r)\left[1-\frac{\delta c^t-\Delta r}{\delta(b+h)}\right] = \bar{c}^F. \tag{A2}$$

If c^e is above the threshold $\bar{c}^F$, the physician does not exert high effort for any patient within this range of priors.

ii. If $\frac{\delta c^t-\Delta r}{\delta(b+h)} < p \le \frac{\theta(\delta c^t-\Delta r)}{\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r]}$, the physician exerts high effort if and only if

$$[\theta p+(1-\theta)(1-p)]\cdot\left[r^t+\delta\cdot\frac{\theta p}{\theta p+(1-\theta)(1-p)}\cdot b-\delta c^t\right] + [(1-\theta)p+\theta(1-p)]\cdot\left[r^n-\delta\cdot\frac{(1-\theta)p}{(1-\theta)p+\theta(1-p)]}\cdot h\right]-c^e > r^t+\delta pb-\delta c^t,$$

which gives

$$p < \frac{\theta(\delta c^t-\Delta r)-c^e}{\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r]} = p_2^F.$$

We can easily verify that

$$p_2^F < \frac{\theta(\delta c^t-\Delta r)}{\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r]}.$$

Moreover, we have

$$p_2^F \ge \frac{\delta c^t-\Delta r}{\delta(b+h)},$$

if and only if

$$c^e \le (2\theta-1)(\delta c^t-\Delta r)\left[1-\frac{\delta c^t-\Delta r}{\delta(b+h)}\right] = \bar{c}^F,$$

which coincides with (A2). If c^e is above the threshold, the physician exerts low effort for any patient within this range of priors. *Q.E.D.*

Proof of Proposition 3 We denote by $a^F(p)$ (resp. $a^S(p)$) the diagnostic accuracy for a patient with a prior p under fee-for-service (resp. in the social optimum). To prove the result, we proceed by stating and proving three intermediary results. We first establish the following corollary:

Corollary A1. *Under fee-for-service, the diagnostic accuracy is as follows:*
(i) If $\Delta r \le \delta(c^t - b - h)$, then $a^F(p) = 1 - p$;
(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e > \bar{c}^F$, then

$$a^F(p) = \begin{cases} 1-p & \text{if } p < \frac{\delta c^t - \Delta r}{\delta(b+h)} \\ 1 & \text{else;} \end{cases}$$

(iii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e \le \bar{c}^F$, then

$$a^F(p) = \begin{cases} 1-p & \text{if } p \le p_1^F \\ 1-p(1-\theta) & \text{if } p_1^F < p \le p_2^F \\ 1 & \text{if } p > p_2^F; \end{cases}$$

(iv) If $\Delta r \ge \delta c^t$, then $a^F(p) = 1$.

Proof of Corollary A1. The four parts of the corollary correspond to those in Proposition 1. Note from Proposition 1 that in case (i), the physician always reaches a mild diagnosis after exerting low effort. Thus, the diagnostic accuracy corresponds to the likelihood that the patient's true state is negative; that is, $1-p$. Similarly, in case (iv), the physician always orders the test. Because the test can perfectly reveal the patient's true state, the diagnostic accuracy is 1.

In case (ii), the physician exerts low effort for all patients and orders a test if and only if $p \ge \frac{\delta c^t - \Delta r}{\delta(b+h)}$. Thus, if $p \ge \frac{\delta c^t - \Delta r}{\delta(b+h)}$, the diagnostic accuracy is 1; otherwise, the physician reaches a mild diagnosis, so the diagnostic accuracy is the likelihood that the patient's true state is negative, that is, $1-p$.

In case (iii), if $p \le p_1^F$, the physician exerts low effort and reaches a mild diagnosis; thus, the diagnostic accuracy is $1-p$. Next, if $p_1^F < p \le p_2^F$, the physician exerts high effort and follows the signal obtained; thus, the diagnostic accuracy is

$$\begin{aligned} &\underbrace{[p\theta + (1-p)(1-\theta)]}_{\Pr(\sigma=\bar{\sigma})} \cdot 1 + \underbrace{[p(1-\theta) + (1-p)\theta]}_{\Pr(\sigma=\underline{\sigma})} \cdot \underbrace{\frac{(1-p)\theta}{p(1-\theta)+p(1-\theta)}}_{\Pr(s=\underline{s}|\sigma=\underline{\sigma})} \\ &= p\theta + (1-p)(1-\theta) + (1-p)\theta \\ &= 1 - p(1-\theta). \end{aligned}$$

Finally, if $p > p_2^F$, the physician orders a test, so the diagnostic accuracy is 1. *Q.E.D.*

We next establish the following result, where we define $Q(x) = \int_0^x t f(t) dt$ and $\mu = Q(1)$.

Proposition A1. *Under FFS, the average population diagnostic accuracy is as follows:*
(i) If $\Delta r \le \delta(c^t - b - h)$, then $\mathbb{E}_p[a^F(p)] = 1 - \mu$;
(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e > \bar{c}^F$, then

$$\mathbb{E}_p[a^F(p)] = 1 - Q\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right);$$

(iii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e \le \bar{c}^F$, then

$$\mathbb{E}_p[a^F(p)] = 1 - \theta Q(p_1^F) - (1-\theta) Q(p_2^F);$$

(iv) If $\Delta r \geq \delta c^t$, then $\mathbb{E}_p[a^F(p)] = 1$.

In addition, the average population diagnostic accuracy is continuous and non-decreasing in Δr.

Proof of Proposition A1 The expression of $\mathbb{E}_p[a^F(p)]$ follows directly from Corollary A1. It remains to show the monotonicity. For $\Delta r \leq \delta(c^t - b - h)$, $\mathbb{E}_p[a^F(p)]$ is a constant (case (i)). For $\delta(c^t - b - h) < \Delta r < \delta c^t$, at the left extreme of the range, $\delta(c^t - b - h)^+$, we have $\bar{c}^F = 0^+$, and thus $c^e > \bar{c}^F$, so we are in case (ii). Moreover, at this extreme, $\mathbb{E}_p[a^F(p)] = (1-\mu)^+$, so the expected accuracy is continuous. Because $\Delta r < \delta c^t$ in this range,

$$\frac{\partial \mathbb{E}_p[a^F(p)]}{\partial \Delta r} = \frac{\delta c^t - \Delta r}{\delta^2 (b+h)^2} f\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right) > 0,$$

so the expected accuracy is increasing while in case (ii). If $c^e > (2\theta - 1)\delta(b+h)/4$ (which is the maximum value taken by $\bar{c}^F$), we remain in case (ii) over the whole range, and at $(\delta c^t)^-$, $\mathbb{E}_p[a^F(p)] = 1^-$, so the aggregate accuracy is continuous and non-decreasing. Otherwise, we switch to case (iii) within the range $\delta(c^t - b - h) < \Delta r < \delta c^t$, and then back to case (ii) because $\bar{c}^F$ is unimodal. The switch occurs when Δr is such that $c^e = \bar{c}^F$. When $c^e = \bar{c}^F$, we have (see proof of Lemma 3) $p_1^F = p_2^F = (\delta c^t - \Delta r)/(\delta(b+h))$; thus, the aggregate accuracy is continuous. Monotonicity in case (iii) follows from p_1^F and p_2^F decreasing in Δr (see proof of Corollary 1). *Q.E.D.*

We next establish the following result:

Proposition A2. *In the social optimum, the average population diagnostic accuracy is as follows:*

(i) If $b + h \leq C^t$, then $\mathbb{E}_p[a^S(p)] = 1 - \mu$;

(ii) If $b + h > C^t$ and $c^e > \bar{c}^S$, then

$$\mathbb{E}_p[a^S(p)] = 1 - Q\left(\frac{C^t}{b+h}\right) \quad (> 1 - \mu)\,;$$

(iii) If $b + h > C^t$ and $c^e \leq \bar{c}^S$, then

$$\mathbb{E}_p[a^S(p)] = 1 - \theta Q(p_1^S) - (1-\theta) Q(p_2^S) \quad \left(\geq 1 - Q\left(\frac{C^t}{b+h}\right) > 1 - \mu\right).$$

Proof of Proposition A2. The result follows similarly to Proposition A1 after noting the socially optimal policy is as follows:

(i) If $b + h \leq C^t$, then $a^S(p) = 1 - p$.

(ii) If $b + h > C^t$ and $c^e > \bar{c}^S$, then

$$a^S(p) = \begin{cases} 1 - p & \text{if } p < \frac{C^t}{b+h} \\ 1 & \text{else.} \end{cases}$$

(iii) If $b + h > C^t$ and $c^e \leq \bar{c}^S$, then

$$a^S(p) = \begin{cases} 1 - p & \text{if } p \leq p_1^S \\ 1 - p(1-\theta) & \text{if } p_1^S < p \leq p_2^S \\ 1 & \text{if } p > p_2^S. \end{cases}$$

Q.E.D.

Proposition 3 then follows from the above intermediary results. *Q.E.D.*

Proof of Corollary 1. We first establish a preliminary result: if $\delta(c^t-b-h)<\Delta r<\delta c^t$, then p_1^F and p_2^F are decreasing in Δr and $\bar{c}^F$ is unimodal in Δr, reaching a maximum equal to $(2\theta-1)\delta(b+h)/4$ at $\Delta r=\delta(c^t-(b+h)/2)$.

To show this preliminary result, note we have

$$\begin{aligned}\frac{\partial p_1^F}{\partial \Delta r}&=\frac{-(1-\theta)((1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r])-[(1-\theta)(\delta c^t-\Delta r)+c^e](2\theta-1)}{((1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r])^2}\\&=\frac{-(1-\theta)\theta\delta(b+h)-c^e(2\theta-1)}{((1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r])^2}<0,\end{aligned}$$

where the last inequality follows from $\theta>1/2$. Similarly,

$$\begin{aligned}\frac{\partial p_2^F}{\partial \Delta r}&=\frac{-\theta(\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r])-[\theta(\delta c^t-\Delta r)-c^e](1-2\theta)}{(\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r])^2}\\&=\frac{-\theta(1-\theta)\delta(b+h)-c^e(2\theta-1)}{(\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r])^2}<0.\\\frac{\partial \bar{c}^F}{\partial \Delta r}&=(2\theta-1)\left(-1+\frac{\delta c^t-\Delta r}{\delta(b+h)}+(\delta c^t-\Delta r)\frac{1}{\delta(b+h)}\right)\\&=(2\theta-1)\left(-1+2\frac{\delta c^t-\Delta r}{\delta(b+h)}\right).\end{aligned}$$

Therefore, $\bar{c}^F$ increases in Δr when $\Delta r<\delta(c^t-\frac{b+h}{2})$, and $\bar{c}^F$ decreases in Δr when $\Delta r>\delta(c^t-\frac{b+h}{2})$. By substitution, we find it reaches a maximum equal to $(2\theta-1)\delta(b+h)/4$ at $\Delta r=\delta(c^t-(b+h)/2)$. This concludes the proof of the preliminary result.

Hence, $c^e\le\bar{c}^F$ can occur only when c^e is below the maximum value reached by $\bar{c}^F$. In that case, $c^e\le\bar{c}^F$ occurs whenever Δr is between the two roots of the quadratic equation $c^e=\bar{c}^F$; that is, denoting $x=\delta c^t-\Delta r$,

$$\begin{aligned}c^e&=(2\theta-1)x\left(1-\frac{x}{\delta(b+h)}\right)\\\Leftrightarrow &x^2-\delta(b+h)x+\frac{c^e\delta(b+h)}{2\theta-1}=0.\end{aligned}$$

The result follows from the standard solution of a quadratic equation. *Q.E.D.*

Proof of Corollary 2. When $\delta(c^t-b-h)<\Delta r<\delta c^t$ and $c^e\le\bar{c}^F$, we show from the proof of Proposition 4 that the range of the priors that correspond to high effort is

$$\begin{aligned}p_2^F-p_1^F&=\frac{\theta(\delta c^t-\Delta r)-c^e}{\theta(\delta c^t-\Delta r)+(1-\theta)[\delta(b+h-c^t)+\Delta r]}-\frac{(1-\theta)(\delta c^t-\Delta r)+c^e}{(1-\theta)(\delta c^t-\Delta r)+\theta[\delta(b+h-c^t)+\Delta r]}\\&=\frac{\bar{c}^F-c^e}{\theta(1-\theta)\delta(b+h)+(2\theta-1)\bar{c}^F}\end{aligned}$$

after simplifications. Moreover,

$$\begin{aligned}\frac{\partial(p_2^F-p_1^F)}{\partial\bar{c}^F}&=\frac{\theta(1-\theta)\delta(b+h)+(2\theta-1)\bar{c}^F-(2\theta-1)(\bar{c}^F-c^e)}{(\theta(1-\theta)\delta(b+h)+(2\theta-1)\bar{c}^F)^2}\\&=\frac{\theta(1-\theta)\delta(b+h)+(2\theta-1)c^e}{(\theta(1-\theta)\delta(b+h)+(2\theta-1)\bar{c}^F)^2}>0.\end{aligned}$$

Thus, it suffices to use the monotonicity results on $\bar{c}^F$. From the above preliminary result, we know that $\bar{c}^F$ increases in Δr when $\Delta r<\delta(c^t-\frac{b+h}{2})$, and $\bar{c}^F$ decreases in Δr when $\Delta r>\delta(c^t-\frac{b+h}{2})$. By substitution, we obtain that for $\Delta r=\delta(c^t-\frac{b+h}{2})$, the range $p_2^F-p_1^F$ equals $2\theta-1-4c^e/(\delta(b+h))$. *Q.E.D.*

Proof of Proposition 4. From the proof of Corollary 1, the widest range of high effort under the physician's optimal strategy is obtained for $\Delta r = \delta(c^t - \frac{b+h}{2})$, where $p_2^F - p_1^F = 2\theta - 1 - 4c^e/(\delta(b+h))$ and where $\bar{c}^F = (2\theta-1)\delta(b+h)/4$. Therefore, we want to determine the sign of $p_2^S - p_1^S - (2\theta - 1 - 4c^e/(\delta(b+h)))$. Using derivations from the proof of Corollary 1, we have

$$p_2^S - p_1^S = \frac{\bar{c}^S - c^e}{\theta(1-\theta)(b+h) + (2\theta-1)\bar{c}^S}.$$

As shown in the proof of Corollary 1, the expression for $p_2^S - p_1^S$ is increasing in $\bar{c}^S$. Thus, the socially optimum range of priors leading to high effort reaches a maximum (over C^t) when $\bar{c}^S$ is at its maximum, that is, for $C^t = (b+h)/2$, and $\bar{c}^S$ then equals $(2\theta-1)(b+h)/4$, and $p_2^S - p_1^S$ then equals $2\theta - 1 - 4c^e/(b+h)$.

When $\delta > 1$, the biggest FFS range of priors leading to high effort is bigger than the socially optimal largest range of priors leading to high effort; thus, it is bigger than the socially optimum range of priors leading to high effort for any C^t.

When $\delta \leq 1$, the biggest FFS range of priors leading to high effort is bigger than the social optimal range of priors leading to high effort when C^t is outside the two roots of the equation $p_2^S - p_1^S - (2\theta - 1 - 4c^e/(\delta(b+h))) = 0$, which is quadratic in C^t. The result follows from the standard solution of a quadratic equation. *Q.E.D.*

Proof of Proposition 5 To prove this result, we start with showing an intermediate result, where $\bar{F}(\cdot) = 1 - F(\cdot)$ and $Q(x) = \int_0^x tf(t)dt$:

Proposition A3. *Under fee-for-service, the average population social welfare is as follows:*

(i) If $\Delta r \leq \delta(c^t - b - h)$, *then* $\mathbb{E}_p[SW^F(p)] = -h\mu$;

(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ *and* $c^e > \bar{c}^F$, *then*

$$\mathbb{E}_p[SW^F(p)] = b\mu - C^t \bar{F}\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right) - (b+h)Q\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right);$$

(iii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ *and* $c^e \leq \bar{c}^F$, *then*

$$\begin{aligned}\mathbb{E}_p[SW^F(p)] = {} & b\mu - C^t - c^e[F(p_2^F) - F(p_1^F)] + C^t[\theta F(p_2^F) + (1-\theta)F(p_1^F)] \\ & - C^t\left[\theta Q(p_2^F) + (1-\theta)Q(p_1^F)\right] - (b+h-C^t)\left[(1-\theta)Q(p_2^F) + \theta Q(p_1^F)\right];\end{aligned}$$

(iv) If $\Delta r \geq \delta c^t$, *then* $\mathbb{E}_p[SW^F(p)] = b\mu - C^t$.

Proof of Proposition A3 The result follows from noting that for a patient with given prior p, the social welfare is as follows:

(i) If $\Delta r \leq \delta(c^t - b - h)$, then $SW^F(p) = -ph$;

(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e > \bar{c}^F$, then

$$SW^F(p) = \begin{cases} -ph & \text{if } p < \frac{\delta c^t - \Delta r}{\delta(b+h)} \\ pb - C^t & \text{else;} \end{cases}$$

(iii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ and $c^e \leq \bar{c}^F$, then

$$SW^F(p) = \begin{cases} -ph & \text{if } p \leq p_1^F \\ -c^e - C^t(p\theta + (1-p)(1-\theta)) + bp\theta - hp(1-\theta) & \text{if } p_1^F < p \leq p_2^F \\ pb - C^t & \text{if } p > p_2^F; \end{cases},$$

(iv) If $\Delta r \geq \delta c^t$, then $SW^F(p) = pb - C^t$. *Q.E.D.*

We now return to the proof of Proposition 5. If $c^e > (2\theta - 1)\delta(b+h)/4$, case (iii) does not occur. As Δr increases from $-\infty$ to $+\infty$, we go from case (i) to case (ii) to case (iv) of Proposition A3. Taking the derivative of $\mathbb{E}_p[SW^F(p)]$ w.r.t. Δr in case (ii), we obtain

$$\frac{-\Delta r - \delta(C^t - c^t)}{\delta^2(b+h)} f\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right).$$

It follows that $\mathbb{E}_p[SW^F(p)]$ is unimodal in case (ii), reaching a maximum for $\Delta r = -\delta(C^t - c^t)$. This value is in the range of case (ii) if $b + h > C^t$. (Otherwise, social welfare is decreasing over case (ii).) By simple substitution, we find the value reached by $\mathbb{E}_p[SW^F(p)]$ at $\Delta r = -\delta(C^t - c^t)$ is

$$b\mu - C^t \bar{F}\left(\frac{C^t}{b+h}\right) - (b+h)Q\left(\frac{C^t}{b+h}\right).$$

We next prove this quantity is greater than both (1) $-h\mu$ (value in case (i)) and (2) $b\mu - C^t$ (value in case (iv)):

- Proof of (1): We have

$$xF(x) = \int_0^x xf(t)dt > \int_0^x tf(t)dt = Q(x).$$

The result follows after substituting x with $C^t/(b+h)$.

- Proof of (2): We have

$$Q(1) - Q(x) = \int_x^1 tf(t)dt > \int_x^1 xf(t)dt = x\bar{F}(x).$$

The result follows after substituting x with $C^t/(b+h)$.

The socially optimal social welfare can be obtained using Proposition A3 with the values $\delta = 1$ and $\Delta r = -(C^t - c^t)$. Hence, if $b + h \leq C^t$, the maximum social welfare is $-h\mu$, which matches the FFS case. If $b + h > C^t$, we are in case (ii) when $c^e > \bar{c}^S$, and the maximum social welfare matches FFS. *Q.E.D.*

Proof of Lemma 4. Note that, for $\delta c^t = \Delta r$, $\bar{c}^F = 0 < c^e$. Moreover, we obtain

$$\frac{\partial \bar{c}^F}{\partial \delta} = c^t\left(1 - \frac{c^t}{b+h}\right) + \frac{\Delta r^2}{\delta^2(b+h)} > 0;$$
$$\lim_{\delta \to \infty} \bar{c}^F = \infty.$$

Hence, when $\Delta r > 0$, it can be seen from Proposition A1 that, as δ increases from 0, we go from case (iv) (where the diagnostic accuracy is constant) to case (ii) (as δ passes the value $\Delta r/c^t$) to case (iii) (as $\bar{c}^F$ passes the value c^e). In case (ii), using the expression in Proposition A1(ii), the diagnostic accuracy is decreasing in δ, because $Q(\cdot)$ is increasing. For case (iii), we need to consider how p_1^F and p_2^F vary with δ. We find

$$\frac{\partial p_1^F}{\partial \delta} = \frac{\theta(1-\theta)\Delta r(b+h) - c^e[(1-\theta)c^t + \theta(b+h-c^t)]}{((1-\theta)(\delta c^t - \Delta r) + \theta[\delta(b+h-c^t) + \Delta r])^2},$$

which is positive under the assumption that $c^e < \theta(1-\theta)\Delta r(b+h)/((1-\theta)c^t + \theta(b+h-c^t))$. Moreover,

$$\frac{\partial p_2^F}{\partial \delta} = \frac{\theta(1-\theta)\Delta r(b+h) + c^e[\theta c^t + (1-\theta)(b+h-c^t)]}{(\theta(\delta c^t - \Delta r) + (1-\theta)[\delta(b+h-c^t) + \Delta r])^2} > 0.$$

Therefore, in the situation considered in the lemma, both p_1^F and p_2^F are increasing in δ. Using the expression in Proposition A1(iii), because $Q(\cdot)$ is increasing, it follows that in case (iii), the diagnostic accuracy is decreasing in δ. *Q.E.D.*

Proof of Lemma 5. The proof of Lemma 4 shows $\bar{c}^F$ increases from zero to infinity as δ increases. Hence, when $\Delta r = 0$, Proposition A1 shows that as δ increases from 0, we go from case (ii) to case (iii) (as $\bar{c}^F$ passes the value c^e). In case (ii), using the expression in Proposition A1(ii), the diagnostic accuracy is independent of δ. In case (iii), $Q(\cdot)$ is increasing, but we now have p_1^F decreasing in δ while p_2^F is increasing in δ. We thus obtain the derivative of the diagnostic accuracy (denoting $D_1 \triangleq (1-\theta)c^t + \theta(b+h-c^t)$ and $D_2 \triangleq \theta c^t + (1-\theta)(b+h-c^t)$):

$$\begin{aligned}\frac{\partial \mathbb{E}_p[a^F(p)]}{\partial \delta} =& -\theta p_1^F f(p_1^F)\frac{\partial p_1^F}{\partial \delta} - (1-\theta)p_2^F f(p_2^F)\frac{\partial p_2^F}{\partial \delta} \\ =&\theta \frac{c^e}{\delta^2}\frac{(1-\theta)c^t + c^e/\delta}{D_1^2} f(p_1^F) - (1-\theta)\frac{c^e}{\delta^2}\frac{\theta c^t - c^e/\delta}{D_2^2} f(p_2^F) \\ =&\frac{c^e}{\delta^2}\theta(1-\theta)c^t\left(\frac{f(p_1^F)}{D_1^2} - \frac{f(p_2^F)}{D_2^2}\right) + \frac{(c^e)^2}{\delta^3}\left(\frac{\theta f(p_1^F)}{D_1^2} + \frac{(1-\theta)f(p_2^F)}{D_2^2}\right).\end{aligned}$$

The diagnostic accuracy is increasing in case (iii) when $f(p_1^F)/D_1^2 \geq f(p_2^F)/D_2^2$.

Otherwise, the derivative of the accuracy has the sign of

$$-\theta(1-\theta)c^t\left(\frac{f(p_2^F)}{D_2^2} - \frac{f(p_1^F)}{D_1^2}\right) + \frac{c^e}{\delta}\left(\frac{\theta f(p_1^F)}{D_1^2} + \frac{(1-\theta)f(p_2^F)}{D_2^2}\right),$$

which is positive if and only if

$$\delta < \frac{c^e}{\theta(1-\theta)c^t}\left(\frac{\theta f(p_1^F)}{D_1^2} + \frac{(1-\theta)f(p_2^F)}{D_2^2}\right)\frac{1}{\frac{f(p_2^F)}{D_2^2} - \frac{f(p_1^F)}{D_1^2}}.$$

Q.E.D.

Proof of Proposition 6. To prove this result, we establish a more detailed result below.

Proposition A4. *The range of priors leading to high effort under fee-for-service is monotonically increasing in δ if and only if*

1. $\Delta r = 0$, *or*
2. $\Delta r > 0$ *and* $K \geq 0$, *or*
3. $\Delta r > 0$ *and* $K < 0$ *and*

$$-\frac{\theta(1-\theta)(b+h)}{c^e} + \frac{1}{\Delta r}\sqrt{\frac{-K(b+h)}{c^e}} \leq 0,$$

where

$$K \triangleq -\frac{\theta^2(1-\theta)^2(\Delta r)^2(b+h)}{c^e} + \frac{c^e}{b+h}(2\theta-1)^2 c^t(b+h-c^t) + c^e\theta(1-\theta)(b+h) + \theta(1-\theta)(2\theta-1)\Delta r(b+h-2c^t).$$

Otherwise, the range of priors leading to high effort is unimodal in δ, increasing if and only if

$$\delta < \frac{2\theta-1}{-\frac{\theta(1-\theta)(b+h)}{c^e} + \frac{1}{|\Delta r|}\sqrt{\frac{-K(b+h)}{c^e}}}.$$

Proof of Proposition A4. The derivative of p_1^F w.r.t. δ has the sign of

$$\theta(1-\theta)\Delta r(b+h) - c^e((1-\theta)c^t + \theta(b+h-c^t)).$$

The derivative of p_2^F w.r.t. δ has the sign of

$$\theta(1-\theta)\Delta r(b+h) + c^e(\theta c^t + (1-\theta)(b+h-c^t)).$$

The derivative of $\bar{c}^F$ w.r.t. δ has the sign of

$$\left(\frac{\Delta r}{\delta}\right)^2 + c^t(b+h-c^t) > 0.$$

From the proof of Corollary 1, we have

$$p_2^F - p_1^F = \frac{\bar{c}^F - c^e}{\theta(1-\theta)\delta(b+h) + (2\theta-1)\bar{c}^F}.$$

Thus, $\partial(p_2^F - p_1^F)/\partial\delta$ has the sign of

$$\begin{aligned}
&\frac{\partial\bar{c}^F}{\partial\delta}[\theta(1-\theta)\delta(b+h) + (2\theta-1)\bar{c}^F] - (\bar{c}^F - c^e)\left[\theta(1-\theta)(b+h) + (2\theta-1)\frac{\partial\bar{c}^F}{\partial\delta}\right]\\
=&\frac{\partial\bar{c}^F}{\partial\delta}[\theta(1-\theta)\delta(b+h) + \bar{c}^F(2\theta-1)] - (\bar{c}^F - c^e)\theta(1-\theta)(b+h)\\
=&\frac{2\theta-1}{b+h}\left[\left(\frac{\Delta r}{\delta}\right)^2 + c^t(b+h-c^t)\right][\theta(1-\theta)\delta(b+h) + \bar{c}^F(2\theta-1)] + c^e\theta(1-\theta)(b+h)\\
&-\theta(1-\theta)(2\theta-1)(b+h)\delta\left(c^t - \frac{\Delta r}{\delta}\right)\left[b+h-c^t + \frac{\Delta r}{\delta}\right]\\
=&\frac{c^e}{b+h}(2\theta-1)^2\left(\frac{\Delta r}{\delta}\right)^2 + 2(2\theta-1)\theta(1-\theta)\delta\left(\frac{\Delta r}{\delta}\right)^2 + \frac{c^e}{b+h}(2\theta-1)^2c^t(b+h-c^t) + c^e\theta(1-\theta)(b+h)\\
&+\theta(1-\theta)(2\theta-1)\Delta r(b+h-2c^t).
\end{aligned}$$

This expression is positive when

$$c^e > \frac{(b+h)\theta(1-\theta)(2\theta-1)\Delta r\left[-(b+h-2c^t) - 2\frac{\Delta r}{\delta}\right]}{\theta(1-\theta)(b+h)^2 + (2\theta-1)^2\left[\left(\frac{\Delta r}{\delta}\right)^2 + c^t(b+h-c^t)\right]}. \tag{A3}$$

The above is necessarily true when $\Delta r = 0$.

We can write differently that $\partial(p_2^F - p_1^F)/\partial\delta$ is positive when

$$\frac{c^e}{b+h}(\Delta r)^2\left(\frac{2\theta-1}{\delta} + \frac{\theta(1-\theta)(b+h)}{c^e}\right)^2 + K > 0,$$

where

$$K \triangleq -\frac{\theta^2(1-\theta)^2(\Delta r)^2(b+h)}{c^e} + \frac{c^e}{b+h}(2\theta-1)^2c^t(b+h-c^t) + c^e\theta(1-\theta)(b+h) + \theta(1-\theta)(2\theta-1)\Delta r(b+h-2c^t).$$

Hence, if $K \geq 0$, $p_2^F - p_1^F$ is increasing in δ. If $K < 0$, $p_2^F - p_1^F$ is increasing in δ if and only if:

$$-\frac{\theta(1-\theta)(b+h)}{c^e} + \frac{1}{|\Delta r|}\sqrt{\frac{-K(b+h)}{c^e}} > 0 \text{ and } \delta < \frac{2\theta-1}{-\frac{\theta(1-\theta)(b+h)}{c^e} + \frac{1}{|\Delta r|}\sqrt{\frac{-K(b+h)}{c^e}}}.$$

Q.E.D.

Proposition 6 directly follows from Proposition A4. *Q.E.D.*

Proof of Corollary 3 If $c^t \leq (b+h)/2$, the right-hand-side in (A3) is negative (because the numerator is negative), and thus, the inequality (A3) holds.

Note if $c^t = 0$, from Proposition 1, regardless of δ, the physician will exert a low diagnostic effort level and order a test for all patients, so the range will remain empty. *Q.E.D.*

Proof of Lemma 6 As detailed in the proof of Lemma 5, we go from case (ii) to case (iii) as δ increases. Using Proposition A3, we observe that social welfare is independent of δ in case (ii) when $\Delta r = 0$. We now focus on case (iii). We denote $D_1 \triangleq (1-\theta)c^t + \theta(b+h-c^t)$ and $D_2 \triangleq \theta c^t + (1-\theta)(b+h-c^t)$, which are both positive. We obtain

$$\frac{\partial \mathbb{E}_p[SW^F(p)]}{\partial \delta} = \frac{c^e}{\delta^2}\Bigg[(\theta C^t - c^e)\frac{f(p_2^F)}{D_2} - ((1-\theta)C^t + c^e)\frac{f(p_1^F)}{D_1} - (\theta C^t + (1-\theta)(b+h-C^t))\frac{\theta c^t - c^e/\delta}{D_2}\frac{f(p_2^F)}{D_2} + ((1-\theta)C^t + \theta(b+h-C^t))\frac{(1-\theta)c^t + c^e/\delta}{D_1}\frac{f(p_1^F)}{D_1}\Bigg]$$

When priors have a uniform distribution, because $f(.) = 1$, the result simplifies into

$$\frac{\partial \mathbb{E}_p[SW^F(p)]}{\partial \delta} = \frac{c^e}{\delta^3 D_1^2 D_2^2}(u + v\delta),$$

where

$$u \triangleq c^e C^t[\theta D_1^2 + (1-\theta)D_2^2] + c^e(b+h-C^t)[(1-\theta)D_1^2 + \theta D_2^2],$$

$$v \triangleq -c^e(b+h)D_1D_2 + (2\theta-1)\theta(1-\theta)(b+h)^2(C^t - c^t)(b+h-2c^t).$$

Because the partial derivative is proportional to a linear expression in δ, it can change sign at most once. Furthermore, the slope is negative if and only if $v < 0$. When the slope is negative, the partial derivative in case (iii) either remains negative, or is positive and then negative as δ increases. If $b+h-2c^t \leq 0$, then $v < 0$. When $b+h-2c^t > 0$, we have $v < 0$ if and only if

$$c^e > \frac{(2\theta-1)\theta(1-\theta)(b+h)(C^t-c^t)(b+h-2c^t)}{D_1D_2}.$$

When $b+h-2c^t > 0$ and c^e is less than the above threshold, the slope is positive, so the derivative either remains positive or is first negative then positive as δ increases. *Q.E.D.*

Proof of Proposition 7. Let $A = C^t,\ B = b+h-C^t,\ A' = \delta c^t - \Delta r$ and $B' = \delta(b+h-c^t) + \Delta r$. Then, adjusting parameters $b, h, c^t, \delta, \Delta r$ means we seek A' and B' so that $p_1^F = p_1^S$ and $p_2^F = p_2^S$, that is,

$$\begin{cases} \frac{(1-\theta)A + c^e}{(1-\theta)A + \theta B} = \frac{(1-\theta)A' + c^e}{(1-\theta)A' + \theta B'} \\ \frac{\theta A - c^e}{\theta A + (1-\theta)B} = \frac{\theta A' - c^e}{\theta A' + (1-\theta)B'}. \end{cases}$$

After simplification, we find this system is equivalent to

$$\begin{cases} B - B' = A - A' \\ \theta(1-\theta)(AB' - A'B) = c^e[\theta(B-B') + (1-\theta)(A-A')] \end{cases}$$

$$\Leftrightarrow \quad A - A' = B - B' = 0 \quad \text{or} \quad \begin{cases} B - B' = A - A' \neq 0 \\ c^e = \theta(1-\theta)(b+h-2C^t). \end{cases}$$

$A - A' = B - B' = 0$ means $C^t = \delta c^t - \Delta r$ and $b+h-C^t = \delta(b+h-c^t) + \Delta r$, which is equivalent to $\delta = 1$ and $\Delta r = c^t - C^t$. In this case, it is easy to check that we also have $\bar{c}^F = \bar{c}^S$.

Now consider the other solution, which is possible when $c^e = \theta(1-\theta)(b+h-2C^t)$. In this case, let D such that $D = B - B' = A - A'$. Combining the equations $D = C^t - \delta c^t + \Delta r$ and $D = b+h-C^t - \delta(b+h-c^t) - \Delta r$, it follows that $D = (b+h)(1-\delta)/2$. Plugging into the expression of $\bar{c}^F$, we find that $\bar{c}^F = \bar{c}^S$ implies $\delta = 1$, which implies $D = 0$ and thus reduces to the previous solution. *Q.E.D.*

Proof of Proposition 8. We start with the case in which the physician exerts low effort. In this case, the physician should order a test if and only if

$$pr^+ + (1-p)r^- + \delta pb - \delta c^t > r^n - \delta ph,$$

which is equivalent to

$$p > \frac{\delta c^t - (r^- - r^n)}{\delta(b+h) + r^+ - r^-}.$$

and provide a mild diagnosis otherwise. To ensure $0 \le \frac{\delta c^t - (r^- - r^n)}{\delta(b+h) + r^+ - r^-} < 1$, we need

$$r^+ - r^n > \delta(c^t - b - h). \tag{A4}$$

If $r^+ - r^n \le \delta(c^t - b - h)$, the physician never orders a test, regardless of p. Different from the case of the fee-for-service payment scheme, *no* case exists in which the physician orders a test for all patients.

Next, we consider the case in which the physician exerts effort in the consultation process. Depending on the private signal, two scenarios exists:

(i) If the consultation generates a positive signal, the physician updates the patient's probability of suffering from the positive condition:

$$\Pr(s = \bar{s} | \sigma = \bar{\sigma}) = \frac{\theta p}{\theta p + (1-\theta)(1-p)}.$$

The physician should order a test if and only if

$$\frac{\theta p}{\theta p + (1-\theta)(1-p)} \cdot (r^+ + \delta b) + \frac{(1-\theta)(1-p)}{\theta p + (1-\theta)(1-p)} \cdot r^- - \delta c^t > r^n - \delta \cdot \frac{\theta p}{\theta p + (1-\theta)(1-p)} \cdot h,$$

which gives

$$p > \frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]}.$$

To ensure $0 \le \frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]} \le 1$, we need $r^+ - r^n > \delta(c^t - b - h)$, which coincides with (A4). If $r^+ - r^n \le \delta(c^t - b - h)$, the physician never orders a test, regardless of p. No case exists in which the physician orders a test for all patients.

(ii) If the consultation generates a positive signal, the physician updates the patient's probability of suffering from the positive condition:

$$\Pr(s = \bar{s} | \sigma = \underline{\sigma}) = \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)}.$$

The physician should order a test if and only if

$$\frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)} \cdot (r^+ + \delta b) + \frac{\theta(1-p)}{(1-\theta)p + \theta(1-p)} \cdot r^- - \delta c^t > r^n - \delta \cdot \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)} \cdot h,$$

which is equivalent to

$$p > \frac{\theta[\delta c^t - (r^- - r^n)]}{\theta[\delta c^t - (r^- - r^n)] + (1-\theta)[\delta(b+h-c^t) + r^+ - r^n]}.$$

To ensure $0 \leq \frac{\theta[\delta c^t - (r^- - r^n)]}{\theta[\delta c^t - (r^- - r^n)] + (1-\theta)[\delta(b+h-c^t) + r^+ - r^n]} < 1$, we need $r^+ - r^n > \delta(c^t - b - h)$, which coincides with (A4). If $r^+ - r^n \leq \delta(c^t - b - h)$, the physician never orders a test, regardless of p. No case exists in which the physician orders a test for all patients.

Finally, we compare the two cases (i.e., the physician exerts low vs. high effort) and analyze the physician's effort decision.

(i) If $r^+ - r^n \leq \delta(c^t - b - h)$, the physician should choose a low effort level.

(ii) If $r^+ - r^n > \delta(c^t - b - h)$, the physician's effort decision can be determined as follows:

(1) If $\frac{\theta[\delta c^t - (r^- - r^n)]}{\theta[\delta c^t - (r^- - r^n)] + (1-\theta)[\delta(b+h-c^t) + r^+ - r^n]} < p \leq 1$, the expert always orders the test regardless of the effort or the signal. Thus, the expert exerts low effort.

(2) If $0 < p \leq \frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]}$, the expert always provides a mild diagnosis regardless of the effort or the signal. Thus, the expert exerts low effort.

(3) If $\frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]} < p \leq \frac{\theta[\delta c^t - (r^- - r^n)]}{\theta[\delta c^t - (r^- - r^n)] + (1-\theta)[\delta(b+h-c^t) + r^+ - r^n]}$, the expert follows the signal if exerting effort. The condition for the physician to exert effort is as follows:

(a) If $\frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]} < p \leq \frac{\delta c^t - (r^- - r^n)}{\delta(b+h) + r^+ - r^-}$, the physician exerts effort if and only if

$$\begin{aligned}
&[\theta p + (1-\theta)(1-p)] \cdot \left[\frac{\theta p}{\theta p + (1-\theta)(1-p)} \cdot (r^+ + \delta b) + \frac{(1-\theta)(1-p)}{\theta p + (1-\theta)(1-p)} \cdot r^- - \delta c^t\right] \\
&+ [(1-\theta)p + \theta(1-p)] \cdot \left[r^n - \delta \cdot \frac{(1-\theta)p}{(1-\theta)p + \theta(1-p)} \cdot h\right] - c^e \\
&> r^n - \delta p h,
\end{aligned}$$

which is equivalent to

$$p > \frac{(1-\theta)[\delta c^t - (r^- - r^n)] + c^e}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]}.$$

We can easily verify that

$$\frac{(1-\theta)[\delta c^t - (r^- - r^n)] + c^e}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]} > \frac{(1-\theta)[\delta c^t - (r^- - r^n)]}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]}.$$

To ensure

$$\frac{(1-\theta)[\delta c^t - (r^- - r^n)] + c^e}{(1-\theta)[\delta c^t - (r^- - r^n)] + \theta[\delta(b+h-c^t) + r^+ - r^n]} \leq \frac{\delta c^t - (r^- - r^n)}{\delta(b+h) + r^+ - r^-},$$

we need

$$c^e \leq \bar{c}^d \triangleq \frac{(2\theta-1)[\delta c_t - (r^- - r^n)][\delta(b+h-c_t) + r^+ - r^n]}{\delta(b+h) + r^+ - r^-}. \tag{A5}$$

If c^e is above the threshold, the physician exerts low effort for any patient within this range of priors.

(b) If $\frac{\delta c^t-(r^- - r^n)}{\delta(b+h)+r^+ - r^-} < p \leq \frac{\theta[\delta c^t-(r^- - r^n)]}{\theta[\delta c^t-(r^- - r^n)]+(1-\theta)[\delta(b+h-c^t)+r^+ - r^n]}$, the physician exerts effort if and only if

$$\begin{aligned}&[\theta p+(1-\theta)(1-p)]\cdot\left[\frac{\theta p}{\theta p+(1-\theta)(1-p)}\cdot(r^+ +\delta b)+\frac{\theta p}{\theta p+(1-\theta)(1-p)}r^- - \delta c^t\right]\\&+[(1-\theta)p+\theta(1-p)]\cdot\left[r^n-\delta\cdot\frac{(1-\theta)p}{(1-\theta)p+\theta(1-p)]}\cdot h\right]-c^e\\&> pr^+ +(1-p)r^- +\delta pb-\delta c^t,\end{aligned}$$

which yields

$$p<\frac{\theta[\delta c^t-(r^- - r^n)]-c^e}{\theta[\delta c^t-(r^- - r^n)]+(1-\theta)[\delta(b+h-c^t)+r^+ - r^n]}.$$

We can easily verify that

$$\frac{\theta[\delta c^t-(r^- - r^n)]-c^e}{\theta[\delta c^t-(r^- - r^n)]+(1-\theta)[\delta(b+h-c^t)+r^+ - r^n]}<\frac{\theta[\delta c^t-(r^- - r^n)]}{\theta[\delta c^t-(r^- - r^n)]+(1-\theta)[\delta(b+h-c^t)+r^+ - r^n]}.$$

To ensure

$$\frac{\theta[\delta c^t-(r^- - r^n)]-c^e}{\theta[\delta c^t-(r^- - r^n)]+(1-\theta)[\delta(b+h-c^t)+r^+ - r^n]}\geq\frac{\delta c^t-(r^- - r^n)}{\delta(b+h)+r^+ - r^-},$$

we need

$$c^e\leq\frac{(2\theta-1)[\delta c_t-(r^- - r^n)][\delta(b+h-c_t)+r^+ - r^n]}{\delta(b+h)+r^+ - r^-}=\bar{c}^d,$$

which is identical to (A5). If c^e is above the threshold, the physician exerts low effort for any patient within this range of priors. *Q.E.D.*

Proof of Proposition 9. Setting $p_1^d=p_1^S$ and $p_2^d=p_2^S$ gives

$$\begin{aligned}&r^+ - r^n=b-C^t+h-\delta(b-c^t+h)=\delta c^t-C^t+(1-\delta)(b+h)\\&r^- - r^n=\delta c^t-C^t.\end{aligned}$$

We can verify that under the above solution,

$$\begin{aligned}\bar{c}^d&=\frac{(2\theta-1)[\delta c_t-(r^- - r^n)][\delta(b+h-c_t)+r^+ - r^n]}{\delta(b+h)+r^+ - r^-}\\&=\frac{C^t(2\theta-1)(b+h-C^t)}{b+h},\end{aligned}$$

which is equal to $\bar{c}^S$. The proof is complete. *Q.E.D.*

A3: Alternate Definition of Social Welfare

In this section, we consider a definition of social welfare that differs from that used in the main body. Specifically, we define an alternative social welfare as the combination of expected patient utility, payer expenditure, and physician utility:

$$SW=\Pi_{\text{payer}}+U_{\text{physician}}+\mathbb{E}[U_{\text{patient}}].$$

This definition contrasts with that used in the main body of the paper, in that we use as the second term of social welfare the physician utility $U_{\text{physician}} = \Pi_{\text{physician}} + \delta\mathbb{E}[U_{\text{patient}}]$ instead of physician's payoff $\Pi_{\text{physician}}$. This new definition of social welfare implies

$$SW = \Pi_{\text{payer}} + \Pi_{\text{physician}} + (\delta+1)\mathbb{E}[U_{\text{patient}}];$$

that is, the expected patient utility has a weight $\delta+1$, whereas in the main body of the paper, this term has a coefficient equal to 1, which changes the weight of parameters b, h, and c^t in social welfare. More fundamentally, the weight of the patient utility in social welfare $(\delta+1)$ is now guaranteed to exceed the weight of the patient utility in the physician's objective (δ). By contrast, in our base model, the weight of the patient utility in social welfare is equal to 1, which could be lower than the weight of the patient utility in the physician's objective (if $\delta > 1$).

The results focused on the physician's optimal policy are unaffected. Only the results that make use of the socially optimal policy are affected. Namely, Proposition 2 is modified as follows (the proofs of all modified results are omitted for the sake of brevity; they are very similar to the proof of the corresponding results presented in Appendix A2):

Let

$$\bar{c}^S \triangleq (2\theta-1)(C^t+\delta c^t)\left(1-\frac{C^t+\delta c^t}{(\delta+1)(b+h)}\right)$$
$$p_1^S \triangleq \frac{(1-\theta)(C^t+\delta c^t)+c^e}{(1-\theta)(C^t+\delta c^t)+\theta((\delta+1)(b+h)-C^t-\delta c^t)}$$
$$p_2^S \triangleq \frac{\theta(C^t+\delta c^t)-c^e}{\theta(C^t+\delta c^t)+(1-\theta)((\delta+1)(b+h)-C^t-\delta c^t)}.$$

PROPOSITION A5. *The socially optimal policy is as follows:*

(i) If $(\delta+1)(b+h) \le C^t+\delta c^t$, the physician exerts low effort and does not order a test for any patient.

(ii) If $(\delta+1)(b+h) > C^t+\delta c^t$ and $c^e > \bar{c}^S$, the physician exerts low effort for all patients and orders a test if and only if $p \ge (C^t+\delta c^t)/((\delta+1)(b+h))$.

(iii) If $(\delta+1)(b+h) > C^t+\delta c^t$ and $c^e \le \bar{c}^S$, the socially optimal policy depends on the patient's prior:

(a) if $p \le p_1^S$, the physician exerts low effort and does not order a test;

(b) if $p_1^S < p \le p_2^S$, the physician exerts high effort and follows the signal (i.e., if the signal is positive, order a test; if the signal is negative, do not order a test);

(c) if $p > p_2^S$, the physician exerts low effort and orders a test.

Although the thresholds for the different cases are slightly modified, the general structure of the socially optimal policy remains unchanged.

Proposition 4 is modified as follows:

PROPOSITION A6. *When Δr is such that the range of priors with high effort is at its widest under the fee-for-service physician's optimal strategy, that range is wider than the socially optimal range if and only if either $C^t+\delta c^t < c_0$ or $C^t+\delta c^t > c_1$, where*

$$c_0 \triangleq \frac{(\delta+1)(b+h)}{2}(1-\sqrt{q}); \quad c_1 \triangleq \frac{(\delta+1)(b+h)}{2}(1+\sqrt{q}), \quad q \triangleq \frac{c^e}{(2\theta-1)(1+\delta)[c^e(2\theta-1)+\theta(1-\theta)\delta(b+h)]}.$$

Otherwise (i.e., $c_0 \le C^t+\delta c^t \le c_1$), the socially optimal range of high effort is wider than the fee-for-service range of high effort for all Δr.

The definitions of c_0 and c_1 are modified, but the main difference from the result in the base model is that $\delta > 1$ no longer guarantees fee-for-service leads to a wider range of priors leading to high effort than the social optimum. The reason is that in the base model, $\delta > 1$ corresponds to a situation where the social planner weighs the patient utility less than the physician—a case that can no longer occur with this alternate definition. However, the insight that fee-for-service may lead to a wider range of priors leading to high effort than the social optimum remains true. It would be the case for sufficiently low or sufficiently high cost of the test (condition similar to that obtained in the base model for $\delta \leq 1$).

The intermediate result Proposition A3 is modified as follows:

Proposition A7. *Under fee-for-service, the average population social welfare is as follows:*

(i) If $\Delta r \leq \delta(c^t - b - h)$*, then* $\mathbb{E}_p[SW^F(p)] = -(\delta+1)h\mu$*;*

(ii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ *and* $c^e > \bar{c}^F$*, then*

$$\mathbb{E}_p[SW^F(p)] = b(\delta+1)\mu - (C^t + \delta c^t)\bar{F}\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right) - (b+h)(\delta+1)Q\left(\frac{\delta c^t - \Delta r}{\delta(b+h)}\right);$$

(iii) If $\delta(c^t - b - h) < \Delta r < \delta c^t$ *and* $c^e \leq \bar{c}^F$*, then*

$$\begin{aligned}\mathbb{E}_p[SW^F(p)] = \; & b(\delta+1)\mu - C^t - \delta c^t - c^e(F(p_2^F) - F(p_1^F)) + (C^t + \delta c^t)(\theta F(p_2^F) + (1-\theta)F(p_1^F)) \\ & - (C^t + \delta c^t)\left(\theta Q(p_2^F) + (1-\theta)Q(p_1^F)\right) - ((b+h)(\delta+1) - C^t - \delta c^t)\left((1-\theta)Q(p_2^F) + \theta Q(p_1^F)\right);\end{aligned}$$

(iv) If $\Delta r \geq \delta c^t$*, then* $\mathbb{E}_p[SW^F(p)] = (\delta+1)b\mu - C^t - \delta c^t$*.*

Using this intermediate result, Proposition 5 is modified as follows:

Proposition A8. *If* $c^e > (2\theta - 1)\delta(b+h)/4$*, the value of* Δr *that maximizes the average population social welfare under fee-for-service is*

$$\Delta r = \begin{cases} -\delta(C^t - c^t)/(\delta+1) & \text{if } (\delta+1)(b+h) > C^t + \delta c^t \\ \text{any value within } (-\infty, -\delta(b+h-c^t)] & \text{otherwise.} \end{cases}$$

The above result is similar to that in the base model (with slightly modified optimal Δr and associated condition) and leads to the same insights.

Lemma 6 is modified as follows: suppose $\Delta r = 0$ and the patient priors are uniformly distributed. For δ below a threshold, social welfare is monotonic with respect to δ, with a slope equal to $b/2 - c^t + (c^t)^2/(2(b+h))$, which is positive if $b + h - c^t > \sqrt{h(b+h)}$, and negative otherwise. This property means social welfare may be decreasing in δ in this region (when $0 < b + h - c^t < \sqrt{h(b+h)}$). Above the threshold, analytically studying the sign of the derivative of social welfare with respect to δ is intractable: the derivative can no longer be written as a linear expression of δ divided by δ^3. Instead, it is a polynomial of degree 3 divided by δ^3. However, we find numerically that in this region, social welfare may also be decreasing with δ.

Let us now focus on the incentive-alignment results (in Section 6.4). Proposition 9 is modified as follows:

Proposition A9. *The physician's effort and testing decisions maximize social welfare under a diagnosis-based payment scheme with* $r^- - r^n = -C^t$ *and* $r^+ - r^n = b + h - C^t$*. In particular,* $r^- - r^n < 0$ *and, if* $b + h > C^t$*, then* $r^+ - r^n > 0$*.*

The result indicates that, even with the alternate definition of social welfare, a diagnosis-based payment scheme can align the physician's decisions to the social optimum. The main difference from the result obtained in the main body of the paper is that there is no longer any condition on δ to ensure $r^+ - r^n > 0$.