Online Supplemental Materials to
Auditing with a Chance of Whistleblowing

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1 Firm strategic misreporting

In our main model, we assume the bad firm always misrepresents its type to focus on how whistleblowing intensity impacts the auditor and the enforcer. In the context of whistleblowing, another important consideration is intentional misreporting by firms.[1] In this section, we study an extension in which the firm engages in strategic misreporting to shed light on this issue.

We assume that before date 0, the firm privately observes its financial condition and may misrepresent its true type to the auditor. If the financial condition is good (denoted by θ_G), the investment will generate a positive cash flow $X > I$; if the condition is bad (denoted by θ_B), the investment will generate zero cash flow. The common prior belief is that the probability of financial condition being good is q, and the default action is to invest, i.e., $qX > I$. While it is straightforward that a good firm always truthfully reports its financial condition, a bad firm may misreport its condition. Specifically, we assume the bad firm chooses an unobservable misreporting effort $m \in [0, 1]$, and the cost of misreporting is $\frac{1}{2}\rho m^2$.

[1] Specifically, if a financial misstatement is due to an unintentional mistake, an insider may report it internally and correct the mistake, making external whistleblowing unnecessary. We thank the Associate Editor for this suggestion.

We assume the misreporting cost is not trivial (i.e., $\rho > I$) so misreporting is not always successful. With probability m, the bad firm successfully misreports its financial condition to the auditor, which corresponds to the B firm in our main setting. With probability $1-m$, the bad firm fails to misrepresent its bad condition and is directly revealed to the auditor. We further assume that a bad firm's objective is to maximize the expected investment subject to the misreporting cost, which may arise due to the manager's "empire building" incentives (Baldenius, 2003; Ewert and Wagenhofer, 2019). A bad firm's objective, therefore, is

$$\max_{m} \Pr\left[Investment|\theta_B\right] - \frac{1}{2}\rho m^2.$$

After receiving the firm's unaudited report claiming its condition to be good, the auditor exerts effort a to detect potential misreporting. With probability a, the auditor can successfully detect the misreporting. The rest of the model is the same as our main setting.

In this setting, the equilibrium comprises the bad firm's misreporting decision m^*, the auditor's audit effort a_m^*, and the enforcer's investigative efforts e_{m1}^* and e_{m0}^*. Given the new ingredient introduced in this section pertains to a bad firm's misreporting decision, we will focus on the impact of whistleblowing intensity on the firm's misreporting behavior. We present the results in the following Proposition.

Proposition OA1 *Considering firm's strategic misreporting: Increasing whistleblowing intensity aggravates firms' misreporting behavior if, and only if* $\frac{R}{\gamma} > r_m^*$ *and* $\frac{D}{k} < d_m^*$.

The threshold values d_m^* *and* r_m^* *are derived in the proof.*

2 Auditor making type I errors

In our main model, we follow prior research and assume that when the firm type is G, the auditor never obtains the signal b or reaches the false conclusion that the firm is bad. This means that the auditor only makes type II errors and does not make type I errors. Previous research has emphasized type II errors, as clients are more likely to object to an incorrect rejection and, when shown evidence, auditors may revise their opinion (Antle and Nalebuff, 1991). However, recent studies have started to investigate type I errors (Chen et al., 2019; Deng et al., 2012) in the auditing context. Therefore, this section analyzes whether including type I errors will impact our main findings.

Specifically, the auditor exerts effort $a \in [0,1]$ to find audit evidence regarding the potential misstatement, leading to a binary signal $\{g, b\}$. We assume the mapping from the firm's type to the auditor's signal is as follows:

$$\Pr(b|B) = \Pr(g|G) = \frac{1+a}{2}.$$

In particular, the auditor may fail to detect a misstatement made by a bad firm and issue a clean audit opinion. Similarly, the auditor may falsely accuse a good firm of misreporting its financial conditions and issue a qualified audit opinion. Therefore, increasing the level of audit effort not only boosts the likelihood of detecting accounting misstatements (i.e., reduces the type II error) but also decreases the probability of making false accusations (i.e., reduces the type I error). Furthermore, to highlight the important role of auditing, we focus on the case where the prior belief p is such that investors will invest in the firm if and only if the auditor issues a clean audit opinion.

We find that our main results continue to hold even if the auditor may falsely accuse a good firm of misstatements. These results are described in the following proposition.

Proposition OA2 *Suppose the auditor may make both type I and type II errors. Increasing whistleblowing intensity impairs audit quality if, and only if* $\frac{R}{\gamma} > r_t^*$ *and* $\frac{D}{k} < d_t^*$.

The threshold values d_t^* *and* r_t^* *are derived in the proof.*

Proposition OA2 confirms the robustness of our key findings, that increasing whistleblowing intensity hinders audit quality when the enforcer's effort incentive is strong but the auditor's incentive is weak. In this alternative setting, the impact of whistleblowing intensity on audit quality depends on the same trade-off between the direct and the indirect effects, leading to similar results as in our main model. More importantly, audit quality plays an even more crucial role in enhancing investment efficiency in this setting. This is because a false accusation by the auditor may cause a profitable firm to lose the opportunity of being funded, leading to an additional source of investment inefficiency. Consequently, improving audit quality enhances investment efficiency not only by reducing the likelihood of audit failure and thus the investment loss in case of audit failure, but also by making a good firm more likely to receive funding—the latter effect is unique to this alternative setting, therefore making our results even more compelling.

3 Impact of meritless whistleblowing

Our main model assumes that the whistleblower's signal is informative about the firm's type and the precision of the signal is β. To provide a more comprehensive analysis, in this section, we address another aspect of whistleblowing, where individuals may make groundless allegations against firms without any evidence. This type of meritless whistleblowing can be concerning because resource-constrained regulators may miss the opportunity to investigate legitimate claims.

Specifically, we assume there are two types of whistleblowers: an informative type (I-type) and an uninformative type (U-type).[2] The whistleblower's type is unobservable, and outsiders hold a prior belief that the type is I with probability $\lambda \in (0, 1)$. The whistleblower obtains a signal $s \in \{s_h, s_l\}$ about the firm's type, but the precision of the signal depends

[2] The existence of the informative type of whistleblowers is necessary to study the issue of meritless whistleblowing. Without them, promoting whistleblowing would have no effect, as the enforcer would ignore all whistleblowing allegations due to the lack of informational value.

on the whistleblower's type:

$$\begin{aligned}\Pr(s_l|B,I) &= \Pr(s_h|G,I)=\beta;\\ \Pr(s_l|B,U) &= \Pr(s_h|G,U)=\frac{1}{2}.\end{aligned}$$

That is, the I-type's signal is noisy but informative about the firm's true type with the precision $\beta \in \left(\frac{1}{2},1\right)$, whereas the U-type's signal is pure noise. Similar to the main model, the whistleblower is eligible to blow the whistle only when receiving the signal s_l, but the enforcer cannot distinguish the whistleblower's type and thus is uncertain about the precision of the signal. To focus on the effects of meritless whistleblowing, we hold the I-type's decision constant and assume that the I-type always blows the whistle upon receiving s_l, while we vary the U-type's behavior by allowing the U-type to blow the whistle with a certain probability y, with a larger y indicating more severe meritless whistleblowing. Our goal is to examine how changes in y affect the equilibrium audit quality. The rest of the model is the same as our main setting.

We find that, just like in the main setting, increasing y has both a *direct effect* and an *indirect effect* on the equilibrium audit quality. To demonstrate this, we break down the impact of y on a^*, similar to (12), as follows:

$$\frac{da^*}{dy} \propto \underbrace{\frac{\partial \Pr(W_1|B,g)}{\partial y}\left[e_1^* - e_0^*\right]}_{\text{direct effect}} + \underbrace{\Pr(W_1|B,g)\frac{de_1^*}{dy} + \Pr(W_0|B,g)\frac{de_0^*}{dy}}_{\text{indirect effect}}.$$

First, there is a positive *direct effect* of increasing y on audit quality. Although the enforcer cannot differentiate the whistleblower's type, the enforcer's investigative effort upon receiving a whistleblowing tip is always higher than that without whistleblowing. As increasing y triggers the high-level investigative effort more often, the auditor has stronger incentives to improve audit quality. Second, increasing y also imposes an *indirect effect* on audit quality by influencing the enforcer's effort, e_1^* and e_0^*. As y increases, the enforcer rationally anticipates that a whistleblowing tip is more likely to originate from the U-type, and thus becomes less certain about the misstatement. This reduced information content of whistleblowing causes the enforcer to exert less effort, ultimately resulting in a decline in audit quality. Additionally, if there is no whistleblowing, the enforcer believes the silence is more likely driven by the I-type receiving the signal s_h, which also reduces the investigative effort.

Therefore, the overall impact of y on a^* is determined by the trade-off between the direct and the indirect effects, which can be positive or negative depending on the parameters. We present the results in the following proposition.

Proposition OA3 *Considering meritless whistleblowing:*

1. *If $\lambda < \lambda_y$, meritless whistleblowing impairs audit quality.*
2. *If $\lambda > \lambda_y$, meritless whistleblowing improves audit quality when p is sufficiently small.*

Proposition OA3 suggests that meritless whistleblowing decreases audit quality if the likelihood of an informed whistleblower is low. Intuitively, when the whistleblower is highly likely to be uninformed, the enforcer can gain little insight about firms' misstatements from whistleblowing or the lack thereof. Consequently, although more meritless whistleblowing can still trigger high-level regulatory efforts more frequently, the difference between e_1^* and e_0^* is minimum. Therefore, the positive direct effect is only marginal, resulting in an overall negative impact.

Interestingly, our findings indicate that if the likelihood of an informed whistleblower is high, meritless whistleblowing could actually enhance audit quality. This scenario is more likely to occur when the prior probability of misstatement is high. In contrast to the previous case, the positive direct effect is significant because the whistleblower is likely informed. Furthermore, when the prior probability of misstatements is already high, although increasing y can reduce the enforcer's investigative effort, such reduction is limited. Consequently, the direct effect outweighs the indirect effect, resulting in an overall positive impact on audit quality.

4 Alternative objective function: Comparative Statics

In section 4.2, we consider an alternative objective function of the social planner as follows:

$$\max_{\tau} \Omega_1 \equiv \underbrace{IE - \frac{1}{2}k\left(a^*\right)^2}_{IE_1} - \frac{1}{2}n\tau^2. \tag{A.1}$$

We also examine the comparative statics regarding the optimal whistleblowing intensity under this alternative objective function. Similar to those in Section 3.4, these analyses are largely intractable due to the algebraic complexity. However, using numerical simulations, we have confirmed that most of the results established in our main setting continue to hold. We have included an illustration of our numerical analyses in Figure 1, using the same parameters in the main setting. In addition, we have observed that these patterns are highly robust across a wide range of numerical values.

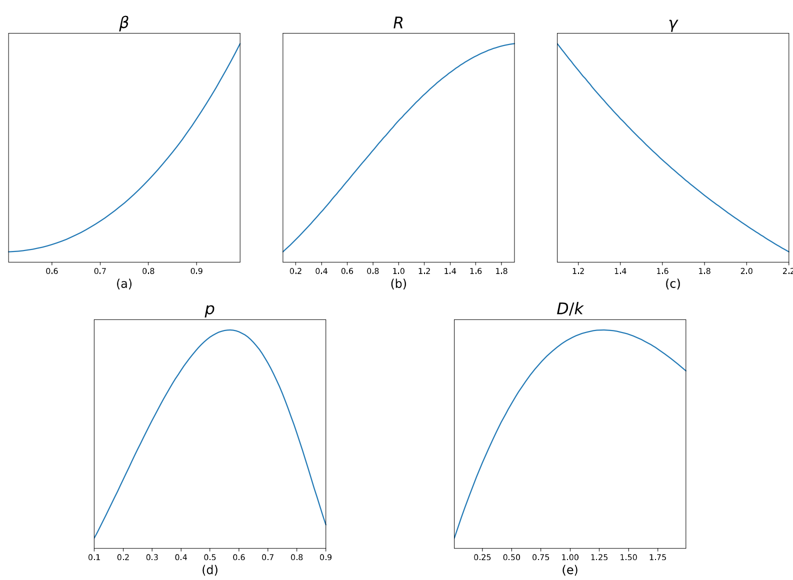

Figure 1: The effects of β, R, γ, p, and $\frac{D}{k}$ on the optimal whistleblowing intensity τ_1^o. The following parameters are used: In panel (a), $p = 0.5, R = 1.1, \gamma = 1, I = 2, b = 4, \frac{D}{k} = 1$. In panel (b), $p = 0.5, \gamma = 1, I = 2, b = 4, \frac{D}{k} = 1, \beta = 0.8$. In panel (c), $p = 0.5, R = 1.1, I = 2, b = 4, \frac{D}{k} = 1, \beta = 0.8$. In panel (d), $R = 1.1, \gamma = 1, I = 2, b = 4, \frac{D}{k} = 1, \beta = 0.8$. In panel (e), $p = 0.5, R = 1.1, \gamma = 1, I = 2, b = 4, \beta = 0.8$.

Specifically, our numerical analysis shows that, similar to the main setting, the optimal whistleblowing intensity τ_1^o increases with the whistleblower's information quality β and the liquidation value of the investment R, decreases with the cost parameter of the enforcer's investigative effort γ, and exhibits a non-monotonic relationship with the business risk of the firm $1-p$. Nevertheless, by comparing Figure 3 in the main setting with Figure 1 in this extension, we observe that the comparative statics regarding the auditor's effort incentive becomes different. In particular, in our main setting, numerical simulations suggest that τ^o increases with $\frac{D}{k}$. This is because, as $\frac{D}{k}$ becomes larger, increasing whistleblowing intensity is more effective in motivating the auditor to exert effort and thus improve investment efficiency. However, in this alternative setting where the social planer fully internalizes the auditor's effort cost, τ_1^o becomes non-monotonic in $\frac{D}{k}$ and may exhibit an inverse-U shape relationship. To see this, consider the scenario where $\frac{D}{k}$ is sufficiently large. As $\frac{D}{k}$ further increases, using whistleblowing to motivate audit quality and improve investment efficiency incurs an overwhelmingly high auditing cost. As a result, the marginal benefit of τ decreases with $\frac{D}{k}$, resulting in a decline in the optimal whistleblowing intensity τ_1^o. Therefore, the comparative statics regarding $\frac{D}{k}$ may vary when the social planner's objective function is defined differently.

5 Auditor moves after whistleblowing

In our main setting, we assume that auditors take action before whistleblowers to investigate how whistleblowing impacts audit quality. This timeline is consistent with many high-profile whistleblowing cases, such as Worldcom, Enron, and the recent Wirecard scandal.[3] In this section, we explore a different scenario in which the whistleblower takes action before the auditor to examine how the auditor responds to publicly observable whistleblowing.[4]

[3] Wirecard, a German payment processor and financial services provider, filed for insolvency in 2020 after revealing that 1.9 billion Euro was unaccounted for. Despite this, Wirecard's auditor, Ernst & Young, had been issuing unqualified opinions on its financial statements for several years. The fraud was ultimately exposed by Pav Gil, Wirecard's senior legal counsel, who blew the whistle in 2019. In 2023, Ernst & Young was fined 500,000 Euro and prohibited from auditing public interest companies in Germany for two years.

[4] In practice, the enforcer is typically the only party aware of whistleblowing tips or their absence. For example, the SEC states that "the SEC treats all tips, complaints and referrals as confidential and nonpublic,

Specifically, at date 0, a potential whistleblower receives a signal $s \in \{s_l, s_h\}$ about the firm's true financial conditions. We assume the whistleblower with the signal s_l blows the whistle with probability τ. At date 1, depending on whether there is a whistleblowing allegation, the auditor performs auditing procedures and issues the audit report. The auditor's effort upon whistleblowing and no whistleblowing, denoted by a_1 and a_0, respectively, determines the likelihood of detecting misreporting. To shed light on the audit fees, we assume the audit market is competitive and the auditor earns zero profit in equilibrium, and the audit fees are denoted by F_1 and F_0. At date 2, after observing whistleblowing (or no whistleblowing) and receiving the auditor's report, investors decide whether to fund the investment. At date 3, the enforcer investigates the firm and imposes an enforcement action if an audit failure is revealed. To make sure the auditor is actively involved, we concentrate on situations in which investors will only fund the company if the auditor provides a clean report. The rest of the model is the same as our main setting.

To compare the results in our main setting, we focus on how whistleblowing intensity affects audit quality and the enforcer's effort in the absence of whistleblowing (i.e., a_0^* and e_0^*). We present the results in this alternative setting in the proposition below.

Proposition OA4 *In the alternative setting in which the whistleblower moves before the auditor, the following results hold:*

1. a_0^* *and* e_0^* *strictly decrease with* τ.

2. $a_1^* > a_0^*$ *and* $F_1^* > F_0^*$.

First, increasing whistleblowing intensity reduces the auditor's effort and the enforcer's effort when there is no whistleblowing allegation. These results are driven by the similar

and does not disclose such information to third parties, except in limited circumstances authorized by statute, rule, or other provisions of law." The PCAOB states that "...federal law prohibits the PCAOB and its staff from publicly disclosing information about the Board's investigative and enforcement activities unless and until the information results in a public proceeding..." In this section, we focus on the scenario in which the whistleblowing allegations are publicly observable, with the aim of examining the auditor's response to whistleblowing. This could happen if the whistleblower files a claim with an external regulatory agency, such as the Occupational Safety and Health Administration (Kuang et al. (2021)).

effects as in our main setting. Specifically, for the impact of τ on e_0^*, a larger τ has a *silence effect* and an *audit quality effect* on the enforcer's effort, both of which are negative, as follows:

$$\frac{de_0^*}{d\tau} \propto \underbrace{\frac{\partial \Pr(B|g, W_0)}{\partial \tau}}_{\text{silence effect}} + \underbrace{\frac{\partial \Pr(B|g, W_0)}{\partial a^*}\frac{da_0^*}{d\tau}}_{\text{audit quality effect}}$$

Similar to the analysis in our main setting, a larger τ has a *direct effect* on the auditor's incentive: holding the enforcer's effort constant, the auditor anticipates that no whistleblowing indicates the firm is more likely to be good and thus reduces the audit effort. Additionally, increasing τ has an *indirect effect* on audit quality: a larger τ reduces the the enforcer's investigative effort, which decreases the auditor's effort accordingly.

$$\frac{da_0^*}{d\tau} \propto \underbrace{\frac{\partial \Pr(B|W_0)}{\partial \tau} e_0^*}_{\text{direct effect}} + \underbrace{\Pr(B|W_0)\frac{de_0^*}{d\tau}}_{\text{indirect effect}}.$$

Part 2 of Proposition OA4 suggests that, compared with the case without a whistleblowing allegation, the auditor exerts greater effort and charges higher audit fees when such an allegation is present. This follows because whistleblowing allegations inform the auditor of the increased risk of misreporting, motivating the auditor to improve audit quality. As a result, the auditor charges a higher audit fee to account for the higher expected damage payment and greater effort required. These results are consistent with the empirical findings of Kuang et al. (2021).

In summary, with the alternative timeline in which the auditor moves after the whistleblower, we find that the main effects and trade-offs are similar to those in the main setting in terms of how whistleblowing intensity affects audit quality and the enforcer's effort.

6 Continuous setting discussion

In our main setting, we adopt a binary structure to maintain tractability and to capture auditing practices, that is, the auditor issuing opinions on whether the financial statements are materially misstated or not. Our results imply that increasing whistleblowing intensity may increase or decrease the likelihood of the auditor detecting material misstatements. However, we acknowledge that the binary setting also has its limitations, and therefore, in this section, we provide a discussion about the main trade-offs in a continuous setting.

We assume a firm's fundamental, denoted by $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, follows a probability density function $f(\theta)$. The firm reports $\theta + x$, where the parameter $x \in [0, \bar{x}]$ captures the reporting bias. To prevent the bias from being fully unraveled, we assume that x is random with a probability density function $g(x)$. This randomness may reflect that the manager's reporting objective is uncertain, as suggested by Fischer and Verrecchia (2000), or that the costs of misreporting are unknown to outsiders due to the firm's internal controls and/or reporting environment, as suggested by Dye and Sridhar (2008).

At date 0, the auditor exerts effort a to collect audit evidence and certify the report. We assume that with probability a, the auditor observes the true state θ and thus reveals the bias x. Otherwise, the bias is not detected by the auditor, and the auditor may face penalties if the enforcer subsequently detects the bias. The penalty increases with the magnitude of the bias. At date 1, a potential whistleblower observes a signal s about the firm's reporting bias. We assume the conditional density $s|x$ satisfies the monotone likelihood ratio property, so that a higher s indicates statistically more severe bias. To capture whistleblowing intensity parsimoniously, we assume the whistleblower follows a threshold strategy; that is, the whistleblower reports if and only if the signal indicates the bias is severe enough (i.e., $s > s_T$). Therefore, when the threshold s_T decreases, the whistleblower is more willing to blow the whistle, representing more intensified whistleblowing provisions. We aim to investigate the effects of increasing whistleblowing intensity by decreasing the threshold s_T. At date 2, the enforcer exerts effort e to detect misreporting: with probability e, the enforcer observes the true state θ and thus uncovers the audit failure. We further assume that a larger reporting bias leads to a greater investment loss, which makes the enforcer's detection more valuable.

While integrating the continuous setting may render the model intractable, we believe that the primary trade-offs of our study should persist. In particular, we conjecture the following results in this continuous setting.

(1) First, whistleblowing always induces the enforcer to exert greater effort compared with the case without whistleblowing (i.e., $e_1 > e_0$). This is because a whistleblowing allegation implies that the whistleblower's signal is high (i.e., $s > s_T$), whereas no whistleblowing means the whistleblower's signal is low (i.e., $s < s_T$). By the MLRP, the expected reporting bias is greater when there is a whistleblowing allegation, i.e., $E\left[x|s > s_T\right] > E\left[x|s < s_T\right]$, suggesting the enforcer's benefit of exerting effort is greater.

(2) When whistleblowing intensity is increased (i.e., s_T decreases), a firm is more likely to receive a whistleblowing allegation. Hence, ex ante, the enforcer exerts the high effort e_1 more frequently and the low effort e_0 less frequently. This, in turn, motivates the auditor to improve audit quality, consistent with the direct effect in the main model.

Appendix: Proofs

Proof. of Proposition OA1: The proof proceeds as follows. In step 1, we derive the equilibrium when considering the firm's misreporting decision. In step 2, we study how the firm's misreporting is affected by whistleblowing intensity.

Step 1) First, it is easy to confirm that when the investor observes the auditor's report g, the expected investment return is

$$\begin{aligned} & \Pr(G|g)\,X + \Pr(r_b, B|g)\,(R+D) + \Pr(r_g, B|g) * 0 - I \\ > \ & \Pr(G|g)\,X - I = \frac{q}{q+(1-q)\,\hat{m}\,(1-\hat{a})}X - I \\ > \ & qX - I > 0. \end{aligned}$$

Anticipating investors' investment decision, the θ_B firm's objective becomes

$$\max_{m} \Pr[Investment|\theta_B] - \frac{1}{2}\rho m^2 \Leftrightarrow m \Pr(g|\theta_B)\, I - \frac{1}{2}\rho m^2.$$

Denote $i = \frac{I}{\rho}$, and the firm's optimal misreporting decision is thus

$$m = i \Pr(g|\theta_B) = i\,(1-\hat{a})\,.$$

where $\hat{a}$ denotes the conjectured audit quality. Next, anticipating the θ_b firm's misreporting decision $\hat{m}$, the auditor understands that a firm claiming to be good is bad with probability $\frac{(1-q)\hat{m}}{q+(1-q)\hat{m}}$. Hence, the auditor's optimal effort decision is given by

$$a = d\frac{(1-q)\,\hat{m}}{q+(1-q)\,\hat{m}} \Pr(r_b|B, g) = d\frac{(1-q)\,\hat{m}}{q+(1-q)\,\hat{m}} \left[\beta\tau\hat{e}_1 + (1-\beta\tau)\,\hat{e}_0\right].$$

Lastly, the enforcer's posterior beliefs depend on the conjecture about the firm's misreporting and audit quality, as follows:

$$\begin{aligned}\Pr(B|g,W_1) &= \frac{(1-q)\,\hat{m}\,(1-\hat{a})\,\beta}{(1-q)\,(1-\hat{a})\,\beta+q\,(1-\beta)};\\ \Pr(B|g,W_0) &= \frac{(1-p)\,\hat{m}\,(1-\hat{a})\,(1-\beta\tau)}{(1-q)\,\hat{m}\,(1-\hat{a})\,(1-\beta\tau)+q\,[1-(1-\beta)\,\tau]}.\end{aligned}$$

By rational expectations, the equilibrium consists of a quadruplet $\{m^*, a_m^*, e_{m1}^*, e_{m0}^*\}$ satisfying the following system of equations:

$$\begin{aligned}m^* &= i\,(1-a_m^*);\\ a_m^* &= d\frac{(1-q)\,m^*}{q+(1-q)\,m^*}\left[\beta\tau e_{m1}^*+(1-\beta\tau)\,e_{m0}^*\right];\\ e_{m1}^* &= \min\left\{1, r\frac{(1-q)\,m^*\,(1-a_m^*)\,\beta}{(1-q)\,m^*\,(1-a_m^*)\,\beta+p\,(1-\beta)}\right\};\\ e_{m0}^* &= r\frac{(1-q)\,m^*\,(1-a_m^*)\,(1-\beta\tau)}{(1-q)\,m^*\,(1-a_m^*)\,(1-\beta\tau)+p\,[1-(1-\beta)\,\tau]}.\end{aligned}$$

Step 2) The proof is similar to that of Proposition 2. Because $m^* = i\,(1-a_m^*)$, a^*, e_{m1}^*, e_{m0}^* can be expressed as functions of m^* and τ, denoted by $a(m^*,\tau)$, $e_{m1}(m^*,\tau)$ and $e_{m0}(m^*,\tau)$, respectively. Substituting them into the second equation yields the following

$$W_m(m^*) \equiv a(m^*,\tau) - \frac{(1-q)\,m^*}{q+(1-q)\,m^*}\left[\beta\tau e_{m1}(m^*,\tau)+(1-\beta\tau)\,e_{m0}(m^*,\tau)\right].$$

It is easy to verify that the following result must hold:

$$\frac{\partial W_m(m^*)}{\partial m^*} = \frac{\partial a(m^*,\tau)}{\partial m^*} - (1-p)\,d\left[\beta\tau\frac{\partial e_{m1}(m^*,\tau)}{\partial m^*}+(1-\beta\tau)\frac{\partial e_{m0}(m^*,\tau)}{\partial m^*}\right] < 0.$$

Therefore, the implicit function theorem implies

$$\frac{dm^*}{d\tau} = -\frac{\frac{\partial W_m(m^*)}{\partial\tau}}{\frac{\partial W_m(m^*)}{\partial m}} \propto \frac{\partial W_m(m^*)}{\partial\tau}.$$

We first consider the case that $d < d_m^*$ and $r > r_m^*$. In this case, we conjecture $e_{m1}^* = 1$ so that $\frac{\partial W_m(m^*)}{\partial \tau} > 0$ is equivalent to

$$r > V_m(m^*) = \frac{\beta\left[iq((\beta-1)\tau+1) + (m^*)^2(q-1)(\beta\tau-1)\right]^2}{(q-1)(\beta\tau-1)(m^*)^2\left[iq\left(\beta^2\tau-\beta(\tau-3)-1\right)+\beta(m^*)^2(q-1)(\beta\tau-1)\right]}.$$

It is easy to verify that $V_m(m^*)$ is decreasing in m^*. In addition, when $m^* \to 1$, we have

$$\lim_{m^*\to 1} V_m(m^*) = r_m^* = \frac{\beta\left[iq((\beta-1)\tau+1)+(q-1)(\beta\tau-1)\right]^2}{(q-1)(\beta\tau-1)\left[iq\left(\beta^2\tau-\beta(\tau-3)-1\right)+\beta(q-1)(\beta\tau-1)\right]}.$$

In other words, if $r < r_m^*$, it must hold that $r < \lim_{m^*\to 1} V_m(m^*) < V_m(m^*)$, and thus $\frac{\partial W_m(m^*)}{\partial \tau} < 0$ is satisfied. Instead, if $r > r_m^*$, $r > V_m(m^*)$ is equivalent to $m^* > m_1$, where m_1 is the second root of the following equation of x:

$$-\beta i^2 q^2\left[(\beta-1)\tau+1\right]^2 + \left\{i(q-1)q(\beta\tau-1)\left(r\left(\beta^2\tau-\beta(\tau-3)-1\right)-2((\beta-1)\beta\tau+\beta)\right)\right\}x^2$$
$$+\beta x^4(q-1)^2(r-1)(\beta\tau-1)^2 = 0.$$

Because m^* decreases with d and m_1 is independent of d, there exists a unique threshold d_m^*, such that $m^* > m_1$ if and only if $d < d_m^*$. Therefore, if $d < d_m^*$ and $r > r_m^*$, it must hold that $\frac{\partial W_m(m^*)}{\partial \tau} > 0$ and thus $\frac{dm^*}{d\tau} > 0$. Otherwise, if $d > d_m^*$ or $r < r_m^*$, we can easily show $\frac{\partial W_m(m^*)}{\partial \tau} < 0$ always holds so that $\frac{dm^*}{d\tau} < 0$.

Taken together, we have shown that m^* increases with τ if and only if $\frac{D}{k} < d_m^*$ and $\frac{R}{\gamma} > r_m^*$. ■

Proof. of Proposition OA2: The proof proceeds as follows. In step 1, we derive the equilibrium and how audit quality varies with τ. In step 2, we examine investment efficiency in this extension, which sheds light on the incremental role of audit quality in enhancing investment efficiency.

Step 1) If the auditor can make a type I error, the equilibrium consists of a triplet

$\{a_t^*, e_{t1}^*, e_{t0}^*\}$ such that

$$\begin{aligned} a_t^* &= \frac{(1-p)\,d}{2}\left[\beta\tau e_{t1}^* + (1-\beta\tau)\,e_{t0}^*\right] \\ e_{t1}^* &= \min\left\{1, r\frac{(1-p)\,(1-a_t^*)\,\beta}{(1-p)\,(1-a_t^*)\,\beta + p\,(1+a_t^*)\,(1-\beta)}\right\}; \\ e_{t0}^* &= r\frac{(1-p)\,(1-a_t^*)\,(1-\beta\tau)}{(1-p)\,(1-a_t^*)\,(1-\beta\tau) + p\,(1+a_t^*)\left[1-(1-\beta)\,\tau\right]}. \end{aligned}$$

Similar to the proof of Proposition 2, we next show a_t^* decreases with τ if and only if $d < d_t^*$ and $r > r_t^*$. We express the equilibrium e_{t1}^* and e_{t0}^* as functions of a_t^* and τ, denoted by $e_{t1}(a_t^*, \tau)$ and $e_{t0}(a_t^*, \tau)$, respectively. Substituting them into Equation (29) yields the following:

$$V_t(a_t^*) \equiv a_t^* - (1-p)\,d\left[\beta\tau e_{t1}(a_t^*, \tau) + (1-\beta\tau)\,e_{t0}(a_t^*, \tau)\right].$$

We can easily show that

$$\frac{\partial V_t(a_t^*)}{\partial a_t^*} = 1-(1-p)\,d\left[\beta\tau\frac{\partial e_{t1}(a_t^*, \tau)}{\partial a_t^*} + (1-\beta\tau)\frac{\partial e_{t0}(a_t^*, \tau)}{\partial a_t^*}\right] > 0.$$

The implicit function theorem implies $\frac{da_t^*}{d\tau} \propto -\frac{\partial V_t(a_t^*)}{\partial \tau}$. When $d < d_t^*$ and $r > r_t^*$, it is easy to show that $\frac{\partial V_t(a_t^*)}{\partial \tau}$ is decreasing in a_t^* and

$$\lim_{a_t^* \to 0} \frac{\partial V_t(a_t^*)}{\partial \tau} > 0 \Leftrightarrow r > r_t^* = \frac{\beta\left[\tau(\beta - 2\beta p + p) - 1\right]^2}{(p-1)(\beta\tau - 1)\left[\beta - \beta^2\tau + (2\beta - 1)p(\beta\tau + 1)\right]}.$$

In other words, if $r < r_t^*$, it must hold that $\frac{\partial V_t(a_t^*)}{\partial \tau} < 0$ and thus $\frac{da_t^*}{d\tau} > 0$. Otherwise, if $r > r_t^*$, we can show that there exists a threshold a_{2t} such that $\frac{\partial V_t(a_t^*)}{\partial a_t^*} > 0$ if and only if $a < a_{2t}$, where a_{2t} is independent of d. Because a_t^* increases with d, there exists a threshold d_t^* such that $a_t^* < a_t$ is equivalent to $d < d_t^*$. When $d > d_t^*$ or $r < r_t^*$, we can easily show that $\frac{\partial V_t(a_t^*)}{\partial \tau} < 0$ always holds.

Taken together, we have shown a_t^* decreases with τ and only if $d < d_t^*$ and $r > r_t^*$.

Step 2) Investment efficiency can be written as

$$IE_t = p\left(\frac{1+a_t^*}{2}\right)(X-I)-(1-p)\left(\frac{1-a_t^*}{2}\right)\left\{I-\Pr\left(r_b|g,B\right)R\right\}.$$

From step 1, we know that the equilibrium audit quality satisfies

$$a_t^* = \frac{(1-p)\,d}{2}\Pr\left(r_b|B,g\right) \Rightarrow \Pr\left(r_b|B,g\right) = \frac{2a_t^*}{(1-p)\,d}.$$

Substituting into the above equation for investment efficiency yields

$$IE_t = p\left(\frac{1+a_t^*}{2}\right)(X-I)-(1-p)\left(\frac{1-a_t^*}{2}\right)\left[I-\frac{2a_t^*}{(1-p)\,d}R\right],$$

which increases with a_t^*. ■

Proof. of Proposition OA3: The enforcer's posterior beliefs are:

$$\begin{aligned}
\Pr\left(B|g,W_1\right) &= \frac{(1-p)(1-\hat{a})\left[\lambda\beta+(1-\lambda)\frac{y}{2}\right]}{(1-p)(1-\hat{a})\left[\lambda\beta+(1-\lambda)\frac{y}{2}\right]+p\left[\lambda(1-\beta)+(1-\lambda)\frac{y}{2}\right]};\\
\Pr\left(B|g,W_0\right) &= \frac{(1-p)(1-\hat{a})\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]}{(1-p)(1-\hat{a})\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]+p\left[\lambda\beta+(1-\lambda)\left(1-\frac{y}{2}\right)\right]}.
\end{aligned}$$

The auditor's first-order condition is

$$a^* = (1-p)\,d\left[\left(\lambda\beta+(1-\lambda)\frac{y}{2}\right)\hat{e}_1+\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]\hat{e}_0\right].$$

Therefore, the equilibrium is jointly determined by the following system of equations

$$\begin{aligned}
a^* &= (1-p)\,d\left[\left(\lambda\beta+(1-\lambda)\frac{y}{2}\right)e_1^*+\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]e_0^*\right];\\
e_1^* &= \min\left\{1, r\frac{(1-p)(1-a^*)\left[\lambda\beta+(1-\lambda)\frac{y}{2}\right]}{(1-p)(1-a^*)\left[\lambda\beta+(1-\lambda)\frac{y}{2}\right]+p\left[\lambda(1-\beta)+(1-\lambda)\frac{y}{2}\right]}\right\};\\
e_0^* &= r\frac{(1-p)(1-a^*)\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]}{(1-p)(1-a^*)\left[\lambda(1-\beta)+(1-\lambda)\left(1-\frac{y}{2}\right)\right]+p\left[\lambda\beta+(1-\lambda)\left(1-\frac{y}{2}\right)\right]}.
\end{aligned}$$

To examine how y affects the equilibrium audit quality a^*, we substitute e_1^* and e_0^* into the auditor's decision, yielding an equation $Q(a)$. It can be easily verified that $Q(a)$ is strictly increasing in a, with $Q(a=0) < 0$ and $Q(a=1) > 0$. Therefore, there exists a unique solution such that $Q(a^*) = 0$. By the implicit function theorem, we have

$$\frac{\partial a^*}{\partial y} = -\frac{\frac{\partial Q(a)}{\partial y}}{\frac{\partial Q(a)}{\partial a}}|_{a=a^*} \propto -\frac{\partial Q(a)}{\partial y}|_{a=a^*}.$$

For convenience, we define $\frac{\partial Q(a)}{\partial y}$ as Q_y. After some tedious algebra, we find a sufficient condition for $Q_y > 0$ is

$$\lambda < \lambda_y = \frac{1-y}{2\beta - y}.$$

In this case, we have shown $\frac{\partial a^*}{\partial y} < 0$. Instead, if $\lambda > \lambda_y$, we show that

$$\begin{aligned}
\lim_{p\to 0} Q_y &= \lim_{p\to 0} \frac{\partial Q_y}{\partial p} = 0; \\
\lim_{p\to 0} \frac{\partial^2 Q_y}{\partial p^2} &= -\frac{16d(\lambda-1)r(\lambda-2\beta\lambda)^2(-2\beta\lambda+(\lambda-1)y+1)}{(a-1)^2((\lambda-1)y-2\beta\lambda)^2(-2\beta\lambda+(\lambda-1)y+2)^2} < 0.
\end{aligned}$$

By continuity, when p is sufficiently small, $\frac{\partial Q_y}{\partial p}$ is decreasing in p, suggesting $\frac{\partial Q_y}{\partial p} < \lim_{p\to 0} \frac{\partial Q_y}{\partial p} = 0$. Because Q_y is decreasing in p, it must hold true that $Q_y < \lim_{p\to 0} Q_y = 0$, suggesting that $Q_y < 0$ when p is sufficiently small. Taken together, we have shown that $\frac{\partial a^*}{\partial y} > 0$ if $\lambda > \lambda_y$ and p is sufficiently small. ■

Proof. of Proposition OA4: The auditor's beliefs conditional on whistleblowing and no whistleblowing are

$$\begin{aligned}
\Pr(B|W_1) &= \frac{(1-p)\beta}{(1-p)\beta + p(1-\beta)}; \\
\Pr(B|W_0) &= \frac{(1-p)(1-\beta\tau)}{(1-p)(1-\beta\tau) + p[1-(1-\beta)\tau]}.
\end{aligned}$$

Denote the auditor's decision by a_1 and a_0, respectively. After observing a clean audit opinion, the enforcer's beliefs are

$$\begin{aligned}
\Pr\left(B|W_1,g\right) &= \frac{(1-p)\left(1-\hat{a}_1\right)\beta}{(1-p)\left(1-\hat{a}_1\right)\beta+p\left(1-\beta\right)}; \\
\Pr\left(B|W_0,g\right) &= \frac{(1-p)\left(1-\hat{a}_0\right)\left(1-\beta\tau\right)}{(1-p)\left(1-\hat{a}_0\right)\left(1-\beta\tau\right)+p\left[1-\left(1-\beta\right)\tau\right]}.
\end{aligned}$$

Therefore, the equilibrium consists of $\{a_1^*, a_0^*, e_1^*, e_0^*\}$ satisfying the following system of equations:

$$a_1^* = d\frac{(1-p)\beta}{(1-p)\beta+p(1-\beta)}e_1^* \tag{A.2}$$

$$a_0^* = d\frac{(1-p)(1-\beta\tau)}{(1-p)(1-\beta\tau)+p\left[1-(1-\beta)\tau\right]}e_0^* \tag{A.3}$$

$$e_1^* = \min\left\{r\frac{(1-p)\left(1-a_1^*\right)\beta}{(1-p)\left(1-a_1^*\right)\beta+p(1-\beta)},1\right\} \tag{A.4}$$

$$e_0^* = r\frac{(1-p)\left(1-a_0^*\right)(1-\beta\tau)}{(1-p)\left(1-a_0^*\right)(1-\beta\tau)+p\left[1-(1-\beta)\tau\right]}. \tag{A.5}$$

Substituting ($A.5$) into ($A.3$) leads to

$$a_0^* - dr\frac{(1-p)(1-\beta\tau)}{(1-p)(1-\beta\tau)+p\left[1-(1-\beta)\tau\right]}\frac{(1-p)\left(1-a_0^*\right)(1-\beta\tau)}{(1-p)\left(1-a_0^*\right)(1-\beta\tau)+p\left[1-(1-\beta)\tau\right]}=0. \tag{A.6}$$

Because the above equation is increasing in τ and a_0^*, the implicit function theorem suggests $\frac{da_0^*}{d\tau}<0$. Similarly, substituting ($A.3$) into ($A.5$) and applying the implicit function theorem yields $\frac{de_0^*}{d\tau}<0$.

Next, we compare a_0^* with a_1^*. Because $\frac{da_0^*}{d\tau}<0$, the maximum of a_0^* is reached when $\tau\to 0$, which satisfies the following equation:

$$a_0^* - dr\frac{(1-p)^2\left(1-a_0^*\right)}{(1-p)\left(1-a_0^*\right)+p}=0.$$

In addition, a_1^* solves the following equation:

$$a_1^* - d\frac{(1-p)\beta}{(1-p)\beta + p(1-\beta)} \min\left\{ r\frac{(1-p)(1-a_1^*)\beta}{(1-p)(1-a_1^*)\beta + p(1-\beta)}, 1 \right\} = 0. \tag{A.7}$$

If $e_1^* = 1$, because $\frac{(1-p)\beta}{(1-p)\beta+p(1-\beta)} > (1-p)$, we have

$$a_1^* = d\frac{(1-p)\beta}{(1-p)\beta + p(1-\beta)} > d(1-p)\frac{(1-p)(1-a_0^*)}{(1-p)(1-a_0^*) + p} = a_0^*.$$

If $e_1^* < 1$, the implicit function theorem implies $\frac{da_1^*}{d\beta} > 0$. When $\beta \to \frac{1}{2}$, a_1^* solves the following equation:

$$a_1^* - dr\frac{(1-p)^2(1-a_1^*)}{(1-p)(1-a_1^*) + p} = 0 \Rightarrow a_1^* > \lim_{\beta \to \frac{1}{2}} a_1^* = a_0^*.$$

Therefore, we have shown that $a_1^* > a_0^*$ always holds.

Lastly, in a competitive audit market, audit fees solve

$$\begin{aligned} F_1 &= \Pr(B|W_1)(1-\hat{a}_1)\hat{e}_1 D + \frac{1}{2}k\hat{a}_1^2; \\ F_0 &= \Pr(B|W_0)(1-\hat{a}_0)\hat{e}_0 D + \frac{1}{2}k\hat{a}_0^2. \end{aligned}$$

By rational expectations, the conjectures must be consistent. Substituting $\Pr(B|W_1)\, e_1^* D = ka_1^*$ into F_1 yields

$$F_1 = \Pr(B|W_1)(1-a_1^*)e_1^* D + \frac{1}{2}k(a_1^*)^2 = ka_1^* - \frac{1}{2}k(a_1^*)^2.$$

Similarly, when there is no whistleblowing allegation, the audit fees are

$$F_0 = ka_0^* - \frac{1}{2}k(a_0^*)^2.$$

Therefore, we have

$$
\begin{aligned}
F_1 - F_0 &= k\left(a_1^* - a_0^*\right) - \frac{1}{2}k\left(a_1^* - a_0^*\right)\left(a_1^* + a_0^*\right) \\
&= \frac{1}{2}k\left(a_1^* - a_0^*\right)\left[2 - \left(a_1^* + a_0^*\right)\right] > 0.
\end{aligned}
$$

■

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