Online Supplement for "Index-Based Yield Protection for Smallholder Farmers"

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Appendix A Proofs of Analytical Results

Appendix A.1 Proofs of Analytical Results in §3 and §4

Proof of Lemma 1. According to the formulation of farmers' utility $u(i|x)$ defined in §3, for any given planting amount x, a higher production cost h_i leads to a lower utility. Therefore, in equilibrium, it must be farmers with lower h_i who choose to plant, and those with higher h_i who choose to not plant. Then, if $x = 0$ is an equilibrium planting amount, we must have $u(0|0) \leq 0$ (i.e., the utility of the lowest cost farmer is non-positive when no one plants); if $x = 1$ is an equilibrium planting amount, we must have $u(1|1) \geq 0$ (i.e., the utility of the highest cost farmer is non-negative when everyone plants); if $x \in (0,1)$ is an equilibrium planting amount, we must have $u(x|x) = 0$ (i.e., the farmer whose planting cost is xc is indifferent between planting and not planting). We derive the equilibrium planting amount and its closed-form expression in the following two steps:

Step 1: We show that there is a unique positive solution to $u(x|x) = 0$, which lies in $(0,1)$. To do so, it is sufficient to prove the following three parts: (i) $u(x|x)$ strictly decreases in x for $x \geq 0$, (ii) $u(0|0) > 0$, and (iii) $u(1|1) < 0$.

First, for part (i), recall that the yield of any farmer i is $Y_i = Y + \epsilon_i$, where ϵ_i has mean zero and is independent of Y, and Y takes values $\mu + \sigma$ and $\mu - \sigma$, each with probability $\frac{1}{2}$. Besides, recall that $R_H(x) = [a - bx(\mu + \sigma)](\mu + \sigma)$ (expected revenue from selling the crop when $Y = \mu + \sigma$) and $R_L(x) = [a - bx(\mu - \sigma)](\mu - \sigma)$ (expected revenue from selling the crop when $Y = \mu - \sigma$). Then,

$$\mathbb{E}[\pi(x|x)] = \frac{1}{2}R_H(x) + \frac{1}{2}R_L(x) - xc = a\mu - bx(\mu^2 + \sigma^2) - xc$$

$$\begin{aligned}\mathbb{V}\text{ar}[\pi(x|x)] &= \frac{1}{4}(R_H(x) - R_L(x))^2 + \mathbb{V}\text{ar}[(a - bxY)\epsilon_i] \\ &= (a\sigma - 2bx\mu\sigma)^2 + \left(a^2 - 2abx\mu + b^2x^2(\mu^2 + \sigma^2)\right)\sigma_\epsilon^2. \end{aligned} \tag{A.1}$$

Plugging these into the expression of $u(x|x)$ introduced in §3, we have

$$\begin{aligned}u(x|x) &= a\mu - bx(\mu^2 + \sigma^2) - xc - \lambda\left[(a\sigma - 2bx\mu\sigma)^2 + \left(a^2 - 2abx\mu + b^2x^2(\mu^2 + \sigma^2)\right)\sigma_\epsilon^2\right] \\ &= -\alpha_0 x^2 - \beta_0 x - \gamma_0, \end{aligned} \tag{A.2}$$

where $\alpha_0 = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2 + \sigma^2)\sigma_\epsilon^2$, $\beta_0 = b(\mu^2 + \sigma^2) + c - 4\lambda ab\mu\sigma^2 - 2\lambda ab\mu\sigma_\epsilon^2$, and $\gamma_0 = \lambda a^2(\sigma^2 + \sigma_\epsilon^2) - a\mu$.

Clearly, we have $\alpha_0 > 0$. Thus, from Equation (A.2), $u(x|x)$ is a quadratic concave function in x. Moreover, since $c > c_1 \geq 4\lambda ab\sigma^2\mu + 2\lambda ab\mu\sigma_\epsilon^2 - b(\mu^2+\sigma^2)$ as discussed in §3, we have $\beta_0 > 0$. Hence, $u(x|x)$ strictly decreases in x for $x \geq 0$.

Second, for part (ii), since $\lambda < \frac{\mu}{a(\sigma^2+\sigma_\epsilon^2)}$ as discussed in §3, we have $u(0|0) = -\gamma_0 > 0$.

Finally, for part (iii), since $c > c_1 \geq a\mu - b(\mu^2+\sigma^2) - \lambda((a\sigma - 2b\mu\sigma)^2 + (a^2 - 2ab\mu + b^2(\mu^2+\sigma^2))\sigma_\epsilon^2)$, we have $u(1|1) < 0$.

Collectively, we conclude that there must exist a unique positive solution to $u(x|x) = 0$ in $(0,1)$, which we denote as $\hat{x}_0$. From Equation (A.2), we have $\hat{x}_0 = \frac{-\beta_0 + \sqrt{\beta_0^2 - 4\alpha_0\gamma_0}}{2\alpha_0} \in (0,1)$.

Step 2: We show that there is a unique equilibrium planting amount $x_0^* = \hat{x}_0 \in (0,1)$, where $\hat{x}_0$ is the unique positive solution to $u(x|x) = 0$. Since we have shown in Step 1 that $u(0|0) > 0$ and $u(1|1) < 0$, based on the discussion at the beginning of this proof, neither $x = 0$ nor $x = 1$ is an equilibrium planting amount. Moreover, we have shown in Step 1 that $\hat{x}_0 \in (0,1)$ is the unique positive solution to $u(x|x) = 0$. Therefore, $\hat{x}_0$ is the only possible candidate for an equilibrium amount. It remains to prove that $\hat{x}_0$ is an equilibrium planting amount.

Suppose the planting amount is given by $x = \hat{x}_0$ (i.e., all farmers whose production cost is lower than $\hat{x}_0 c$ plant, while all farmers whose production cost is higher than $\hat{x}_0 c$ do not plant). Then, because $u(\hat{x}_0|\hat{x}_0) = 0$, no farmer with a production cost lower than $\hat{x}_0 c$ would deviate from planting because they have a positive utility from planting; similarly, no farmer with a production cost higher than $\hat{x}_0 c$ would deviate from not planting because they have a negative utility from planting. Hence, $\hat{x}_0$ is an equilibrium planting amount, which completes the proof. $\square$

Proof of Lemma 2. We first prove this lemma by considering $s \in [0, 2a\sigma]$, and at the end of the proof, we show $\bar{s} \leq 2a\sigma$. Following the same logic as in the proof of Lemma 1, given $s \in [0, 2a\sigma]$, if $x = 0$, $x = 1$, or $x \in (0,1)$ is an equilibrium planting amount, we must have $u(0|0,s) \leq 0$, $u(1|1,s) \geq 0$ or $u(x|x,s) = 0$ respectively. We derive the equilibrium planting amount and its closed-form expression in the following two steps:

Step 1: We show that, given $s \in [0, 2a\sigma]$, there is a unique positive solution to $u(x|x,s) = 0$. To do so, it is sufficient to prove the following three parts for any given $s \in [0, 2a\sigma]$: (i) $u(x|x,s)$ strictly decreases in x for $x \geq 0$, (ii) $u(0|0,s) > 0$ and (iii) $u(x|x,s) \leq 0$ for some $x > 0$.

First, for part (i), recall that $R_H(x) = [a - bx(\mu+\sigma)](\mu+\sigma)$ (expected revenue from selling the crop when $Y = \mu+\sigma$) and $R_L(x) = [a - bx(\mu-\sigma)](\mu-\sigma)$ (expected revenue from selling the crop when $Y = \mu - \sigma$). Then, based on the expression of $\pi(i|x,s)$ in §3, we have

$$\mathbb{E}[\pi(x|x,s)] = \frac{1}{2}R_H(x) + \frac{1}{2}R_L(x) - xc + \frac{1}{2}s = a\mu - bx(\mu^2+\sigma^2) - xc + \frac{1}{2}s$$

$$\mathbb{V}\text{ar}[\pi(x|x,s)] = \mathbb{V}\text{ar}[(a - bxY)Y + sI - xc] + \mathbb{V}\text{ar}[(a - bxY)\epsilon_i]$$

$$
\begin{aligned}
&+2\mathrm{Cov}\left((a-bxY)Y+sI-xc,(a-bxY)\epsilon_i\right)\\
=&\mathbb{V}\mathrm{ar}[(a-bxY)Y]+\mathbb{V}\mathrm{ar}[sI]+2\mathrm{Cov}((a-bxY)Y,sI)+\mathbb{V}\mathrm{ar}[(a-bxY)\epsilon_i]\\
=&\frac{1}{4}(R_H(x)-R_L(x))^2+\frac{1}{4}s^2-s\left(r-\frac{1}{2}\right)(R_H(x)-R_L(x))\\
&+\left(a^2-2abx\mu+b^2x^2(\mu^2+\sigma^2)\right)\sigma_\epsilon^2 \qquad\qquad (A.3)\\
=&(a\sigma-2bx\mu\sigma)^2+\frac{1}{4}s^2-s\left(r-\frac{1}{2}\right)(2a\sigma-4bx\mu\sigma)\\
&+\left(a^2-2abx\mu+b^2x^2(\mu^2+\sigma^2)\right)\sigma_\epsilon^2, \qquad\qquad (A.4)
\end{aligned}
$$

where the second equality holds because ϵ_i is independent of Y and I. Plugging these into the expression of $u(x|x,s)$ in §3, we have

$$
\begin{aligned}
u(x|x,s)=&a\mu-bx(\mu^2+\sigma^2)+\frac{1}{2}s-xc\\
&-\lambda\left[(a\sigma-2bx\mu\sigma)^2+\frac{1}{4}s^2-s\left(r-\frac{1}{2}\right)(2a\sigma-4bx\mu\sigma)+\left(a^2-2abx\mu+b^2x^2(\mu^2+\sigma^2)\right)\sigma_\epsilon^2\right]\\
=&-\alpha x^2-\beta(s)x-\gamma(s), \qquad\qquad (A.5)
\end{aligned}
$$

where $\alpha=4\lambda b^2\mu^2\sigma^2+\lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$, $\beta(s)=b(\mu^2+\sigma^2)+c-4\lambda ab\mu\sigma^2-2\lambda ab\mu\sigma_\epsilon^2+4\lambda b(r-\frac{1}{2})\mu\sigma s$, and $\gamma(s)=\lambda a^2(\sigma^2+\sigma_\epsilon^2)-a\mu-2\lambda a(r-\frac{1}{2})\sigma s-\frac{1}{2}s+\frac{1}{4}\lambda s^2$.

Clearly, we have $\alpha=\alpha_0>0$. Thus, from Equation (A.5), $u(x|x,s)$ is a quadratic concave function in x. Moreover, we have $\beta(s)=\beta_0+4\lambda b(r-\frac{1}{2})\mu\sigma s>0$, where the inequality holds because we consider $r\geq\frac{1}{2}$ and we have shown $\beta_0>0$ in the proof of Lemma 1. Therefore, we must have that $u(x|x,s)$ strictly decreases in x for $x\geq 0$ for any given $s\in[0,2a\sigma]$.

Second, for part (ii), we have $u(0|0,s)=-\gamma(s)$ from Equation (A.5). Thus, for any given $s\in[0,2a\sigma]$, in order to prove that $u(0|0,s)>0$, it is equivalent to proving that $\gamma(s)<0$. Further, since we have that $\gamma(s)$ is a quadratic convex function of s and we also have that $\gamma(0)=\gamma_0<0$ from the proof of Lemma 1, in order to prove that $\gamma(s)<0$ for any given $s\in[0,2a\sigma]$, it remains to prove that $\gamma(2a\sigma)<0$. From the expression of $\gamma(s)$, we have

$$
\begin{aligned}
\gamma(2a\sigma)&=\lambda a^2(\sigma^2+\sigma_\epsilon^2)-a\mu-2\lambda a(r-\frac{1}{2})\sigma(2a\sigma)-\frac{1}{2}(2a\sigma)+\frac{1}{4}\lambda(2a\sigma)^2\\
&\leq 2\lambda a^2\sigma^2+\lambda a^2\sigma_\epsilon^2-a\mu-a\sigma\\
&<0,
\end{aligned}
$$

where the first inequality holds because we consider $r\geq\frac{1}{2}$, and the second inequality holds because we consider $\lambda<\frac{\mu+\sigma}{2a\sigma^2+a\sigma_\epsilon^2}$. Therefore, for any given $s\in[0,2a\sigma]$, we have $u(0|0,s)>0$.

Finally, for part (iii), from Equation (A.5), we have that $u(x|x,s)$ is a quadratic concave function. Thus, there must exist $x>0$ such that $u(x|x,s)\leq 0$.

Collectively, we conclude that given $s \in [0, 2a\sigma]$, there must exist a unique positive solution to $u(x|x,s) = 0$, which we denote as $\hat{x}(s)$. From Equation (A.5), we have

$$\hat{x}(s) = \frac{-\beta(s) + \sqrt{\beta^2(s) - 4\alpha\gamma(s)}}{2\alpha}. \tag{A.6}$$

Step 2: We show that given $s \in [0, 2a\sigma]$, there is a unique equilibrium planting amount $x^*(s) = \min\{\hat{x}(s), 1\} \in (0,1]$, where $\hat{x}(s)$ is the unique positive solution to $u(x|x,s) = 0$. For any given $s \in [0, 2a\sigma]$, we have shown in Step 1 that $u(0|0,s) > 0$, so based on the discussion at the beginning of this proof, $x = 0$ cannot be an equilibrium planting amount. Then, it remains to check whether $x = 1$ or $x = \hat{x}(s)$ is an equilibrium amount.

Consider any $s \in [0, 2a\sigma]$. If $\hat{x}(s) \geq 1$, then, since $u(x|x,s)$ strictly decreases in x, we have $u(1|1,s) \geq 0$. In this case, the equilibrium planting amount is given by $x^*(s) = 1$ because all farmers have a positive utility from planting given that all other farmers also plant, and thus no one would deviate. On the other hand, if $\hat{x}(s) \in (0,1)$, then, following the same logic as in Step 2 of the proof of Lemma 1, we have that the equilibrium planting amount is $x^*(s) = \hat{x}(s)$. Collectively, we have that there is a unique equilibrium planting amount given by $x^*(s) = \min\{\hat{x}(s), 1\} \in (0,1]$.

Next, we prove that $\bar{s} \leq 2a\sigma$. Recall that $\bar{s}$ is defined as the lowest subsidy amount that satisfies $R_L(x^*(s)) + s \geq R_H(x^*(s))$. Thus, for any $s \in [0, \bar{s}]$, we have $R_L(x^*(s)) + s \leq R_H(x^*(s))$, and thus

$$s \leq R_H(x^*(s)) - R_L(x^*(s)) = 2a\sigma - 4b\mu\sigma x^*(s) \leq 2a\sigma,$$

where the last inequality holds because $x^*(s) \geq 0$. Hence, we have $\bar{s} \leq 2a\sigma$.

Finally, we prove that there exists a threshold c_2 such that if $c \geq c_2$, then for any given $s \in [0, \bar{s}]$, the equilibrium planting amount is $x^*(s) = \hat{x}(s) \in (0,1)$. To do so, based on the discussion in Step 2, it is sufficient to prove that there exists a threshold c_2 such that if $c \geq c_2$, then $u(1|1,s) < 0$ for any $s \in [0, \bar{s}]$. From Equation (A.5), we have

$$\begin{aligned} u(1|1,s) =& a\mu - b(\mu^2 + \sigma^2) + \frac{1}{2}s - c \\ &- \lambda\left[(a\sigma - 2b\mu\sigma)^2 + \frac{1}{4}s^2 - s\left(r - \frac{1}{2}\right)(2a\sigma - 4b\mu\sigma) + \left(a^2 - 2ab\mu + b^2(\mu^2 + \sigma^2)\right)\sigma_\epsilon^2\right]. \end{aligned}$$

From its expression, we have that $u(1|1,s)$ linearly decreases in c. Moreover, it is easy to check that for any $s \in [0, \bar{s}]$, $u(1|1,s)$ is bounded above by a positive constant independent of s. Therefore, there must exist a threshold c_2 such that if $c \geq c_2$, then $u(1|1,s) < 0$ for any $s \in [0, \bar{s}]$. $\square$

Before we proceed to prove Proposition 1, we first prove two auxiliary lemmas which serve as important building blocks for proving the rest of our results. Lemma A.1 introduces a connection between the first derivative of the equilibrium planting amount $x^*(s)$ and the first derivative of farmers' utility $u(x|x,s)$ at $x = x^*(s)$, with respect to any parameter in our model. Lemma A.2 characterizes the concavity of the equilibrium planting amount $x^*(s)$ in the subsidy s.

Lemma A.1. *Let η be one of the parameters in the model (η can represent s, r, σ, λ, etc.). For any $s \in [0, \bar{s}]$, $\frac{\partial}{\partial \eta} x^*(s)$ has the same sign as $\frac{\partial}{\partial \eta} u(x|x,s)|_{x=x^*(s)}$.*

Proof of Lemma A.1. Consider a fixed $s \in [0, \bar{s}]$. Since we consider $c \geq c_2$, from Lemma 2, we know that $x^*(s)$ satisfies the equation $u(x^*(s)|x^*(s), s) = 0$. By taking the derivative of both sides of this equation with respect to η, we have

$$\frac{\partial}{\partial x} u(x|x,s)\Big|_{x=x^*(s)} \times \frac{\partial}{\partial \eta} x^*(s) + \frac{\partial}{\partial \eta} u(x|x,s)\Big|_{x=x^*(s)} = 0. \tag{A.7}$$

From Equation (A.5), we know that $u(x|x,s)$ is a quadratic concave function in x. Since $x^*(s)$ is the larger root of $u(x^*(s)|x^*(s), s) = 0$ by Lemma 2, we have $\frac{\partial}{\partial x} u(x|x,s)|_{x=x^*(s)} < 0$. Putting this into Equation (A.7), we conclude that $\frac{\partial}{\partial \eta} u(x|x,s)|_{x=x^*(s)}$ has the same sign as $\frac{\partial}{\partial \eta} x^*(s)$. $\square$

Lemma A.2. *For any $s \in [0, \bar{s}]$, $\frac{\partial^2}{\partial s^2} x^*(s) < 0$. Moreover, $\frac{\partial}{\partial s} x^*(s)|_{s=0} > 0$.*

Proof of Lemma A.2. First, we prove that $\frac{\partial^2}{\partial s^2} x^*(s) < 0$. Since we consider $c \geq c_2$, by Lemma 2, we have $x^*(s) = \hat{x}(s) = \frac{-\beta(s) + \sqrt{\beta^2(s) - 4\alpha\gamma(s)}}{2\alpha}$. Based on the expressions of α, $\beta(s)$ and $\gamma(s)$ in Lemma 2, we have that α is independent of s, $\beta(s)$ is linear in s and $\beta^2(s) - 4\alpha\gamma(s)$ is a quadratic concave function in s as we consider $r \in [\frac{1}{2}, 1]$. Moreover, the second derivative of $\beta^2(s) - 4\alpha\gamma(s)$ implies that it is not the square of an affine function in s. Since the square root function (i.e., $\sqrt{\cdot}$) is a concave and increasing function, $x^*(s)$ is a strictly concave function in s. Thus, $\frac{\partial^2}{\partial s^2} x^*(s) < 0$.

Next, we prove that $\frac{\partial}{\partial s} x^*(s)|_{s=0} > 0$. By Lemma 1, we have $x^*(0) = x_0^* = \hat{x}(0)$. Then, according to Lemma A.1, $\frac{\partial}{\partial s} x^*(s)|_{s=0}$ has the same sign as $\frac{\partial}{\partial s} u(x|x,s)|_{x=x_0^*, s=0}$. We have

$$\frac{\partial}{\partial s} u(x|x,s)\Big|_{x=x_0^*, s=0} = \frac{1}{2} + \lambda \left(r - \frac{1}{2} \right) (R_H(x_0^*) - R_L(x_0^*)).$$

Since we consider $r \geq \frac{1}{2}$ and $R_H(x_0^*) > R_L(x_0^*)$ (i.e., high-yield revenue is not lower than low-yield revenue without any subsidy), we have $\frac{\partial}{\partial s} u(x|x,s)|_{x=x_0^*, s=0} > 0$. Thus, $\frac{\partial}{\partial s} x^*(s)|_{s=0} > 0$. $\square$

Proof of Proposition 1. According to Lemma A.2, $x^*(s)$ is strictly concave in s. Since $s_1 = \arg\max_{s \in [0, \bar{s}]} x^*(s)$, we have that $x^*(s)$ increases in s if and only if $s \leq s_1$. Then, it is sufficient to prove that there exists a threshold r_1 such that if $r < r_1$, we have $s_1 < \bar{s}$ and s_1 increases in r; if $r \geq r_1$, we have $s_1 = \bar{s}$. We prove these results in the following three steps:

Step 1: We prove that for any given $r \in [\frac{1}{2}, 1]$, if $s_1 < \bar{s}$, then s_1 strictly increases in r. According to Lemma A.2, $x^*(s)$ is strictly concave in s. Thus, given $s_1 < \bar{s}$, we must have $\frac{\partial}{\partial s} x^*(s)|_{s=s_1} = 0$. Then, to prove that s_1 increases in r, it is sufficient to prove that $\frac{\partial^2}{\partial s \partial r} x^*(s)|_{s=s_1} > 0$.

By taking the derivative of both sides of Equation (A.7) with respect to r and substituting the subsidy amount s for η, we have

$$\left(\frac{\partial^2}{\partial x^2} u(x|x,s)\Big|_{x=x^*(s_1), s=s_1} \frac{\partial}{\partial r} x^*(s)\Big|_{s=s_1} + \frac{\partial^2}{\partial x \partial r} u(x|x,s)\Big|_{x=x^*(s_1), s=s_1} \right) \frac{\partial}{\partial s} x^*(s)\Big|_{s=s_1}$$

$$+\frac{\partial}{\partial x}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\frac{\partial^2}{\partial s\partial r}x^*(s)\Big|_{s=s_1}+\frac{\partial^2}{\partial x\partial s}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\frac{\partial}{\partial r}x^*(s)\Big|_{s=s_1}$$
$$+\frac{\partial^2}{\partial s\partial r}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}=0.$$

Because $\frac{\partial}{\partial s}x^*(s)|_{s=s_1}=0$, the first term vanishes and the above equation is reduced to

$$\frac{\partial}{\partial x}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\frac{\partial^2}{\partial s\partial r}x^*(s)\Big|_{s=s_1}+\frac{\partial^2}{\partial x\partial s}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\frac{\partial}{\partial r}x^*(s)\Big|_{s=s_1}$$
$$+\frac{\partial^2}{\partial s\partial r}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}=0. \tag{A.8}$$

In addition, as discussed in the proof of Lemma A.1, we have $\frac{\partial}{\partial x}u(x|x,s)|_{x=x^*(s)}<0$. Therefore, in order to prove that $\frac{\partial^2}{\partial s\partial r}x^*(s)|_{s=s_1}>0$, it is sufficient to prove that $\frac{\partial^2}{\partial x\partial s}u(x|x,s)|_{x=x^*(s_1),s=s_1}\frac{\partial}{\partial r}x^*(s)|_{s=s_1}+\frac{\partial^2}{\partial s\partial r}u(x|x,s)|_{x=x^*(s_1),s=s_1}>0$.

From Equation (A.5), we can get $\frac{\partial^2}{\partial x\partial s}u(x|x,s)=-4\lambda b(r-\frac{1}{2})\sigma\mu$ and $\frac{\partial^2}{\partial s\partial r}u(x|x,s)=\lambda(R_H(x)-R_L(x))$. From Equation (A.7), we have

$$\frac{\partial}{\partial r}x^*(s)=-\frac{\partial u(x|x,s)/\partial r}{\partial u(x|x,s)/\partial x}\Big|_{x=x^*(s)}=\frac{\lambda s[R_H(x^*(s))-R_L(x^*(s))]}{2\alpha x^*(s)+\beta(s)}=\frac{\lambda s[R_H(x^*(s))-R_L(x^*(s))]}{\sqrt{\beta^2(s)-4\alpha\gamma(s)}},$$

where the third equality follows from substituting the expression in Equation (A.6) for $x^*(s)$. Then,

$$\frac{\partial^2}{\partial x\partial s}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\frac{\partial}{\partial r}x^*(s)\Big|_{s=s_1}+\frac{\partial^2}{\partial s\partial r}u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}$$
$$=\lambda[R_H(x^*(s_1))-R_L(x^*(s_1))]\left[1-\frac{4\lambda b(r-\frac{1}{2})\sigma\mu s_1}{\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)}}\right]. \tag{A.9}$$

Since we consider $s\in[0,\bar{s}]$ and $\bar{s}$ is the lowest subsidy amount that satisfies $R_L(x^*(s))+s\geq R_H(x^*(s))$, we have $R_H(x^*(s))-R_L(x^*(s))\geq s\geq 0$ for all $s\in[0,\bar{s}]$. Then, the right hand side of Equation (A.9) is positive if and only if $\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)}>4\lambda b(r-\frac{1}{2})\sigma\mu s_1$.

By the definition of s_1, we have

$$\frac{\partial}{\partial s}x^*(s)\Big|_{s=s_1}=\frac{1}{2\alpha}\left(-4\lambda b(r-\frac{1}{2})\sigma\mu+\frac{\partial(\beta^2(s)-4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s)-4\alpha\gamma(s)}}\Big|_{s=s_1}\right)=0,$$

and thus

$$4\lambda b(r-\frac{1}{2})\sigma\mu=\frac{\partial(\beta^2(s)-4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s)-4\alpha\gamma(s)}}\Big|_{s=s_1}.$$

Multiplying both sides of the equation above by s_1, we have

$$4\lambda b(r-\frac{1}{2})\sigma\mu s_1=\frac{\partial(\beta^2(s)-4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s)-4\alpha\gamma(s)}}\Big|_{s=s_1}\times s_1. \tag{A.10}$$

Further, as shown in the proof of Lemma A.2, $\beta^2(s)-4\alpha\gamma(s)$ is concave in s and increasing in s at $s=0$, which implies that

$$[\beta^2(s_1)-4\alpha\gamma(s_1)]-[\beta^2(0)-4\alpha\gamma(0)]\geq\frac{\partial}{\partial s}\left(\beta^2(s)-4\alpha\gamma(s)\right)\Big|_{s=s_1}\times s_1.$$

We also have $\beta^2(0)-4\alpha\gamma(0)=\beta_0^2-4\alpha_0\gamma_0>0$ as shown in the proof of Lemma 1. Therefore,

$$\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)} > \frac{\partial(\beta^2(s)-4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s)-4\alpha\gamma(s)}}\Big|_{s=s_1}\times s_1. \tag{A.11}$$

By combining (A.10) and (A.11), we have $\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)} > 4\lambda b(r-\frac{1}{2})\mu\sigma s_1$, and thus $\frac{\partial^2}{\partial s\partial r}x^*(s)|_{s=s_1}>0$, which implies that s_1 strictly increases in r.

Step 2: We prove that if $s_1=\bar{s}$ under some $r\in[\frac{1}{2},1]$, then we have $s_1=\bar{s}$ under any higher index accuracy. Let $r_1\in[\frac{1}{2},1]$ denote the lowest index accuracy under which $s_1=\bar{s}$ (we prove the existence of such r_1 in Step 3). Then, it is sufficient to prove that we have $s_1=\bar{s}$ for any $r\geq r_1$.

Recall from Lemma 2 that $x^*(s)=\hat{x}(s)=\frac{-\beta(s)+\sqrt{\beta^2(s)-4\alpha\gamma(s)}}{2\alpha}$. As shown in the proof of Lemma A.2, we have that $\beta(s)$ is linearly increasing in s, $\beta^2(s)-4\alpha\gamma(s)$ is a quadratic concave function in s, and $\hat{x}(s)$ is increasing in s when $s=0$. Therefore, there must exist a threshold, denoted as $\hat{s}$, such that $\frac{\partial}{\partial s}\hat{x}(s)|_{s=\hat{s}}=0$ and $\hat{x}(s)$ increases in s if and only if $s\leq\hat{s}$. By the definition of r_1, when $r<r_1$, we must have $s_1<\bar{s}$. Therefore, we have $s_1=\hat{s}<\bar{s}$ when $r<r_1$.

Then, to prove the conclusion of this step, it is sufficient to prove that if $r\geq r_1$, then $\hat{s}\geq\bar{s}$. Therefore, it is sufficient to prove that for any given r such that $\hat{s}=\bar{s}$, we have $\frac{\partial}{\partial r}\hat{s}>0$ and $\frac{\partial}{\partial r}\bar{s}\leq 0$.

Following the analysis in Step 1, we have that, whenever $\hat{s}\leq\bar{s}$, $\hat{s}$ strictly increases in r. Thus, it remains to prove $\frac{\partial}{\partial r}\bar{s}\leq 0$ when $\hat{s}=\bar{s}$. Since $x^*(s)$ is continuous in s, $R_H(x^*(s))-R_L(x^*(s))$ must also be continuous in s. Moreover, when $s=0$, we assumed that $R_H(x^*(s))-R_L(x^*(s))>s$ (i.e., $R_H(x^*(0))-R_L(x^*(0))>0$); when $s=2a\sigma$, we have shown in the proof of Lemma 2 that $R_H(x^*(s))-R_L(x^*(s))\leq s$ (i.e., $R_H(x^*(2a\sigma))-R_L(x^*(2a\sigma))\leq 2a\sigma$). Recall that $\bar{s}$ is the lowest subsidy amount that satisfies $R_L(x^*(s))+s\geq R_H(x^*(s))$. Thus, we have

$$\bar{s}=R_H(x^*(\bar{s}))-R_L(x^*(\bar{s}))=2a\sigma-4b\mu\sigma x^*(\bar{s}). \tag{A.12}$$

Then, we take the derivative of both sides of Equation (A.12) with respect to r, and have

$$\frac{\partial}{\partial r}\bar{s}=-4b\mu\sigma\left(\frac{\partial}{\partial r}x^*(s)\Big|_{s=\bar{s}}+\frac{\partial}{\partial s}x^*(s)\Big|_{s=\bar{s}}\frac{\partial}{\partial r}\bar{s}\right). \tag{A.13}$$

When $\hat{s}=\bar{s}$, we have $\frac{\partial}{\partial s}x^*(s)|_{s=\bar{s}}=0$. Thus, in order to prove that $\frac{\partial}{\partial r}\bar{s}\leq 0$ when $\hat{s}=\bar{s}$, it remains to prove that $\frac{\partial}{\partial r}x^*(s)|_{s=\bar{s}}\geq 0$.

For any $s\in[0,\bar{s}]$, by Lemma A.1, $\frac{\partial}{\partial r}x^*(s)$ and $\frac{\partial}{\partial r}u(x|x,s)|_{x=x^*(s)}$ have the same sign. We have

$$\frac{\partial}{\partial r}u(x|x,s)\Big|_{x=x^*(s)}=\lambda s[R_H(x^*(s))-R_L(x^*(s))].$$

Since we consider $s\in[0,\bar{s}]$, we have $R_H(x^*(s))-R_L(x^*(s))\geq s\geq 0$. Thus, $\frac{\partial}{\partial r}u(x|x,s)|_{x=x^*(s)}\geq 0$, which implies

$$\frac{\partial}{\partial r}x^*(s)\geq 0. \tag{A.14}$$

Then, we have $\frac{\partial}{\partial r}x^*(s)|_{s=\bar{s}} \geq 0$. Therefore, by Equation (A.13), $\frac{\partial}{\partial r}\bar{s} \leq 0$ when $\hat{s} = \bar{s}$.

Step 3: We prove the existence of $r_1 \in [\frac{1}{2}, 1]$, where r_1 is defined in Step 2, and we also prove that $r_1 < 1$. We first prove that such $r_1 \in [\frac{1}{2}, 1]$ exists. To do so, it is sufficient to prove that when $r = 1$, we have $s_1 = \bar{s}$. We prove this result by contradiction. Consider $r = 1$. Suppose $s_1 < \bar{s}$. Then, based on the definition of $\bar{s}$, we have $s_1 < R_H(x^*(s_1)) - R_L(x^*(s_1))$. On the other hand, since $s_1 < \bar{s}$, as discussed in Step 1, we have $\frac{\partial}{\partial s}x^*(s)|_{s=s_1} = 0$. Then, according to Lemma A.1, we have $\frac{\partial}{\partial s}u(x|x,s)|_{x=x^*(s_1),s=s_1} = 0$, which is equivalent to

$$s_1 = \frac{1}{\lambda} + 2(r - \frac{1}{2})[R_H(x^*(s_1)) - R_L(x^*(s_1))]. \tag{A.15}$$

Plugging $r = 1$ into the above inequality, we have $s_1 = \frac{1}{\lambda} + R_H(x^*(s_1)) - R_L(x^*(s_1)) > R_H(x^*(s_1)) - R_L(x^*(s_1))$, which leads to a contradiction. Thus, we have $s_1 = \bar{s}$ when $r = 1$.

Next, we prove that $r_1 < 1$ by contradiction. Suppose $r_1 = 1$. Then, as discussed in Step 2, we have $s_1 = \hat{s} = \bar{s}$ when $r = r_1 = 1$. By the definition of $\hat{s}$ in Step 2, we have $\frac{\partial}{\partial s}x^*(s)|_{s=s_1} = 0$. Then, as shown above, we have $s_1 > R_H(x^*(s_1)) - R_L(x^*(s_1))$, which contradicts $s_1 = \bar{s}$.

Collectively, putting together all three steps, we have that, there exists a threshold $r_1 \in [\frac{1}{2}, 1)$ such that when $r < r_1$, we have $s_1 < \bar{s}$ and s_1 increases in r; when $r \geq r_1$, we have $s_1 = \bar{s}$. □

Proof of Proposition 2. To prove this proposition, we first prove that the variance of farmer income is strictly convex in s. From Equation (A.3), the variance of farmer income is independent of h_i and thus is the same for all farmers who plant. Thus, we only need to consider the farmer with production cost $x^*(s)c$ and show that $\mathbb{V}\text{ar}[\pi(x^*(s)|x^*(s), s)]$ is strictly convex in s.

According to Lemma 2, when $c \geq c_2$, we have $u(x^*(s)|x^*(s), s) = 0$. By taking the second derivative of both sides of $u(x^*(s)|x^*(s), s) = 0$ with respect to the subsidy amount s, we have

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathbb{E}[\pi(x^*(s)|x^*(s), s)] - \lambda\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathbb{V}\text{ar}[\pi(x^*(s)|x^*(s), s)] = 0,$$

which implies that $\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathbb{V}\text{ar}[\pi(x^*(s)|x^*(s), s)]$ and $\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathbb{E}[\pi(x^*(s)|x^*(s), s)]$ have the same sign. We also have

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathbb{E}[\pi(x^*(s)|x^*(s), s)] = -[b(\mu^2 + \sigma^2) + c]\frac{\partial^2}{\partial s^2}x^*(s) > 0,$$

where the inequality holds because, as shown in Lemma A.2, $x^*(s)$ is strictly concave in s. Therefore, the variance of farmer income $\mathbb{V}\text{ar}[\pi(i|x^*(s), s)]$ is strictly convex in s.

Then, there must exist a unique threshold $s_2 \in [0, \bar{s}]$ that minimizes $\mathbb{V}\text{ar}[\pi(i|x^*(s), s)]$ among all $s \in [0, \bar{s}]$. To prove the proposition, it is sufficient to show that (i) $\mathbb{V}\text{ar}[\pi(i|x^*(s), s)]$ decreases in s at $s = 0$; and (ii) $s_2 \leq s_1$. From Equation (A.3), we have

$$\frac{\mathrm{d}}{\mathrm{d}s}\mathbb{V}\text{ar}[\pi(i|x^*(s), s)] = \frac{1}{2}[R_H(x^*(s)) - R_L(x^*(s))]\frac{\partial}{\partial s}[R_H(x^*(s)) - R_L(x^*(s))] + \frac{1}{2}s$$

$$
\begin{aligned}
&- \left(r - \frac{1}{2}\right)[R_H(x^*(s)) - R_L(x^*(s))] - s\left(r - \frac{1}{2}\right)\frac{\partial}{\partial s}[R_H(x^*(s)) - R_L(x^*(s))] \\
&+ 2b\sigma_\epsilon^2(-a\mu + bx^*(s)(\mu^2 + \sigma^2))\frac{\partial}{\partial s}x^*(s) \\
= &- 2b\mu\sigma[R_H(x^*(s)) - R_L(x^*(s))]\frac{\partial}{\partial s}x^*(s) + \frac{1}{2}s \\
&- \left(r - \frac{1}{2}\right)[R_H(x^*(s)) - R_L(x^*(s))] + 4\left(r - \frac{1}{2}\right)b\mu\sigma s\frac{\partial}{\partial s}x^*(s) \\
&+ 2b\sigma_\epsilon^2(-a\mu + bx^*(s)(\mu^2 + \sigma^2))\frac{\partial}{\partial s}x^*(s).
\end{aligned} \tag{A.16}
$$

From Equation (A.16), at $s = 0$, we have

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}s}\mathbb{V}\mathrm{ar}[\pi(i|x^*(s), s)]\Big|_{s=0} = &- 2b\mu\sigma[R_H(x_0^*) - R_L(x_0^*)]\frac{\partial}{\partial s}x^*(s)\Big|_{s=0} - \left(r - \frac{1}{2}\right)[R_H(x_0^*) - R_L(x_0^*)] \\
&+ 2b\sigma_\epsilon^2(-a\mu + bx_0^*(\mu^2 + \sigma^2))\frac{\partial}{\partial s}x^*(s)\Big|_{s=0}.
\end{aligned}
$$

By Lemma A.2, we know $\frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$. Further, since $r \in [\frac{1}{2}, 1]$, $R_H(x_0^*) > R_L(x_0^*)$, $x_0^* \leq 1$, and $a > 2b\mu$, the income variance must strictly decrease in s at $s = 0$.

Next, we prove that $s_2 \leq s_1$. Based on Proposition 1, we know that if $r \geq r_1$, then $s_1 = \bar{s}$. In this case, we clearly have $s_2 \leq s_1$. Therefore, it is sufficient to focus on the case where $r < r_1$. In this case, to prove $s_2 \leq s_1$, it is sufficient to prove that the income variance increases in s at $s = s_1$.

Since $x^*(s)$ is a strictly concave function in s (Lemma A.2) and $s_1 < \bar{s}$, by the definition of s_1, we have $\frac{\partial}{\partial s}x^*(s)|_{s=s_1} = 0$. Plugging it into Equation (A.16), we have

$$
\frac{\mathrm{d}}{\mathrm{d}s}\mathbb{V}\mathrm{ar}[\pi(i|x^*(s), s)]\Big|_{s=s_1} = \frac{1}{2}s_1 - \left(r - \frac{1}{2}\right)[R_H(x^*(s_1)) - R_L(x^*(s_1))].
$$

By Equation (A.15), we have $s_1 = \frac{1}{\lambda} + 2(r - \frac{1}{2})[R_H(x^*(s_1)) - R_L(x^*(s_1))]$. Therefore, $\frac{\mathrm{d}}{\mathrm{d}s}\mathbb{V}\mathrm{ar}[\pi(i|x^*(s), s)]|_{s=s_1} = \frac{1}{2}s_1 - (r - \frac{1}{2})[R_H(x^*(s_1)) - R_L(x^*(s_1))] = \frac{1}{2\lambda} > 0$, which completes the proof. $\square$

We next study how the expected farmer income varies in s in Proposition A.1.

Proposition A.1. *There exists a threshold b_1 such that if $b \leq b_1$, then the expected farmer income $\mathbb{E}[\pi(i|x^*(s), s)]$ increases in s.*

Proof of Proposition A.1. First, we show that the expected farmer income is convex in s. In equilibrium, given a subsidy amount s, the expected income of the farmer i is $\mathbb{E}[\pi(i|x^*(s), s)] = a\mu - bx^*(s) \times (\mu^2 + \sigma^2) + \frac{1}{2}s - h_i$. By taking the second derivative of $\mathbb{E}[\pi(i|x^*(s), s)]$ with respect to s, we have $\frac{\partial^2}{\partial s^2}\mathbb{E}[\pi(i|x^*(s), s)] = -b(\mu^2 + \sigma^2)\frac{\partial^2}{\partial s^2}x^*(s) > 0$, where the inequality holds because we have shown in Lemma A.2 that $\frac{\partial^2}{\partial s^2}x^*(s) < 0$. Thus, the expected farmer income is convex in s.

Then, due to the convexity, in order to study how the expected income changes in s, we focus on how it changes in s at $s = 0$. By taking the first derivative of $\mathbb{E}[\pi(i|x^*(s), s)]$ with respect to s at

$s=0$, we have $\frac{\partial}{\partial s}\mathbb{E}[\pi(i|x^*(s),s)]|_{s=0} = -b(\mu^2+\sigma^2)\frac{\partial}{\partial s}x^*(s)|_{s=0} + \frac{1}{2}$. In Lemma A.2, we have shown that $\frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$. Moreover, for any $b \in (0, \frac{a}{2\mu})$, $\frac{\partial}{\partial s}x^*(s)|_{s=0}$ is bounded above by a positive constant independent of b because (1) $\frac{\partial}{\partial s}x^*(s)|_{s=0}$ is continuous in b; and (2) $\frac{\partial}{\partial s}x^*(s)|_{s=0}$ is bounded by a positive constant when b approaches 0 and when b approaches $\frac{a}{2\mu}$. Therefore, there must exist $b_1 > 0$ such that for any $b \le b_1$, $\frac{\partial}{\partial s}\mathbb{E}[\pi(i|x^*(s),s)]|_{s=0} \ge 0$, which completes the proof. □

Proof of Corollary 1. By Equation (A.14), we have shown that $x^*(s)$ increases in r. It remains to prove that the variance of farmer income $\mathbb{V}\mathrm{ar}[\pi(i|x^*(s),s)]$ decreases in r.

From Equation (A.3), we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}r}\mathbb{V}\mathrm{ar}[\pi(i|x^*(s),s)] =& \frac{1}{2}[R_H(x^*(s)) - R_L(x^*(s))] \times \frac{\partial}{\partial r}[R_H(x^*(s)) - R_L(x^*(s))] \\
& - s[R_H(x^*(s)) - R_L(x^*(s))] - s\left(r-\frac{1}{2}\right) \times \frac{\partial}{\partial r}[R_H(x^*(s)) - R_L(x^*(s))] \\
& + 2b\sigma_\epsilon^2(-a\mu + bx^*(s)(\mu^2+\sigma^2))\frac{\partial}{\partial r}x^*(s) \\
=& \frac{1}{2}[R_H(x^*(s)) - R_L(x^*(s))] \times \left(-4b\mu\sigma\frac{\partial}{\partial r}x^*(s)\right) \\
& - s[R_H(x^*(s)) - R_L(x^*(s))] - s\left(r-\frac{1}{2}\right) \times \left(-4b\mu\sigma\frac{\partial}{\partial r}x^*(s)\right) \\
& + 2b\sigma_\epsilon^2(-a\mu + bx^*(s)(\mu^2+\sigma^2))\frac{\partial}{\partial r}x^*(s) \\
=& -2b\mu\sigma\left(\frac{\partial}{\partial r}x^*(s)\right)\left([R_H(x^*(s)) - R_L(x^*(s))] - 2\left(r-\frac{1}{2}\right)s\right) \\
& - s[R_H(x^*(s)) - R_L(x^*(s))] \\
& + 2b\sigma_\epsilon^2(-a\mu + bx^*(s)(\mu^2+\sigma^2))\frac{\partial}{\partial r}x^*(s) \\
\le& 0,
\end{aligned}$$

where the inequality holds because $r \in [\frac{1}{2}, 1]$ and we have shown in the proof of Proposition 1 that $\frac{\partial}{\partial r}x^*(s) \ge 0$, $R_H(x^*(s)) - R_L(x^*(s)) \ge s \ge 0$, and $-a\mu + bx^*(s)(\mu^2+\sigma^2) \le -a\mu + 2b\mu^2 \le 0$. □

Proof of Theorem 1. Recall that the net benefit $v(s)$ is given by

$$\begin{aligned}
v(s) &= \mathbb{E}\left[\int_0^{x^*(s)} ((a - bx^*(s)Y)Y - xc + sI)\,\mathrm{d}x\right] - x^*(s)\mathbb{E}[sI] \\
&= v_1 \times x^*(s) - v_2 \times (x^*(s))^2. \qquad \text{(A.17)}
\end{aligned}$$

where $v_1 = a\mu$ and $v_2 = b(\mu^2+\sigma^2) + \frac{1}{2}c$. Since $v_1 > 0$ and $v_2 > 0$, $v(s)$ is a quadratic concave function in $x^*(s)$. Then, the value of $x^*(s)$ that maximizes $v(s)$ is defined as follows:

$$x^{opt} = \frac{v_1}{2v_2} = \frac{a\mu}{2b(\mu^2+\sigma^2)+c}. \qquad \text{(A.18)}$$

As the government's objective is to find the subsidy amount s to maximize $v(s)$ under the budget constraint, the optimal s^* must make $x^*(s)$ as close to x^{opt} as possible. Let $\mathcal{S} = [0, \bar{s}] \cap \{s :$

$x^*(s)\mathbb{E}[sI] \le B\}$. Then, $\mathcal{S}$ denotes the feasible set of s. (Note that the set $\mathcal{S}$ depends on the index accuracy r; for notational simplicity, we omit this dependence.) Since we assume $x_0^* \le x^{opt}$ and we have shown that $x^*(s)$ is strictly concave in s, we can uniquely characterize $s^* \in \mathcal{S}$ with one of the following two conditions: (i) $s^* = \arg\max_{s\in\mathcal{S}} x^*(s)$ and $x^*(s^*) < x^{opt}$; (ii) $s^* = \inf\{s \in \mathcal{S} : x^*(s) = x^{opt}\}$.

We define $r_c \ge \frac{1}{2}$ as the smallest index accuracy such that $x^*(s) = x^{opt}$ for some $s \in \mathcal{S}$. (If no such index accuracy exists, then let $r_c = \infty$.) We separate the rest of the proof into two cases:

Case 1: We consider the scenario where $r \ge r_c$. First, we prove that for any $r \ge r_c$, there exists a subsidy amount $s'' \in \mathcal{S}$ such that $x^*(s'') = x^{opt}$. When $r = r_c$, by the definition of r_c, we have $x^*(s) = x^{opt}$ for some $s \in \mathcal{S}$. Let $s' = \inf\{s \in \mathcal{S} : x^*(s) = x^{opt}\}$ when $r = r_c$. Given that $s' \in \mathcal{S}$, we must have $s' \le R_H(x^*(s')) - R_L(x^*(s')) = R_H(x^{opt}) - R_L(x^{opt})$ and $x^*(s')\mathbb{E}[s'I] = \frac{1}{2}x^{opt}s' \le B$. Next, consider any $r \ge r_c$. We must have $x^*(s) = x^{opt}$ for some $s \le s'$, because we consider $x_0^* \le x^{opt}$ and we have that $x^*(s)$ increases in r by Corollary 1. For any given $r \ge r_c$, let $s'' = \inf\{s \le s' : x^*(s) = x^{opt}\}$ (for notational simplicity, we omit the dependence of s'' on r). Since $s'' \le s'$, we have $s'' \le R_H(x^{opt}) - R_L(x^{opt})$ and $x^*(s'')\mathbb{E}[s''I] = \frac{1}{2}x^{opt}s'' \le B$. Therefore, we have $s'' \in \mathcal{S}$, and thus $s^* = s''$.

Second, we prove that $s^* = s''$ decreases in r. Since we consider $x_0^* \le x^{opt}$ and s'' is the lowest subsidy amount that satisfies $x^*(s'') = x^{opt}$, we must have that $x^*(s)$ increases in s at $s = s''$. Moreover, x^{opt} is independent of r and we showed in Corollary 1 that $x^*(s)$ increases in r. Hence, s'' must decrease in r to maintain $x^*(s'') = x^{opt}$. Therefore, when $r \ge r_c$, s^* decreases in r.

Case 2: We consider the scenario where $r < r_c$. By the definition of r_c, when $r < r_c$, the optimal subsidy s^* must satisfy condition (i) above, i.e., $s^* = \arg\max_{s\in\mathcal{S}} x^*(s)$ and $x^*(s^*) < x^{opt}$.

Next, we study how s^* changes in r. Recall from Proposition 1 that if $r < r_1$, then we have $\frac{\partial x^*(s)}{\partial s}|_{s=s_1} = 0$ and s_1 increases in r, where s_1 is the maximizer of $x^*(s)$ among all $s \in [0, \bar{s}]$. We also know from Corollary 1 that $x^*(s)$ increases in r. Then, when $r < r_1$, $x^*(s_1)\mathbb{E}[s_1 I]$ must increase in r. Then, there must exist $r_b \in [\frac{1}{2}, r_1]$ such that for $r \in [\frac{1}{2}, r_b)$, we have $x^*(s_1)\mathbb{E}[s_1 I] < B$, and for $r \in [r_b, r_1)$, we have $x^*(s_1)\mathbb{E}[s_1 I] \ge B$. We next show how s^* changes in r by considering three sub-cases: (a) $r \in [\frac{1}{2}, r_b)$, (b) $r \in [r_b, r_1)$, (c) $r \in [r_1, r_c)$ (this sub-case does not exist if $r_1 > r_c$).

(a): We consider the scenario where $r \in [\frac{1}{2}, r_b)$. In this case, by the definition of r_b, we have $x^*(s_1)\mathbb{E}[s_1 I] < B$. Then, s_1 satisfies condition (i) discussed above, and thus we have $s^* = s_1$. Moreover, we have shown in Proposition 1 that s_1 increases in r. Thus, s^* increases in r.

(b): We consider the scenario where $r \in [r_b, r_1)$. By the definition of r_b, we have $x^*(s_1)\mathbb{E}[s_1 I] \ge B$ for all $r \in [r_b, r_1)$. Therefore, by condition (i) above, s^* must satisfy $x^*(s^*)\mathbb{E}[s^* I] = B$. Since $x^*(s)$ increases in r (Corollary 1), s^* must decrease in r to maintain $x^*(s^*)\mathbb{E}[s^* I] = B$.

(c): We consider the scenario where $r \in [r_1, r_c)$. In this case, $x^*(s)$ is increasing in s for all $s \in \mathcal{S}$. Therefore, by condition (i) above, s^* must be the highest $s \in \mathcal{S}$, which is either $\bar{s}$ or the subsidy amount that satisfies $s < \bar{s}$ and $x^*(s)\mathbb{E}[sI] = B$ (if exists). We next prove s^* decreases in r.

First, when $r \geq r_1$, by Proposition 1, we have $\frac{\partial}{\partial s}x^*(s)|_{s=\bar{s}} \geq 0$, which, by Equation (A.13), implies $\frac{\partial}{\partial r}\bar{s} \leq 0$. Second, since $x^*(s)$ increases in r, if there exist $r \in [r_1, r_c)$ and $s' < \bar{s}$ such that $x^*(s')\mathbb{E}[s'I] = B$, then for any higher index accuracy, there must exist $s'' \leq s'$ that satisfies $x^*(s'')\mathbb{E}[s''I] = B$. Moreover, s'' must decrease in r. Therefore, collectively, we conclude that when $r \in [r_1, r_c)$, s^* decreases in r.

Let $r_2 := \min\{r_b, r_c\}$. Then, combining Cases 1 and 2, s^* increases in r if and only if $r < r_2$. □

Proof of Corollary 2. Since s_A^* is independent of r and we know from Theorem 1 that s^* first increases and then decreases in r, to prove the corollary, it is sufficient to prove that $s^* \geq s_A^*$ when $r = 1$ (i.e., when the index can perfectly predict the area-level yield Y). From the proof of Theorem 1 and Equation (A.12), we know that when $r = 1$, the optimal subsidy s^* under the index-based policy must satisfy either (1) s^* is the lowest subsidy that satisfies $x^*(s^*) = x^{opt}$, or (2) s^* is the highest feasible subsidy, i.e., s^* satisfies $x^*(s^*)\mathbb{E}[s^*I] = B$ or $s^* = \bar{s} = 2a\sigma - 4b\mu\sigma x^*(s^*)$.

In the proof of Lemma 2, we have derived the equilibrium planting amount $x^*(s)$ under the index-based policy for any $s \in [0, 2a\sigma]$, where $2a\sigma \geq \bar{s}$. Let $x_A^*(s)$ denote the equilibrium planting amount under the actual-yield-based policy for any $s \in [0, 2a\sigma]$.

Consider $r = 1$. We next prove that, if $x^*(s) \leq x_A^*(s)$ for all $s \in [0, 2a\sigma]$, then we have $s^* \geq s_A^*$. This is because if $x^*(s) \leq x_A^*(s)$ for all $s \in [0, 2a\sigma]$, then we have the following: Suppose s^* is the lowest subsidy that satisfies $x^*(s^*) = x^{opt}$ under the index-based policy. Then under the actual-yield-based policy, a lower subsidy can achieve x^{opt} and thus a lower subsidy is optimal. Suppose s^* is the highest feasible subsidy under the index-based policy, i.e., s^* satisfies $x^*(s^*)\mathbb{E}[s^*I] = B$ or $s^* = \bar{s} = 2a\sigma - 4b\mu\sigma x^*(s^*)$. Then, under the actual-yield-based policy, s^* is no longer feasible (i.e., we have either $x_A^*(s^*)\mathbb{E}[s^*I] > B$ or $s^* > 2a\sigma - 4b\mu\sigma x_A^*(s^*)$) and therefore the optimal subsidy must be lower than s^*.

Therefore, it is sufficient to prove $x^*(s) \leq x_A^*(s)$ for all $s \in [0, 2a\sigma]$. Consider a fixed subsidy amount $s \in [0, 2a\sigma]$. To prove $x^*(s) \leq x_A^*(s)$, following the logic in the proof of Lemma 2, it is sufficient to prove that for any planting amount $x \in [0, 1]$, the utility of each farmer i is higher under the actual-yield-based policy. For $i \in [0, 1]$, let $I_A = 1$ if $Y_i < \mu$ and $I_A = 0$ otherwise. Given the planting amount x, we denote the profit of farmer i under the actual-yield-based policy as

$$\pi_A(i|x, s) = (a - bxY)Y_i + sI_A - h_i.$$

Then, we have

$$\mathbb{E}[\pi_A(i|x, s)] = a\mu - bx(\mu^2 + \sigma^2) - h_i + \frac{1}{2}s = \mathbb{E}[\pi(i|x, s)], \tag{A.19}$$

where the second equality has been shown in the proof of Lemma 2. We also have

$$\mathbb{V}\mathrm{ar}[\pi_A(i|x, s)] = \mathbb{V}\mathrm{ar}[(a - bxY)Y + sI_A - h_i] + \mathbb{V}\mathrm{ar}[(a - bxY)\epsilon_i]$$

$$
\begin{aligned}
&+2\mathrm{Cov}\left((a-bxY)Y+sI_A-h_i,(a-bxY)\epsilon_i\right)\\
=&\mathbb{V}\mathrm{ar}[(a-bxY)Y]+\mathbb{V}\mathrm{ar}[sI_A]+2\mathrm{Cov}((a-bxY)Y,sI_A)+\mathbb{V}\mathrm{ar}[(a-bxY)\epsilon_i]\\
&+2\mathrm{Cov}((a-bxY)\epsilon_i,sI_A)\\
=&\frac{1}{4}(R_H(x)-R_L(x))^2+\frac{1}{4}s^2+2\mathrm{Cov}((a-bxY)Y,sI)+\mathbb{V}\mathrm{ar}[(a-bxY)\epsilon_i]\\
&+[2\mathrm{Cov}((a-bxY)Y,sI_A)+2\mathrm{Cov}((a-bxY)\epsilon_i,sI_A)-2\mathrm{Cov}((a-bxY)Y,sI)]\\
=&\mathbb{V}\mathrm{ar}[\pi(i|x,s)]+2s\left[\mathrm{Cov}((a-bxY)Y_i,I_A)-\mathrm{Cov}((a-bxY)Y,I)\right],
\end{aligned}
\tag{A.20}
$$

where the last equality has been shown in the proof of Lemma 2. From Equation (A.19) and (A.20), to prove that the utility is higher under the actual-yield-based policy, it remains to prove that $\mathrm{Cov}((a-bxY)Y_i,I_A)-\mathrm{Cov}((a-bxY)Y,I)\le 0$. Let $q=\mathbb{P}(\epsilon_i<-\sigma)$. Then, we have

$$
\begin{aligned}
&\mathrm{Cov}((a-bxY)Y_i,I_A)-\mathrm{Cov}((a-bxY)Y,I)\\
=&\mathbb{E}[(a-bxY)Y_iI_A]-\mathbb{E}[(a-bxY)YI]\\
=&\frac{1}{2}\mathbb{P}(\epsilon_i<-\sigma)\mathbb{E}[(a-bx(\mu+\sigma))\left(\mu+\sigma+\epsilon_i\right)|\,\epsilon_i<-\sigma]+\frac{1}{2}\mathbb{P}(\epsilon_i<\sigma)\mathbb{E}[(a-bx(\mu-\sigma))\left(\mu-\sigma+\epsilon_i\right)|\,\epsilon_i<\sigma]\\
&-\frac{1}{2}\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)\\
=&\frac{1}{2}q\left(a-bx(\mu+\sigma)\right)\left(\mu+\sigma\right)+\frac{1}{2}\left(1-q\right)\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)\\
&+\frac{1}{2}q(a-bx(\mu+\sigma))\mathbb{E}[\epsilon_i\,|\,\epsilon_i<-\sigma]+\frac{1}{2}(1-q)(a-bx(\mu-\sigma))\mathbb{E}[\epsilon_i\,|\,\epsilon_i<\sigma]-\frac{1}{2}\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)\\
=&\frac{1}{2}q\left(a-bx(\mu+\sigma)\right)\left(\mu+\sigma\right)-\frac{1}{2}q\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)+q(a-bx\mu)\mathbb{E}[\epsilon_i\,|\,\epsilon_i<-\sigma]\\
\le&\frac{1}{2}q\left(a-bx(\mu+\sigma)\right)\left(\mu+\sigma\right)-\frac{1}{2}q\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)-q(a-bx\mu)\sigma,
\end{aligned}
\tag{A.21}
$$

where the fourth equality holds because ϵ_i follows a normal distribution with mean of zero, which implies $q\mathbb{E}[\epsilon_i\,|\,\epsilon_i<-\sigma]=(1-q)\mathbb{E}[\epsilon_i\,|\,\epsilon_i<\sigma]$, and the inequality holds because $\mathbb{E}[\epsilon_i\,|\,\epsilon_i<-\sigma]<-\sigma$. Since the right hand side of Equation (A.21) is linear in q and we have $q\in[0,\frac{1}{2})$, we have

$$
\begin{aligned}
&\frac{1}{2}q\left(a-bx(\mu+\sigma)\right)\left(\mu+\sigma\right)-\frac{1}{2}q\left(a-bx(\mu-\sigma)\right)\left(\mu-\sigma\right)-q(a-bx\mu)\sigma\\
\le&\max\left\{0,\frac{1}{4}\left(2a\sigma-4bx\mu\sigma\right)-\frac{1}{2}(a-bx\mu)\sigma\right\}\\
=&0.
\end{aligned}
$$

Thus, we have $\mathrm{Cov}((a-bxY)Y_i,I_A)-\mathrm{Cov}((a-bxY)Y,I)\le 0$, which completes the proof. $\square$

Appendix A.2 Proofs of Analytical Results in §5

We first present four auxiliary lemmas to derive the closed-form expression for $x^*(m,s)$ and to analyze how it varies with key parameters.

Lemma A.4 (Equilibrium planting amount with price and yield protection)**.** *Consider any $m \in [0, \bar{m}]$ and $s \in [0, \bar{s}]$. Let $\hat{x}(m,s)$ denote the unique positive solution to $u(x|x,m,s) = 0$. Then, the equilibrium planting amount is $x^*(m,s) = \min\{\hat{x}(m,s), 1\} \in (0,1]$. Moreover, there exist two thresholds m_1 and m_2 (which depend on s but not m) such that $0 \le m_1 \le m_2 \le \bar{m}$ and*

(i) if $m \in [0, m_1]$, then $\hat{x}(m,s) = \hat{x}(s)$, where $\hat{x}(s)$ is defined in Lemma 2;

(ii) if $m \in (m_1, m_2]$, then $\hat{x}(m,s) = \frac{-\beta(m,s)+\sqrt{\beta^2(m,s)-4\alpha_m\gamma(m,s)}}{2\alpha_m}$, where $\alpha_m = \frac{1}{4}\lambda b^2(\mu-\sigma)^4 + \frac{1}{2}\lambda b^2(\mu-\sigma)^2\sigma_\epsilon^2$, $\beta(m,s) = \frac{1}{2}b(\mu-\sigma)^2 + c + \frac{1}{2}\lambda b(\mu-\sigma)^2(m(\mu+\sigma) - a(\mu-\sigma)) - \lambda ab(\mu-\sigma)\sigma_\epsilon^2 - \lambda b(r-\frac{1}{2})s(\mu-\sigma)^2$, and $\gamma(m,s) = \frac{1}{4}\lambda(m(\mu+\sigma) - a(\mu-\sigma))^2 + \frac{1}{2}\lambda(a^2+m^2)\sigma_\epsilon^2 - \frac{1}{2}m(\mu+\sigma) - \frac{1}{2}a(\mu-\sigma) - \lambda(r-\frac{1}{2})s(m(\mu+\sigma) - a(\mu-\sigma)) - \frac{1}{2}s + \frac{1}{4}\lambda s^2$;

(iii) if $m \in (m_2, \bar{m}]$, then $\hat{x}(m,s) = \frac{m\mu - \lambda m^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r-\frac{1}{2})sm}{c}$.

Further, there exists a threshold c_3 such that if $c \ge c_3$, then $x^*(m,s) = \hat{x}(m,s) \in (0,1)$.

Proof of Lemma A.4. Since we have shown in the proof of Lemma 2 that $\bar{s} \le 2a\sigma$, it is sufficient to prove this lemma by considering $s \in [0, 2a\sigma]$. Moreover, since $\bar{m} \le a$, we prove this lemma by considering $m \in [0,a]$. For any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, recall that $\pi(i|x,m,s) = \max\{(a - bxY), m\}Y_i + sI - h_i$ is the net income of the farmer i whose production cost is h_i, and $u(i|x,m,s) = \mathbb{E}[\pi(i|x,m,s)] - \lambda \mathbb{V}\mathrm{ar}[\pi(i|x,m,s)]$ is the utility of this farmer. Following the same logic as in the proof of Lemma 1, given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, if $x = 0$, $x = 1$, or $x \in (0,1)$ is an equilibrium planting amount, then we must have $u(0|0,m,s) \le 0$, $u(1|1,m,s) \ge 0$ or $u(x|x,m,s) = 0$ respectively.

Step 1: We show that for any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, there is a unique positive solution to $u(x|x,m,s) = 0$. To do so, it is sufficient to prove the following three parts: (1) $u(x|x,m,s)$ strictly decreases in x for $x \ge 0$, (2) $u(0|0,m,s) > 0$ and (3) $u(x|x,m,s) \le 0$ for some $x > 0$.

First, let $x_1 = \frac{a-m}{b(\mu+\sigma)}$ (i.e., $a - bx_1(\mu+\sigma) = m$) and $x_2 = \frac{a-m}{b(\mu-\sigma)}$ (i.e., $a - bx_2(\mu-\sigma) = m$). We prove that $u(x|x,m,s)$ strictly decreases in x for $x \ge 0$ separately on three intervals of x: (a) $[0, x_1]$, (b) $(x_1, x_2]$, and (c) (x_2, ∞).

(a): For $x \in [0, x_1]$, by the definition of x_1, we have $a - bx(\mu-\sigma) > a - bx(\mu+\sigma) \ge m$. Thus, from the expression of $u(x|x,m,s)$, we have that $u(x|x,m,s) = u(x|x,s)$, which is farmers' utility with only yield protection. We already know from the proof of Lemma 2 that $u(x|x,s)$ strictly decreases in x for $x \ge 0$. Therefore, $u(x|x,m,s)$ strictly decreases in x for $x \in [0, x_1]$.

(b): For $x \in (x_1, x_2]$, by the definition of x_1 and x_2, we have $a - bx(\mu-\sigma) \ge m \ge a - bx(\mu+\sigma)$. Recall that $R_L(x) = (a - bx(\mu-\sigma))(\mu-\sigma)$ denotes the expected revenue when $Y = \mu - \sigma$. Then,

$$
\begin{aligned}
u(x|x,m,s) =& \mathbb{E}[\max\{(a-bxY), m\}Y_i + sI - xc] - \lambda \mathbb{V}\mathrm{ar}[\max\{(a-bxY), m\}Y_i + sI - xc] \\
=& \frac{1}{2}[m(\mu+\sigma) + R_L(x) + s] - xc \\
& - \lambda\left(\frac{1}{4}(m(\mu+\sigma) - R_L(x))^2 + \frac{1}{4}s^2 - s\left(r - \frac{1}{2}\right)(m(\mu+\sigma) - R_L(x))\right)
\end{aligned}
$$

$$-\lambda\left(\frac{1}{2}a^2-abx(\mu-\sigma)+\frac{1}{2}b^2x^2(\mu-\sigma)^2+\frac{1}{2}m^2\right)\sigma_\epsilon^2. \tag{A.22}$$

By taking the derivative of $u(x|x,m,s)$ with respect to x, we have

$$\begin{aligned}\frac{\partial}{\partial x}u(x|x,m,s)=&-\frac{1}{2}b(\mu-\sigma)^2-c-\lambda b(\mu-\sigma)^2\left(\frac{1}{2}(m(\mu+\sigma)-R_L(x))-s\left(r-\frac{1}{2}\right)\right)\\&-\lambda b(\mu-\sigma)\sigma_\epsilon^2\left(bx(\mu-\sigma)-a\right).\end{aligned}$$

Since we consider $r\leq 1$ and $s\leq 2a\sigma$, we have

$$\begin{aligned}\frac{1}{2}(m(\mu+\sigma)-R_L(x))-s\left(r-\frac{1}{2}\right)&\geq\frac{1}{2}(m(\mu+\sigma)-R_L(x_1))-a\sigma\\&=\frac{1}{2}\left(m(\mu+\sigma)-a(\mu-\sigma)+\frac{(a-m)(\mu-\sigma)^2}{\mu+\sigma}\right)-a\sigma\\&=(m-a)\frac{2\mu\sigma}{\mu+\sigma}\\&\geq -a\frac{2\mu\sigma}{\mu+\sigma}.\end{aligned}$$

Clearly, we also have $bx(\mu-\sigma)-a\geq -a$. Since $c>c_1\geq 2\lambda ab(\mu-\sigma)^2\frac{\mu\sigma}{\mu+\sigma}+\lambda ab(\mu-\sigma)\sigma_\epsilon^2-\frac{1}{2}b(\mu-\sigma)^2$ (as discussed in §3), we have $\frac{\partial}{\partial x}u(x|x,m,s)<0$, i.e., $u(x|x,m,s)$ strictly decreases in x for $x\in(x_1,x_2]$.

(c): For $x>x_2$, by the definition of x_2, we have $m>a-bx(\mu-\sigma)>a-bx(\mu+\sigma)$. Then,

$$\begin{aligned}u(x|x,m,s)&=\mathbb{E}[mY_i+sI-xc]-\lambda\mathbb{V}\mathrm{ar}[mY_i+sI-xc]\\&=m\mu+\frac{1}{2}s-xc-\lambda\left(m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{4}s^2-2\sigma\left(r-\frac{1}{2}\right)sm\right).\end{aligned} \tag{A.23}$$

Clearly, $u(x|x,m,s)$ strictly decreases in x for $x>x_2$. Combining all three cases and considering that $u(x|x,m,s)$ is continuous in x, we conclude that $u(x|x,m,s)$ strictly decreases in x.

Second, we prove $u(0|0,m,s)>0$. When no farmer plants (i.e., $x=0$), the market-clearing price is given by a for both realized values of Y. As we consider $m\leq a$, we must have $u(0|0,m,s)=u(0|0,s)=-\gamma(s)>0$, where the last inequality is shown in the proof of Lemma 2.

Finally, we prove $u(x|x,m,s)<0$ for some $x>0$. Since we have shown earlier that $u(x|x,m,s)$ strictly decreases in x for $x\geq 0$ and is linear in x for $x>x_2$, there must exist $x\geq 0$ such that $u(x|x,m,s)<0$.

Collectively, we conclude that for any given $m\in[0,a]$ and $s\in[0,2a\sigma]$, there must exist a unique positive solution to $u(x|x,m,s)=0$, which we denote as $\hat{x}(m,s)$.

Step 2: We show that for any given $m\in[0,a]$ and $s\in[0,2a\sigma]$, there is a unique equilibrium planting amount $x^*(m,s)=\min\{\hat{x}(m,s),1\}$, where $\hat{x}(m,s)$ is the unique positive solution to $u(x|x,m,s)=0$. Since we have shown that $u(0|0,m,s)>0$ in Step 1, $x=0$ cannot be an equilibrium planting amount. Then, based on the discussion at the beginning of the proof, it remains to check whether $x = 1$ or $x = \hat{x}(m,s)$ is an equilibrium planting amount.

If $\hat{x}(m,s) \in (0,1)$, then, following the same logic as in Step 2 of the proof of Lemma 1, we have that the equilibrium planting amount is $x^*(m,s) = \hat{x}(m,s)$. If $\hat{x}(m,s) \geq 1$, then, as we just showed that $u(x|x,m,s)$ strictly decreases in x, we must have $u(1|1,m,s) \geq 0$. In this case, we have that the equilibrium planting amount is given by $x^*(m,s) = 1$. Hence, there is a unique equilibrium planting amount given by $x^*(m,s) = \min\{\hat{x}(m,s), 1\} \in (0,1]$.

Step 3: For any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, we derive the expression of $\hat{x}(m,s)$.

(a): Suppose $\hat{x}(m,s) \leq x_1$. Then, as shown earlier, we have $u(x|x,m,s) = u(x|x,s)$ at $x = \hat{x}(m,s)$ and thus $\hat{x}(m,s) = \hat{x}(s)$ defined in Lemma 2. Therefore, we conclude that whenever $\hat{x}(m,s)$ satisfies $\hat{x}(m,s) \leq x_1$, we have $\hat{x}(m,s) = \hat{x}(s)$.

(b): Suppose $\hat{x}(m,s) \in (x_1, x_2]$. Then, based on Equation (A.22), $\hat{x}(m,s)$ must be the solution to the following equation:

$$\begin{aligned} u(x|x,m,s) =& \frac{1}{2}[m(\mu+\sigma) + R_L(x) + s] - xc \\ & - \lambda\left(\frac{1}{4}(m(\mu+\sigma) - R_L(x))^2 + \frac{1}{4}s^2 - s\left(r - \frac{1}{2}\right)(m(\mu+\sigma) - R_L(x))\right) \\ & - \lambda\left(\frac{1}{2}a^2 - abx(\mu-\sigma) + \frac{1}{2}b^2x^2(\mu-\sigma)^2 + \frac{1}{2}m^2\right)\sigma_\epsilon^2 \\ =& -\alpha_m x^2 - \beta(m,s)x - \gamma(m,s) \\ =& 0, \end{aligned}$$

where $\alpha_m = \frac{1}{4}\lambda b^2(\mu-\sigma)^4 + \frac{1}{2}\lambda b^2(\mu-\sigma)^2\sigma_\epsilon^2$, $\beta(m,s) = \frac{1}{2}b(\mu-\sigma)^2 + c + \frac{1}{2}\lambda b(\mu-\sigma)^2(m(\mu+\sigma) - a(\mu-\sigma)) - \lambda ab(\mu-\sigma)\sigma_\epsilon^2 - \lambda b(r-\frac{1}{2})s(\mu-\sigma)^2$ and $\gamma(m,s) = \frac{1}{4}\lambda(m(\mu+\sigma) - a(\mu-\sigma))^2 + \frac{1}{2}\lambda(a^2+m^2)\sigma_\epsilon^2 - \frac{1}{2}m(\mu+\sigma) - \frac{1}{2}a(\mu-\sigma) - \lambda(r-\frac{1}{2})s(m(\mu+\sigma) - a(\mu-\sigma)) - \frac{1}{2}s + \frac{1}{4}\lambda s^2$. Then, we have

$$\hat{x}(m,s) = \frac{-\beta(m,s) + \sqrt{\beta^2(m,s) - 4\alpha_m\gamma(m,s)}}{2\alpha_m}. \tag{A.24}$$

Therefore, we conclude that whenever $\hat{x}(m,s)$ satisfies $\hat{x}(m,s) \in (x_1, x_2]$, we have $\hat{x}(m,s) = \frac{-\beta(m,s) + \sqrt{\beta^2(m,s) - 4\alpha_m\gamma(m,s)}}{2\alpha_m}$.

(c): Suppose $\hat{x}(m,s) > x_2$. Then, based on Equation (A.23), $\hat{x}(m,s)$ must be the solution to the following equation:

$$u(x|x,m,s) = m\mu + \frac{1}{2}s - xc - \lambda\left(m^2(\sigma^2 + \sigma_\epsilon^2) + \frac{1}{4}s^2 - 2\sigma\left(r - \frac{1}{2}\right)sm\right) = 0.$$

Then, we have

$$\hat{x}(m,s) = \frac{m\mu - \lambda m^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r-\frac{1}{2})sm}{c}. \tag{A.25}$$

Therefore, we conclude that whenever $\hat{x}(m,s)$ satisfies $\hat{x}(m,s) > x_2$, we have $\hat{x}(m,s) = (m\mu - \lambda m^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r-\frac{1}{2})sm)/c$.

Step 4: We prove that for any given $s \in [0, 2a\sigma]$, there exist two thresholds $0 \le m_1 \le m_2 \le a$ such that if $m \le m_1$, then $\hat{x}(m,s) \le x_1$; if $m_1 < m \le m_2$, then $x_1 \le \hat{x}(m,s) \le x_2$; and if $m_2 < m \le a$, then $\hat{x}(m,s) > x_2$. (For the purpose of the lemma, we can redefine the thresholds by truncating them at $\bar{m}$.)

Consider a fixed $s \in [0, 2a\sigma]$. To prove the existence of such m_1 and m_2, it is sufficient to prove that (1) if $\hat{x}(m,s) \le x_1$ for some floor price m, then $\hat{x}(m,s) \le x_1$ for any lower floor price; and (2) if $\hat{x}(m,s) > x_2$ for some floor price m, then $\hat{x}(m,s) > x_2$ for any higher floor price.

For (1), first, we prove that for any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, $\hat{x}(m,s) \le x_1$ if and only if $\hat{x}(s) \le x_1$, where $\hat{x}(s)$ is defined in Lemma 2 and $x_1 = \frac{a-m}{b(\mu+\sigma)}$. By Step 3, we have shown that if $\hat{x}(m,s) \le x_1$, then $\hat{x}(m,s) = \hat{x}(s)$. Thus, if $\hat{x}(m,s) \le x_1$, then we have $\hat{x}(s) = \hat{x}(m,s) \le x_1$. On the other hand, if $\hat{x}(s) \le x_1$, then we have

$$\begin{aligned}
u(\hat{x}(s)|\hat{x}(s),m,s) =& \mathbb{E}[\max\{(a - b\hat{x}(s)Y), m\}Y_i + sI - \hat{x}(s)c] \\
&- \lambda\mathbb{V}\text{ar}[\max\{(a - b\hat{x}(s)Y), m\}Y_i + sI - \hat{x}(s)c] \\
=& \mathbb{E}[(a - b\hat{x}(s)Y)Y_i + sI - \hat{x}(s)c] - \lambda\mathbb{V}\text{ar}[(a - b\hat{x}(s)Y)Y_i + sI - \hat{x}(s)c] \\
=& u(\hat{x}(s)|\hat{x}(s),s) \\
=& 0,
\end{aligned}$$

where the second equality holds because, by the definition of x_1, we have $a - b\hat{x}(s)(\mu-\sigma) > a - b\hat{x}(s)(\mu+\sigma) \ge m$, and the third and fourth equalities follow from the definitions of $u(x|x,s)$ and $\hat{x}(s)$ in Lemma 2. Thus, $\hat{x}(s) > 0$ is a solution to the equation $u(x|x,m,s) = 0$. Moreover, we have shown in Step 1 that $\hat{x}(m,s)$ is the unique positive solution to $u(x|x,m,s) = 0$. Hence, we have $\hat{x}(m,s) = \hat{x}(s)$, which implies $\hat{x}(m,s) \le x_1$. Collectively, we have $\hat{x}(m,s) \le x_1$ if and only if $\hat{x}(s) \le x_1$.

Then, to prove (1), it is sufficient to prove that if $\hat{x}(s) \le x_1$ for some floor price m, then $\hat{x}(s) \le x_1$ for any lower m. This is true because x_1 strictly decreases in m while $\hat{x}(s)$ is independent of m.

For (2), let $\check{x}(m,s) = (m\mu - \lambda m^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r-\frac{1}{2})sm)/c$. First, we prove that for any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$, $\hat{x}(m,s) > x_2$ if and only if $\check{x}(m,s) > x_2$, where $x_2 = \frac{a-m}{b(\mu-\sigma)}$. By Step 3, we have shown that if $\hat{x}(m,s) > x_2$, then $\hat{x}(m,s) = \check{x}(m,s)$. Thus, if $\hat{x}(m,s) > x_2$, then we have $\check{x}(m,s) = \hat{x}(m,s) > x_2$. On the other hand, if $\check{x}(m,s) > x_2$, then we have

$$\begin{aligned}
u(\check{x}(m,s)|\check{x}(m,s),m,s) =& \mathbb{E}[\max\{(a - b\check{x}(m,s)Y), m\}Y_i + sI - \check{x}(m,s)c] \\
&- \lambda\mathbb{V}\text{ar}[\max\{(a - b\check{x}(m,s)Y), m\}Y_i + sI - \check{x}(m,s)c] \\
=& \mathbb{E}[mY_i + sI - \check{x}(m,s)c] - \lambda\mathbb{V}\text{ar}[mY_i + sI - \check{x}(m,s)c] \\
=& 0,
\end{aligned}$$

where the second equality holds because, by the definition of x_2, we have $m > a - b\check{x}(m,s)(\mu - \sigma) > a - b\check{x}(m,s)(\mu + \sigma)$, and the third equality has been shown in (c) of Step 3. Thus, $\check{x}(m,s) > x_2 \geq 0$ is a solution to the equation $u(x|x,m,s) = 0$. Moreover, we have shown in Step 1 that $\hat{x}(m,s)$ is the unique positive solution to $u(x|x,m,s) = 0$. Hence, we have $\hat{x}(m,s) = \check{x}(m,s)$, which implies $\check{x}(m,s) > x_2$. Collectively, we have $\check{x}(m,s) > x_2$ if and only if $\hat{x}(m,s) > x_2$.

Then, in order to prove (2), it is sufficient to prove that if $\check{x}(m,s) > x_2$ for some floor price m, then $\check{x}(m,s) > x_2$ for any higher floor price. That is, it is sufficient to prove that if $\check{x}(m,s) - x_2 > 0$ for some floor price m, then $\check{x}(m,s) - x_2 > 0$ for any higher floor price. Since x_2 is linear in m and $\check{x}(m,s)$ is a quadratic concave function in m, $\check{x}(m,s) - x_2$ must be a quadratic concave function in m. Recall that we consider $m \leq a$. Then, it is sufficient to prove that when $m = a$, we have $\check{x}(m,s) - x_2 = \check{x}(m,s) > 0$. Since $s \in [0, 2a\sigma]$ and $\check{x}(m,s)$ is a quadratic concave function of s, it is sufficient to prove when $m = a$, we have $\check{x}(m,0) > 0$ and $\check{x}(m,2a\sigma) > 0$. When $m = a$, we have

$$\check{x}(m,0) = \frac{a\mu - \lambda a^2(\sigma^2 + \sigma_\epsilon^2)}{c} > \frac{1}{c}\left(a\mu - \frac{\mu}{a(\sigma^2 + \sigma_\epsilon^2)} a^2(\sigma^2 + \sigma_\epsilon^2)\right) = 0,$$

where the first inequality holds because we consider $\lambda < \frac{\mu}{a(\sigma^2 + \sigma_\epsilon^2)}$; we also have

$$\begin{aligned}
\check{x}(m, 2a\sigma) &= \frac{1}{c}\left(a\mu - \lambda a^2(\sigma^2 + \sigma_\epsilon^2) + a\sigma - \lambda a^2\sigma^2 + 4\lambda\sigma^2\left(r - \frac{1}{2}\right)a^2\right) \\
&\geq \frac{1}{c}\left(a(\mu + \sigma) - 2\lambda a^2\sigma^2 - \lambda a^2\sigma_\epsilon^2\right) \\
&> \frac{1}{c}\left(a(\mu + \sigma) - \left(\frac{\mu + \sigma}{2a\sigma^2 + a\sigma_\epsilon^2}\right)(2a^2\sigma^2 + a^2\sigma_\epsilon^2)\right) \\
&= 0,
\end{aligned}$$

where the first inequality holds because $r \geq \frac{1}{2}$ and the second inequality holds because $\lambda < \frac{\mu+\sigma}{2a\sigma^2 + a\sigma_\epsilon^2}$. Hence, we have $\check{x}(m,s) > x_2$ when $m = a$. We conclude that if $\hat{x}(m,s) > x_2$ for some floor price m, then $\hat{x}(m,s) > x_2$ for any higher floor price.

Combining all four steps completes the derivation of $x^*(m,s)$ for any given $m \in [0,a]$ and $s \in [0, 2a\sigma]$.

Finally, we prove that there exists a threshold c_3 such that if $c \geq c_3$, then the equilibrium planting amount under given floor price $m \in [0, \bar{m}]$ and subsidy amount $s \in [0, \bar{s}]$ is $x^*(m,s) = \hat{x}(m,s) \in (0,1)$. To do so, it is sufficient to prove that there exists a threshold c_3 such that if $c \geq c_3$, then $u(1|1,m,s) < 0$ for any $m \in [0, \bar{m}]$ and $s \in [0, \bar{s}]$. By definition, we have

$$u(1|1,m,s) = \mathbb{E}[\max\{(a - bY), m\}Y_i + sI] - c - \lambda \mathbb{V}\text{ar}[\max\{(a - bY), m\}Y_i + sI].$$

From this expression, we have that $u(1|1,m,s)$ linearly decreases in c. Moreover, it is easy to check that for any $m \in [0, \bar{m}]$ and $s \in [0, \bar{s}]$, $u(1|1,m,s)$ is bounded above by a positive constant independent of m and s. Therefore, there must exist a threshold c_3 such that if $c \geq c_3$, then $u(1|1,m,s) < 0$ for any $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$. $\square$

As discussed in §5, we consider the case where $c \geq c_3$ in the remainder of our analysis, which allows us to avoid unnecessary technical complexities arising from $x^*(m,s)$ reaching one.

Lemma A.5. *For any $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, we have $s \leq \max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma)$.*

Proof of Lemma A.5. Consider a fixed $m \in [0,\bar{m}]$ and a fixed $s \in [0,\bar{s}]$. For $s \leq \bar{s}$, we have $s \leq R_H(x^*(0,s)) - R_L(x^*(0,s))$. Then, to prove this lemma, it is sufficient to prove that $R_H(x^*(0,s)) - R_L(x^*(0,s)) \leq \max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma)$. Similar to the proof of Lemma A.4, let $x_1 = \frac{a-m}{b(\mu+\sigma)}$ and $x_2 = \frac{a-m}{b(\mu-\sigma)}$, and we separate the analysis into three cases:

Case 1: Suppose $x^*(m,s) \leq x_1$. Then, by Lemma A.4, we have $x^*(m,s) = x^*(0,s)$ and

$$\begin{aligned}
&\max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma) \\
=&(a - bx^*(m,s)(\mu+\sigma))(\mu+\sigma) - (a - bx^*(m,s)(\mu-\sigma))(\mu-\sigma) \\
=&R_H(x^*(0,s)) - R_L(x^*(0,s)).
\end{aligned}$$

Case 2: Suppose $x_1 < x^*(m,s) \leq x_2$. We have $\max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma) = m(\mu+\sigma) - R_L(x^*(m,s))$. Thus, it is sufficient to prove $[m(\mu+\sigma) - R_L(x^*(m,s))] - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \geq 0$. On the one hand, if $x^*(m,s) \geq x^*(0,s)$, we have $R_L(x^*(0,s)) - R_L(x^*(m,s)) \geq 0$ because $R_L(x)$ decreases in x. Moreover, we have $m(\mu+\sigma) - R_H(x^*(0,s)) > 0$ because, by Step 4 of the proof of Lemma A.4, $x^*(m,s) > x_1$ implies $x^*(0,s) > x_1$ and thus $m > P_H(x^*(0,s))$, where $P_H(x) = a - bx(\mu+\sigma)$. Therefore, we have

$$\begin{aligned}
&[m(\mu+\sigma) - R_L(x^*(m,s))] - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \\
=&[m(\mu+\sigma) - R_H(x^*(0,s))] + [R_L(x^*(0,s)) - R_L(x^*(m,s))] \\
>&0.
\end{aligned}$$

On the other hand, if $x^*(m,s) < x^*(0,s)$, we have

$$\begin{aligned}
&[m(\mu+\sigma) - R_L(x^*(m,s))] - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \\
=&[m(\mu+\sigma) - R_H(x^*(m,s))] + [R_H(x^*(m,s)) - R_L(x^*(m,s))] - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \\
=&[m(\mu+\sigma) - R_H(x^*(m,s))] + 4b\mu\sigma\,[x^*(0,s) - x^*(m,s)] \\
>&0,
\end{aligned}$$

where the inequality holds because $x^*(m,s) > x_1$ and $x^*(m,s) < x^*(0,s)$. Hence, we can conclude that $m(\mu+\sigma) - R_L(x^*(m,s)) \geq R_H(x^*(0,s)) - R_L(x^*(0,s))$ in this case.

Case 3: Suppose $x^*(m,s) > x_2$. We have $\max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma) = 2m\sigma$. Then, we have

$$\begin{aligned}
&2m\sigma - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \\
>&2P_H(x^*(0,s))\sigma - [R_H(x^*(0,s)) - R_L(x^*(0,s))] \\
=&[2a\sigma - 2b\sigma x^*(0,s)(\mu+\sigma)] - [2a\sigma - 4b\mu\sigma x^*(0,s)] \\
=&2b\sigma x^*(0,s)(\mu-\sigma) \\
>&0,
\end{aligned}$$

where the first inequality holds because, by Step 4 of the proof of Lemma A.4, $x^*(m,s) > x_1$ implies $x^*(0,s) > x_1$ and thus $m > P_H(x^*(0,s))$.

Combining all three cases, we conclude that for any $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, $s \leq R_H(x^*(0,s)) - R_L(x^*(0,s)) \leq \max\{a - bx^*(m,s)(\mu+\sigma), m\}(\mu+\sigma) - \max\{a - bx^*(m,s)(\mu-\sigma), m\}(\mu-\sigma)$. $\square$

Lemma A.6. *(i) For any $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, $x^*(m,s)$ increases in the index accuracy r.*
(ii) For any $m \in [0,\bar{m}]$, $\frac{\partial}{\partial s}x^*(m,s)|_{s=0} > 0$.

Proof of Lemma A.6. (i) Consider a fixed $m \in [0,\bar{m}]$ and a fixed $s \in [0,\bar{s}]$. Let m_1 and m_2 be the two thresholds defined in Lemma A.4. Accordingly, we separate the analysis into three cases:

Case 1: Suppose $m \leq m_1$. In this case, according to Lemma A.4, we have $x^*(m,s) = x^*(s)$. We have already proven that $x^*(s)$ increases in r in Corollary 1. Therefore, $x^*(m,s)$ increases in r.

Case 2: Suppose $m \in (m_1, m_2]$. Following the same logic as in the proof of Lemma A.1, $\frac{\partial}{\partial r}x^*(m,s)$ and $\frac{\partial}{\partial r}u(x|x,m,s)|_{x=x^*(m,s)}$ have the same sign. Then, from Equation (A.22), we have $\frac{\partial}{\partial r}u(x|x,m,s)|_{x=x^*(m,s)} = \lambda s[m(\mu+\sigma) - R_L(x^*(m,s))] \geq 0$, where the inequality holds because, based on Lemma A.5, given $s \in [0,\bar{s}]$, farmers' expected revenue conditional on $Y = \mu - \sigma$, plus s, does not exceed their expected revenue conditional on $Y = \mu + \sigma$, i.e., $m(\mu+\sigma) - R_L(x^*(m,s)) \geq s \geq 0$. Therefore, $x^*(m,s)$ increases in r.

Case 3: Suppose $m > m_2$. By Lemma A.4, we have $x^*(m,s) = (m\mu - \lambda m^2(\sigma^2 + \sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r - \frac{1}{2})sm)/c$. Therefore, we have $\frac{\partial}{\partial r}x^*(m,s) = \frac{2\lambda\sigma sm}{c} \geq 0$.

Combining all three cases and considering that $x^*(m,s)$ remains continuous when an increase in r results in a switch between cases, we conclude that $x^*(m,s)$ increases in r.

(ii) Consider a fixed $m \in [0,\bar{m}]$. Recall that given $m \in [0,\bar{m}]$, x_1 and x_2 are defined in Step 1 of the proof of Lemma A.4, and the relationship between the equilibrium planting amount and x_1 and x_2 determines whether the floor price m takes effect. Accordingly, we separate the analysis into the following three cases: If $x^*(m,0) < x_1$, we have $x^*(m,0) = x^*(0)$ and $\frac{\partial}{\partial s}x^*(m,s)|_{s=0} = \frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$ by Lemma A.2. If $x^*(m,0) \in [x_1, x_2]$, following the same logic as in the proof of Lemma A.1, $\frac{\partial}{\partial s}x^*(m,s)|_{s=0}$ and $\frac{\partial}{\partial s}u(x|x,m,s)|_{s=0,x=x^*(m,0)}$ have the same sign. From Equation (A.22), we have

$$\frac{\partial}{\partial s}u(x|x,m,s)\Big|_{s=0,x=x^*(m,0)} = \frac{1}{2} + \lambda\left(r - \frac{1}{2}\right)(m(\mu+\sigma) - R_L(x^*(m,0))) > 0,$$

where the inequality holds because $r \in [\frac{1}{2}, 1]$ and we have shown that $m(\mu+\sigma) - R_L(x^*(m,0)) \geq s \geq 0$. Thus, $\frac{\partial}{\partial s}x^*(m,s)|_{s=0} > 0$. If $x^*(m,0) > x_2$, then from Equation (A.25), we have

$$\frac{\partial}{\partial s}x^*(m,s)\Big|_{s=0} = \frac{\frac{1}{2} + 2\lambda\sigma(r-\frac{1}{2})m}{c} > 0.$$

Hence, combining all three cases, we have that $\frac{\partial}{\partial s}x^*(m,s)|_{s=0} > 0$ for any given $m \in [0, \bar{m}]$. □

Before we proceed, let $s_{m,1} := \arg\max_{s\in[0,\bar{s}]} x^*(m,s)$ for any given $m \in [0,\bar{m}]$.[1] Then, we have:

Lemma A.7. *Consider any fixed $m \in [0,\bar{m}]$. There exist two thresholds $\frac{1}{2} \leq r_{m,1} \leq r_{m,2} \leq 1$ such that when the index accuracy is low ($r < r_{m,1}$), $x^*(m,s)$ increases in s if and only if $s \leq s_{m,1}$; furthermore, we have $\frac{\partial}{\partial m}s_{m,1} \geq 0$ and $\frac{\partial}{\partial r}s_{m,1} \geq 0$. When the index accuracy is high ($r \geq r_{m,2}$), $x^*(m,s)$ increases in s.*

Proof of Lemma A.7. For ease of exposition, we first prove this lemma by considering $\sigma_\epsilon = 0$. (We will consider $\sigma_\epsilon > 0$ at the end of the proof.) We prove this lemma in the following four steps:

Step 1: We prove that given $m \in [0,\bar{m}]$ and $r \in [\frac{1}{2}, 1]$, $x^*(m,s)$ increases in s if and only if $s \leq s_{m,1}$, where $s_{m,1}$ is defined before the statement of Lemma A.7. First, we introduce a few building blocks for further analysis. Let $x_1 = \frac{a-m}{b(\mu+\sigma)}$ (i.e., $a - bx_1(\mu+\sigma) = m$) and $x_2 = \frac{a-m}{b(\mu-\sigma)}$ (i.e., $a - bx_2(\mu-\sigma) = m$). From the proof of Lemma A.4, we know that if $x^*(m,s) \leq x_1$, then $x^*(m,s) = x^*(s)$ as characterized in Lemma 2; if $x^*(m,s) \in (x_1, x_2]$, then $x^*(m,s) = \frac{-\beta(m,s)+\sqrt{\beta^2(m,s)-4\alpha_m\gamma(m,s)}}{2\alpha_m}$; if $x^*(m,s) > x_2$, then $x^*(m,s) = (m\mu - \lambda m^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2 + 2\lambda\sigma(r-\frac{1}{2})sm)/c$. From these results, we can see that within each of the three cases, the expression of $x^*(m,s)$ is strictly concave in s. Also, it is straightforward to check that $x^*(m,s)$ is continuous in s on $[0,\bar{s}]$.

Based on the above observations, we next prove that $x^*(m,s)$ increases in s if and only if $s \leq s_{m,1}$ (note that the three cases are divided based on the value of $x^*(m,s)$ rather than that of s). Suppose there exists $s' \in [0, s_{m,1})$ such that $x^*(m,s)$ strictly decreases in s at $s = s'$. If $x^*(m,s') \leq x_1$, then, by the definition of $s_{m,1}$ and the continuity of $x^*(m,s)$, there must exist $s'' \in (s', s_{m,1})$ such that $x^*(m,s'') \leq x_1$ and $x^*(m,s)$ strictly increases in s at $s = s''$. The existence of such s' and s'' contradicts the concavity of $x^*(m,s)$ when $x^*(m,s) \leq x_1$. If $x^*(m,s') \in (x_1,x_2]$ or $x^*(m,s') > x_2$, we

[1] To ensure that $s_{m,1}$ is well-defined, we define it as the smallest s that maximizes $x^*(m,s)$ in case of ties. (However, we show within the proof of Lemma A.7 that the s that maximizes $x^*(m,s)$ is unique.)

can reach a contradiction by using the same argument (note that both x_1 and x_2 are independent of s). Using the same argument, we also conclude that $x^*(m,s)$ decreases in s for $s \in (s_{m,1}, \bar{s}]$.

Furthermore, we prove that $\arg\max_{s\in[0,\bar{s}]} x^*(m,s)$ is unique. As shown above, $x^*(m,s)$ increases in s if and only if $s \leq s_{m,1}$; moreover, for any $s \in [0,\bar{s}]$, we have $\frac{\partial^2}{\partial s^2}x^*(m,s) < 0$ when $x^*(m,s) < x_1$, $x^*(m,s) \in (x_1,x_2)$ or $x^*(m,s) > x_2$. Therefore, $\arg\max_{s\in[0,\bar{s}]} x^*(m,s)$ must be unique.

Step 2: We prove that given $m \in [0,\bar{m}]$ and $r \in [\frac{1}{2},1]$, if $s_{m,1} < \bar{s}$, then $\frac{\partial}{\partial m}s_{m,1} \geq 0$ and $\frac{\partial}{\partial r}s_{m,1} \geq 0$. We divide the analysis into three cases based on the value of $x^*(m,s_{m,1})$.

Case 1: Suppose $x^*(m,s_{m,1}) \leq x_1$. Then, by the definition of $s_{m,1}$, we have $x^*(m,s) \leq x_1$ and thus $x^*(m,s) = x^*(s)$ for all $s \in [0,\bar{s}]$. Thus, the floor price m does not play any role, and we have $s_{m,1} = s_1$ as defined in Proposition 1. Since s_1 increases in r when $s_1 < \bar{s}$ (as shown in Proposition 1), we have that $s_{m,1}$ increases in r when $s_{m,1} < \bar{s}$. Further, $s_{m,1}$ is independent of m.

Case 2: Suppose $x^*(m,s_{m,1}) \in (x_1,x_2]$. Then, according to Lemma A.4, we have $x^*(m,s_{m,1}) = \frac{-\beta(m,s_{m,1})+\sqrt{\beta^2(m,s_{m,1})-4\alpha_m\gamma(m,s_{m,1})}}{2\alpha_m}$. We first prove that if $s_{m,1} < \bar{s}$, then $s_{m,1}$ increases in r. If $s_{m,1} < \bar{s}$, since $\frac{-\beta(m,s)+\sqrt{\beta^2(m,s)-4\alpha_m\gamma(m,s)}}{2\alpha_m}$ is strictly concave in s, we have $\frac{\partial}{\partial s}x^*(m,s)|_{s=s_{m,1}} = 0$. Then, following the same logic as in the proof of Lemma A.1, we have

$$\frac{\partial}{\partial s}u(x|x,m,s)\Big|_{x=x^*(m,s),s=s_{m,1}} = 0.$$

Then, based on the expression of $u(x|x,m,s)$ in Equation (A.22), we have

$$s_{m,1} = \frac{1}{\lambda} + 2(r-\frac{1}{2})[m(\mu+\sigma) - R_L(x^*(m,s_{m,1}))]. \tag{A.26}$$

Taking the derivative of $s_{m,1}$ with respect to r, we have

$$\begin{aligned}\frac{\partial}{\partial r}s_{m,1} =& 2(r-\frac{1}{2})b(\mu-\sigma)^2\left(\frac{\partial}{\partial r}x^*(m,s)\Big|_{s=s_{m,1}} + \frac{\partial}{\partial s}x^*(m,s)\Big|_{s=s_{m,1}}\frac{\partial}{\partial r}s_{m,1}\right)\\ &+ 2[m(\mu+\sigma) - R_L(x^*(m,s_{m,1}))]\\ =& 2(r-\frac{1}{2})b(\mu-\sigma)^2\frac{\partial}{\partial r}x^*(m,s)\Big|_{s=s_{m,1}} + 2[m(\mu+\sigma) - R_L(x^*(m,s_{m,1}))],\end{aligned}$$

where the second equality holds because $\frac{\partial}{\partial s}x^*(m,s)|_{s=s_{m,1}} = 0$. We have shown in part (i) of Lemma A.6 that $x^*(m,s)$ increases in r. Moreover, we have shown in the proof of part (i) of Lemma A.6 that $m(\mu+\sigma) - R_L(x^*(m,s)) \geq s \geq 0$ for any $s \in [0,\bar{s}]$. Thus, we have $\frac{\partial}{\partial r}s_{m,1} \geq 0$.

Next, we prove that if $s_{m,1} < \bar{s}$, then $s_{m,1}$ increases in m. To do so, as we already know that $x^*(m,s)$ increases in s if and only if $s \leq s_{m,1}$ and that $\frac{\partial}{\partial s}x^*(m,s)|_{s=s_{m,1}} = 0$ if $s_{m,1} < \bar{s}$, it is sufficient to prove that if $s_{m,1} < \bar{s}$, then $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}} \geq 0$. Let $\Delta(m,s) = \beta^2(m,s) - 4\alpha_m\gamma(m,s)$. By taking the derivative of $x^*(m,s)$ with respect to m and s, we have

$$\frac{\partial^2}{\partial m \partial s}x^*(m,s)\Big|_{s=s_{m,1}}$$

$$
\begin{aligned}
=&\frac{1}{4\alpha_m}\left(\Delta^{-\frac{1}{2}}(m,s_{m,1})\times\frac{\partial^2}{\partial m\partial s}\Delta(m,s)\Big|_{s=s_{m,1}}+\frac{\partial}{\partial m}\Delta(m,s)\Big|_{s=s_{m,1}}\frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)\Big|_{s=s_{m,1}}\right)\\
=&\frac{1}{4\alpha_m}\times\frac{\partial}{\partial m}\Delta(m,s)\Big|_{s=s_{m,1}}\frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)\Big|_{s=s_{m,1}},
\end{aligned}\tag{A.27}
$$

where the second equality holds because, when $\sigma_\epsilon=0$, through algebraic calculation, we have that $\frac{\partial^2}{\partial m\partial s}\Delta(m,s)=2\lambda^2(r-\frac{1}{2})b^2(\mu+\sigma)(\mu-\sigma)^2\sigma_\epsilon^2=0$. Moreover, we have $\frac{1}{4\alpha_m}>0$ and, when $\sigma_\epsilon=0$, we have $\frac{\partial}{\partial m}\Delta(m,s)=\lambda b(\mu-\sigma)^2(\mu+\sigma)(c+b(\mu-\sigma)^2)>0$. Therefore, by Equation (A.27), to prove that $\frac{\partial^2}{\partial m\partial s}x^*(m,s)|_{s=s_{m,1}}\ge 0$, it remains to show that $\frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)|_{s=s_{m,1}}\ge 0$.

As discussed above, we have $\frac{\partial}{\partial s}x^*(m,s)|_{s=s_{m,1}}=0$ when $s_{m,1}<\bar{s}$. Then, we have

$$
\frac{\partial}{\partial s}x^*(m,s)\Big|_{s=s_{m,1}}=\frac{\partial}{\partial s}\frac{-\beta(m,s)+\Delta^{\frac{1}{2}}(m,s)}{2\alpha_m}\Big|_{s=s_{m,1}}=0.
$$

From the expressions stated in Lemma A.4, we know that $\frac{\partial}{\partial s}(-\beta(m,s))\ge 0$ and α_m is independent of s. Then, we must have $\frac{\partial}{\partial s}\Delta^{\frac{1}{2}}(m,s)|_{s=s_{m,1}}\le 0$. Therefore, we have $\frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)|_{s=s_{m,1}}\ge 0$, which implies that $\frac{\partial^2}{\partial m\partial s}x^*(m,s)|_{s=s_{m,1}}\ge 0$. Therefore, $s_{m,1}$ increases in m if $s_{m,1}<\bar{s}$.

Case 3: Suppose $x^*(m,s_{m,1})>x_2$. Then, according to Lemma A.4, we have $x^*(m,s_{m,1})=(m\mu-\lambda m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}s_{m,1}-\frac{1}{4}\lambda s_{m,1}^2+2\lambda\sigma(r-\frac{1}{2})s_{m,1}m)/c$. Since $s_{m,1}<\bar{s}$ and $(m\mu-\lambda m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}s-\frac{1}{4}\lambda s^2+2\lambda\sigma(r-\frac{1}{2})sm)/c$ is a quadratic concave function in s, we have

$$
\frac{\partial}{\partial s}x^*(m,s)\Big|_{s=s_{m,1}}=\frac{\partial}{\partial s}\frac{m\mu-\lambda m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}s-\frac{1}{4}\lambda s^2+2\lambda\sigma(r-\frac{1}{2})sm}{c}\Big|_{s=s_{m,1}}=0.
$$

Thus, we have

$$
s_{m,1}=\frac{1}{\lambda}+4(r-\frac{1}{2})m\sigma,\tag{A.28}
$$

which increases in m and r.

Combining all three cases, we conclude that if $s_{m,1}<\bar{s}$, then $s_{m,1}$ increases in both m and r.

Step 3: We prove that given $m\in[0,\bar{m}]$, there exists a threshold $r_{m,1}$ such that $s_{m,1}<\bar{s}$ if $r<r_{m,1}$. To construct the threshold $r_{m,1}$, we let $r'_{m,1}\in[\frac{1}{2},1]$ denote the lowest index accuracy such that $s_{m,1}=\bar{s}$. Such $r'_{m,1}$ exists because we have $s_{m,1}=\bar{s}$ when $r=1$ (see Step 4). Therefore, for any $r<r'_{m,1}$, we have $s_{m,1}<\bar{s}$, and thus letting $r_{m,1}=r'_{m,1}$ is sufficient for the conclusion in Step 3 to hold. To gain further insights into when $r'_{m,1}$ is guaranteed to be strictly higher than $\frac{1}{2}$, we next study the relationship between $s_{m,1}$ and $\bar{s}$ when $r=\frac{1}{2}$. If $s_{m,1}<\bar{s}$ when $r=\frac{1}{2}$, then $r'_{m,1}>\frac{1}{2}$; If $s_{m,1}=\bar{s}$ when $r=\frac{1}{2}$, then $r'_{m,1}=\frac{1}{2}$. Given $r=\frac{1}{2}$, consider the following three cases:

Case 1: Suppose $x^*(m,s_{m,1})\le x_1$. In this case, as shown in Step 2, the floor price m is inactive for any $s\in[0,\bar{s}]$, and we have $s_{m,1}=s_1$ as defined in Proposition 1. Given $r=\frac{1}{2}$, we know from the proof of Proposition 1 that $s_1=\frac{1}{\lambda}<\bar{s}$ if and only if $r_1>\frac{1}{2}$. Therefore, we have $s_{m,1}<\bar{s}$ if and only if $r_1>\frac{1}{2}$. That is, we have $r'_{m,1}>\frac{1}{2}$ if and only if $r_1>\frac{1}{2}$.

Case 2: Suppose $x^*(m, s_{m,1}) \in (x_1, x_2]$. Given $r = \frac{1}{2}$, if $s_{m,1} < \bar{s}$, we know from Equation (A.26) that $s_{m,1} = \frac{1}{\lambda} + 2(r - \frac{1}{2})[m(\mu+\sigma) - R_L(x^*(m, s_{m,1}))] = \frac{1}{\lambda}$. Then, similar to Case 1, we have $r'_{m,1} > \frac{1}{2}$ if and only if $r_1 > \frac{1}{2}$.

Case 3: Suppose $x^*(m, s_{m,1}) > x_2$. Given $r = \frac{1}{2}$, if $s_{m,1} < \bar{s}$, we know from Equation (A.28) that $s_{m,1} = \frac{1}{\lambda} + 4(r - \frac{1}{2})m\sigma = \frac{1}{\lambda}$. Then, similar to Case 1, we have $r'_{m,1} > \frac{1}{2}$ if and only if $r_1 > \frac{1}{2}$.

Step 4: We prove that given $m \in [0, \bar{m}]$, there exists a threshold $r_{m,2}$ such that $s_{m,1} = \bar{s}$ if $r \geq r_{m,2}$. Let $r'_{m,2} \in [\frac{1}{2}, 1]$ denote the lowest index accuracy such that $s_{m,1} = \bar{s}$ for any $r \geq r'_{m,2}$. If such $r'_{m,2}$ exists, then letting $r_{m,2} = r'_{m,2}$ is sufficient for the conclusion in Step 4 to hold. We next prove the existence of $r'_{m,2}$ and further show that $r'_{m,2} < 1$. First, we prove the existence of $r'_{m,2} \in [\frac{1}{2}, 1]$. To do so, it is sufficient to prove that when $r = 1$, we have $s_{m,1} = \bar{s}$. Given $r = 1$, consider the following three cases:

Case 1: Suppose $x^*(m, s_{m,1}) \leq x_1$. In this case, as shown in Step 2, the floor price m is inactive for any $s \in [0, \bar{s}]$, and we have $s_{m,1} = s_1$ as defined in Proposition 1. Given $r = 1$, we know from the proof of Proposition 1 that $s_1 = \bar{s}$. Thus, we have $s_{m,1} = \bar{s}$.

Case 2: Suppose $x^*(m, s_{m,1}) \in (x_1, x_2]$. We prove $s_{m,1} = \bar{s}$ by contradiction. Suppose $s_{m,1} < \bar{s}$. Then, by Lemma A.5, we have $s_{m,1} < m(\mu+\sigma) - R_L(x^*(m, s_{m,1}))$. On the other hand, given $r = 1$ and $s_{m,1} < \bar{s}$, by Equation (A.26), we have $s_{m,1} = \frac{1}{\lambda} + 2(r - \frac{1}{2})[m(\mu+\sigma) - R_L(x^*(m, s_{m,1}))] > m(\mu+\sigma) - R_L(x^*(m, s_{m,1}))$, which leads to a contradiction. Hence, we have $s_{m,1} = \bar{s}$.

Case 3: Suppose $x^*(m, s_{m,1}) > x_2$. We prove $s_{m,1} = \bar{s}$ by contradiction. Suppose $s_{m,1} < \bar{s}$. Then, by Lemma A.5, we have $s_{m,1} < m(\mu+\sigma) - m(\mu-\sigma) = 2m\sigma$. On the other hand, given $r = 1$ and $s_{m,1} < \bar{s}$, by Equation (A.28), we have $s_{m,1} = \frac{1}{\lambda} + 4(r - \frac{1}{2})m\sigma > 2m\sigma$, which leads to a contradiction. Hence, we have $s_{m,1} = \bar{s}$.

Combining all three cases, we conclude that $s_{m,1} = \bar{s}$ when $r = 1$. This result implies the existence of $r'_{m,2} \in [\frac{1}{2}, 1]$.

Next, we prove that $r'_{m,2} < 1$. Recall from the proof of Lemma A.4 that when $c \geq c_3$, the equilibrium planting amount is given by $x^*(m, s) = \hat{x}(m, s)$. Following a similar analysis as in the proof of Proposition 1, it can be shown that for any given $m \in [0, \bar{m}]$, there exists a threshold $\hat{s} > 0$ such that $\frac{\partial}{\partial s}\hat{x}(m, s)|_{s=\hat{s}} = 0$. Then, we have $s_{m,1} = \hat{s}$ if $\hat{s} < \bar{s}$, and we have $s_{m,1} = \bar{s}$ if $\hat{s} \geq \bar{s}$.

Suppose $r'_{m,2} = 1$. Since $\hat{s}$ and $\bar{s}$ are continuous in r, by the definition of $r'_{m,2}$, we have $s_{m,1} = \hat{s} = \bar{s}$ when $r = r'_{m,2} = 1$. Then, we have $\frac{\partial}{\partial s}x^*(m, s)|_{s=s_{m,1}} = 0$ when $r = 1$. Then, following a similar analysis as in the proof of Proposition 1, for the three cases where $x^*(m, s_{m,1}) \leq x_1$, $x^*(m, s_{m,1}) \in (x_1, x_2]$, and $x^*(m, s_{m,1}) > x_2$, we have Equations (A.15), (A.26), and (A.28), respectively. In each case, the corresponding equation implies $s_{m,1} > \bar{s}$, which leads to a contradiction. Therefore, we have $r'_{m,2} < 1$.

Combining all four steps completes the proof of the lemma when $\sigma_\epsilon = 0$. We next prove the lemma when $\sigma_\epsilon > 0$.

When $\sigma_\epsilon > 0$, since the expression of $x^*(m,s)$ remains strictly concave in s in each of the three cases where $x^*(m,s) \le x_1$, $x^*(m,s) \in (x_1, x_2]$, and $x^*(m,s) > x_2$, respectively, it is straightforward to check that Step 1 of the proof remains valid. Moreover, our analysis in Step 4 for $s_{m,1} = \bar{s}$ when $r = 1$ continues to hold for $\sigma_\epsilon > 0$. Therefore, $r'_{m,1}$ (defined in Step 3) and $r'_{m,2}$ (defined in Step 4) remain well-defined, and thus Steps 3 and 4 of the proof remain valid.

It remains to prove Step 2 for $\sigma_\epsilon > 0$. Consider any given $m \in [0, \bar{m}]$ and $r \in [\frac{1}{2}, 1]$. Suppose $s_{m,1} < \bar{s}$. If $x^*(m, s_{m,1}) \le x_1$, as discussed in Case 1 of Step 2 above, we have $s_{m,1} = s_1$, which increases in r and is independent of m. If $x^*(m, s_{m,1}) > x_2$, Equation (A.28) continues to hold for $\sigma_\epsilon > 0$, and thus $s_{m,1}$ increases in r and m. If $x^*(m, s_{m,1}) \in (x_1, x_2]$, Equation (A.26) continues to hold for $\sigma_\epsilon > 0$, and thus $s_{m,1}$ increases in r; it remains to study the sign of $\frac{\partial}{\partial m} s_{m,1}$. As discussed earlier in Step 2 of the proof, given $s_{m,1} < \bar{s}$, to show $\frac{\partial}{\partial m} s_{m,1} \ge 0$, it is sufficient to show $\frac{\partial^2}{\partial m \partial s} x^*(m,s)|_{s=s_{m,1}} \ge 0$. When $x^*(m, s_{m,1}) \in (x_1, x_2]$, by Equation (A.27), we have

$$\begin{aligned}
&\frac{\partial^2}{\partial m \partial s} x^*(m,s)\Big|_{s=s_{m,1}} \\
=&\frac{1}{4\alpha_m}\left(\Delta^{-\frac{1}{2}}(m, s_{m,1}) \times \frac{\partial^2}{\partial m \partial s}\Delta(m,s)\Big|_{s=s_{m,1}} + \frac{\partial}{\partial m}\Delta(m,s)\Big|_{s=s_{m,1}} \frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)\Big|_{s=s_{m,1}}\right).
\end{aligned}$$

Since $\Delta(m,s) = \beta^2(m,s) - 4\alpha_m \gamma(m,s)$, we have

$$\frac{\partial^2}{\partial m \partial s}\Delta(m,s) = 2\lambda^2 (r - \frac{1}{2}) b^2 (\mu + \sigma)(\mu - \sigma)^2 \sigma_\epsilon^2 \ge 0,$$

which implies that

$$\Delta^{-\frac{1}{2}}(m,s) \times \frac{\partial^2}{\partial m \partial s}\Delta(m,s) \ge 0. \tag{A.29}$$

Through the same analysis in Case 2 of Step 2 above, we also have $\frac{\partial}{\partial s}\Delta^{-\frac{1}{2}}(m,s)|_{s=s_{m,1}} \ge 0$. Moreover, since we have

$$\begin{aligned}
\frac{\partial}{\partial m}\Delta(m,s)\Big|_{s=s_{m,1}} =& 2\lambda b (\mu - \sigma)^2 \left[\frac{1}{2}c(\mu+\sigma) + \frac{1}{2}b(\mu+\sigma)(\mu-\sigma)^2\right. \\
&\left. + b\sigma_\epsilon^2 \left(\frac{1}{2}(\mu+\sigma) - \lambda m(\mu^2+\sigma^2) + \lambda(\mu+\sigma)\left(r - \frac{1}{2}\right) s_{m,1} - \lambda m \sigma_\epsilon^2\right)\right],
\end{aligned}$$

there must exist $\bar{\sigma}_\epsilon > 0$ (independent of r) such that if $\sigma_\epsilon < \bar{\sigma}_\epsilon$, we have $\frac{\partial}{\partial m}\Delta(m,s)|_{s=s_{m,1}} \ge 0$. Thus, if $\sigma_\epsilon < \bar{\sigma}_\epsilon$, we have $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}} \ge 0$, which implies $\frac{\partial}{\partial m}s_{m,1} \ge 0$. If $\sigma_\epsilon \ge \bar{\sigma}_\epsilon$, then analytically characterizing the sign of $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}}$ becomes challenging. To address this, we next define a new threshold $r_{m,1}$ for the lemma to hold for a general σ_ϵ. In particular, let $r''_{m,1}$ denote the lowest index accuracy (or the infimum if the lowest is not attained) within $[\frac{1}{2}, r'_{m,1}]$ such that $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}} < 0$ (if no such index accuracy exists, we let $r''_{m,1} = r'_{m,1}$). Then, we have $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}} \ge 0$ if $r < r''_{m,1}$. Thus, letting $r_{m,1} := r''_{m,1}$ and $r_{m,2} := r'_{m,2}$ is sufficient for the lemma to hold. Further, based on our analysis above, we know that $r''_{m,1} = r'_{m,1}$ when $\sigma_\epsilon < \bar{\sigma}_\epsilon$.

When σ_ϵ is higher, we may have $r''_{m,1} < r'_{m,1}$. However, we next prove that $r''_{m,1}$ is lower bounded by another threshold.

For any $r < r'_{m,1}$, we have $\frac{\partial}{\partial s}x^*(m,s)|_{s=s_{m,1}} = 0$. Thus,

$$\frac{\mathrm{d}x^*(m,s_{m,1})}{\mathrm{d}r} = \frac{\partial x^*(m,s)}{\partial s}\Big|_{s=s_{m,1}}\frac{\partial s_{m,1}}{\partial r} + \frac{\partial x^*(m,s)}{\partial r}\Big|_{s=s_{m,1}} = \frac{\partial x^*(m,s)}{\partial r}\Big|_{s=s_{m,1}} \geq 0,$$

where the inequality follows from part (i) of Lemma A.6. Moreover, x_1 is independent of r. Thus, there exists a threshold $r'''_{m,1} \in [\frac{1}{2}, r'_{m,1}]$ such that $x^*(m,s_{m,1}) < x_1$ for any $r < r'''_{m,1}$. When $x^*(m,s_{m,1}) < x_1$, we have $\frac{\partial^2}{\partial m \partial s}x^*(m,s)|_{s=s_{m,1}} = 0$. Therefore, we must have $r''_{m,1} \geq r'''_{m,1}$. Moreover, since $x^*(m,s_{m,1})$ decreases in λ while x_1 is independent of λ, if λ is higher than a certain threshold, then we have $x^*(m,s_{m,1}) < x_1$ at $r = \frac{1}{2}$. By continuity, this inequality continues to hold for r sufficiently close to $\frac{1}{2}$, implying $r'''_{m,1} > \frac{1}{2}$.

Before concluding the proof, we note that, in order to use the same thresholds $r_{m,1}$ and $r_{m,2}$ in this lemma and Proposition 3, we will update the definitions of $r_{m,1}$ and $r_{m,2}$ within the proof of Proposition 3. $\square$

Proof of Proposition 3. By the expression of the net benefit $v(m,s)$ introduced in §5, given $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, we have

$$\begin{aligned}
v(m,s) =& \mathbb{E}\left[\int_0^{x^*(m,s)} (\max\{(a - bx^*(m,s)Y), m\} \times Y + sI - xc)\,\mathrm{d}x\right] - x^*(m,s)\mathbb{E}[sI] \\
&- x^*(m,s)\mathbb{E}[\max\{m - (a - bx^*(m,s)Y), 0\} \times Y] \\
=& v_1 \times x^*(m,s) - v_2 \times (x^*(m,s))^2.
\end{aligned}$$

where $v_1 = a\mu$ and $v_2 = b(\mu^2 + \sigma^2) + \frac{1}{2}c$. We have $x^{opt} = \frac{v_1}{2v_2}$ as the value of $x^*(m,s)$ that maximizes $v(m,s)$. Thus, given m, the optimal $s^*(m)$ must make $x^*(m,s)$ as close to x^{opt} as possible.

Given $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, let $\omega(m,s)$ denote the total expected government expenditure. That is, $\omega(m,s) = x^*(m,s)\mathbb{E}[sI] + x^*(m,s)\mathbb{E}[\max\{m - (a - bx^*(m,s)Y), 0\} \times Y]$. With a slight abuse of notation, let $\mathcal{S} = [0,\bar{s}] \cap \{s : \omega(m,s) \leq B\}$ denote the feasible set of s. (Note that the set $\mathcal{S}$ depends on r and m; for notational simplicity, we omit these dependencies.) Without loss of generality, we focus on floor prices m for which $\mathcal{S}$ is non-empty. Since we consider m such that $x^*(m,0) \leq x^{opt}$, $s^*(m)$ must satisfy either (i) $s^*(m) = \arg\max_{s \in \mathcal{S}} x^*(m,s)$ and $x^*(m,s^*(m)) < x^{opt}$, or (ii) $s^*(m) = \inf\{s \in \mathcal{S} : x^*(m,s) = x^{opt}\}$.

Step 1: We prove that there exists a threshold $r_{m,1} \in [\frac{1}{2}, 1]$ such that $\frac{\partial}{\partial m}s^*(m) \geq 0$ if $r < r_{m,1}$. First, we construct the threshold $r_{m,1}$. As shown in the proof of Lemma A.7, when $r < r''_{m,1}$, we have $\frac{\partial x^*(m,s)}{\partial s}|_{s=s_{m,1}} = 0$ and $s_{m,1}$ increases in r, where $s_{m,1}$ is defined as the maximizer of $x^*(m,s)$ among all $s \in [0,\bar{s}]$. Moreover, as shown in part (i) of Lemma A.6, $x^*(m,s)$ increases in r. Thus, when $r < r''_{m,1}$, we have that $\omega(m, s_{m,1})$ increases in r. Then, there must exist $r_{m,b} \in [\frac{1}{2}, r''_{m,1}]$ such that for any $r < r_{m,b}$, we have $\omega(m, s_{m,1}) < B$.

Recall that the equilibrium planting amount is increasing in the index accuracy by part (i) of Lemma A.6. Let $r_{m,c} \in [\frac{1}{2}, 1]$ denote the lowest index accuracy under which there exists some $s \in \mathcal{S}$ such that $x^*(m,s) = x^{opt}$. (Let $r_{m,c} = 1$ if no such index accuracy exists.) Define $r_{m,1} := \min\{r_{m,b}, r_{m,c}\}$.

Consider $r < r_{m,1}$. Since $r < r_{m,1} \le r_{m,c}$, $s^*(m)$ must satisfy $x^*(m, s^*(m)) < x^{opt}$ and $s^*(m) = \arg\max_{s \in \mathcal{S}} x^*(m,s)$. Moreover, since $r < r_{m,1} \le r_{m,b}$, we have $s^*(m) = s_{m,1}$. We have also shown in Lemma A.7 that $\frac{\partial}{\partial m} s_{m,1} \ge 0$ when $r < r_{m,1} \le r''_{m,1}$. Therefore, we have $\frac{\partial}{\partial m} s^*(m) \ge 0$ if $r < r_{m,1}$.

Step 2: We prove that there exists a threshold $r_{m,2} \in [\frac{1}{2}, 1]$ such that $\frac{\partial}{\partial m} s^*(m) \le 0$ if $r > r_{m,2}$. In order to construct the threshold $r_{m,2}$, we let $r''_{m,2} \in [\frac{1}{2}, 1]$ denote the highest index accuracy (or the supremum if the highest is not attained) such that $\frac{\partial}{\partial m} x^*(m,s)|_{s=s^*(m)} < 0$ (let $r''_{m,2} = \frac{1}{2}$ if no such index accuracy exists). Then, for any $r > r''_{m,2}$, we have $\frac{\partial}{\partial m} x^*(m,s)|_{s=s^*(m)} \ge 0$. We later discuss in detail when $r''_{m,2}$ is guaranteed to be strictly less than 1.

Recall that the equilibrium planting amount is increasing in the index accuracy by part (i) of Lemma A.6. Let $r'_{m,c} \in [\frac{1}{2}, 1]$ denote the lowest index accuracy under which there exists some $s \in \mathcal{S}$ such that $x^*(m,s) = x^{opt}$ for all $r > r'_{m,c}$. Define $r_{m,2} := \max\left\{r''_{m,2}, r'_{m,c}\right\}$. We now prove when $r > r_{m,2}$, we have $\frac{\partial}{\partial m} s^*(m) \le 0$. Given $r > r_{m,2} \ge r'_{m,c}$, following similar logic as in Case 1 of the proof of Theorem 1, we have that $s^*(m)$ must satisfy $s^*(m) = \inf\{s : x^*(m,s) = x^{opt}\}$. Since we consider $x^*(m,0) \le x^{opt}$ and $s^*(m)$ is the lowest subsidy amount that satisfies $x^*(m, s^*(m)) = x^{opt}$, we must have $x^*(m,s)$ increases in s at $s = s^*(m)$. Moreover, since x^{opt} is independent of m and we have $\frac{\partial}{\partial m} x^*(m,s)|_{s=s^*(m)} \ge 0$ when $r > r_{m,2} \ge r''_{m,2}$, $s^*(m)$ must decrease in m to maintain $s^*(m) = \inf\{s : x^*(m,s) = x^{opt}\}$.

Before concluding the proof, we now discuss when $r''_{m,2}$ is guaranteed to be strictly less than 1. In particular, we show that $r''_{m,2} < 1$ when λ is low or σ_ϵ is low. We note that it is reasonable to construct $r''_{m,2} < 1$ only for relatively small σ_ϵ. Specifically, recall that the key insight for the second part of the proposition is that price and yield protection are substitutes (i.e., $\frac{\partial}{\partial m} s^*(m) \le 0$) when the index is sufficiently accurate. If σ_ϵ is too large, the index does not accurately predict yields even when $r = 1$; in such cases, the two policies may function as complements even when $r = 1$.

To prove $r''_{m,2} < 1$ when λ is low or σ_ϵ is low, it is sufficient to prove that when λ is low or σ_ϵ is low, we have $\frac{\partial}{\partial m} x^*(m,s) \ge 0$ for any $r \in [\frac{1}{2}, 1]$ and $s \in [0, \bar{s}]$. Since if $m = 0$, $x^*(m,s) = x^*(s)$ and the conclusion is immediate, we only consider $m > 0$ in the following. Specifically, consider $\lambda \le \min\{\frac{\mu+\sigma}{m(\mu+\sigma)^2 + 2m\sigma_\epsilon^2}, \frac{\mu}{2m\sigma^2 + 2m\sigma_\epsilon^2}\}$ (note that this condition is easier to satisfy when σ_ϵ is lower). Fix an $r \in [\frac{1}{2}, 1]$ and an $s \in [0, \bar{s}]$. By Lemma A.4, there exist two thresholds m_1 and m_2 that indicate whether the floor price m is active. Accordingly, we consider the following three cases:

Case 1: Suppose $m \le m_1$. Then, by Lemma A.4, we have $x^*(m,s) = x^*(s)$ and $\frac{\partial}{\partial m} x^*(m,s) = 0$.

Case 2: Suppose $m \in (m_1, m_2]$. To prove $\frac{\partial}{\partial m} x^*(m,s) \ge 0$, following the same logic as in the proof of Lemma A.1, it is sufficient to prove that $\frac{\partial}{\partial m} u(x|x,m,s)|_{x=x^*(m,s)} \ge 0$. By Equation (A.22), we have

$$\begin{aligned}\frac{\partial}{\partial m} u(x|x,m,s)\Big|_{x=x^*(m,s)} &= (\mu+\sigma)\left(\frac{1}{2} - \frac{1}{2}\lambda(m(\mu+\sigma) - R_L(x^*(m,s))) + \lambda\left(r - \frac{1}{2}\right)s\right) - \lambda m\sigma_\epsilon^2 \\ &\ge (\mu+\sigma)\left(\frac{1}{2} - \frac{1}{2}\lambda m(\mu+\sigma)\right) - \lambda m\sigma_\epsilon^2 \\ &\ge 0,\end{aligned}$$

where the first inequality holds because $R_L(x^*(m,s)) \ge 0$ and $\lambda(r - \frac{1}{2})s \ge 0$, and the second inequality holds given that $\lambda \le \frac{\mu+\sigma}{m(\mu+\sigma)^2 + 2m\sigma_\epsilon^2}$.

Case 3: Suppose $m > m_2$. By Equation (A.25), we have

$$\frac{\partial}{\partial m} x^*(m,s) = \frac{\mu - 2\lambda m(\sigma^2 + \sigma_\epsilon^2) + 2\lambda\sigma\left(r - \frac{1}{2}\right)s}{c} \ge \frac{\mu - 2\lambda m(\sigma^2 + \sigma_\epsilon^2)}{c} \ge 0,$$

where the last inequality holds given that $\lambda \le \frac{\mu}{2m\sigma^2 + 2m\sigma_\epsilon^2}$. Combining all three cases, we conclude that when λ is low or σ_ϵ is low, if r is higher than a threshold strictly less than 1, then we have $\frac{\partial}{\partial m} x^*(m,s) \ge 0$ for any $s \in [0, \bar{s}]$. $\square$

Before the proof of Theorem 2, we prove the following auxiliary lemma as a building block. Let $(\hat{m}, \hat{s}) \in \underset{m\in[0,\bar{m}], s\in[0,\bar{s}]}{\arg\max}\, x^*(m,s)$. (If there are multiple joint optimizers, we adopt the same tie-breaking rule as for (m^*, s^*).) Let $\hat{m}_0 = \inf\{\underset{m\in[0,\bar{m}]}{\arg\max}\, x^*(m,0)\}$, and let $\hat{s}_0 = \underset{s\in[0,\bar{s}]}{\arg\max}\, x^*(0,s)$. Note that $\hat{s}_0$ is the same as s_1 defined in Proposition 1, and we use $\hat{s}_0$ here to align with the notation for $\hat{m}_0$. For any given planting amount $x \in (0,1)$, let $P_H(x) = a - bx(\mu+\sigma)$ and $P_L(x) = a - bx(\mu-\sigma)$ denote the market prices when $Y = \mu+\sigma$ and $Y = \mu-\sigma$, respectively.

Lemma A.8. *(i) For any $r \in [\frac{1}{2}, 1]$ such that $\hat{s} < \bar{s}$ and $\hat{s}_0 < \bar{s}$, we have $\hat{s} \ge \hat{s}_0$.*
(ii) When $r = \frac{1}{2}$, if $\hat{m} = 0$, we have $\hat{m}_0 = 0$.
(iii) When $r = \frac{1}{2}$, if $\hat{s} < \bar{s}$, $\hat{s}_0 < \bar{s}$, and $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$, then we have $c < 2\alpha x^*(0,\hat{s}_0) + \beta_0$, where $\alpha = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$ and $\beta_0 = b(\mu^2+\sigma^2) + c - 4\lambda ab\mu\sigma^2 - 2\lambda ab\mu\sigma_\epsilon^2$ are defined in Lemma 2.
(iv) For any $r \ge \frac{1}{2}$ such that $\hat{s} < \bar{s}$, $\hat{s}_0 < \bar{s}$, $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$, and $\hat{m} \ge P_L(x^*(0,\hat{s}_0))$, we have $c < 2\alpha x^*(0,\hat{s}_0) + \beta(\hat{s}_0)$.

Proof of Lemma A.8. (i) Given that $\hat{s}_0 < \bar{s}$, by Equation (A.15), we have

$$\hat{s}_0 = \frac{1}{\lambda} + 2\left(r - \frac{1}{2}\right)[R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))] \le \frac{1}{\lambda} + 2\left(r - \frac{1}{2}\right)[R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s}))],$$

where $R_H(x) = P_H(x)(\mu+\sigma)$, $R_L(x) = P_L(x)(\mu-\sigma)$, and the inequality holds because $R_H(x) - R_L(x) = 2a\sigma - 4bx\mu\sigma$ is a decreasing function of x and we have $x^*(0,\hat{s}_0) \geq x^*(0,\hat{s})$ by the definition of $\hat{s}_0$. Moreover, given that $\hat{s} < \bar{s}$, by Equations (A.15), (A.26), and (A.28), we have

$$\hat{s} = \frac{1}{\lambda} + 2\left(r - \frac{1}{2}\right)[\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma)].$$

Thus, to prove that $\hat{s} \geq \hat{s}_0$, it is sufficient to prove that $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) \geq R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s}))$. To do so, we consider the following three cases:

If $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$, then we have $x^*(\hat{m},\hat{s}) = x^*(0,\hat{s})$. Therefore, we have $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) = P_H(x^*(\hat{m},\hat{s}))(\mu+\sigma) = R_H(x^*(\hat{m},\hat{s})) = R_H(x^*(0,\hat{s}))$. Similarly, we have $\max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) = R_L(x^*(0,\hat{s}))$. Hence, we have

$$\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) = R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s})).$$

If $P_L(x^*(\hat{m},\hat{s})) \geq \hat{m} > P_H(x^*(\hat{m},\hat{s}))$, then we must have $\hat{m} > P_H(x^*(0,\hat{s}))$ (because in Step 4 of the proof of Lemma A.4, we have proven that for any given $m \in [0,\bar{m}]$ and $s \in [0,\bar{s}]$, we have $m \leq P_H(x^*(0,s))$ if and only if $m \leq P_H(x^*(m,s))$; thus, if $\hat{m} \leq P_H(x^*(0,\hat{s}))$, we would have $\hat{m} \leq P_H(x^*(\hat{m},\hat{s}))$). Therefore, we have $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) = \hat{m}(\mu+\sigma) > P_H(x^*(0,\hat{s}))(\mu+\sigma) = R_H(x^*(0,\hat{s}))$. Moreover, we have $x^*(\hat{m},\hat{s}) \geq x^*(0,\hat{s})$ and thus $\max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) = P_L(x^*(\hat{m},\hat{s}))(\mu-\sigma) = R_L(x^*(\hat{m},\hat{s})) \leq R_L(x^*(0,\hat{s}))$. Hence, we have

$$\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) > R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s})).$$

If $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$, then we consider the following two cases: On the one hand, if $\hat{m} \leq P_L(x^*(0,\hat{s}))$, then we have $\max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) = \hat{m}(\mu-\sigma) \leq R_L(x^*(0,\hat{s}))$. Moreover, as discussed above, given that $\hat{m} > P_H(x^*(\hat{m},\hat{s}))$, we have $\hat{m} > P_H(x^*(0,\hat{s}))$, and thus $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) = \hat{m}(\mu+\sigma) > R_H(x^*(0,\hat{s}))$. Hence, we have

$$\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) > R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s})).$$

On the other hand, if $\hat{m} > P_L(x^*(0,\hat{s}))$, then we have

$$\begin{aligned}
&\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma)\\
=&\hat{m}(\mu+\sigma) - \hat{m}(\mu-\sigma)\\
>&P_L(x^*(0,\hat{s}))(\mu+\sigma) - P_L(x^*(0,\hat{s}))(\mu-\sigma)\\
>&R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s})),
\end{aligned}$$

where the second inequality holds because $P_L(x^*(0,\hat{s})) > P_H(x^*(0,\hat{s}))$.

Combining all three cases, we have $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma)-\max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma)\geq R_H(x^*(0,\hat{s}))-R_L(x^*(0,\hat{s}))$, and thus $\hat{s}\geq\hat{s}_0$. This completes the proof of part (i).

(ii) Consider $r=\frac{1}{2}$. Suppose $\hat{m}=0$. In order to prove $\hat{m}_0=0$, it is sufficient to prove that neither $P_L(x^*(\hat{m}_0,0))\geq\hat{m}_0>P_H(x^*(\hat{m}_0,0))$ nor $\hat{m}_0>P_L(x^*(\hat{m}_0,0))>P_H(x^*(\hat{m}_0,0))$ can hold.

First, we prove by contradiction that we cannot have $P_L(x^*(\hat{m}_0,0))\geq\hat{m}_0>P_H(x^*(\hat{m}_0,0))$. By the definition of $\hat{m}_0$, we must have $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0}\geq 0$. Suppose $P_L(x^*(\hat{m}_0,0))\geq\hat{m}_0>P_H(x^*(\hat{m}_0,0))$. We next prove that $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0}<0$, which leads to a contradiction.

Given that $\hat{m}=0$, by the definition of $\hat{m}$, we have $x^*(\hat{m},\hat{s})=x^*(0,\hat{s})\geq x^*(m,\hat{s})$ for any $m\in[0,\bar{m}]$. More specifically, let $m'=P_H(x^*(0,\hat{s}))$, then for $m\in[0,m']$, $x^*(m,\hat{s})=x^*(0,\hat{s})$ is independent of m. When m further increases from $m=m'$, $x^*(m,\hat{s})$ must start to decrease, i.e., the (right) derivative of $x^*(m,s)$ with respect to m must satisfy $\frac{\partial}{\partial m}x^*(m,s)|_{m=m',s=\hat{s}}\leq 0$; otherwise, if $\frac{\partial}{\partial m}x^*(m,s)|_{m=m',s=\hat{s}}>0$, then there must exist an $m>m'$ such that $x^*(m,\hat{s})>x^*(0,\hat{s})$, which contradicts $\hat{m}=0$. Therefore, following the same logic as in the proof of Lemma A.1 and by Equation (A.22), the (right) derivative of $u(x|x,m,s)$ with respect to m must satisfy

$$\begin{aligned}
&\frac{\partial}{\partial m}u(x|x,m,s)\Big|_{x=x^*(m,s),m=m',s=\hat{s}}\\
=&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(m'(\mu+\sigma)-R_L(x^*(m',\hat{s})))\right)-\lambda m'\sigma_\epsilon^2\\
=&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(R_H(x^*(0,\hat{s}))-R_L(x^*(m',\hat{s})))\right)-\lambda m'\sigma_\epsilon^2\\
=&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(R_H(x^*(0,\hat{s}))-R_L(x^*(0,\hat{s})))\right)-\lambda m'\sigma_\epsilon^2\\
\leq&0,
\end{aligned}\tag{A.30}$$

where the last equality holds because we have $x^*(m,\hat{s})=x^*(0,\hat{s})$ for any $m\in[0,m']$ (as discussed above). Further, because $x^*(0,\hat{s})=x^*(\hat{m},\hat{s})\geq x^*(0,0)$, we have $R_H(x^*(0,\hat{s}))-R_L(x^*(0,\hat{s}))\leq R_H(x^*(0,0))-R_L(x^*(0,0))$. Therefore, we have

$$R_H(x^*(0,\hat{s}))-R_L(x^*(0,\hat{s}))\leq R_H(x^*(0,0))-R_L(x^*(0,0))<\hat{m}_0(\mu+\sigma)-R_L(x^*(\hat{m}_0,0)),\tag{A.31}$$

where the second inequality holds because (1) as discussed in the proof of part (i), given $\hat{m}_0>P_H(x^*(\hat{m}_0,0))$, we must have $\hat{m}_0>P_H(x^*(0,0))$ and thus $\hat{m}_0(\mu+\sigma)>P_H(x^*(0,0))(\mu+\sigma)=R_H(x^*(0,0))$, and (2) $x^*(\hat{m}_0,0)\geq x^*(0,0)$, and thus $R_L(x^*(\hat{m}_0,0))\leq R_L(x^*(0,0))$. Moreover, since $x^*(0,\hat{s})\geq x^*(\hat{m}_0,0)$, we have $m'=P_H(x^*(0,\hat{s}))\leq P_H(x^*(\hat{m}_0,0))<\hat{m}_0$. Hence,

$$\begin{aligned}
&\frac{\partial}{\partial m}u(x|x,m,s)\Big|_{x=x^*(m,s),m=\hat{m}_0,s=0}\\
=&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(\hat{m}_0(\mu+\sigma)-R_L(x^*(\hat{m}_0,0)))\right)-\lambda\hat{m}_0\sigma_\epsilon^2\\
<&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(R_H(x^*(0,\hat{s}))-R_L(x^*(0,\hat{s})))\right)-\lambda m'\sigma_\epsilon^2\\
\leq&0,
\end{aligned}$$

where the first inequality follows from Equation (A.31), and the second inequality follows from Equation (A.30). Therefore, we have $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0} < 0$, which leads to a contradiction.

Next, we prove by contradiction that we cannot have $\hat{m}_0 > P_L(x^*(\hat{m}_0,0)) > P_H(x^*(\hat{m}_0,0))$. Suppose $\hat{m}_0 > P_L(x^*(\hat{m}_0,0)) > P_H(x^*(\hat{m}_0,0))$. Then, for any $s \in [0,\hat{s}]$, we must have $\hat{m}_0 > P_L(x^*(\hat{m}_0,s)) > P_H(x^*(\hat{m}_0,s))$ (because given $r = \frac{1}{2}$ and $\hat{m} = 0$, we know from Equation (A.15) that the maximizer of $x^*(0,s)$ is $\hat{s} = \min\{\frac{1}{\lambda}, \bar{s}\}$; then, from Equation (A.25), we can see that the equilibrium planting amount $x^*(\hat{m}_0,s)$ increases in s when $s \in [0,\hat{s}]$). Therefore, for any $s \in [0,\hat{s}]$,

$$x^*(\hat{m}_0,s) = \frac{\hat{m}_0\mu - \lambda\hat{m}_0^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s - \frac{1}{4}\lambda s^2}{c}. \tag{A.32}$$

Thus, we have

$$\frac{\partial}{\partial s}x^*(\hat{m}_0,s) = \frac{\frac{1}{2} - \frac{1}{2}\lambda s}{c}.$$

On the other hand, from Equation (A.7), we have, when $r = \frac{1}{2}$,

$$\begin{aligned}\frac{\partial}{\partial s}x^*(0,s) &= -\frac{\partial u(x|x,0,s)/\partial s}{\partial u(x|x,0,s)/\partial x}\Big|_{x=x^*(0,s)} \\ &= \frac{\frac{1}{2} - \frac{1}{2}\lambda s}{2\alpha x^*(0,s) + \beta_0},\end{aligned}$$

where $\alpha = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$ and $\beta_0 = b(\mu^2+\sigma^2) + c - 4\lambda ab\mu\sigma^2 - 2\lambda ab\mu\sigma_\epsilon^2$ are defined in Lemma 2.

Since $\hat{m} = 0$ and $\hat{m}_0 > P_L(x^*(\hat{m}_0,0))$, we must have $x^*(\hat{m}_0,\hat{s}) \le x^*(0,\hat{s})$ and $x^*(\hat{m}_0,0) > x^*(0,0)$. Then, given the continuity of $x^*(\hat{m}_0,s)$ and $x^*(0,s)$ in s, there must exist $s \in (0,\hat{s}]$ such that $x^*(\hat{m}_0,s) = x^*(0,s)$. Let s_c be the smallest such value, and denote $\check{x} = x^*(\hat{m}_0,s_c) = x^*(0,s_c)$. Suppose $s_c < \frac{1}{\lambda}$. We next prove that $\frac{\partial}{\partial s}x^*(\hat{m}_0,s)|_{s=s_c} > \frac{\partial}{\partial s}x^*(0,s)|_{s=s_c}$, which contradicts the minimality of s_c.

To prove $\frac{\partial}{\partial s}x^*(\hat{m}_0,s)|_{s=s_c} > \frac{\partial}{\partial s}x^*(0,s)|_{s=s_c}$, it is sufficient to prove that $c < 2\alpha\check{x} + \beta_0$. Since $u(\check{x}|\check{x},0,s_c) = 0$ and $r = \frac{1}{2}$, by Equation (A.5), we have

$$\alpha\check{x}^2 + \beta_0\check{x} + \gamma_0 - \frac{1}{2}s_c + \frac{1}{4}\lambda s_c^2 = 0$$

where $\gamma_0 = \lambda a^2(\sigma^2+\sigma_\epsilon^2) - a\mu$. We also have, by Equation (A.32),

$$c\check{x} = \hat{m}_0\mu - \lambda\hat{m}_0^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}s_c - \frac{1}{4}\lambda s_c^2.$$

Then, combining two equations above, we have

$$\begin{aligned}(2\alpha\check{x} + \beta_0 - c)\check{x} &= \alpha\check{x}^2 - \gamma_0 - \hat{m}_0\mu + \lambda\hat{m}_0^2(\sigma^2+\sigma_\epsilon^2) \\ &= \alpha\check{x}^2 + (a-\hat{m}_0)\mu - \lambda(a^2-\hat{m}_0^2)(\sigma^2+\sigma_\epsilon^2) \\ &= \alpha\check{x}^2 + (a-\hat{m}_0)\left(\mu - \lambda(a+\hat{m}_0)(\sigma^2+\sigma_\epsilon^2)\right) \\ &= \alpha\check{x}^2 + (a-\hat{m}_0)\left(\mu - 2\lambda\hat{m}_0(\sigma^2+\sigma_\epsilon^2)\right) - \lambda(a-\hat{m}_0)^2(\sigma^2+\sigma_\epsilon^2)\end{aligned}$$

Thus, to prove $c < 2\alpha\check{x} + \beta_0$, it is sufficient to prove that $\mu - 2\lambda\hat{m}_0(\sigma^2 + \sigma_\epsilon^2) \geq 0$ and $\alpha\check{x}^2 - \lambda(a - \hat{m}_0)^2(\sigma^2 + \sigma_\epsilon^2) > 0$. Due to the optimality of $\hat{m}_0$, we have

$$\frac{\partial}{\partial m} x^*(m, 0)\Big|_{m=\hat{m}_0} = \frac{\mu - 2\lambda\hat{m}_0(\sigma^2 + \sigma_\epsilon^2)}{c} \geq 0,$$

which implies $\mu - 2\lambda\hat{m}_0(\sigma^2 + \sigma_\epsilon^2) \geq 0$. Moreover, we have

$$\begin{aligned}
\alpha\check{x}^2 - \lambda(a - \hat{m}_0)^2(\sigma^2 + \sigma_\epsilon^2) &> \alpha\check{x}^2 - \lambda b^2\check{x}^2(\mu - \sigma)^2(\sigma^2 + \sigma_\epsilon^2) \\
&= \left(4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2 + \sigma^2)\sigma_\epsilon^2\right)\check{x}^2 - \lambda b^2\check{x}^2(\mu - \sigma)^2(\sigma^2 + \sigma_\epsilon^2) \\
&= \lambda b^2\check{x}^2\left(4\mu^2\sigma^2 + (\mu^2 + \sigma^2)\sigma_\epsilon^2 - (\mu - \sigma)^2(\sigma^2 + \sigma_\epsilon^2)\right) \\
&= \lambda b^2\check{x}^2\sigma(3\mu^2\sigma + 2\mu\sigma_\epsilon^2 + 2\mu\sigma^2 - \sigma^3) \\
&> 0.
\end{aligned}$$

Therefore, we have $c < 2\alpha\check{x} + \beta_0$.

Suppose $s_c = \frac{1}{\lambda}$. We have $\frac{\partial}{\partial s} x^*(\hat{m}_0, s)|_{s=s_c} = \frac{\partial}{\partial s} x^*(0, s)|_{s=s_c}$, and

$$\frac{\partial^2}{\partial s^2} x^*(\hat{m}_0, s)\Big|_{s=s_c} = -\frac{\lambda}{2c} < -\frac{\lambda}{2(2\alpha\check{x} + \beta_0)} = \frac{\partial^2}{\partial s^2} x^*(0, s)\Big|_{s=s_c}.$$

Then, we must have $x^*(\hat{m}_0, s) < x^*(0, s)$ for all s sufficiently close to s_c from the left, which contradicts the minimality of s_c. This completes the proof of part (ii).

(iii) Since $r = \frac{1}{2}$ and $\hat{s} < \bar{s}$, $\hat{s}_0 < \bar{s}$, by Equations (A.15) and (A.28), we have $\hat{s} = \hat{s}_0 = \frac{1}{\lambda}$. First, as a building block, we prove that $\hat{m} > P_L(x^*(0, \hat{s}_0))$. Since $\hat{m} > P_L(x^*(\hat{m}, \hat{s}))$, following the analysis in Step 4 of the proof of Lemma A.4, we cannot have $\hat{m} \leq P_H(x^*(0, \hat{s}_0))$. Therefore, it is sufficient to prove that we cannot have $P_H(x^*(0, \hat{s}_0)) < \hat{m} \leq P_L(x^*(0, \hat{s}_0))$ by contradiction. Suppose $P_H(x^*(0, \hat{s}_0)) < \hat{m} \leq P_L(x^*(0, \hat{s}_0))$. Then, by Equation (A.22), when $r = \frac{1}{2}$, we have

$$\begin{aligned}
&u(x^*(0, \hat{s}_0) \mid x^*(0, \hat{s}_0), \hat{m}, \hat{s}_0) - u(x^*(0, \hat{s}_0) \mid x^*(0, \hat{s}_0), 0, \hat{s}_0) \\
=&\frac{1}{2}\Big(\hat{m}(\mu + \sigma) - R_H(x^*(0, \hat{s}_0))\Big) \\
&- \frac{\lambda}{4}\Big(\big(\hat{m}(\mu + \sigma) - R_L(x^*(0, \hat{s}_0))\big)^2 - \big(R_H(x^*(0, \hat{s}_0)) - R_L(x^*(0, \hat{s}_0))\big)^2\Big) \\
&- \frac{\lambda}{2}\Big(\hat{m}^2 - P_H(x^*(0, \hat{s}_0))^2\Big)\sigma_\epsilon^2.
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
&\big(\hat{m}(\mu + \sigma) - R_L(x^*(0, \hat{s}_0))\big)^2 - \big(R_H(x^*(0, \hat{s}_0)) - R_L(x^*(0, \hat{s}_0))\big)^2 \\
=&2\big(R_H(x^*(0, \hat{s}_0)) - R_L(x^*(0, \hat{s}_0))\big)\big(\hat{m}(\mu + \sigma) - R_H(x^*(0, \hat{s}_0))\big) \\
&+ \big(\hat{m}(\mu + \sigma) - R_H(x^*(0, \hat{s}_0))\big)^2 \\
>&\frac{2}{\lambda}\big(\hat{m}(\mu + \sigma) - R_H(x^*(0, \hat{s}_0))\big) + \big(\hat{m}(\mu + \sigma) - R_H(x^*(0, \hat{s}_0))\big)^2,
\end{aligned}$$

where the inequality holds because by Equation (A.15) and Lemma A.5, we have $R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0)) > \hat{s}_0 = \frac{1}{\lambda}$. Therefore, combining the two equations above, we have

$$\begin{aligned} &u(x^*(0,\hat{s}_0) \mid x^*(0,\hat{s}_0), \hat{m}, \hat{s}_0) - u(x^*(0,\hat{s}_0) \mid x^*(0,\hat{s}_0), 0, \hat{s}_0) \\ &< -\frac{\lambda}{4}\big(\hat{m}(\mu+\sigma) - R_H(x^*(0,\hat{s}_0))\big)^2 - \frac{\lambda}{2}\big(\hat{m}^2 - P_H(x^*(0,\hat{s}_0))^2\big)\sigma_\epsilon^2 < 0. \end{aligned}$$

By the definition of $x^*(0,\hat{s}_0)$, we have $u(x^*(0,\hat{s}_0) \mid x^*(0,\hat{s}_0), 0, \hat{s}_0) = 0$. Thus we have $u(x^*(0,\hat{s}_0) \mid x^*(0,\hat{s}_0), \hat{m}, \hat{s}_0) < 0$, which implies $x^*(0,\hat{s}_0) > x^*(\hat{m},\hat{s}_0) = x^*(\hat{m},\hat{s})$. It contradicts the definition of $\hat{m}$ and $\hat{s}$.

Next, we prove that $c < 2\alpha x^*(0,\hat{s}_0) + \beta_0$. Since $\hat{s} = \hat{s}_0 = \frac{1}{\lambda}$ and $x^*(\hat{m},\hat{s}) > x^*(0,\hat{s}_0)$, by Equation (A.25) we have

$$cx^*(0,\hat{s}_0) < cx^*(\hat{m},\hat{s}) = \hat{m}\mu - \lambda\hat{m}^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{4\lambda}.$$

On the other hand, since $u(x^*(0,\hat{s}_0) \mid x^*(0,\hat{s}_0), 0, \hat{s}_0) = 0$ and $\hat{s}_0 = \frac{1}{\lambda}$, Equation (A.5) gives

$$\alpha(x^*(0,\hat{s}_0))^2 + \beta_0 x^*(0,\hat{s}_0) + \gamma_0 - \frac{1}{4\lambda} = 0,$$

where $\gamma_0 = \lambda a^2(\sigma^2+\sigma_\epsilon^2) - a\mu$. Combining the two equations above, we obtain

$$\begin{aligned} \big(2\alpha x^*(0,\hat{s}_0) + \beta_0 - c\big)x^*(0,\hat{s}_0) &> \alpha(x^*(0,\hat{s}_0))^2 - \gamma_0 - \hat{m}\mu + \lambda\hat{m}^2(\sigma^2+\sigma_\epsilon^2) \\ &= \alpha(x^*(0,\hat{s}_0))^2 + (a-\hat{m})\mu - \lambda(a^2-\hat{m}^2)(\sigma^2+\sigma_\epsilon^2) \\ &= \alpha(x^*(0,\hat{s}_0))^2 + (a-\hat{m})\big(\mu - \lambda(a+\hat{m})(\sigma^2+\sigma_\epsilon^2)\big) \\ &= \alpha(x^*(0,\hat{s}_0))^2 + (a-\hat{m})\big(\mu - 2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)\big) - \lambda(a-\hat{m})^2(\sigma^2+\sigma_\epsilon^2). \end{aligned}$$

By similar logic as in the end of the proof of part (ii), we have $\mu - 2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2) \geq 0$ because $\frac{\partial}{\partial m}x^*(m,\hat{s}_0)|_{m=\hat{m}} = \frac{\mu-2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)}{c} \geq 0$. We also have proven that $\hat{m} > P_L(x^*(0,\hat{s}_0)) = a - bx^*(0,\hat{s}_0)(\mu-\sigma)$. Hence, we have

$$\begin{aligned} \big(2\alpha x^*(0,\hat{s}_0) + \beta_0 - c\big)x^*(0,\hat{s}_0) &> \alpha(x^*(0,\hat{s}_0))^2 - \lambda b^2(x^*(0,\hat{s}_0))^2(\mu-\sigma)^2(\sigma^2+\sigma_\epsilon^2) \\ &= \lambda b^2(x^*(0,\hat{s}_0))^2\Big(4\mu^2\sigma^2 + (\mu^2+\sigma^2)\sigma_\epsilon^2 - (\mu-\sigma)^2(\sigma^2+\sigma_\epsilon^2)\Big) \\ &= \lambda b^2(x^*(0,\hat{s}_0))^2\,\sigma\big(3\mu^2\sigma + 2\mu\sigma^2 + 2\mu\sigma_\epsilon^2 - \sigma^3\big) > 0, \end{aligned}$$

which implies $c < 2\alpha x^*(0,\hat{s}_0) + \beta_0$. This completes the proof of part (iii).

(iv) Similar to the logic in the proof of part (iii), by Equation (A.25), we have

$$cx^*(0,\hat{s}_0) \leq cx^*(\hat{m},\hat{s}) = \hat{m}\mu - \lambda\hat{m}^2(\sigma^2+\sigma_\epsilon^2) + \frac{1}{2}\hat{s} - \frac{1}{4}\lambda\hat{s}^2 + 2\lambda\sigma\left(r - \frac{1}{2}\right)\hat{s}\hat{m}.$$

Moreover, by Equation (A.5), we have

$$\alpha(x^*(0,\hat{s}_0))^2 + \beta(\hat{s}_0)x^*(0,\hat{s}_0) + \gamma(\hat{s}_0) = 0.$$

Therefore, we have

$$\begin{aligned}(2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)-c)\,x^*(0,\hat{s}_0) =&\alpha\left(x^*(0,\hat{s}_0)\right)^2-\gamma(\hat{s}_0)-cx^*(0,\hat{s}_0)\\ \geq&\alpha\left(x^*(0,\hat{s}_0)\right)^2-\gamma(\hat{s}_0)-cx^*(\hat{m},\hat{s})\\ =&\alpha\left(x^*(0,\hat{s}_0)\right)^2-\gamma(\hat{s}_0)\\ &-\left[\hat{m}\mu-\lambda\hat{m}^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}\hat{s}-\frac{1}{4}\lambda\hat{s}^2+2\lambda\sigma\left(r-\frac{1}{2}\right)\hat{s}\hat{m}\right].\end{aligned}$$

Since we have $\hat{s}<\bar{s}$ and $\hat{s}_0<\bar{s}$, by Equations (A.15) and (A.28), we have $\hat{s}_0=\frac{1}{\lambda}+4\left(r-\frac{1}{2}\right)\sigma\left(a-2b\mu x^*(0,\hat{s}_0)\right)$ and $\hat{s}=\frac{1}{\lambda}+4\left(r-\frac{1}{2}\right)\sigma\hat{m}$. Then, the inequality above becomes

$$\begin{aligned}(2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)-c)\,x^*(0,\hat{s}_0)\geq (a-\hat{m})&\left[\mu-2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)+2\left(r-\frac{1}{2}\right)\sigma+8\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\hat{m}\right]\\ &+\alpha\left(x^*(0,\hat{s}_0)\right)^2-\lambda(a-\hat{m})^2(\sigma^2+\sigma_\epsilon^2)\\ &+4\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\left[(a-\hat{m})^2-4b^2\mu^2\left(x^*(0,\hat{s}_0)\right)^2\right].\end{aligned}$$

The term in the first bracket is non-negative because we have

$$\begin{aligned}\frac{\partial x^*(m,\hat{s})}{\partial m}\Big|_{m=\hat{m}}&=\frac{\mu-2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)+2\lambda\sigma\left(r-\frac{1}{2}\right)\hat{s}}{c}\\ &=\frac{\mu-2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)+2\left(r-\frac{1}{2}\right)\sigma+8\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\hat{m}}{c}\\ &\geq 0,\end{aligned}$$

where the second equality holds because $\hat{s}=\frac{1}{\lambda}+4\left(r-\frac{1}{2}\right)\sigma\hat{m}$. Then, to prove $2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)-c>0$, it remains to prove that $\alpha\left(x^*(0,\hat{s}_0)\right)^2-\lambda(a-\hat{m})^2(\sigma^2+\sigma_\epsilon^2)+4\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\left[(a-\hat{m})^2-4b^2\mu^2\left(x^*(0,\hat{s}_0)\right)^2\right]\geq 0$.

To do so, given $\hat{m}\geq P_L(x^*(0,\hat{s}_0))$, we have $b(\mu-\sigma)x^*(0,\hat{s}_0)\geq a-\hat{m}\geq 0$. Then, we have

$$\begin{aligned}&\alpha\left(x^*(0,\hat{s}_0)\right)^2-\lambda(a-\hat{m})^2(\sigma^2+\sigma_\epsilon^2)+4\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\left[(a-\hat{m})^2-4b^2\mu^2\left(x^*(0,\hat{s}_0)\right)^2\right]\\ \geq&\lambda b^2\left(x^*(0,\hat{s}_0)\right)^2\left\{4\mu^2\sigma^2+(\mu^2+\sigma^2)\sigma_\epsilon^2-(\mu-\sigma)^2(\sigma^2+\sigma_\epsilon^2)+4\left(r-\frac{1}{2}\right)^2\sigma^2\left[(\mu-\sigma)^2-4\mu^2\right]\right\}\\ =&\lambda b^2\left(x^*(0,\hat{s}_0)\right)^2\left\{\left[1-4\left(r-\frac{1}{2}\right)^2\right]\sigma^2\left[4\mu^2-(\mu-\sigma)^2\right]+\sigma_\epsilon^2\left[\mu^2+\sigma^2-(\mu-\sigma)^2\right]\right\}\\ \geq&0.\end{aligned}$$

Note that the equality to 0 holds if and only if $r=1$ and $\sigma_\epsilon=0$. If $r=1$ and $\sigma_\epsilon=0$, then we have both $\mu-2\lambda\hat{m}(\sigma^2+\sigma_\epsilon^2)+2\left(r-\frac{1}{2}\right)\sigma+8\lambda\left(r-\frac{1}{2}\right)^2\sigma^2\hat{m}>0$ and $a-\hat{m}>0$ because $\hat{s}<\bar{s}\leq 2a\sigma$ and $\hat{s}=\frac{1}{\lambda}+2\sigma\hat{m}$. Hence, in all cases, we must have $c<2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)$. This completes the proof of part (iv). □

Proof of Theorem 2. Following similar logic as in the proof of Proposition 3, the joint optimal m^* and s^* must make $x^*(m,s)$ as close to x^{opt} as possible. Let $\omega(m,s) = x^*(m,s)\mathbb{E}[sI] + x^*(m,s)\mathbb{E}[\max\{m-(a-bx^*(m,s)Y),0\}\times Y]$ denote the expected expenditure, and let $\mathcal{F} = ([0,\bar{m}]\times[0,\bar{s}])\cap\{m,s:\omega(m,s)\le B\}$ be the feasible set of (m,s). Then, since we consider $x^*(0,0)=x_0^*\le x^{opt}$, m^* and s^* must satisfy either (i) $(m^*,s^*)\in\arg\max_{(m,s)\in\mathcal{F}} x^*(m,s)$ and $x^*(m^*,s^*)<x^{opt}$, or (ii) $x^*(m^*,s^*)=x^{opt}$.

We now introduce some notation used in the proof. Let $(\hat{m},\hat{s})\in\arg\max_{m\in[0,\bar{m}],s\in[0,\bar{s}]} x^*(m,s)$. If there are multiple joint optimizers, we adopt the same tie-breaking rule as for (m^*,s^*). Let $\hat{m}_0 = \inf\{\arg\max_{m\in[0,\bar{m}]} x^*(m,0)\}$, and let $\hat{s}_0 = \arg\max_{s\in[0,\bar{s}]} x^*(0,s)$. Note that $\hat{s}_0$ is the same as s_1 defined in Proposition 1, and we use $\hat{s}_0$ here to align with the notation for $\hat{m}_0$. Given a planting amount $x\in(0,1)$, let $P_H(x)=a-bx(\mu+\sigma)$ and $P_L(x)=a-bx(\mu-\sigma)$ denote the market prices when $Y=\mu+\sigma$ and $Y=\mu-\sigma$, respectively. We next prove the theorem in the following three steps.

Step 1: We prove that there exists a threshold $r'_{m,3}$ such that if $r<r'_{m,3}$, then both $\hat{m}$ and $\hat{s}$ increase in r. Recall that $(\hat{m},\hat{s})\in\arg\max_{m\in[0,\bar{m}],s\in[0,\bar{s}]} x^*(m,s)$. Then, we must have either $\hat{s}<\bar{s}$ or $\hat{s}=\bar{s}$. As an intermediate step for constructing $r'_{m,3}$, let $r''_{m,3}\in[\frac{1}{2},1]$ denote the lowest index accuracy such that $\hat{s}=\bar{s}$. Following a similar analysis as in Step 3 of the proof of Lemma A.7, such an $r''_{m,3}$ must exist (because when $r=1$, for any given m, the subsidy amount s that maximizes $x^*(m,s)$ must be $\bar{s}$). Next, we prove by contradiction that for any $r<r''_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s}))\ge\hat{m}>P_H(x^*(\hat{m},\hat{s}))$.

Consider any fixed $r<r''_{m,3}$. By the definition of $\hat{m}$, we have $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}}\ge 0$. Suppose $(\hat{m},\hat{s})$ satisfies $P_L(x^*(\hat{m},\hat{s}))\ge\hat{m}>P_H(x^*(\hat{m},\hat{s}))$. We next prove that $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}}<0$, which leads to a contradiction. Since $P_L(x^*(\hat{m},\hat{s}))\ge\hat{m}>P_H(x^*(\hat{m},\hat{s}))$, by Equation (A.22),

$$\begin{aligned}\frac{\partial}{\partial m}u(x|x,m,s)\Big|_{x=x^*(\hat{m},\hat{s}),m=\hat{m},s=\hat{s}} =&(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s})))+\lambda\left(r-\frac{1}{2}\right)\hat{s}\right)\\ &-\lambda\hat{m}\sigma_\epsilon^2.\end{aligned}\tag{A.33}$$

Since $r<r''_{m,3}$, we have $\hat{s}<\bar{s}$, which by Lemma A.5 implies that $\hat{s}<\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s}))$. Moreover, since $\hat{s}<\bar{s}$, we have $\hat{s}=\frac{1}{\lambda}+2(r-\frac{1}{2})[\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s}))]$ by Equation (A.26). Hence, we have

$$\begin{aligned}\frac{1}{2}\lambda(\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s})))>&\frac{1}{2}\lambda\hat{s}\\ &=\frac{1}{2}\lambda\times\left(\frac{1}{\lambda}+2\left(r-\frac{1}{2}\right)[\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s}))]\right)\\ &=\frac{1}{2}+\lambda\left(r-\frac{1}{2}\right)[\hat{m}(\mu+\sigma)-R_L(x^*(\hat{m},\hat{s}))]\\ &\ge\frac{1}{2}+\lambda\left(r-\frac{1}{2}\right)\hat{s}.\end{aligned}$$

Plugging this into Equation (A.33), we have

$$\frac{\partial}{\partial m}u(x|x,m,s)\Big|_{x=x^*(\hat{m},\hat{s}),m=\hat{m},s=\hat{s}}<0.$$

Then, following the same logic as in the proof of Lemma A.1, we have $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}}<0$, which leads to a contradiction. Hence, for any $r<r''_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s}))\geq\hat{m}>P_H(x^*(\hat{m},\hat{s}))$.

Define $r'_{m,3}:=\min\{r''_{m,3},r_1\}$, where r_1 is defined in Proposition 1. We next prove that both $\hat{m}$ and $\hat{s}$ increase in r when $r<r'_{m,3}$. Without loss of generality, we consider settings where $r'_{m,3}>\frac{1}{2}$ (otherwise, the claim is trivial). Given that $\frac{1}{2}<r'_{m,3}\leq r''_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s}))\geq\hat{m}>P_H(x^*(\hat{m},\hat{s}))$ when $r=\frac{1}{2}$. Thus, we separate further analysis into the following two cases:

Case 1: Suppose $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$ when $r=\frac{1}{2}$. Then, by Lemma A.4, when $r=\frac{1}{2}$, we have

$$x^*(\hat{m},\hat{s})=\frac{\hat{m}\mu-\lambda\hat{m}^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}\hat{s}-\frac{1}{4}\lambda\hat{s}^2+2\lambda\sigma(r-\frac{1}{2})\hat{s}\hat{m}}{c}. \tag{A.34}$$

We first prove that for any $r<r'_{m,3}$, we have $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$. Since for any $r<r'_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s}))\geq\hat{m}>P_H(x^*(\hat{m},\hat{s}))$, it is sufficient to prove that for any $r<r'_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))\geq\hat{m}$. To do so, it is sufficient to prove that for any $r<r'_{m,3}$, we have $x^*(\hat{m},\hat{s})>x^*(0,\hat{s}_0)$ (because if $P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))\geq\hat{m}$, then the floor price is inactive, and thus we must have $x^*(\hat{m},\hat{s})=x^*(0,\hat{s}_0)$). We prove this by contradiction. Suppose for some $r<r'_{m,3}$, we have $x^*(\hat{m},\hat{s})\leq x^*(0,\hat{s}_0)$. Since we have $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$ and thus $x^*(\hat{m},\hat{s})>x^*(0,\hat{s}_0)$ when $r=\frac{1}{2}$, there must exist some other $r<r'_{m,3}$ such that $x^*(\hat{m},\hat{s})>x^*(0,\hat{s}_0)$ and $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s})<\frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0)$. We next prove that for any $r<r'_{m,3}$ such that $x^*(\hat{m},\hat{s})>x^*(0,\hat{s}_0)$, we have $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s})>\frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0)$, which leads to a contradiction.

Consider some $r<r'_{m,3}$ such that $x^*(\hat{m},\hat{s})>x^*(0,\hat{s}_0)$. Then, we have $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$. By the definition of $\hat{s}$, we cannot have $\hat{s}=0$ because we have shown in part (ii) of Lemma A.6 that for any $m\in[0,\bar{m}]$, $\frac{\partial}{\partial s}x^*(m,s)|_{s=0}>0$; we also cannot have $\hat{s}=\bar{s}$ because we consider $r<r'_{m,3}\leq r''_{m,3}$. Therefore, we must have $\frac{\partial}{\partial s}x^*(m,s)|_{m=\hat{m},s=\hat{s}}=0$. Further, we cannot have $\hat{m}=0$ as it contradicts $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$. Therefore, we must have either $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}}=0$ or $\hat{m}=\bar{m}$. With these properties of $\hat{m}$ and $\hat{s}$, we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s})&=\frac{\partial}{\partial r}x^*(m,s)\Big|_{m=\hat{m},s=\hat{s}}+\frac{\partial}{\partial m}x^*(m,s)\Big|_{m=\hat{m},s=\hat{s}}\frac{\partial}{\partial r}\hat{m}+\frac{\partial}{\partial s}x^*(m,s)\Big|_{m=\hat{m},s=\hat{s}}\frac{\partial}{\partial r}\hat{s}\\&=\frac{\partial}{\partial r}x^*(m,s)\Big|_{m=\hat{m},s=\hat{s}}\\&=\frac{2\lambda\sigma\hat{s}\hat{m}}{c},\end{aligned}$$

where the last equality follows from Equation (A.34) (which holds because we have $\hat{m}>P_L(x^*(\hat{m},\hat{s}))>P_H(x^*(\hat{m},\hat{s}))$). Note that even when $\hat{m}$ or $\hat{s}$ is not continuous or differentiable in

r, we have $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s}) \geq \frac{\partial}{\partial r}x^*(m,s)\Big|_{m=\hat{m},s=\hat{s}} = \frac{2\lambda\sigma\hat{s}\hat{m}}{c}$ because considering how $\hat{m}$ and $\hat{s}$ change in r could only further increase $x^*(\hat{m},\hat{s})$. We also have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0) &= \frac{\partial}{\partial r}x^*(0,s)\Big|_{s=\hat{s}_0} + \frac{\partial}{\partial s}x^*(0,s)\Big|_{s=\hat{s}_0}\frac{\partial}{\partial r}\hat{s}_0 \\ &= \frac{\partial}{\partial r}x^*(0,s)\Big|_{s=\hat{s}_0},\end{aligned} \tag{A.35}$$

where the second equality holds because, given $r < r'_{m,3} \leq r_1$, we have $\hat{s}_0 < \bar{s}$ and thus $\frac{\partial}{\partial s}x^*(0,s)|_{s=\hat{s}_0} = 0$. Moreover, from Equation (A.7), we have

$$\begin{aligned}\frac{\partial}{\partial r}x^*(0,s)\Big|_{s=\hat{s}_0} &= -\frac{\partial u(x|x,0,s)/\partial r}{\partial u(x|x,0,s)/\partial x}\Big|_{x=x^*(0,s),s=\hat{s}_0} \\ &= \frac{\lambda[R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))]\hat{s}_0}{2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)},\end{aligned}$$

where $\alpha = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$ and $\beta(s) = b(\mu^2+\sigma^2)+c-4\lambda ab\mu\sigma^2-2\lambda ab\mu\sigma_\epsilon^2+4\lambda b(r-\frac{1}{2})\mu\sigma s$, as defined in Lemma 2. Then, to prove $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s}) > \frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0)$, it remains to prove $\frac{2\lambda\hat{s}\hat{m}\sigma}{c} > \frac{\lambda\hat{s}_0[R_H(x^*(0,\hat{s}_0))-R_L(x^*(0,\hat{s}_0))]}{2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)}$.

To do so, we first prove $c < 2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)$. We begin by proving this inequality for $r=\frac{1}{2}$. When $r = \frac{1}{2} < r'_{m,3}$, we have $\hat{s} < \bar{s}$ and $\hat{s}_0 < \bar{s}$. Moreover, when $r = \frac{1}{2}$, we have $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$. Then, by part (iii) of Lemma A.8, we have $c < 2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)$ when $r=\frac{1}{2}$. Further, since $x^*(0,\hat{s}_0)$ increases in r when $r < r'_{m,3}$ by Equation (A.35) and Corollary 1, and $\beta(\hat{s}_0)$ is minimized at $r = \frac{1}{2}$ among all $r \in [\frac{1}{2}, 1]$, we have $c < 2\alpha x^*(0,\hat{s}_0)+\beta(\hat{s}_0)$ when $r < r'_{m,3}$.

To prove $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s}) > \frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0)$, it remains to prove $2\lambda\hat{s}\hat{m}\sigma \geq \lambda\hat{s}_0[R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))]$. Since we consider $r < r'_{m,3}$, by part (i) of Lemma A.8, we have $\hat{s} \geq \hat{s}_0$. Moreover, in the proof of part (i) of Lemma A.8, we have proven that $\max\{P_H(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu+\sigma) - \max\{P_L(x^*(\hat{m},\hat{s})),\hat{m}\}(\mu-\sigma) \geq R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s}))$. Then, given $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$, we have $2\hat{m}\sigma \geq R_H(x^*(0,\hat{s})) - R_L(x^*(0,\hat{s})) \geq R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))$ because $x^*(0,\hat{s}_0) \geq x^*(0,\hat{s})$. Hence, we have $2\lambda\hat{s}\hat{m}\sigma \geq \lambda\hat{s}_0[R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))]$.

Therefore, for any $r < r'_{m,3}$ such that $x^*(\hat{m},\hat{s}) > x^*(0,\hat{s}_0)$, we have $\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s}) > \frac{\mathrm{d}}{\mathrm{d}r}x^*(0,\hat{s}_0)$. This implies that for any $r < r'_{m,3}$, we have $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$.

We now prove that both $\hat{m}$ and $\hat{s}$ increase in r when $r < r'_{m,3}$. Consider any $r < r'_{m,3}$. We have shown earlier that $\frac{\partial}{\partial s}x^*(m,s)|_{m=\hat{m},s=\hat{s}} = 0$, and that either $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}} = 0$ or $\hat{m} = \bar{m}$. Consider the following two sub-cases:

(a): Suppose $\frac{\partial}{\partial m}x^*(m,s)|_{m=\hat{m},s=\hat{s}} = 0$ and $\frac{\partial}{\partial s}x^*(m,s)|_{m=\hat{m},s=\hat{s}} = 0$. Since we have shown that $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$ for any $r < r'_{m,3}$, by Equation (A.34), we have

$$\hat{m} = \frac{\mu + 2\sigma(r-\frac{1}{2})}{2\lambda(\sigma^2+\sigma_\epsilon^2) - 8\lambda\sigma^2(r-\frac{1}{2})^2}$$

$$\hat{s} = \frac{1}{\lambda} + 4\sigma(r - \frac{1}{2})\hat{m}.$$

Thus, $\hat{m}$ and $\hat{s}$ both increase in r.

(b): Suppose $\hat{m} = \bar{m}$ and $\frac{\partial}{\partial s}x^*(m,s)|_{m=\hat{m},s=\hat{s}} = 0$. Since we have shown that $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$ for any $r < r'_{m,3}$, by Equation (A.34), we have $\hat{s} = \frac{1}{\lambda} + 4\sigma(r - \frac{1}{2})\bar{m}$. Since $\bar{m}$ is independent of r, we have that $\hat{s}$ increases in r. Moreover, $\hat{m} = \bar{m}$ is independent of r.

Combining the two sub-cases, we conclude that if $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$ when $r = \frac{1}{2}$, then $\hat{m}$ and $\hat{s}$ increase in r for any $r < r'_{m,3}$.

Case 2: Suppose $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$ when $r = \frac{1}{2}$. We have shown earlier that for any $r < r'_{m,3}$, we cannot have $P_L(x^*(\hat{m},\hat{s})) \geq \hat{m} > P_H(x^*(\hat{m},\hat{s}))$. Consider the first switch point in index accuracy $r > \frac{1}{2}$, if any, where $(\hat{m},\hat{s})$ moves from $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$ to $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$. By continuity of the equilibrium planting amount, at the switch point there is a tie between two solutions: one with an inactive floor price and one with a floor price that is active at both high and low area-yield realizations. As r approaches the switch point from below, the limiting planting amount is $x^*(\hat{m},\hat{s}) = x^*(0,\hat{s}_0)$. For index accuracies approaching this switch point from above, we have $\hat{m} > P_L(x^*(\hat{m},\hat{s}))$. Taking the right limit gives that the limiting value of $\hat{m}$ from above satisfies $\hat{m} \geq P_L(x^*(0,\hat{s}_0))$. Then, by part (iv) of Lemma A.8, we can extend the reasoning from Case 1 above and conclude that, once $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$ holds for some $r < r'_{m,3}$, it must also hold for any higher $r < r'_{m,3}$. Therefore, it is sufficient to consider the following two sub-cases:

(a): Suppose for any $r < r'_{m,3}$, we have $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$. Then, we have $\hat{m} = 0$ and $\hat{s} = \hat{s}_0$. Since $r < r'_{m,3} \leq r_1$, we have $\hat{s} < \bar{s}$ and $\hat{s}$ increases in r by Proposition 1.

(b): Suppose there exists $r' \in (\frac{1}{2}, r'_{m,3})$ such that we have $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$ when $r \leq r'$, and we have $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$ when $r \in (r', r'_{m,3})$. In this case, we have $\hat{m} = 0$ for any $r \leq r'$. Moreover, based on the analysis in Case 1, $\hat{m}$ increases in r for any $r \in (r', r'_{m,3})$. Thus, $\hat{m}$ increases in r for any $r < r'_{m,3}$. In addition, we have $\hat{s} = \hat{s}_0$ increases in r for any $r \leq r'$. Moreover, based on the analysis in Case 1, $\hat{s}$ increases in r for any $r \in (r', r'_{m,3})$. By part (i) of Lemma A.8, we have $\hat{s} \geq \hat{s}_0$ for any $r < r'_{m,3}$. Hence, $\hat{s}$ increases in r for any $r < r'_{m,3}$.

Combining the two sub-cases, we conclude that if $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$ when $r = \frac{1}{2}$, then $\hat{m}$ and $\hat{s}$ increase in r for any $r < r'_{m,3}$.

Step 2: We prove that there exists a threshold $r_{m,3}$ such that if $r < r_{m,3}$, then $s^* \geq s^*_0$ and $m^* \geq m^*_0$. To do so, we first prove that when the index accuracy is lower than a threshold, we have $s^* = \hat{s}$ and $m^* = \hat{m}$. Based on the analysis in Step 1, we know that for any $r < r'_{m,3}$, we have

$$\frac{\mathrm{d}}{\mathrm{d}r}x^*(\hat{m},\hat{s}) \geq \frac{\partial}{\partial r}x^*(m,s)\big|_{m=\hat{m},s=\hat{s}} \geq 0,$$

where the second inequality holds because $x^*(m,s)$ increases in r by part (i) of Lemma A.6. Then, for any $r < r'_{m,3}$, $\omega(\hat{m},\hat{s})$ increases in r. Therefore, there must exist $r_{m,b} \in [\frac{1}{2}, r'_{m,3}]$ (with a slight abuse of notation) such that for any $r < r_{m,b}$, we have $\omega(\hat{m},\hat{s}) < B$. With a slight abuse of notation, let $r_{m,c}$ be the lowest index accuracy such that $x^*(m,s) = x^{opt}$ for some $(m,s) \in \mathcal{F}$ (let $r_{m,c} = 1$ if no such index accuracy exists).

Define $r_{m,3} := \min\{r_{m,b}, r_{m,c}\}$. Consider any $r < r_{m,3}$. Since $r < r_{m,c}$, we have $(m^*, s^*) = \arg\max_{(m,s)\in\mathcal{F}} x^*(m,s)$ and $x^*(m^*,s^*) < x^{opt}$. Further, since $r < r_{m,b}$, we have $m^* = \hat{m}$ and $s^* = \hat{s}$.

We now prove that if $r < r_{m,3}$, then $s^* \geq s_0^*$ and $m^* \geq m_0^*$. First, we prove that for any $r < r_{m,3}$, we have $s^* \geq s_0^*$. Consider any given $r < r_{m,3}$. By part (i) of Lemma A.8, we have $\hat{s} \geq \hat{s}_0$. Since $s^* = \hat{s}$, to prove $s^* \geq s_0^*$, it remains to prove $s_0^* = \hat{s}_0$. Since $\hat{s}_0 \leq \hat{s}$, $x^*(0,\hat{s}_0) \leq x^*(\hat{m},\hat{s})$, and $r < r_{m,3} \leq r_{m,b}$, we have $\omega(0,\hat{s}_0) \leq \omega(\hat{m},\hat{s}) < B$. In addition, since $r < r_{m,3} \leq r_{m,c}$, we have $x^*(0,\hat{s}_0) \leq x^*(\hat{m},\hat{s}) < x^{opt}$. Hence, by condition (i) at the beginning of the proof of Theorem 1, we have $s_0^* = \hat{s}_0$. Therefore, we have $s^* \geq s_0^*$ when $r < r_{m,3}$.

Next, we prove that for any $r < r_{m,3}$, we have $m^* \geq m_0^*$. Let $\mathcal{M} = [0,\bar{m}] \cap \{m : \omega(m,0) \leq B\}$ be the feasible set of m when $s = 0$. Then, since we consider $x^*(0,0) = x_0^* \leq x^{opt}$, m_0^* must satisfy either $m_0^* \in \arg\max_{m\in\mathcal{M}} x^*(m,0)$ and $x^*(m_0^*,0) < x^{opt}$, or $m_0^* = \inf\{m : x^*(m,0) = x^{opt}\}$. Without loss of generality, we consider settings where $r_{m,3} > \frac{1}{2}$ (otherwise, the claim is trivial).

Since m_0^* is independent of r and we have shown in Step 1 that m^* increases in r for any $r < r_{m,3}$, to prove $m^* \geq m_0^*$ when $r < r_{m,3}$, it is sufficient to prove $m^* \geq m_0^*$ when $r = \frac{1}{2}$.

Consider $r = \frac{1}{2}$. We have shown in Step 1 that we cannot have $P_L(x^*(\hat{m},\hat{s})) \geq \hat{m} > P_H(x^*(\hat{m},\hat{s}))$. Thus, it is sufficient to consider the following two cases:

Case 1: Suppose $\hat{m} > P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s}))$. Then, by Case 1 of Step 1, we have

$$\hat{m} = \min\{\frac{\mu}{2\lambda(\sigma^2+\sigma_\epsilon^2)}, \bar{m}\}.$$

To prove $m^* \geq m_0^*$, we first prove $\hat{m} \geq \hat{m}_0$. Consider the following three scenarios: If $\hat{m}_0 > P_L(x^*(\hat{m}_0,0)) > P_H(x^*(\hat{m}_0,0))$, then following a similar analysis as in Case 1 of Step 1, we have $\hat{m}_0 = \min\{\frac{\mu}{2\lambda(\sigma^2+\sigma_\epsilon^2)}, \bar{m}\} = \hat{m}$. If $P_L(x^*(\hat{m}_0,0)) > P_H(x^*(\hat{m}_0,0)) \geq \hat{m}_0$, then $\hat{m}_0 = 0 < \hat{m}$. Finally, we prove that we cannot have $P_L(x^*(\hat{m}_0,0)) \geq \hat{m}_0 > P_H(x^*(\hat{m}_0,0))$. By the definition of $\hat{m}_0$, we have $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0} \geq 0$. Suppose $P_L(x^*(\hat{m}_0,0)) \geq \hat{m}_0 > P_H(x^*(\hat{m}_0,0))$. We next prove that $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0} < 0$, which leads to a contradiction. Since $\hat{m}_0 > P_H(x^*(\hat{m}_0,0))$, we must have $\hat{m}_0 > P_H(x^*(0,0))$; otherwise, if $\hat{m}_0 \leq P_H(x^*(0,0))$, then we would have $x^*(\hat{m}_0,0) = x^*(0,0)$, which implies $\hat{m}_0 \leq P_H(x^*(\hat{m}_0,0))$. Moreover, given $r = \frac{1}{2} < r_{m,3} \leq r_1$, it follows from Equation (A.15) that $\hat{s}_0 = \frac{1}{\lambda}$. Hence, we have

$$\frac{\partial}{\partial m}u(x|x,m,s)\Big|_{x=x^*(\hat{m}_0,0),m=\hat{m}_0,s=0}$$

$$
\begin{aligned}
&=(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(\hat{m}_0(\mu+\sigma)-R_L(x^*(\hat{m}_0,0)))\right)-\lambda\hat{m}_0\sigma_\epsilon^2\\
&<(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(R_H(x^*(0,0))-R_L(x^*(0,0)))\right)-\lambda P_H(x^*(0,0))\sigma_\epsilon^2\\
&<(\mu+\sigma)\left(\frac{1}{2}-\frac{1}{2}\lambda(R_H(x^*(0,\hat{s}_0))-R_L(x^*(0,\hat{s}_0)))\right)-\lambda P_H(x^*(0,0))\sigma_\epsilon^2\\
&<0,
\end{aligned}
$$

where the first inequality holds because $\hat{m}_0 > P_H(x^*(0,0))$ and $x^*(\hat{m}_0,0) \geq x^*(0,0)$, and second inequality holds because $x^*(0,0) < x^*(0,\hat{s}_0)$, and the last inequality holds because given $r < r_{m,3} \leq r_1$, we have $\hat{s}_0 = \frac{1}{\lambda} < R_H(x^*(0,\hat{s}_0)) - R_L(x^*(0,\hat{s}_0))$. Then, following the same logic as in the proof of Lemma A.1, we must have $\frac{\partial}{\partial m}x^*(m,0)|_{m=\hat{m}_0} < 0$, which leads to a contradiction. Combining all three scenarios, we conclude that $\hat{m} \geq \hat{m}_0$.

Since $m^* = \hat{m}$, to prove $m^* \geq m_0^*$, it remains to prove $m_0^* = \hat{m}_0$. Since $\hat{m}_0 \leq \hat{m}$ and $x^*(\hat{m}_0,0) \leq x^*(\hat{m},\hat{s})$, we have $\omega(\hat{m}_0,0) \leq \omega(m^*,s^*) < B$ and $x^*(\hat{m}_0,0) < x^{opt}$. Hence, we have $m_0^* = \hat{m}_0$.

Case 2: Suppose $P_L(x^*(\hat{m},\hat{s})) > P_H(x^*(\hat{m},\hat{s})) \geq \hat{m}$. Then, we have $m^* = \hat{m} = 0$. Then, by part (ii) of Lemma A.8, we have $\hat{m}_0 = 0$. In this case, we have $m_0^* = \hat{m}_0 = 0$.

Combining both cases, we have $m^* \geq m_0^*$ when $r = \frac{1}{2}$. Therefore, $m^* \geq m_0^*$ when $r < r_{m,3}$.

Step 3: We prove that there exists a threshold $r_{m,4}$ such that if $r > r_{m,4}$, then $s^* \leq s_0^*$ and $m^* \leq m_0^*$. Recall from the proof of Theorem 1 that r_c is defined as the lowest index accuracy at which x^{opt} is achievable in the absence of a floor price. Suppose $r_c < 1$ (otherwise, let $r_{m,4} = 1$). Then, following the analysis in the proof of Theorem 1, x^{opt} must be achievable for any $r > r_c$. This implies that for any $r > r_c$, (m^*,s^*) must satisfy condition (ii) presented at the beginning of the proof. Further, for $r > r_c$, we must have $s^* \leq s_0^*$ because (1) we have $x^*(0,s_0^*) = x^{opt}$ when $r > r_c$, and (2) for any $s > s_0^*$ and $m \in [0,\bar{m}]$ that satisfy $x^*(m,s) = x^{opt}$, we have $\omega(m,s) > \omega(0,s_0^*)$.

It remains to prove that there exists a threshold $r_{m,4} \in [r_c, 1)$ such that if $r > r_{m,4}$, then $m^* \leq m_0^*$. For notational simplicity, we let s' denote the value of s_0^* when $r = \frac{r_c+1}{2}$. As a building block of our analysis, we next prove that if the index accuracy is sufficiently high, we have that the variance of each farmer's income, $\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i + sI - x^{opt}c]$, strictly increases in m and strictly decreases in s for any $m \in (P_H(x^{opt}),\bar{m}]$ and $s \in [0,s']$. Consider the following two cases:

Suppose $m \in (P_H(x^{opt}), P_L(x^{opt})]$. Then, for any given $s \in [0,s']$, we have

$$
\begin{aligned}
&\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i + sI - x^{opt}c]\\
=&\frac{1}{4}(m(\mu+\sigma)-R_L(x^{opt}))^2+\frac{1}{4}s^2-s\left(r-\frac{1}{2}\right)(m(\mu+\sigma)-R_L(x^{opt}))\\
&+\left(\frac{1}{2}a^2-abx^{opt}(\mu-\sigma)+\frac{1}{2}b^2(x^{opt})^2(\mu-\sigma)^2+\frac{1}{2}m^2\right)\sigma_\epsilon^2.
\end{aligned}
$$

Then, we have

$$\frac{\partial}{\partial m}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]$$
$$=(\mu+\sigma)\left(\frac{1}{2}(m(\mu+\sigma)-R_L(x^{opt}))-s\left(r-\frac{1}{2}\right)\right)+m\sigma_\epsilon^2,$$

and

$$\frac{\partial}{\partial s}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]=\frac{1}{2}s-\left(r-\frac{1}{2}\right)(m(\mu+\sigma)-R_L(x^{opt})).$$

Based on the proof of Theorem 1, we know that $s_0^*\le R_H(x^{opt})-R_L(x^{opt})$ when $r=r_c$ and s_0^* strictly decreases in r when $r>r_c$. Since s' denotes the value of s_0^* when $r=\frac{r_c+1}{2}>r_c$, we have $s'<R_H(x^{opt})-R_L(x^{opt})=P_H(x^{opt})(\mu+\sigma)-R_L(x^{opt})<m(\mu+\sigma)-R_L(x^{opt})$. Therefore, for any $s\le s'$ and $r\in[\frac{1}{2},1]$, we have $\frac{\partial}{\partial m}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]>0$. Further, let $r'_{m,4}:=\frac{1}{2}+\frac{1}{2}\times\frac{s'}{R_H(x^{opt})-R_L(x^{opt})}<1$. Then, for any $s\le s'$ and $r>r'_{m,4}$, we have $\frac{\partial}{\partial s}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]<0$.

Suppose $m\in(P_L(x^{opt}),\bar{m}]$. Then, for any given $s\in[0,s']$, we have

$$\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]=m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{4}s^2-2\sigma\left(r-\frac{1}{2}\right)sm.$$

Then, we have

$$\frac{\partial}{\partial m}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]=2\sigma\left(m\sigma-\left(r-\frac{1}{2}\right)s\right)+2m\sigma_\epsilon^2,$$

and

$$\frac{\partial}{\partial s}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]=\frac{1}{2}s-2\left(r-\frac{1}{2}\right)m\sigma.$$

Since $s'<R_H(x^{opt})-R_L(x^{opt})<P_L(x^{opt})(\mu+\sigma)-P_L(x^{opt})(\mu-\sigma)<2m\sigma$, for any $s\le s'$ and $r\in[\frac{1}{2},1]$, we must have $\frac{\partial}{\partial m}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]>0$. Further, let $r''_{m,4}:=\frac{1}{2}+\frac{1}{2}\times\frac{s'}{2P_L(x^{opt})\sigma}<1$. Then, for any $s\le s'$ and $r>r''_{m,4}$, we have $\frac{\partial}{\partial s}\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]<0$. Moreover, because $2P_L(x^{opt})\sigma>R_H(x^{opt})-R_L(x^{opt})$, we have $r'_{m,4}>r''_{m,4}$.

Combining the above two cases, for any $r>r'_{m,4}$, we have $\mathbb{V}\mathrm{ar}[\max\{(a-bx^{opt}Y),m\}Y_i+sI-x^{opt}c]$ strictly increases in m and strictly decreases in s for any $m\in(P_H(x^{opt}),\bar{m}]$ and $s\in[0,s']$.

Define $r_{m,4}:=\max\{r'_{m,4},\frac{r_c+1}{2}\}$. We now prove by contradiction that for any $r>r_{m,4}$, we have $m^*=0\le m_0^*$ and $s^*=s_0^*$. Consider any $r>r_{m,4}$. Suppose there exists $(\check{m},\check{s})\in\mathcal{F}$ such that $x^*(\check{m},\check{s})=x^{opt}$, $\omega(\check{m},\check{s})\le\omega(0,s_0^*)$, and $(\check{m},\check{s})\ne(0,s_0^*)$. Then, it is sufficient to consider $\check{m}>P_H(x^{opt})$ because if $\check{m}\le P_H(x^{opt})$, then the floor price is always below the market price, and thus $(\check{m},\check{s})$ is dominated by $(0,s_0^*)$. Further, we must have $\check{s}<s_0^*$; otherwise, $(\check{m},\check{s})$ would lead to a strictly higher expenditure than $(0,s_0^*)$. Further, given $r>r_{m,4}\ge\frac{r_c+1}{2}$, we have $s_0^*<s'$ because we

know from Theorem 1 that s_0^* strictly decreases in r when $r > r_c$. For any $(m,s) \in \mathcal{F}$, recall that

$$u(x^*(m,s)|x^*(m,s),m,s) = \mathbb{E}[\pi(x^*(m,s)|x^*(m,s),m,s)] - \lambda \mathbb{V}\mathrm{ar}[\pi(x^*(m,s)|x^*(m,s),m,s)], \tag{A.36}$$

where $\pi(i|x,m,s) = \max\{(a-bxY),m\}Y_i + sI - h_i$. Since $x^*(\check{m},\check{s}) = x^*(0,s_0^*) = x^{opt}$ and $\omega(\check{m},\check{s}) \le \omega(0,s_0^*)$, the expected subsidy amount (including both price and yield protection subsidies) received by each farmer who plants under $(\check{m},\check{s})$ must be no more than that under $(0,s_0^*)$. Therefore,

$$\mathbb{E}[\pi(x^*(\check{m},\check{s})|x^*(\check{m},\check{s}),\check{m},\check{s})] \le \mathbb{E}[\pi(x^*(0,s_0^*)|x^*(0,s_0^*),0,s_0^*)]. \tag{A.37}$$

Moreover, given $r > r_{m,4}$, $\check{m} > P_H(x^{opt})$, and $\check{s} < s_0^* < s'$, we have

$$\begin{aligned}\mathbb{V}\mathrm{ar}[\pi(x^*(\check{m},\check{s})|x^*(\check{m},\check{s}),\check{m},\check{s})] >& \mathbb{V}\mathrm{ar}[\pi(x^*(P_H(x^{opt}),s_0^*)|x^*(P_H(x^{opt}),s_0^*),P_H(x^{opt}),s_0^*)] \\ =& \mathbb{V}\mathrm{ar}[\pi(x^*(0,s_0^*)|x^*(0,s_0^*),0,s_0^*)],\end{aligned} \tag{A.38}$$

Thus, by Equations (A.36), (A.37), and (A.38), we have

$$u(x^*(\check{m},\check{s})|x^*(\check{m},\check{s}),\check{m},\check{s}) < u(x^*(0,s_0^*)|x^*(0,s_0^*),0,s_0^*).$$

Based on the derivation of the equilibrium planting amount in the proof of Lemma A.4, we have $u(x^*(\check{m},\check{s})|x^*(\check{m},\check{s}),\check{m},\check{s}) = u(x^*(0,s_0^*)|x^*(0,s_0^*),0,s_0^*) = 0$, which leads to a contradiction. $\square$

Appendix A.3 Proofs of Analytical Results in §6

Proof of Lemma 3. As discussed in §6, we have

$$\begin{aligned}v(s,r) &= \mathbb{E}\left[\int_0^{x^*(s,r)} ((a - bx^*(s,r)Y)Y + sI - xc)\,\mathrm{d}x\right] - \psi(s,r) - \phi(r) \\ &= \mathbb{E}\left[\int_0^{x^*(s,r)} ((a - bx^*(s,r)Y)Y + sI - xc)\,\mathrm{d}x\right] - x^*(s,r)\mathbb{E}[sI] - \kappa(r-r_0)^2 \\ &= v_1 \times x^*(s,r) - v_2 \times (x^*(s,r))^2 - \kappa(r-r_0)^2,\end{aligned} \tag{A.39}$$

where $v_1 = a\mu$ and $v_2 = b(\mu^2+\sigma^2) + \frac{1}{2}c$.

We now prove that for any given budget $B > 0$, there exists a threshold $\sigma_1 \in [0,\mu]$ such that if $\sigma < \sigma_1$, then $r^* = r_0$; moreover, σ_1 increases in B. Consider a fixed $B > 0$. Similar to before, we say that the planting amount x^{opt} is achievable if there exists $s \in [0,\bar{s}]$ that satisfies $x^*(s,r) \ge x^{opt}$ and $\psi(s,r) + \phi(r) \le B$. Let $\sigma_1 \in [0,\mu]$ be the largest area-yield variability such that, for any given $\sigma \le \sigma_1$, the planting amount x^{opt} is achievable when $r = r_0$ (let $\sigma_1 = 0$ if no such threshold exists). It can be seen from Equation (A.39) that if x^{opt} is achievable when $r = r_0$, we must have $r^* = r_0$. Thus, for any $\sigma < \sigma_1$, we have $r^* = r_0$. Further, by definition, σ_1 must increase in B (because given $r = r_0$, if x^{opt} is achievable under a lower budget, it must be achievable under a higher budget).

Before concluding the proof, we show that σ_1 is guaranteed to be strictly positive provided that σ_ϵ is not too high (if σ_ϵ is too high, then intuitively, x^{opt} would not be achievable even when $\sigma = 0$). We first consider the case where $\sigma_\epsilon = 0$. In this case, it is straightforward to verify that when σ approaches zero, we have $x_0^* > x^{opt}$, which implies $\sigma_1 > 0$ (note that we focus on parameter ranges where $x_0^* \leq x^{opt}$; since x_0^* and x^{opt} are both continuous in σ, for σ sufficiently small but within this range, x_0^* is only slightly below x^{opt}, so x^{opt} is achievable). Since x_0^* is continuous in σ_ϵ and x^{opt} is independent of σ_ϵ, it follows that if σ_ϵ is lower than a certain threshold, we still have $x_0^* > x^{opt}$ when $\sigma = 0$. Therefore, as long as σ_ϵ is lower than the threshold, we still have $\sigma_1 > 0$. □

Proof of Theorem 3. By Lemma 3, under any given budget B, there exists σ_1 such that if $\sigma < \sigma_1$, then $r^*(B) = r_0$ and $x^*\big(s^*(B), r^*(B)\big) = x^{opt}$. Let $\sigma_1(B)$ denote the value of σ_1 under a given budget B.

Consider two budget levels B' and B such that $0 < B' < B$. By the proof of Lemma 3, we have $\sigma_1(B') \leq \sigma_1(B)$. Let $\sigma_2 = \sigma_1(B')$ and $\sigma_3 = \sigma_1(B)$. We next prove that if $\sigma \in (\sigma_2, \sigma_3)$, then $r^*(B') \geq r^*(B)$ and $s^*(B') \leq s^*(B)$.

First, we prove that $r^*(B') \geq r^*(B)$ if $\sigma \in (\sigma_2, \sigma_3)$. Given $\sigma < \sigma_3 \leq \sigma_1(B)$, we have $r^*(B) = r_0$. Moreover, given $\sigma > \sigma_2 \geq \sigma_1(B')$, we have $r^*(B') \geq r_0$. Hence, we have $r^*(B') \geq r^*(B)$.

Second, if $\sigma \in (\sigma_2, \sigma_3)$, we also have $x^*\big(s^*(B), r^*(B)\big) = x^{opt}$. We next prove $s^*(B') \leq s^*(B)$ by using contradiction. Suppose $s^*(B') > s^*(B)$. Consider the following two cases:

Suppose $x^*\big(s^*(B'), r^*(B')\big) \geq x^*\big(s^*(B), r^*(B)\big)$. In this case, $(s^*(B'), r^*(B'))$ must not be optimal under budget B' because $(s^*(B'), r^*(B'))$ is dominated by $(s^*(B), r^*(B))$, with the latter resulting in an equilibrium planting amount x^{opt} and a lower investment in index accuracy. (Note that $(s^*(B), r^*(B))$ must be feasible under budget B' given $s^*(B') > s^*(B)$ and $x^*\big(s^*(B'), r^*(B')\big) \geq x^*\big(s^*(B), r^*(B)\big)$.)

Suppose $x^*\big(s^*(B'), r^*(B')\big) < x^*\big(s^*(B), r^*(B)\big)$. Since $r^*(B') \geq r^*(B)$, and the equilibrium planting amount is increasing in the index accuracy (as shown in Corollary 1), we have $x^*\big(s^*(B'), r^*(B')\big) < x^*\big(s^*(B), r^*(B)\big) \leq x^*\big(s^*(B), r^*(B')\big)$. This implies that given the index accuracy $r^*(B')$, a higher subsidy $s^*(B')$ leads to a lower equilibrium planting amount than a lower subsidy $s^*(B)$. From the proof of Theorem 1, for any given index accuracy, the optimal subsidy level must not be one at which a higher subsidy reduces the equilibrium planting amount. Therefore, $s^*(B')$ must not be optimal.

Since both cases result in a contradiction, we must have $s^*(B') \leq s^*(B)$. □

Appendix A.4 Proofs of Analytical Results in §8

Proof of Proposition 4. (i) Recall that when x farmers plant, the net income of farmer i who plants and enrolls in the policy is given by $(a - bxY)Y_i + sI - h_i - \xi$. Similarly, the net income of farmer i who plants but does not enroll in the policy is given by $(a-bxY)Y_i-h_i$. For any given ξ, s, and planting amount x, farmers' utility differs only through their production cost h_i. Thus, either all farmers have a higher utility from planting and enrolling than from planting but not enrolling, or all farmers have a higher utility from planting but not enrolling than from planting and enrolling. (We note that we can incorporate a scenario where some farmers have a higher utility from planting and enrolling, while others have a higher utility from planting but not enrolling, by introducing another dimension of farmer heterogeneity, such as heterogeneity in farmer risk aversion. The key mechanisms underlying our insights remain valid under this extension, and we numerically verify that our insights continue to hold; details are available upon request.)

For any (ξ,s) under which all farmers have a higher utility from planting but not enrolling than from planting and enrolling, setting $\xi=0$ and $s=0$ results in the same equilibrium outcomes. Therefore, it is sufficient to focus on (ξ,s) under which all farmers have a higher utility from planting and enrolling than from planting but not enrolling. For any such (ξ,s), let $x^*(\xi,s)$ denote the equilibrium planting amount. Then, the net benefit is given as follows:

$$\begin{aligned} v(\xi,s) &= \mathbb{E}\left[\int_0^{x^*(\xi,s)} ((a-bx^*(\xi,s)Y)Y - xc + sI - \xi)\,\mathrm{d}x\right] - (\mathbb{E}[sI]-\xi)x^*(\xi,s) \\ &= v_1 \times x^*(\xi,s) - v_2 \times (x^*(\xi,s))^2, \end{aligned}$$

where $v_1 = a\mu$ and $v_2 = b(\mu^2+\sigma^2)+\frac{1}{2}c$. Clearly, x^{opt} defined in Equation (A.18) is still the value of $x^*(\xi,s)$ that maximizes $v(\xi,s)$. Then, following the same logic as in the proof of Theorem 1, we have that the joint optimal ξ^* and s^* must make $x^*(\xi,s)$ as close to x^{opt} as possible. With a slight abuse of notation, let $\mathcal{F}=[0,\infty)\times[0,\bar{s}]\cap\{(\xi,s):(\mathbb{E}[sI]-\xi)x^*(\xi,s)\le B\}$ denote the feasible set of (ξ,s). Since we assume $x^*(0,0)\le x^{opt}$, ξ^* and s^* must satisfy either (a) $(\xi^*,s^*)\in \underset{(\xi,s)\in\mathcal{F}}{\arg\max}\, x^*(\xi,s)$ and $x^*(\xi^*,s^*)<x^{opt}$, or (b) $x^*(\xi^*,s^*)=x^{opt}$.

First, we prove that there exists a threshold $r_{\xi,1}$ such that if $r<r_{\xi,1}$, then ξ^*, s^*, and $\mathbb{E}[s^*I]-\xi^*$ increase in r. Let s_0^* denote the optimal s given $\xi=0$. Then, recall from Theorem 1 that r_2 denotes the threshold such that s_0^* increases in r if and only if $r<r_2$. We next prove that if $r<r_2$, then $\xi^*=0$ and $s^*=s_0^*$. To see this, consider a fixed $r<r_2$. As shown in the proof of Theorem 1, if $r<r_2$, we have $s_0^*=\underset{s\in[0,\bar{s}]}{\arg\max}\, x^*(0,s)$ and $x^*(0,s_0^*)\mathbb{E}[s_0^*I]<B$. Thus, we have $(0,s_0^*)\in\mathcal{F}$. Moreover, it is straightforward to verify that $x^*(\xi,s)$ strictly decreases in ξ. Therefore, $(0,s_0^*)=\underset{(\xi,s)\in\mathcal{F}}{\arg\max}\, x^*(\xi,s)$ is unique maximizer of $x^*(\xi,s)$. Since $r<r_2$, we have $x^*(0,s_0^*)<x^{opt}$ from the proof of Theorem 1, and thus, by condition (a) above, we have $\xi^*=0$ and $s^*=s_0^*$. Further, by Theorem 1, s_0^* increases in r if $r<r_2$. Hence, ξ^*, s^*, and $\mathbb{E}[s^*I]-\xi^*$ (weakly) increase in r if $r<r_2$. It implies that there must exist an $r_{\xi,1}\in[r_2,1]$ such that for any $r<r_{\xi,1}$, ξ^*, s^*, and $\mathbb{E}[s^*I]-\xi^*$ increase in r.

Next, we prove that there exists a threshold $r_{\xi,2}$ such that if $r>r_{\xi,2}$, then ξ^*, s^*, and $\mathbb{E}[s^*I]-\xi^*$ decrease in r. Recall from the proof of Theorem 1 that r_c denotes the lowest index accuracy such that $x^*(0,s) = x^{opt}$ for some $(0,s) \in \mathcal{F}$. We next prove that if $r > r_c$, then $\xi^* = 0$ and $s^* = s_0^*$. Consider any fixed $r > r_c$. As shown in the proof of Theorem 1, if $r > r_c$, we have $x^*(0,s_0^*) = x^{opt}$ and $x^*(0,s_0^*)\mathbb{E}[s_0^* I] < B$. Thus, we have $(0,s_0^*) \in \{(\xi,s) \in \mathcal{F} : x^*(\xi,s) = x^{opt}\}$. Then, by condition (b) above and the tie-breaking rule discussed in §8.3, we have $\xi^* = 0$ and $s^* = s_0^*$. Further, as shown in the proof of Theorem 1, s_0^* decreases in r if $r > r_c$. Hence, if $r > r_c$, ξ^*, s^*, and $\mathbb{E}[s^* I] - \xi^*$ (weakly) decrease in r. It implies that there must exist an $r_{\xi,2} \in [\frac{1}{2}, r_c]$ such that for any $r > r_{\xi,2}$, ξ^*, s^*, and $\mathbb{E}[s^* I] - \xi^*$ decrease in r.

(ii) As before, let s_0^* denote the optimal s given $\xi = 0$. By a similar argument to the proof of part (i), it follows that if $x^*(0,s_0^*)\mathbb{E}[s_0^* I] < B$, then $\xi^* = 0$ and $s^* = s_0^*$. Suppose $x^*(0,s_0^*)\mathbb{E}[s_0^* I] < B$ when $r = \frac{1}{2}$. Then, due to the continuity of $x^*(0,s)$ and s_0^* in r, there must exist some $r_{\xi,b} \in (\frac{1}{2}, 1]$ such that for any $r \le r_{\xi,b}$, we have $x^*(0,s_0^*)\mathbb{E}[s_0^* I] < B$. Then, letting $r_{\xi,3} = r_{\xi,b}$ completes the proof. Suppose $x^*(0,s_0^*)\mathbb{E}[s_0^* I] = B$ when $r = \frac{1}{2}$. To prove part (ii) of the proposition, it is sufficient to prove that when $r = \frac{1}{2}$, we have $\xi^* = 0$ and $s^* = s_0^*$.

Consider $r = \frac{1}{2}$ and suppose $x^*(0,s_0^*)\mathbb{E}[s_0^* I] = B$. We next prove that $\xi^* = 0$ and $s^* = s_0^*$. From the proof of Theorem 1, we know that $x^*(0,s_0^*) \le x^{opt}$. Then, to prove that $\xi^* = 0$ and $s^* = s_0^*$, it is sufficient to prove that $(0,s_0^*) = \arg\max_{(\xi,s)\in\mathcal{F}} x^*(\xi,s)$ is unique maximizer of $x^*(\xi,s)$. Since we have $x^*(0,s_0^*)\mathbb{E}[s_0^* I] = B$, it is sufficient to prove that for any $\xi > 0$ and $s \in [0,\bar{s}]$ such that $x^*(\xi,s) \ge x^*(0,s_0^*)$, we have $\mathbb{E}[sI] - \xi > \mathbb{E}[s_0^* I]$ and thus $(\mathbb{E}[sI] - \xi)x^*(\xi,s) > B$. To do so, it is equivalent to proving that for any $\xi > 0$ and $s \in [0,\bar{s}]$ such that $\mathbb{E}[sI] - \xi \le \mathbb{E}[s_0^* I]$, we have $x^*(\xi,s) < x^*(0,s_0^*)$.

Consider any fixed $\xi' > 0$ and $s' \in [0,\bar{s}]$ such that $\mathbb{E}[s' I] - \xi' \le \mathbb{E}[s_0^* I]$. If $s' \le s_0^*$, then, since $x^*(\xi,s)$ strictly decreases in ξ and $x^*(0,s)$ increases in s for any $s \in [0,s_0^*]$, we have that $x^*(\xi',s') < x^*(0,s') \le x^*(0,s_0^*)$. Thus, it remains to consider the scenario where $s' > s_0^*$. Suppose $s' > s_0^*$. To prove that $x^*(\xi',s') < x^*(0,s_0^*)$, following the proof of Lemma 2, it is sufficient to prove that for any $x \in [0,1]$, $u(x|x,\xi',s') < u(x|x,0,s_0^*)$, where

$$u(i|x,\xi,s) = \mathbb{E}[(a - bxY)Y_i + sI - h_i - \xi] - \lambda \mathbb{V}\text{ar}[(a - bxY)Y_i + sI - h_i - \xi].$$

Then, when $r = \frac{1}{2}$, for any given $x \in [0,1]$, we have

$$\begin{aligned}
u(x|x,\xi',s') =& \mathbb{E}[(a - bxY)Y_i + s'I - xc - \xi'] - \lambda \mathbb{V}\text{ar}[(a - bxY)Y_i + s'I - xc - \xi'] \\
=& \mathbb{E}[(a - bxY)Y_i - xc] + \mathbb{E}[s'I] - \xi' - \lambda\left(\mathbb{V}\text{ar}[(a - bxY)Y_i] + \mathbb{V}\text{ar}[s'I]\right) \\
<& \mathbb{E}[(a - bxY)Y_i - xc] + \mathbb{E}[s_0^* I] - \lambda\left(\mathbb{V}\text{ar}[(a - bxY)Y_i] + \mathbb{V}\text{ar}[s_0^* I]\right) \\
=& u(x|x,0,s_0^*),
\end{aligned}$$

where the inequality holds because $\mathbb{E}[s'I] - \xi' \le \mathbb{E}[s_0^* I]$ and $s' > s_0^*$. Thus, we conclude that $s^* = s_0^*$ and $\xi^* = 0$ when $r = \frac{1}{2}$, which completes the proof. □

Appendix B Numerical Calibration

In this section, we describe in detail the steps for estimating the model parameters used in our numerical study in §7. At the end of this section, we also present additional numerical results to provide further insights.

Estimation of Model Parameters. We first describe how we estimate the mean μ and the standard deviation σ of area-level yield Y, by using population-level data. Recall that the crop yield of farmer i is denoted as $Y_i = Y + \epsilon_i$, where $Y \sim \mathcal{N}(\mu, \sigma^2)$ and $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Based on the annual average corn yield in Indonesia from 2010 to 2024, which is available from the USDA's Grain and Feed Annual,[2] we estimate that $\mu = 3.07$ tons per hectare and $\sigma = 0.3$ tons per hectare.

With the value of σ, to estimate σ_ϵ, it is sufficient to estimate the value of $\rho = \mathrm{corr}(Y_i, Y_j)$ for $i \neq j$. According to Wang et al. (1998), the correlation between an individual farmer's corn yield and the county-level average corn yield is approximately 0.8. That is, if we consider a county with n corn farmers and denote the county-level average corn yield as $\bar{Y}_n$, we have $\mathrm{Corr}(Y_i, \bar{Y}_n) = \frac{\mathrm{Cov}(Y_i, \bar{Y}_n)}{\sigma^2\sqrt{\rho + \frac{1-\rho}{n}}} = \frac{\frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2}{\sigma^2\sqrt{\rho + \frac{1-\rho}{n}}} \approx \frac{\rho}{\sqrt{\rho}} = 0.8$, where the approximation holds because n is large relative to ρ and $1-\rho$. Therefore, we obtain that ρ is approximately 0.6. We next estimate σ_ϵ based on the values of σ and ρ. For any $i \neq j$, we have $\mathrm{Corr}(Y_i, Y_j) = \frac{\mathrm{Cov}(Y_i, Y_j)}{\sigma^2 + \sigma_\epsilon^2} = \frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2} = \rho$. Thus, we obtain $\sigma_\epsilon = \sigma\sqrt{(1-\rho)/\rho} \approx 0.245$ tons per hectare.

We next describe how we estimate the production cost parameter c and the market price parameters a, b, and σ_p. To estimate c, we obtain from the USDA's website that the average corn production cost is \$875.1 per hectare (USDA 2023).[3] Letting $\frac{c}{2} = 875.1$, we have $c = \$1750.2$ per hectare. We next estimate a and b. From the USDA's Grain and Feed Annual mentioned earlier, we also obtain corn market prices and total corn supplies in Indonesia from 2010 to 2024. After adjusting these prices to 2021 values (to match the year of our estimated production cost) using inflation rates from the World Bank's website,[4] we fit a linear model to the adjusted price (AP, \$ per ton) and the total corn supply (TS, per ton) and obtain AP $= \$467.68 - \$1.04 \times 10^{-5} \times$ TS. Since there are approximately 7.6×10^6 farmers involved in cultivating corn in Indonesia (Nazmi et al. 2021), while our numerical study considers $n = 100,000$ farmers, we scale the slope of the price function accordingly to reflect the smaller total supply in our setting. That is, we estimate $a = \$467.68$ per ton and $b = \$1.04 \times 10^{-5} \times \frac{7.6\times10^6}{n} \approx 7.9 \times 10^{-4}$ per ton-squared. To estimate σ_p,

[2] The USDA's Grain and Feed Annual can be obtained through https://gain.fas.usda.gov/#/search. On this website, click "Advanced Search." In the search page, under "Countries," select "Indonesia;" under "Categories," check "Grain and Feed;" and under "Report Name" at "Additional Key Word Filter," type in "Grain and Feed Annual." Finally, click "Search." In the search results, for each year, there is a link to the corresponding report.

[3] This is obtained from the URL provided in the citation by clicking "Corn" under "Recent Cost and Returns" and downloading the Excel file. The file reports an operating cost of \$354.14 per acre for corn production in 2021, equivalent to \$875.1 per hectare.

[4] See the "Inflation, consumer prices (annual %)" tab on https://data.worldbank.org/country/indonesia.

we note that the ratio of the standard deviation to the mean of crop prices is typically small (e.g., approximately 0.1 in the setting considered by Chintapalli and Tang 2021). Accordingly, given that the corn price in Indonesia in 2021 is \$311, we set $\sigma_p = \$30$ per ton for the base scenario and later consider additional values to examine how price variability affects the optimal subsidy design.

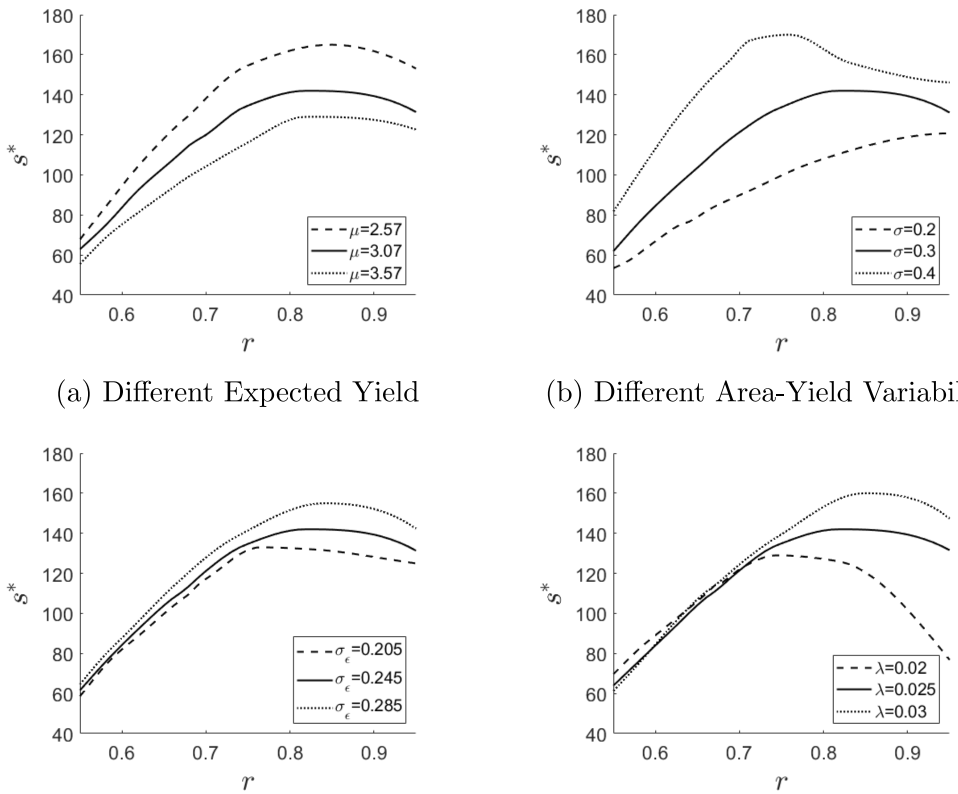
We now describe how we estimate the government's budget level B. We begin by estimating the budget when the government only offers an index-based yield protection policy. Based on design data for a rainfall index-based yield protection policy in Indonesia, farmers can receive up to approximately \$124 (Stoppa and Manuamorn 2017, p. 22) when the index predicts a low yield situation. Recall that the budget constraint in our setup is given by $\frac{sx^*(s)}{2} \leq B$. By assuming that the government targets the theoretically optimal planting level x^{opt} and scaling the planting level by the farmer population n, we estimate the government's budget to be $B = \frac{1}{2} \times \$124 \times n \times x^{opt} \approx \$2.8$ million. Since detailed data on both yield and price protection from a single setting are limited, we next draw on available data from a different context to estimate the budget for price protection. In Brazil, the support price policy spent approximately \$103 million subsidizing the price of 5.77 million tons of corn in one year (FAO 2014). Since we consider $n = 100,000$ farmers and, each farmer on average produces $\mu = 3.07$ tons of corn annually, we estimate the corresponding minimum price guarantee expenditure to be approximately \$5.5 million. Therefore, for the scenario where the government provides both price and yield protection, we set $B = 2.8 + 5.5 = \$8.3$ million.

We now describe how we estimate farmers' risk aversion coefficient λ. Recall from §3 that, to focus on the most relevant range of model parameters, we assume that λ is not too high, so that a positive number of farmers will plant without subsidies (i.e., $x_0^* > 0$), and that it is not too low, so that there is no over-planting without subsidies (i.e., $x_0^* \leq x^{opt}$). Based on the estimated values of other parameters, λ must fall within the interval [0.015, 0.035]. Therefore, for our numerical study, we set $\lambda = 0.025$.

Next, we describe how we estimate the cost of sampling each farmer to construct the area-yield index. Based on a recent study on yield estimation using crop cutting, assessing the yield of a smallholder corn farmer takes around 3.5 hours on average when carried out by 12 to 20 laborers (Kosmowski et al. 2021). The average hourly wage in Indonesia is approximately \$4 per hour (Economic Research Institute 2025). Thus, we estimate the cost of sampling each farmer as $\$4 \times 3.5 \times \frac{12+20}{2} \approx \$220$.

Additional Numerical Results. Using these estimates, we now present additional numerical results to provide further insights. First, we vary several key parameters, including the expected yield (μ), the area-yield variability (σ), the individual yield noise (σ_ϵ), and the farmer risk-aversion coefficient (λ), and plot the optimal subsidies under different index accuracy (see Figure B.1). We make the following two observations. First, the non-monotonic relationship between the optimal

Figure B.1 **Optimal Subsidy Amount under Different Index Accuracy and Other Key Parameters**



(a) Different Expected Yield

(b) Different Area-Yield Variability

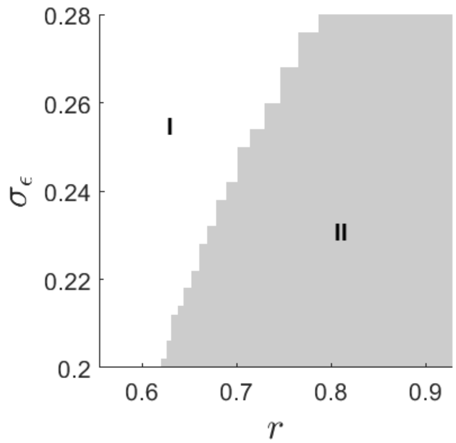
(c) Different Individual Yield Noise

(d) Different Risk-Aversion Coefficient

subsidy and index accuracy remains robust: as accuracy improves, subsidies should initially increase and then decrease once accuracy becomes sufficiently high. Second, the comparative statics with respect to the key parameters result in additional policy insights. In particular, governments should set higher subsidies when expected yield is low, or when either area-yield variability or individual yield noise is high. The effect of risk aversion, however, is more nuanced. When index accuracy is high, greater farmer risk aversion leads to a higher optimal subsidy, since risk-averse farmers particularly benefit from compensation in low-yield scenarios. By contrast, when index accuracy is low, greater risk aversion can lead to a lower optimal subsidy, because, as shown in §4, higher subsidies under low index accuracy can reduce worst-case income and increase income variability, which is detrimental for risk-averse farmers.

Second, we examine how subsidies should be adjusted relative to the actual-yield-based benchmark under different combinations of index accuracy r and individual yield noise σ_ϵ (both of which affect how well the index reflects individual yields). Specifically, we vary both r and σ_ϵ and generate a region plot to compare the optimal subsidies under index-based and actual-yield-based policies (see Figure B.2). We find that governments should set a lower subsidy than the actual-yield-based policy when r is low or σ_ϵ is high, and a higher subsidy otherwise. These results provide guidance on

Figure B.2 Comparison between Optimal Subsidies under Index-Based and Actual-Yield-Based Policies



Notes: In Region I, the optimal subsidy under the index-based policy is lower than that under the actual-yield-based policy; in Region II, it is higher.

how both sources of index inaccuracy—systematic mismatch at the area level (r) and idiosyncratic noise at the individual level (σ_ϵ)—jointly determine how subsidies should be adjusted.

Appendix C Extensions

In this appendix, we present several extensions and show that our key insights remain valid under alternative model setups. The proofs of all analytical results are provided in Appendix D.

C.1 Conditional Value at Risk (CVaR) for Farmer Risk Aversion

In §8.1, we used an alternative risk measure, the conditional value at risk (CVaR) to model farmers' risk aversion. We now show that our key insights continue to hold.

In practice, smallholder farmers are often highly vulnerable to income variability and exhibit high levels of risk aversion. Accordingly, for ease of exposition, we consider $\alpha \in (0, \frac{1}{4})$ in our analysis. Under the CVaR model, each farmer's utility no longer has a closed-form expression, which significantly complicates the analysis. For analytical tractability and to illustrate the robustness of our insights, we consider a special case with no individual noise in crop yields (i.e., $\sigma_\epsilon = 0$). Then, if $r \le 1 - 2\alpha$ (which is equivalent to $\frac{1-r}{2} \ge \alpha$), the expression of $\mathrm{CVaR}_\alpha(i|x, s)$ can be simplified as $\mathrm{CVaR}_\alpha(i|x, s) = (a - bx(\mu - \sigma))(\mu - \sigma) - h_i$. If $r > 1 - 2\alpha$ (which is equivalent to $\frac{1-r}{2} < \alpha$), we have $\mathrm{CVaR}_\alpha(i|x, s) = (a - bx(\mu - \sigma))(\mu - \sigma) - h_i + \frac{(\alpha - \frac{1-r}{2})s}{\alpha}$. The next proposition shows that the optimal subsidy amount remains non-monotonic in index accuracy under the CVaR model.

Proposition C.1. *When farmers make planting decisions based on their conditional value at risk (CVaR), the optimal subsidy amount first increases and then decreases in the index accuracy r.*

Proposition C.1 states that our non-monotonicity result presented in Theorem 1 continues to hold under the CVaR model, which demonstrates that our insights are not driven by the particular risk-averse model used in the main text (i.e., the mean-variance utility model). Further, since the combination of price and yield protection subsidies can more effectively increase farmers' lower-tail payoff, we numerically verify that our insights on the complementarity between these policies under low index accuracy continue to hold under the CVaR model.

C.2 Alternative Government Objectives

In our main analysis, we considered a government objective that maximizes the net benefit, defined as the expected total farmer surplus minus the expected total government expenditure. We now consider two alternative government objectives to demonstrate the robustness of our insights.

First, in §8.2, we extended our model to consider the scenario where the government incorporates consumer surplus into its objective. The next proposition shows that our key insights continue to hold under this extension.

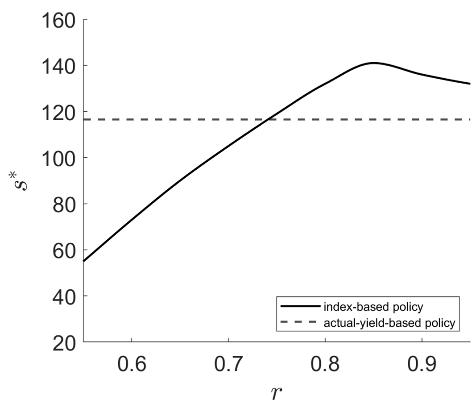
Proposition C.2. *When the government incorporates consumer surplus into the objective function, Theorems 1, 2, and 3 continue to hold. Moreover, the optimal subsidy amount is higher than when consumer surplus is not considered.*

As shown in Proposition C.2, our key insights remain valid when the government also considers consumer surplus when designing an index-based yield protection policy. Proposition C.2 also suggests that the optimal subsidy amount is higher when consumer surplus is included in the government's objective function than when it is not. Moreover, the inequality is strict when the index accuracy is higher than a certain threshold (see the proof of Proposition C.2 in §D). This is because when index accuracy is high, a higher subsidy can effectively incentivize more planting, which results in a lower market price and thus benefits consumers. When index accuracy is low, however, as shown in Propositions 1 and 2, an increase in subsidy can increase farmers' income variance and is less effective in incentivizing planting. In this case, even if consumer surplus is included in the government's objective, a low subsidy amount remains optimal.

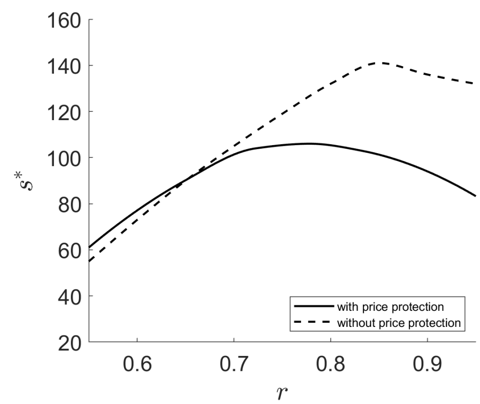
Next, we extend our model to consider another scenario where the government explicitly incorporates farmers' income variability into its objective. Recall that in our base model, the government's objective has incorporated the total farmer surplus, which is the aggregation of all farmers' income. This approach is consistent with existing studies on agricultural subsidies. For example, Chintapalli and Tang (2021) consider two government objectives: one that maximizes total farmer surplus minus government expenditure, and another that maximizes the sum of total farmer surplus and total consumer surplus minus government expenditure. Similarly, Alizamir et al. (2019) compare the performance of two subsidy policies based on total farmer surplus, total consumer surplus, and government expenditure. To demonstrate the robustness of our insights in a broader range of settings, we now consider an extension with the government objective

$\int_0^{x^*(s)} \left(\mathbb{E}[(a - bx^*(s)Y)Y + sI - xc] - \theta \times \mathbb{V}\text{ar}\left[(a - bx^*(s)Y)(Y + \epsilon_0) + sI - xc\right]\right) \mathrm{d}x \quad - \quad x^*(s)\mathbb{E}[sI]$, where we use ϵ_0 because the individual yield noise for all farmers has identical distribution. In this objective, the government explicitly incorporates farmers' income variability, with $\theta \geq 0$ capturing the weight the government places on farmers' income variability. Under this new government objective, the optimal subsidy decisions become analytically intractable due to the presence of higher-order terms of the equilibrium planting amount $x^*(s)$ in the objective function. Nevertheless, we numerically verify that our key insights continue to hold under this extension (see Figure C.1 for an illustration).

Figure C.1 Optimal Subsidy Decisions when the Government Incorporates Farmer Income Variability into Its Objective



(a) Index-based vs. actual-yield-based policy



(b) With vs. without price protection

Notes: We use the same parameter values as considered in §7, except for considering a two-point distribution for the area-level yield Y for computational simplicity. In addition, we consider a relatively small value of $\theta = 0.002$ given that the variance of farmers' income is much larger than their expected income.

C.3 Farmer Premiums in Government Programs

In §8.3, we extended our model to consider a scenario where farmers may be required to contribute a premium, and analyzed the jointly optimal payment amount s and farmer premium ξ. In this section, we analyze the government's decision on the payment amount s when the farmer premium ξ is set as a fixed fraction of the actuarially fair premium (i.e., a fixed fraction of $\mathbb{E}[sI]$). This setup is relevant when the government faces institutional constraints and has limited flexibility in adjusting how much premium it can require farmers to contribute. The following proposition shows that the key insights from our main analysis continue to hold.

Proposition C.3. *Suppose the farmer premium ξ is set as a fixed fraction of the actuarially fair level and the government optimizes the payment amount s. Then, Theorems 1, 2, and 3 continue to hold.*

Proposition C.3 shows that our key results regarding the government's choice of payment amount remain valid when the farmer premium is set as any fixed fraction of the actuarially fair level. This is because the premium each farmer pays is deterministic, and its effect is to reduce farmers' utility by a fixed amount that is linear in the payment amount s. As a result, all structural results are preserved under this extension (see §D for a formal proof of the proposition).

C.4 The Role of Insurers in Government Programs

In §8.4, we extended our model to incorporate insurer implementation in government yield protection programs, where the government first specifies two policy parameters: the payment amount s (the amount insurers pay to farmers when the index indicates a low yield) and the farmer premium ξ, and insurers then submit premium bids to compete for contracts. We now show that our key insights continue to hold and derive additional insights on how insurer implementation affects policy design, in particular how higher insurer administrative costs or information rents may affect the optimal subsidy level.

We start by analyzing insurers' premium decisions under given government decisions s and ξ. To facilitate the analysis, we introduce the following tie-breaking rule for insurers' decisions. Recall that if insurer j is awarded the contract, it earns an expected profit of $\eta_j - f_j - \mathbb{E}[sI]$ per farmer. When an insurer is indifferent between (i) a bid that loses and (ii) a bid that wins with zero margin (i.e., $\eta_j = \mathbb{E}[sI] + f_j$), it chooses the latter. This is a standard assumption in price competition, which does not affect our insights regarding the government's decisions and serves only to ensure the uniqueness of the insurers' equilibrium bidding decisions.

We first analyze the case where all insurers incur the same publicly known administrative cost f. The following lemma characterizes the insurers' equilibrium bidding decisions.

Lemma C.1. *When each insurer's administrative cost $f_j = f$ is public information, the premium bid of each insurer in equilibrium is given by $\eta_j = \mathbb{E}[sI] + f$.*

Lemma C.1 shows that in the full information setting where all insurers incur the same administrative cost f, each insurer bids $\mathbb{E}[sI] + f$. Intuitively, any higher bid would lose the contract, while any lower bid would result in a negative margin. As a result, competition drives the premium to the actuarially fair level plus the administrative cost, leaving insurers with zero expected profit.[5]

We now analyze the case where administrative costs are heterogeneous and private information. Specifically, each insurer j has an administrative cost f_j, which is drawn uniformly from $[0, \bar{f}]$, with the realization known only to the insurer. In this case, insurers' bidding decisions form a Bayesian game, and the following result characterizes the symmetric Bayesian Nash equilibrium.

[5] Here, zero expected profit implies that the insurer is compensated for their costs, where the cost parameter f may be interpreted broadly to include not only administrative expenses but also insurers' reservation earnings or opportunity costs.

Lemma C.2. *When each insurer's administrative cost f_j is uniformly distributed on $[0, \bar{f}]$ and is private information, the premium bid of each insurer in the symmetric Bayesian Nash equilibrium is given by $\eta_j = \mathbb{E}[sI] + \frac{J-1}{J} f_j + \frac{1}{J}\bar{f}$.*

Lemma C.2 shows that in the asymmetric information setting, each insurer bids a premium strictly higher than the actuarially fair level plus its administrative cost. Thus, in equilibrium, the winning insurer extracts an information rent and earns a strictly positive profit. This equilibrium outcome reflects the trade-off each insurer faces between the profit margin conditional on winning and the probability of winning. In addition, the lemma shows that insurers with lower costs submit more competitive bids, while those with higher costs bid higher premiums.

Having characterized the insurers' equilibrium bidding in both the public- and private-information cases, we now turn to the government's decisions. We first analyze the government's decision on the payment amount s when the farmer premium ξ is set as a fixed fraction of the actuarially fair premium (in which case ξ is proportional to s). This setup includes the fully subsidized case (i.e., $\xi = 0$) as a special case, and it is relevant when the government faces institutional constraints and has limited flexibility in adjusting how much premium it can require farmers to contribute. The following result holds for both the public- and private-information cases.

Proposition C.4. *Suppose the farmer premium ξ is set as a fixed fraction of the actuarially fair level and the government optimizes the payment amount s. Then, Theorems 1, 2, and 3 continue to hold.*

Proposition C.4 shows that the key insights derived from our main model that assumes centralized decision-making remain valid when insurer implementation is incorporated. This is because, in both the public- and private-information cases, the government's expected expenditure per farmer equals the actuarially fair premium plus insurers' administrative costs (and possible information rents), minus the farmer's premium contribution. This mirrors the centralized case, except that administrative costs and information rents (when present) are modeled explicitly. Although explicitly modeling these costs increases the government's expenditure per farmer, the structure of the government's problem remains unchanged, and thus our key insights remain valid.

We next explore how higher insurer administrative costs or information rents affect policy design. For the public-information case, let $\hat{f} = f$. For the private-information case, define $\hat{f} = \mathbb{E}[\min_{j \in \{1,\ldots,J\}} \frac{J-1}{J} f_j + \frac{1}{J}\bar{f}] = \frac{J-1}{J}\frac{\bar{f}}{J+1} + \frac{1}{J}\bar{f} = \frac{2\bar{f}}{J+1}$. Then, in both cases, the expected premium of the winning bid is $\mathbb{E}[sI] + \hat{f}$, where $\hat{f}$ can be interpreted as the sum of the administrative cost of the winning insurer and the information rent (when it exists).

Proposition C.5. *Suppose the farmer premium ξ is set as a fixed fraction of the actuarially fair level and the government optimizes the payment amount s. Then, the optimal payment amount s^* decreases in $\hat{f}$.*

Proposition C.5 shows that higher insurer administrative costs or information rents (reflected by a higher $\hat{f}$) result in a lower optimal payment amount s^*. Since the farmer premium is set as a fixed fraction of the actuarially fair level, this implies a lower optimal subsidy level. The intuition behind this result is that a higher $\hat{f}$ makes subsidies less cost-effective, so the government reduces s^* to balance farmer protection against administrative costs. This result formalizes how information rents undermine the effectiveness of subsidies: part of the government's expenditure is absorbed as insurer profits rather than translated into farmer protection. It underscores the importance of designing bidding mechanisms or regulatory measures that limit such rents to preserve the effectiveness of yield protection policies.

We next analyze the case where the government jointly optimizes the payment amount s and the farmer premium ξ. The following result holds for both the public- and private-information cases.

Proposition C.6. *Suppose the government jointly optimizes the payment amount s and the farmer premium ξ. Then, Proposition 4 continues to hold.*

Proposition C.6 shows that our insights in Proposition 4 regarding the joint design of the payment amount and farmer premium continue to hold when insurer implementation is incorporated. As discussed above, under insurer implementation, the only difference from the centralized case is that administrative costs (and possibly information rents) are made explicit. These terms increase the government's expected expenditure per farmer but do not alter the structure of the government's problem. Hence, the characterization of the optimal (s, ξ) remains valid.

Moreover, while Proposition 4 establishes the non-monotonicity of the optimal subsidy in index accuracy, it also shows that the optimal farmer premium is $\xi = 0$ (i.e., fully subsidized) when index accuracy is sufficiently low. Thus, by showing that these results carry over under insurer implementation, Proposition C.6 implies that the complementarity between price protection and index-based yield protection at low index accuracy, established in the main model, also remains valid.

C.5 Alternative Modeling of Farmers' Planting Decisions

In our main analysis, we considered scenarios where farmers decide whether to plant a specific crop. In this section, we present two alternative setups for farmers' planting decisions.

First, we extend our model to consider farmers' planting decisions for two crops. We denote the two crops as Crop 1 and Crop 2, where Crop 1 generates a higher expected revenue but entails greater risk. In particular, we use the notation from our base model to denote the parameters for Crop 2, while Crop 1 has a higher maximum possible price, ua, and a higher standard deviation of the area-level yield, $v\sigma$, where $u, v > 1$. Further, since high-revenue, high-risk crops often require more inputs (e.g., fertilizer, irrigation, pest control) and are more sensitive to production conditions, they typically involve higher production costs. Accordingly, we assume that the production cost of farmer $i \in [0,1]$ for Crop 1 is given by wh_i, where $h_i = ic$ and $w > 1$. Each farmer decides whether to grow one of these crops or pursue outside options. Since crops with higher yield variability pose greater income risk to farmers, governments often prioritize such crops for yield protection. Accordingly, we assume that the government provides an index-based yield protection policy for Crop 1. Let s denote the subsidy payment each farmer who plants Crop 1 receives when the index predicts a low yield. The following lemma establishes that, for any given subsidy amount s, farmers' equilibrium planting decisions exhibit a threshold structure.

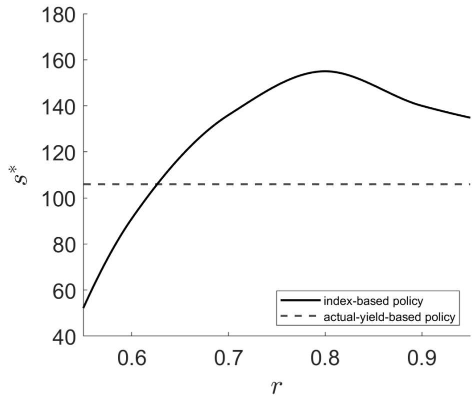
Lemma C.3. *For any given subsidy amount $s \geq 0$, there exist two thresholds τ_1, τ_2, with $\tau_1 \leq \tau_2$, such that in equilibrium, farmer $i \in [0,1]$ plants Crop 1 if $h_i \leq \tau_1$, plants Crop 2 if $\tau_1 < h_i \leq \tau_2$, and does not plant if $h_i > \tau_2$.*

Lemma C.3 characterizes an intuitive structure of farmers' equilibrium planting decisions: farmers with high production efficiencies (i.e., $h_i \leq \tau_1$) plant the high-revenue but riskier crop, i.e., Crop 1, farmers with moderate production efficiencies (i.e., $\tau_1 < h_i \leq \tau_2$) plant the low-revenue but less risky crop, i.e., Crop 2, while farmers with low production efficiencies (i.e., $h_i > \tau_2$) do not plant.

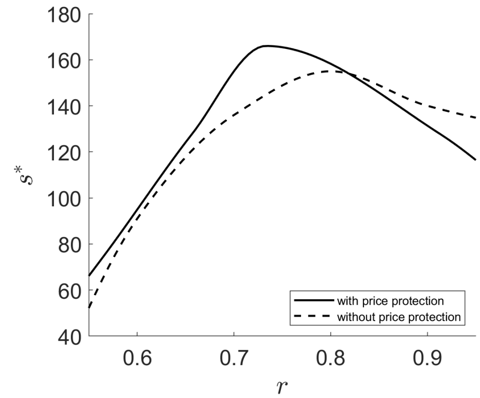
Let $x_1^*(s)$ and $x_2^*(s)$ denote the equilibrium planting amounts for Crop 1 and Crop 2, respectively, under a given subsidy amount s. Let Y_1 denote the area-level yield for Crop 1, where $Y_1 = \mu - v\sigma$ with probability $\frac{1}{2}$ and $Y_1 = \mu + v\sigma$ with probability $\frac{1}{2}$, and let Y_2 denote the area-level yield for Crop 2, where $Y_2 = \mu - \sigma$ with probability $\frac{1}{2}$ and $Y_2 = \mu + \sigma$ with probability $\frac{1}{2}$. Then, similar to the base model, the net benefit is $v(s) = \mathbb{E}[\int_0^{x_1^*(s)} ((ua - bx_1^*(s)Y_1)Y_1 + sI - wxc)\,\mathrm{d}x] - x_1^*(s)\mathbb{E}[sI] + \mathbb{E}[\int_{x_1^*(s)}^{x_1^*(s)+x_2^*(s)} ((a - bx_2^*(s)Y_2)Y_2 - xc)\,\mathrm{d}x]$. Under this setup, since the equilibrium planting amounts take a more complex form, and the net benefit involves interaction terms between the two planting amounts, such as $x_1^*(s)x_2^*(s)$, a full characterization of the government's optimal subsidy decisions becomes analytically intractable. Nevertheless, we numerically verify that our qualitative insights, including how subsidies should be adjusted for index inaccuracy and the complementarity between price and yield protection under low index accuracy, continue to hold (see Figure C.2 for an illustration).

Next, we extend our model to consider another setting where each farmer i makes a continuous planting quantity choice $q_i \in [0,1]$, with a production cost cq_i^2. The quadratic cost function captures increasing marginal costs faced by smallholder farmers when expanding production. Each farmer receives a subsidy payment s for each unit of planting quantity when the index indicates a low area-level yield. That is, farmer i receives a subsidy payment sq_i when $I = 1$. We continue to let Y

Figure C.2 Optimal Subsidy Decisions under an Alternative Planting Model with Two Crops



(a) Index-based vs. actual-yield-based policy



(b) With vs. without price protection

Notes: We use the same parameter values as considered in §7, except for considering a two-point distribution for the area-level yield Y for computational simplicity. In addition, we consider $u, v, w = 1.1$.

and Y_i denote the area-level yield and the yield of farmer i, respectively. Suppose the total planting quantity by all farmers (i.e., the integral of q_i for all i) is x. Then, given the subsidy amount s, the net income of farmer i who plants q_i is given by $\pi(q_i, s) = (a - bxY)q_iY_i + sq_iI - cq_i^2$. Each farmer chooses their planting quantity to maximize their mean-variance utility. Under this setup, the mean-variance utility of farmer i involves $x^2q_i^2$, which significantly complicates the analytical characterization of the equilibrium planting quantities and makes a full characterization of the government's optimal subsidy decisions analytically intractable. Nevertheless, we numerically verify that our qualitative insights continue to hold (see Figure C.3 for an illustration).

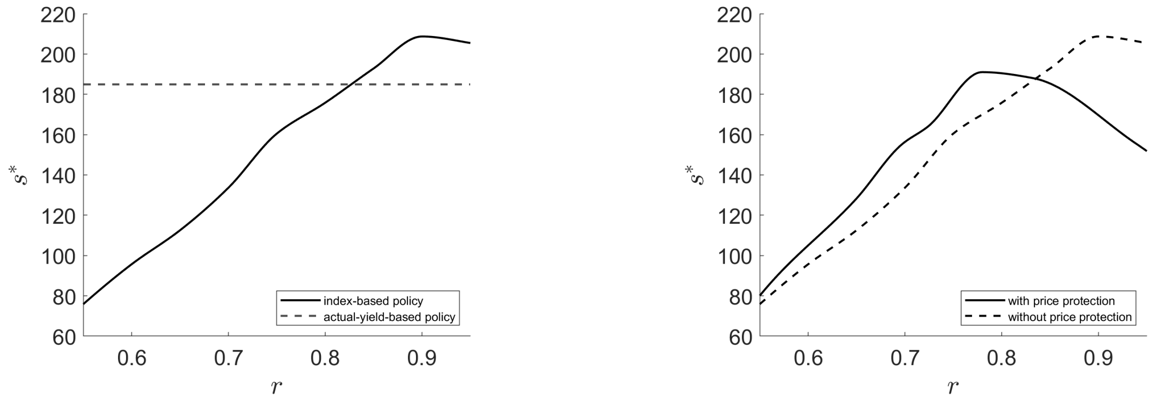
C.6 Alternative Distributions for Yield and Index

In our main analysis, for analytical tractability, we used a relatively simple representation with binary distributions for both the area-level yield Y and the index I to capture two key features: yield uncertainty and the fact that the index is an imperfect proxy of the area-level yield. In this section, we present two extensions with more realistic distributions for crop yields and the index to demonstrate the robustness of our insights.

Continuous Distributions for Y and I. First, we extend our model to consider a normal distribution for the area-level yield, $Y \sim \mathcal{N}(\mu, \sigma^2)$, and introduce the following two representations of the index I to capture two commonly used types of indices.

To model commonly used weather indices (such as a rainfall index), we let R denote the rainfall level (or other similar features) and assume that $Y = f(R) + \eta$, where the error term η is independent of R and follows a normal distribution, $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$. We define the index $I = f(R)$ as the predicted yield based on rainfall. Then, we have $Y = I + \eta$. This model allows for a potentially non-linear

Figure C.3 Optimal Subsidy Decisions under an Alternative Planting Model with Continuous Quantity Choice



(a) Index-based vs. actual-yield-based policy (b) With vs. without price protection

Notes: We use the same parameter values as in §7, except that we consider a two-point distribution for the area-level yield Y for computational simplicity and recalibrate the risk aversion coefficient λ for the new setup. Specifically, recall from Appendix B that we calibrated the value of λ such that the equilibrium planting amount without subsidies is neither too high nor too low. For the current setup, we follow the same approach as in Appendix B and choose $\lambda = 0.035$ so that the equilibrium planting amount without subsidies falls within a reasonable range.

relationship between rainfall and yield. For instance, we may have $f(R) = -\alpha R^2 + \beta R + \gamma$, which captures a scenario where both very high and very low rainfall lead to low crop yields.

To model another commonly used index, the area-yield index (i.e., the sample estimate of the average yield of an area), we specify the relationship between the area-level yield (i.e., Y) and the index (i.e., I) as $I = Y + \eta$, where the error term η is independent of Y and follows a normal distribution, $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$. In other words, the index I is a noisy observation of Y. We note that this setup includes the setup in §7 as a special case. Specifically, if I is the sample average estimate of Y based on k samples, then $I = \frac{1}{k}\sum_{i=1}^{k} Y_i = Y + \frac{1}{k}\sum_{i=1}^{k}\epsilon_i$. In this case, $\eta = \frac{1}{k}\sum_{i=1}^{k}\epsilon_i$.

In both of the above representations, given the distribution of Y, the accuracy of the index I is uniquely determined by the variability of the error term, i.e., σ_η. When $\sigma_\eta = 0$, the index I provides a perfect prediction of Y; as σ_η increases, I becomes a noisier prediction of Y. In line with stepwise payment functions used in practice (Elabed et al. 2013), we consider that farmers receive a subsidy payment s when the predicted yield (i.e., I) falls below the expected yield μ. We later demonstrate that our qualitative insights continue to hold under alternative payment functions.

In addition to considering more realistic distributions for the yield Y and the index I, we further extend our model by introducing a price noise term to capture additional variability in market price beyond supply effects. For any given planting amount x, we now assume that the market-clearing price is given by $a - bxY + \epsilon_p$, where $\epsilon_p \sim \mathcal{N}(0, \sigma_p^2)$ represents the price noise. With ϵ_p, we assume $\lambda < \frac{a\mu}{a^2(\sigma^2+\sigma_\epsilon^2)+(\mu^2+\sigma^2+\sigma_\epsilon^2)\sigma_p^2}$ to avoid the extreme case where no one plants without subsidies.

In addition, as before, we assume that farmers' maximum production cost c is high such that for any given subsidy within the budget, the equilibrium planting amount is strictly less than one.

Under this setup, the following proposition shows that our insight regarding the non-monotonic effect of index accuracy on the optimal subsidy continues to hold.

Proposition C.7. *Given $Y \sim \mathcal{N}(\mu, \sigma^2)$, under both representations of the index I, the optimal subsidy amount first increases and then decreases as index accuracy increases (i.e., as σ_η decreases).*

Proposition C.7 shows that our insight on how the optimal subsidy varies with index accuracy remains robust under continuous yield and index distributions and in the presence of price noise (see §D for the proof of the proposition). With this result, it is also straightforward to verify that our insights regarding how subsidies should be set differently to account for index inaccuracy when using predicted rather than actual yields, as presented in Corollary 2, continue to hold.

With continuous distributions, analytically establishing our other insights, such as the complementarity between price and yield protection under low index accuracy, becomes intractable. However, in §7, we have numerically verified that these insights continue to hold.

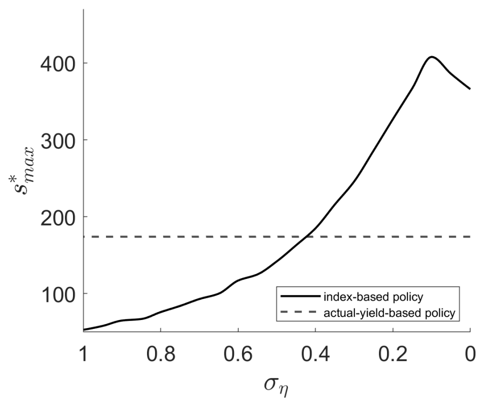
We further note that our qualitative insights are not driven by the stepwise payment function considered in our analysis. To illustrate this, we numerically verify that our insights remain valid under an alternative payment function that is piecewise linear in the predicted yield (see Figure C.4 for an illustration).

Alternative Payment Probability. In our main analysis, we assumed that the area-level yield Y and the index I have the same marginal distributions. This implies that the probability of triggering a payment is equal to the probability that the actual area-level yield is low, i.e., $\mathbb{P}(I=1)=\mathbb{P}(Y=\mu-\sigma)$, which simplifies the exposition and allows us to highlight our key insights more clearly. We now extend our model to allow different payment probabilities and show that our key insights continue to hold.

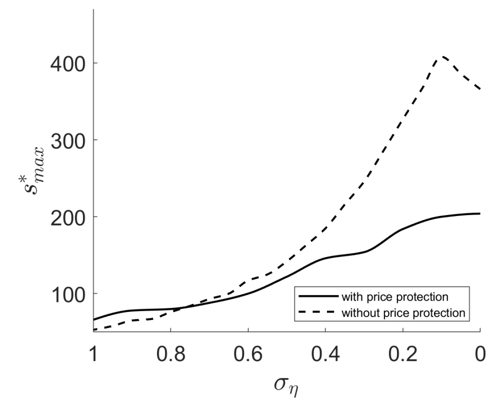
Specifically, we now consider a more general joint distribution of Y and I, as shown in Table C.1. This distribution is characterized by two parameters: p and r. Here, $p \in (0,1)$ denotes the probability of a low-yield prediction, $\mathbb{P}(I=1)$, which we refer to as the payment probability. In practice, this probability can be adjusted by changing the threshold of the predicted yield below which a payment is triggered. For any given payment probability p, a higher value of r implies a higher probability of receiving payment when the actual yield is low. Accordingly, we continue to use r to represent index accuracy. When $p=\frac{1}{2}$, the joint distribution reduces to the same as in our base model. To ensure that all probabilities are non-negative, we assume that $rp \le \frac{1}{2}$ and $(1-r)p \le \frac{1}{2}$.

Proposition C.8. *When the joint distribution of the area-level yield Y and the index I is as shown in Table C.1, Theorems 1, 2, and 3 continue to hold.*

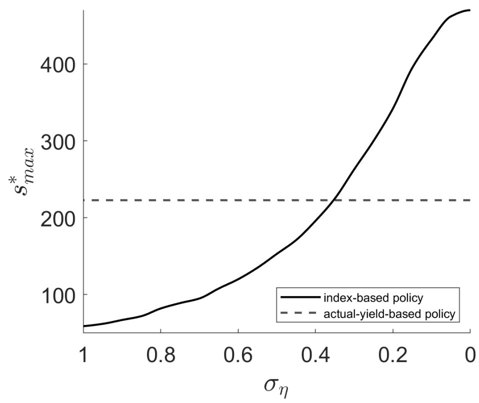
Figure C.4 Optimal Subsidy Decisions under an Alternative Payment Function



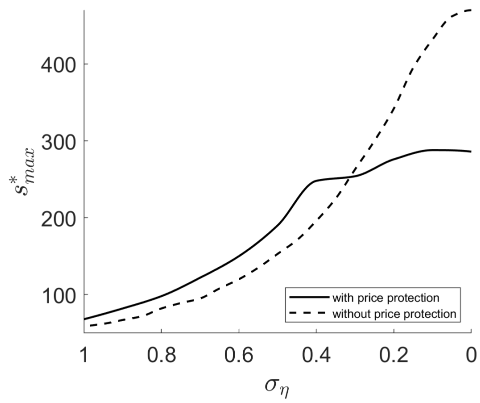
(a) Index-based vs. actual-yield-based policy ($\sigma_\epsilon = 0.2$)



(b) With vs. without price protection ($\sigma_\epsilon = 0.2$)



(c) Index-based vs. actual-yield-based policy ($\sigma_\epsilon = 0.245$)



(d) With vs. without price protection ($\sigma_\epsilon = 0.245$)

Notes: We use the same setup and parameter values as in §7, except that we consider an alternative payment function: given the index $I = Y + \eta$, where $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$ and is independent of Y, each farmer receives a payment $s_{\max} \times \max\{\mu - I, 0\}$. In addition, we consider two different values of σ_ϵ. Our qualitative insights continue to hold. Moreover, the range of conditions in which the optimal subsidy under an index-based policy is below that under an actual-yield-based policy, as well as the range of conditions in which the presence of price protection leads to a higher index-based yield protection subsidy, expands as the variability of individual yield noise σ_ϵ increases.

Table C.1 New Joint Probability Table for Index-based Yield Protection

	$I = 0$	$I = 1$
$Y = \mu + \sigma$	$\frac{1}{2} - (1-r)p$	$(1-r)p$
$Y = \mu - \sigma$	$\frac{1}{2} - rp$	rp

Proposition C.8 shows that our key insights continue to hold under a more general joint distribution of Y and I, where the payment probability p can take any value in [0,1]. The proof of the proposition is included in Appendix D. Moreover, we numerically observe that a lower payment probability p expands the range of index accuracy r over which (i) the optimal subsidy increases with index accuracy, and (ii) price and yield protection act as strategic complements.

C.7 Comparison with Actual-Yield-Based Policy

In Corollary 2, we compared the optimal subsidy amount under an index-based policy and that of an actual-yield-based policy. To isolate the impact of index inaccuracy, we conducted the comparison by considering the same budget for subsidy payments under the two policies. In this section, we generalize the analysis by considering that an actual-yield-based policy incurs additional costs for assessing individual farmers' yields, which reduces the available budget for subsidy payment. We show that our insights presented in Corollary 2 continue to hold after yield assessment costs are incorporated. Moreover, we derive additional insights on when each policy leads to a higher net benefit.

Let w denote the cost of assessing each farmer's yield. Recall that under an actual-yield-based policy, farmers receive a subsidy payment s if their actual yield is below its expected value μ. Let $x_A^*(s)$ denote the equilibrium planting amount under this policy. Then, since we have $\mathbb{E}[s\mathbb{1}(Y_i < \mu)] = \frac{1}{2}s$ for any farmer $i \in [0,1]$, the subsidy amount under this policy must satisfy $x_A^*(s) \times (\frac{1}{2}s + w) \leq B$. That is, the sum of the total expected payment to farmers and the total cost of yield assessment must not exceed the budget. Without loss of generality, we assume $wx_0^* \leq B$; otherwise, there will be zero budget left for subsidizing farmers. As in the index-based policy, we restrict attention to subsidy levels such that the expected income conditional on Y being low plus the subsidy does not exceed that conditional on Y being high. The government chooses the subsidy amount s to maximize the net benefit $v_A(s,w) = \mathbb{E}[\int_0^{x_A^*(s)} \left((a - bx_A^*(s)Y)Y + \frac{1}{2}s - xc\right) \mathrm{d}x] - \frac{1}{2}sx_A^*(s) - wx_A^*(s) = v_1 x_A^*(s) - v_2(x_A^*(s))^2 - wx_A^*(s)$, where $v_1 = a\mu$ and $v_2 = b(\mu^2 + \sigma^2) + \frac{1}{2}c$. Let $x_A^{opt} = \frac{v_1 - w}{2v_2}$. To focus our analysis on relevant scenarios, we assume that $w \leq v_1$. Then, the optimal subsidy under the actual-yield-based policy, s_A^*, must satisfy that the corresponding equilibrium planting amount $x_A(s_A^*)$ is as close to x_A^{opt} as possible. Further, as in our base model, to avoid an uninteresting case where planting has already been sufficient without any subsidy, we focus on scenarios with $x_0^* \leq x_A^{opt}$ (it is straightforward to verify that $x_A^{opt} \leq x^{opt}$). Then, we have the following result.

Proposition C.9. *There exists a threshold r_3 such that if the index accuracy is low ($r < r_3$), the optimal subsidy under an index-based policy is lower than that of an actual-yield-based policy (i.e., $s^* < s_A^*$). Otherwise ($r \geq r_3$), the optimal subsidy under an index-based policy is higher than that of an actual-yield-based policy (i.e., $s^* \geq s_A^*$). Moreover, r_3 decreases in w.*

Proposition C.9 shows that our insights regarding how subsidies should be set differently when using predicted rather than actual yields, as presented in Corollary 2, continue to hold when the cost of yield assessment is incorporated. Moreover, it shows that a higher yield assessment cost w expands the range of parameters for which the optimal subsidy under the index-based policy is higher than under the actual-yield-based policy (i.e., r_3 decreases in w). This is because, when w is higher, a lower budget remains for subsidizing farmers under the actual-yield-based policy, which necessitates a lower subsidy amount under that policy.

Our analysis highlights an important trade-off between index-based and actual-yield-based policies: the former avoids the cost of yield assessment but is subject to the risk of failing to compensate farmers when their actual yield is low. To further examine this trade-off, the next proposition shows when each of the two policies leads to a higher net benefit.

Proposition C.10. *There exist two thresholds w_1 and w_2, with $w_1 \leq w_2$, such that if the yield assessment cost is low ($w \leq w_1$), the actual-yield-based policy always performs better; if the yield assessment cost is high ($w \geq w_2$), the index-based policy always performs better; otherwise ($w_1 < w < w_2$), the index-based policy performs better if and only if the index accuracy exceeds a threshold.*

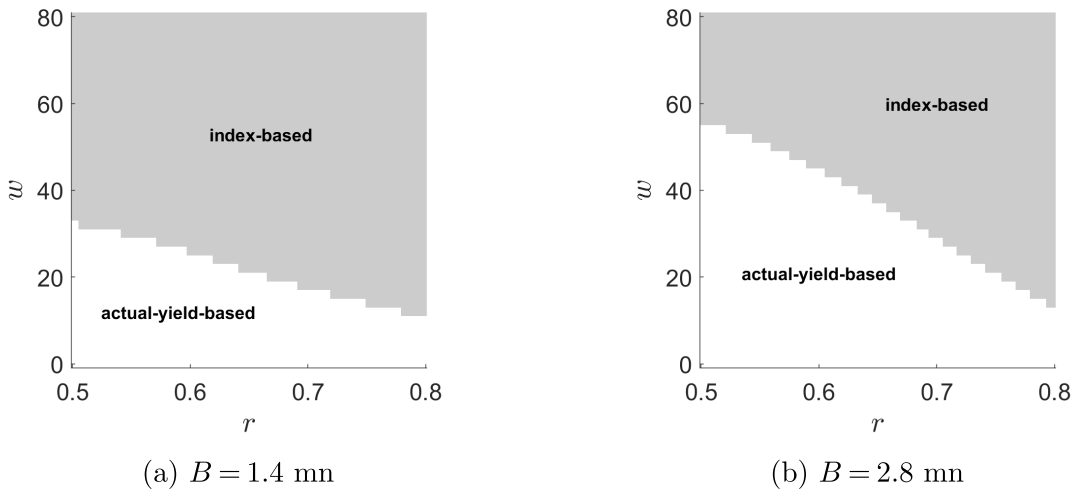
Proposition C.10 shows that the relative performance of index-based and actual-yield-based policies critically depends on the magnitude of the yield assessment cost and the index accuracy. Intuitively, if the yield assessment cost is very low ($w \leq w_1$), the cost savings under the index-based policy are minimal, making the actual-yield-based policy more favorable (note that even if the index perfectly predicts the area-level yield Y, i.e., $r = 1$, it may still fail to reflect individual farmers' actual yields). On the other hand, if the yield assessment cost is very high ($w \geq w_2$), the cost savings under the index-based policy outweigh the potential losses from index inaccuracy, making the index-based policy more favorable. Finally, if the yield assessment cost is moderate ($w_1 < w < w_2$), the index-based policy is preferred if and only if the index accuracy is sufficiently high. Moreover, our numerical analysis shows that the range of conditions under which the index-based policy is preferred expands when the budget is tighter (see Figure C.5 for an illustration).

We conclude this section with a discussion of additional factors that may favor index-based over actual-yield-based policies in practice. Yield assessment is not only costly but also time-consuming. By avoiding this, an index-based approach can streamline the payment distribution process and ensure that farmers receive payments in a timely manner. In addition, since individual farmers typically cannot influence the index, an index-based approach effectively minimizes moral hazard issues (Carter et al. 2017). Accounting for these factors can further expand the range of conditions under which index-based policies are preferred over actual-yield-based policies.

Appendix D Proofs of Analytical Results in Appendix C

Proof of Proposition C.1. As discussed in §8.1, if $r \leq 1 - 2\alpha$ (which is equivalent to $\frac{1-r}{2} \geq \alpha$), we have $\mathrm{CVaR}_\alpha(i|x,s) = (a - bx(\mu - \sigma))(\mu - \sigma) - h_i$. If $r > 1 - 2\alpha$ (which is equivalent to $\frac{1-r}{2} < \alpha$),

Figure C.5 Comparison between an Index-Based Policy and an Actual-Yield-Based Policy



Notes: We use the same parameter values as considered in §7, except for considering a two-point distribution for the area-level yield Y for computational simplicity.

we have $\mathrm{CVaR}_\alpha(i|x,s) = (a - bx(\mu-\sigma))(\mu-\sigma) - h_i + \frac{(\alpha - \frac{1-r}{2})s}{\alpha}$. We next characterize the optimal subsidy amount by considering $r \leq 1-2\alpha$ and $r > 1-2\alpha$, respectively. For brevity, we prove the result without explicitly incorporating the budget constraint; the analysis can be extended to include it following the approach in Theorem 1.

First, consider $r \leq 1-2\alpha$. In this case, because there is a significant probability that farmers suffer from low crop yield without receiving subsidy, increasing the subsidy amount s cannot increase the lower-tail conditional expected payoff. Therefore, in this range of r, we have $s^* = 0$.

Second, consider $r > 1-2\alpha$. In this case, following a similar analysis as for the base model, it can be shown that the equilibrium planting amount $x^*(s)$ must satisfy either $x^*(s) = 1$ or

$$(a - bx^*(s)(\mu-\sigma))(\mu-\sigma) - cx^*(s) + \frac{\left(\alpha - \frac{1-r}{2}\right)s}{\alpha} = 0.$$

Then, the equilibrium planting amount is given by

$$x^*(s) = \min\left\{\frac{a(\mu-\sigma) + \frac{(\alpha - \frac{1-r}{2})s}{\alpha}}{b(\mu-\sigma)^2 + c}, 1\right\}.$$

From this expression, it is clear that $x^*(s)$ increases in s, and it increases at a faster rate when r is higher. Further, since $x^*(s)$ strictly increases in s until it hits one, we must have $x^*(s) \geq x^{opt}$ when s is sufficiently high. Recall that the optimal subsidy amount s^* must satisfy that the corresponding equilibrium planting amount $x^*(s^*)$ is equal to or as close to x^{opt} as possible. Therefore, for $r > 1-2\alpha$, the optimal subsidy amount s^* must decrease in r to ensure that $x^*(s^*) = x^{opt}$.

To summarize, when r increases from a small number, the optimal subsidy amount s^* initially remains zero, then jumps to a positive number, and then decreases in r, which demonstrates that the non-monotonic relationship between optimal subsidy and index accuracy continues to hold. Further, we remark that the main reason why there is a jump in the optimal subsidy amount as a function of r is that the yield Y follows a two-point distribution. Following the same setup as in Section 7, we numerically verify that when the yield follows a continuous distribution, the optimal subsidy also becomes continuous in r, and it first increases and then decreases in r. $\square$

Proof of Proposition C.2. Since including the consumer surplus in the net benefit does not affect the farmers' utility, the expression of the equilibrium planting amount remains unchanged under this extension. Thus, all our results presented before Theorem 1 continue to hold.

For Theorem 1, by incorporating consumer surplus, the government's objective is as follows:

$$\begin{aligned} v_c(s) =& \mathbb{E}\left[\int_0^{x^*(s)} ((a - bx^*(s)Y)Y + sI - xc)\, \mathrm{d}x\right] + \mathbb{E}\left[\frac{1}{2}(bx^*(s)Y) \times (x^*(s)Y)\right] - x^*(s)\mathbb{E}[sI] \\ =& a\mu \times x^*(s) - \frac{1}{2}(c + b(\mu^2 + \sigma^2)) \times (x^*(s))^2 \end{aligned}$$

Let $x_c^{opt} = \frac{a\mu}{b(\mu^2+\sigma^2)+c}$ be the planting amount that maximizes $v_c(s)$, and let $\mathcal{S} = [0, \bar{s}] \cap \{s : x^*(s)\mathbb{E}[sI] \leq B\}$. By replacing $v(s)$ with $v_c(s)$ and x^{opt} with x_c^{opt} in the proof of Theorem 1, we can show that there must exist a threshold $r_2 = \min\{r_b, r_c'\}$ such that the optimal subsidy, which we now denote as s_c^*, increases in r if and only if $r \leq r_2$, where r_b is defined in the proof of Theorem 1 and r_c' is the lowest index accuracy under which $x^*(s) = x_c^{opt}$ for some $s \in \mathcal{S}$. (Let $r_c' = \infty$ if no such index accuracy exists.) Further, since $x_c^{opt} = \frac{a\mu}{b(\mu^2+\sigma^2)+c} > \frac{a\mu}{2b(\mu^2+\sigma^2)+c} = x^{opt}$, we have $s_c^* \geq s^*$.

Similarly, since the expression of the equilibrium planting amount remains unchanged, our proofs for Theorems 2 and 3 remain valid by replacing $v(s)$ with $v_c(s)$ and x^{opt} with x_c^{opt}. $\square$

Proof of Proposition C.3. Consider any given $s \in [0, 2a\sigma]$ (recall from our main analysis that $\bar{s} \leq 2a\sigma$). Since the farmer premium ξ is a fixed fraction of the actuarially fair level, we let $\xi = \eta\mathbb{E}[sI]$, where $\eta \in [0,1]$. Suppose x farmers plant. Then, the utility of farmer i from planting and enrolling is $u(i|x,\eta,s) = \mathbb{E}[(a - bxY)Y_i + sI - h_i - \eta\mathbb{E}[sI]] - \lambda\mathbb{V}\mathrm{ar}[(a - bxY)Y_i + sI - h_i - \eta\mathbb{E}[sI]]$. Then,

$$\begin{aligned} u(x|x,\eta,s) &= \mathbb{E}[(a - bxY)Y_i + sI - xc - \eta\mathbb{E}[sI]] - \lambda\mathbb{V}\mathrm{ar}[(a - bxY)Y_i + sI - xc - \eta\mathbb{E}[sI]] \\ &= -\alpha x^2 - \beta(s)x - \gamma(\eta,s), \end{aligned}$$

where $\alpha = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$, $\beta(s) = b(\mu^2+\sigma^2) + c - 4\lambda ab\mu\sigma^2 - 2\lambda ab\mu\sigma_\epsilon^2 + 4\lambda b(r - \frac{1}{2})\mu\sigma s$, and $\gamma(\eta,s) = \lambda a^2(\sigma^2+\sigma_\epsilon^2) - a\mu - 2\lambda a(r-\frac{1}{2})\sigma s - \frac{1}{2}(1-\eta)s + \frac{1}{4}\lambda s^2$.

Further, as discussed in the proof of Proposition 4, it is sufficient to focus on the scenario where all farmers have a higher utility from planting and enrolling than from planting but not enrolling. In this case, following the same analysis as in the proof of Lemma 2, the equilibrium planting amount is given by $x^*(\eta,s) = \max\{\min\{\hat{x}(\eta,s), 1\}, 0\}$, where $\hat{x}(\eta,s) = \frac{-\beta(s)+\sqrt{\beta(s)^2-4\alpha\gamma(\eta,s)}}{2\alpha}$.

Clearly, the equilibrium planting amount has the same structure as before. Further, the government's objective function remains unchanged. Thus, following the same steps of analysis as before, the results presented in §4 continue to hold.

Next, with a floor price $m \in [0,a]$ and an index-based yield protection subsidy $s \in [0, 2a\sigma]$, the utility of farmer i from planting is as follows:

$$\begin{aligned} u(i|x,\eta,m,s) =& \mathbb{E}[\max\{(a-bxY),m\}Y_i + sI - h_i - \eta\mathbb{E}[sI]] \\ & - \lambda\mathbb{V}\text{ar}[\max\{(a-bxY),m\}Y_i + sI - h_i - \eta\mathbb{E}[sI]]. \end{aligned}$$

Following the same analysis in the proof of Lemma A.4, the equilibrium planting amount is given by $x^*(\eta,m,s) = \max\{\min\{\hat{x}(\eta,m,s),1\},0\}$, where $\hat{x}(\eta,m,s)$ can be obtained by considering the following three cases. Specifically, there exist two thresholds m_1 and m_2 such that

(i): If $m \in [0,m_1]$, then we have $\hat{x}(\eta,m,s) = \hat{x}(\eta,s)$.

(ii): If $m \in (m_1,m_2]$, then we have $\hat{x}(\eta,m,s) = \frac{-\beta(m,s)+\sqrt{\beta^2(m,s)-4\alpha_m\gamma(\eta,m,s)}}{2\alpha_m}$, where $\alpha_m = \frac{1}{4}\lambda b^2(\mu-\sigma)^4 + \frac{1}{2}\lambda b^2(\mu-\sigma)^2\sigma_\epsilon^2$, $\beta(m,s) = \frac{1}{2}b(\mu-\sigma)^2 + c + \frac{1}{2}\lambda b(\mu-\sigma)^2(m(\mu+\sigma)-a(\mu-\sigma)) - \lambda ab(\mu-\sigma)\sigma_\epsilon^2 - \lambda b(r-\frac{1}{2})s(\mu-\sigma)^2$ and $\gamma(\eta,m,s) = \frac{1}{4}\lambda(m(\mu+\sigma)-a(\mu-\sigma))^2 + \frac{1}{2}\lambda(a^2+m^2)\sigma_\epsilon^2 - \frac{1}{2}m(\mu+\sigma) - \frac{1}{2}a(\mu-\sigma) - \lambda(r-\frac{1}{2})s(m(\mu+\sigma)-a(\mu-\sigma)) - \frac{1}{2}(1-\eta)s + \frac{1}{4}\lambda s^2$.

(iii): If $m > m_2$, then we have $\hat{x}(\eta,m,s) = \frac{m\mu-\lambda m^2(\sigma^2+\sigma_\epsilon^2)+\frac{1}{2}(1-\eta)s-\frac{1}{4}\lambda s^2+2\lambda\sigma(r-\frac{1}{2})sm}{c}$.

Clearly, the equilibrium planting amount has the same structure as before. The government's objective function also remains unchanged. Thus, following the same steps of analysis as before, the results of Proposition 3 and Theorem 2 continue to hold.

Finally, since $x^*(\eta,s)$ remains structurally the same as $x^*(s)$, and the government's objective remains unchanged, the results of Lemma 3 and Theorem 3 continue to hold. $\square$

Proof of Lemma C.1. To prove the lemma, we first note that the number of enrolled farmers is determined by the policy parameters (s,ξ), while each insurer's bidding decisions affect its margin per farmer and the probability of winning the bid. If insurer j is awarded the contract (i.e., η_j is the lowest among all bids), it earns an expected margin of $\eta_j - f - \mathbb{E}[sI]$ per farmer. Otherwise (i.e., η_j is not the lowest among all bids), it earns a zero profit. We next prove that the unique equilibrium of the insurers' bidding decisions is given by $\eta_j = \mathbb{E}[sI] + f$.

First, we prove that no insurer will bid $\eta_j < \mathbb{E}[sI] + f$ in equilibrium. If some insurer bids $\eta_j < \mathbb{E}[sI] + f$, then $\eta_j - f - \mathbb{E}[sI] < 0$, and thus it earns a negative margin per farmer when it is awarded the contract. Regardless of other insurers' bids, the best response is therefore to bid at least $\mathbb{E}[sI] + f$. Hence, we must have $\eta_j \geq \mathbb{E}[sI] + f$ in equilibrium for all $j \in \{1,\ldots,J\}$.

Next, we prove that we must have $\min_{j\in\{1,\ldots,J\}} \eta_j = \mathbb{E}[sI] + f$ in equilibrium. We prove this by contradiction. Suppose $\min_{j\in\{1,\ldots,J\}} \eta_j > \mathbb{E}[sI] + f$. If we have $\eta_{j'} > \min_{j\in\{1,\ldots,J\}} \eta_j$ for some

$j' \in \{1, ..., J\}$, then insurer j' will not be awarded the contract. Thus, the insurer can profitably deviate to a premium that is slightly lower than $\min_{j\in\{1,...,J\}} \eta_j$, so that it will be awarded the contract with probability one and earn a strictly positive profit. If we have $\eta_{j'} = \min_{j\in\{1,...,J\}} \eta_j$ for all $j' \in \{1, ..., J\}$, then each insurer will be awarded the contract with probability $\frac{1}{J}$. In this case, any insurer can profitably deviate to a premium that is slightly lower than $\min_{j\in\{1,...,J\}} \eta_j$, so that it will be awarded the contract with probability one and earn a strictly higher profit. Therefore, we must have $\min_{j\in\{1,...,J\}} \eta_j = \mathbb{E}[sI] + f$ in equilibrium.

Finally, given $\min_{j\in\{1,...,J\}} \eta_j = \mathbb{E}[sI] + f$, by the tie-breaking rule, any insurer that sets $\eta_j > \mathbb{E}[sI] + f$ will deviate to $\mathbb{E}[sI] + f$. It remains to prove that $\eta_j = \mathbb{E}[sI] + f$ for all $j \in \{1, ..., J\}$ can be sustained as an equilibrium. Consider any insurer $j \in \{1, ..., J\}$. When all other insurers set their premium as $\mathbb{E}[sI] + f$, the best response of this insurer is also to set the premium at $\mathbb{E}[sI] + f$. Any downward deviation leads to a negative profit upon winning, and any upward deviation leads to a zero profit upon losing. This completes the proof. □

Proof of Lemma C.2. To prove the lemma, we first note that the number of enrolled farmers is determined by the policy parameters (s, ξ), while each insurer's bidding decisions affect its margin per farmer and the probability of winning the bid. Given (s, ξ), we now derive the symmetric Bayesian Nash equilibrium for insurers' bidding decisions, characterized by a bidding function $g(f)$ for $f \in [0, \bar{f}]$, which maps from insurer costs to premium bids. From established results on first-price auctions, it is sufficient to focus on non-decreasing and continuous bidding functions (see Proposition 2.2 in Krishna 2009).

Consider any insurer $j \in \{1, ..., J\}$. Suppose all other insurers follow the bidding function $g(\cdot)$. Then, for any $f_j \in [0, \bar{f}]$, it is sufficient to consider premium bids in $[g(0), g(\bar{f})]$, since other bids are weakly dominated (a bid lower than $g(0)$ reduces the margin without increasing the probability of winning, while a bid higher than $g(\bar{f})$ results in a zero probability of winning). Suppose insurer j's cost is f_j and that it bids $g(z)$ for some $z \in [0, \bar{f}]$. Since each insurer's cost is uniform on $[0, \bar{f}]$, the probability that insurer j wins the bid (i.e., all other insurers have costs at least z) is given by $(1 - \frac{z}{\bar{f}})^{J-1}$. Therefore, its expected profit per farmer is given by $(g(z) - f_j - \mathbb{E}[sI])(1 - \frac{z}{\bar{f}})^{J-1}$.

If the bidding function $g(\cdot)$ constitutes a Bayesian Nash equilibrium, then given that all other insurers follow this bidding function, it must be the best response for insurer j to bid $g(f_j)$. From insurer j's profit function, its best response must satisfy the first-order condition with respect to z. By differentiating the expected profit with respect to z and setting the derivative to zero at $z = f_j$, we have

$$g'(f_j)\Big(1 - \frac{f_j}{\bar{f}}\Big)^{J-1} - \big(g(f_j) - f_j - \mathbb{E}[sI]\big)\,\frac{J-1}{\bar{f}}\Big(1 - \frac{f_j}{\bar{f}}\Big)^{J-2} = 0,$$

which simplifies to

$$g'(f_j) = \frac{J-1}{\bar{f} - f_j}\big(g(f_j) - f_j - \mathbb{E}[sI]\big).$$

Solving the above linear differential equation, we obtain

$$g(f_j) = \mathbb{E}[sI] + \frac{J-1}{J} f_j + \frac{1}{J}\bar{f} + C\,(\bar{f} - f_j)^{-(J-1)},$$

where C is a constant. Since it is sufficient to consider non-decreasing $g(\cdot)$, it must be bounded on $[0, \bar{f}]$; this rules out $C \neq 0$ because the last term diverges as f_j approaches $\bar{f}$. Thus, $C = 0$ and

$$g(f_j) = \mathbb{E}[sI] + \frac{J-1}{J} f_j + \frac{1}{J}\bar{f},\ \ f_j \in [0, \bar{f}],$$

which completes the proof. $\square$

Proof of Proposition C.4. The proof of this proposition is similar to that of Proposition C.3. Consider any given $s \in [0, 2a\sigma]$ (recall from our main analysis that $\bar{s} \leq 2a\sigma$). Since the farmer premium ξ is a fixed fraction of the actuarially fair level, we let $\xi = \eta\mathbb{E}[sI]$, where $\eta \in [0, 1]$.

Note that farmers' decisions in the setups considered in §8.3 and §8.4 are the same. Thus, given the government decision on the payment amount s (with $\xi = \eta\mathbb{E}[sI]$), the equilibrium planting amount, denoted by $x^*(\eta, s)$, is the same as that characterized in the proof of Proposition C.3.

For the public-information case, let $\hat{f} = f$. For the private-information case, define $\hat{f} = \mathbb{E}[\min_{j\in\{1,\ldots,J\}} \frac{J-1}{J} f_j + \frac{1}{J}\bar{f}] = \frac{J-1}{J}\frac{\bar{f}}{J+1} + \frac{1}{J}\bar{f} = \frac{2\bar{f}}{J+1}$. Then, in both cases, the expected premium of the winning bid is $\mathbb{E}[sI] + \hat{f}$, where $\hat{f}$ can be interpreted as the sum of the administrative cost of the winning insurer and the information rent (when it exists). Then, under given (η, s), the net benefit is as follows:

$$\begin{aligned} v(\eta, s) &= \mathbb{E}\left[\int_0^{x^*(\eta,s)} ((a - bx^*(\eta, s)Y)Y - xc + sI - \eta\mathbb{E}[sI])\,\mathrm{d}x\right] - (\mathbb{E}[sI] + \hat{f} - \eta\mathbb{E}[sI])x^*(\eta, s) \\ &= v_1 \times x^*(\eta, s) - v_2 \times (x^*(\eta, s))^2, \end{aligned}$$

where $v_1 = a\mu - \hat{f}$ and $v_2 = b(\mu^2 + \sigma^2) + \frac{1}{2}c$. Therefore, the government's problem remains structurally the same as the centralized case in our main model and in §8.3, except that v_1 becomes smaller due to the explicit incorporation of administrative costs and information rents. The value of v_1 affects the value of x^{opt} defined in §3 but not the structure of the problem. Thus, following the proof of Proposition C.3, the results of Theorems 1, 2, and 3 continue to hold. $\square$

Proof of Proposition C.5. As discussed in the proof of Proposition C.4, the government's problem is to choose s to maximize $v(\eta, s) = v_1 \times x^*(\eta, s) - v_2 \times (x^*(\eta, s))^2$, where $x^*(\eta, s)$ denotes the equilibrium planting amount and is characterized in the proof of Proposition C.3. With a slight abuse of notation, let $x^{opt} = \frac{v_1}{2v_2} = \frac{a\mu - \hat{f}}{2b(\mu^2+\sigma^2)+c}$. Let $\mathcal{S} = [0, \bar{s}] \cap \{s : (\mathbb{E}[sI] + \hat{f} - \eta\mathbb{E}[sI])x^*(\eta, s) \leq B\}$. Then, following the analysis in the proof of Theorem 1, the optimal payment amount s^* must satisfy either (i) $s^* = \arg\max_{s\in\mathcal{S}} x^*(\eta, s)$ and $x^*(\eta, s^*) < x^{opt}$, or (ii) $s^* = \inf\{s : x^*(\eta, s) = x^{opt}\}$.

Since the value of $\hat{f}$ does not affect farmers' decisions, a higher $\hat{f}$ affects the government's decisions only through (1) a lower value of x^{opt}; and (2) a smaller feasible set of s. For a fixed feasible set of s, a lower value of x^{opt} clearly leads to a lower optimal payment amount. Accounting for the smaller feasible set from the higher cost reinforces the conclusion that the optimal payment amount is lower. Therefore, a higher $\hat{f}$ must lead to a lower optimal payment amount. $\square$

Proof of Proposition C.6. The proof of this proposition is similar to that of Proposition 4. Farmers' decisions in the setups considered in §8.3 and §8.4 are the same. Thus, given the government decision on the payment amount s and the farmer premium ξ, the equilibrium planting amount, denoted by $x^*(\xi, s)$, is the same as that considered in Proposition 4. Since the government incurs an expected expenditure of $\mathbb{E}[sI] + \hat{f} - \xi$ per farmer, the net benefit is given as follows:

$$\begin{aligned} v(\xi, s) &= \mathbb{E}\left[\int_0^{x^*(\xi,s)} ((a - bx^*(\xi, s)Y)Y - xc + sI - \xi)\,\mathrm{d}x\right] - (\mathbb{E}[sI] + \hat{f} - \xi)x^*(\xi, s) \\ &= v_1 \times x^*(\xi, s) - v_2 \times (x^*(\xi, s))^2, \end{aligned}$$

where $v_1 = a\mu - \hat{f}$ and $v_2 = b(\mu^2 + \sigma^2) + \frac{1}{2}c$. With a slight abuse of notation, let $x^{opt} = \frac{v_1}{2v_2} = \frac{a\mu - \hat{f}}{2b(\mu^2+\sigma^2)+c}$. Let $\mathcal{F} = [0, \infty) \times [0, \bar{s}] \cap \{(\xi, s) : (\mathbb{E}[sI] + \hat{f} - \xi)x^*(\xi, s) \leq B\}$ denote the feasible set of (ξ, s). Then, following the analysis in the proof of Proposition 4, the government's optimal decisions ξ^* and s^* must satisfy either (i) $(\xi^*, s^*) \in \arg\max_{(\xi,s)\in\mathcal{F}} x^*(\xi, s)$ and $x^*(\xi^*, s^*) < x^{opt}$, or (ii) $x^*(\xi^*, s^*) = x^{opt}$.

Since the value of $\hat{f}$ does not affect farmers' decisions, a higher $\hat{f}$ affects the government's decisions only through (1) a lower value of x^{opt}; and (2) a smaller feasible set of (ξ, s). These changes do not alter the structure of the government's problem, and the analysis in the proof of Proposition 4 remains valid. $\square$

Proof of Lemma C.3. Consider a fixed $s \geq 0$. Suppose x_1 farmers plant Crop 1 and x_2 farmers plant Crop 2. Then, following the same analysis as in the proof of Lemma 1, the mean and variance of the net income of farmer $i \in [0, 1]$ from planting Crop 1, denoted as $\pi_1(i|x_1, s)$, are as follows:

$$\mathbb{E}[\pi_1(i|x_1, s)] = ua\mu - bx_1(\mu^2 + v^2\sigma^2) - wh_i + \frac{1}{2}s,$$

$$\begin{aligned} \mathbb{V}\mathrm{ar}[\pi_1(i|x_1, s)] =& (ua - 2bx_1\mu)^2v^2\sigma^2 + \left(u^2a^2 - 2uabx_1\mu + b^2x_1^2(\mu^2 + v^2\sigma^2)\right)\sigma_\epsilon^2 \\ &+ \frac{1}{4}s^2 - s\left(r - \frac{1}{2}\right)(2ua - 4bx_1\mu)v\sigma. \end{aligned}$$

The utility of the farmer i is given by

$$u_1(i|x_1, s) = \mathbb{E}[\pi_1(i|x_1, s)] - \lambda\mathbb{V}\mathrm{ar}[\pi_1(i|x_1, s)] = R_1(x_1, s) - wh_i, \tag{C.1}$$

where we use $R_1(x_1, s)$ to denote the value of $u_1(i|x_1, s) + wh_i$, which is independent of i.

Similarly, let $\pi_2(i|x_2)$ denote the net income of a farmer $i \in [0,1]$ from planting Crop 2. Then,

$$\mathbb{E}[\pi_2(i|x_2)] = a\mu - bx_2(\mu^2 + \sigma^2) - h_i,$$

$$\mathbb{V}\mathrm{ar}[\pi_2(i|x_2)] = (a - 2bx_2\mu)^2\sigma^2 + \left(a^2 - 2abx_2\mu + b^2x_2^2(\mu^2 + \sigma^2)\right)\sigma_\epsilon^2.$$

The utility of the farmer i is given by

$$u_2(i|x_2) = \mathbb{E}[\pi_2(i|x_2)] - \lambda\mathbb{V}\mathrm{ar}[\pi_2(i|x_2)] = R_2(x_2) - h_i, \tag{C.2}$$

where we use $R_2(x_2)$ to denote the value of $u_2(i|x_2) + h_i$, which is independent of i.

To prove the lemma, we first observe that the farmers who choose not to plant must be those with higher production costs. Therefore, it is sufficient to prove that, among the farmers who plant, those who choose to plant Crop 1 must be the ones with lower production costs. To do so, it is sufficient to prove that, for any two farmers, i_1 who plants Crop 1 and i_2 who plants Crop 2, we have $h_{i_1} \leq h_{i_2}$. We prove this by contradiction. Suppose there exist two farmers i_1 and i_2, where i_1 plants Crop 1, i_2 plants Crop 2, and $h_{i_1} > h_{i_2}$. Let $x_1^* \geq 0$ and $x_2^* \geq 0$ denote the equilibrium planting amounts for Crop 1 and Crop 2, respectively. Since farmer i_2 plants Crop 2, by Equations (C.1) and (C.2), we must have $R_2(x_2^*) - h_{i_2} \geq R_1(x_1^*, s) - wh_{i_2}$. Then, since $w > 1$, for any farmer whose production cost satisfies $h > h_{i_2}$, we have $R_2(x_2^*) - h > R_1(x_1^*, s) - wh$. This implies that farmer i_1 prefers Crop 2 over Crop 1 (because $h_{i_1} > h_{i_2}$), which leads to a contradiction. □

Proof of Proposition C.7. We first prove the proposition by considering the relationship between the index and area-level yield as $I = Y + \eta$, one of the two representations discussed in Appendix C.6. Throughout the proof, we highlight all steps that rely on this specific representation. At the end of the proof, we demonstrate that our results hold under the other representation (i.e., $Y = f(R) + \eta$ and $I = f(R)$). We prove this proposition in the following four steps:

Step 1: We characterize the equilibrium planting amount $x^*(s)$ for any given subsidy amount $s \geq 0$. Let $\zeta := \mathbb{1}(I < \mu)$. For any $s \geq 0$, if x farmers plant, the utility of farmer x is given by $\pi(x|x, s) = (a - bxY + \epsilon_p)Y_i - xc + s\zeta$. Then, $\mathbb{E}[\pi(x|x, s)] = a\mu - bx(\mu^2 + \sigma^2) - xc + \frac{1}{2}s$, and

$$\begin{aligned}
&\mathbb{V}\mathrm{ar}[\pi(x|x, s)] \\
=&\mathbb{V}\mathrm{ar}[(a - bxY)Y] + \mathbb{V}\mathrm{ar}[s\zeta] + 2\mathrm{Cov}((a - bxY)Y, s\zeta) + \mathbb{V}\mathrm{ar}[(a - bxY)\epsilon_i] + \mathbb{V}\mathrm{ar}[\epsilon_p(Y + \epsilon_i)] \\
=&\left(a^2\sigma^2 - 4ab\mu\sigma^2 x + b^2x^2t_1\right) + \frac{1}{4}s^2 + 2s\left(at_2 - \frac{1}{2}a\mu - bxt_3 + \frac{1}{2}bx(\mu^2 + \sigma^2)\right) \\
&+ \left(a^2 - 2abx\mu + b^2x^2(\mu^2 + \sigma^2)\right)\sigma_\epsilon^2 + (\mu^2 + \sigma^2 + \sigma_\epsilon^2)\sigma_p^2,
\end{aligned}$$

where $t_1 = \mathbb{V}\mathrm{ar}[Y^2]$, $t_2 = \mathbb{E}[Y\zeta]$ and $t_3 = \mathbb{E}[Y^2\zeta]$. Then, with a slight abuse of notation, we have

$$u(x|x,s) = \mathbb{E}[\pi(x|x,s)] - \lambda \mathbb{V}\mathrm{ar}[\pi(x|x,s)] = -\alpha x^2 - \beta(s)x - \gamma(s), \tag{C.3}$$

where $\alpha = \lambda b^2 t_1 + \lambda b^2(\mu^2+\sigma^2)\sigma_\epsilon^2$, $\beta(s) = b(\mu^2+\sigma^2)+c-4\lambda ab\mu\sigma^2-2\lambda ab\mu\sigma_\epsilon^2-2\lambda bt_3 s+\lambda b(\mu^2+\sigma^2)s$, and $\gamma(s) = \lambda a^2(\sigma^2+\sigma_\epsilon^2) - a\mu + \lambda a(2t_2-\mu)s - \frac{1}{2}s + \frac{1}{4}\lambda s^2 + \lambda(\mu^2+\sigma^2+\sigma_\epsilon^2)\sigma_p^2$.

Following the analysis in the proof of Lemma 2, to obtain the equilibrium planting amount, it is sufficient to analyze the solutions of $u(x|x,s) = 0$. As a building block, we first prove $\beta(s) > 0$ for any $s \geq 0$. Since we have proven in the proof of Lemma 2 that $b(\mu^2+\sigma^2)+c-4\lambda ab\mu\sigma^2-2\lambda ab\mu\sigma_\epsilon^2 > 0$, it remains to prove $-2\lambda bt_3 s + \lambda b(\mu^2+\sigma^2)s \geq 0$, which is equivalent to proving $t_3 \leq \frac{1}{2}(\mu^2+\sigma^2)$. To do so, let $Z = Y - \mu$. Then, we have $Z \sim \mathcal{N}(0,\sigma^2)$. Let $f_Z(\cdot)$ denote the density function of Z. Since $I = Y + \eta$, and Y and η are independent, by the definition of t_3, we have

$$\begin{aligned}
t_3 &= \mathbb{E}[Y^2\zeta] = \mathbb{E}[(Z+\mu)^2 \times \mathbb{1}(Z+\mu+\eta<\mu)] \\
&= \int_{-\infty}^{\infty} (z+\mu)^2 \times \mathbb{P}(\eta<-z) \times f_Z(z)\,\mathrm{d}z \\
&= \int_{0}^{\infty} \big((z+\mu)^2 \times \mathbb{P}(\eta<-z) + (-z+\mu)^2 \times \mathbb{P}(\eta<z)\big) \times f_Z(z)\,\mathrm{d}z \\
&= \int_{0}^{\infty} [(z^2+\mu^2)\left(\mathbb{P}(\eta<-z)+\mathbb{P}(\eta<z)\right) + 2z\mu(\mathbb{P}(\eta<-z)-\mathbb{P}(\eta<z))] \times f_Z(z)\,\mathrm{d}z \\
&\leq \int_{0}^{\infty} (z^2+\mu^2) \times f_Z(z)\,\mathrm{d}z \\
&= \frac{1}{2}(\mu^2+\sigma^2),
\end{aligned} \tag{C.4}$$

where the inequality holds because we have $\mathbb{P}(\eta<-z)+\mathbb{P}(\eta<z) = 1$ and $\mathbb{P}(\eta<-z) \leq \mathbb{P}(\eta<z)$ for any $z \in [0,\infty)$. Thus, we conclude that $\beta(s) > 0$ for any $s \geq 0$.

Since we also have $\alpha > 0$, if $\gamma(s) < 0$, then following the analysis in the proof of Lemma 2, there must exist a unique positive solution to the equation $u(x|x,s) = 0$, which is given by $\hat{x}(s) = \frac{-\beta(s)+\sqrt{\beta^2(s)-4\alpha\gamma(s)}}{2\alpha}$. In this case, we have $x^*(s) = \hat{x}(s)$ (recall that we assume c is sufficiently high such that the equilibrium planting amount is strictly less than one). On the other hand, if $\gamma(s) \geq 0$, then $u(x|x,s) \leq 0$ for any $x \geq 0$, which implies $x^*(s) = 0$. Since we assume $\lambda < \frac{a\mu}{a^2(\sigma^2+\sigma_\epsilon^2)+(\mu^2+\sigma^2+\sigma_\epsilon^2)\sigma_p^2}$, we have $\gamma(0) = \lambda a^2(\sigma^2+\sigma_\epsilon^2) - a\mu + \lambda(\mu^2+\sigma^2+\sigma_\epsilon^2)\sigma_p^2 < 0$ and thus $x^*(0) > 0$.

Step 2: We prove that (i) if $x^*(s) > 0$, then $\frac{\partial^2}{\partial s^2}x^*(s) < 0$, and (ii) $\frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$.

We first prove $\frac{\partial^2}{\partial s^2}x^*(s) < 0$ when $x^*(s) > 0$. Based on our analysis in Step 1, this is equivalent to proving $\frac{\partial^2}{\partial s^2}\hat{x}(s) < 0$ when $\gamma(s) < 0$. To do so, since α is independent of s, $\beta(s)$ is linear in s, and $\beta^2(s) - 4\alpha\gamma(s)$ is a quadratic function in s, it is sufficient to prove that the coefficient of the s^2 term in $\beta^2(s) - 4\alpha\gamma(s)$ is negative, which is equivalent to proving that $\lambda^2 b^2 t_1 + \lambda^2 b^2(\mu^2+\sigma^2)\sigma_\epsilon^2 > \lambda^2 b^2(\mu^2+\sigma^2-2t_3)^2$. Therefore, it is sufficient to prove that $t_1 > (\mu^2+\sigma^2-2t_3)^2$.

By the definition of t_1, we have $t_1 = \mathbb{V}\mathrm{ar}[Y^2] = \mathbb{V}\mathrm{ar}[(Z+\mu)^2] = \mathbb{V}\mathrm{ar}[Z^2] + 4\mu^2\mathbb{V}\mathrm{ar}[Z] > 4\mu^2\sigma^2$, where the third equality holds because $Z \sim \mathcal{N}(0,\sigma^2)$ and thus $\mathrm{Cov}(Z^2, 2\mu Z) = 2\mu(\mathbb{E}[Z^3] - \mathbb{E}[Z^2]\mathbb{E}[Z]) = 2\mu(0-0) = 0$. On the other hand, since $I = Y + \eta$, by Equation (C.4), we have $\mu^2 + \sigma^2 - 2t_3 = \int_0^\infty 4z\mu(\mathbb{P}(\eta < z) - \mathbb{P}(\eta < -z)) \times f_Z(z)\,\mathrm{d}z \le \int_0^\infty 4z\mu \times f_Z(z)\,\mathrm{d}z = \frac{4}{\sqrt{2\pi}}\mu\sigma < 2\mu\sigma$. Therefore, we have $t_1 > (\mu^2 + \sigma^2 - 2t_3)^2$.

We next prove $\frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$. We have shown in Step 1 that $x^*(0) > 0$. Therefore, following the analysis in the proof of Lemma A.2, it is sufficient to prove $\frac{\partial}{\partial s}u(x|x,s)|_{x=x^*(0),s=0} > 0$. We have

$$\frac{\partial}{\partial s}u(x|x,s)\Big|_{x=x^*(0),s=0} = \frac{1}{2} - 2\lambda\left(at_2 - \frac{1}{2}a\mu - bx^*(0)t_3 + \frac{1}{2}bx^*(0)(\mu^2+\sigma^2)\right).$$

Thus, to prove $\frac{\partial}{\partial s}u(x|x,s)|_{x=x^*(0),s=0} > 0$, it remains to prove that $at_2 - \frac{1}{2}a\mu - bx^*(0)t_3 + \frac{1}{2}bx^*(0)(\mu^2+\sigma^2) = a(t_2 - \frac{1}{2}\mu) - bx^*(0)(t_3 - \frac{1}{2}(\mu^2+\sigma^2)) < 0$. Since $I = Y + \eta$, and Y and η are independent, by the definition of t_2, we have

$$\begin{aligned}
t_2 &= \mathbb{E}[Y\zeta] = \mathbb{E}[(Z+\mu)\times \mathbb{1}(Z+\mu+\eta<\mu)] \\
&= \int_{-\infty}^{\infty} (z+\mu)\times \mathbb{P}(\eta<-z)\times f_Z(z)\,\mathrm{d}z \\
&= \int_0^{\infty} ((z+\mu)\times \mathbb{P}(\eta<-z) + (-z+\mu)\times\mathbb{P}(\eta<z))\times f_Z(z)\,\mathrm{d}z \\
&= \int_0^{\infty} [\mu\,(\mathbb{P}(\eta<-z)+\mathbb{P}(\eta<z)) + z(\mathbb{P}(\eta<-z)-\mathbb{P}(\eta<z))]\times f_Z(z)\,\mathrm{d}z \\
&= \frac{1}{2}\mu + \int_0^\infty z(\mathbb{P}(\eta<-z)-\mathbb{P}(\eta<z))\times f_Z(z)\,\mathrm{d}z,
\end{aligned} \tag{C.5}$$

which implies $t_2 < \frac{1}{2}\mu$. By Equations (C.4) and (C.5), we have

$$\begin{aligned}
t_3 - \frac{1}{2}(\mu^2+\sigma^2) &= -\int_0^\infty 2z\mu(\mathbb{P}(\eta<z)-\mathbb{P}(\eta<-z))\times f_Z(z)\,\mathrm{d}z \\
&= -2\mu\left[\int_0^\infty z(\mathbb{P}(\eta<z)-\mathbb{P}(\eta<-z))\times f_Z(z)\,\mathrm{d}z\right] \\
&= 2\mu(t_2 - \frac{1}{2}\mu).
\end{aligned}$$

Thus, we have

$$a(t_2 - \frac{1}{2}\mu) - bx^*(0)(t_3 - \frac{1}{2}(\mu^2+\sigma^2)) = (t_2 - \frac{1}{2}\mu)(a - 2b\mu x^*(0)) < 0, \tag{C.6}$$

where the inequality holds because we have proven that $t_2 < \frac{1}{2}\mu$ and we consider $a > 2b\mu$. Hence, we conclude that $\frac{\partial}{\partial s}u(x|x,s)|_{x=x^*(0),s=0} > 0$, and thus $\frac{\partial}{\partial s}x^*(s)|_{s=0} > 0$.

Step 3: Let $\mathcal{S} = \{s \ge 0 : x^*(s)\mathbb{E}[s\zeta] \le B\}$, and let $s_1 = \arg\max_{s\in\mathcal{S}} x^*(s)$; we now prove that if $x^*(s_1)\mathbb{E}[s_1\zeta] < B$, then s_1 strictly increases in index accuracy (i.e., it strictly decreases in σ_η). Given $x^*(s_1)\mathbb{E}[s_1\zeta] < B$ and the conclusion in Step 2, we must have $\frac{\partial}{\partial s}x^*(s)|_{s=s_1} = 0$. Then, to prove that

s_1 strictly decreases in σ_η, it is sufficient to prove $\frac{\partial^2}{\partial s \partial \sigma_\eta} x^*(s)|_{s=s_1} < 0$. Following a similar analysis as in Step 1 of the proof of Proposition 1, it is sufficient to prove that

$$\frac{\partial^2}{\partial x \partial s} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} \frac{\partial}{\partial \sigma_\eta} x^*(s)\Big|_{s=s_1} + \frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} < 0.$$

To do so, by Equation (C.3), we have

$$\frac{\partial^2}{\partial x \partial s} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} = 2\lambda b t_3 - \lambda b(\mu^2+\sigma^2),$$

$$\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} = -2\lambda \frac{\partial}{\partial \sigma_\eta}\left(a t_2 - \frac{1}{2}a\mu - b x t_3 + \frac{1}{2} b x(\mu^2+\sigma^2)\right)\Big|_{x=x^*(s_1)}. \tag{C.7}$$

Moreover, by Equation (A.7), we have

$$\begin{aligned}
\frac{\partial}{\partial \sigma_\eta} x^*(s)\Big|_{s=s_1} &= -\frac{\partial u(x|x,s)/\partial \sigma_\eta}{\partial u(x|x,s)/\partial x}\Big|_{x=x^*(s),s=s_1} \\
&= \frac{-2\lambda \frac{\partial}{\partial \sigma_\eta}\left(a t_2 - \frac{1}{2}a\mu - b x t_3 + \frac{1}{2} b x(\mu^2+\sigma^2)\right)|_{x=x^*(s_1)} \times s_1}{2\alpha x^*(s_1) + \beta(s_1)} \\
&= \frac{s_1}{\sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)}} \times \frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1},
\end{aligned} \tag{C.8}$$

where the third equality holds due to Equation (C.7). Therefore, we have

$$\begin{aligned}
&\frac{\partial^2}{\partial x \partial s} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} \frac{\partial}{\partial \sigma_\eta} x^*(s)\Big|_{s=s_1} + \frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1} \\
=&\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s_1),s=s_1}\left(1 + \frac{s_1 \times (2\lambda b t_3 - \lambda b(\mu^2+\sigma^2))}{\sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)}}\right).
\end{aligned} \tag{C.9}$$

To prove that Equation (C.9) is negative, it is sufficient to prove that $1 + \frac{s_1 \times (2\lambda b t_3 - \lambda b(\mu^2+\sigma^2))}{\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)}} > 0$ and $\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)|_{x=x^*(s_1),s=s_1} < 0$. We first prove that $1 + \frac{s_1 \times (2\lambda b t_3 - \lambda b(\mu^2+\sigma^2))}{\sqrt{\beta^2(s_1)-4\alpha\gamma(s_1)}} > 0$. Since we have shown that $2t_3 \leq \mu^2 + \sigma^2$ in Equation (C.4) and thus we have $2\lambda b t_3 - \lambda b(\mu^2+\sigma^2) \leq 0$, it is sufficient to prove that $s_1 \times (\lambda b(\mu^2+\sigma^2) - 2\lambda b t_3) < \sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)}$. We prove this by following a similar analysis as in Step 1 of the proof of Proposition 1. Specifically, by the definition of s_1, we have

$$\frac{\partial}{\partial s} x^*(s)\Big|_{s=s_1} = \frac{1}{2\alpha}\left(-(\lambda b(\mu^2+\sigma^2) - 2\lambda b t_3) + \frac{\partial(\beta^2(s) - 4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s) - 4\alpha\gamma(s)}}\Big|_{s=s_1}\right) = 0,$$

and thus

$$\lambda b(\mu^2+\sigma^2) - 2\lambda b t_3 = \frac{\partial(\beta^2(s) - 4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s) - 4\alpha\gamma(s)}}\Big|_{s=s_1}.$$

Multiplying both sides of the equation above by s_1, we have

$$s_1 \times (\lambda b(\mu^2+\sigma^2) - 2\lambda b t_3) = \frac{\partial(\beta^2(s) - 4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s) - 4\alpha\gamma(s)}}\Big|_{s=s_1} \times s_1. \tag{C.10}$$

Further, as shown in Step 2, $\beta^2(s) - 4\alpha\gamma(s)$ is concave in s and increasing in s at $s = 0$, which implies that

$$[\beta^2(s_1) - 4\alpha\gamma(s_1)] - [\beta^2(0) - 4\alpha\gamma(0)] > \frac{\partial}{\partial s}\left(\beta^2(s) - 4\alpha\gamma(s)\right)\Big|_{s=s_1} \times s_1.$$

We also have $\beta^2(0) - 4\alpha\gamma(0) > 0$ because we have shown $\gamma(0) < 0$ in Step 1. Therefore, we have

$$\sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)} > \frac{\partial(\beta^2(s) - 4\alpha\gamma(s))/\partial s}{2\sqrt{\beta^2(s) - 4\alpha\gamma(s)}}\Big|_{s=s_1} \times s_1. \tag{C.11}$$

By combining Equations (C.10) and (C.11), we have $s_1 \times (\lambda b(\mu^2 + \sigma^2) - 2\lambda b t_3) < \sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)}$, which implies $1 + \frac{s_1 \times (2\lambda b t_3 - \lambda b(\mu^2 + \sigma^2))}{\sqrt{\beta^2(s_1) - 4\alpha\gamma(s_1)}} > 0$.

We next prove that $\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)|_{x=x^*(s_1), s=s_1} < 0$. To do so, it is sufficient to prove that $\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)|_{x=x^*(s)} < 0$ for any $s \geq 0$ such that $x^*(s) > 0$. Let $\Phi(\cdot)$ denote the cumulative distribution function of a standard normal random variable, and let $\phi(\cdot)$ denote the probability density function. Since $I = Y + \eta$, by Equation (C.7), we have

$$\begin{aligned}
\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s)} &= -2\lambda \frac{\partial}{\partial \sigma_\eta}\left(a t_2 - \frac{1}{2} a\mu - b x t_3 + \frac{1}{2} b x(\mu^2 + \sigma^2)\right)\Big|_{x=x^*(s)} \\
&= -2\lambda \frac{\partial}{\partial \sigma_\eta}\left((t_2 - \frac{1}{2}\mu)(a - 2b\mu x)\right)\Big|_{x=x^*(s)} \\
&= -2\lambda(a - 2b\mu x^*(s)) \frac{\partial}{\partial \sigma_\eta}\left(t_2 - \frac{1}{2}\mu\right) \\
&= 2\lambda(a - 2b\mu x^*(s)) \frac{\partial}{\partial \sigma_\eta}\left(\int_0^\infty z(\mathbb{P}(\eta < z) - \mathbb{P}(\eta < -z)) \times f_Z(z)\,\mathrm{d}z\right) \\
&= 2\lambda(a - 2b\mu x^*(s)) \int_0^\infty \frac{\partial}{\partial \sigma_\eta}\left(\Phi(\frac{z}{\sigma_\eta}) - \Phi(\frac{-z}{\sigma_\eta})\right) \times z f_Z(z)\,\mathrm{d}z \\
&< 0,
\end{aligned}$$

where the second equality holds due to Equation (C.6), the fourth equality holds due to Equation (C.5), the fifth equality holds because $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$ and $\int_0^\infty \left|\frac{\partial}{\partial \sigma_\eta}(\Phi(\frac{z}{\sigma_\eta}) - \Phi(\frac{-z}{\sigma_\eta})) \times z f_Z(z)\right| \mathrm{d}z = \int_0^\infty \left|(-\phi(\frac{z}{\sigma_\eta}) \times \frac{2z}{\sigma_\eta^2}) \times z f_Z(z)\right| \mathrm{d}z \leq \frac{2\phi(0)}{\sigma_\eta^2} \int_0^\infty z^2 f_Z(z)\,\mathrm{d}z = \frac{\phi(0)\sigma^2}{\sigma_\eta^2} < \infty$ (which justifies the interchange of differentiation and integration), and the inequality holds because $a > 2b\mu$ and $\frac{\partial}{\partial \sigma_\eta}(\Phi(\frac{z}{\sigma_\eta}) - \Phi(\frac{-z}{\sigma_\eta})) = -\phi(\frac{z}{\sigma_\eta}) \times \frac{2z}{\sigma_\eta^2} < 0$ for any $z \in (0, \infty)$. Therefore, our conclusion in Step 3 holds.

Step 4: We prove that s^* first increases and then decreases as the index accuracy increases. Since index accuracy is uniquely determined by σ_η in this extension, where a higher σ_η implies a lower index accuracy, we use a higher (lower) index accuracy and a lower (higher) σ_η interchangeably.

Since we have $\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)|_{x=x^*(s)} < 0$ for any $s \geq 0$ such that $x^*(s) > 0$ (as shown in Step 3), by Equation (C.8), we have $\frac{\partial}{\partial \sigma_\eta} x^*(s) < 0$ for any $s \geq 0$ such that $x^*(s) > 0$. Therefore, for any $s \geq 0$, $x^*(s)$ increases in the index accuracy. Moreover, we have shown in Step 3 that if $x^*(s_1)\mathbb{E}[s_1\zeta] < B$, then s_1 strictly increases in the index accuracy. Given these results, we must have $x^*(s_1)\mathbb{E}[s_1\zeta] < B$

if and only if the index accuracy is lower than a threshold. We must also have that x^{opt} is achievable (i.e., $x^*(s_1) \geq x^{opt}$) if and only if the index accuracy is higher than a threshold. Following a similar analysis as in the proof of Theorem 1, it can be shown that when $x^*(s_1)\mathbb{E}[s_1\zeta] < B$ and $x^*(s_1) < x^{opt}$, then we have $s^* = s_1$, which increases in the index accuracy; otherwise (i.e., $x^*(s_1)\mathbb{E}[s_1\zeta] = B$ or $x^*(s_1) \geq x^{opt}$, then s^* decreases in the index accuracy. Thus, our conclusion in Step 4 holds.

It remains to prove that all the steps above that rely on the representation where $I = Y + \eta$ also hold under the alternative representation where $Y = f(R) + \eta$ and $I = f(R)$.

First, we prove that $t_3 \leq \frac{1}{2}(\mu^2 + \sigma^2)$. Since η and R are independent, $I = f(R)$ and η must also be independent. Then, since $Y \sim \mathcal{N}(\mu, \sigma^2)$, $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$, and $Y = f(R) + \eta = I + \eta$, by Theorem 1 in Chapter XV.8 of Feller (1991), we have $I \sim \mathcal{N}(\mu, \sigma^2 - \sigma_\eta^2)$. By the definition of t_3, we have

$$
\begin{aligned}
t_3 =& \mathbb{E}[Y^2\zeta] = \mathbb{E}[(I+\eta)^2 \mathbb{1}(I < \mu)] = \mathbb{E}[(I+\eta)^2 \,|\, I < \mu]\mathbb{P}(I < \mu) \\
=& \frac{1}{2}\left(\mathbb{E}[I^2 \,|\, I < \mu] + 2\mathbb{E}[I\eta \,|\, I < \mu] + \mathbb{E}[\eta^2 \,|\, I < \mu]\right) \\
\overset{(a)}{=}& \frac{1}{2}\left(\mathbb{E}[I^2 \,|\, I < \mu] + \sigma_\eta^2\right) \\
=& \frac{1}{2}\left(\mu^2 + (\sigma^2 - \sigma_\eta^2) - 2\mu\sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}} + \sigma_\eta^2\right) \\
=& \frac{1}{2}\left(\mu^2 + \sigma^2 - 2\mu\sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}}\right) \\
\leq& \frac{1}{2}(\mu^2 + \sigma^2),
\end{aligned}
\tag{C.12}
$$

where (a) holds because I and η are independent, and the inequality holds because $\mu > \sigma$.

Next, we prove that $t_1 > (\mu^2 + \sigma^2 - 2t_3)^2$. We have shown in Step 2 that $t_1 = \mathbb{V}\text{ar}[Y^2] > 4\mu^2\sigma^2$. Moreover, by Equation (C.12), we have $\mu^2 + \sigma^2 - 2t_3 \geq 0$ and $\mu^2 + \sigma^2 - 2t_3 = 2\mu\sqrt{\frac{2(\sigma^2-\sigma_\eta^2)}{\pi}} \leq 2\mu\sigma\sqrt{\frac{2}{\pi}} < 2\mu\sigma$. Thus, we have $t_1 > (\mu^2 + \sigma^2 - 2t_3)^2$.

We now prove that $a(t_2 - \frac{1}{2}\mu) - bx^*(0)(t_3 - \frac{1}{2}(\mu^2 + \sigma^2)) < 0$. By the definition of t_2, we have

$$
\begin{aligned}
t_2 =& \mathbb{E}[Y\zeta] = \mathbb{E}[Y \mathbb{1}(I < \mu)] = \mathbb{E}[I + \eta \,|\, I < \mu]\mathbb{P}(I < \mu) = \frac{1}{2}\mathbb{E}[I \,|\, I < \mu] \\
=& \frac{1}{2}\left(\mu - \sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}}\right).
\end{aligned}
\tag{C.13}
$$

Then, by Equations (C.12) and (C.13), we have

$$
a(t_2 - \frac{1}{2}\mu) - bx^*(0)(t_3 - \frac{1}{2}(\mu^2 + \sigma^2)) = -\frac{1}{2}\sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}}\,(a - 2b\mu x^*(0)) \leq 0,
$$

where the last inequality holds because we consider $a > 2b\mu$.

Finally, we prove that $\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)|_{x=x^*(s)} < 0$ for any $s \geq 0$ such that $x^*(s) > 0$. By Equation (C.7), we have

$$
\begin{aligned}
\frac{\partial^2}{\partial s \partial \sigma_\eta} u(x|x,s)\Big|_{x=x^*(s)} &= -2\lambda \frac{\partial}{\partial \sigma_\eta} \left(at_2 - \frac{1}{2} a\mu - bxt_3 + \frac{1}{2} bx(\mu^2 + \sigma^2) \right) \Big|_{x=x^*(s)} \\
&= -2\lambda \frac{\partial}{\partial \sigma_\eta} \left(-\frac{1}{2}(a - 2b\mu x) \sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}} \right) \Big|_{x=x^*(s)} \\
&= \lambda(a - 2bx^*(s)\mu) \frac{\partial}{\partial \sigma_\eta} \sqrt{\frac{2(\sigma^2 - \sigma_\eta^2)}{\pi}} \\
&< 0,
\end{aligned}
$$

which completes the proof. □

Proof of Proposition C.8. Consider any fixed $p \in (0,1)$ and $s \in [0, 2a\sigma]$. Suppose x farmers plant. Let $u_p(i|x,s)$ denote the utility of farmer i from planting. Then, we have

$$
u_p(x|x,s) = \mathbb{E}[(a - bxY)Y_i + sI] - xc - \lambda \mathbb{V}\text{ar}[(a - bxY)Y_i + sI] = -\alpha x^2 - \beta_p(s)x - \gamma_p(s),
$$

where $\alpha = 4\lambda b^2\mu^2\sigma^2 + \lambda b^2(\mu^2 + \sigma^2)\sigma_\epsilon^2$, $\beta_p(s) = b(\mu^2 + \sigma^2) + c - 4\lambda ab\sigma^2\mu - 2\lambda ab\mu\sigma_\epsilon^2 + 4\lambda b(2rp - p)\sigma\mu s$ and $\gamma_p(s) = \lambda a^2(\sigma^2 + \sigma_\epsilon^2) - a\mu - 2\lambda a(2rp - p)\sigma s - ps + \lambda p(1-p)s^2$.

Then, following the same analysis as in the proof of Lemma 2, the equilibrium planting amount is given by $x_p^*(s) = \max\{\min\{\hat{x}_p(s), 1\}, 0\}$, where $\hat{x}_p(s) = \frac{-\beta_p(s) + \sqrt{\beta_p^2(s) - 4\alpha\gamma_p(s)}}{2\alpha}$.

Clearly, the equilibrium planting amount has the same structure as before. Further, the government's objective function remains unchanged. Therefore, following the same analysis as in Appendix A, all of our results presented in §4 continue to hold.

Next, with a floor price $m \in [0, a]$ and an index-based yield protection subsidy $s \in [0, 2a\sigma]$, following the same analysis in the proof of Lemma A.4, the equilibrium planting amount is given by $x_p^*(m,s) = \max\{\min\{\hat{x}_p(m,s), 1\}, 0\}$, where $\hat{x}_p(m,s)$ can be obtained by considering the following three cases. Specifically, there exist two thresholds m_1 and m_2 such that

(i): If $m \in [0, m_1]$, then we have $\hat{x}_p(m,s) = \hat{x}_p(s)$.

(ii): If $m \in (m_1, m_2]$, then we have $\hat{x}_p(m,s) = \frac{-\beta_p(m,s) + \sqrt{\beta_p^2(m,s) - 4\alpha_m\gamma_p(m,s)}}{2\alpha_m}$, where $\alpha_m = \frac{1}{4}\lambda b^2(\mu - \sigma)^4 + \frac{1}{2}\lambda b^2(\mu - \sigma)^2\sigma_\epsilon^2$, $\beta_p(m,s) = \frac{1}{2}b(\mu - \sigma)^2 + c + \frac{1}{2}\lambda b(\mu - \sigma)^2(m(\mu + \sigma) - a(\mu - \sigma)) - \lambda ab(\mu - \sigma)\sigma_\epsilon^2 - \lambda b(2rp - p)s(\mu - \sigma)^2$ and $\gamma_p(m,s) = \frac{1}{4}\lambda(m(\mu + \sigma) - a(\mu - \sigma))^2 + \frac{1}{2}\lambda(a^2 + m^2)\sigma_\epsilon^2 - \frac{1}{2}m(\mu + \sigma) - \frac{1}{2}a(\mu - \sigma) - \lambda(2rp - p)s(m(\mu + \sigma) - a(\mu - \sigma)) - ps + \lambda p(1-p)s^2$.

(iii): If $m > m_2$, then we have $\hat{x}_p(m,s) = \frac{m\mu - \lambda m^2(\sigma^2 + \sigma_\epsilon^2) + ps - \lambda p(1-p)s^2 + 2\lambda\sigma(2rp - p)sm}{c}$.

Clearly, the equilibrium planting amount has the same structure as before. Further, the government's objective function remains unchanged. Therefore, following the same analysis as in Appendix A, our results presented in Proposition 3 and Theorem 2 continue to hold.

Finally, since $x_p^*(s)$ remains structurally the same as $x^*(s)$, and the government's objective function remains unchanged, our proofs for Lemma 3 and Theorem 3 remain valid. $\square$

Proof of Proposition C.9. We have shown in Corollary 2 that when $w=0$, there exists such a threshold r_3 such that $s^* \leq s_A^*$ if and only if $r \leq r_3$. Moreover, we have shown in Theorem 1 that s^* first increases and then decreases in r. Therefore, to prove that such an r_3 exists for all $w \geq 0$ and decreases in w, it is sufficient to prove that s_A^* decreases in w.

Recall that s_A^* must satisfy that the corresponding equilibrium planting amount $x_A^*(s_A^*)$ is as close to x_A^{opt} as possible. Moreover, following a similar analysis as in the proof of Theorem 1, we must have $\frac{\partial}{\partial s} x_A^*(s)|_{s=s_A^*} \geq 0$. Then, since s_A^* satisfies $x_A^*(s_A^*) \times (\frac{1}{2}s_A^* + w) \leq B$, and x_A^{opt} decreases in w, we must have that s_A^* decreases in w, which completes the proof. $\square$

Proof of Proposition C.10. To prove the proposition, we first prove that $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$ and $\frac{\mathrm{d}}{\mathrm{d}w} v_A(s_A^*, w) \leq 0$. We begin by proving $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$. By Equation (A.17), we have

$$\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) = \frac{\mathrm{d}}{\mathrm{d}r}(v_1 \times x^*(s^*) - v_2 \times (x^*(s^*))^2) = (v_1 - 2v_2 x^*(s^*)) \left(\frac{\partial}{\partial r} x^*(s) \Big|_{s=s^*} + \frac{\partial}{\partial s} x^*(s) \Big|_{s=s^*} \frac{\partial}{\partial r} s^* \right).$$

Based on the proof of Theorem 1, we have $x^*(s^*) \leq x^{opt} = \frac{v_1}{2v_2}$. Moreover, if $x^*(s^*) = x^{opt}$, we have $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) = 0$. Thus, to prove $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$, it remains to consider the scenario where $x^*(s^*) < x^{opt}$ and thus $v_1 - 2v_2 x^*(s^*) > 0$. Based on the proof of Theorem 1, consider the following three cases:

Suppose s^* satisfies $s^* < \bar{s}$ and $\frac{\partial}{\partial s} x^*(s)|_{s=s^*} = 0$. Then, by Corollary 1, we have

$$\frac{\partial}{\partial r} x^*(s) \Big|_{s=s^*} + \frac{\partial}{\partial s} x^*(s) \Big|_{s=s^*} \frac{\partial}{\partial r} s^* = \frac{\partial}{\partial r} x^*(s) \Big|_{s=s^*} \geq 0.$$

Thus, we have $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$.

Suppose s^* satisfies $s^* < \bar{s}$, $\frac{\partial}{\partial s} x^*(s)|_{s=s^*} > 0$, and $x^*(s^*)\mathbb{E}[s^* I] = B$. By taking the derivative of both sides of $x^*(s^*)\mathbb{E}[s^* I] = B$ with respect to r, we have

$$\frac{1}{2} x^*(s^*) \frac{\partial}{\partial r} s^* + \frac{1}{2} s^* \left(\frac{\partial}{\partial r} x^*(s) \Big|_{s=s^*} + \frac{\partial}{\partial s} x^*(s) \Big|_{s=s^*} \frac{\partial}{\partial r} s^* \right) = 0.$$

We have shown in the proof of Theorem 1 that $\frac{\partial}{\partial r} s^* \leq 0$ in this case. Then, we have $\frac{\partial}{\partial r} x^*(s)|_{s=s^*} + \frac{\partial}{\partial s} x^*(s)|_{s=s^*} \frac{\partial}{\partial r} s^* \geq 0$ and thus $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$.

Suppose s^* satisfies $s^* = \bar{s}$. Then, by Equation (A.13), we have

$$\frac{\partial}{\partial r} s^* = -4b\mu\sigma \left(\frac{\partial}{\partial r} x^*(s) \Big|_{s=s^*} + \frac{\partial}{\partial s} x^*(s) \Big|_{s=s^*} \frac{\partial}{\partial r} s^* \right).$$

We have shown in the proof of Theorem 1 that $\frac{\partial}{\partial r} s^* \leq 0$ in this case. Then, we have $\frac{\partial}{\partial r} x^*(s)|_{s=s^*} + \frac{\partial}{\partial s} x^*(s)|_{s=s^*} \frac{\partial}{\partial r} s^* \geq 0$ and thus $\frac{\mathrm{d}}{\mathrm{d}r} v(s^*) \geq 0$.

Next, we prove that $\frac{\mathrm{d}}{\mathrm{d}w}v_A(s_A^*, w) \le 0$. We have shown in the proof of Proposition C.9 that s_A^* decreases in w and $\frac{\partial}{\partial s}x_A^*(s)|_{s=s_A^*} \ge 0$. Thus, by the expression of $v_A(s, w)$ in Appendix C.7, we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}w}v_A(s_A^*, w) &= (v_1 - 2v_2 x_A^*(s_A^*))\frac{\mathrm{d}}{\mathrm{d}w}x_A^*(s_A^*) - x_A^*(s_A^*) - w\frac{\mathrm{d}}{\mathrm{d}w}x_A^*(s_A^*) \\
&= (v_1 - w - 2v_2 x_A^*(s_A^*))\left(\frac{\partial}{\partial w}x_A^*(s)\Big|_{s=s_A^*} + \frac{\partial}{\partial s}x_A^*(s)\Big|_{s=s_A^*}\frac{\partial}{\partial w}s_A^*\right) - x_A^*(s_A^*) \\
&= (v_1 - w - 2v_2 x_A^*(s_A^*))\frac{\partial}{\partial s}x_A^*(s)\Big|_{s=s_A^*}\frac{\partial}{\partial w}s_A^* - x_A^*(s_A^*) \\
&\le 0,
\end{aligned}$$

where the inequality holds because $x_A^*(s_A^*) \le x_A^{opt} = \frac{v_1 - w}{2v_2}$ and thus $v_1 - w - 2v_2 x_A^*(s_A^*) \ge 0$.

Having established both $\frac{\mathrm{d}}{\mathrm{d}r}v(s^*) \ge 0$ and $\frac{\mathrm{d}}{\mathrm{d}w}v_A(s_A^*, w) \le 0$, we now proceed to proving the proposition. To do so, we first observe that when $w = 0$, we must have $v_A(s_A^*, w) \ge v(s^*)$ for any $r \in [\frac{1}{2}, 1]$. This is because, for any given subsidy amount s, the equilibrium planting amount under the actual-yield-based policy is higher than that under the index-based policy (as shown in the proof of Corollary 2), which implies that, to achieve the same equilibrium planting amount, a lower subsidy is needed under the actual-yield-based policy. Further, when $w = 0$, we have $x_A^{opt} = x^{opt}$. Therefore, when $w = 0$, we have $v_A(s_A^*, w) \ge v(s^*)$ for any $r \in [\frac{1}{2}, 1]$. Let w_1 denote the highest yield assessment cost w such that $v_A(s_A^*, w) \ge v(s^*)$ when $r = 1$. Then, given that $\frac{\mathrm{d}}{\mathrm{d}r}v(s^*) \ge 0$ and $\frac{\mathrm{d}}{\mathrm{d}w}v_A(s_A^*, w) \le 0$, we must have $v_A(s_A^*, w) \ge v(s^*)$ for any $w \le w_1$ and any $r \in [\frac{1}{2}, 1]$.

On the other hand, when w is sufficiently high (e.g., when $w = \frac{B}{x_0^*}$ or $w = v_1$), we clearly have $v_A(s_A^*, w) \le v(s^*)$ for any $r \in [\frac{1}{2}, 1]$. Let w_2 denote the lowest yield assessment cost w such that $v_A(s_A^*, w) \le v(s^*)$ when $r = \frac{1}{2}$. Then, given that $\frac{\mathrm{d}}{\mathrm{d}r}v(s^*) \ge 0$ and $\frac{\mathrm{d}}{\mathrm{d}w}v_A(s_A^*, w) \le 0$, we must have $v_A(s_A^*, w) \le v(s^*)$ for any $w \ge w_2$ and any $r \in [\frac{1}{2}, 1]$.

Finally, when $w_1 < w < w_2$, by construction of w_1 and w_2, we have $v_A(s_A^*, w) > v(s^*)$ when $r = \frac{1}{2}$ and $v_A(s_A^*, w) < v(s^*)$ when $r = 1$. Since $v_A(s_A^*, w)$ is independent of r and $\frac{\mathrm{d}}{\mathrm{d}r}v(s^*) \ge 0$, there must exist a threshold in $(\frac{1}{2}, 1)$ such that $v_A(s_A^*, w) < v(s^*)$ if and only if r exceeds the threshold. □

Appendix E Subsidy Design under Low vs. High Index Accuracy

In this section, we summarize the key policy implications of our analysis under low versus high index accuracy. Table E.1 presents a concise overview of how the optimal yield protection subsidy should be adjusted as index accuracy improves, how it should be set differently relative to an actual-yield benchmark, and how it interacts with a minimum price guarantee. This summary highlights how subsidy design should differ under low versus high index accuracy, underscoring the importance of explicitly accounting for the accuracy (or inaccuracy) of the index in subsidy design.

Table E.1 Policy implications for designing index-based yield protection subsidies

Index Accuracy	**Low**	**High**
Subsidy response to accuracy changes	**Raise** when accuracy improves	**Reduce** when accuracy improves
Subsidy level	Set **lower** than actual-yield benchmark	Set **higher** than actual-yield benchmark
Yield subsidy with vs. without price guarantee	**Raise** yield subsidy when introducing price guarantee (complements)	**Reduce** yield subsidy when introducing price guarantee (substitutes)
Price guarantee with vs. without yield subsidy	**Raise** price guarantee when introducing yield subsidy	**Reduce** price guarantee when introducing yield subsidy

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