Online Appendix for

Sample Range as Subjective Variability and Implications for Experience-Based Decision Making

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Appendix A: Distribution of Sample Range from Uniform Distribution

Given *n* random samples from probability density function (PDF) $f(x)$ and cumulative distribution function (CDF) $F(x)$, by the distribution of order statistics, sample range *w* follows:

PDF:

$$f_{w_n}(w) = n(n-1)\int_{-\infty}^{\infty}(F(w+x)-F(x))^{n-2}f(x)f(x+w)dx$$

CDF:

$$F_{w_n}(w) = P(w_n \le w) = n\int_{-\infty}^{\infty}(F(x+w)-F(x))^{n-1}f(x)dx$$

For a uniform distribution,

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases},$$

and

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x > b. \end{cases}$$

When $0 \le w \le b-a$:

$$\begin{aligned} F_{w_n}(w) &= n\int_a^{b-w}\left(\frac{w}{b-a}\right)^{n-1}\frac{1}{b-a}dx + n\int_{b-w}^{b}\left(1-\frac{x-a}{b-a}\right)^{n-1}\frac{1}{b-a}dx \\ &= n\frac{w^{n-1}}{(b-a)^n}x\Big|_a^{b-w} + \left(-\frac{(b-x)^n}{(b-a)^n}\right)\Big|_{b-w}^{b} \\ &= \frac{(b-a)nw^{n-1}-(n-1)w^n}{(b-a)^n} \end{aligned}$$

And:

$$f_{w_n}(w) = \frac{dF_{w_n}(w)}{dw} = \frac{n(n-1)w^{n-2}(b-a-w)}{(b-a)^n}$$

Therefore, the distribution of *w* is:

PDF:

$$f_{w_n}(w) = \begin{cases} \dfrac{n(n-1)w^{n-2}(b-a-w)}{(b-a)^n} & 0 \leqslant w \leqslant b-a \\ 0 & \text{otherwise ,} \end{cases}$$

CDF:

$$F_{w_n}(w) = \begin{cases} 0 & w < 0 \\ \dfrac{(b-a)nw^{n-1} - (n-1)w^n}{(b-a)^n} & 0 \leqslant w \leqslant b-a \\ 1 & w > b-a \end{cases}$$

The expectation of *w*:

$$\begin{aligned} E[w] &= \int_{-\infty}^{\infty} w f_{w_n}(w) dw \\ &= \int_0^{b-a} \frac{n(n-1)w^{n-1}(b-a-w)}{(b-a)^n} dw \\ &= \frac{n-1}{n+1}(b-a) \end{aligned}$$

Appendix B: Supplementary Items for the experiments

Table B1. Stimuli used in Experiment 4

Stimulus ID	Species	Samples	EV	SD	Sample size
1	A	67, 74, 60	67.58	7.06	3
2	B	123, 194, 219	178.49	48.86	3
3	C	346, 275, 456	356.2	91.73	3
4	D	812, 773, 620	744.6	99.94	3
5	E	1556, 1242, 1241	1328.18	178.92	3
6	A	73, 79, 60, 74, 72, 64, 63, 61, 62, 60	67.58	7.06	10
7	B	213, 196, 262, 240, 179, 146, 138, 114, 175, 137	178.49	48.86	10
8	C	547, 242, 261, 343, 435, 361, 280, 369, 392, 393	356.2	91.73	10
9	D	885, 786, 741, 779, 585, 723, 865, 585, 768, 706	744.6	99.94	10
10	E	1283, 1511, 1065, 1593, 1356, 1093, 1337, 1525, 1198, 1229	1328.18	178.92	10