Appendix

Online Supplement Appendix to Manuscript "Organizational Culture, Innovation, and Competitive Performance: A Multi-Level Dynamic Model"

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A. When idea generators don't always find an idea that works

In section 3.1 we have assumed that idea generators always find a suitable (not necessarily the optimal) solution to the current environment state. Here we relax this assumption, introducing "fallible" generators who successfully find a suitable solution only with a probability $\alpha > 0$. This probability affects the interactions of generators with implementors. For the interactions between implementors we assume that the results disseminate fast within the organization, so they are not affected. The implementor/generator payoff matrix is then shown in Table 5.

	I	G
I	$(1-\gamma)V - C_I$	$\alpha V - C_I$
G	$\alpha V - C_G$	$-C_G$

Table 5 Symmetric normal-form evolutionary game between implementors and generators when generators find a solution with probability α. The payoffs displayed accrue to the player on the left.

The ESS equilibrium x^*_α is then

$$x^*_\alpha = \frac{\alpha V - \Delta c}{(2\alpha + \gamma - 1)V} \tag{A.1}$$

The above equation determines the condition on the parameter α so that the equilibrium x^*_α is in the range $[0,1]$ which becomes $\alpha > \max\left\{\frac{\Delta c}{V}, \frac{1}{2}(1-\gamma), (1-\gamma) - \frac{\Delta c}{V}\right\}$. If this condition is not fulfilled, an unstable equilibrium results with either $x^*_\alpha = 0$ (all generators) or $x^*_\alpha = 1$ (all implementors).

We would intuitively expect that the equilibrium mix should shift toward implementors when generators are fallible and therefore experience lower expected rewards. But this is not the case, as the following equation shows which exhibits the change of x^*_α as a function of a change in α:

$$\frac{\partial x^*_\alpha}{\partial \alpha} = \frac{\Delta c - \frac{(1-\gamma)}{2}V}{V\left[\alpha - \frac{(1-\gamma)}{2}\right]^2} \tag{A.2}$$

This equation indicates that the equilibrium shifts into the expected direction only if $\Delta c > \frac{1}{2}(1-\gamma)V$. The reason is the alluded-to subtle interaction between generators and implementors: implementors *also* suffer when generators are fallible in the interaction, and if the cost differential is too small, then the implementors suffer *more* than the generators, and therefore, the equilibrium shifts the other way. In sum, the results are consistent with the base model, but highlight subtleties of interactions.

B. Continuous Improvement and Experimental IP

This section tests the robustness of the base model results under two modifications: first, we incorporate the idea that pairs of implementors may, in periods of continuity (without environmental change), be able to incrementally improve upon existing products/processes. In a period of environment stability, only incremental ideas are necessary, which implementors are able to produce. We assume that this increases the output value by a small percentage $0 < \epsilon \ll 1$. Second, we acknowledge that in periods of disruption, an idea generator may be able to generate *some* value even when interacting with another generator in spite of their disconnect from the formal procedures; this value is $V(1-\epsilon')$ with $0 \ll \epsilon' \leqslant 1$. For example, they may produce some value from experiments registered as *intellectual property* (IP), but not quite the full value V of a fully implemented idea. When $\epsilon = 0$ and $\epsilon' = 1$, the modified model reduces to the base model from Table 1. The interactions between implementors and generators under these modified assumptions produce the payoffs shown in Table 6.

	I	G
I	$(1-\gamma)(1+\epsilon)V - C_I$	$V - C_I$
G	$V - C_G$	$\gamma(1-\epsilon')V - C_G$

Table 6 Adapted symmetric normal-form evolutionary game between implementors and generators. The payoffs displayed accrue to the player on the left.

The equilibrium, x_+^*, retains the same structure and is now given by:

$$x_+^* = \frac{(1-\gamma+\gamma\epsilon')V - \Delta c}{[(1-\epsilon+\gamma(\epsilon+\epsilon')]V} \tag{B.1}$$

The equilibrium population mix behaves qualitatively in the same way as before: $\partial x_+^*/\partial V > 0$, $\partial x_+^*/\partial\gamma < 0$, and $\partial x_+^*/\partial(\Delta C) < 0$. The equilibrium is shifted versus the base model in a way that is consistent with the logic of the base model: first, if $0 = \epsilon < \epsilon' < 1$ (i.e., the generators can now create at least some output), the equilibrium shifts toward more generators in the population (as their position becomes more attractive). But $\partial x_+^*/\partial\epsilon > 0$ and $\partial x_+^*/\partial\epsilon' > 0$, which means that as ϵ increases (the value of the implementors' incremental improvement) and as ϵ' increases (the value of IP produced deteriorates), the equilibrium mix shifts towards implementors. In both cases, their expected payoff (fitness) increases relative to the generators. Both shifts of the equilibrium support the intuition of the base model: the employee type with the higher payoff (or lower cost in the base model) is more prevalent in the equilibrium, which is *not* normally consistent with maximum performance of the organization. Figure 7 depicts an instance of the dynamics of the model discussed in this section. This extension supports model robustness.

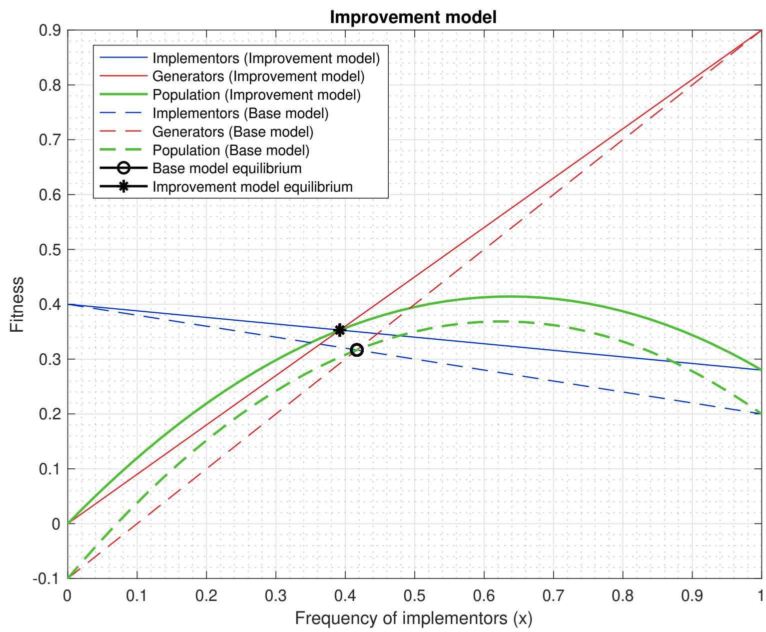

Figure 7 Expected fitness of implementors, generators, and mean population performance as a function of implementor frequency. In this example, $V = 1$, $C_I = 0.6$, $C_G = 0.1$, $\gamma = 0.2$, $\epsilon = 0.1$ (implementor incremental innovation increases the value by 10%), and $\epsilon' = 0.5$ (idea generator interactions produce 50% value).

C. When individuals interact with a silo level m and share value at level δ

In this section we assume that individuals interact with silo level $0 \leqslant m < 1$ and share value at level $\delta \geqslant 0$. The fitness of the idea generators and of the implementors are

$$W_I(m,\delta) = (1-\gamma m)V + (1-m)\delta - C_I - (\delta + \gamma m V)(1-m)x \tag{C.1}$$

$$W_G(m,\delta) = (1-m)(V-\delta)x - C_G \tag{C.2}$$

It is straightforward to show that the equilibrium $x^*(m,\delta)$ now depends on both m and δ and is given by

$$x^*(m,\delta) = \frac{(1-\gamma m)V + (1-m)\delta - \Delta c}{(1+\gamma)(1-m)V} \tag{C.3}$$

When individuals meet randomly ($m = 0$) and share value equally ($\delta = 0$), then the result is identical to the one in section 3.1. Also, when the individuals interact at random ($m = 0$) but share value at a level $\delta > 0$, the result is identical to the one in section 4.2. Finally, when the individuals share value equally but interact with some silo-level $m > 0$, the results are identical to the ones in section 4.1.

In a population where a fraction x behave as implementors and a fraction $1-x$ as idea generators, the mean fitness is $\widehat{W}(m,\delta) = W_I(m,\delta)x + W_G(m,\delta)(1-x)$. Note that $\widehat{W}(m,\delta)$ is maximized at $x^{**}(m,\delta)$ which is given by

$$x^{**}(m,\delta) = \frac{2V - \Delta c - m(1+\gamma)V}{2(1+\gamma)(1-m)V} \tag{C.4}$$

For a given level m of silo behavior, there exists a $\delta^*(m)$ that maximizes the mean population fitness or in other words solves the equation $x^{**}(m,\delta^*) = x^*(m,\delta^*)$. This equation gives

$$\delta^*(m) = \frac{\Delta c - m(1-\gamma)V}{2(1-m)} \tag{C.5}$$

As expected, when $m = 0$, the optimal δ coincides with the result found in section 4.2. It is straightforward to show that $\partial\delta^*/\partial m = [\Delta c - (1-\gamma)V]/[2(1-m)^2]$. When $\Delta c > (1-\gamma)V$ then $\partial\delta^*/\partial m > 0$ and when $\Delta c < (1-\gamma)V$ then $\partial\delta^*/\partial m < 0$.

D. Proof of Proposition 5

All forms with a profit at or above the threshold Π_0 do not change their sharing norm. If there are firms that do not earn the threshold profit, one of them must either be firm j with $\delta_j(t) = \max_i \delta_i(t)$ or firm k with $\delta_k(t) = \min_i \delta_i(t)$, or both: the firms with the sharing norms the furthest away from δ^* have the highest costs. Suppose now the firm with the lowest profit is firm j (the argument for firm k is analogous).

By the construction of the process by which the firm (or its replacement) changes the sharing norm, firm j performs a random draw from a distribution $F(\cdot)$ on the support of feasible sharing norms $[-(V - \Delta c), \gamma V + \Delta c]$. We show below that the profit falls off symmetrically around δ^*, therefore, we can write the probability of drawing a sharing norm with better profit than $\delta_j(t)$, $\delta_j(t+1)$ as: $0 \leqslant u_j = \Pr\{\delta^* - (\delta_j(t) - \delta^*) < \delta_j(t+1) < \delta_j(t)\} \leqslant 1$. With probability $(1 - u_j)$, the chosen new sharing norm will not improve the profit and thus, the firm will choose to stay with sharing norm $\delta_j(t)$. Therefore, the sharing norm of firm j weakly decreases toward δ^*. If all firms earn the threshold profit, then according to our change procedure,none of them will change its sharing norm, and thus the convergence stops.

Now we turn to what happens if the threshold profit Π_0 or the number of firms N increase. A firm's Cournot equilibrium profit is the squared difference of the Cournot price and the cost in Equation (5.3), and Equations (5.1) and (4.7) link the cost to the firm's cultural productivity and the sharing norm. Substituting in, we can write the firm's profit, and the threshold condition that it is equal to Π_0, in the following way:

$$C_0 - \lambda\frac{(V-\delta)(V+\delta-\Delta c)}{(1+\gamma)V} + \lambda C_G = P^* - \sqrt{B\Pi_0} \tag{D.1}$$

where P^* is the price at the Cournot equilibrium. Rearranging terms, we find that the critical values satisfy the equation

$$\delta^2 - (\Delta c)\delta - V^2 + (\Delta c)V + \left(C_0 + \lambda C_G - P^* + \sqrt{B\Pi_0}\right)\frac{1+\gamma}{\lambda}V = 0. \tag{D.2}$$

The roots of this equation yield (5.4) in the proposition. Note that the interval $[\delta_L, \delta_H]$ is symmetric around $\delta^* = \Delta c/2$ and that the interval shrinks if P^* gets smaller and if Π_0 becomes larger.

We finally show that the entry of a new firm causes the equilibrium price to shrink. Suppose that at a certain time period there are N active firms competing with ordered costs $C_1 < C_2 < \cdots < C_N < A$. The equilibrium price, P, is given by $P = (A + \sum_{j=1}^N C_j)/(N+1)$. The fact that all the firms are active implies that $P > C_N$. If a new firm enters with a competitive cost $C' < C_N < P$, then there are $N+1$ firms, the equilibrium price P' becomes

$$P' = \frac{A + \sum_{j=1}^N C_j + C'}{N+2} = \frac{N+1}{N+2}P + \frac{C'}{N+2} \tag{D.3}$$

Note that $P' \geqslant P$ implies that $C' \geqslant P$ which is not possible because we have assumed that the new firm enters if $C' < C_N < P$. Therefore, when the competition involves $N+1$ firms, $P' < P$.

If the competition involves N firms then the new equilibrium price, P', will be $P' = (A + \sum_{j=1}^{N-1} C_j + C')/(N+1)$. Again, $P' < P$ since $C' < C_N$.

If the competition, post entry, involves less than N firms (because the new firm enters at such a low cost that renders some of the high cost previous participant firms inactive) then again the new price $P' < P$, because the new price will have to be smaller than the lowest cost of the rendered inactive firm.

E. Evolutionary Stable Strategies (ESS)

E.1. Cournot Competition

As we have assumed in equation (5.1) a choice, δ, of the value sharing norm maps into the marginal production cost, c, according to the rule $c = c_0 - B(\delta)$, where $B(\delta) = \lambda\widehat{W}(\delta)$.

Assume a population of firms all playing strategy δ with marginal cost $c = c_0 - B(\delta)$ and a small fraction of "invaders" who choose a different strategy $\delta + \epsilon$, with $\epsilon \neq 0$, which results in a different marginal cost $c' = c_0 - B(\delta + \epsilon)$. The firms are paired at random and compete in a manner of Cournot (selecting quantities) in a market with inverse demand function $p = \alpha - \beta Q$, where α and β are exogenous parameters, p is the price, and Q is the total quantity. The normal-form game is given in Table (7).

The payoffs $\Pi(\delta,\delta)$ and $\Pi(\delta+\epsilon,\delta)$ in Table (7) are the symmetric and asymmetric profits at the Cournot equilibrium respectively and are given by

$$\Pi(\delta,\delta) = \frac{1}{9\beta}[\alpha - c_0 + B(\delta)]^2 \quad \text{and} \quad \Pi(\delta+\epsilon) = \frac{1}{9\beta}[\alpha - c_0 - B(\delta) + 2B(\delta+\epsilon)]^2 \tag{E.1}$$

The strategy δ is evolutionary stable (ESS) if $\Pi(\delta,\delta) > \Pi(\delta+\epsilon,\delta)$ for all ϵ. It is straightforward to see that the condition $\Pi(\delta,\delta) > \Pi(\delta+\epsilon,\delta)$ implies that $B(\delta) > B(\delta+\epsilon)$ for all ϵ, which, in turn, implies that $\delta = \delta^* = \arg\max\{B(\delta)\}$.

	δ	$\delta+\epsilon$
δ	$\Pi(\delta,\delta)$	$\Pi(\delta,\delta+\epsilon)$
$\delta+\epsilon$	$\Pi(\delta+\epsilon,\delta)$	$\Pi(\delta+\epsilon,\delta+\epsilon)$

Table 7 Symmetric normal-form game between firms with strategy δ and invaders with strategy $\delta+\epsilon$. The payoffs displayed accrue to the player on the left

E.2. Differentiated Bertrand Competition

Consider the case of two firms $i=\{1,2\}$ that produce differentiated products with costs c_1 and c_2 at prices p_1 and p_2 and compete on price (Betrand). The output of the firms q_1 and q_2 depends on the prices according to

$$q_1 = 1 - p_1 + \alpha p_2 \quad \text{and} \quad q_2 = 1 - p_2 + \alpha p_1 \tag{E.2}$$

where α is an exogenous parameter $0<\alpha<1$ that measures the degree of substitutability of the two products. Each firm chooses price to maximize profits $\Pi_i = (p_i - c_i)q_i$ for $i=\{1,2\}$. The first order conditions $\partial\Pi_1/\partial p_1 = 0$ and $\partial\Pi_2/\partial p_2 = 0$ determine the prices p_1 and p_2 at the Bertrand equilibrium which are given by

$$p_1 = \frac{2+\alpha+2c_1+\alpha c_2}{4-\alpha^2} \quad \text{and} \quad p_2 = \frac{2+\alpha+2c_2+\alpha c_1}{4-\alpha^2} \tag{E.3}$$

The profits of the two firms at the Bertrand equilibrium are then

$$\begin{aligned}\Pi_1(c_1,c_2) &= \frac{(2+\alpha-2c_1+\alpha c_2+\alpha^2 c_1)^2}{(4-\alpha^2)^2}\\ \Pi_2(c_1,c_2) &= \frac{(2+\alpha-2c_2+\alpha c_1+\alpha^2 c_2)^2}{(4-\alpha^2)^2}\end{aligned} \tag{E.4}$$

Note that in the symmetric case when $c_1=c_2=c$, the Bertrand price at equilibrium becomes $p_1=p_2=p=(1+c)/(2-\alpha)$ and the corresponding profits are

$$\Pi(c,c) = \frac{[1-c(1-\alpha)]^2}{(2-\alpha)^2} \tag{E.5}$$

As in the previous section, we assume that a population of firms follows the δ strategy and produce at a marginal cost c that depends on the firm's efficiency as dictated by the benefit gained at the mixed strategies equilibrium. In other words we suppose that $c = c_0 - B(\delta)$. An invader firm chooses a $\delta+\epsilon$ strategy and produces at cost $c' = c_0 - B(\delta+\epsilon)$. The firms are randomly paired and the resulting game is the one displayed in Table 7 where the profits are now given by the differentiated Bertrand formulas E.4 and E.5. For the strategy δ to be an ESS its is required

$$\frac{[1-(c_0-B(\delta))(1-\alpha)]^2}{(2-\alpha)^2} > \frac{[2+\alpha-(c_0-B(\delta+\epsilon))(2-\alpha^2)+\alpha(c_0-B(\delta))]^2}{(2-\alpha)^2(2+\alpha)^2}$$

Rearranging terms, the above inequality reduces to

$$-(2-\alpha^2)(\gamma - B(\delta)) > -(2-\alpha^2)(\gamma - B(\delta+\epsilon))$$

which in turn implies that $B(\delta) > B(\delta+\epsilon)$ for all ϵ. In other words, the strategy δ^* is the ESS strategy.