Online Appendix to "Whiplash: Congestion Dissipation in a Disrupted Circulatory Shipping System"

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A. Definition of the Stochastic Dynamic Process

In this section, we provide the formal definition of the stochastic dynamic process of the discrete model.

We first analyze the outbound goods. We denote by $\bar{\mathcal{Q}}_i = \{\bar{Q}_i(t), t \geq 0\}$ (resp., $\bar{\mathcal{Q}}_j = \{\bar{Q}_j(t), t \geq 0\}$) the outbound backlog process at Port i (resp., j), where $\bar{Q}_i(t)$ (resp., $\bar{Q}_j(t)$) represents the amount of outbound cargoes waiting to be loaded onto ships at Port i (resp., j) at time t.

The sample paths of the outbound backlog processes at Ports i and j satisfy the following relations:

$$\bar{Q}_i(t) = \bar{Q}_i(0) + A_i(t) - \bar{S}_i(\bar{B}_i(t)), \tag{A.1}$$

$$\bar{Q}_j(t) = \bar{Q}_j(0) + A_j(t) - \bar{S}_j(\bar{B}_j(t)), \tag{A.2}$$

with $\bar{B}_i(t) = 0$ if $t \in [0, \tau)$ and $\bar{B}_i(t) = \int_\tau^t \mathbb{1}\{\bar{Q}_i(s) > 0\} ds$ if $t \geq \tau$ and $\bar{B}_j(t) = 0$ if $t \in [0, \tau)$ and $\bar{B}_j(t) = \int_\tau^t \mathbb{1}\{\bar{Q}_j(s) > 0\} ds$, respectively. Note that here to avoid the structural discontinuity, we extend the domain to $(-l, \infty)$. Here, $\bar{Q}_i(0)$ (resp., $\bar{Q}_j(0)$) represents the amount of outbound goods that Port i (resp., j) initially holds at time $t = 0$, $A_i(t)$ (resp., $A_j(t)$) represents the amount of outbound cargoes that have arrived at Port i (resp., j) exogenously during $(0, t]$, and $\bar{S}_i(\bar{B}_i(t))$ (resp., $\bar{S}_j(\bar{B}_j(t))$) represents the amount of outbound cargoes that have been loaded onto ships and have departed from Port i (resp., j) during $(0, t]$.

Following the analysis of the outbound backlog process, we now focus on the dynamics of inbound cargoes within the system. Once loaded onto ships, outbound goods necessitate a travel time of l days to reach their destination port. Therefore, goods that arrive at Port i (resp., j) at any time t correspond precisely to the goods that depart from Port j (resp., i) at time $t - l$. Mathematically, the amount of goods that arrive at Port i (resp., j) at time t is $\bar{S}_j(\bar{B}_j(t-l))$ (resp., $\bar{S}_i(\bar{B}_i(t-l))$).

We denote by $\underline{\mathcal{Q}}_i = \{\underline{Q}_i(t), t \geq 0\}$ (resp., $\underline{\mathcal{Q}}_j = \{\underline{Q}_j(t), t \geq 0\}$) the inbound backlog process at Port i (resp., j), where $\underline{Q}_i(t)$ (resp., $\underline{Q}_j(t)$) represents the amount of inbound goods waiting to be

unloaded at Port i (resp., j) at time t. The sample paths of the inbound backlog processes at Ports i and j satisfy the following relations:

$$\underline{Q}_i(t) = \underline{Q}_i(0) + \bar{S}_j(\bar{B}_j(t-l)) - \underline{S}_i(\underline{B}_i(t)), \tag{A.3}$$

$$\underline{Q}_j(t) = \underline{Q}_j(0) + \bar{S}_i(\bar{B}_i(t-l)) - \underline{S}_j(\underline{B}_j(t)), \tag{A.4}$$

with $\underline{B}_i(t) = \int_0^t \mathbb{1}\{\underline{Q}_i(s) > 0\}ds$ and $\underline{B}_j(t) = \int_0^t \mathbb{1}\{\underline{Q}_j(s) > 0\}ds$, respectively. Here, $\underline{Q}_i(0)$ (resp., $\underline{Q}_j(0)$) represents the amount of inbound goods Port i (resp., j) initially holds at time $t = 0$, $\bar{S}_j(\bar{B}_j(t-l))$ (resp., $\bar{S}_i(\bar{B}_i(t-l))$) represents the amount of inbound cargoes that have arrived at Port i (resp., j) from Port j (resp., i) during $(0, t]$, and $\underline{S}_i(\underline{B}_i(t))$ (resp., $\underline{S}_j(\underline{B}_j(t))$) represents the amount of inbound cargoes that have been unloaded from the arrived ships at Port i (resp., j) during $(0, t]$.

We assume that the interarrival times and service times of both inbound and outbound backlog processes follow the functional strong law of large numbers (FSLLN):

$$A_i(nt)/n \to \alpha_i t, \quad \bar{S}_i(nt)/n \to \bar{\mu}_i t, \quad \underline{S}_i(nt)/n \to \underline{\mu}_i t, \quad \text{u.o.c., as } n \to \infty,$$
$$A_j(nt)/n \to \alpha_j t, \quad \bar{S}_j(nt)/n \to \bar{\mu}_j t, \quad \underline{S}_j(nt)/n \to \underline{\mu}_j t, \quad \text{u.o.c., as } n \to \infty.$$

We note that a sufficient condition for the above FSLLN conditions to hold is that both the arrival and service processes at Ports i and j are renewal processes with finite means. As we scale up the system by letting $n \to \infty$, we allow the initial backlog levels to depend on n, denoted by $\bar{Q}_i^n(0)$, $\bar{Q}_j^n(0)$, $\underline{Q}_i^n(0)$, and $\underline{Q}_j^n(0)$. We assume that almost surely,

$$\frac{\bar{Q}_i^n(0)}{n} \to O_i(0), \quad \frac{\bar{Q}_j^n(0)}{n} \to O_j(0), \quad \underline{Q}_i^n(0) \to I_i(0), \quad \underline{Q}_j^n(0) \to I_j(0), \quad \text{as } n \to \infty,$$

with $O_i(0) \geq 0$, $O_j(0) \geq 0$, $I_i(0) \geq 0$, and $I_j(0) \geq 0$ being constants. We index the performance of the outbound and inbound backlog processes by a superscript n, denoted by $\bar{Q}_i^n$, $\bar{Q}_j^n$, $\underline{Q}_i^n$, and $\underline{Q}_j^n$.

Lemma A.1 (Fluid Approximation). *We assume that the interarrival times and service times of both inbound and outbound backlog processes follow the functional strong law of large numbers. Then, for the outbound and inbound backlog processes at Ports i and j, respectively, we have*

$$\bar{Q}_i(nt)/n \to O_i(t) \text{ and } \bar{Q}_j(nt)/n \to O_j(t) \quad w.p.1 \text{ as } n \to \infty,$$
$$\underline{Q}_i(nt)/n \to I_i(t) \text{ and } \underline{Q}_j(nt)/n \to I_j(t) \quad w.p.1 \text{ as } n \to \infty,$$

where $O_i(t)$ and $O_j(t)$ are the associated fluid processes of the outbound backlog processes at Ports i and j, respectively, and $I_i(t)$ and $I_j(t)$ are the associated fluid processes of the inbound backlog processes at Ports i and j, respectively, with their detailed expressions provided in Online Appendices I and J.

Lemma A.1 establishes fluid approximations of the outbound and inbound backlog processes at both Ports i and j in the two-port system. In particular, the outbound backlog level exhibits a continuous growth at a rate of λ within the disruption period $t \in [0, \tau)$. Figures 3(a)-(d) depict graphical representations illustrating such fluid dynamics associated with both inbound and outbound backlog processes.

A.1. Robustness: Discrete Batching and the Fluid Approximation

While the fluid approximation provides a tractable framework for analyzing systemic recovery, it treats maritime capacity as a continuous flow. In practice, however, transportation capacity is deployed in discrete "batches" corresponding to vessel arrivals. We examine the impact of this discretization on backlog dynamics and its interaction with the whiplash effect.

Let $\bar{Q}(t)$ denote the backlog level in the fluid model, and let $\hat{Q}(t)$ denote the corresponding backlog level under a discrete process:

$$\hat{Q}(t) = Q(0) + \int_0^t \lambda(s)\, ds - \sum_{n=1}^{N(t)} S_n, \tag{A.5}$$

where $N(t)$ is a point process representing vessel arrivals up to time t, and S_n is the capacity of the n-th vessel. Let S denote the maximum vessel capacity, i.e., $S_n \leq S$ for all n.

To facilitate comparison, consider a periodic schedule in which vessels arrive every Δt units of time, with Δt chosen such that the time-averaged service rate matches the fluid capacity:

$$\frac{\mathbb{E}[S_n]}{\Delta t} = K\lambda.$$

Lemma A.2 (Discretization Bound). *For any time $t \geq 0$, the deviation between the fluid backlog $\bar{Q}(t)$ and the discrete-batching backlog $\hat{Q}(t)$ is bounded by the capacity of a single vessel, where $S_n \leq S$ for all n. Then for any $t \geq 0$, we have,*

$$|\hat{Q}(t) - \bar{Q}(t)| \leq S. \tag{A.6}$$

Lemma A.2 establishes that the fluid model provides a uniformly accurate approximation of the discrete system up to an error bounded by the maximum vessel capacity. This implies that the macroscopic whiplash dynamics identified in the fluid model are preserved under batching.

At the same time, discretization introduces a "peak-load" effect. In the discrete system, the backlog level evolves continuously between arrivals but increases discontinuously by S_n upon each vessel's berthing. As a result, the backlog level immediately after an arrival can exceed the fluid backlog by up to S, generating transient congestion spikes. For analytical tractability, we characterize these peak effects using the worst-case batch size S, which provides a uniform upper bound on all shipment sizes.

Proposition A.1 (Peak-Load Amplification and Buffer Requirement). *Let $\bar{Q}(t)$ and $\hat{Q}(t)$ be defined as above. Then:*

1. *(Peak-load amplification) The discrete backlog satisfies*

$$\sup_{t\geq 0}\hat{Q}(t)\leq \sup_{t\geq 0}\bar{Q}(t)+S.$$

2. *(Buffer requirement) A sufficient condition to avoid buffer overflow is*

$$Y\geq \sup_{t\geq 0}\bar{Q}(t)+S+\epsilon,$$

where $\epsilon>0$ is a safety margin.

Proof: By Lemma A.2, we have $|\hat{Q}(t)-\bar{Q}(t)|\leq S$ for all $t\geq 0$, which implies $\hat{Q}(t)\leq \bar{Q}(t)+S$. Taking the supremum over t yields the first result. The second result follows directly, as the buffer must accommodate the maximum realized backlog in the discrete system. □

Proposition A.1 highlights that, while the fluid model captures the average recovery dynamics, discrete batching introduces additional high-frequency fluctuations due to the lumpy nature of vessel arrivals. In particular, even when the system operates in the non-whiplash regime, where the fluid backlog clears monotonically, discrete batching can still generate localized oscillations due to repeated backlog jumps. This distinction is especially relevant in the whiplash regime. When $C<\min\{K,\tau/2l+1\}$, the system already exhibits oscillatory dynamics at the fluid level, and discrete batching further amplifies these fluctuations by increasing peak congestion and tightening buffer requirements.

B. Data, Summary Statistics, and Prediction Results

In this section, we present a comprehensive overview of the dataset's summary statistics used in Section 6 to develop the two-phase prediction model. We also present the predictions from our two-phase prediction model.

Table B.1 Summary statistics of the categorical variables of the Shanghai Pudong International Container Terminal

	Percentage
Load Condition At Arrival	47.50% "Partially Laden", 33.75% "In Ballast", 18.75% "Laden"
Vessel Name	3.75% "ACACIA ARIES", 3.75% "CONSISTENCE ", the rest are other vessels
Destination Port	17.50% "NINGBO", 11.25% "OSAKA", other vessels arrived at other ports
Voyage Origin Port	13.75% "NINGBO", 6.25% "QINGDAO", 6.25% "KOBE", other vessels came from other ports
Port Operation	71.25% "LOAD", 28.75% "DISCHARGE"

Table B.2 Descriptions of the variables in the two data sets used in Section 6

Data set	Variable	Description of the variable
Port congestion	Port	The name of the port
	Week	The number of weeks counting from January 1, 2022
	Med Wait Off Limits	Median waiting time of arrived ships off port limits
Berth call	Current Port Ata	Actual time of arrival of the ship at the current port
	Model Pred Outbound*	The predicted outbound backlog level generated using our model
	Vessel Name	Name of the vessel
	Berth Name	Name of the berth the vessel docked at
	Destination Port	Name of the vessel's destination port of its current voyage
	Load Condition At Arrival	The loading condition of the vessel when it arrives at the port
	Voyage Origin Port	Name of the port from which the voyage originated
	Origin Port Country	Name of the country from which the voyage originated
	Port Operation	The operations that the port took
	Time At Port**	Total time the vessel spent at the current port

*This variable is generated from the "Port congestion" dataset using our model.

**This is the target variable.

Table B.3 Summary statistics of the continuous variables of the Shanghai Pudong International Container Terminal

	Model Pred Outbound	Time At Port
count	80	80
mean	1.167	13.634
std	0.239	5.479
min	0.741	2.600
25%	0.979	10.175
50%	1.113	12.750
75%	1.401	16.300
max	1.566	38.600

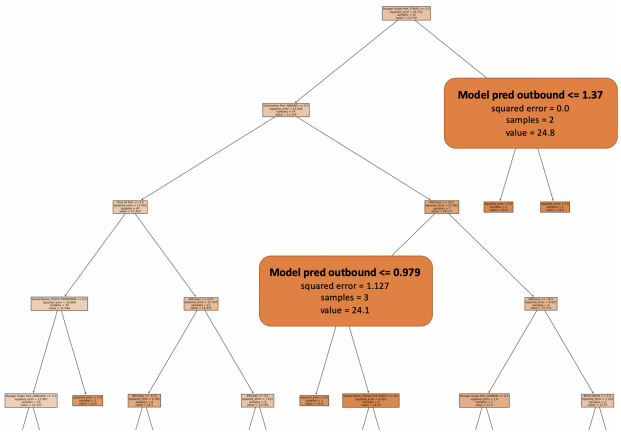
Table B.4 Summary statistics of the categorical variables of the SCT Jungonglu Terminal

	Percentage
Load Condition At Arrival	85.71% "Laden", 8.57% "Partially Laden", 5.71% "In Ballast"
Vessel Name	11.43% "XIN LONG YUN 66", 11.43% "JI FA BEI HAI", the rest are other vessels
Destination Port	22.86% "DALIAN", 20.00% "BAYUQUAN", other vessels arrived at other ports
Voyage Origin Port	28.57% "BAYUQUAN", 17.14% "DALIAN", other vessels came from other ports
Port Operation	88.57% "DISCHARGE", 11.43% "LOAD"

Table B.5 Summary statistics of the continuous variables of the SCT Jungonglu Terminal

	Model Pred Outbound	Time At Port
count	35	35
mean	1.158	16.769
std	0.251	5.247
min	0.700	8.900
25%	1.009	13.050
50%	1.133	14.200
75%	1.422	20.750
max	1.525	28.400

Figure B.1 The first five levels of the trained decision tree from the training set of Shanghai Pudong International Container Terminal



Notes: The "Model pred outbound" variable denotes the outbound backlog predictor generated by our model.

C. Simulation Comparing Shared-Capacity System with Effective-Capacity System

To validate the effective-capacity representation, we compare our analytical model with a more detailed system in which inbound and outbound processes share a common pool of resources.

Simulation setup. We consider a system with two types of jobs, inbound backlog-clearing and outbound backlog-clearing jobs, that draw from a shared resource pool. Each process has a baseline service rate λ under normal operating conditions. When backlogs are present, the system operates at an elevated capacity level, and we use $K \geq 1$ to capture the extent to which processing intensity can be increased under congestion.

A key difference between the shared-capacity system and our reduced-form model lies in how this capacity is utilized. In the shared-capacity system, the total capacity is allocated endogenously between inbound and outbound processes based on their relative backlog levels. Let B_t^{in} and B_t^{out} denote the respective backlogs at time t. The fraction of capacity allocated to inbound processing

is given by

$$\alpha_t = \begin{cases} \frac{B_t^{\text{in}}}{B_t^{\text{in}} + B_t^{\text{out}}}, & \text{if } B_t^{\text{in}} + B_t^{\text{out}} > 0, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

The resulting service rates are therefore

$$\mu_t^{\text{in}} = \alpha_t \cdot 2K\lambda, \tag{C.1}$$

$$\mu_t^{\text{out}} = (1 - \alpha_t) \cdot 2K\lambda. \tag{C.2}$$

Thus, the service rate allocated to each process varies with the relative backlog sizes, reflecting competition for shared resources.

In contrast, our effective-capacity model abstracts from such state-dependent allocation. Instead, each process operates with a fixed congestion-dependent service rate. Specifically, once a backlog is present, the service rates are given by

$$\tilde{\mu}_t^{\text{in}} = K\lambda, \tag{C.3}$$

$$\tilde{\mu}_t^{\text{out}} = K\lambda, \tag{C.4}$$

independent of the relative magnitudes of inbound and outbound backlogs. In this sense, the reduced-form model replaces endogenous capacity allocation with a symmetric and constant allocation across flows.

Dynamics. The backlog evolution in the shared-capacity system follows

$$B_{t+1}^{\text{in}} = \left[B_t^{\text{in}} + A_t^{\text{in}} - \mu_t^{\text{in}}\right]^+, \tag{C.5}$$

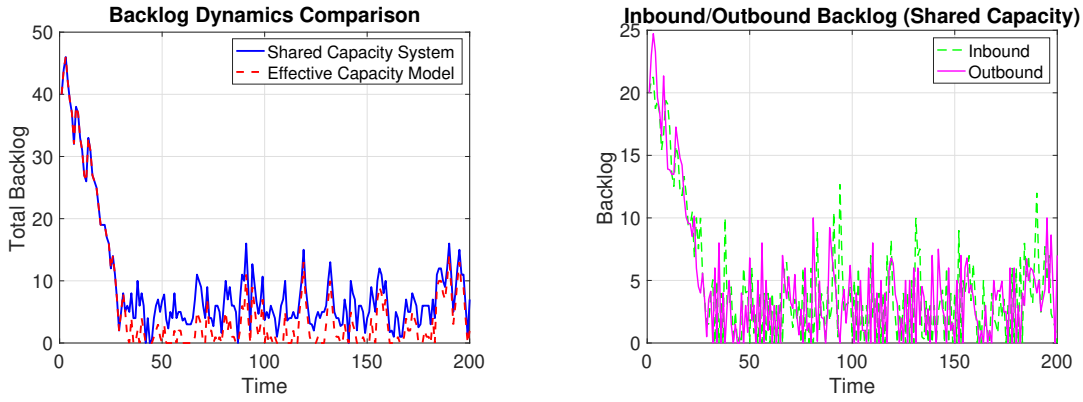
$$B_{t+1}^{\text{out}} = \left[B_t^{\text{out}} + A_t^{\text{out}} - \mu_t^{\text{out}}\right]^+, \tag{C.6}$$

where A_t^{in} and A_t^{out} denote exogenous arrivals.

This comparison highlights the key modeling simplification: the shared-capacity system features endogenous, state-dependent allocation of total capacity across flows, whereas the reduced-form model captures the aggregate effect of congestion through a fixed capacity level $K\lambda$ for each process.

Simulation Results. We implement the simulation in MATLAB to compare the dynamics of the shared-capacity system with those of the effective-capacity model. We simulate both systems under identical arrival processes and initial conditions, and track the evolution of inbound and outbound backlogs over time. Figure C.1 reports the resulting trajectories. The left panel compares the aggregate backlog dynamics across the two systems, while the right panel illustrates the evolution of inbound and outbound backlogs in the shared-capacity system, highlighting the endogenous fluctuations induced by resource competition. These comparisons allow us to assess how well the reduced-form model approximates the behavior of a system with shared resources and to illustrate the underlying mechanism driving the observed dynamics.

Figure C.1 Simulation results comparing a shared-capacity system with the effective-capacity approximation.



(a) Aggregate backlog dynamics: shared-capacity system vs. effective-capacity approximation

(b) Inbound and outbound backlog dynamics under shared capacity

D. Multiple Disruptions

We enrich the baseline scenario by allowing Port i to experience two temporary closures rather than a single one. Throughout, we benchmark against the setting illustrated in Figure 3(c) where $K = 1.5$, $C = 1.2$, $\tau = 5$, $\lambda = 10$, and $l = 6$.

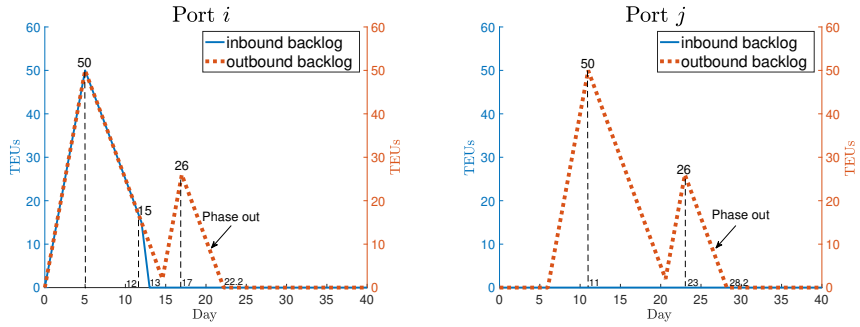
Now we assume that the first disruption begins at $t = 0$ and lasts for $\tau_1 = 5$ days. Port i then reopens, but partway through the recovery at $t = 10$ days, it is hit by a second closure of length $\tau_2 = 3$ days. Operational rules during both recovery windows remain identical to those in the base model: whenever either port faces a backlog, each works at its maximum processing rate $K\lambda$, and every vessel sails at its maximal loading capacity $C\lambda$ until the system fully stabilizes (i.e., until Port j has eliminated all outbound backlogs). Thus, we can still use the fluid approximation to study this new stochastic system, and we present the backlog levels of both Ports i and j in Figure D.1. Specifically, Figure D.1 plots the resulting inbound and outbound backlog trajectories at both ports, illustrating how the second shock re-initiates whiplash dynamics before the system finally converges to steady state.

PROPOSITION D.1 **(Phase-shift invariance of the outbound backlog)**. *Assume that Port i undergoes two non-overlapping closures on the intervals $[t_1, t_1 + \tau_1)$ and $[t_2, t_2 + \tau_2)$, with the second closure starting within the window $t_2 \in \big(t_1 + \tau_1, t_1 + \tau_1 + 2l(C/K)\big)$. A comparison of the outbound-backlog clearing process between a single disruption and a two-sequential disruption yields three key observations:*

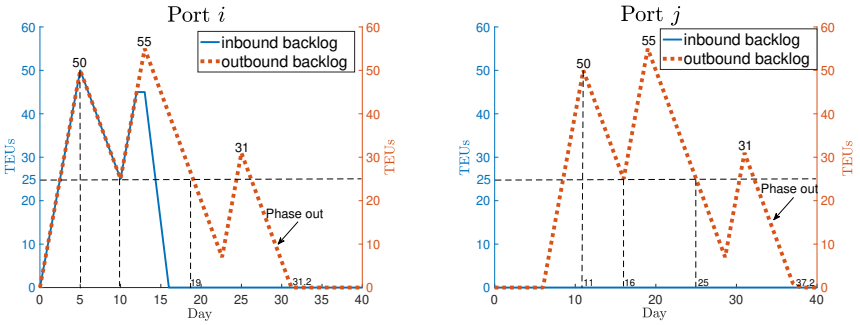
(i) The second disruption introduces exactly one additional oscillation phase (a down-then-up cycle).

(ii) Following the second disruption at Port i, the recovery dynamics mirror those of the single-

Figure D.1 Comparison of the backlog levels at Ports i and j when there is only one disruption and two disruptions



(a) The backlog levels of Ports i and j when there is only one disruption on $t = 0$ and lasts for $\tau_1 = 5$ days



(b) The backlog levels of Ports i and j when there are two disruptions with second disruption is on $t = 10$ and lasts for $\tau_2 = 3$ days

disruption case: each backlog reduction phase (down period) lasts $2l(C/K)$ days, while each backlog accumulation phase (up period) lasts $2l(1 - C/K)$ days (except the phase out period).

(iii) The entire oscillation that follows the second disruption, including all subsequent breakpoints but except the final phase out period, is merely translated rightward by $(t_2 + \tau_2) - (t_1 + \tau_1)$ days.

Hence, an additional disruption inserts one extra oscillation phase without distorting the temporal geometry of the existing phases; it merely delays their occurrence by a constant amount.

To better illustrate Proposition D.1, Figure D.1 presents the visual comparison of the inbound and outbound backlog clearing process for Port i (left column panels) and Port j (right column panels) under single and multiple disruptions. The top row (Panel (a)) details the impact of a single disruption, while the bottom row (Panel (b)) shows the effects of two sequential disruptions. This analysis uses parameters in which the first disruption begins at $t_1 = 0$ with $\tau_1 = 5$ days, and the second occurs at $t_2 = 10$ with $\tau_2 = 3$ days.

During the initial closure window $(0 \leq t \leq 5)$ at Port i, the outbound backlog in both settings accumulates linearly at the outbound goods inflow rate λ, reaching an identical peak of 50 TEUs at

$t = 5$. Upon Port i's reopening following the first disruption, the single-disruption trajectory shows backlog clearance at the maximal goods-unloading rate $K\lambda$ until $t = 14.6$. At this point, vessel availability temporarily constrains further outbound backlog clearance. The port then temporarily "runs out" of available vessels, and no further departures are possible until the next waves of inbound ships arrive. As a result, due to the cessation of vessel arrivals during the remaining sailing time, Port i's backlog increases from $t = 14.6$ to $t = 17$.

Comparatively, in the scenario with two sequential disruptions, a second three-day closure ($10 \leq t \leq 13$) follows the initial recovery. During this second closure, Port i's outbound backlog again increases at rate λ, achieving a secondary peak of 55 TEUs. After Port i reopens from this second disruption (at $t = 13$), its backlog clearance exhibits a "down" period lasting 9.6 days (from $t = 13$ to $t = 22.6$), followed by an "up" period of 2.4 days (from $t = 22.6$ to $t = 25$), before the final phase-out period. These durations precisely match those of the single-disruption scenario. The resultant oscillation is therefore an exact replication of the down-up cycle from the single-disruption case, shifted temporally by $\Delta = (t_2 + \tau_2) - (t_1 + \tau_1) = 8$ days. The corresponding dynamics for Port j (right-hand panels) exhibit the same phase-shifted replication, thereby confirming that an additional disruption introduces a self-contained oscillation without altering the intrinsic timing of the constituent down and up periods.

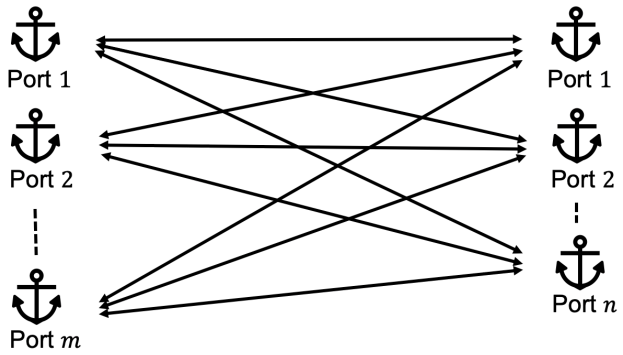
Finally, Figure D.1 illustrates how a second closure reshapes the inbound backlog dynamics at Port i. Figure D.1(a) considers a single disruption at Port i over the interval $t \in [0, 5]$. During this period, inbound backlogs accumulate at rate $\lambda = 10$, reaching a level of $10 \times 5 = 50$ at $t = 5$. Once the port reopens at $t = 5$, it begins clearing the accumulated backlog at a rate $K\lambda = 15$. However, over the interval $t \in [5, 2l] = [5, 12]$, inbound shipments from Port j continue to arrive at rate $\lambda = 10$. As a result, the net backlog reduction rate is $K\lambda - \lambda = 5$. Therefore, the backlog level at $t = 12$ is $50 - 5(12 - 5) = 15$. For $t \geq 12$, no further arrivals occur at Port i, and the backlog is cleared at the full rate $K\lambda = 15$. Therefore, all inbound backlogs are fully cleared by $t = 13$.

Figure D.1(b) shows that when a second closure intervenes at $t_2 = 10$, this pattern breaks. From $t = 10$ to $t = 12$, no cargo can be unloaded since Port i is closed, so the inbound backlog again grows linearly at rate λ. On the final day of the closure ($t = 12$-13), since no vessels arrive, the inbound backlog remains flat. Port i reopens at $t = 13$ and, with no additional ship arrivals, empties the inbound backlogs queue at rate $K\lambda$, eliminating the entire backlog by $t = 16$. Thus, although the second disruption does not alter the service slope ($-K\lambda$) or the growth slope ($+\lambda$), it inserts an extra accumulation plateau followed by a clearing interval, mirroring the phase-shift phenomenon documented for the outbound queue in Proposition D.1.

E. Bipartite Network Systems

In this section, we study three types of bipartite network systems: m-to-1, 1-to-n, and m-to-n. Each system consists of two disjoint sets of ports, denoted by sides $\mathcal{I}$ and $\mathcal{J}$. Note that the two-port system in our base model corresponds to the special case where $m=1$ and $n=1$. For each pair of Ports $i \in \mathcal{I}$ and $j \in \mathcal{J}$, we assume that the daily volume of cargo to be shipped from Port $i \in \mathcal{I}$ to Port $j \in \mathcal{J}$ is the same as the daily volume of cargo to be shipped from Port $j \in \mathcal{J}$ to Port $i \in \mathcal{I}$, denoted by $\lambda_{i,j}$. The travel time between Ports $i \in \mathcal{I}$ and $j \in \mathcal{J}$ is $L_{i,j}$ days.

Figure E.1 Network structure of the multi-port system



In the following three subsections, we begin by analyzing two specific network systems: the m-to-1 and 1-to-n systems. The insights and results obtained from these two special cases serve as building blocks for our analysis of the more general m-to-n network system.

Without loss of generality, we assume that the disruption occurs at Port $i=1$ on side $\mathcal{I}$ and lasts for τ days in each network system. In the m-to-1 network system, the disrupted port is one of m ports on side $\mathcal{I}$, while there is only one port on side $\mathcal{J}$. Thus, the disrupted port is part of a set of ports on one side, each connected to a single port on the opposite side. In contrast, in the 1-to-n network system, the disrupted port is the only port on side $\mathcal{I}$, and it is directly connected to $n>1$ ports on side $\mathcal{J}$. Thus, in the 1-to-n network, the disrupted port on one side is uniquely linked to multiple ports on the other side.

We assume that the maximum port processing capacity for Port $i \in \mathcal{I}$ is K_i and Port $j \in \mathcal{J}$ is K^j for each port on side $\mathcal{I}$ and $\mathcal{J}$, respectively. In addition, we assume that ships on routes $\{(i,j),(j,i)\}$ between Ports $i \in \mathcal{I}$ and $j \in \mathcal{J}$ can increase their carrying capacity levels to $C_{i,j}$ times their carrying rate during the normal time whenever there are backlogs in the system need to be cleared.

E.1. m-to-1 Network

In this subsection, we analyze an m-to-1 network (i.e., $n=1$). Since Port $i=1$ accumulates both inbound and outbound backlogs due to its disruption, the maximum throughput rates for loading

and unloading operations at Port $i=1$ are $K_1\lambda_1$ between Ports $i=1$ and $j=1$ once Port $i=1$ reopens. Thus, $L_{1,1}$ days after the reopening of Port $i=1$, the goods arriving at Port $j=1$ from Port $i=1$ have increased to $K_1\lambda_1$ compared to the pre-disruption level λ_1, thus altering the composition of goods arriving at Port $j=1$ from each Port $i\in\mathcal{I}$. Due to the high centrality of Port $j=1$ in the m-to-1 network, Port $j=1$ must decide how to allocate its processing capacity when the total inbound goods volume exceeds its processing capacity. There are two kinds of capacity allocation policies that Port $j=1$ can choose from: committed and contingent. To capture the heterogeneity in shipping routes, we allow the travel time between each Port $i\in\mathcal{I}$ and $j=1$ to vary, i.e., $L_{1,1}\neq L_{2,1}$. This general modeling framework accommodates differences in geographic distance, sea conditions, and route-specific operational constraints, thereby enhancing the realism and applicability of our analysis.

We describe the *committed* capacity allocation policy at Port $j=1$ in plain words as follows. Under normal conditions, Port $j=1$ dedicates a fixed loading and unloading capacity of λ_i for each shipping route connecting itself with Port $i\in\mathcal{I}$. After a disruption, Port $j=1$ proportionately increases its loading and unloading capacities for all routes between itself and each Port i on side $\mathcal{I}$. Specifically, Port $j=1$ commits to a maximum loading and unloading capacity of $K^1\lambda_i$ for every route linking Port $j=1$ with Port $i\in\mathcal{I}$. This policy guarantees that Port $j=1$ maintains a predetermined capacity level of loading and unloading for each route $(i,1),(1,i)$. However, this committed policy may lead to inefficient outcomes. Due to variability in travel times across routes, there are time periods in which no ships arrive at Port $j=1$ on certain routes, while other routes simultaneously experience a high influx of arriving ships. Despite this, Port $j=1$ continues to reserve its unloading capacity for routes with no ship arrivals, resulting in inefficient utilization and a waste of a portion of its unloading capacity, although the committed policy is more effective in minimizing inbound backlogs.

The *contingent* capacity allocation policy at Port $j=1$ works as follows. After Port $i=1$'s disruption, Port $j=1$ shifts from a fixed proportion of loading and unloading processing capacity for each route to a dynamic allocation policy. That is, on the one hand, Port $j=1$ first allocates its unloading processing capacity, proportional to the current inbound backlog levels for each shipping route between it and each Port $i\in\mathcal{I}$. On the other hand, Port $j=1$ then allocates its loading capacity proportional to its current outbound backlog levels for each shipping route between Ports $j=1$ and $i\in\mathcal{I}$.

We formalize the committed and contingent capacity allocation policy mathematically as follows. Under both capacity allocation policies, for each route between Ports $j=1$ and $i\in\mathcal{I}$, the allocated loading and unloading capacity is fixed at $\mu_i^{committed}(t)=\lambda_i$ under normal conditions. After the disruption, under the committed capacity allocation policy, Port $j=1$ scales all route capacities

proportionally by a factor $K^1 \geq 1$ to $\mu_i^{committed}(t) = K^1 \lambda_i$. In contrast, under the contingent capacity allocation policy, let $Q_i^{in}(t)$ (resp., $Q_i^{out}(t)$) denote the amount of inbound (resp., outbound) backlogs at Port $i \in \mathcal{I}$. And let $\mu_i^{unload}(t)$ (resp., $\mu_i^{load}(t)$) denote the unloading (resp., loading) capacity Port $j = 1$ allocates to the shipping route $(i,1),(1,i)$ under the contingent allocation policy. We have $\mu_i^{unload}(t) = Q_i^{in}(t) / \sum_k Q_k^{in}(t) \cdot K^1 \sum_{n=1}^m \lambda_n$ and $\mu_i^{load}(t) = Q_i^{out}(t) / \sum_k Q_k^{out}(t) \cdot K^1 \sum_{n=1}^m \lambda_n$.

The contingent capacity allocation policy allows Port $j = 1$ to allocate its processing capacity contingently and dynamically, adapting to the current backlog levels across routes. Nevertheless, a drawback of this approach is that certain routes may experience an accumulation of a large number of ships. In such scenarios, Port $j = 1$ allocates a greater-than-usual capacity to the congested routes, thereby reducing the capacity available to other routes. Under the committed policy, inbound backlogs on less-congested routes would typically be cleared. However, because capacity is diverted to accommodate more congested routes, the efficiency of clearing inbound backlogs on less congested routes is adversely affected.

In the following proposition, we analyze the inbound and outbound backlogs in this m-to-1 network under those two capacity allocation policies.

Proposition E.1 (m-to-1 Network: Committed and Contingent Policies). *In an m-to-1 network system in which a disruption occurs at Port $i = 1$, we denote by $\rho \equiv (K_1 \lambda_1 + \sum_{i=2}^m \lambda_i)/(K^1 \sum_{i=1}^m \lambda_i)$ the implied utilization of Port $j = 1$.*

(i) Using the committed and contingent capacity allocation policies leads to the following shared properties: (a) Port $i = 1$ has inbound and outbound backlogs; (b) Port $i = 2, \ldots, m$ never has any inbound backlogs; (c) Port $j = 1$ has outbound backlogs.

(ii) Using a different capacity allocation policy leads to the following different properties: (a) Port $i = 2, \ldots, m$ never has any outbound backlogs under the committed policy. However, Port $i = 2, \ldots, m$ has outbound backlogs if and only if $C_i/\rho < 1$ under the contingent policy; (b) Port $j = 1$ has inbound backlogs if and only if $K_1 > K^1$ under the committed policy. However, Port $j = 1$ has inbound backlogs if and only if $\rho > 1$ under the contingent policy.

Proposition E.1 analyzes the effects of a single port disruption in an m-to-1 network system. Part (i)-(a) shows that the disrupted port, Port $i = 1$, always experiences both inbound and outbound backlogs due to its inability to load or unload cargo during the disruption. This mirrors the behavior in the 1-to-n network and the two-port case. Proposition E.1(i)-(b) states that for those ports on the same side as the disrupted port that are not directly connected to the disrupted port, they will never have any inbound backlogs. This is because after Port $i = 1$ reopens, the throughput rate for routes $\{(1,1),(1,1)\}$ is larger than before the disruption, and the capacity at Port $j = 1$ allocated to other routes will only be reduced (under the contingent policy) or remain unchanged (under the committed policy), resulting in a non-increase rather than a strict increase compared

to the original inbound goods arrival rate. Proposition E.1(i)-(c) shows that the only port at the opposite side of the disrupted port, i.e., Port $j=1$, always has outbound backlogs, consistent with the 1-to-n network system (with the two-port system as a special case). This is because no ships are arriving at Port $j=1$ during Port $i=1$'s disruption for routes $\{(1,1),(1,1)\}$, thus leading to an accumulation of outbound goods at Port $j=1$ with the destination of Port $i=1$.

Proposition E.1(ii) compares the system under committed and contingent allocation policies. Proposition E.1(ii)-(a) shows that, under the committed policy, each shipping route operates independently. Ports $i=2,\ldots,m$ are unaffected by Port $i=1$'s disruption, since Port $j=1$ maintains fixed processing capacities for all routes. The disruption only affects the $(1,1)$ route. In contrast, under the contingent policy, Port $j=1$ dynamically reallocates its processing capacity. As a result, Ports $i=2,\ldots,m$ may experience outbound backlogs if the capacity allocated to their routes is insufficient. This occurs when the total inbound arrival rate at Port $j=1$ exceeds its processing capacity, i.e., when the implied utilization $\rho=\frac{K_1\lambda_1+\sum_{i=2}^m\lambda_i}{K^1\sum_{i=1}^m\lambda_i}>1$. In this case, Port $j=1$ allocates a fraction $\lambda_i/(K_1\lambda_1+\sum_{i=2}^m\lambda_i)$ of its total processing capacity $K^1\sum_{i=1}^m\lambda_i$ to route $(1,i)$. Thus, ships on route $(1,i)$ can carry at most $C_i\cdot\frac{\lambda_i}{K_1\lambda_1+\sum_{i=2}^m\lambda_i}\cdot K^1\sum_{i=1}^m\lambda_i$ from Port $j=1$ to Port i using the ships' maximum carrying capacity. Port i avoids outbound backlogs if and only if this quantity exceeds its outbound arrival rate λ_i. Finally, Part (ii)-(b) describes the conditions under which Port $j=1$ experiences inbound backlogs. Under the committed policy, this occurs only if the processing capacity K^1 is less than that of Port $i=1$, resulting in an unbalanced two-port system. Under the contingent policy, inbound backlogs occur at Port $j=1$ when its implied utilization exceeds 1, i.e., the total inbound load exceeds its processing capacity.

E.2. 1-to-n Network

In this subsection, we analyze a 1-to-n network system (i.e., $m=1$). Similar to E.1, there are two distinct port processing capacity allocation policies that Port $i=1$ can adopt: the committed vs. contingent capacity allocation policies. In the following proposition, we show that the system has the same properties regardless of whether the committed or contingent capacity allocation policy is adopted at Port $i=1$.

Lemma E.1 (1-to-n Network System: Decoupling Under the Committed Policy). *The 1-to-n network system in which a disruption occurs at Port $i=1$ can be decoupled into n general two-port systems with different capacity levels if Port $i=1$ adopts the committed capacity allocation policy.*

Lemma E.1 illustrates a way to analyze the 1-to-n network when Port $i=1$ reserves a fixed loading and unloading capacity for each shipping route between Port $i=1$ and Port $j\in\mathcal{J}$. Specifically, for each Port j on side $\mathcal{J}$, we decompose the 1-to-n network system into a number of n separate two-port systems. Hence, any insights or results obtained from analyzing the base two-port model

can be applied to the 1-to-n network system.

Proposition E.2 (1-to-n Network: Committed and Contingent Policies). *In a 1-to-n network system in which a disruption occurs at Port $i=1$ with different $L_{i,j}$ for $i \in \mathcal{I}$ and $j \in \mathcal{J}$, we have:*

(i) Using the committed and contingent capacity allocation policies leads to the following shared properties: (a) Port $i=1$ always has inbound backlogs; (b) All ports in the system always have outbound backlogs.

(ii) Using the committed or contingent capacity allocation policy leads to the following different properties: (a) The number of clearance days of outbound backlogs for every port is shorter when the contingent policy is adopted than when the committed policy is adopted. (b) The number of clearance days of inbound backlogs for every port is shorter when the committed policy is adopted than when the contingent policy is adopted.

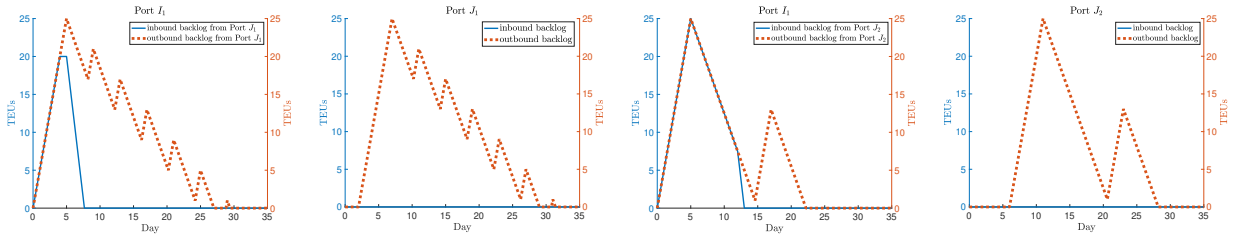
We first show that both committed and contingent policies may result in the same dynamics in the 1-to-n network. Specifically, Proposition E.2(i)-(a) states that Port $i=1$, being the only port on side $\mathcal{I}$, always experiences both inbound and outbound backlogs due to its complete shutdown during the disruption. Proposition E.2(i)-(b) shows that all ports on side $\mathcal{J}$ inevitably experience outbound backlogs. This result arises from the central role of Port $i=1$: Port $i=1$'s disruption halts all incoming shipments to ports in $\mathcal{J}$, leaving them without arriving ships to load outbound cargo between $t=l$ and $t=l+\tau$.

Next, we primarily examine the distinct system dynamics when the disrupted port implements both policies, with different travel times across routes. Specifically, we demonstrate that adopting the committed vs. contingent policy leads to different durations of the clearing process. Proposition E.2(ii)-(a) demonstrates that, as illustrated in Figure E.2, regarding the loading capacity allocation, the adoption of the contingent allocation policy at Port $i=1$ facilitates a faster clearance of outbound backlogs in comparison to employing the committed allocation policy. This is attributed to the more efficient use of loading capacity under the contingent capacity allocation policy, in contrast to reserving the current available capacity for future ships under the committed policy. Nonetheless, this efficiency comes at the cost of creating inbound backlogs at ports that originally had none under the committed policy. This is because using the contingent policy may result in an increase in the throughput rate for a certain route $\{(1,j),(j,1)\}$, exceeding the processing capacity that Port j can handle, thus leading to an accumulation of inbound goods at Port j.

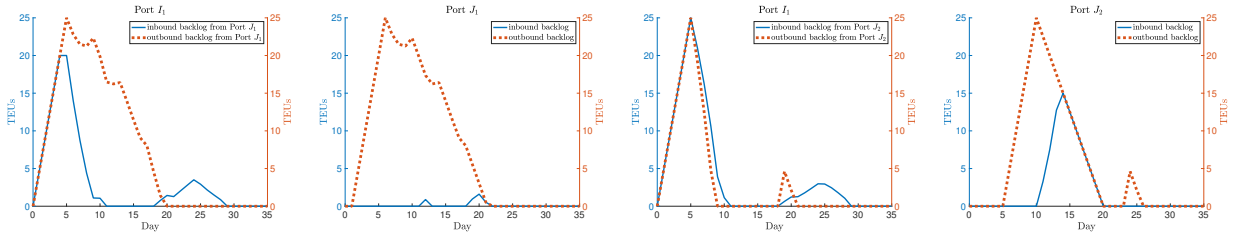
Regarding the unloading capacity allocation, Proposition E.2(ii)-(b) shows that opting for the contingent allocation policy results in slower clearance of inbound backlogs compared to the committed policy. Moreover, for Port $j \in \mathcal{J}$ with the processing capacity level $K^j \geq K_1$, it will never

have any inbound backlogs under the committed policy, but it may have inbound backlogs under the contingent policy. This is because when Port $i = 1$ adopts the contingent capacity allocation policy, it may allocate processing capacity exceeding $K_1\lambda_j$ (the maximum capacity under the committed policy) to shipping routes $\{(1, j), (j, 1)\}$ if these routes have larger proportions of outbound backlogs, exceeds the maximum amount of inbound goods Port j can handle which is $K^j\lambda_j$. Therefore, as the contingent capacity allocation policy does not guarantee a fixed maximum capacity allocated to each route, when the routes $\{(1, j), (j, 1)\}$ have relatively large throughput rates, Port j may experience inbound backlogs under this policy, which would not occur under the committed allocation policy which ensures a fixed maximum capacity allocated to each route.

Figure E.2 **Inbound and outbound backlogs in a 1-to-2 network at Ports $i = 1$ and $j = 1$ under committed and contingent capacity allocation policies when $K_1 = K^1 = K^2 = 1.5$, $C = 1.2$, $\lambda = 5$, $\tau = 5$, $L_{1,1} = 2$, and $L_{1,2} = 6$**



(a) committed policy



(b) contingent policy

E.3. m-to-n Network System

In this subsection, we analyze the general m-to-n network system and show that its properties are a combination of those in the 1-to-n and m-to-1 network systems. Given that all ports on side $\mathcal{J}$ are directly connected to the disrupted port, the composition of goods arriving at Port $j \in \mathcal{J}$ changes after the reopening of Port $i = 1$, compared to the normal time, because Port $i = 1$ increases its capacity to clear the outbound backlogs. Analogous to the 1-to-n and m-to-1 network systems, we examine two types of capacity allocation policies that each Port $j \in \mathcal{J}$ can adopt: committed vs. contingent capacity allocation policies. In the following proposition, we analyze the inbound and

outbound backlogs under these two capacity allocation policies within the m-to-n network system.

PROPOSITION E.3 (m-TO-n NETWORK: COMMITTED AND CONTINGENT POLICIES). *In an m-to-n network system in which a disruption occurs at Port $i=1$, we denote by $\rho_j \equiv (K_1\lambda_{1,j}+\sum_{i=2}^{m}\lambda_{i,j})/(K^j\sum_{i=1}^{m}\lambda_{i,j})$ the maximum implied utilization of Port $j\in\mathcal{J}$.*

(i) Using the committed and contingent capacity allocation policies leads to the following shared properties: (a) Port $i=1$ always has inbound and outbound backlogs; (b) Port $i=2,\ldots,m$ never has any inbound backlogs; (c) Port $j\in\mathcal{J}$ always has outbound backlogs.

(ii) Using the committed or contingent capacity allocation policy leads to the following different properties: (a) Port $i=2,\ldots,m$ never has any outbound backlogs under the committed policy. However, Port $i=2,\ldots,m$ has outbound backlogs if and only if $\min_{j\in\{1,\ldots,n\}}\{C_{i,j}/\rho_j\}<1$ under the contingent policy; (b) Port $j\in\mathcal{J}$ has inbound backlogs if and only if $K_1>K^j$ under the committed policy. However, Port $j\in\mathcal{J}$ has inbound backlogs if and only if $\rho_j>1$ under the contingent policy.

Proposition E.3 provides a comprehensive characterization of the existence of inbound and outbound backlogs within a generalized m-to-n bipartite network system. Proposition E.3(i) shows the common system properties that exist in the m-to-n network regardless of which allocation policy is used by Port $i=1$. Specifically, Proposition E.3(i)-(a) establishes that the disrupted port itself invariably experiences both inbound and outbound backlogs, which aligns with both the 1-to-n and m-to-1 network systems. Proposition E.3(i)-(b) shows that ports located on the same side of the disrupted port never have any inbound backlogs, which aligns with the m-to-1 network. Furthermore, Proposition E.3(ii)-(c) demonstrates that ports located on the opposite side of the disrupted port consistently encounter outbound backlogs, which aligns with the 1-to-n network.

Proposition E.3(ii) compares the different system properties that arise when adopting committed vs. contingent allocation policies. Similar to the m-to-1 network system, Proposition E.3(ii)-(a) states that under the committed policy, those ports on the same side that are not directly connected to the disrupted port will never experience outbound backlogs. However, the conditions under which these ports encounter outbound backlogs under the contingent policy differ between the m-to-n and m-to-1 networks. In the m-to-1 network, one only needs to compare the ship carrying capacity level on routes $\{(i,1),(1,i)\}$ with the maximum implied utilization of Port $j=1$. However, in the m-to-n network, if the maximum implied utilization of any Port $j\in\mathcal{J}$ is larger than the ship carrying capacity level on routes $\{(i,j),(j,i)\}$, Port i will experience outbound backlogs. This result in the m-to-n network can be viewed as a generalization of the m-to-1 network.

Proposition E.3(ii)-(b) shows the conditions to determine when Port $j\in\mathcal{J}$ will experience inbound backlogs. Under the committed policy, each pair of Ports $i\in\mathcal{I}$ and $j\in\mathcal{J}$ forms a generalized two-port system with distinct processing-capacity levels. Thus, whether Port $j\in\mathcal{J}$ will

have inbound backlogs depends on the comparison between the processing capacity levels of Ports $j \in \mathcal{J}$ and $i = 1$. Under the contingent policy, akin to the 1-to-n network, Port $j \in \mathcal{J}$ will encounter inbound backlogs if and only if the total volume of inbound goods arriving at Port $j \in \mathcal{J}$ after the reopening (i.e., $K_1\lambda_{1,j} + \sum_{i=2}^{m} \lambda_{i,j}$) exceeds its aggregate processing capacity levels for all routes after the reopening (i.e., $K^j \sum_{i=1}^{m} \lambda_{i,j}$).

E.4. Persistence of the whiplash Effect in Bipartite Networks

We conclude our analysis of bipartite network systems by highlighting a key structural insight: whiplash effects persist locally along edges of the bipartite graph, even if they are not always observable at the global network level. Specifically, in a 1-to-n bipartite network in which a single disrupted port connects to multiple counterparts, the system can be decomposed into n independent two-port subsystems. Each such subsystem inherits the oscillatory recovery dynamics from the base two-port model: the whiplash effect arises whenever the ship capacity satisfies $C < \min\{K, \tau/2L + 1\}$. Thus, the core logic of the two-port model generalizes naturally and remains valid under this network decomposition. For more complex structures, such as the general m-to-n network system, this effect may propagate and interact across connected ports, depending on the travel times, coupling policies, and asymmetries in port and ship capacities. This structural decomposition reinforces the robustness of the whiplash mechanisms and provides a principled means to anticipate local oscillatory behavior in larger, realistic maritime logistics networks.

F. Cyclic System

Building on our previous discussion, where we first examined a two-port circulatory system and then extended the analysis to a bipartite network, we now consider another extension in which the shipping route may involve multiple ports rather than a simple point-to-point service. In practice, ocean liner operations may follow intra-regional cyclic service routes, involving scheduled rotations among multiple ports of call. To account for such operational realities, in addition to the bipartite network system discussed in Online Appendix E, we extend our analysis of the two-port model to a configuration that could be observed in real practices: an intra-regional shipping system. Specifically, we study a three-port cyclic system. And without loss of generality, we assume that this three-port cyclic system comprises the Port of Shanghai in China, the Port of Long Beach, and the Port of Oakland in the United States. We further assume that all cargo movements between Shanghai and Oakland must pass through the Port of Long Beach, which serves as a central transit hub.

In this system, outbound vessels departing from the Port of Shanghai (resp., the Port of Oakland) offload half of their cargo at the Port of Long Beach and the remaining half at the Port of Oakland (resp., the Port of Shanghai). This symmetric routing structure mirrors operational practices in liner shipping and facilitates analytical tractability. We now examine the system's response to a

temporary disruption at the central node, the Port of Long Beach, with a focus on the network's backlog dynamics. Our analysis reveals several systemic behaviors that differ markedly from those observed in the two-port system.

To more accurately reflect routing behavior, we define the outbound direction as beginning at the Port of Shanghai, transiting through the Port of Long Beach, and terminating at the Port of Oakland. The return route then begins in Oakland, revisits Long Beach, and concludes in Shanghai. This structure allows generalization to longer cyclic routes, where additional intermediate stops can be incorporated without loss of analytical generality. Travel times are parameterized as follows: let L_1 denote the travel time from Shanghai to Long Beach, and L_2 the time from Long Beach to Oakland. Empirically, L_1 ranges from 12 to 18 days, with an average of 15 days, while L_2 spans approximately 21 to 28 hours, with an average of 1 day (24 hours). These values reflect typical vessel speeds and route conditions across the Pacific. On the demand side, we assume symmetric flow rates: ships will load the outbound goods with rate λ at the Port of Shanghai (resp., the Port of Oakland), where cargo is destined for Long Beach with rate $\lambda/2$ and the same rate $\lambda/2$ for Oakland (resp., for Shanghai). All ports are assumed to have identical maximum processing capacities, denoted by $K\lambda$, and each ship has a maximum carrying capacity of $C\lambda$.

To evaluate the effects of the disruption, we consider a scenario in which the Port of Long Beach is fully shut down for a period of τ days. During this period, all vessels bypass Long Beach and offload their entire outbound cargo, previously split between Long Beach and Oakland, exclusively at the Port of Oakland. As a result, the inbound cargo volume at Oakland doubles, reaching the rate λ. When $\lambda > K\lambda/2$, the arrival rate of inbound goods at the Port of Oakland exceeds its maximum unloading capacity, leading to the accumulation of inbound backlogs.

In addition to inbound backlogs, the disruption at Long Beach leads to a shortage of ships available for return voyages. Under normal operations, vessels arrive at Oakland half-loaded (carrying inbound goods with rate $\lambda/2$) and are reloaded to full capacity (with rate λ) for the return trip, with half of the outbound cargo destined for Long Beach and the other half for Shanghai. During the disruption, however, vessels arrive fully loaded (at rate λ) but cannot offload their entire cargo due to Oakland's processing constraint at rate $K\lambda/2$. As a result, cargo remains onboard, vessel turnaround times increase, and the effective supply of ships for outbound transport declines. This ship shortage propagates upstream, causing outbound backlogs at the Port of Shanghai, where cargo accumulates while waiting for delayed or unavailable vessels.

Using the same fluid-approximation approach applied to the two-port system, we analyze the behavior of a three-port cyclic network. Figure F.1 presents the evolution of inbound and outbound backlogs at each port under a disruption at the Port of Long Beach. From Figure F.1, we identify several noteworthy patterns that characterize the system's response when the intermediate port is

temporarily shut down. We formally summarize these properties for the three-port cyclic network below.

Proposition F.1 (Three-Port Cyclic Network). *Consider a three-port cyclic service operated under the same fluid approximation used for the two-port system. Suppose the intermediate port (e.g., Long Beach) is fully shut down for a finite interval, while the other two ports (e.g., Shanghai and Oakland) remain open. Then the system exhibits the following properties:*

(i) The two undisrupted ports have inbound backlogs when $K < 2$.

(ii) Outbound backlogs at the two undisrupted ports follow identical dynamics up to a constant time shift: one port's trajectory is a time-lagged version of the other's.

(iii) The disrupted port itself does not accumulate inbound or outbound backlogs during its disruption: no vessels arrive or depart, so no queue forms locally.

Proposition F.1 demonstrates key structural properties of an intra-regional cyclic system and outlines their associated policy implications.

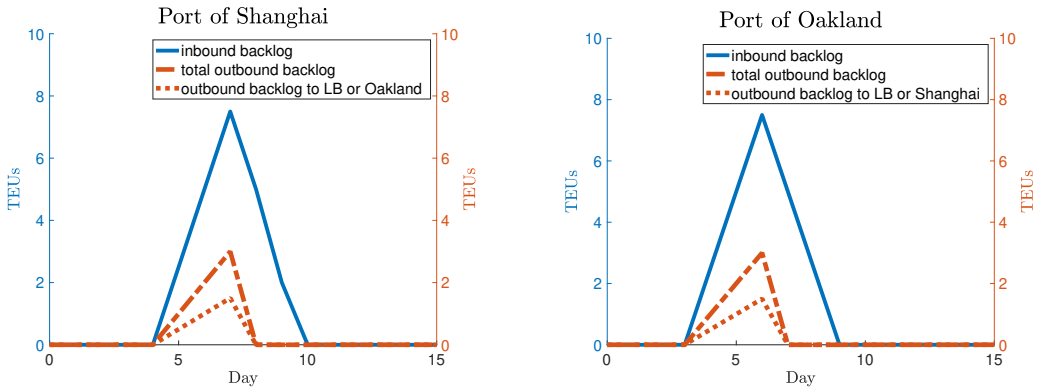
First, we observe that both the Port of Shanghai and the Port of Oakland accumulate inbound backlogs during the recovery period following the disruption at the Port of Long Beach. This stands in sharp contrast to the behavior of a two-port system, where inbound backlogs typically form only at the disrupted port, while the undisrupted port experiences none. The key structural difference lies in the network configuration. In the cyclic system, when the intermediate port is shut down, vessels bypass the disrupted node and unload their cargo at the next available port, either Shanghai or Oakland. As a result, the volume of inbound goods arriving at these end ports can exceed their standard capacity under certain circumstances. Once this volume exceeds the maximum unloading rate, inbound backlogs accumulate at these ports.

Second, we find that the outbound backlogs at the Port of Shanghai and the Port of Oakland exhibit nearly identical dynamics, offset by a consistent time lag. This observation is consistent with the lag behavior associated with the whiplash effect observed in the two-port system.

Third, the disrupted port (i.e., the Port of Long Beach) does not experience any backlog accumulation, either inbound or outbound. Since operations at Long Beach are fully suspended during the disruption period, no vessels arrive at or depart from the port. All maritime traffic bypasses the disrupted node, so no queue forms at that location. Moreover, in our cyclic network formulation, Long Beach is assumed not to have exogenous outbound demand (i.e., no new export cargo enters the system at this node). As a result, there is no inflow that could generate an outbound backlog while the port is closed. Instead, congestion manifests at the adjacent ports, where cargo handling continues. This outcome illustrates that when a central node in a cyclic system becomes completely non-operational, it does not serve as a point of congestion; rather, its functional absence redistributes pressure to other parts of the network.

These system properties demonstrate the importance of network topology in determining system resilience and congestion dynamics. Disruptions to a central node in a cyclic system cannot be fully understood through intuition developed from two-node models. Instead, the feedback and propagation effects across the network introduce qualitatively different behaviors that must be accounted for in operational planning and disruption management.

Figure F.1 **Inbound and outbound backlogs of the two undisrupted ports in the three-port cyclic system when** $L_1 = 12$, $L_2 = 1$, $\tau = 3$, $K = 1.5$, $C = 1.2$, **and** $\lambda = 10$



(a) Port of Shanghai (b) Port of Oakland

G. Proofs of Main Results

LEMMA G.1. *In a two-port system, when Port i is shut down from $t = 0$ to $t = \tau$, there will still be ships arriving at Port i during the period from $t = 0$ to $t = 2l$, but no ships will arrive at Port i during the period from $t = 2l$ to $t = 2l + \tau$, where l is the travel time between Port i and Port j.*

Proof of Lemma G.1. Since ships cannot depart from Port i during $t = 0$ to $t = \tau$, and it takes l (resp. l) days for ships to travel from Port i (resp. Port j) to Port j (resp. Port i), there will be no ships arriving from Port j during $t = l$ to $t = l + \tau$. As a result, no ships will depart from Port j during $t = l$ to $t = l + \tau$. Thus, no ships will arrive at Port i during $t = 2l$ to $t = 2l + \tau$. □

LEMMA G.2 (CONTINUITY OF SHIPS FLOW). *For any $i \in \mathbb{Z}^{0+}$, if during $t \in [\tau + i \cdot 2l, \tau + (i + 1) \cdot 2l]$, there are ships continuously leaving Port i, then during $t \in [\tau + (i + 1) \cdot 2l, \tau + (i + 2) \cdot 2l]$, there are ships continuously arriving at Port i.*

Proof of Lemma G.2. Since ships continuously leave Port i from $t = \tau + i \cdot 2l$ to $t = \tau + (i+1) \cdot 2l$, there are ships continuously arrive at Port j from $t = \tau + i \cdot 2l + l$ to $t = \tau + (i + 1) \cdot 2l + l$. Port j first unloads the goods from the arriving ships and then loads goods onto those ships. Thus, ships will continuously leave Port j from $t = \tau + i \cdot 2l + l$ to $t = \tau + (i+1) \cdot 2l + l$. After l days, these ships

arrive at Port i. Therefore, during $t \in [\tau + (i+1)\cdot 2l, \tau + (i+2)\cdot 2l]$, there are ships continuously arriving at Port i. $\square$

Proof of Proposition 1. First, we consider the scenario when $\tau > 2l$. Consider the period from $t=0$ to $t=\tau+2l$. By Lemma G.1, ships will only arrive at Port i during the period from $t=0$ to $t=2l$. Every day, the arriving ships can carry λ amount of goods in total at Port i. Since $\tau > 2l$, from $t=0$ to $t=\tau$, the total amount of goods waiting to be unloaded at Port i is $2l\lambda$. At $t=\tau$, Port i starts to operate again after the shutdown. It begins to clear the inbound backlogs at the rate $K\lambda$. The total time Port i needs to clear those inbound backlogs is $\frac{2l\lambda}{K\lambda} = \frac{2l}{K}$ days. As a result, the inbound backlogs will be cleared out at $t = \tau + \frac{2l}{K}$. Once the inbound backlogs at Port i have been emptied, there will be no more accumulation of inbound goods at Port i in the future. The reason is simple: the arriving goods are all from Port j, and Port j can only ship as many $K\lambda$ goods per day at most, which is within the processing capacity of Port i. Therefore, Port i needs $\tau + \frac{2l}{K}$ days to clear inbound backlogs. Because we assume $\tau > 2l$, we have $\frac{2l}{K} < \frac{\tau}{K-1}$ for all K. Thus, we prove that Port i needs $\tau + \min\{\frac{2l}{K}, \frac{\tau}{K-1}\}$ days to clear inbound backlogs when $\tau > 2l$.

Next, we consider the scenario when $\tau \le 2l$. Consider the period from $t=0$ to $t=\tau+2l$ again. By Lemma G.1, ships only arrive at Port i from $t=0$ to $t=2l$ during this period, and no ships are arriving at Port i from $t=2l$ to $t=\tau+2l$. Thus, the total amount of inbound backlogs at Port i is $2l\lambda$ during $t \in [0, \tau+2l]$. Starting from $t=\tau$, Port i begins to clear the inbound backlogs at the maximum processing rate $K\lambda$. If $K \ge \frac{2l}{2l-\tau}$, Port i can clear all the inbound backlogs before $t=2l$. Starting from $t=\tau$, the rate of arrival of goods is λ, and the rate at which Port i unloads the arrived goods is $K\lambda$. Thus, the inbound backlogs accumulated during the period from $t=0$ to $t=\tau$ are cleared at a rate of $(K-1)\lambda$. Therefore, Port i needs $\frac{\tau\lambda}{(K-1)\lambda} = \frac{\tau}{K-1}$ days to clear the inbound backlogs accumulated during Port i's shutdown. If $K < \frac{2l}{2l-\tau}$, Port i cannot clear all the inbound backlogs before $t=2l$. From $t=0$ to $t=2l$, the total amount of arrived goods at Port i is $2l\lambda$. The amount of inbound goods Port i can clear during $t=0$ to $t=2l$ is $(2l-\tau)K\lambda$. At $t=2l$, there are $\tau K\lambda - 2l\lambda(K-1)$ inbound backlogs at Port i. Starting from $t=2l$, no more goods are arriving at Port i. Thus, the inbound backlogs can be cleared at a rate of $K\lambda$. Therefore, starting from $t=2l$, Port i needs additional $\frac{\tau K\lambda - 2l\lambda(K-1)}{K\lambda} = \tau - 2l + \frac{2l}{K}$ days to clear all the inbound backlogs. Thus, the total days Port i needs to clear all inbound backlogs is $2l + \tau - 2l + \frac{2l}{K} = \tau + \frac{2l}{K}$. Because when $K \ge \frac{2l}{2l-\tau}$, we have $\frac{\tau}{K-1} \le \frac{2l}{K}$. When $K < \frac{2l}{2l-\tau}$, we have $\frac{2l}{K} < \frac{\tau}{K-1}$. Therefore, we prove that Port i needs $\min\{\frac{2l}{K}, \frac{\tau}{K-1}\}$ days to clear all the inbound backlogs when $\tau \le 2l$.

For Port j, since all arrived goods are loaded from Port i, the maximum arrival rate is $K\lambda$, which is within the processing capacity of Port j. Therefore, Port j can always handle incoming cargoes without accumulating inbound goods. $\square$

Proof of Proposition 2. We first prove that the cumulative amount of inbound backlogs at Port

i is $2\lambda l(\tau - l + \frac{l}{K})$ when $\tau < 2l$ and $K < \frac{2l}{2l-\tau}$.

Using the associated inbound backlog fluid process in Appendix J, when $\tau < 2l$ and $K < \frac{2l}{2l-\tau}$, the cumulative amount of inbound backlogs during $t \in [0, \tau]$ is

$$\int_0^\tau \bar{I}_i(t)dt = \frac{1}{2}\tau \cdot \lambda\tau = \frac{\lambda\tau^2}{2}.$$

During $t \in (\tau, 2l]$, the cumulative amount of inbound backlogs is

$$\int_\tau^{2l} \bar{I}_i(t)dt = \frac{1}{2}(2l-\tau) \cdot (\lambda\tau + \lambda\tau - (K-1)\lambda(2l-\tau)) = \frac{1}{2}(2l-\tau) \cdot (2\lambda\tau - (K-1)\lambda(2l-\tau)).$$

During $t \in (2l, \infty]$, the cumulative amount of inbound backlogs is

$$\int_{2l}^\infty \bar{I}_i(t)dt = \frac{1}{2}(\tau + \frac{2l}{K} - 2l) \cdot (\lambda\tau - (K-1)\lambda(2l-\tau)).$$

Thus, Port i's cumulative amount of inbound backlogs during $t \in [0, \infty)$ is

$$\begin{aligned}
&\int_0^\tau \bar{I}_i(t)dt + \int_\tau^{2l} \bar{I}_i(t)dt + \int_{2l}^\infty \bar{I}_i(t)dt \\
&= \frac{\lambda\tau^2}{2} + \frac{1}{2}(2l-\tau) \cdot (2\lambda\tau - (K-1)\lambda(2l-\tau)) + \frac{1}{2}(\tau + \frac{2l}{K} - 2l) \cdot (\lambda\tau - (K-1)\lambda(2l-\tau)) \\
&= \lambda\tau l + \frac{l}{K}(\lambda\tau - (K-1)\lambda(2l-\tau)) = 2\lambda\tau l + 2\lambda l^2(\frac{1}{K} - 1) = 2\lambda l(\tau - l + \frac{l}{K}).
\end{aligned}$$

Then we prove that the cumulative inbound backlogs at Port i is $\frac{K\lambda\tau^2}{2(K-1)}$, provided that $\tau < 2l$ and $K \geq \frac{2l}{2l-\tau}$, throughout the entire time horizon.

Using the associated inbound backlog fluid process in Appendix J again, when $\tau < 2l$ and $K \geq \frac{2l}{2l-\tau}$, the cumulative amount of inbound backlogs during $t \in [0, \tau]$ is

$$\int_0^\tau \bar{I}_i(t)dt = \frac{1}{2}\tau \cdot \lambda\tau = \frac{\lambda\tau^2}{2}.$$

During $t \in (\tau, \tau + \frac{\tau}{K-1}]$, the cumulative amount of inbound backlogs is

$$\int_\tau^{\tau + \frac{\tau}{K-1}} \bar{I}_i(t)dt = \frac{1}{2}\frac{\tau}{K-1} \cdot \lambda\tau = \frac{\lambda\tau^2}{2(K-1)}.$$

Therefore, Port i's cumulative amount of inbound backlogs during $t \in [0, \infty)$ is

$$\begin{aligned}
\int_0^\tau \bar{I}_i(t)dt + \int_\tau^{\tau + \frac{\tau}{K-1}} \bar{I}_i(t)dt + \int_{\tau + \frac{\tau}{K-1}}^\infty \bar{I}_i(t)dt &= \frac{\lambda\tau^2}{2} + \frac{\lambda\tau^2}{2(K-1)} \\
&= \frac{K\lambda\tau^2}{2(K-1)}.
\end{aligned}$$

Finally, we prove that the cumulative amount of inbound backlogs at Port i is $2\lambda l(\tau - l + \frac{l}{K})$ when $\tau \geq 2l$. According to the associated inbound backlog fluid process in Appendix J again, during $t \in [0, 2l]$, Port i's cumulative amount of inbound backlogs is

$$\int_0^{2l} \bar{I}_i(t)dt = \frac{1}{2}2l \cdot \lambda 2l = 2\lambda l^2.$$

During $t \in (2l, \tau]$, Port i's cumulative amount of inbound backlogs is

$$\int_{2l}^{\tau} \bar{I}_i(t)dt = 2\lambda l \cdot (\tau - 2l) = 2\lambda\tau l - 4\lambda l^2.$$

During $t \in (\tau, \tau + \frac{2l}{K}]$, Port i's cumulative amount of inbound backlogs is

$$\int_{\tau}^{\tau+\frac{2l}{K}} \bar{I}_i(t)dt = \frac{1}{2}2\lambda l \cdot \frac{2l}{K} = \frac{2\lambda l^2}{K}.$$

Therefore, Port i's cumulative amount of inbound backlogs during $t \in [0, \infty)$ is

$$\begin{aligned}\int_0^{2l} \bar{I}_i(t)dt + \int_{2l}^{\tau} \bar{I}_i(t)dt + \int_{\tau}^{\tau+\frac{2l}{K}} \bar{I}_i(t)dt + \int_{\tau+\frac{2l}{K}}^{\infty} \bar{I}_i(t)dt &= 2\lambda l^2 + 2\lambda\tau l - 4\lambda l^2 + \frac{2\lambda l^2}{K} \\ &= 2\lambda l(\tau - l + \frac{l}{K}). \quad \square\end{aligned}$$

Proof of Proposition 3. Note that for both Ports i and j, the goods that need to be loaded at the port are continuously arriving with the rate of λ. Therefore, the outbound goods at Port i accumulate during $t \in [0, \tau]$ because of Port i's shutdown. l days later, Port j will also have exactly the same pattern of outbound goods accumulation during $t \in [l, \tau + l]$ because there are no ships coming out from Port i. Starting from $t = \tau$, Port i begins to clear the outbound backlogs at the rate of $(K-1)\lambda$ by using the maximal carrying capacity of the available ships. Then l days later, those ships will arrive at Port j and are able to clear the outbound backlogs at the rate $(K-1)\lambda$. As a result, the dynamics of Port j's outbound backlog level are the same as those of Port i's outbound backlog level, except with a l days lag. $\square$

Proof of Proposition 4. First we prove that when $C \geq \min\{K, \frac{\tau}{2l} + 1\}$, Port i clears the outbound backlogs within one cycle, and the outbound backlog level at Port i continuously decreases at the rate of $(K-1)\lambda$ without any oscillation. For all the ships that arrive at Port i during $t \in [0, 2l]$, the maximal amount of goods that ships can carry is $C\lambda 2l$ in total. Since the maximal processing capacity of Port i is $K\lambda$, it needs $\frac{C\lambda 2l}{K\lambda} = 2l\frac{C}{K}$ days to load $C\lambda 2l$ goods. When $C \geq K$, $2l\frac{C}{K} \geq 2l$. As a result, ships will continuously come at Port i after $t = \tau$, and therefore, the outbound backlogs at Port i can be cleared within one period. Therefore, we prove that when $C \geq \min\{K, \frac{\tau}{2l} + 1\}$, Port i clears the outbound backlogs within one period of stack accumulation. Since goods arrive at the rate of λ, and Port i clears the goods at the rate of $K\lambda$, the clearance rate at Port i is $(K-1)\lambda$.

Next, we prove that Port i clears the outbound backlogs within at least two periods of stack accumulation when $C < \min\{K, \frac{\tau}{2l} + 1\}$. Let $B_i^{\text{out}}(t)$ denote the outbound backlog at Port i. During the disruption period $t \in [0, \tau)$, outbound goods cannot be processed, so the backlog accumulates at rate λ. Hence,

$$\frac{d}{dt}B_i^{\text{out}}(t) = \lambda, \quad t \in [0, \tau),$$

which implies $B_i^{\text{out}}(t) = \lambda t$. Therefore, the backlog reaches its peak at time $t = \tau$ with

$$B_i^{\text{out}}(\tau) = \lambda\tau.$$

After reopening ($t \geq \tau$), Port i processes outbound goods at rate $K\lambda$. The evolution of the backlog is governed by the difference between the inflow and the processing rate. Due to the transportation delay $2l$, the inflow to the outbound queue depends on the system state at time $t-2l$, which induces a delayed feedback structure. As a result, the inflow is piecewise constant within each interval of length $2l$, leading to oscillatory dynamics.

We partition the time horizon into phases of the form $[\tau + 2nl, \tau + 2(n+1)l)$ for $n = 0, 1, \ldots$. Consider an arbitrary phase $[\tau + 2nl, \tau + 2(n+1)l)$. At the beginning of the phase, i.e., for $t \in [\tau + 2nl, \tau + 2nl + 2l(C/K)]$, the returning flow induced by prior clearing has not yet fully materialized. In this interval, the effective inflow is λ, and thus the net rate of change of the backlog is

$$\frac{d}{dt} B_i^{\text{out}}(t) = \lambda - K\lambda = -(K-1)\lambda.$$

Therefore, the backlog decreases linearly at rate $(K-1)\lambda$ during this period.

After time $t = \tau + 2nl + 2l(C/K)$, the delayed inflow arrives and replenishes the outbound queue. In this interval $t \in [\tau + 2nl + 2l(C/K), \tau + 2(n+1)l)$, the backlog evolves according to

$$\frac{d}{dt} B_i^{\text{out}}(t) = \lambda,$$

which implies that the backlog increases linearly at rate λ. This establishes that within each phase, the backlog first decreases and then increases, yielding the cyclical pattern described in the proposition.

Across phases, the net change in backlog over one complete oscillation phase is given by the reduction during the decreasing interval minus the increase during the subsequent interval. This net change equals $2l\lambda(C-1)$, which is strictly positive under the whiplash regime. Consequently, the backlog is reduced by a constant amount in each phase. Let $N = \left\lfloor \frac{\tau}{2l(C-1)} \right\rfloor$. After N phases, the remaining backlog becomes insufficient to generate another oscillation.

In the final phase $[\tau + 2lN, \tau - 2lN + (\tau + (C-1)2lN)/(K-1)]$, no further delayed replenishment can reverse the decreasing trend. The backlog then evolves according to

$$\frac{d}{dt} B_i^{\text{out}}(t) = -(K-1)\lambda,$$

and decreases monotonically until it reaches zero. This implies that no further oscillation occurs in the last phase.

Therefore, when $C < \min\{K, \frac{\tau}{2l} + 1\}$, the outbound backlog process exhibits cyclical behavior in each oscillation phase, followed by a final phase of monotone decay to zero, which completes the proof. $\square$

Proof of Proposition 5. Since Port i is shut down from $t=0$ to $t=\tau$, then the outbound backlog level is increasing during $t\in[0,\tau]$. At $t=\tau$, the outbound backlog level at Port i is $\lambda\tau$. Starting from $t=\tau$, Port i begins to load goods at the rate of $K\lambda$. If we look at the period of $t\in[0,\tau+2l]$, ships are departing from Port j and arriving at Port i at the rate of λ during $t\in[0,2l]$, and no ships are arriving at Port i during $t\in[2l,\tau+2l]$. We then analyze Port i's outbound backlog level over the period $t\in[\tau,\tau+2l]$. If $\tau<2l$, ships that arrived at Port i can use their maximal carrying capacity of $C\lambda$ to clear Port i's outbound backlogs. Thus, for all the ships that arrive at Port i during $t\in[0,2l]$, the maximal amount of goods they can carry is $C\lambda 2l$. Since the maximal processing capacity of Port i is $K\lambda$, it needs $\frac{C\lambda 2l}{K\lambda}=2l\frac{C}{K}$ days to load $C\lambda 2l$ goods. Thus, Port i is clearing its outbound backlogs with the rate of $(K-1)\lambda$ during $t\in[\tau,\tau+2l\frac{C}{K}]$. If $C<K$, then, during $t\in[\tau+2l\frac{C}{K},\tau+2l]$, since no ships are arriving at Port i, the outbound backlog level begins to increase again with the rate of λ. Therefore, at $t=\tau+2l\frac{C}{K}$, the outbound backlog level at Port i is $\tau\lambda-2l\lambda\frac{(K-1)C}{K}$. At $t=\tau+2l$, the outbound backlog level at Port i is $\tau\lambda-2l\lambda\frac{(K-1)C}{K}+2l\lambda\frac{K-C}{K}=\tau\lambda-(C-1)2l\lambda$. Starting from $t=\tau+2l$, since there are ships that come from Port j arriving at Port i again, the outbound backlog level at Port i will decrease. Therefore, the first peak of the outbound backlog level is $\tau\lambda$ at $t=\tau$, and the second peak of the outbound backlog level is $\tau\lambda-(C-1)2l\lambda<\tau\lambda$, which proves that the peak of the cargoes accumulation is decreasing over time. □

Proof of Proposition 6. First, we calculate the number of days Port i needs to clear all the outbound backlogs when $C\geq K$. To compute that, we first analyze the ships arriving at Port i. We prove that ships arrive at Port i from $t=0$ to $t=2l$ carrying λ cargoes per day. Then, from $t=2l$ to $t=2l+\tau$, there is no ship arriving at Port i. Recall that the travel time is l days. There is no ship leaving Port i from $t=0$ to $t=\tau$ because of Port i's shutdown. Thus, no ship arrives at Port j from $t=l$ to $t=\tau+l$. No incoming ship implies that Port j cannot unload or load any goods. This also implies that no ship is leaving Port j from $t=l$ to $t=\tau+l$. As a result, after l days, no ship arrives at Port i from $t=2l$ to $t=2l+\tau$. Therefore, we prove that ships arrive at Port i from $t=0$ to $t=2l$, and no ship arrives at Port i from $t=2l$ to $t=2l+\tau$.

Next, we prove that ships continuously arrive at Port i after $t=2l+\tau$. We prove it by discussing the relationship between $2l$ and τ. If $2l<\tau$, ships are arriving at Port i from $t=0$ to $t=2l$, carrying λ amount of outbound goods per day, then waiting at Port i to be unloaded. From $t=2l$ to $t=\tau$, there is no ship arriving at Port i. Therefore, when $t=\tau$, the total amount of cargo carried by those ships waiting to enter Port i is $2l\lambda$. When $t=\tau$, Port i is reopen. To clear outbound backlogs, ships operate at their maximum capacity to load cargo. Therefore, the maximum amount of cargo that can be loaded by all ships waiting at Port i when $t=\tau$ is $C\cdot 2l\lambda$. Starting from $t=\tau$, Port i uses its maximum capacity, $K\lambda$, to load the outbound goods. From $t=\tau$ to $t=\tau+2l$, the total amount of cargo loaded in Port i is $K\cdot 2l\lambda$. When $C\geq K$, the amount of cargo actually loaded in

Port i does not exceed the total amount of cargo that those waiting ships can carry. The ships will not be used up at $t=\tau+2l$. Therefore, Port i continuously loads goods with a rate of $K\lambda$ from $t=\tau$ to $t=\tau+2l$. If $2l\geq\tau$, ships continuously arrive at Port i from $t=0$ to $t=2l$. Starting from $t=\tau$, Port i begins to load and unload cargoes. The maximum amount of cargo that can be loaded by all ships waiting at Port i when $t=\tau$ is $C\cdot\tau\lambda$. From $t=\tau$ to $t=2l$, there are ships continuously arriving at Port i, and therefore, there are also ships continuously leaving Port i. Starting from $t=2l$, there is no ship arriving at Port i. However, Port i could use those waiting ships to load goods. During $t\in[2l,\tau+2l]$, the maximum cargoes Port i can load is $K\tau\lambda$, because Port i operates τ days and each day Port i use its maximum loading rate which is $K\lambda$ TEUs per day. At the same time, when we consider all the available ships at Port i, the amount of goods those available ships can carry is $2lC\lambda-(2l-\tau)K\lambda$, which is larger than $K\tau\lambda$, and it implies that the ships are enough to carry all the loaded goods during $t\in[\tau,\tau+2l]$ at Port i. Therefore, we prove that there are ships continuously leaving Port i during $t\in[\tau,\tau+2l]$.

Since we have proved that there are ships continuously leaving Port i during $t\in[\tau,\tau+2l]$, whether $2l<\tau$ or $2l\geq\tau$, by Lemma G.2, there are ships continuously arriving at Port i during $t\in[\tau+2l,\tau+4l]$. We could use this result to iterate on and then prove there are ships continuously leaving Port i after $t=\tau+2l$. From $t=0$ to $t=\tau$, Port i is shut down and unable to load any goods. However, goods continuously arrive at Port i with a rate of λ waiting to be loaded. Therefore, the outbound backlog level at Port i increases with the rate of λ. When $t=\tau$, the total amount of outbound backlogs at Port i is $\lambda\tau$. Starting from $t=\tau$, Port i begins to load goods and then ship those goods to Port j using the maximal port capacity and the maximal ship capacity. Since the total outbound backlogs needs to clear is $\lambda\tau$, Port i needs $\frac{\lambda\tau}{(K-1)\lambda}=\frac{\tau}{K-1}$ days to clear the total outbound backlogs. Also, because $C>K$, the bottleneck is the port processing capacity, and shipping capacity is enough. Therefore, we prove that when $\min\{K,C\}>1$ and $K<C$, Port i needs $\frac{\tau}{K-1}$ days to clear all the outbound backlogs.

Next, we calculate the number of days Port i needs to clear all the outbound backlogs when $1<C<K$. To calculate the outbound backlog clearance days for Port i, we first define a new period concept. We define the first period as $t\in(\tau,\tau+2l]$, the second period as $t\in(\tau+2l,\tau+4l],\ldots$, the n^{th} period as $t\in(\tau+2(n-1)l,\tau+2nl]$. From $t=0$ to $t=\tau$, Port i cannot load any goods, and outbound cargoes accumulate at a rate of λ. Therefore, at $t=\tau$, the total amount of outbound backlogs at Port i is $\tau\lambda$. Starting from $t=\tau$, Port i begins to load goods and then ship those goods to Port j. Since $C<K$, the shipping capacity is now the bottleneck. We calculate the amount of outbound backlog that Port i can clear in the first period. Normally, the amount of outbound goods Port i can load is λ. However, since Port i has outbound backlogs during the first period, ships can increase their capacity to load $C\lambda$ amount of outbound goods. Note that ships arrive

at Port i from $t=0$ to $t=2l$. Therefore, the amount of outbound goods Port i can load during the first period is $2l \cdot C\lambda$. Because of outbound backlogs, Port i also uses its maximum carrying capacity in the first period, K. Since $C < K$, the time for Port i to actually load the outbound cargoes is $\frac{2l \cdot C\lambda}{K\lambda} = 2l\frac{C}{K}$ days during the first period. Therefore, Port i load the outbound goods at the rate $K\lambda$ during $t \in (\tau, \tau + 2l\frac{C}{K}]$. Since there are no available ships during $t \in (\tau + 2l\frac{C}{K}, \tau + 2l]$, Port i cannot load any outbound goods during that period. Note that those outbound goods that depart from Port i will arrive at Port j after l days. Therefore, the arrival rate of goods at Port j is $K\lambda$ during $t \in (\tau + l, \tau + 2l\frac{C}{K} + l]$ and 0 during $t \in (\tau + 2l\frac{C}{K} + l, \tau + 3l]$. Since there are no ship departs from Port i during $t \in (0, \tau]$, there are no arrived ships at Port j during $t \in (l, \tau + l]$. As a result, Port j will accumulate outbound goods during $t \in (l, \tau + l]$. At $t = \tau + l$, the amount of outbound backlogs at Port j is $\tau\lambda$. Because Port j has outbound backlogs during $t \in (\tau + l, \tau + 3l]$, Port j also loads at maximum rate $K\lambda$ as Port i loads during $t \in (\tau, \tau + 2l]$. Therefore, Port j loads outbound goods at the rate $K\lambda$ during $t \in (\tau + l, \tau + 2l\frac{C}{K} + l]$ and do not load any outbound goods during $t \in (\tau + 2l\frac{C}{K} + l, \tau + 3l]$. After l days, those outbound goods will arrive at Port i. Therefore, the arrival rate of goods at Port i is $K\lambda$ during $t \in (\tau + 2l, \tau + 2l\frac{C}{K} + 2l]$ and 0 TEU per day during $t \in (\tau + 2l\frac{C}{K} + 2l, \tau + 4l]$. Thus, during the second period, Port i can also load $2l \cdot C\lambda$ outbound goods. Using this logic, we can prove that during each period, the amount of outbound goods Port i can load at maximum is $2l \cdot C\lambda$. Because the goods will also come in continuously at the rate of λ in each period, the amount of outbound backlogs that Port i can reduce in each period is $2l \cdot C\lambda - 2l\lambda = 2l\lambda(C-1)$.

Finally, we provide the proof establishing that the duration required for Port j to clear all of its outbound backlogs exceeds that of Port i by l days. It is important to note that goods loaded at Port i will reach Port j after l days. Given that the maximum quantity of goods departing from Port i per day is $K\lambda$, Port j is capable of unloading goods from Port i without encountering any backlog, as its maximum unloading rate matches the daily departing volume of Port i, i.e., $K\lambda$. Furthermore, between the time interval $t=0$ to $t=\tau$, no vessels depart from Port i. Similarly, after l days, no vessels arrive at Port j. Thus, within the time frame of $t=l$ to $t=\tau+l$, there is an absence of vessel arrivals at Port j, which prevents it from engaging in unloading and loading operations. By $t=\tau+l$, the cumulative quantity of outbound backlogs at Port j amounts to $\tau\lambda$. It should be emphasized that Port j's rate of unloading inbound goods is identical to Port i's rate of loading outbound goods l days earlier. Thus, starting from $t=\tau+l$, Port j initiates the clearance of its outbound backlogs, accomplishing this task at a rate equivalent to that of Port i in the period before l days. Hence, Port j discharges its outbound backlogs in a manner analogous to Port i before the lapse of l days. Hence, the duration required for Port j to clear all of its outbound backlogs exceeds that of Port i by l days. □

Proof of Proposition 7. We first prove that the cumulative amount of outbound backlogs at Port i is $\frac{K\lambda\tau^2}{2(K-1)}$ when $C \geq \min\{K, \frac{\tau}{2l}+1\}$. According to the associated outbound backlog fluid process in Appendix I, when $C \geq \min\{K, \frac{\tau}{2l}+1\}$, the cumulative amount of outbound backlogs during $t \in [0,\tau]$ is $\frac{1}{2}\tau \cdot \lambda\tau = \frac{\lambda\tau^2}{2}$. During $t \in (\tau, \tau + \frac{\tau}{K-1}]$, the cumulative amount of outbound backlogs is $\frac{1}{2}\frac{\tau}{K-1} \cdot \lambda\tau = \frac{\lambda\tau^2}{2(K-1)}$. Therefore, Port i's cumulative amount of outbound backlogs during $t \in [0,\infty)$ is $\frac{\lambda\tau^2}{2} + \frac{\lambda\tau^2}{2(K-1)} = \frac{K\lambda\tau^2}{2(K-1)}$.

Then we compute the cumulative amount of outbound backlogs at Port i when $C < \min\{K, \frac{\tau}{2l}+1\}$ and $\frac{\tau}{2l(C-1)} = \mathbb{Z}^+$. According to the associated outbound backlog fluid process in Appendix I, when $C < \min\{K, \frac{\tau}{2l}+1\}$ and $\frac{\tau}{2l(C-1)} = \mathbb{Z}^+$, the cumulative amount of outbound backlogs during $t \in [0,\tau]$ is $\frac{1}{2}\tau \cdot \lambda\tau = \frac{\lambda\tau^2}{2}$. Then, for a given $n = \{0, 1, \ldots, N-1\}$, where $N \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$, we compute the cumulative amount of outbound backlogs when $t \in (\tau + 2nl, \tau + 2(n+1)l]$.

During $t \in (\tau + 2nl, \tau + 2nl + 2\frac{C}{K}l]$, the cumulative amount of outbound backlogs is

$$\int_{\tau+2nl}^{\tau+2nl+2\frac{C}{K}l} \bar{O}_i(t)dt = \frac{1}{2}(\bar{O}_i(\tau+2nl) + \bar{O}_i(\tau+2nl+2\frac{C}{K}l)) \cdot 2\frac{C}{K}l.$$

During $t \in (\tau + 2nl + 2l\frac{C}{K}, \tau + 2nl + 2l]$, the cumulative amount of outbound backlogs is

$$\int_{\tau+2nl+2l\frac{C}{K}}^{\tau+2(n+1)l} \bar{O}_i(t)dt = \frac{1}{2}(\bar{O}_i(\tau+2nl+2l\frac{C}{K}) + \bar{O}_i(\tau+2(n+1)l)) \cdot (2l - 2\frac{C}{K}l).$$

Therefore, the cumulative amount of outbound backlogs during $t \in (\tau + 2nl, \tau + 2nl + 2l]$ is

$$\begin{aligned}\int_{\tau+2nl}^{\tau+2nl+2\frac{C}{K}l} \bar{O}_i(t)dt + \int_{\tau+2nl+2l\frac{C}{K}}^{\tau+2(n+1)l} \bar{O}_i(t)dt =& \frac{C}{K}l\bar{O}_i(\tau+2nl) + (l - \frac{C}{K}l)\bar{O}_i(\tau+2nl+2l) \\ &+ l\bar{O}_i(\tau+2nl+2\frac{C}{K}l). \end{aligned} \tag{G.1}$$

We have

$$\begin{aligned}\bar{O}_i(\tau+2nl) &= \lambda\tau + 2(1-C)n\lambda l, \\ \bar{O}_i(\tau+2nl+2l) &= \lambda\tau + 2(1-C)(n+1)\lambda l, \\ \bar{O}_i(\tau+2nl+2\frac{C}{K}l) &= \lambda\tau + 2\lambda l((1-C)n + \frac{C}{K} - C).\end{aligned}$$

Thus, equation G.1 equals to $2\lambda l^2(1 - 2C + 2(1-C)n + \frac{C^2}{K}) + 2\lambda\tau l$.

Since we have $n = 0, 1, \ldots, N-1$, the cumulative outbound backlogs during $t \in (\tau, \tau + 2lN]$ is

$$\sum_{n=0}^{n=N-1} \int_{\tau+2nl}^{\tau+2nl+2l} \bar{O}_i(t)dt = 2N\lambda l^2(1-2C+\frac{C^2}{K}) + 2N\lambda\tau l + 2(1-C)N(N-1)\lambda l^2 = 2N\lambda l((1-C)Nl + \tau - Cl + \frac{C^2}{K}$$

Thus, when $C < \min\{K, \frac{\tau}{2l}+1\}$ and $\frac{\tau}{2l(C-1)} = \mathbb{Z}^+$, the cumulative outbound backlogs during the whole time horizon is $\frac{\lambda\tau^2}{2} + 2N\lambda l((1-C)Nl + \tau - Cl + \frac{C^2}{K}l)$, where $N \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$.

Finally, we compute the cumulative amount of outbound backlogs at Port i when $C < \min\{K, \frac{\tau}{2l} + 1\}$ and $\frac{\tau}{2l(C-1)} \neq \mathbb{Z}^+$. According to the associated outbound backlog fluid process in Appendix I, we only need to compute the additional cumulative amount of outbound backlogs during $t \in (\tau + 2Nl, \tau + 2Nl + \frac{\tau - 2(C-1)Nl}{(K-1)}]$ plus the cumulative amount of outbound backlogs we have already computed during $t \in [0, \tau + 2Nl]$. Thus, we have

$$\int_{\tau+2Nl}^{\tau+2Nl+\frac{\tau-2(C-1)Nl}{(K-1)}} \bar{O}_i(t)dt = \frac{1}{2}\frac{\tau - 2(C-1)Nl}{K-1}(\lambda\tau - 2(C-1)N\lambda l) = \frac{\lambda[\tau - 2(C-1)Nl]^2}{2(K-1)}.$$

Therefore, when $C < \min\{K, \frac{\tau}{2l} + 1\}$ and $\frac{\tau}{2l(C-1)} \neq \mathbb{Z}^+$, the cumulative amount of outbound backlogs during the whole horizon is $\frac{\lambda\tau^2}{2} + 2N\lambda l((1-C)Nl + \tau - Cl + \frac{C^2}{K}l) + \frac{\lambda[\tau-2(C-1)Nl]^2}{2(K-1)}$.

By combining the results under the conditions $\frac{\tau}{2l(C-1)} = \mathbb{Z}^+$ and $\frac{\tau}{2l(C-1)} \neq \mathbb{Z}^+$, we prove that when $C < \min\{K, \frac{\tau}{2l} + 1\}$, the cumulative amount of outbound backlogs at Port i throughout the whole horizon is

$$\frac{\lambda\tau^2}{2} + 2N\lambda l((1-C)Nl + \tau - Cl + \frac{C^2}{K}l) + \mathbb{1}_{M>0}\frac{\lambda[\tau - 2(C-1)Nl]^2}{2(K-1)},$$

where $N \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$ and $M \equiv \tau - 2lN(C-1)$.

Based on the outbound backlog fluid process presented in Appendix I, it is observed that the pattern of the outbound backlogs in Port j is identical to that of Port i, albeit with a lag of l days. Thus, the cumulative amount of outbound backlogs in Port j is equivalent to that of Port i. □

Proof of Corollary 1. We first prove that the number of days required for Port i to recover to its original state decreases in K. Let us assume that after the disruption of τ days, Port i needs T_i days to be back to normal. Using the result in Proposition 6, we have

$$T_i = \frac{K\tau}{K-1} + 2l \cdot \frac{K - \bar{K}}{K-1}(N - 1 + \mathbb{1}_{M>0}),$$

where $N \equiv \lfloor \frac{\tau}{2l(\bar{K}-1)} \rfloor$, $M \equiv \tau - 2lN(\bar{K}-1)$, and $\bar{K} \equiv \min\{K, C\}$. Next, we discuss the value of T_i based on the size relationship between K and C.

When $K < C$, we have $\bar{K} = K$. Thus, $T_i = \frac{K\tau}{K-1}$. Taking the first-order derivative of T_i with respect to K, we have $\frac{\partial T_i}{\partial K} = -\frac{\tau}{(K-1)^2} < 0$. As a result, T_i is decreasing in K.

When $K \geq C$, we have $\bar{K} = C$. Thus, we have

$$T_i = \frac{K\tau}{K-1} + 2l \cdot \frac{K - C}{K-1}(\lfloor \frac{\tau}{2l(C-1)} \rfloor - 1 + \mathbb{1}_{\tau - 2l(C-1)\lfloor \frac{\tau}{2l(C-1)} \rfloor > 0}). \tag{G.2}$$

Let us denote $S \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor - 1 + \mathbb{1}_{\tau - 2l(C-1)\lfloor \frac{\tau}{2l(C-1)} \rfloor > 0}$.

Next, we discuss (G.2) based on the size relationship between τ and $2l(C-1)$.

Define $Z \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$. When $\frac{\tau}{2l(C-1)} = Z$, we have $\mathbb{1}_{\tau - 2l(C-1)\lfloor \frac{\tau}{2l(C-1)} \rfloor > 0} = 0$. Thus, we have

$$S = \frac{\tau}{2l(C-1)} - 1,$$

$$T_i = \frac{K\tau}{K-1} + 2l(Z-1)\frac{K-C}{K-1} = \frac{K\tau + 2l(Z-1)K - 2l(Z-1)C}{K-1}.$$

Taking the first-order derivative of T_i with respect to K, we have

$$\frac{\partial T_i}{\partial K} = \frac{2l(Z-1)(C-1)-\tau}{(K-1)^2} = -\frac{2l(C-1)}{(K-1)^2} < 0.$$

Therefore, T_i is decreasing in K when $\frac{\tau}{2l(C-1)} = Z$.

When $\frac{\tau}{2l(C-1)} \neq Z$, we have $\mathbb{1}_{\tau - 2l(C-1)\lfloor \frac{\tau}{2l(C-1)} \rfloor > 0} = \mathbb{1}_{\tau - 2l(C-1)(Z-1)>0} = 1$. Thus, we have

$$T_i = \frac{K\tau}{K-1} + 2lZ\frac{K-C}{K-1} = \frac{K\tau + 2lZK - 2lZC}{K-1}.$$

Taking the first-order derivative of T_i with respect to K, we have

$$\frac{\partial T_i}{\partial K} = \frac{2lZ(C-1)-\tau}{(K-1)^2} < 0$$

because now we have $Z \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor < \frac{\tau}{2l(C-1)}$. Therefore, T_i is decreasing in K when $\frac{\tau}{2l(C-1)} \neq Z$.

As a result, we prove that the number of days Port i needs to clear all the outbound backlogs is decreasing in the port's maximal processing capacity K.

Next, we prove that the number of days required for Port j to recover to its original state is decreasing in K. By Proposition 6, Port j needs

$$T_j = \frac{K\tau}{K-1} + 2l \cdot \frac{K-\bar{K}}{K-1}(N - 1 + \mathbb{1}_{M>0}) + l$$

days to clear all the outbound backlogs. We have $T_j = T_i + l$. We show that T_j is decreasing in K because T_i is decreasing in K and l is constant. Thus, we prove that the number of days Port j needs to clear all the outbound backlogs is decreasing in the port's maximal processing capacity K. □

Proof of Corollary 2. Using Proposition 6, when $C < K$, we have

$$C^* = \arg\min_C \{\frac{K\tau}{K-1} + 2l \cdot \frac{K-C}{K-1}(\lfloor \frac{\tau}{2l(C-1)} \rfloor - 1 + \mathbb{1}_{M>0})\}. \tag{G.3}$$

We first compute the value of $N - 1 + \mathbb{1}_{M>0}$ under different value of C, where $N \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$. When $2l(C-1) < \tau$, $N = \lfloor \frac{\tau}{2l(C-1)} \rfloor \geq 1$, $M = \tau - 2lN(C-1) > 0$, $N - 1 + \mathbb{1}_{M>0} > 0$. When $2l(C-1) = \tau$, $N = \lfloor \frac{\tau}{2l(C-1)} \rfloor = 1$, $M = \tau - 2lN(C-1) = 2l(C-1) - 2l(C-1) = 0$, $N - 1 + \mathbb{1}_{M>0} = 0$. When $2l(C-1) > \tau$, $N = \lfloor \frac{\tau}{2l(C-1)} \rfloor = 0$, $M = \tau - 2lN(C-1) = \tau$, $N - 1 + \mathbb{1}_{M>0} = 0$. Thus, when $2l(C-1) \geq \tau$, the second term in (G.3) is zero, and the first term in (G.3) is not related to C. As a result, when $C < K$, $C^* = \arg\min_C \{2l(C-1) > \tau\} = \frac{\tau}{2l} + 1$.

When $C \geq K$, we have

$$\begin{aligned} C^* &= \arg\min_C \{\frac{K\tau}{K-1} + 2l \cdot \frac{K-K}{K-1}(\lfloor \frac{\tau}{2l(C-1)} \rfloor - 1 + \mathbb{1}_{M>0})\} \\ &= \arg\min_C \frac{K\tau}{K-1} = K. \end{aligned}$$

Therefore, combining the two circumstances, we have $C^* = \min\{K, \frac{\tau}{2l} + 1\}$. □

Proof of Lemma A.2. Let $D(t) = \int_0^t K\,ds$ and $\hat{D}(t) = \sum_{n=1}^{N(t)} S_n$ be the cumulative discharge in the fluid and discrete models, respectively. Since the cumulative demand $\int_0^t \lambda(s)\,ds$ is identical in both models, the error is determined solely by the discharge process: $|\hat{B}(t) - B(t)| = |\hat{D}(t) - D(t)|$. In a periodic schedule, $D(t)$ is the linear interpolant of the discrete step function $\hat{D}(t)$. Because each jump in $\hat{D}(t)$ is exactly S, the continuous line $D(t)$ can never deviate from the "staircase" $\hat{D}(t)$ by more than S. Thus, the fluid model serves as the central tendency for the discrete system. □

Proof of Lemma E.1. Based on the definition of the base policy governing the allocation of loading and unloading capacities at Port $i = 1$, it can be observed that for the shipping route connecting Ports $i = 1$ and $j \in \mathcal{J}$, Port $i = 1$ assigns a maximum of $K_1\lambda_j$ as the daily capacity for unloading and loading operations. Similarly, Port j has a processing capability of up to $K^j\lambda_j$. Because the n shipping routes are independent, the entire system can be decomposed into n individual two-port systems. These systems correspond to the relationships between Ports $i = 1$ and $j = 1$, Ports $i = 1$ and $j = 2$, and so on, up to Ports $i = 1$ and $j = n$. □

Proof of Proposition D.1. First, we compare the outbound backlog clearing process of Port i under a single closure and under two non-overlapping closures. In the single-closure case, once Port i reopens at time $t_1 + \tau_1$, the backlog enters a recovery pattern consisting of alternating down phases and up phases. Each down phase, during which the backlog decreases, lasts $2l(C/K)$ days, while each up phase, during which the backlog increases, lasts $2l(1 - C/K)$ days, except for the final phase-out period. This cycle repeats periodically until the backlog is fully cleared.

Now suppose that Port i experiences a second closure from t_2 to $t_2 + \tau_2$, where t_2 lies within the first down phase after the first reopening, that is, $t_2 \in \big(t_1 + \tau_1,\, t_1 + \tau_1 + 2l(C/K)\big)$. During the single-closure recovery, the backlog would have continued to decrease from $t_1 + \tau_1$ until $t_1 + \tau_1 + 2l(C/K)$. However, with the second closure, the decrease in backlog is interrupted at t_2. From t_2 to $t_2 + \tau_2$, no ships can be loaded, and the backlog therefore increases. Once Port i reopens at $t_2 + \tau_2$, the down phase resumes. This interruption splits the original down phase into three segments: a partial down phase before the second closure, an up phase during the closure, and a resumed down phase after the closure. As a result, exactly one new oscillation phase (a down-then-up cycle) is created.

After the second reopening at $t_2 + \tau_2$, the recovery process follows the same temporal pattern as in the single-closure case. Each down phase still lasts $2l(C/K)$ days, and each up phase still lasts $2l(1 - C/K)$ days, because these durations are determined by the fleet rotation and port processing rates, which are unaffected by the timing of the second closure. The only difference is that the entire sequence of breakpoints after $t_2 + \tau_2$ occurs later than in the single-closure case. The delay is exactly $(t_2 + \tau_2) - (t_1 + \tau_1)$ days, which is the additional "pause" inserted into the cycle by the second closure.

Therefore, the second disruption adds exactly one oscillation phase without changing the shape or duration of the subsequent phases. The whole oscillation pattern after the second reopening is simply shifted to the right by a constant amount, establishing the phase-shift invariance attribute. □

Proof of Proposition E.1. We begin by proving that adopting committed and contingent capacity allocation policies yields the following shared system properties in an m-to-1 network system. First, we prove that Port $i=1$ has inbound and outbound backlogs under both capacity allocation policies. This is because when Port $i=1$ is shut down, it cannot unload or load any goods, resulting in the accumulation of inbound and outbound goods. Therefore, Port $i=1$ will have inbound and outbound backlogs regardless of the allocation policy adopted. Second, we prove that Port $i=2,\ldots,m$ never has any inbound or outbound backlogs under the committed policy. This is because Port $j=1$ commits a fixed processing capacity to the shipping route between Ports $i=2,\ldots,m$ and $j=1$. Thus, Port $i=1$'s disruption will not affect the goods throughput rate between Ports $i=2,\ldots,m$ and $j=1$. Third, we prove that Port $j=1$ always has outbound backlogs, regardless of the allocation policy adopted. This is because no ships are departing from Port $i=1$ to arrive at Port $j=1$ during Port $i=1$'s closure. Thus, Port $j=1$ has no ships to load its outbound goods. As a result, Port $j=1$ will have outbound backlogs due to the disruption of Port $i=1$, regardless of the allocation policy adopted.

Then, we prove that adopting committed and contingent capacity allocation policies yields the following distinct system properties in an m-to-1 network. We already showed that Port $i=2,\ldots,m$ never has any outbound backlogs under the committed policy. Then we prove that Port $i=2,\ldots,m$ has outbound backlogs if and only if $C_i/\rho<1$ under the contingent policy. Because we have $C_i>1$, when $C_i/\rho<1$, it implies that $\rho>1$. Thus, when $\rho>1$, Port $j=1$ allocates its processing capacity according to the volume of goods arriving at Port $j=1$ from each Port $i\in\mathcal{I}$. Specifically, Port $j=1$ allocates $\frac{\lambda_i}{K_1\lambda_1+\sum_{i=2}^m\lambda_i}$ proportions of its processing capacity to each Port $i\in\mathcal{I}$. Note that the maximum processing capacity Port $j=1$ has is $K^1\sum_{i=1}^m\lambda_i$. Therefore, there will be $\frac{\lambda_i}{K_1\lambda_1+\sum_{i=2}^m\lambda_i}K^1\sum_{i=1}^m\lambda_i=\lambda_i/\rho$ amount of goods arriving at each Port i on side $\mathcal{I}$. Since we have $\lambda_i/\rho<\lambda_i$ when $\rho>1$, there will be no inbound backlogs at Port $i=2,\ldots,m$. However, when ships need to load outbound goods at Port $i=2,\ldots,m$ to transport them to Port $j=1$, the number of ships is not enough to load λ_i amount of goods. Since ships can increase their carrying capacity up to C_i times the current carrying capacity on shipping routes $\{(i,1),(1,i)\}$, the maximal amount of goods those ships can carry is λ_iC_i/ρ. Thus, when $\lambda_iC_i/\rho<\lambda_i$, Port $i=2,\ldots,m$ will have outbound backlogs.

We proceed by proving that Port $j=1$ has inbound backlogs if and only if $K_1>K^1$ under the committed policy. In the committed policy framework, Port $j=1$ dedicates a fixed processing capacity to each shipping route it serves. Upon the reopening of Port $i=1$, an amount of outbound

goods, $K_1\lambda_1$, is loaded at Port $i=1$ and directed towards Port $j=1$ to alleviate Port $i=1$'s outbound backlogs. With a processing capacity of $K^1\lambda_1$, Port $j=1$ will have inbound backlogs if and only if the inbound goods arrival rate exceeds its processing capacity, i.e., $K_1\lambda_1 > K^1\lambda_1$. Hence, it follows that Port $j=1$ experiences inbound backlogs only when $K_1 > K^1$ under the committed policy framework.

Finally, we prove that Port $j=1$ has inbound backlogs if and only if $\rho > 1$ under the contingent policy, where $\rho \equiv \frac{K_1\lambda_1+\sum_{i=2}^m \lambda_i}{K^1\sum_{i=1}^m \lambda_i}$. Recall that Port $i=1$ remains closed during $t \in [0,\tau]$. Upon reopening at $t=\tau$, Port $i=1$ handles goods at a rate of $K_1\lambda_1$ to clear its inbound and outbound backlogs. Thus, by $t=\tau+l_1$, goods departing from Port $i=1$ arrive at Port $j=1$ at a rate of $K_1\lambda_1$. Therefore, starting from $t=\tau+l_1$, the total goods arriving at Port $j=1$ sum up to $K_1\lambda_1+\sum_{i=2}^m \lambda_i$. However, the maximum unloading capacity of Port $j=1$ is $K^1\sum_{i=1}^m \lambda_i$. Thus, if $K^1\sum_{i=1}^m \lambda_i < K_1\lambda_1+\sum_{i=2}^m \lambda_i$, Port $j=1$ cannot process all inbound goods, resulting in inbound backlogs. □

Proof of Proposition E.2. We begin by proving part (i)-(a), which states that Port $i=1$ always experiences inbound backlogs. During the disruption, Port $i=1$ is unable to unload any inbound cargo from arriving ships. As a result, all inbound cargo directed to Port $i=1$ accumulates, leading to persistent inbound backlogs throughout the shutdown period.

Next, we prove part (i)-(b), which concerns outbound backlogs. Since Port $i=1$ cannot load outbound cargo during the disruption, goods accumulate in its yard, resulting in outbound backlogs at the disrupted port. For each Port $j \in \mathcal{J}$, which is directly connected to Port $i=1$, no ships depart from Port $i=1$ during the shutdown. As a result, after a certain time, no ships arrive at any Port $j \in \mathcal{J}$, and outbound cargo at these ports cannot be loaded. This leads to the accumulation of outbound backlog at all ports on side $\mathcal{J}$ connected to the disrupted node.

Finally, we prove the two claims in parts (ii)-(a) and (ii)-(b). Proposition E.2(ii)-(a) shows that adopting the contingent allocation policy at Port $i=1$ leads to faster clearance of outbound backlogs across all ports. This improvement arises from the more efficient utilization of available loading capacity under the contingent policy. Specifically, the contingent policy dynamically reallocates loading capacity across routes based on current outbound backlog levels, allowing heavily congested routes to receive more capacity in the short term. In contrast, the committed policy reserves a fixed proportion of capacity for each route, including routes with no immediate demand, resulting in underutilization of total available capacity. Therefore, the number of days required to clear outbound backlogs is strictly shorter under the contingent policy than under the committed policy for all affected ports.

Proposition E.2(ii)-(b) demonstrates that inbound backlog clearance is slower under the contingent policy compared to the committed policy. Under the committed policy, each route is

guaranteed a maximum unloading capacity of $K_1\lambda_j$, thereby ensuring predictable service rates. In contrast, the contingent policy allocates unloading capacity proportionally based on the current distribution of inbound backlogs across routes. While this approach may accelerate backlog clearance on highly congested routes, it can result in insufficient allocation to other routes. In particular, if the throughput rate on a route $\{(1,j),(j,1)\}$ exceeds the processing capacity at Port j (i.e., exceeds $K^j\lambda_j$), then inbound backlogs will accumulate at Port j, even if $K^j \geq K_1$, a scenario that does not arise under the committed policy. Hence, although the contingent policy may temporarily prioritize high-backlog routes, it lacks the safeguards of the committed policy and can lead to persistent delays in inbound backlog clearance. □

Proof of Proposition E.3. We first prove that using the committed and contingent capacity allocation policies leads to the following shared system properties. Using the similar proof steps employed in the proofs of Propositions E.1 and E.2, it can be established that Port $i=1$ experiences both inbound and outbound backlogs whether using the committed and contingent allocation policies, as Port $i=1$ is incapable of unloading or loading goods during its disruption.

Next, we prove that Port $i=2,\ldots,m$ never has inbound backlogs, regardless of the allocation policy adopted. Since the goods arrival rate at each Port $i=2,\ldots,m$ is $\lambda_i=\sum_{j=1}^{n}\lambda_{i,j}$, which is always equal to or less than its maximal processing capacity $K_i\lambda_i$, Port $i=2,\ldots,m$ will not have any inbound backlogs.

We then prove that Port $j\in\mathcal{J}$ always has outbound backlogs, regardless of whether the committed or contingent allocation policy is adopted. Considering the period $t\in[0,\tau]$, no ships are departing from Port $i=1$ as a consequence of its temporary closure. Thus, $L_{1,j}$ days later, within the timeframe $t\in[L_{1,j},\tau+L_{1,j}]$, no ships will arrive at Port $j\in\mathcal{J}$ from Port $i=1$. Therefore, the outbound goods at Port j accumulate specifically because goods destined for Port $i=1$ cannot be loaded due to the absence of ships on the $(1,j)$ route under both capacity allocation policies.

We then prove that using the committed or contingent capacity allocation policy yields the following distinct system properties. First, we prove that Port $i=2,\ldots,m$ never has outbound backlogs under the committed-capacity allocation policy. Under the committed capacity allocation policy, each shipping route is independent. After Port $i=1$ reopens from its disruption, the processing capacity Port $j\in\mathcal{J}$ allocates to each shipping route $\{(i,j),(j,i)\}$ remains unchanged for $i=2,\ldots,m$. Thus, under the committed policy, Port $i=2,\ldots,m$ never has any outbound backlogs.

Second, we prove that Port $i=2,\ldots,m$ has outbound backlogs if and only if $\min_j\{C_{i,j}/\rho_j\}<1,\forall j\in\mathcal{J}$ under the contingent capacity allocation policy. By Proposition E.1, if there is only one port on side $\mathcal{J}$, we only need to compare the size relationship between $C_{i,j}/\rho_j$ and 1 when Port $j=1$ uses the contingent allocation policy. However, when there is more than one port on side $\mathcal{J}$, we need to compare the size relationships between $C_{i,j}/\rho_j$ and 1 for each pair of shipping routes

between Port $i=2,\ldots,m$ and each Port $j\in\mathcal{J}$. If there exists one Port j on the side $\mathcal{J}$ such that on the corresponding shipping routes, we have $C_{i,j}/\rho_j<1$, then Port i will have outbound backlogs. Thus, under the contingent capacity allocation policy, if $\min_j\{C_{i,j}/\rho_j\}<1,\forall j\in\mathcal{J}$, Port i will have outbound backlogs.

Finally, we prove that Port $j\in\mathcal{J}$ has inbound backlogs if and only if $K_1>K^j$ under the committed policy. Because under the committed policy, Port j commits a fixed processing capacity to each shipping route $\{(i,j),(j,i)\}$ for all $i\in\mathcal{I}$. Thus, when Port $i=1$ reopens after disruption, only ships that depart from Port $i=1$ will experience an increase relative to the normal time. As a result, Port $j\in\mathcal{J}$ has inbound backlogs if and only if the maximum inbound goods arrival rate exceeds Port j's processing capacity. After Port $i=1$ reopens, Port $i=1$ loads $K_1\lambda_{1,j}$ amount of outbound goods for the shipping routes $\{(1,j),(j,1)\}$. Note that Port j's processing capacity for the shipping routes $\{(1,j),(j,1)\}$ is $K^j\lambda_{1,j}$. Then, Port j will have inbound backlogs if and only if $K_1\lambda_{1,j}>K^j\lambda_{1,j}$.

We then prove that Port $j\in\mathcal{J}$ has inbound backlogs if and only if $\rho_j>1$ under the contingent policy, where $\rho_j\equiv\frac{K_1\lambda_{1,j}+\sum_{i=2}^m\lambda_{i,j}}{K^j\sum_{i=1}^m\lambda_{i,j}}$. Recall that Port $i=1$ is closed from $t=0$ to $t=\tau$. When Port $i=1$ reopens at $t=\tau$, for each routes $\{(1,j),(j,1)\}$ where $j=1,\ldots,n$, Port $i=1$ will unload and load goods at the rate of $K_1\lambda_{1,j}$ in order to clear its inbound and outbound backlogs. Therefore, at $t=\tau+L_{1,j}$, the rate of inbound goods arriving at Port j which departed from Port $i=1$ at $t=\tau$ is $K_1\lambda_{1,j}$. Thus, the total goods that arrived at Port j is $K_1\lambda_{1,j}+\sum_{i=2}^m\lambda_{i,j}$ starting from $t=\tau+L_{1,j}$. However, the maximal amount of goods that Port j could unload per day is $K^j\sum_{i=1}^m\lambda_{i,j}$. Therefore, if $K^j\sum_{i=1}^m\lambda_{i,j}<K_1\lambda_{1,j}+\sum_{i=2}^m\lambda_{i,j}$, Port j cannot process all the inbound goods and will have inbound backlogs. □

Proof of Proposition F.1. We begin by comparing how cargo flows are rerouted when the intermediate port is shut down. In a steady state, each port receives and handles a share of the arriving vessels according to a fixed rotation. When the intermediate port becomes non-operational, vessels do not stop there; instead, they proceed to the next scheduled port of call and unload there. This detour increases the effective inbound volume arriving at the undisrupted ports beyond that handled in the steady state. Whenever this increased arrival volume exceeds the local maximum unloading rate, an inbound queue forms. Hence, even though the ports that are directly connected with the intermediate port are uninterrupted, they accumulate inbound backlogs during the recovery period. This is precisely what we observe in Figure F.1: the inbound trajectories at Shanghai and Oakland begin to climb once ships start bypassing Long Beach and offloading extra cargo at the next available stop.

Next, we compare the outbound backlog trajectories of the two undisrupted end ports in the three-port cyclic system, both of which are directly connected to the disrupted intermediate port.

Because the vessels cycle through the same route and follow the same service rules at both undisrupted ports, the sequence of "backlog builds while ships are away" and "backlog is reduced when ships return" repeats with the same shape at each port. The only difference is timing. The bypass of the closed port inserts a fixed delay in the rotation, so the outbound backlog trajectory at one undisrupted port is a time-shifted replica of the other. In the fluid picture, that shift equals the intermediate port's downtime (the window during which the rotation is effectively paused at that node). As a result, the two outbound trajectories exhibit the same oscillations with a constant lag, mirroring the lagged symmetry seen in the two-port case but now coordinated by the disruption window rather than a single sailing leg.

Finally, we explain why no queue forms at the disrupted port while it is closed. During the intermediate port's shutdown, there are no arrivals or departures at that node: ships do not call, no unloading occurs, and no loading is attempted. With no incoming service and no local processing, there is no mechanism for a backlog to appear or grow at the closed port. The congestion, therefore, is not "stored" at the disruption site; it is redistributed to the adjacent ports that continue to handle cargo. This is again visible in Figure F.1, where both inbound and outbound curves at the closed port remain flat during the shutdown while the peripheral ports carry the system's adjustment.

Taken together, these observations establish that a temporary shutdown at the intermediate node of a three-port cycle redistributes inbound pressure to the two open ends, synchronizes their outbound dynamics up to a fixed time shift determined by the closure window, and leaves the disrupted node without any backlog accumulation while it is closed. □

H. Oblique Reflection Mapping

We are interested in deriving the fluid approximation for the two-port system. Specifically, the fluid approximation is concerned with the limits of $\bar{Q}_i^n(nt)/n$, $\bar{Q}_j^n(nt)/n$, $\underline{Q}_i^n(nt)/n$, and $\underline{Q}_j^n(nt)/n$ as $n \to \infty$.

We assume that the arrival and service processes of outbound cargoes, as well as the service processes of inbound cargoes under the usual scaling, follow the strong law of large numbers that, with probability one,

$$\begin{aligned} \frac{1}{t}A_i(t) \to \alpha_i, \qquad & \frac{1}{t}A_j(t) \to \alpha_j, && \text{as } t \to \infty, \\ \frac{1}{t}\bar{S}_i(t) \to \bar{\mu}_i, \quad \frac{1}{t}\bar{S}_j(t) \to \bar{\mu}_j, \quad & \frac{1}{t}\underline{S}_i(t) \to \underline{\mu}_i, \ \frac{1}{t}\underline{S}_j(t) \to \underline{\mu}_j, && \text{as } t \to \infty. \end{aligned}$$

Then, we could apply a "centering" operation to the outbound and inbound backlog process and rewrite (A.1) and (A.2) for the outbound backlog process as follows,

$$\bar{Q}_i(t) = \bar{X}_i(t) + \bar{Y}_i(t), \qquad \bar{Q}_j(t) = \bar{X}_j(t) + \bar{Y}_j(t),$$

where

$$\bar{X}_i(t) = \bar{Q}_i(0) + (\alpha_i - \bar{\mu}_i)t + [A_i(t) - \alpha_i t] - [\bar{S}_i(\bar{B}_i(t)) - \bar{\mu}_i \bar{B}_i(t)],$$
$$\bar{X}_j(t) = \bar{Q}_j(0) + (\alpha_j - \bar{\mu}_j)t + [A_j(t) - \alpha_j t] - [\bar{S}_j(\bar{B}_j(t)) - \bar{\mu}_j \bar{B}_j(t)],$$
$$\bar{Y}_i(t) = \bar{\mu}_i[t - \bar{B}_i(t)] = \bar{\mu}_i \int_0^t \mathbf{1}\{\bar{Q}_i(s) = 0\} ds, \quad \bar{Y}_j(t) = \bar{\mu}_j[t - \bar{B}_j(t)] = \bar{\mu}_j \int_0^t \mathbf{1}\{\bar{Q}_j(s) = 0\} ds.$$

Similarly, we can rewrite (A.3) and (A.4) for inbound backlog process as follows,

$$\underline{Q}_i(t) = \underline{X}_i(t) + \underline{Y}_i(t) - \bar{Y}_j(t-l), \qquad \underline{Q}_j(t) = \underline{X}_j(t) + \underline{Y}_j(t) - \bar{Y}_i(t-l),$$

where

$$\underline{X}_i(t) = \underline{Q}_i(0) + (\bar{\mu}_j - \underline{\mu}_i)t + [\bar{S}_j(\bar{B}_j(t-l)) - \bar{\mu}_j \bar{B}_j(t-l)] - [\underline{S}_i(\underline{B}_i(t)) - \underline{\mu}_i \underline{B}_i(t)] - \bar{\mu}_j l,$$
$$\underline{X}_j(t) = \underline{Q}_j(0) + (\bar{\mu}_i - \underline{\mu}_j)t + [\bar{S}_i(\bar{B}_i(t-l)) - \bar{\mu}_i \bar{B}_i(t-l)] - [\underline{S}_j(\underline{B}_j(t)) - \underline{\mu}_j \underline{B}_j(t)] - \bar{\mu}_i l,$$
$$\underline{Y}_i(t) = \underline{\mu}_i[t - \underline{B}_i(t)] = \underline{\mu}_i \int_0^t \mathbf{1}\{\underline{Q}_i(s) = 0\} ds, \quad \underline{Y}_j(t) = \underline{\mu}_j[t - \underline{B}_j(t)] = \underline{\mu}_j \int_0^t \mathbf{1}\{\underline{Q}_j(s) = 0\} ds.$$

Thus, we have the following equations hold for all $t > 0$,

$$\bar{Q}_i(t) \geq 0, \quad \bar{Q}_j(t) \geq 0, \quad \underline{Q}_i(t) \geq 0, \quad \underline{Q}_j(t) \geq 0, \tag{H.1}$$
$$d\bar{Y}_i(t) \geq 0, \quad d\bar{Y}_j(t) \geq 0, \quad d\underline{Y}_i(t) \geq 0, \quad d\underline{Y}_j(t) \geq 0 \text{ and } \bar{Y}_i(0) = \bar{Y}_j(0) = \underline{Y}_i(0) = \underline{Y}_j(0) = 0, \tag{H.2}$$
$$\bar{Q}_i(t) d\bar{Y}_i(t) = \bar{Q}_j(t) d\bar{Y}_j(t) = \underline{Q}_i(t) d\underline{Y}_i(t) = \underline{Q}_j(t) d\underline{Y}_j(t) = 0. \tag{H.3}$$

Therefore, equations (H.1)–(H.3) uniquely determine a pair of mappings from $\bar{X}$ to $\bar{Q} := \bar{\Phi}(\bar{X})$ and $\bar{Y} := \bar{\Psi}(\bar{X})$ for outbound backlog process, and a pair of mappings from $\underline{X}$ to $\underline{Q} := \underline{\Phi}(\underline{X})$ and $\underline{Y} := \underline{\Psi}(\underline{X})$ for inbound backlog process. Furthermore, when we take the limit $n \to \infty$, we allow the initial outbound (resp., inbound) backlog level to depend on n, which are $\bar{Q}_i^n(0)$ (resp., $\underline{Q}_i^n(0)$) in Port i and $\bar{Q}_j^n(0)$ (resp., $\underline{Q}_j^n(0)$) in Port j. Accordingly, we index the performance of the outbound (resp., inbound) backlog process by a superscript n, denoted by $\bar{Q}_i^n$ (resp., $\underline{Q}_i^n$) for Port i, and $\bar{Q}_j^n$ (resp., $\underline{Q}_j^n$) for Port j. We concentrate on analyzing the limits of $\frac{1}{n}\bar{Q}_i^n(nt)$ and $\frac{1}{n}\bar{Q}_j^n(nt)$ in relation to the outbound backlog process, and the limits of $\frac{1}{n}\underline{Q}_i^n(nt)$ and $\frac{1}{n}\underline{Q}_j^n(nt)$ in relation to the inbound backlog process as $n \to \infty$. We assume that, almost surely, we have

$$\frac{1}{n}\bar{Q}_i^n(0) \to \bar{Q}_i^0, \qquad \frac{1}{n}\bar{Q}_j^n(0) \to \bar{Q}_j^0, \qquad \text{as } n \to \infty,$$
$$\frac{1}{n}\underline{Q}_i^n(0) \to \underline{Q}_i^0, \qquad \frac{1}{n}\underline{Q}_j^n(0) \to \underline{Q}_j^0, \qquad \text{as } n \to \infty,$$

with $\bar{Q}_i^0$, $\bar{Q}_j^0$, $\underline{Q}_i^0$, and $\underline{Q}_j^0$ are constant numbers.

I. Outbound Backlog Fluid Process

The associated fluid process $O_i(t)$ of the outbound backlog process at Port i can be written as follows:

(i) If $C \geq \min\{K, \frac{\tau}{2l}+1\}$,

$$O_i(t) = \begin{cases} \lambda t & t \in [0,\tau], \\ \lambda\tau - (K-1)\lambda(t-\tau) & t \in (\tau, \tau + \dfrac{\tau}{K-1}], \\ 0 & t \in (\tau + \dfrac{\tau}{K-1}, \infty). \end{cases}$$

(ii) Consider the situation in which $C < \min\{K, \frac{\tau}{2l}+1\}$.

If $\frac{\tau}{2l(C-1)} = \mathbb{Z}^+$, we have

$$O_i(t) = \begin{cases} \lambda t & t \in [0,\tau], \\ K\lambda\tau + 2(K-C)n\lambda l - (K-1)\lambda t & t \in (\tau + 2nl, \tau + 2\dfrac{C}{K}l + 2nl], \\ \lambda t - 2C(n+1)\lambda l & t \in (\tau + 2\dfrac{C}{K}l + 2nl, \tau + 2(n+1)l], \\ 0, & t \in (\tau + 2Nl, \infty). \end{cases}$$

If $\frac{\tau}{2l(C-1)} \neq \mathbb{Z}^+$, we have

$$O_i(t) = \begin{cases} \lambda t & t \in [0,\tau], \\ K\lambda\tau + 2(K-C)n\lambda l - (K-1)\lambda t & t \in (\tau + 2nl, \tau + 2\dfrac{C}{K}l + 2nl], \\ \lambda t - 2C(n+1)\lambda l & t \in (\tau + 2\dfrac{C}{K}l + 2nl, \tau + 2(n+1)l], \\ \lambda\tau - 2(C-1)N\lambda l - (K-1)\lambda(t-\tau-2Nl) & t \in (\tau + 2Nl, \tau + 2Nl + \dfrac{\tau - 2(C-1)Nl}{(K-1)}], \\ 0 & t \in (\tau + 2Nl + \dfrac{\tau - 2(C-1)Nl}{(K-1)}, \infty), \end{cases}$$

where $n \equiv \lfloor \frac{t-\tau}{2l} \rfloor$ and $n \leq N-1$, with $N \equiv \lfloor \frac{\tau}{2l(C-1)} \rfloor$. Note that although we do allow for a general case of $C=1$ in Proposition 6(ii), here we only focus on scenarios that $C > 1$.

The associated fluid process $O_j(t)$ of the outbound backlog process at Port j is

$$O_j(t) = \begin{cases} 0 & t \in [0,l], \\ O_i(t-l) & t \in (l, \infty). \end{cases}$$

J. Inbound Backlog Fluid Process

The associated fluid process $I_i(t)$ of the inbound backlog process at Port i can be written as follows,

(i) If $\tau < 2l$ and $K < \frac{2l}{2l-\tau}$,

$$I_i(t) = \begin{cases} \lambda t & t \in [0, \tau], \\ \lambda\tau - (K-1)\lambda(t-\tau) & t \in (\tau, 2l], \\ \lambda\tau - (K-1)\lambda(2l-\tau) - K\lambda(t-2l) & t \in (2l, \tau + \dfrac{2l}{K}], \\ 0 & t \in (\tau + \dfrac{2l}{K}, \infty). \end{cases}$$

(ii) If $\tau < 2l$ and $K \geq \frac{2l}{2l-\tau}$,

$$I_i(t) = \begin{cases} \lambda t & t \in [0, \tau], \\ \lambda\tau - (K-1)\lambda(t-\tau) & t \in (\tau, \tau + \dfrac{\tau}{K-1}], \\ 0 & t \in (\tau + \dfrac{\tau}{K-1}, \infty). \end{cases}$$

(iii) If $\tau \geq 2l$,

$$I_i(t) = \begin{cases} \lambda t & t \in [0, 2l], \\ 2\lambda l & t \in (2l, \tau], \\ 2\lambda l - K\lambda(t-\tau) & t \in (\tau, \tau + \dfrac{2l}{K}], \\ 0 & t \in (\tau + \dfrac{2l}{K}, \infty). \end{cases}$$

The associated fluid process $I_j(t)$ of the inbound backlog process at Port j is

$$I_j(t) = 0, \quad t \in [0, \infty).$$