

An Information-Based Theory of Auditor Switching

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Appendix A. Properties of Mixed Strategy Bidding Equilibrium

1 Intuition Behind Mixed Strategy Bidding Equilibrium

We follow the first-price auction with asymmetrically informed bidders in Engelbrecht-Wiggans, Milgrom, and Weber (1983) and establish several properties of the bidding strategies in such an auction. We first show that the equilibrium strategies are mixed strategies, rather than pure strategies, because the probability density functions (pdfs) for these equilibrium bidding strategies have no atoms (except possibly at the upper bound) and no gaps (i.e., positive, non-zero values throughout the bidding range). We then present the cumulative distribution function (cdf) and pdf of the randomized mixed bidding strategies. Finally, we illustrate the characteristics of these density functions and provide the intuition for why only mixed strategies exist in this type of bidding game, followed by detailed step-by-step proofs in the next subsection.

(1) No pure strategy bidding equilibrium exists for the incumbent auditor in the low-cost state and for the prospective auditor

First, we need to establish that the bidding range is $[\bar{w}, w^h]$, where \bar{w} is the expected total audit cost, defined as $\bar{w} \equiv \lambda w^h + (1 - \lambda)w^l$. The outside prospective auditor adopts a bidding strategy b_O . The informed incumbent auditor adopts a bidding strategy b_I^l in the low-cost state and b_I^h in the high-cost state. If one bids over w^h and makes a profit for sure, the other can bid slightly

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less than w^h . The competition between the bidders will drive away any profit, resulting in w^h as the bidding upper bound. The outside prospective auditor will not bid less than \bar{w} , because the expected profit is negative for $b_O < \bar{w}$. The informed incumbent auditor will also not bid less than \bar{w} , because this will decrease his revenue without increasing the probability of winning.

Second, we need to argue that the outside prospective auditor will adopt a randomized bidding strategy over the bidding range. For any pure strategy bid $b_O \in [\bar{w}, w^h)$ by the outside prospective auditor, the informed incumbent auditor can respond with a strategy of bidding $b_I^l = b_O$ in the low-cost state and $b_I^h = w^h$ in the high-cost state. Given that the tie-breaking rule favors the incumbent by our assumption, the outside prospective auditor will only win in the high-cost state and thus suffer a loss of $b_O - w^h$. The winner's curse will drive the prospective auditor to deviate from adopting this pure strategy in bidding. For a pure strategy bid $b_O = w^h$ by the prospective auditor, the incumbent auditor will respond by bidding $b_I^l = w^h$ in the low-cost state and $b_I^h = w^h$ in the high-cost state. The prospective auditor will then have an incentive to deviate, because she can bid $b_O \in (\bar{w}, w^h)$, win the audit engagement all the time, and earn a profit of $b_O - \bar{w}$. Consequently, the uninformed prospective auditor will randomize her bid b_O over the bidding range $[\bar{w}, w^h)$. Similarly, we can show that the informed incumbent auditor will also randomize her bids over the same bidding range.

(2) The mixed bidding strategies have no atoms in the bidding range (except at the upper bound)

An atom in a mixed strategy refers to a bid value that is assigned a positive probability (i.e. a non-zero probability mass). Suppose that there is an atom in the outside prospective auditor's mixed bidding strategy b_O . Then, this outside prospective auditor would choose a bid $a \in [\bar{w}, w^h)$ with a strictly positive probability, i.e., $b_O(a) = \Pr(b_O = a) > 0$. Given that the tie-breaking rule is in favor of the incumbent, there exists a small neighborhood $\varepsilon > 0$, in which the low-cost informed incumbent auditor will do better by bidding $b_I^l = a$ rather than $b_I^l \in (a, \varepsilon]$. Relative to a slightly higher bid $b_I^l = a + \varepsilon$, the lower bid $b_I^l = a$ allows the incumbent auditor to win with a strictly positive probability (i.e. the value of atom $b_O(a)$) and achieve higher expected profits. Then, the outside prospective auditor can do better by bidding

$b_O = a + \varepsilon$ instead of $b_O = a$. This contradiction demonstrates that having an atom in the outside prospective auditor’s bidding strategy b_O provides bidders incentives to deviate and undermines the definition of equilibrium bidding strategies. In Lemma 4, we demonstrate in detail that the outside prospective auditor’s bidding strategy b_O has no atoms on $[\bar{w}, w^h)$. Analogously, in Lemma 5, we demonstrate in detail that the informed incumbent auditor’s bidding strategy in the low-cost state b_I^l has no atoms on $[\bar{w}, w^h)$. Overall, in a Bayesian Nash equilibrium, each auditor must be indifferent among all bids they place within the bidding range of the mixed strategy. This indifference is maintained because the expected payoff from each bid must be the same. An atom would provide a focal point that the opponent could exploit, causing the expected payoff to be higher at the atom point than at its closest neighboring points. Consequently, no atoms can exist in the bidding range except at the upper bound w^h .

The atom at the bidding upper bound w^h is driven by the binomial structure in this model — i.e., the fact that a high audit cost occurs with a positive probability of λ . When privately observing a high audit cost, the informed incumbent will always bid the upper bound w^h . This pure strategy results in an atom of λ in the informed incumbent’s expected bidding strategy. Consequently, the prospective auditor’s bidding strategy also has an atom of λ at the upper bound w^h , to avoid excessive losses and ensure zero expected profits. This finding is consistent with the statement by Engelbrecht-Wiggans, Milgrom, and Weber (EMW hereafter, 1983, footnote 6, pp. 164-165) that when the informed’s bidding strategy H has an atom at the lower bound $H(0)$, the uninformed’s bidding strategy G also has an atom at the lower bound $H(0)$. Note that bidding at the upper bound in our paper corresponds to bidding at the lower bound in EMW. Unlike the highest-value bidding in EMW, we use the lowest-cost bidding to model audit retendering.

(3) The mixed strategy bidding equilibrium has no gaps in the bidding range.

In equilibrium, an auditor using a mixed strategy must be indifferent to all bids in the bidding range, which means that any bid in the bidding range provides the same expected payoff. A gap would imply that a specific bidding range is never used. Suppose that there is a gap $[a', a'']$ in the outside prospective auditor’s bidding strategy b_O . Then, this outside prospective auditor would not bid in the given range $[a', a'']$, i.e., $\Pr(b_O \in [a', a'']) = 0$. As a response, the incumbent

auditor in the low-cost state would bid $b_I^l = a''$ rather than any point in the range $[a', a'']$, i.e., $\Pr(b_I^l \in [a', a'']) = 0$. This would not affect the incumbent auditor's probability of winning in the low-cost state, but it would increase the audit fee when the incumbent auditor wins. In return, instead of bidding slightly less than a' or $b_O \in (a' - \eta, a')$, the uninformed prospective auditor would do better by bidding an interior point in the range $[a', a'']$, such as $b_O = \frac{a' + a''}{2}$. Since there is no atom at point a' , this would reduce the prospective auditor's winning probability by a very small amount, but it would increase the audit fee by a significant amount when the prospective auditor wins. This contradiction demonstrates that the outside prospective auditor's bidding strategy b_O has no gaps. In Lemma 6, we demonstrate in detail that the outside prospective auditor's bidding strategy b_O has no gaps, resulting in a positive density function. Analogously, in Lemma 7, we demonstrate in detail that the informed incumbent auditor's bidding strategy in the low-cost state b_I^l has no gaps, resulting in a positive pdf.

(4) The mixed strategy bidding equilibrium is over a continuum based on the support for the cdfs.

Our version of EMW is a first-price auction with two types under information asymmetry. In Lemma 4, we rule out the existence of an atom in the outside prospective auditor's bidding function b_O (except possibly at the upper bound w^h). This is the same as ruling out a pure strategy equilibrium or, in other words, showing that the prospective auditor must adopt a mixed strategy equilibrium. In Lemma 6, we rule out the existence of a gap in the outside prospective auditor's bidding function b_O , which is the same as showing that b_O has a positive density function. Lemma 4 and Lemma 6 together demonstrate that the outside prospective auditor adopts a mixed strategy b_O over a continuum $[\bar{w}, w^h]$ (the support for the cdf of b_O). Analogously, Lemma 5 and Lemma 7 together show that the informed incumbent auditor adopts a mixed strategy b_I^l over a continuum $[\bar{w}, w^h]$ (the support for the cdf of b_I^l). Please note that Lemma 3 shows that the incumbent in the high-cost state adopts a pure strategy and always bids the upper bound $b_I^h = w^h$. Collectively, we show that the equilibrium bidding strategies in the auction are mixed strategies, whose density functions are smooth without gaps and atoms (except possibly at the upper bound w^h).

(5) Steps for solving bidding strategies.

The next subsection provides step-by-step proofs and completes the solution of bidding strategies.

- Lemmas 1-3 prove that no pure bidding strategy exists and establish the bidding range.
- Lemmas 4 and 5 prove that the mixed bidding strategy has no atoms except possibly at the upper bound w^h .
- Lemmas 6 and 7 show that both the incumbent's and the prospective auditor's bidding strategies have no gaps in the bidding density function.

Having established the above results, we can now solve the equilibrium strategies for the uninformed prospective auditor and for the informed incumbent auditor when the client type is low. Let $\{\mathcal{F}_O(b), \mathcal{F}_I^l(b), \mathcal{F}_I^h(b)\}$ denote the bidding strategies (i.e., cdfs) of the prospective auditor, the incumbent auditor with a low audit cost, and the incumbent auditor with a high audit cost, respectively. With a mixed bidding strategy, the prospective auditor's expected payoff, given $b_O = b$, is

$$\begin{aligned} E[U_O(b)] &= \lambda \Pr(\tilde{b}_I^h > b)(b - w^h) + (1 - \lambda) \Pr(\tilde{b}_I^l > b)(b - w^l), \\ &= \lambda(1 - \mathcal{F}_I^h(b))(b - w^h) + (1 - \lambda)(1 - \mathcal{F}_I^l(b))(b - w^l), \end{aligned} \tag{1}$$

where $w^r = (1 - q_r)(1 - p)L + \frac{1}{2}c_r q^2$ for $r \in \{l, h\}$. The first term in (1) represents the case where given a high audit cost w^h , the prospective auditor wins the auction if the incumbent's bid \tilde{b}_I^h exceeds the prospective auditor's bid b . The prospective auditor wins the auction with probability $\Pr(\tilde{b}_I^h > b)$ and thus realizes an payoff $b - w^h$ (which is negative unless $b = w^h$). Similarly, the second term in (1) shows that if the audit cost is w^l , the prospective auditor wins the auction with probability $\Pr(\tilde{b}_I^l > b)$ and earns a positive payoff $b - w^l$ ($b \geq \bar{w} > w^l$).

With the mixed strategy, the prospective auditor must be indifferent to all bid b and expects to break even $E[U_O(b)] = 0$. Since the incumbent always bids $b_I^h = w^h$ given a high audit cost, we

obtain

$$\mathcal{F}_I^l(b) = \frac{b - \bar{w}}{(1 - \lambda)(b - w^l)}.$$

When the client's audit cost is low, the incumbent auditor randomizes and has the same expected profit over the bidding range. Thus, for $b_I^l = b \in [\bar{w}, w^h)$, the incumbent's expected payoff is

$$E[U_I^l(b)] = \Pr(b > \tilde{b}_O)(b - w^l), \quad (2)$$

$$= (1 - \mathcal{F}_O(b))(b - w^l) \quad (3)$$

$$= E[U_I^l(\bar{w})] = (\bar{w} - w^l).$$

Note that the third, last equation follows from the fact that the low-cost incumbent's profit at any point of the bidding range is the same as his profit at the lower bound. Thus we have

$$\mathcal{F}_O(b) = \frac{b - \bar{w}}{b - w^l},$$

It follows that $f_O(b)$ and $f_I^l(b)$ represent the p.d.f of the prospective auditor's and the incumbent's bidding strategy, respectively,

$$f_O(b) = \frac{\lambda(w^h - w^l)}{(b - w^l)^2}, \quad (4)$$

$$f_I^l(b) = \frac{\lambda(w^h - w^l)}{(1 - \lambda)(b - w^l)^2},$$

for $b \in [\bar{w}, w^h)$. We can verify that $\lim_{b \rightarrow w^h} \mathcal{F}_I^l(b) = 1$. Thus, \mathcal{F}_I^l is continuous on all of its domain $[\bar{w}, w^h]$.

It can be shown that there is an atom (mass point) in the prospective auditor's bidding strategy $f_O(b)$ at the upper bound w^h . Since $F_O(w^h) = \lim_{b \rightarrow w^h} F_O(b) = 1 - \lambda$, the atom (mass point) at the bidding upper bound w^h is $\Pr(b_O = w^h) = 1 - F_O(w^h) = 1 - (1 - \lambda) = \lambda$.

From Lemma 3, when the client's audit cost is high, the incumbent always bids w^h . Thus, we obtain a unique set of bidding strategies for the uninformed prospective auditor (i.e., $\mathcal{F}_O(b)$) and for the informed incumbent (i.e., $\mathcal{F}_I^l(b)$ for the low-cost client and $\mathcal{F}_I^h(b)$ for the high-cost client).

Neither auditor has an incentive to defect from the equilibrium because any deviation will result in a lower payoff.

2 Detailed Step-by-Step Proofs

This subsection provides step-by-step proofs of the existence and properties of mixed bidding strategies. Lemmas 1-3 prove that no pure bidding strategy exists and establish the bidding range. Lemmas 4 and 5 prove that the mixed bidding strategy has no atoms except at the upper bound w^h . Lemmas 6 and 7 show that both the incumbent's and the prospective auditor's bidding strategies have no gaps in the bidding density function.

We denote the prospective auditor's bid by b_O . We also denote the incumbent auditor's bid when the client type is low and high by b_I^l and b_I^h , respectively. Let $\mathcal{F}_O(b)$ denote the cumulative distribution function (c.d.f) of the bid $b = b_O$ from the uninformed prospective (outside) auditor, and $\mathcal{F}_I^r(b)$ denote the c.d.f. of the bid $b = b_I^r$ from the incumbent auditor with the client's type $r \in \{l, h\}$. The incumbent has private information about the client's audit cost r through its prior engagement experience and thus observes the ex-post realization of r . The prospective auditor cannot observe the client's audit cost r . As a prior, the audit cost r can be either high ($r = h$) with ex-ante probability $\lambda \in (0, 1)$, or low ($r = l$) with probability $1 - \lambda$. The proof proceeds with the following lemmas.

Lemma 1 *Let $[b, \bar{b}]$ denote the bidding range. Then the bidding lower bound $\underline{b} = \bar{w}$ and the bidding upper bound $\bar{b} = w^h$.*

Let w^h (w^l) denote the prospective auditor's total audit cost when the client type is high (low). The expected total audit cost is $\bar{w} \equiv \lambda w^h + (1 - \lambda)w^l$, where $\lambda \in (0, 1)$. It is clear that both the prospective auditor and incumbent auditors should have identical bidding ranges.

First, since the prospective auditor will incur a loss if $\underline{b} < \bar{w}$, we establish that $\underline{b} \geq \bar{w}$. Suppose $\underline{b} > \bar{w}$. Then the incumbent will not bid less than \underline{b} , since that will decrease his revenue without increasing the probability of winning. Thus, the prospective can win the bid with certainty and make a positive profit by bidding $\underline{b} = \bar{w} - \varepsilon$, where $\varepsilon \in (0, \underline{b} - \bar{w})$. This is a contradiction to the fact that \underline{b} is the bidding lower bound. Hence, the bidding lower bound is $\underline{b} = \bar{w}$.

Next, we show that the bidding upper bound $\bar{b} = w^h$. If $\bar{b} < w^h$, the informed incumbent will lose money when the firm has high control risk. If $\bar{b} > w^h$, the two auditors can undercut each

other's bid by a small amount, which drives away the profit. This indicates that the bidding upper bound is $\bar{b} = w^h$.

Lemma 2 *Let \tilde{b}_I^h and \tilde{b}_I^l be the incumbent's equilibrium bidding strategies (possibly mixed strategies) when the client's audit cost is high and low, respectively. Then $w^h > w^l$ implies $\tilde{b}_I^h \geq \tilde{b}_I^l$.*

Let \bar{b}_I^h (\bar{b}_I^l) be an arbitrary deterministic selection from \tilde{b}_I^h (\tilde{b}_I^l). We need to show that $\bar{b}_I^h \geq \bar{b}_I^l$. Suppose not. Then $\bar{b}_I^h < \bar{b}_I^l$. This implies that $\Pr(\bar{b}_I^h\text{-wins}) > \Pr(\bar{b}_I^l\text{-wins})$. If not, then $\Pr(\bar{b}_I^h\text{-wins}) \leq \Pr(\bar{b}_I^l\text{-wins})$. Thus, the incumbent's expected payoff is

$$(\bar{b}_I^h - w^h) \Pr(\bar{b}_I^h\text{-wins}) < (\bar{b}_I^l - w^h) \Pr(\bar{b}_I^l\text{-wins}),$$

which contradicts the fact that \bar{b}_I^h is an optimal bid when the client's type is high. Thus, given $\Pr(\bar{b}_I^h\text{-wins}) > \Pr(\bar{b}_I^l\text{-wins})$, we have

$$\begin{aligned} & (w^h - w^l) \Pr(\bar{b}_I^h\text{-wins}) > (w^h - w^l) \Pr(\bar{b}_I^l\text{-wins}) \\ \Rightarrow & \left[-(\bar{b}_I^h - w^h) + (\bar{b}_I^h - w^l) \right] \Pr(\bar{b}_I^h\text{-wins}) > \left[-(\bar{b}_I^l - w^h) + (\bar{b}_I^l - w^l) \right] \Pr(\bar{b}_I^l\text{-wins}) \\ \Rightarrow & (\bar{b}_I^h - w^l) \Pr(\bar{b}_I^h\text{-wins}) - (\bar{b}_I^l - w^l) \Pr(\bar{b}_I^l\text{-wins}) \\ > & (\bar{b}_I^h - w^h) \Pr(\bar{b}_I^h\text{-wins}) - (\bar{b}_I^l - w^h) \Pr(\bar{b}_I^l\text{-wins}) \geq 0, \end{aligned}$$

where the last weak inequality follows the definition that \bar{b}_I^h is an optimal bidding strategy for the high-type client. This inequality further implies that

$$(\bar{b}_I^h - w^l) \Pr(\bar{b}_I^h\text{-wins}) > (\bar{b}_I^l - w^l) \Pr(\bar{b}_I^l\text{-wins}).$$

Thus, \bar{b}_I^l cannot be an optimal bidding strategy for the low-type client because the incumbent can obtain a higher expected payoff by bidding \bar{b}_I^h . Hence, a contradiction is obtained. Since \bar{b}_I^h and \bar{b}_I^l are arbitrarily chosen, the proof follows.

Lemma 3 *For the high-type client, the incumbent adopts a pure strategy and always bids the upper bound $b_I^h = w^h$.*

Given the high-type client, the incumbent will not bid lower than w^h because the expected payoff is negative. Suppose that the incumbent does not always bid $b_I^h = w^h$. Then he bids $b_I^h \in [w'_h, w^h)$ with positive probability, where $w'_h < w^h$. Lemma 2 shows $b_I^h \geq b_I^l$. This implies that when the client's audit cost is low, the incumbent will not bid in $(w'_h, w^h]$ or $b_I^l \notin (w'_h, w^h]$. Thus if the prospective auditor submits any bid $b_O \in [w'_h, w^h)$, then the prospective auditor's expected payoff is

$$U_O(b_O) = \lambda \Pr(b_I^h > b_O)(b_O - w^h) + (1 - \lambda) \Pr(b_I^l > b_O)(b_O - w^l) \quad (5)$$

$$= \lambda \Pr(b_I^h > b_O)(b_O - w^h) + (1 - \lambda) \times 0 \times (b_O - w^l) \quad (6)$$

$$< 0,$$

where $\Pr(b_I^h > b_O) > 0$, or otherwise w^h is not an upper bound. Because $b_I^l \notin (w'_h, w^h]$, the prospective auditor has no chance of winning when the incumbent has the low-cost client, or $\Pr(b_I^l > b_O) = 0$. However, the prospective auditor wins with a positive probability when the incumbent has the high-cost client, $\Pr(b_I^h > b_O) > 0$ because w^h is the upper bound by Lemma 1. Due to the winner's curse, the prospective auditor will suffer a loss, if she bids $b_O \in [w'_h, w^h)$. Thus, the prospective auditor will not submit any bid in the range of $[w'_h, w^h)$, i.e., $b_I^l \notin [w'_h, w^h)$, but bids w^h to break even. In response, instead of bidding $b_I^h \in [w'_h, w^h)$, the incumbent should also bid $b_I^h = w^h$ because he will win with the same probability and earn more if he wins. This gives a contradiction.

Lemma 4 $\mathcal{F}_O(b)$ is atomless in $[\bar{w}, w^h)$, except possibly at the upper bound w^h .

Suppose that there exists an atom $a \in [\bar{w}, w^h)$ such that the prospective auditor bids a with probability $\Pr(b_O = a) = f_O(a) > 0$. First, we will prove that when the client type is low, the incumbent's bidding function has a gap; that is, there exists $\varepsilon > 0$ such that $\Pr(b_I \in (a, a + \varepsilon]) = 0$.

Specifically, given the low-type client, the incumbent's expected payoff for any $b_I^l = b \in (a, a + \varepsilon]$ is

$$U_I^l(b) = \Pr(b_O \geq b)(b - w^l), \quad (7)$$

$$\leq \Pr(b_O \geq b)(a + \varepsilon - w^l), \quad (8)$$

$$\leq (\Pr(b_O \geq a) - f_O(a))(a - w^l + \varepsilon), \quad (9)$$

$$< (\Pr(b_O \geq a) - f_O(a)) \left[a - w^l + \frac{f_O(a)}{\Pr(b_O > a) - f_O(a)} (a - w^l) \right], \quad (10)$$

$$= \Pr(b_O \geq a)(a - w^l), \quad (11)$$

$$= U_I^l(b = a),$$

for an $\varepsilon \in \left(0, \min \left\{ \frac{f_O(a)(a-w^l)}{\Pr(b_O > a) - f_O(a)}, w^h - a \right\}\right)$. As $U_I^l(b = a) > U_I^l(b)$, when the client type is low, the incumbent is better off by bidding $b = a$ than $b \in (a, a + \varepsilon]$. This implies $\Pr(b_I^l > a) = \Pr(b_I^l > a + \varepsilon)$.

Second, consider the case where the prospective auditor's expected payoff, given a bid $b_O = b = a + \varepsilon$,

$$U_O(b_O = a + \varepsilon) = (1 - \lambda) \Pr(b_I^l > a + \varepsilon)(b - w^l) + \lambda(a + \varepsilon - w^h), \quad (12)$$

$$= (1 - \lambda) \Pr(b_I^l > a)(b - w^l) + \lambda(a + \varepsilon - w^h), \quad (13)$$

$$> (1 - \lambda) \Pr(b_I^l > a)(b - w^l) + \lambda(a - w^h), \quad (14)$$

$$= U_O(b_O = a).$$

Therefore, the prospective auditor is better off bidding $b_O = a + \varepsilon$ rather than $b_O = a$. This is a contradiction to $\Pr(b_O = a) = f_O(a) > 0$.

Lemma 5 $\mathcal{F}_I^l(b)$ is atomless in $[\bar{w}, w^h)$.

We will consider two cases. First, suppose that there is an atom at the lower bound $a = \bar{w}$, such that $f_I^l(a = \bar{w}) > 0$. We need to show that there is a gap in the prospective auditor's bidding function. For any $b_O = b \in [\bar{w}, \bar{w} + \varepsilon)$, $\varepsilon \in \left(0, \min \left\{ \frac{(1-\lambda)\mathcal{F}_I^l(\bar{w})(\bar{w}-w^l)}{[1-\mathcal{F}_I^l(\bar{w})(1-\lambda)]}, w^h - \bar{w} \right\}\right)$, the prospective

auditor's expected payoff is

$$U_O(b) = (1 - \lambda) \Pr(b_I^l > b)(b - w^l) + \lambda(b - w^h), \quad (15)$$

$$\leq (1 - \lambda) \Pr(b_I^l > \bar{w})(b - w^l) + \lambda(b - w^h), \quad (16)$$

$$= (1 - \lambda)(1 - \mathcal{F}_I^l(\bar{w}))(b - w^l) + \lambda(b - w^h), \quad (17)$$

$$< (1 - \lambda)(1 - \mathcal{F}_I^l(\bar{w}))(\bar{w} + \varepsilon - w^l) + \lambda(\bar{w} + \varepsilon - w^h), \quad (18)$$

$$= (1 - \lambda)(1 - \mathcal{F}_I^l(\bar{w}))(\bar{w} - w^l) + \lambda(\bar{w} - w^h) + \varepsilon[1 - \mathcal{F}_I^l(\bar{w})(1 - \lambda)], \quad (19)$$

$$< (1 - \lambda)(1 - \mathcal{F}_I^l(\bar{w}))(\bar{w} - w^l) + \lambda(\bar{w} - w^h) + \frac{(1 - \lambda)\mathcal{F}_I^l(\bar{w})(\bar{w} - w^l)}{[1 - \mathcal{F}_I^l(\bar{w})(1 - \lambda)]}[1 - \mathcal{F}_I^l(\bar{w})(1 - \lambda)] \quad (20)$$

$$= (1 - \lambda)(\bar{w} - w^l) + \lambda(\bar{w} - w^h) = 0.$$

Thus the prospective auditor will not bid in $[\bar{w}, \bar{w} + \varepsilon)$, suggesting that $\Pr(b_O \geq \bar{w}) = \Pr(b_O \geq \bar{w} + \varepsilon) = 1$.

For the incumbent auditor with the low-type client, the expected payoff is

$$U_I^l(b = \bar{w} + \varepsilon) = \Pr(b_O \geq \bar{w} + \varepsilon)(\bar{w} + \varepsilon - w^l), \quad (21)$$

$$> \Pr(b_O \geq \bar{w})(\bar{w} - w^l), \quad (22)$$

$$= U_I^l(b = \bar{w}).$$

Thus, the incumbent is better off bidding $\bar{w} + \varepsilon$ instead of \bar{w} . This is a contradiction that $f_I^l(a = \bar{w}) > 0$.

Second, consider the case where the atom $a \in (\bar{w}, w^h)$. Let $\Delta = \min\{a - \bar{w}, w^h - a\}$. We will show that there exists $\varepsilon \in (0, \Delta)$, such that for any $b' \in [a, a + \varepsilon)$ and $b'' \in (a - \varepsilon, a)$, $U_O(b') < U_O(b'')$.

Note that for $b_O = b' \in [a, a + \varepsilon)$, the prospective auditor's expected payoff is

$$U_O(b') = (1 - \lambda) \Pr(b_I^l > b')(b' - w^l) + \lambda(b' - w^h) \quad (23)$$

$$\leq (1 - \lambda) \Pr(b_I^l > a)(b' - w^l) + \lambda(b' - w^h), \quad (24)$$

$$< (1 - \lambda) \Pr(b_I^l > a)(a + \varepsilon - w^l) + \lambda(a + \varepsilon - w^h), \quad (25)$$

$$= (1 - \lambda) \Pr(b_I^l > a)(a - w^l) + \lambda(a - w^h) + \varepsilon[(1 - \lambda) \Pr(b_I^l > a) + \lambda].$$

Similarly, for $b'' \in (a - \varepsilon, a)$, the prospective auditor's expected payoff is

$$U_O(b'') = (1 - \lambda) \Pr(b_I^l > b'')(b'' - w^l) + \lambda(b'' - w^h) \quad (26)$$

$$\geq (1 - \lambda) \Pr(b_I^l \geq a)(b'' - w^l) + \lambda(b'' - w^h), \quad (27)$$

$$> (1 - \lambda) \Pr(b_I^l \geq a)(a - \varepsilon - w^l) + \lambda(a - \varepsilon - w^h) \quad (28)$$

$$= (1 - \lambda) \Pr(b_I^l \geq a)(a - w^l) + \lambda(a - w^h) - \varepsilon[(1 - \lambda) \Pr(b_I^l \geq a) + \lambda].$$

To ensure that $U_O(b') < U_O(b'')$,

$$\varepsilon \leq \frac{(1 - \lambda)(a - w^l)f_I^l(a)}{2\hat{\lambda} + (1 - \lambda)[2\Pr(b_I^l \geq a) - f_I^l(a)]}.$$

Thus, for any $\varepsilon \in \left(0, \min \left\{ \frac{(1 - \lambda)(a - w^l)f_I^l(a)}{2\hat{\lambda} + (1 - \lambda)[2\Pr(b_I^l \geq a) - f_I^l(a)]}, \Delta \right\} \right)$, $U_O(b') < U_O(b'')$ suggesting that the prospective auditor will be better off bidding b'' instead of b' . As a result, there is a gap in the prospective auditor's bidding function, i.e., $\Pr(b_O \in [a, a + \varepsilon)) = 0$ and it follows

$$U_I^l(b_I^l = a + \varepsilon) = \Pr(b_O \geq a + \varepsilon)(a + \varepsilon - w^l) \quad (29)$$

$$= \Pr(b_O \geq a)(a + \varepsilon - w^l), \quad (30)$$

$$> \Pr(b_O \geq a)(a - w^l), \quad (31)$$

$$= U_I^l(b = a).$$

Thus, given the low-type client, the incumbent can do better by bidding $a + \varepsilon$ instead of a . This

is a contradiction to $f_I^l(a) > 0$.

Lemma 6 *The prospective auditor's bid b_O has no gaps.*

Suppose there is a gap. Then there exists an interval $[a', a''] \subset [\bar{w}, w^h]$ such that $b_O \notin [a', a'']$. We will consider two cases. First, suppose $a' = \bar{w}$. The incumbent will bid a'' because he can increase his payoff without decreasing the probability of winning. Then the prospective auditor will bid $a'' - \varepsilon$, where $\varepsilon \in (0, a'' - a')$ and make a positive profit of $[a'' - \varepsilon - \bar{w}]$. This is a contradiction to $b_O \notin [a', a'']$.

Second, suppose $a' > \bar{w}$. Then there exists $\Delta \in (0, a' - \bar{w})$ such that the prospective auditor bids with positive probability in every subset of $(a' - \Delta, a')$; otherwise, the first case will be repeated. Note that for $b_I^l = b \in [a', a'']$, $\Pr(b_O \geq b) = \Pr(b_O \geq a'')$.

$$U_I^l(b_I^l = b) = \Pr(b_O \geq F)(b - w^l), \quad (32)$$

$$= \Pr(b_O \geq a'')(b - w^l), \quad (33)$$

$$< \Pr(b_O \geq a'')(a'' - w^l), \quad (34)$$

$$= U_I^l(b_I^l = a'').$$

As a result, the incumbent will not bid in $[a', a'']$, suggesting that $\Pr(b_I^l \geq a') = \Pr(b_I^l \geq a'') = \Pr(b_I^l \geq (a' + a'')/2)$. We next show that there exists $\eta \in (0, \Delta)$ such that for $b_O = b \in (a' - \eta, a')$, $U_O(b) < U_O((a' + a'')/2)$. Let

$$\varepsilon^* = \frac{\frac{a'' - a'}{2} \left[\lambda + (1 - \lambda) \Pr(b_I^l \geq \frac{a' + a''}{2}) \right]}{(1 - \lambda)(a' - w^l)}.$$

Recall that $\mathcal{F}_I^l(b)$ is atomless on $[\bar{w}, w^h]$, which implies that $\Pr(b_I^l \geq b)$ is also continuous. Following the property of continuity, for $\varepsilon^* > 0$, there exists $\delta^* > 0$, such that $|b - a'| < \delta^*$ implies $|\Pr(b_I^l \geq b) - \Pr(b_I^l \geq a')| < \varepsilon^*$. Given that $\Pr(b_I^l \geq b)$ decreases in b , it follows that $0 < b - a' < \delta^*$ implies $(\Pr(b_I^l \geq b) - \Pr(b_I^l \geq a')) < \varepsilon^*$ or $b \in (a' - \delta^*, a')$ implies that $\Pr(b_I^l \geq b) < \delta^* + \Pr(b_I^l \geq a')$. Take

a $\eta \in (0, \min(\delta^*, \Delta))$. For $b_O = b \in (a' - \eta, a')$,

$$U_O(b) = (1 - \lambda) \Pr(b_I^l > b)(b - w^l) + \lambda(b - w^h), \quad (35)$$

$$< (1 - \lambda)(\varepsilon^* + \Pr(b_I^l \geq a'))(a' - w^l) + \lambda(a' - w^h), \quad (36)$$

$$= (1 - \lambda)(a' - w^l) \left[\frac{\frac{a'' - a'}{2} \left[\lambda + (1 - \lambda) \Pr(b_I^l \geq \frac{a' + a''}{2}) \right]}{(1 - \lambda)(a' - w^l)} + \Pr(b_I^l \geq a') \right] + \lambda(a' - w^h), \quad (37)$$

$$= \frac{a'' - a'}{2} \left[\lambda + (1 - \lambda) \Pr(b_I^l \geq \frac{a' + a''}{2}) \right] + (1 - \lambda)(a' - w^l) \Pr(b_I^l \geq a') + \lambda(a' - w^h) \quad (38)$$

$$= (1 - \lambda) \left(\frac{a' + a''}{2} - w^l \right) \Pr(b_I^l \geq \frac{a' + a''}{2}) + \lambda \left(\frac{a' + a''}{2} - w^h \right), \quad (39)$$

$$= U_O\left(\frac{a' + a''}{2}\right),$$

where $\Pr(b_I^l \geq a') = \Pr(b_I^l \geq (a' + a'')/2)$. Thus, the prospective auditor will be better off bidding $b_O = (a' + a'')/2$, a contradiction to $b_O \notin [a', a'']$.

Lemma 7 *When the client's audit cost is low, the incumbent auditor's bid b_I^l has no gaps.*

Suppose there is a gap. Then there exists an interval $[a', a''] \subset [\bar{w}, w^h]$ such that $b_I^l \notin [a', a'']$. We will consider two cases. First, suppose $a' = \bar{w}$. The incumbent can make a profit by bidding $b_I^l = a'' - \varepsilon$ where $\varepsilon \in (0, a'' - a')$ because he will win for sure.

Second, suppose $a' > \bar{w}$. Then there exists $\Delta \in (0, a' - \bar{w})$ such that the incumbent bids with positive probability in every subset of $(a' - \Delta, a')$; otherwise, the first case will be repeated. Note that the prospective auditor will not bid in $[a', a'']$. Instead, he will bid a'' because this will increase the profit without reducing the probability of winning, which suggests that $\Pr(b_O \geq a') = \Pr(b_O \geq a'')$. We will need to show that there exists $\eta \in (0, \Delta)$ such that for $b_I^l = b \in (a' - \eta, a')$, $U_I^l(b) < U_I^l(a'')$. Let

$$\varepsilon^* = \frac{(a'' - a') [\Pr(b_O \geq a'')]}{(a' - w^l)}.$$

Recall that $\mathcal{F}_O(b)$ is atomless in $[\bar{w}, w^h]$, which implies that $\Pr(b_O \geq b)$ is also continuous. Then following the property of continuity, for $\varepsilon^* > 0$, there exists $\delta^* > 0$, such that $|b - a'| < \delta^*$ implies $|\Pr(b_O \geq b) - \Pr(b_O \geq a')| < \varepsilon^*$. As $\Pr(b_O \geq b)$ decreases in b , $0 < a' - b < \delta^*$ implies $\Pr(b_O \geq b) -$

$\Pr(b_O \geq a') < \varepsilon^*$ or $b \in (a' - \delta^*, a')$ implies that $\Pr(b_O \geq b) < \Pr(b_O \geq a') + \varepsilon^*$. Take a $\eta \in (0, \min(\delta^*, \Delta))$. For $b_I^l = b \in (a' - \eta, a')$,

$$U_I^l(b_I^l = b) = \Pr(b_O \geq b)(b - w^l), \quad (40)$$

$$< (a' - w^l)[\varepsilon^* + \Pr(b_O \geq a')], \quad (41)$$

$$= (a' - w^l) \left[\frac{(a'' - a') [\Pr(b_O \geq a'')]}{(a' - w^l)} + \Pr(b_O \geq a'') \right], \quad (42)$$

$$= (a'' - w^l) \Pr(b_O \geq a'') = U_I^l(b_I^l = a''),$$

where the equality holds because $\Pr(b_O \geq a') = \Pr(b_O \geq a'')$. Thus, the incumbent will be better off bidding a'' instead of bidding in $b \in (a' - \eta, a')$. This is a contradiction to $b_I^l \notin [a', a'']$.

Lemma 8 $\mathcal{F}_O(b)$ and $\mathcal{F}_I^l(b)$ are strictly increasing in $[\bar{w}, w^h)$, respectively.

The atomless result in Lemma 4 and the gapless result in Lemma 6 imply that $\mathcal{F}_O(b)$ are strictly increasing in $[\bar{w}, w^h)$. Analogously, Lemma 5 and Lemma 7 imply that $\mathcal{F}_I^l(b)$ are strictly increasing in $[\bar{w}, w^h)$.

References

- [1] Engelbrecht-Wiggans, R., P. R. Milgrom, and R. J. Weber. 1983. Competitive bidding and proprietary information. *Journal of Mathematical Economics* 11 (2): 161–169.

Appendix B. Proofs of Propositions and Lemmas in Section 5

Extensions

Proof of Proposition 5

After observing the public signal $m \in \{H, L\}$, the prospective auditor's updated belief about the high-cost type is λ_m . The prospective auditor's expected total audit cost is then given by,

$$\bar{w}_m \equiv \lambda_m w^h + (1 - \lambda_m) w^l,$$

which represents the audit fee to break even from the prospective auditor's perspective. Thus the bidding strategy conditional on the public signal follows the same type of equilibrium as in Proposition 1, with the prior λ now being replaced by the updated λ_m conditional on public signal m and \bar{w} being replaced by \bar{w}_m .

Following Corollary 1, a high-cost client's probability of switching conditional on public signal m is $\phi_m^h = 1 - \lambda_m$ and a low-cost client's probability of switching is $\phi_m^l = \frac{1 - \lambda_m}{2}$. The overall probability of switching conditional on signal m is $\phi_m = \frac{1 - \lambda_m^2}{2}$.

1) Given the bidding equilibrium conditional on public signals (H, L) , we calculate the expected probability of auditor switching as

$$\begin{aligned} E[\phi] &= Pr(H)\phi_H + Pr(L)\phi_L = Pr(H)\frac{1 - \lambda_H^2}{2} + Pr(L)\frac{1 - \lambda_L^2}{2} \\ &= \frac{(1 - \lambda)[\theta(1 - \theta) + \lambda(1 - 3\theta(1 - \theta))] - \lambda^2(1 - 2\theta)^2}{2[\lambda(1 - \theta) + \theta(1 - \lambda)][(1 - \lambda)(1 - \theta) + \theta\lambda]}. \end{aligned}$$

Taking the first order derivative with respect to θ , we obtain that for $\theta \geq \frac{1}{2}$,

$$\frac{dE[\phi]}{d\theta} = \frac{(1 - 2\theta)(1 - \lambda^2)\lambda^2}{2(\theta + \lambda - 2\theta\lambda)^2(1 - \lambda - \theta(1 - 2\lambda))^2} \leq 0.$$

2) Following Corollary 2, the information content of switching conditional on public signal m is

given by $Pr(r = h|O, m) = \frac{2\lambda_m}{(1+\lambda_m)}$. Thus, taking the expectation over the public signal, we have

$$E[Pr(h|O)] = (\lambda\theta + (1-\lambda)(1-\theta))\frac{2\lambda_H}{(1+\lambda_H)} + (\lambda(1-\theta) + (1-\lambda)\theta)\frac{2\lambda_L}{(1+\lambda_L)}.$$

Taking the first order derivative, we obtain that for $\theta \geq \frac{1}{2}$,

$$\frac{dE[Pr(h|O)]}{d\theta} = \frac{1(1-2\theta)(1-\lambda)^2\lambda^2(1+\lambda)}{(\theta(1-\lambda) + 2\lambda(1-\theta))^2((1-\lambda)(1-\theta) - 2\lambda\theta)^2} \leq 0.$$

3) The information rent earned by the incumbent auditor, conditional on public signal, is given by $(1-\lambda_m)(\bar{w}_m - w_l)$. Substituting λ_m and \bar{w}_m into the rent function, we obtain that the expected rent is $\frac{(1-\theta)(1-\lambda)\lambda\theta(w_h-w_l)}{(1-\lambda-\theta+2\lambda\theta)(\lambda+\theta-2\lambda\theta)}$. Taking the first order derivative with respect to θ , we have

$$\frac{E[Rent]}{d\theta} = \frac{1(1-2\theta)(1-\lambda)^2\lambda^2(1+\lambda)}{(\theta(1-\lambda) + 2\lambda(1-\theta))^2((1-\lambda)(1-\theta) - 2\lambda\theta)^2} \leq 0. \blacksquare$$

Proof of Lemma 1

Given the incumbent's bidding strategy $\mathcal{F}_I^r(b)$, the uninformed prospective auditor's expected payoff, when submitting a bid b , is

$$U_O(b, k) = \lambda(1 - \mathcal{F}_I^h(b))(b - w^h - k) + (1 - \lambda)(1 - \mathcal{F}_I^l(b))(b - w^l - k),$$

where $\mathcal{F}_I^r(\cdot)$ represents the incumbent's bidding strategy conditional on the client's cost type $r \in \{h, l\}$. If the client switches to the prospective auditor, there is an extra switching cost k that reduces the auditor's payoff.

In contrast, the incumbent's expected payoff when submitting a bid b for the firm r is given by

$$U_I^r(b) = (1 - \mathcal{F}_O(b))(b - w^r),$$

where, similarly, $1 - \mathcal{F}_O(b)$ is the probability that the incumbent auditor wins the bid, given that the prospective auditor's bid follows the distribution $\mathcal{F}_O(b)$.

Each auditor chooses the bidding strategy to maximize their own payoff, taking the other auditor's optimal response as given. The bidding equilibrium is similar to that of the main setting. However, since the incumbent and outsider have different costs, we need to discuss the bidding strategies in more detail.

First, any pure strategy, again, cannot be sustainable in equilibrium. To see this, suppose that the prospective auditor adopts a pure strategy to bid in the range $b \in [w^h, w^h + k]$. Then, given the prospective auditor's pure strategy, the incumbent will always bid a price slightly below $b - \epsilon$ and always wins the engagement, leaving the prospective auditor zero profit. Suppose that the prospective auditor always bids $b \in [\bar{w} + k, w^h]$, the incumbent always bids $b = w^h$ when the audit cost is high and bids the same w as the prospective auditor when the audit cost is low. In this case, the prospective auditor can win only when the audit cost is high, leading to a loss (the winner's curse). Thus, any pure strategy by the prospective auditor is not sustainable in equilibrium. Therefore, the prospective auditor will play a mixed strategy in equilibrium.

Second, we will show that the prospective auditor's randomization range is $[\bar{w} + k, w^h + k]$. First, the prospective bidder will never bid below $\bar{w} + k$, which is the break-even audit cost with a switching cost of k . Otherwise, he will make a loss for the engagement upon winning. Next, the prospective auditor will only bid up to $w^h + k$ because otherwise, the prospective auditor may win the client with a strictly negative profit. However, the prospective auditor will not bid more than $w^h + k$, because both auditors can always undercut the bid by a small amount and win with a strictly positive profit with certainty. Thus, the prospective auditor's randomization range is exactly $[\bar{w} + k, w^h + k]$. If the incumbent knows the client is low cost, the incumbent will adopt a mixed strategy and bid over the same range as the prospective auditor for the same argument we provide in the proof of Proposition 1.

Third, we show the incumbent's strategy for the high-cost client is always to bid exactly $w^h + k$. That is, although the incumbent's lowest possible bid for the high-cost firm is w^h , he will put zero density over the range $[w^h, w^h + k)$. The reason is as follows. Suppose the incumbent bids a fee $b \in [w^h, w^h + k)$ with positive probability. Then given that the incumbent's bid for the high-cost firm always exceeds that for the low-cost firm, the incumbent will not bid over $[w^h, w^h + k)$ if

the client is low-cost. Then the prospective auditor is not likely to win a low-cost client, i.e., the prospective auditor suffers a winner's curse, if she bids with positive probability over $[w^h, w^h + k)$. Therefore, the prospective auditor will not submit any bid within this range, $[w^h, w^h + k)$ if the incumbent bids $b \in [w^h, w^h + k)$ with a positive probability for the high-cost client. As a response, the incumbent finds that he can increase the profit strictly, without decreasing the probability of winning the prospective auditor, by bidding at the highest possible fee $w^h + k$ for the high-cost client. Therefore, the incumbent always bids $w^h + k$ for the high-cost client.

Finally, we solve the prospective auditor's mixed bidding strategy $\mathcal{F}_O(b)$ by setting $U_I^l(b) = \bar{w} + k - w^l$, and solve the incumbent's mixed bidding strategy in the low-cost firm $\mathcal{F}_I^l(b)$ by setting $U_O(b, k) = 0$, following the same approach as outlined in the proof of Proposition 1. ■

Proof of Proposition 6

Given the bidding strategies in Lemma 1, the switching probability for a high-cost client is

$$\phi^h = \mathcal{F}_O(w^h + k) = (1 - \lambda) \frac{w^h - w^l}{w^h - w^l + k}.$$

It is easy to see that $\partial\phi^h(k)/\partial k < 0$. In addition, when the transaction cost approaches zero ($k \rightarrow 0$), the switching probability $\phi^h(k) \rightarrow 1 - \lambda$.

The probability of switching for a low-cost client is

$$\begin{aligned} \phi^l &= \int_{\bar{w}+k}^{w^h+k} \mathcal{F}_O(b) f_I^l(b) db \\ &= \int_{\bar{w}+k}^{w^h+k} \frac{b - \bar{w} - k}{b - w^l} \frac{\lambda(b - \bar{w} - k)}{(1 - \lambda)(b - w^l - k)^2} db \\ &= \frac{\lambda(w^h - w^l)}{k^2(1 - \lambda)} \left\{ \begin{array}{l} k\hat{\lambda} + (k + \lambda(w^h - w^l)) [\ln(w^h - w^l) - \ln(k + w^h - w^l)] - \\ [k + (k + \lambda(w^h - w^l)) [\ln(\lambda(w^h - w^l)) - \ln(k + \lambda(w^h - w^l))]] \end{array} \right\}. \end{aligned}$$

To show $\partial\phi^l/\partial k < 0$, taking a derivative with respect to k yields

$$\frac{\partial\phi^l}{\partial k} = \frac{\lambda(w^h - w^l)}{k^2(1 - \lambda)} \left\{ \begin{array}{l} (k + \lambda(w^h - w^l)) [\ln(w^h - w^l) - \ln(k + w^h - w^l)] \\ + [(k + \lambda(w^h - w^l)) [\ln(\lambda(w^h - w^l)) - \ln(k + \lambda(w^h - w^l))] \\ - k(1 - \lambda) \end{array} \right\}.$$

Given that $\ln(k + w^h - w^l) > \ln(w^h - w^l)$ and $\ln(k + \lambda(w^h - w^l)) > \ln(\lambda(w^h - w^l))$ for any $k > 0$, it follows that $\partial\phi^l/\partial k < 0$.

Second, we prove that $\phi^l \rightarrow (1 - \lambda)/2$ when $k \rightarrow 0$. To prove this, we apply L'Hospital's Rule and *twice* differentiate the numerator and the denominator of ϕ^l with respect to k :

$$\begin{aligned} \lim_{k \rightarrow 0} \phi^l &= \frac{\lambda(w^h - w^l)}{(1 - \lambda)} \frac{1}{2} \frac{(1 - \lambda)^2 (w^h - w^l)^2}{(k + w^h - w^l)^2 (k + \lambda(w^h - w^l))} \\ &= \frac{\lambda(w^h - w^l)}{(1 - \lambda)} \frac{1}{2} \frac{(1 - \lambda)^2}{\lambda(w^h - w^l)} = \frac{1 - \lambda}{2}. \end{aligned}$$

Third, we prove $\phi^h > \phi^l$. We can rewrite

$$\begin{aligned} \phi^l &= \frac{\lambda(w^h - w^l)}{k^2(1 - \lambda)} \left\{ \begin{array}{l} k\hat{\lambda} + (k + \lambda(w^h - w^l)) [\ln(w^h - w^l) - \ln(k + w^h - w^l)] - \\ [k + (k + \lambda(w^h - w^l)) [\ln(\lambda(w^h - w^l)) - \ln(k + \lambda(w^h - w^l))] \end{array} \right\} \\ &= \frac{\lambda(w^h - w^l)}{k^2(1 - \lambda)} [k + \lambda(w^h - w^l)] \ln\left(\frac{k + \lambda(w^h - w^l)}{\lambda(k + w^h - w^l)}\right) - \frac{\lambda(w^h - w^l)}{k} \\ &= \frac{\lambda(w^h - w^l)}{k} \left[\frac{k + \lambda(w^h - w^l)}{k(1 - \lambda)} \ln\left(1 + \frac{(1 - \lambda)k}{\lambda(k + w^h - w^l)}\right) - 1 \right]. \end{aligned}$$

Since $\ln(1 + x) < x$ for any $x > 0$, it follows that

$$\begin{aligned} \phi^l &< \frac{\lambda(w^h - w^l)}{k} \left[\frac{k + \lambda(w^h - w^l)}{k(1 - \lambda)} \frac{(1 - \lambda)k}{\lambda(k + w^h - w^l)} - 1 \right] \\ &= (1 - \lambda) \frac{w^h - w^l}{w^h - w^l + k} = \phi^h. \end{aligned}$$

■

Proof of Proposition 7

We first derive equation (20). Suppose the client has chosen the incumbent for all previous $n - 1$ periods, the updated belief by the prospective auditor is given by Λ_{n-1} . Then in the n^{th} if the incumbent continues to win the auction, the prospective auditor's updated belief Λ_n becomes

$$\Lambda_n = \frac{\Lambda_{n-1}(1 - \phi_{n-1}^h)}{\Lambda_{n-1}(1 - \phi_{n-1}^h) + (1 - \Lambda_{n-1})(1 - \phi_{n-1}^l)} = \frac{2\Lambda_{n-1}^2}{1 + \Lambda_{n-1}^2}$$

by substituting in $\phi_{n-1}^h = 1 - \Lambda_{n-1}$ and $\phi_{n-1}^l = \frac{1 - \Lambda_{n-1}}{2}$. Thus

$$\Lambda_n - \Lambda_{n-1} = -\frac{\Lambda_{n-1}(2 + (1 - \Lambda_{n-1})^2)}{1 + \Lambda_{n-1}^2}.$$

Therefore for any n , we can show that $\Lambda_n < \Lambda_{n-1}$, which implies that Λ_n decreases with n .

To show Proposition 7, given Λ_n decreases with n , we have the following.

- (i) The overall switching probability, $\phi_n = (1 - \Lambda_{n-1}^2)/2$, increases with n .
- (ii) The informativeness of a good report when the incumbent auditor has engaged with the client for n periods is calculated as

$$\begin{aligned} Pr(G|g, n) &= \frac{p(1 - \phi_n)}{\Lambda_n(1 - \phi_n^h)p_g^h + (1 - \Lambda_n)(1 - \phi_n^l)p_g^l}, \\ &= \frac{p(1 + \Lambda_n^2)}{\Lambda_n^2(2p_g^h - p_g^l) + p_g^l} \end{aligned}$$

Taking the derivative of $Pr(G|g, n)$ w.r.t. Λ_n , we have

$$D[Pr(G|g, n), \Lambda_n] = \frac{4p\Lambda_n(p_g^l - p_g^h)}{(\Lambda_n^2(2p_g^h - p_g^l) + p_g^l)^2} < 0.$$

Since $p_g^l < p_g^h$, the informativeness of good report $Pr(G|g, n)$ increases with n .

- (iii) From the proof of Proposition 2, the expected audit fee charged by the incumbent is given by

$$E[b_I(n)] - E[b_O(n)] = \Lambda_n^2 w^h$$

It can be shown that the fee difference increases with Λ_n , which implies that $E[b_I(n)]$ decreases with n , i.e., the longer the audit tenure, the lower the expected audit fee cut post auditor switching.