Online Appendix for 'Harvesting Ratings'

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Appendix A Proofs

A.1 Primitives

Towards proving our main Proposition 4, we first show a series of primitives that hold beyond our baseline model: in particular, they hold (i) for any finite $T \geq 2$ periods, (ii) for any q^l such that $q^l < q^A$, (iii) if firm B sells following a positive or no rating. We use them to prove our main proposition and extensions.

In the proofs, we use $\mathbb{H}_t$ to denote histories until t.

Lemma 3. *If the high-quality firm faces no demand in a given period t, the low-quality firm faces no demand in period t.*

Proof of Lemma 3.
Suppose towards a contradiction that in some period t, h is inactive and l is active. Then for any equilibrium price where l sells in t, we have beliefs $E[q_t^B|p] = q^l$. Because $q^l < q^A$, l only sells in t if it charges a price below costs, and since $q^l - q^A < q^l$, it gets no rating with probability strictly less than one in any $t < T$. This is clearly suboptimal in $t = T$, so we must have $t < T$. Next, we show that l earns weakly negative profits for subsequent periods if it gets a positive or a negative rating in period $t < T$. To see this, note that if l sells in t and receives a positive or negative rating, it is identified as a low-quality firm in period $t + 1$ and any subsequent period. Thus, since $q^l < q^A$, following a history with $R_t \in \{1, -1\}$, firm l has zero demand and is inactive after any such histories. But if l earns non-positive profits after $R_t \in \{1, -1\}$, and since it sells below cost in t, it must earn strictly positive profits after $R_t = 0$. But then l has a profitable deviation to not selling in t to get $R_t = 0$ with probability one, contradicting that l is active in period t. □

Lemma 4. *If the low-quality firm sells in period $t + 1$, it must sell in period t.*

Proof of Lemma 4.
Towards a contradiction, suppose l sells in $t + 1$ but not in t.

First consider the case where h is also inactive in period t. But then beliefs are the same in $t+1$ as they were in t, contradicting that l sells in $t+1$.

Next, consider the case where h sells in period t, setting some price p_t with strictly positive probability. Since h sells, we must have $p_t \geq 0$. Because the high-quality firm sells, it obtains $R_t = 1$ with some probability. Since only h gets $R_t = 1$, this rating perfectly identifies h for subsequent periods. If the low-quality firm does not sell in period t, it receives $R_t = 0$ with probability one. Thus, observing $R_t = 0$, and since l has that rating with probability one and h only with probability strictly less than one, beliefs after $R_t = 0$ in $t+1$ must be strictly lower than beliefs in period t. Hence, prices in $t+1$ following $R_t = 0$ must be strictly lower than prices the high-quality firm sets in period t.

We now distinguish two cases. First, if p_t is above q^l then l has a profitable deviation to set p_t in period t and sell, obtaining $R_t \in \{0, -1\}$. Recalling that p_t must be strictly greater than any price following no rating in period $t+1$ if it did not sell, it must be a strictly profitable deviation for the low-quality firm to set p_t, since it sells at a larger price and, since it did not yet get $R_t = -1$, has a larger demand than in $t+1$, contradicting that l does not sell in t.

Second, if p_t is below q^l, then l has a profitable deviation by starting to set p_t in period t and sell. This deviation induces $R_t \in \{1, 0\}$. Again recalling that p_t must be strictly greater than any price following no rating in period $t+1$ if it did not sell, it must be a strictly profitable deviation for the low-quality firm to set p_t. Additionally, since $R_t = 1$ perfectly identifies a high-quality firm, l also earns strictly larger profits after such histories. This contradicts that l does not sell in period t.

We conclude that if the low-quality firm sells in period $t+1$ it must sell in period t. □

Corollary 5. *If the low-quality firm has positive demand in period t, then the high-quality firm must have positive demand in all periods up to and including period t.*

Proof of Corollary 5.
Suppose the low-quality firm receives demand in period t. From Lemma 4 it also faces demand in period $t-1$. Then from Lemma 3 the high-quality firm faces demand in every period for which the low-quality does. Hence, it must be that the high-quality firm faces demand in both period t and $t-1$. Induction then implies the claim. □

Corollary 6. *If the high-quality firm sells in period t, it only obtains a good rating or no rating with strictly positive probability.*

Proof of Corollary 6.
Note that ratings only follow from a sale, and sales require winning the competition against A which has quality $q^A > 0$. This implies that in any period t where a rating could occur for h, we have $E[q_t^B|\mathbb{H}_t] \geq p_t$. Further, observe that expected quality of firm B is a convex combination of q^h and q^l, which is weakly less than q^h. Therefore, due to competition with A, it must be that $q^h > p_t$

and the high-quality firm obtains a good rating or no rating with strictly positive probability, and cannot obtain a bad rating. □

Corollary 7. *In any history $\mathbb{H}_t$ where firm B obtains at least one bad rating, consumer beliefs are the lowest possible, $E[q_t^B|\mathbb{H}_t] = q^l$.*[1]

Proof of Corollary 7.
By Corollary 6, h cannot receive bad ratings. Hence, for any equilibrium history with a bad rating the firm is perfectly identified as l. □

Lemma 5. *Firm A does not employ a mixed strategy in any period t.*

Proof of Lemma 5.
If firm A has zero demand after some histories, then by Selection Assumption 2, firm A plays $p_t = 0$ with probability 1 following those histories. To see this, note that by Selection Assumption 2, some consumers are indifferent, so offers with zero demand must be optimal. Prices below cost would be suboptimal if they would induce sales; prices above costs would be suboptimal if some indifferent consumers purchased, since consumers are indifferent between both firms, a marginally lower price would induce a discrete jump in demand. Thus, if A has zero demand, it charges marginal cost.

Next, consider histories after which firm A sells with strictly positive probability.

Note first that firm A cannot charge multiple mass points with strictly positive probability. If it sells, it must earn strictly positive profits at some of these mass points. Applying standard Bertrand arguments, either firm B earns strictly positive profits, then either firm can profitably deviate by shifting probability mass for some of its mass points downwards. Or firm B earns zero profits and by Selection Assumption 2, consumers must be indifferent between both firms. Then by Selection Assumptions 2, such offers must be best responses to A's offers, which is why B sets price at marginal cost with probability one (by the same argument used in the first paragraph of this proof). But then firm A cannot be indifferent between multiple mass points and shifts probability mass away from the least profitable ones. We conclude that A does not have multiple mass points.

We show next that both firm A and firm B cannot mix over intervals. Towards a contradiction, suppose firm A mixes over an interval of prices. Since firm A mixes over an interval, firm B must also mix over an interval; otherwise firm A would sell with probability zero (which we ruled out above), or with probability one (in which case mixing over an interval is clearly suboptimal).

Now take one such price $p^A > 0$. Note that since A must earn weakly positive profits in t, and therefore does not charge prices below cost, such a $p^A > 0$ must exist whenever A mixes over an interval. By our Selection Assumption 2, consumers are indifferent between both firms, implying that all prices of B, p^B, must be such that consumers are indifferent between p_A and p^B. Thus, all

[1] For the two-period model, possible equilibrium histories with negative ratings are $\mathbb{H}_t \in \{\{-1\}\}$. For the three period model, they are $\mathbb{H}_t \in \{\{-1\}, \{1, -1\}, \{0, -1\}, \{-1, 1\}, \{-1, 0\}, \{-1, -1\}\}$.

prices of B must induce the same expected utility with firm B as p^A does from firm A. But then deviating to a marginally smaller price induces a discrete jump in demand for firm A while only marginally reducing margins. Thus, such a deviation must be strictly profitable, contradicting that A mixes over an interval. This concludes the proof. □

Lemma 6. *When firm B enters and is high-quality, it sets a unique price, $\overline{p}_t$ conditional on the history if in period t:*

- *Good ratings are not beneficial (i.e. conditional on the same history, $R_t = 1$ leads to lower future profits than $R_t = 0$); or*
- *Good ratings are beneficial and* (1) *holds.*

The final period is a special case of obtaining good ratings being not beneficial.

Proof of Lemma 6.
We show that a high-quality firm B sets a unique price in each period conditional on the history.

First, suppose firm h earns zero profits in period t. By our Selection Assumption 2, consumers must be indifferent between h and A and by the same selection assumption, offers with zero demand of h must be optimal even if some indifferent consumers purchase, implying that h charges a price at marginal cost (by the same argument we used in the proof of Lemma 5). Thus, if h earns zero profits, it sets a unique price.

Next, suppose h earns strictly positive profits. Then h must have strictly positive demand for all its prices. Additionally, by Corollary 6, h gets good and no ratings with strictly positive probability. By Selection Assumption 2, consumers are ex-ante indifferent between h and A, which is why firm A must earn zero profits in period t; otherwise A could strictly increase profits by marginally reducing its price. Thus, since A earns zero profits from selling in t and h earns strictly positive profits, our Selection Assumption 2 implies that h sells with probability one for all prices it charges in period t.

We now distinguish two cases, whether good ratings are beneficial and raise continuation profits, or whether they are not beneficial.

Suppose first that good ratings are not beneficial such that obtaining a good rating in period t does not improve the expected continuation payoff. In other words, the sum of expected future profits conditional on obtaining a good rating in period t is weakly less than the sum of expected future profits conditional on obtaining no rating in period t. Then, since all prices in t induce the same demand in t, firm h has a profitable deviation and moves all probability mass in t to its highest price in t from the candidate equilibrium. This strictly raises its current profits and its future profits, contradicting that h plays a mixed strategy. Therefore, when good ratings are not beneficial firm h sets a unique price in period t.

Note the special case of the last period. Since ratings from the last period do not influence any future decision, following a history of ratings, if the firm sells in the last period, it sets the highest possible price which induces demand.

Suppose next that, following a history $\mathbb{H}_t$, good ratings are beneficial such that obtaining a good rating in period t improves the expected continuation payoff. In other words, the sum of expected future profits conditional on obtaining a good rating in period t is strictly larger than the sum of expected future profits conditional on obtaining no rating in period t. Here, we have to consider that although firm B would receive the same demand at any price over which it mixes in period t, shifting probability from a lower price to a higher price reduces the probability of receiving a good rating, reducing continuation profits. Therefore, the distribution of effort to leave ratings has to be sufficiently flat such that the probability of receiving a good rating does not decrease by too much if the firm shifts probability mass to the higher price. To derive such a condition, note the total profit of the firm is $p + F(q^h - p)\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) + (1 - F(q^h - p))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, where $\pi_{t+1}(\{\mathbb{H}_t, R_t\})$ is the expected continuation profit from obtaining R_t. Note $\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) > \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$ for ratings to be beneficial. Then a marginal price increase raises profits if for all $p \in (-q^h, q^h)$, we have

$$
\begin{aligned}
1 - f(q^h - p)(\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow \\
f(q^h - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})}
\end{aligned}
\tag{1}
$$

Therefore if (1) holds for all prices $p \in (-q^h, q^h)$, firm h earns strictly larger profits from its larger candidate equilibrium prices, contradicting that h plays a mixed strategy. The same condition also implies that if h sets a unique price that satisfies Selection Assumption 2, lowering the price cannot increase profits.

Therefore, we conclude that a high-quality firm B sets a unique price $\overline{p}_t$ if in period t good ratings are not beneficial or (1) holds on the support of f. □

Lemma 7. *Suppose firm l enters.*

Firm l charges the following prices when ratings are beneficial. If $\overline{p}_t > q^l$, and (2) *and* (3) *hold in period t:*

- *Firm l charges only $\overline{p}_t$ or $\underline{p}_t$ with positive probability in period t.*
- *Firm l charges $\overline{p}_t$ with probability $\delta_t \in [0, 1]$, obtaining a bad rating with probability $F(\overline{p}_t - q^l)$ and no rating otherwise.*
- *And $\underline{p}_t = q^l - q^A \leq q^l$ with probability $1 - \delta_t$, obtaining a good rating with probability $F(q^l - \underline{p}_t)$ and no rating otherwise.*

If $\overline{p}_t \leq q^l$ and (1) holds, $\delta_t = 1$ and firm l obtains either a good rating with probability $F(q^l - \overline{p}_t)$ or no rating otherwise.

If good ratings are not beneficial, then $\delta_t = 1$.

Proof of Lemma 7.
Selection Assumption 2 implies that consumers are, in expectation, indifferent between firm A and B. Hence, the highest price firm l can set in each period is one that leads to this indifference. This price is the unique price, $\overline{p}_t$, from Lemma 6 that also h sets. We start by assuming good ratings are beneficial in period t, showing the low-quality firm B mixes between this price and a unique lower price in period t. This proof follows in two parts. First, by considering $\overline{p}_t > q^l$. Then we consider $\overline{p}_t \leq q^l$.

Consider first the scenario where $\overline{p}_t > q^l$ and good ratings are beneficial in period t. When the low-quality firm sets $\overline{p}_t$, because $\overline{p}_t > q^l$ the firm receives a bad rating with probability $F(\overline{p}_t - q^l)$ and no rating with probability $1 - F(\overline{p}_t - q^l)$. If the firm deviates to a price above $\overline{p}_t$ it never makes a sale and gets no rating with probability 1. If the firm deviates to a price between q^l and $\overline{p}_t$ it sells at most with probability one and gets a bad rating with a lower probability. Hence, by deviating to a lower price it may be possible to increase the continuation payoff. Such a deviation is not profitable if f is sufficiently flat such that the higher probability of receiving the continuation payoff is dominated by the lower profits the firm earns in period t. In other words, for prices above q^l at which firm l sells, the derivative of the total expected profit, $p + F(p - q^l)\pi_{t+1}(\{\mathbb{H}_t, R_t = -1\}) + (1 - F(p - q^l))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, from playing the price p must be positive for all $p - q^l$ on the support of f. This holds if for all such prices, we have

$$1 + f(p - q^l)(\pi_{t+1}(\{\mathbb{H}_t, R_t = -1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow f(p - q^l) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})}. \tag{2}$$

Where the equivalence follows since by Corollary 7, negative ratings perfectly identify l and induce zero continuation profits. If (2) holds for all prices above $p - q^l$ on the support of f, the low-quality firm sets $\overline{p}_t$ in period t if it sets a price above q^l.

Next consider when the low-quality firm B may receive a good rating with some positive probability in period t. For this to occur the firm has to set prices weakly below q^l. Suppose the low-quality firm B sets more than one such price. This implies that it makes a positive profit at all such prices. However, if f is sufficiently flat such that an increase in price only changes the probability of receiving a rating by a small amount, it must be that raising prices in period t is beneficial as long as firm l continues to sell. In other words, the firm l gathers all its probability mass for prices below q^l at a mass point. This is true if the derivative of the total expected profit of setting a $p \leq q^l$, $p + F(q^l - p)\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) + (1 - F(q^l - p))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, is positive for all $q^l - p$

on the support of f:

$$1 - f(q^l - p)(\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow$$
$$f(q^l - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})}. \tag{3}$$

If (3) holds for all $q^l - p$ on the support of f in period t then the low-quality firm sets a single price $\underline{p}_t$ below q^l in period t.

The low-quality mixes over $\overline{p}_t$ and $\underline{p}_t$ in period t such that it is indifferent between the two expected continuation payoffs. This concludes that if $\overline{p}_t > q^l$ and good ratings are beneficial in period t, a low-quality firm B mixes between $\overline{p}_t$ and some unique $\underline{p}_t$ below q^l in period t.

Next consider the scenario where $\overline{p}_t \leq q^l$ and good ratings are beneficial. The same argument in the previous paragraph implies that l sets a unique price. In this scenario a low-quality firm B would receive a good rating with some strictly positive probability. When $\overline{p}_t \leq q^l$, then at any price above $\overline{p}_t$ consumers must have beliefs such that deviating to such prices induces no sales. This rules out any upward deviation by firm B. Moreover, no firm receives bad ratings. However, there are potential downward deviations from $\overline{p}_t$, since firm l can set lower prices and receive more good ratings. Such a deviation would not be profitable if f is sufficiently flat such that (3) holds.

Next consider the scenario where ratings are not beneficial. Then to prevent receiving good ratings the low-quality firm trivially plays the highest price at which it is able to sell in period t, playing $\overline{p}$ with probability 1.

Finally, because the high-quality firm B never sets $\underline{p}_t$, then it must be that following any price $\underline{p}_t$ the low-quality firm is perfectly identified. To make a sale, the firm has to set a price which provides at least as much utility as firm B, that is to say $\underline{p}_t \leq q^l - q^A$, and because the firm prefers to set the highest possible price following (3), $\underline{p}_t = q^l - q^A$. This concludes the proof. □

We refer to equations (1), (2) and (3) as sufficiently flat conditions for f.

Lemma 8. *If firm B enters, it plays a unique price in period T, which depends on its rating history.*

Proof of Lemma 8.
Given the consumer's information set in the final period, they condition their beliefs only on historical ratings and current prices. Note first that since there is no future period, future ratings do not affect profits. Selection Assumption 2 implies that consumers are ex-ante indifferent between firms A and B. Additionally, one of the firms must earn zero profits. Otherwise, if both firms earn strictly positive profits, firm A can marginally decrease its price to increase demand by a discrete amount, strictly increasing profits. By Selection Assumption 2, the firm earning zero profits charges a price at costs and earns zero profits (using the same argument as in Lemma 5). Thus, if firm B does not sell, it sets a unique price at marginal cost. If firm B sells, by Selection Assumption

2, consumers must be indifferent between both firms, and by the above result firm A earns zero profits, so firm A will set the largest price at which it can sell. This pins down the price of firm B uniquely. We conclude that firm B sets a unique price in period T, conditional on its rating history. $\square$

Corollary 8. *Suppose h and l enter. In period t, firm A sets $p_t^A = max\{q^A - E[q_t^B|\mathbb{H}_t, p_t], 0\}$. Firm B sets either $\overline{p}_{\mathbb{H}_t} = max\{E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A, 0\}$ and $\underline{p}_{\mathbb{H}_t} = q^l - q^A$.*

Proof of Corollary 8.
First recall from Lemma 7 that $\underline{p}_{\mathbb{H}_t} = q^l - q^A$, for any history $\mathbb{H}_t$. Next, consider firm B playing $\overline{p}_{\mathbb{H}_t}$ in each period t. Selection Assumption 2 implies that consumers are ex-ante indifferent between firms A and B. Additionally, one of the firms must earn zero profits. Otherwise, if both firms earn strictly positive profits, firm A can marginally decrease its price to increase demand by a discrete amount, strictly increasing profits. By Selection Assumption 2, the firm earning zero profits charges a price at costs and earns zero profits (using the same argument as in Lemma 5). Thus, if $E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] \geq q^A$, firm A charges a price at cost and earns zero profits in that period and firm B charges $\overline{p}_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A$. If, instead, $E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] < q^A$, then firm B sets the price $\overline{p}_{\mathbb{H}_t} = 0$ and firm A sets the price $q^A - E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}]$. This concludes the proof. $\square$

Lemma 9. *Suppose h and l enter. If* (1), (2) *and* (3) *hold, first period beliefs are without loss of generality*

$$E[q_1^A|p] = q^A \quad \forall p, \qquad\qquad E[q_1^B|p] = \begin{cases} \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*} & \forall p \ \textit{if}\ \overline{p}_1 \leq q^l \\ \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*} & \forall p > q^l \ \textit{if}\ \overline{p}_1 > q^l \\ q^l & \forall p \leq q^l \ \textit{if}\ \overline{p}_1 > q^l. \end{cases}$$

Proof of Lemma 9.
Recall that (1), (2) and (3) are conditions such that the high-quality firm B sets a unique price $\overline{p}_t$ and the low-quality firm B sets either $\overline{p}_t$ or $\underline{p}_t$. The rest of the proof focuses on period 1.

Since firm A's quality is common knowledge, its quality is known to be q^A regardless of price.

Consider the case where $\overline{p}_1 \leq q^l$. Then firms charge $\overline{p}_1$ with probability one. It is straightforward to check that for the equilibrium price $\overline{p}_1$, the above expectations apply Bayes rule. Additionally, $E[q_1^B|\overline{p}_1]$ is such that consumers believe that firm B provides just as much utility as firm A. At $\underline{p}_1$, consumers would buy from a low-quality firm B. This holds, since $E[q_1^B|\underline{p}_1]$ in expectation provides consumers at least as much utility as firm A. Since $q^l < \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*}$, consumers buy at both prices if $E[q_1^B|p] = \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*}$ $\forall p$. For all other off-equilibrium prices, the above beliefs are consistent with our selection assumptions and the necessary equilibrium conditions we derived so far, since deviations to prices above the equilibrium price induce zero demand. Thus, these off-equilibrium beliefs are without loss of generality.

Next, consider the case where $\overline{p}_1 > q^l$. Recall from Lemma 7 that $\underline{p}_1 < q^l$. Applying Bayes rule shows that the beliefs are correct for equilibrium prices $\overline{p}_1$ and $\underline{p}_1$. For all off-equilibrium prices, the above beliefs are consistent with our selection assumptions and the necessary equilibrium conditions we derived so far, since deviations to prices in $(\underline{p}_1, q^l)$ and to prices above $\overline{p}_1$ strictly reduce demand. Thus, these off-equilibrium beliefs are without loss of generality. $\square$

A.2 Proof of Lemmas 1 and 2

We show a more general result than what we discuss in the main text. Proposition 4 fully characterizes equilibrium. In particular, we also allow for the case where the pricing strategy of low-quality newcomers is in pure strategies. But to simplify exposition, we focus on the pure-strategy pricing equilibria where the low-quality newcomer enters either with probability one or zero. This is the case if (4) holds. Intuitively, in the pure-strategy equilibrium, low-quality newcomers earn negative profits in period 1 and positive profits in period 2. The condition ensures that overall profits are weakly positive. This is the case if q^h is sufficiently large such that reputation is sufficiently valuable. Without this assumption, low-quality newcomers may enter with strictly positive probability less than one, but our results remain qualitatively unaffected.

As discussed in the main text, we also assume that silence is bad news such that a newcomer who receives no rating in period 1 does not attract demand in period 2. This is Condition (5), where δ^* is pinned down in equilibrium by parameters. As $q^h - \overline{p}$ approaches $\overline{e}$, the right-hand-side goes to infinity, and the left-hand-side is bounded away from infinity. If q^h is sufficiently large, $q^h - \overline{p}$ increases and the condition holds. Thus, also this condition holds for sufficiently large q^h. Below, we also discuss the case if the condition does not hold. Then, a qualitatively similar equilibrium exists but the newcomer may also sell after $R = 0$.

Proposition 4. *Suppose the following conditions hold:*

$$\gamma q^h - q^A + F(q^A - \gamma q^h)\left[\frac{\gamma F(q^A + (1-\gamma)q^h)}{\gamma F(q^A + (1-\gamma)q^h) + (1-\gamma)F(q^A - \gamma q^h)} q^h - q^A\right] \geq 0 \tag{4}$$

and

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)\left[\delta^*(1 - F(|\overline{p}|)) + (1-\delta^*)(1 - F(q^A))\right]}{\gamma(1 - F(q^h - \overline{p}))}. \tag{5}$$

Then an equilibrium exists that is unique up to off-path beliefs. There exists a unique $\underline{\delta} \in (0,1)$ such that both types of newcomers enter if and only if $\delta^* > \underline{\delta}$; otherwise, neither enters. If newcomers enter:

1. ***Ratings build reputation:*** $E[q_2^B \mid R = 1] > E[q_2^B \mid R = 0] > E[q_2^B \mid R = -1]$.
2. ***Ratings are valuable:*** $p_2^B = E[q_2^B \mid R = 1] - q^A > 0$. *Firm A sells in period 2 if and only if $R \in \{-1, 0\}$.*

Furthermore, in period 1:

3. ***Firm** A sets $p_1^A = 0$ and gets no demand.*
4. ***Firm** h charges $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$ with probability 1 and receives $R \in \{0, 1\}$.*
5. ***Firm** l randomizes over prices in period 1 if*

$$\frac{\gamma(q^h - q^A)}{(1-\gamma)q^A} > \underline{\delta} \quad \text{and} \quad F(q^A) > \frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2}. \tag{6}$$

In that case:

   (a) It charges $\overline{p} > 0$ with probability δ^* and receives $R \in \{-1, 0\}$, where $\delta^* \in \left(\underline{\delta}, \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}\right)$.

   (b) It charges $\underline{p} \equiv -q^A < 0$ with probability $1 - \delta^*$ and receives $R \in \{0, 1\}$.

Otherwise, firm l sets $\delta^* = 1$ such that $\overline{p} \le 0$ and receives $R \in \{0, 1\}$.

Lemma 1 follows directly from statement 1 and 2 of Proposition 4.

In Lemma 2, the pricing strategy of firm A and firm h follows directly from statements 3 and 4 of Proposition 4; and the pricing strategy of firm l follows directly from the mixed strategy, where $\delta^* \in (\underline{\delta}, \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A})$, in statements 5(a) and 5(b) in Proposition 4.

We prove Proposition 4 in a series of Lemmas and Corollaries.

To apply the previous Lemmas and Corollaries, it is useful to make the following observations: (i) In a two-period model $\mathbb{H} = R$, i.e. all relevant histories are period 1 ratings of the newcomer. (ii) We apply that $q^l = 0$ to save on notation. (iii) In a model where firm B sells only following a good rating, this occurs when $E[q_2^B | R = 0] \le q^A$ which implies $\pi_2(R = 0) = 0$. Then (2) always holds. And (3) evaluates to $f(q^l - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\})}$. Towards proving the proposition, we establish further results in the following Lemmas.

Lemma 10. *Firm B's decision to sell in period 1 is independent of its quality realization. Thus, in period 1, a high-quality firm B sells if and only if a low-quality firm B also sells. If firm B sells in the second period, it must sell in the first period. Thus, whenever newcomers enter, they sell in period* 1.

Proof of Lemma 10.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now consider firm B selling only when it is high-quality.

Suppose instead firm B is only selling when it is high-quality. This means $E[q^B|p] = q^h$ $\forall p$. The high-quality firm B receives a good rating with some positive probability at all prices at which it sells. This allows it to to charge $q^h - q^A > 0$ in both periods. But then l has a profitable deviation to enter the market with strictly positive profits in period 1. Hence it cannot be that only the high-quality firm sells in the market in period 1.

We show next that if firm B sells in period 2, it also sells in period 1. Suppose instead a high-quality firm B sells only in period 2 and not period 1. Then also firm l cannot sell in period 1. But then period 2 is the same as period 1 but without continuation profits, so since both firms did not sell in period 1, they will not sell in period 2, a contradiction. Similarly, if l sold only in period 2, then either h sold in period 1, in which case consumer beliefs about newcomers would be q^h and also l would deviate and mimic h in period 1, or h did not sell in period 1, in which case both firms B did not sell in period 1 and the above result implies that h also does not sell in period 2, contradicting that l sells in period 2. We conclude that if firm B sells in period 2, it also sells in period 1.

Therefore, we can conclude that if firm B enters, it sells in the first period and its entry decision is independent of its quality realization. $\square$

This implies that we cannot have efficient entry - that is there is no situation where all high-quality firm Bs enter the market and low-quality firm Bs do not.

Lemma 11. *If* (1), (3) *hold, and* $\overline{p} > 0$*, second period beliefs over firm* B*'s quality are*

$$E[q_2^B|R] = \begin{cases} \frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*) F(q^A)} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))]} & \text{if } R = 0 \\ 0 & \text{if } R = -1 \end{cases}$$

where δ^* *is the equilibrium probability with which a low-quality firm* B *plays* $\overline{p}$ *in period 1.*

If (1) *and* (3) *hold, and instead* $\overline{p} \leq 0$*, second period beliefs are*

$$E[q_2^B|R] = \begin{cases} \frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma) F(-\overline{p})} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)(1-F(-\overline{p}))} & \text{if } R = 0 \\ 0 & \text{if } R = -1. \end{cases}$$

In either case, ratings are informative in equilibrium as $E[q_2^B|R = 1] > E[q_2^B|R = 0] > E[q_2^B|R = -1]$*.*

These are the beliefs after both newcomer types enter. After other histories, consumers believe the quality of newcomers is q^l *in both periods.*

Proof of Lemma 11.
We first describe how consumer beliefs are constructed when $\overline{p} > 0$, then we show a good rating is beneficial. Then describe beliefs when $\overline{p} \leq 0$.

It is immediate to see consumer beliefs result from applying Bayes rule to the pricing strategies from Lemmas 6 and 7. A high-quality firm always gets good ratings with some positive probability, this probability depends on the price it sets. Conversely, a low-quality firm B obtains a good rating with some positive probability only if it sets a negative price (which occurs with probability $(1-\delta^*)$ or if $\overline{p} \leq 0$). Otherwise, it obtains no rating.

To see that good ratings induce higher beliefs than no rating, we apply $\underline{p} = -q^A$ to the above expectations and get,

$$\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} > \frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)(\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))}$$
$$\Leftrightarrow F(q^h - \overline{p}) - F(q^A) > (F(q^h - \overline{p})F(\overline{p}) - F(q^A))\delta^*,$$

We now argue that this always holds. Lemma 7 tells us $\underline{p}$ is the maximum price that induces demand for l if consumers know the firm's type. This price provides exactly q^A utility to consumers. Hence the firm receives a good rating with probability $F(q^A)$. Moreover, to obtain any demand, a firm h must provide at least expected utility q^A, which is why the ex-post utility satisfies $q^h - \overline{p} \geq q^A$ and therefore $F(q^h - \overline{p}) \geq F(q^A)$. Hence, we know that $F(q^h - \overline{p}) \geq F(q^A)$. This implies $F(q^h - \overline{p}) - F(q^A) > (F(q^h - \overline{p})F(\overline{p}) - F(q^A))\delta^*$ because $\delta^* \in (0, 1]$ and $F(\overline{p}) < 1$. It is straightforward to show that no rating induces higher expectations than a bad rating.

When $\overline{p} \leq 0$, firm B sets a single price in period 1. Hence, there is no mixed strategy involved, and both high- and low-quality firm B would receive good ratings with some positive probability. Note that because $q^h > 0$, obtaining a good rating must be beneficial in equilibrium. It is straightforward to show that no rating induces higher expectations than a bad rating.

Finally, these beliefs apply if both newcomers enter, i.e. if newcomers sell in period 1. By Lemma 10, all other histories are off the path of play, and we set beliefs to q^l. □

Lemma 12. *Suppose h and l enter. Both firms A and B receive some positive demand if and only if firm A sells in period 2, and firm B sells in period 1 with probability* 1*. Both firms sell in period 2 after some ratings if* (7) *holds and* $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$.

Proof of Lemma 12.
Recall that by Lemma 10 the low-quality firm B sells in period 1 if and only if the high-quality firm B also sells in period 1. Furthermore, by Lemma 10 if firm B sells in period 2, it must sell in period 1. This implies the high-quality firm B must sell in period 1 for firm B to sell at all. If the high-quality firm B sells in period 1, the low-quality firm B must also sell in period 1. Hence, firm B must sell with probability 1 in period 1. This means the only possibility for firm A to sell is in period 2.

Thus, we need to check (i) under which conditions firm A sells in period 2; and (ii) under which conditions a high-quality firm B sells in period 2. (i) holds if firm A sells if it faces a rival without rating, i.e. that $q^A > E[q_2^B|R=0] > E[q_2^B|R=-1]$. Using the expression of $E[q_2^B|R=0]$ and rearranging, this condition becomes

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A)))}{\gamma(1-F(q^h-\overline{p}))} \text{ when } \overline{p} > 0 \text{ and}$$

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(1-F(-\overline{p}))}{\gamma(1-F(q^h-\overline{p}))} \text{ when } \overline{p} \leq 0.$$

Observing that when $\overline{p} \leq 0$, $\delta^* = 1$ we can equivalently combine and more generally state that $q^A > E[q_2^B|R=0]$ whenever

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(\delta^*(1-F(|\overline{p}|)) + (1-\delta^*)(1-F(q^A)))}{\gamma(1-F(q^h-\overline{p}))}. \tag{7}$$

Finally, (ii) requires that a firm with a good rating $R=1$ in period 2 sells when competing against firm A and earns a positive profit, i.e. if $E[q_2^B|R=1] > q^A$. Using the expression for the conditional expectation and rearranging leads to $\delta^* > 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{q^A(1-\gamma)F(q^A)}$. Moreover, because δ^* is a probability, $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ for firm B to sell in period 2 after a good rating. ☐

Corollary 9. *Good ratings are instrumental (i.e. affect beliefs and outcomes on the path of play) if and only if a high-quality firm B enters, sells in period 2 and firm B sells in period 1 with probability 1. A high-quality firm B sells after a good rating if $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$.*

Proof of Corollary 9.
If a high-quality firm B sells in period 2, we know from Lemma 12 that both high- and low-quality firm B must have sold in period 1. Then we also know from Lemma 11 that good ratings cause consumers to positively update beliefs.

In turn, if good ratings are instrumental, a high-quality firm B must sell after a good rating, which by Lemma 10 implies it sold in 1.

Thus, good ratings are instrumental if and only if a high-quality firm B enters and sells in period 2, which is equivalent to $E[q_2^B|R=1] > q^A$, from Lemma 12, $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$. ☐

Corollary 10. *Suppose h and l enter and $\overline{p} \leq 0$. Then $\overline{p} = \gamma q^h - q^A < 0$ and $p_1^A = 0$, and (4) implies that firms h and l attract demand and earn positive total profits at this price and therefore enter.*

Proof of Corollary 10.
This is immediate because Lemmas 6 and 7 show that if firm B sells and $\overline{p} \leq 0$, it always sets $\overline{p}$. In other words, $\delta^* = 1$ and consumer beliefs in period 1 about firm B are γq^h (Lemma 9). Therefore the consumer only buys from firm B if it offers as much surplus as firm A does, $\overline{p} = \gamma q^h - q^A$.

Because firms compete in a Bertrand fashion, the firm not selling charges price at marginal cost and the firm selling in period 1 charges a price equal to the expected difference in quality.

Finally, for firm l to want to sell it must face a positive total profit (across both periods), which is the case if (4) holds. Since h has a higher probability to get a positive rating and therefore earns strictly higher profits, the condition implies that h and l enter for the above prices. $\square$

Corollary 11. *If $\overline{p} > 0$ and $\frac{q^h - q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$, we have $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$, $\underline{p} = -q^A$, $p_1^A = 0$ and $\delta^* \in (\max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$. Both newcomer types attract demand and enter. If $\frac{q^h-q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$ is violated, either we must have $\overline{p} \leq 0$, or $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ is violated and newcomers do not enter.*

Proof of Corollary 11.
Note that if $\overline{p} > 0$ it must be that firm B prefers to sell in period 1 with probability one. This occurs if $\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A > 0$.

Rearranging leads to

$$\delta^* < \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}.$$

Therefore, combined with Corollary 9, we know $\delta^* \in (\max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$, and this is possible only when $\frac{q^h-q^A}{q^h} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$.

In this case, firms h and l sell at $\overline{p} > 0$ in period 1 and therefore attract demand and enter.

If $\overline{p} \leq 0$, Corollary 10 implies that B still sells. If $\overline{p} > 0$ holds, but $\frac{q^h-q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$ is violated, either $\delta^* < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$, and therefore we must have $\overline{p} \leq 0$, or δ^* is too small for newcomers to sell after a good rating so that there is no entry. $\square$

Corollary 12. *Suppose h and l enter. In period 2, firm A sets $\pi_2^A = p_2^A = max\{q^A - E[q_2^B|R], 0\}$ and firm B sets $\pi_2^B(R) = p_2^B = max\{E[q_2^B|R] - q^A, 0\}$.*

Proof of Corollary 12.
This follows from Lemma 8. And profits in period 2 are equivalent to the price set in period 2. $\square$

Next, we prove the existence and uniqueness of a mixed strategy.

Given (7) and supposing h and l enter, we can characterize an equilibrium δ^*, which satisfies the following: A low-quality firm B must be indifferent between setting $\overline{p}$ in period 1 and getting no or negative rating, obtaining a profit of $\overline{p} + 0$, and setting $\underline{p}$ in period 1 and getting a good rating with some positive probability, obtaining a profit of $\underline{p} + F(q^A)(E[q_2^B|R=1] - q^A)$.

Lemma 13. *Suppose h and l enter, (4), (7), and (1) and (3) hold. If $\delta^* \geq \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$, newcomers have strictly positive demand and there exists a unique δ^* such that:*

- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} < F(q^A)$ *and* $\overline{p} > 0$, $\delta^* \in (\frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$.
- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} < F(q^A)$ *and* $\overline{p} \leq 0$, $\delta^* = 1$,
- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} \geq F(q^A)$, $\delta^* = 1$.

Proof of Lemma 13.
We first characterize the profits that a low-quality firm B would receive if it plays $\overline{p}$ in period 1, then its profits when playing $\underline{p}$ in period 1. We show the former is decreasing in δ^* while the latter is increasing. Then, we characterize when $\delta^* \in (0, 1)$.

Total profits of the low-quality firm B setting $\overline{p}$ is $\overline{p}$. We know from above that a mixed-strategy equilibrium requires $\overline{p} > 0$. Therefore, the low-quality firm B earns $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$. It is immediate to see that this is decreasing in δ^*.

Total profits of l setting $\underline{p}$ is $\underline{p}$ plus a probability of obtaining a good rating and selling in period 2. Therefore, l earns $-q^A + F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} - q^A)$. This profit is increasing in δ^*. To see this, an increase in δ^* places less emphasis on the firm being low-quality following a good rating. Additionally, an increase in δ^* has an indirect effect of decreasing $\overline{p}$, which increases $F(q^h - \overline{p})$, placing a higher emphasis on the firm being high-quality following a good rating.

In mixed-strategy equilibria, low-quality firms must be indifferent between both strategies, which requires

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} = F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A). \tag{8}$$

Note that $\delta^* \geq \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ ensures that newcomers with a good rating sell in period 2. Then, since for all $\overline{p} > 0$, firms earn attract strictly positive demand and earn positive profits.

To see when the solution is interior to $\delta^* \in (0, 1)$, consider $\delta^* = 0$. Then, evaluating equation (8), we get $\frac{\gamma}{\gamma+(1-\gamma)\delta^*}q^h = q^h$ and $F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} - q^A) = F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)F(q^A)} - q^A)$. It is then immediate to see

$$q^h > F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)F(q^A)} - q^A).$$

This always holds since the term in brackets on the right-hand-side is strictly lower that q^h, and multiplied by a term that is strictly less than one.

Next, note that by Corollary 11, δ^* is bound above, i.e. $\delta^* < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$. Note that $\frac{\gamma(q^h-q^A)}{(1-\gamma)q^A} \leq 1$ if $\gamma q^h \leq q^A$, which we assume throughout. In particular, when $\delta^* = \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$ this implies $\overline{p} = 0 \Leftrightarrow \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} = q^A$, which can be rearranged to $(1-\gamma)(1-\delta^*) = \frac{q^A-\gamma q^h}{q^A}$. Hence, evaluating (8) at this upper bound, a mixed-strategy equilibrium requires that

$$q^A < F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + \frac{q^A - \gamma q^h}{q^A} F(q^A)} - q^A).$$

Recall that F is uniform such that $F(x) = x/\overline{e}$. Then we can replace F to obtain

$$q^A < F(q^A)(\frac{\gamma q^h q^h}{q^A} - q^A).$$

Then recall that the terms in the bracket are the second period profits of firm B conditional on a good rating which must be positive when firm B enters. Therefore,

$$\frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2} < F(q^A)$$

must hold in an interior solution. Otherwise, if this condition is violated, l prefers to set $\delta^* = 1$.

We now show that when (7) holds, $\overline{p} > 0$ and $\frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2} < F(q^A)$ holds such that there is an interior solution, we must have $\delta^* > \frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)}$. Evaluating (8) at $\delta^* = \frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)}$, we show that the left-hand side is greater than the right-hand side:

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)}} > F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1 - \frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)})F(q^A)} - q^A)$$
$$\gamma q^h(F(q^A) + F(q^h - \overline{p}))(1 - F(q^A)) > -q^A F(q^A)(\gamma F(q^h - \overline{p}) + F(q^A))$$

which is always true. Recall that the left-hand side is decreasing in δ^* and the right-hand side increasing. Therefore, it must be that $\delta^* > \frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)}$.

Next, we show that the interval where $\delta^* \in (\frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)}, \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A})$ can exist. This is the case if $\frac{F(q^A)}{F(q^h - \overline{p}) + F(q^A)} < \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}$. Since F is uniform, we can rewrite this as $\frac{q^A}{q^h - \overline{p} + q^A} < \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}$.

To see existence, note, $\frac{q^A}{q^h + q^A} < \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}$ holds if $(q^A)^2 < \gamma(q^h)^2$, and therefore the range can exist if q^h is sufficiently large.

Next, if (7) holds but $\overline{p} \leq 0$, from Lemma 7 we know firm B plays $\overline{p}$ regardless of its type. In other words, the low-quality firm B never mixes and $\delta^* = 1$.

Next, if $\frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2} \geq F(q^A)$, then low-quality firm strictly prefers to mimic prices and we get $\delta^* = 1$.

Finally, we have already shown above that in the mixed-strategy equilibrium, newcomers sell and therefore enter. If $\delta^* = 1$, (4) ensures that newcomers attract strictly positive demand and enter. Thus, newcomers enter with probability one. This concludes the proof. □

We can now use the above results to prove each statement in Proposition 4 .

Proof of Proposition 4.
Suppose (4), (7), and (1) and (3) hold. It remains to show that there exists a unique $\underline{\delta} \in (0,1)$ such that newcomers enter if and only if $\delta^* > \underline{\delta}$. We know already from Lemma 10 that either both types of newcomers enter or none. By our Selection Assumption 1, newcomers enter whenever an equilibrium exists where they enter. By Lemma 13, they enter if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$, i.e. if newcomers with a good rating sell in period 2. Clearly, if they do not sell after a good rating, they never sell. Thus, they enter if and only if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$.

To see that $\underline{\delta} > 0$, note that for $\delta^* = 0$, $1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A} > 0$ if and only if $q^A > \gamma q^h$, which holds by assumption. Thus, the condition is violated for $\delta^* = 0$, implying no entry and $\underline{\delta} > 0$. Next, we know from (4) that even l earns weakly positive profits and attracts demand for $\delta^* = 1$. Since h must earn strictly larger profits than l, both types enter, implying that $\underline{\delta} < 1$. Finally, note that the left-hand-side of $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$ strictly increases in δ^*, while the right-hand-side decreases in δ^*, implying that $\underline{\delta}$ is unique.

Next, note that Lemma 13 shows that conditional on entry, strategies are unique up to off-path beliefs. Additionally, Selection Assumption 1 implies that the same holds conditional on no entry.

We can now prove each statement about the newcomer entry in turn:

- Statement 1 follows directly from Lemma 11.
- Statement 2 follows directly from Corollary 9.
- Statement 3 follows directly from Corollaries 10 and 11.
- Statement 4 follows directly from Corollaries 8 and 6 and Lemma 9.
- The conditions in statement 5 follow from Lemma 13 together with our result that newcomers firms enter if and only if $\delta^* > \underline{\delta}$.
- The prices in statement 5 follow from Corollary 11 and the ratings of the low-quality firm from Lemma 7.
- The equilibrium level of δ^* and its support comes from Lemma 13 and our result that newcomers firms enter if and only if $\delta^* > \underline{\delta}$.

This concludes the proof. □

A.3 Remaining proofs for the main text.

Proof of Proposition 1.
We define the informativeness of ratings as a good rating being able to identify high-quality firms. This way, a good rating becomes more informative if $E[q_2^B | R = 1]$ increases in δ^* when $\delta^* > \underline{\delta}$. To

see this is true, first consider the case where $\overline{p} > 0$ then

$$E[q_2^B|R=1] = \frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)}$$

then note an increase in δ^* decreases $\overline{p}$ which means $\gamma F(q^h - \overline{p})$ increases. Hence, the expectation increases in δ^* and good ratings become more informative.

Suppose instead $\overline{p} \leq 0$, i.e. that δ^* is sufficiently large, and consider the more general beliefs where the low-quality firm may choose to mix between $\overline{p}$ and $\underline{p}$, then we have

$$E[q_2^B|R=1] = \frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)\left[\delta^* F(|\overline{p}|) + (1-\delta^*)F(q^A)\right]}.$$

Observe that $F(q^A) > F(|\overline{p}|)$ because $-(\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A) < q^A \Leftrightarrow 0 < \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*}$ which is always true. Further, because $\overline{p}$ decreases in δ^*, then it must be that any increase in δ^* reduces $\delta^* F(|\overline{p}|) + (1-\delta^*)F(q^A)$ and increases $F(q^h - \overline{p})$. Therefore, $E[q_2^B|R=1]$ is increasing in δ^*. Overall, good ratings become more informative in δ^*. □

Proof of Proposition 3.

To show this, recall that the indifference condition for the low-quality firm must hold in a mixed-strategy equilibrium, define

$$G = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - F(q^A)\left(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A\right).$$

Then applying the uniform distribution, G becomes

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - \frac{q^A}{\overline{e}}\left[\frac{\gamma(q^h - \overline{p})q^h}{\gamma(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)q^A} - q^A\right].$$

We know from the proof of Lemma 13 that G is strictly decreasing in δ^*. Next, we consider the derivative of G with respect to $\overline{e}$, which is clearly strictly increasing. Then, using the implicit-function Theorem, it follows that $\frac{\partial \delta^*}{\partial \overline{e}} = -\frac{\frac{\partial G}{\partial \overline{e}}}{\frac{\partial G}{\partial \delta^*}} > 0$. This concludes the proof. □

Proof of Corollary 1.

First, we show that $\underline{\delta}$ is independent of $\overline{e}$. We know from the proof of Proposition 4 that (i) $\underline{\delta} \in (0,1)$, and (ii) that newcomers enter if and only if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$. Thus, $\underline{\delta}$ is implicitly defined by $\underline{\delta} = 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}$, where also $\overline{p} = \frac{\gamma q^h}{\gamma + (1-\gamma)\underline{\delta}} - q^A$. Using that on its support, $F(x) = \frac{x}{\overline{e}}$, it follows that $\underline{\delta}$ is independent of $\overline{e}$.

Second, we know from Proposition 3 that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$. Together with our results of Proposition 4 that newcomers enter if and only if $\delta^* > \underline{\delta}$, and since $\underline{\delta}$ is independent of $\overline{e}$, this implies that newcomers enter if and only if $\bar{e}$ is above some constant (which has to be strictly positive since we focus on $\bar{e}$ where newcomers get no rating with strictly positive probability). The result follows. $\square$

Proof of Corollary 2.
We start by showing the results on profits.

First suppose $\delta^* \leq \underline{\delta}$. From Proposition 4 we know firm B is inactive and equivalently makes the profit $\pi^B = 0$. Hence, firm A charges monopoly prices, allowing it to extract the full surplus from consumers, q^A, in each period, making a total profit of $\pi^A = 2q^A$.

Now suppose $\delta^* > \underline{\delta}$. Firm B sells and its expected profit is

$$\gamma\left[\overline{p} + F(q^h - \overline{p})\pi_2(R=1)\right] + (1-\gamma)\left[\delta^*\overline{p} - (1-\delta^*)q^A + (1-\delta^*)F(q^A)\pi_2(R=1)\right],$$

where π_2 represents firm B's period 2 profit following the rating $R = 1$. Then substituting $\overline{p}$ and $\pi_2(R=1)$,

$$\gamma q^h(1 + F(q^h - \overline{p})) - q^A\left[1 + \gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)\right].$$

Note that $\overline{p}$ is decreasing in δ^*, which means $F(q^h - \overline{p})$ is increasing in δ^*. Then increases in δ^* increases $\gamma F(q^h - \overline{p})(q^h - q^A)$ and decreases $(1-\gamma)(1-\delta^*)F(q^A)$. Therefore, for increases in δ^*, the expected profits of firm B is increasing.

Firm A's expected profits are

$$\left[\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)\left[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))\right]\right]\left[q^A - E[q_2^B|R=0]\right] + \\ (1-\gamma)\delta^* F(\overline{p})\left[q^A - E[q_2^B|R=-1]\right],$$

and substituting the expectations from Lemma 11, we get

$$\gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)\left[1 - F(q^A)(1-\delta^*)\right]q^A.$$

Observing that an increase in δ^* decreases $\overline{p}$, it must be that $\gamma(1 - F(q^h - \overline{p}))(q^A - q^h)$ becomes larger (since $q^A - q^h < 0$), and also $(1 - F(q^A)(1-\delta^*))$ becomes larger. Therefore the profits of firm A increase in δ^*.

Note that firm A's profits are strictly below monopoly level when firm B sells.

$$2q^A > \gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)\left[1 - F(q^A)(1-\delta^*)\right]q^A \Leftrightarrow \\ q^A(1 + \gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)) > -\gamma(1 - F(q^h - \overline{p}))q^h$$

which is always true.

Therefore the profits of firm A are first flat, then discontinuously decreases in δ^* as firm B begins to sell, and then increasing but remains strictly lower than when it was a monopolist. This concludes the proof.

We now show the results on consumer surplus.

First suppose $\delta^* > \underline{\delta}$. To derive consumer surplus, note that total surplus is

$$\gamma q^h + \gamma \left[F(q^h - \overline{p})q^h + (1 - F(q^h - \overline{p}))q^A\right] + (1-\gamma)\left[\delta^* q^A + (1-\delta^*)(1 - F(q^A))q^A\right].$$

Then consumer surplus is given by total surplus minus the expected profits of A and B. Rearranging leads to the consumer surplus

$$q^A + \left[\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)\right] q^A + \gamma(1 - F(q^h - \overline{p}))q^h.$$

We can further simplify this to

$$q^A + \gamma q^h + \gamma F(q^h - \overline{p})(q^A - q^h) + (1-\gamma)(1-\delta^*)F(q^A)q^A.$$

Note that an increase in δ^* leads to a decrease in $\overline{p}$. Thus, since $(q^A - q^h) < 0$, it is immediate to see that consumers are worse off when δ^* increases.

Suppose now that $\delta^* \leq \underline{\delta}$. Then firm B does not enter and firm A is a monopolist. This way firm A is able to extract all surplus and consumer surplus is zero. □

Proof of Corollary 3.

Suppose $\delta^* > \underline{\delta}$ and consider an increase in $\overline{e}$ such that there is a first order stochastic dominant shift in the uniform rating cost distribution.

We first show the effect of a change in $\overline{e}$ on π^B, recall from the previous corollary that the profit function is $\gamma q^h(1 + \frac{q^h - \overline{p}}{\overline{e}}) - q^A$. Then its derivative w.r.t. $\overline{e}$ is $-\gamma q^h \left[\frac{q^h - \overline{p}}{\overline{e}^2} + \frac{\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}}}{\overline{e}}\right]$. Since the first term in squared brackets is positive and the second one negative, the effect on π^B is ambiguous.

We next look at the effect on π^l. Recall that the low-quality firm B is indifferent between the payoff from setting $\overline{p}$ and $\underline{p}$. Hence, it suffices to show that the profits $\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A$ is decreasing in $\overline{e}$. Notice that the derivative is $-\frac{\gamma(1-\gamma)q^h}{(\gamma + (1-\gamma)\delta^*)^2}\frac{\partial \delta^*}{\partial \overline{e}} < 0$.

We now look at the effect on π^h. Observe that the profits of the high-quality firm B is

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A + \frac{q^h - \overline{p}}{\overline{e}}\left[\frac{\gamma(q^h - \overline{p})q^h}{\gamma(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)(q^A)} - q^A\right].$$

We can see that the term in squared brackets increases in $\overline{e}$ since it increases δ^*. But a larger $\overline{e}$ also decreases the probability that h gets a rating $\frac{q^h - \overline{p}}{\overline{e}}$. Thus, the overall effect is ambiguous.

We now argue that a change in $\overline{e}$ has an ambiguous effect on consumer surplus. To see this, observe that consumer surplus, using the term from the proof of Corollary 2, is

$$q^A + \gamma q^h + \gamma F(q^h - \overline{p})(q^A - q^h) + (1-\gamma)(1-\delta^*)F(q^A)q^A.$$

Then recall from Proposition 3 that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$ and from above note $\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}} < 0$. Then the first and second terms are constant in $\overline{e}$. In the third term, $q^h - \overline{p}$ increase in $\overline{e}$, but $F(q^h - \overline{p})$ decreases in $\overline{e}$, so the effect is ambiguous. In the final term, an increase in $\overline{e}$ decreases $1-\delta^*$ and $F(q^A)$. Therefore, any increase in $\overline{e}$ can have an ambiguous effect on consumer surplus.

Finally, we show $\overline{e}$ has an ambiguous effect on π^A. To see this, consider firm A's profit

$$\gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)[\delta^* + (1-\delta^*)(1 - F(q^A))]q^A$$

Again apply that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$ and $\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}} < 0$. Then we know the first term increases in $q^h - \overline{p}$, but $\overline{e}$ has a direct negative effect in F, so the overall effect is ambiguous. The second term becomes more positive as δ^* increases, but also via the direct effect of $\overline{e}$ on $F(q^A)$. Hence, the overall effect is ambiguous. □

Proof of Corollary 4.
By Corollary 1, newcomers enter if and only if $\overline{e}$ is sufficiently large. Additionally, by Corollary 2 consumer surplus is zero without entry and strictly positive with entry, the result follows. □

Appendix B Extensions

B.1 Selling after no rating

In the base model we focus on the scenario where firm B does not sell after receiving no rating. We establish two results. First, we show that a qualitatively identical equilibrium arises. Second, we show that our comparative statics of $\frac{\partial \delta^*}{\partial \overline{e}}$ are also robust, that is $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Note that if firm B does not sell only following a negative rating, this means $E[q_2^B|R = -1] = 0 \leq q^A < E[q_2^B|R = 0]$.

Proposition 5. *Suppose the following condition holds:*

$$\frac{q^h - q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1 - F(|\overline{p}|)) + (1-\delta^*)(1 - F(q^A))]}{\gamma(1 - F(q^h - \overline{p}))}. \tag{9}$$

Then in equilibrium both newcomers enter:

1. ***Ratings build reputation:*** $E[q_2^B \mid R = 1] > E[q_2^B \mid R = 0] > E[q_2^B \mid R = -1]$.
2. ***Ratings are valuable:*** $p_2^B(R = 1) = E[q_2^B \mid R = 1] - q^A > 0$ *and* $p_2^B(R = 0) = E[q_2^B \mid R = 0] - q^A > 0$. *If* $R = -1$, *firm* A *sells in period 2.*

Furthermore, in period 1:

3. ***Firm*** *A sets $p_1^A = 0$ and faces no demand.*
4. ***Firm*** *h charges $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$ with probability 1 and receives $R \in \{0, 1\}$.*
5. ***Firm*** *l randomizes over prices in period 1 if $\frac{\gamma(q^h-2q^A)}{2(1-\gamma)q^A} > 0$ and (11) hold. In that case:*
   - *(a) It charges $\overline{p} > 0$ with probability δ^* and receives $R \in \{-1, 0\}$, where $\delta^* \in \left(\underline{\delta}, \frac{\gamma(q^h-2q^A)}{2(1-\gamma)q^A}\right)$.*
   - *(b) It charges $\underline{p} \equiv -q^A < 0$ with probability $1 - \delta^*$ and receives $R \in \{0, 1\}$.*

   Otherwise, firm l sets $\delta^* = 1$ such that $\overline{p} \leq 0$ and receives $R \in \{0, 1\}$.

Finally, if (12) holds, this equilibrium is unique up to off-path beliefs.

(12) is a technical condition. It ensures that if the mixed-strategy equilibrium is played, it is unique. Otherwise, there might be multiple $\delta^* \in (0, 1)$ that induce a mixed-strategy equilibrium.

Recall that our results from Appendix A.1 still apply here, i.e. Lemmas 3 to 9 and Corollary 5 to 8 hold also for the two-period model in which the firm B sells after receiving no rating in period 1. We proceed by establishing Lemmas and Corollaries as we did towards the proof of Proposition 4.

Lemma 14. *Suppose both types of newcomer enter. If (1), (2) and (3) hold, and $\overline{p} > 0$, second period beliefs over firm B's quality are*

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]} & \text{if } R = 0 \\ 0 & \text{if } R = -1 \end{cases}$$

where δ^* is the equilibrium probability with which a low-quality firm B plays $\overline{p}$ in period 1.

If (1), (2) and (3) hold, and instead $\overline{p} \leq 0$, second period beliefs are

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)F(|\overline{p}|)} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)(1-F(|\overline{p}|))} & \text{if } R = 0 \\ 0 & \text{if } R = -1. \end{cases}$$

In either case, ratings are beneficial in equilibrium as $E[q_2^B|R = 1] > E[q_2^B|R = 0] > E[q_2^B|R = -1]$.

If not both types of newcomers enter, consumers believe the quality of newcomers is q^l.

Proof of Lemma 14.
Suppose that both newcomer types enter.

When $\overline{p} > 0$, the argument for the case with $R = 1$ follows directly from Lemma 11. When $\overline{p} \leq 0$, the proofs for $R = 1$ and $R = 0$ follow directly from Lemma 11.

What remains to show is consumer beliefs when $\overline{p} > 0$ for $R = 0$ and $R = -1$, and when $\overline{p} \leq 0$ for $R = -1$.

Suppose first $\overline{p} > 0$. Then when $R = 0$, Corollary 6 and 7 show that both high and low-quality firms can get no rating. This occurs if prices are set sufficiently low but with some probability $1-F(q^B-\overline{p})$ consumers face a high cost of rating and do not want to rate or in the case of the low-quality firm setting a sufficiently high price with some probability $1-F(\overline{p})$ consumers face a high cost of ratings and do not want to rate. Therefore, $E[q_2^B|R=0] = \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]}$.

Whenever $R = -1$, Corollary 7 shows only low-quality firms may receive bad ratings. In other words, high-quality firms receive bad ratings with probability 0, and $R = -1$ clearly identifies low-quality firms. Therefore, for any p, $E[q_2^B|R=-1] = 0$.

Finally, to show that ratings are beneficial, we have to show $E[q_2^B|R = 1] > E[q_2^B|R = 0] > E[q_2^B|R = -1]$. The second inequality is immediately satisfied. To see the first inequality, consider first $\overline{p} > 0$:

$$\begin{aligned}
&\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} \\
> &\frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))]} \\
&\delta^* F(q^h-\overline{p})(1-F(\overline{p})) + (1-\delta^*)(F(q^h-\overline{p}) - F(q^A)) > 0.
\end{aligned}$$

This inequality always holds because $F(\cdot) \in [0,1]$, $\delta^* \in [0,1]$, $q^h - \overline{p} > q^A$ imply that $F(q^h-\overline{p}) > F(q^A)$. Consider next $\overline{p} \leq 0$:

$$\begin{aligned}
\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)F(|\overline{p}|)} &> \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)(1-F(|\overline{p}|))} \\
(1-F(|\overline{p}|))F(q^h-\overline{p})q^h &> (1-F(q^h-\overline{p}))F(|\overline{p}|)q^h \\
\frac{F(q^h-\overline{p})}{F(|\overline{p}|)} &> \frac{1-F(q^h-\overline{p})}{1-F(|\overline{p}|)}.
\end{aligned}$$

This inequality always holds as $q^h > 0 \to 1 > F(q^h-\overline{p}) > F(|\overline{p}|) > 0$. Hence, $\frac{F(q^h-\overline{p})}{F(|\overline{p}|)} > 1 > \frac{1-F(q^h-\overline{p})}{1-F(|\overline{p}|)}$.

Finally, for the off-equilibrium case after not both types of newcomers enter, consumers believe the quality of newcomers is q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in period 1. □

Lemma 15. *Firm B's decision to sell in period 1 is independent of its quality realization. Thus, in period 1, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in the second period, it must also sell in the first period.*

Proof of Lemma 15.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now consider firm B selling only when it is high-quality.

Suppose instead firm B only sells when it is high-quality. This means $E[q^B|p] = q^h \; \forall p$. The high-quality firm B receives a positive rating with some positive probability at all prices at which it sells. This allows it to get a continuation profit of $q^h - q^A > 0$ in period 2. But then there is a profitable deviation from a low-quality firm B to enter the market with positive prices in period 1. Hence it cannot be that only the high-quality firm sells in the market in period 1.

Suppose instead a high-quality firm B chooses to enter the market only in period 2 and not period 1. Then it faces the same payoffs as entering in the first period, while forgoing to continuation profits it may get from entering the market in period 1. Thus, h has a profitable deviation to enter in period 1.

Therefore, we can conclude that if firm B enters, it enters the market in the first period and its entry decision is independent of its quality realization. □

Lemma 16. *Suppose h and l enter. Both firms receive some positive demand only if firm A sells in period 2, and firm B sells in period 1 with probability* 1*. Firm A sells in period 2 if and only if* $\overline{p} > 0$ *(such that consumers give negative ratings) and firm B sells in period 2 if* (9) *holds.*

Proof of Lemma 16.
The proof takes two parts. First, proving that if firm B enters, it sells in period 1 with probability 1 and firm A only sells in period 2. Second, we derive the conditions for firm A to sell in period 2 and firm B with no rating to sell in period 2.

To show the first statement, recall that by Lemma 15 the low-quality firm B sells in period 1 if and only if the high-quality firm B sells in period 1. Furthermore, by Lemma 15 if firm B sells in period 2, it must also sell in period 1 if it enters. This implies the high-quality firm B must sell in period 1 if it enters. If the high-quality firm B sells in period 1, the low-quality firm B must also sell in period 1. Hence, firm B must sell with probability 1 in period 1 if it enters. This means the only possibility for firm A to sell is in period 2.

Thus, we need to check (i) firm A sells in period 2, (ii) firm B sells in period 2 if it has no rating. To show (i) is immediate as $q^A > 0$ implies that firm A sells if and only if firm B gets a negative rating. Note firm B only gets negative ratings if $\overline{p} > 0$.

To check for (ii) to hold,

$$\frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]} > q^A$$
$$\Leftrightarrow \frac{q^h-q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]}{\gamma(1-F(q^h-\overline{p}))} \tag{9}$$

□

Corollary 13. *Suppose h and l enter. Ratings are instrumental (i.e. affect beliefs and outcomes) if and only if a high-quality firm B sells in period 2 and firm B sells in period 1 with probability 1. A high-quality firm B sells in period 2 if (9) holds.*

Proof of Corollary 13.
If a high-quality firm B sells in period 2, we know from Lemma 16 that both high- and low-quality firm B must have sold in period 1. Then we also know from Lemma 14 that ratings change consumer beliefs.

In turn, if ratings are instrumental, a high-quality firm B must sell in period 1, which requires that it sells in period 2.

Thus, ratings are instrumental if and only if a high-quality firm B sells in period 2, which holds if $E[q_2^B|R=1] > q^A$, i.e. if (9) is satisfied. □

Lemma 17. *Suppose (4) and (9) hold, as well as (1), (2) and (3). Then*

1. *If $\overline{p} > 0$ and (11) hold, there exists a mixed strategies where $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$.*
2. *If $\overline{p} > 0$ and (11) hold, (12) is a sufficient condition for a unique mixed strategy exists where $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$.*
3. *If $\overline{p} > 0$ holds and (11) is violated, then there exists a unique $\delta^* = 1$.*
4. *If $\overline{p} \leq 0$, then newcomers sell in period 1 and $\delta^* = 1$.*

Proof of Lemma 17.
We first characterize the total profits that a low-quality firm B would receive if it plays $\overline{p}$ in period 1. Then its total profits when playing $\underline{p}$ in period 1. We then find the conditions for an interior solution $\delta^* \in (0,1)$ exists, and when such a solution is unique.

We now characterize the profits the low-quality firm B would receive if it plays $\overline{p}$. When the firm does so, it receives a profit of $\overline{p}$ in the first period, and with probability $F(\overline{p})$ a bad rating and with complementary probability $(1-F(\overline{p}))$ no rating. This leaves the firm with a continuation profit of 0 with probability $F(\overline{p})$ and $\pi_2(R=0)$ with probability $(1-F(\overline{p}))$. Note that the firm is able to sell at a strictly positive price with strictly positive probability and therefore enters.

We next characterize the profits the low-quality firm B would receive if it plays $\underline{p}$. When the firm does so, it receives a profit of $\underline{p}$ in the first period, and with probability $F(-\underline{p}) = F(q^A)$ it receives a good rating and with complementary probability $(1-F(q^A))$ it receives no rating. Hence leaving the firm with a continuation profit of $\pi_2(R=1)$ with probability $F(q^A)$ and $\pi_2(R=0)$ with probability $(1-F(q^A))$.

Recall from Lemma 7 that $\delta^* = 1$ if $\overline{p} \leq 0$. Additionally, note that by (4), l gets strictly positive demand if it only sells after a positive rating, so this condition implies it also gets strictly positive demand if it sells after a positive and no rating. This proves statement 4. Hence, for the remainder of the proof we focus on the situation where $\overline{p} > 0$.

Note the following 3 conditions must be satisfied for an interior solution $\delta^* \in (0,1)$ to exist: (i) At any $\delta^* \in (0,1)$, the low-quality firm B has to be indifferent between setting $\overline{p}$ obtaining either no or bad ratings, and setting $\underline{p}$ obtaining either good or no ratings. (ii) At $\delta^* = 0$, the benefit from setting $\overline{p}$ must be strictly larger than the benefit of setting $\underline{p}$. (iii) At $\delta^* = 1$, the benefit of setting $\underline{p}$ must be strictly larger than the benefit of setting $\overline{p}$.

Thus, for an interior solution, (i) implies the following equation must hold:

$$\begin{aligned}
&\overline{p} + (1-F(\overline{p}))\pi_2(R=0) = \underline{p} + F(q^A)\pi_2(R=1) + (1-F(q^A))\pi_2(R=0)\\
&\Leftrightarrow \overline{p} - F(\overline{p})\pi_2(R=0) = \underline{p} + F(q^A)\pi_2(R=1) - F(q^A)\pi_2(R=0)\\
&\Leftrightarrow \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} + (F(q^A) - F(\overline{p}))\pi_2(R=0) - F(q^A)\pi_2(R=1) = 0,
\end{aligned} \tag{10}$$

defining the LHS of (10) as K. Where

$$\begin{aligned}
\pi_2(R=0) &= \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))]} - q^A,\\
\pi_2(R=1) &= \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A.
\end{aligned}$$

Then (ii) means $K > 0$ at $\delta^* = 0$ is required for an interior solution. Note that at $\delta^* = 0$, $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A = q^h - q^A$. Then, evaluated at $\delta^* = 0$

$$\begin{aligned}
K =& q^h - F(q^A)\left(\frac{\gamma F(q^A)q^h}{\gamma F(q^A) + (1-\gamma)F(q^A)} - q^A\right)\\
&+ (F(q^A) - F(q^h-q^A))\left(\frac{\gamma(1-F(q^A))q^h}{\gamma(1-F(q^A)) + (1-\gamma)(1-F(q^A))} - q^A\right)\\
=& q^h - F(q^A)(\gamma q^h - q^A) + (F(q^A) - F(q^h-q^A))(\gamma q^h - q^A)\\
=& q^h - F(q^h-q^A)(\gamma q^h - q^A),
\end{aligned}$$

because $F(\cdot) \in (0,1)$ and $q^h > \gamma q^h > 0$, when evaluated at $\delta^* = 0$, $K > 0$.

Then (iii) means $K < 0$ at $\delta^* = 1$ is required for an interior solution. Note that at $\delta^* = 1$, $\overline{p} = \gamma q^h - q^A$. Then, evaluated at $\delta^* = 1$

$$K = \gamma q^h + (F(q^A) - F(\overline{p}))(\frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1 - \gamma)(1 - F(\overline{p}))} - q^A) < 0, \tag{11}$$

is required for an interior solution. This implies $F(\overline{p}) > F(q^A)$ is a necessary condition for an interior solution. Note that since $\overline{p}$decreases in δ^*, (11) implies that $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0, 1]$.

Hence two conditions are required for an interior solution: $\overline{p} > 0$ and (11)$\Rightarrow$ $F(\overline{p}) > F(q^A)$.

To show that the interior solution is unique, it suffices to show that K is decreasing in δ^*.

$$\begin{aligned}
\frac{\partial K}{\partial \delta^*} &= \frac{\partial \overline{p}}{\partial \delta^*} - F(q^A)\frac{\partial \pi_2(R=1)}{\partial \delta^*} - f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*}(F(\overline{p}) - F(q^A)), \\
\frac{\partial \overline{p}}{\partial \delta^*} &= -\frac{\gamma(1-\gamma)q^h}{(\gamma + (1-\gamma)\delta^*)^2} < 0, \\
\frac{\partial \pi_2(R=1)}{\partial \delta^*} &= \frac{F(q^A)\gamma(1-\gamma)q^h)(F(q^h - \overline{p}) - (1-\delta^*)f(q^h - \overline{p})\frac{\partial \overline{p}}{\partial \delta^*})}{(\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A))^2} > 0, \\
\frac{\partial \pi_2(R=0)}{\partial \delta^*} &= \frac{\gamma(1-\gamma)q^h\frac{\partial \overline{p}}{\partial \delta^*}(\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A)))f(q^h - \overline{p})}{(\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))])^2} \\
&\quad + \frac{\gamma(1-\gamma)q^h(\delta^* f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*} + F(\overline{p}) - F(q^A))(1 - F(q^h - \overline{p}))}{(\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))])^2}.
\end{aligned}$$

Recall that (11) is required for an interior solution, which implies $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0, 1]$. Then $\frac{\partial \pi_2(R=0)}{\partial \delta^*} > 0$ is a sufficient but not necessary condition for $\frac{\partial K}{\partial \delta^*} < 0$. Rewriting this sufficient condition,

$$\begin{aligned}
&\frac{\partial \pi_2(R=0)}{\partial \delta^*} \geq 0 \Leftrightarrow \\
&F(\overline{p}) - F(q^A) \geq \frac{-\frac{\partial \overline{p}}{\partial \delta^*}((1 - F(q^A))f(q^h - \overline{p}) + (1 - F(q^h - \overline{p}))\delta^* f(\overline{p}))}{1 - F(q^h - \overline{p}) - f(q^h - \overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\delta^*} > 0.
\end{aligned} \tag{12}$$

Finally, because (11) implies $F(\overline{p}) > F(q^A)$, then it must be that $\overline{p} > q^A$, which means $\frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)} > \delta^*$.

With these conditions, we may now show the statements in the Lemma.

Statement 1. If $\overline{p} > 0$ and (11) holds, then there exists solutions to (10) such that all solutions are interior, $\delta^* \in (0, \frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)})$.

Statement 2. A sufficient condition for these interior solutions to be unique is (12).

Statement 3. If $\overline{p} > 0$ and (11) is violated, then there exists a unique $\delta^* = 1$.

Statement 4. If $\overline{p} \leq 0$ there exists a unique $\delta^* = 1$.

□

Proof of Proposition 5.

Suppose (4) and (9) hold, as well as (1), (2) and (3).

Note first that by Lemma 17, (4) and (9) imply that newcomers always have strictly positive demand. Together with our Selection Assumption 1, this implies newcomers enter with probability one.

Statement 1 follows directly from Lemma 14.

Statement 2 follows directly from Corollary 8.

Statement 3 follows directly from Corollary 13 and Lemma 5.

Statement 4 follows directly from Lemma 9, Corollary 6 and 8

The conditions in statement 5 follow from Lemma 17.

The prices in statement 5 follow from the proof of Lemma 17 and the ratings of the low-quality firm from Lemma 7.

The equilibrium level of δ^* and its support follow from Lemma 17.

□

B.1.1 Comparative statics

Again, we focus on mixed-strategy equilibria. We also impose (12) to ensure it is unique. If the mixed-strategy equilibrium is not unique, then we cannot ensure that changes in parameters induce a jump to another mixed-strategy equilibrium.

Corollary 14. $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Proof of Corollary 14.
We apply the uniform distribution to (10), which leads to

$$\overline{p} + q^A + \frac{q^A - \overline{p}}{\overline{e}} \pi_2(R=0) - \frac{q^A}{\overline{e}} \pi_2(R=1) = 0.$$

Now, we calculate the total derivative with respect to $\overline{e}$ as

$$-\frac{\gamma(1-\gamma)q^h \frac{\partial \delta^*}{\partial \overline{e}}}{(\gamma + (1-\gamma)\delta^*)^2} + \frac{q^A}{\overline{e}^2} \left[\pi_2(R=1) - \pi_2(R=0)\right] + \frac{q^A}{\overline{e}} \left[\frac{\partial \pi_2(R=0)}{\partial \delta^*} - \frac{\partial \pi_2(R=1)}{\partial \delta^*}\right] \frac{\partial \delta^*}{\partial \overline{e}} - \left[\frac{\partial \overline{p}}{\partial \delta^*} \frac{\partial \delta^*}{\partial \overline{e}} - \frac{\overline{p}}{\overline{e}^2}\right] \pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*} \frac{\partial \delta^*}{\partial \overline{e}} \frac{\overline{p}}{\overline{e}} = 0.$$

Rearranging leads to

$$\frac{\partial \delta^*}{\partial \overline{e}}\left[\frac{\partial \overline{p}}{\partial \delta^*}\left[1-\frac{\pi_2(R=0)}{\overline{e}}\right]+\frac{\partial \pi_2(R=0)}{\partial \delta^*}\frac{q^A-\overline{p}}{\overline{e}}-\frac{\partial \pi_2(R=1)}{\partial \delta^*}\frac{q^A}{\overline{e}}\right]=\frac{q^A-\overline{p}}{\overline{e}^2}\pi_2(R=0)-\frac{q^A}{\overline{e}^2}\pi_2(R=1).$$

Recall from the proof of Lemma 17 that $\frac{\partial \overline{p}}{\partial \delta^*}=-\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*)^2}<0$, $\frac{\partial \pi_2(R=1)}{\partial \delta^*}>0$, and by (12) also $\frac{\partial \pi_2(R=0)}{\partial \delta^*}>0$.

Observe that the right side of this equation is negative. Applying (2), $1-\frac{\partial \pi_2(R=0)}{\partial \overline{e}}>0$ and because $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$, by the proof of Lemma 17, in mixed-strategy equilibria we must have $q^A-\overline{p}<0$.

Therefore $\frac{\partial \overline{p}}{\partial \delta^*}\left[1-\frac{\pi_2(R=0)}{\overline{e}}\right]<0$, $\frac{\partial \pi_2(R=0)}{\partial \delta^*}\frac{q^A-\overline{p}}{\overline{e}}<0$ and $\frac{\partial \pi_2(R=1)}{\partial \delta^*}\frac{q^A}{\overline{e}}>0$, which implies the term in the large squared brackets is negative. Hence, $\frac{\partial \delta^*}{\partial \overline{e}}>0$. □

B.2 Three-period model

We extend our main model and allow for three periods. To start, we explain how we change the main model.

We introduce notation to keep track of histories, denoting $\mathbb{H}_t$ a history of the game until period t, where a history is characterized by the historic ratings until and not including t. To simplify notation, we may suppress the subscript t. We may also use histories as subscripts to clarify arguments, i.e. $\overline{p}_{\{0,1\}}$ is the large newcomer price in period 3 for a history with $R_1=0$ and $R_2=1$, or $\overline{p}_{\{0\}}$ is the large newcomer price in period 2 for a history with $R_1=0$. Throughout, we denote $\emptyset$ as the history in period 1 when the game begins.

Entry. The newcomer B chooses to enter/exit in each period 1 and 2 and they do so if and only if their subsequent demand is non-zero. Due to this assumption, this extension also captures the possibility of exit better than our baseline model.

Low-quality newcomers enter with probability 1 or 0. We also have to adjust Condition (4) to this setting and replace it with Conditions (17) and (18). The first one ensures that low-quality newcomers enter with probability 1 or 0 in period 1, and the second one ensures this for period 2.

Silence is bad news. We extend Condition (5) to this setting in the following way. We focus on equilibria where firm B sells in period 2 and 3 if and only if it has a history of exclusively good ratings. Again, this is in line with evidence we discuss in the main text that silence is bad news. In other words, firm B only sells if $\mathbb{H}_t \in \{\emptyset, \{1\}, \{1,1\}\}$. A sufficient condition is that $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$. I.e. if the newcomer does not sell after a history of one good and no rating, and not after a history of no rating, they also do not sell after other histories that involve no rating or a negative rating, since expectations must be lower in such histories. Intuitively, this is satisfied if q^h is sufficiently large, since then the high-quality firm is unlikely to receive no rating, lowering expectations for every history with no rating.

Other conditions and restrictions. The conditions that the PDF of F is sufficiently flat are the same as in the proof of Proposition 4. Also the equilibrium-selection assumptions are the same as in the main text. We also continue to assume $\gamma q^h < q^A$.

Proposition 6. *If f is sufficiently flat (i.e. (1), (2) and (3) hold),* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$*, and (17) and (18) hold. There exists an equilibrium that is unique up to off-path beliefs. In this equilibrium, there exist unique values $\underline{\delta}_\emptyset \in (0,1)$ and $\underline{\delta}_{\{1\}} \in (0,1)$ such that the following holds. Both types of newcomer B enter in period 1 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$; otherwise none enters. Both types of newcomer B enter in period 2 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ hold; otherwise none enters in period 2.*

Furthermore, if the newcomer enters in period t:

1. ***Ratings build reputation:*** *For $t > 1$, $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] \geq E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] \geq E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$, where the inequalities are strict if $\mathbb{H}_{t-1}$ only includes positive ratings.*
2. ***Ratings are valuable:*** *$p_{\mathbb{H}_3} = E[q_3^B|\mathbb{H}_3] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings, and for $t < 3$, we have $p_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, p_{\mathbb{H}_t}] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings. Otherwise, $p_{\mathbb{H}_t} = 0$ and firm A sells in period t.*

Furthermore, in period $t < 3$ for any history $\mathbb{H}_t = \{\emptyset, \{1\}\}$:

3. ***Firm** A sets $p_t^A = 0$ and faces no demand.*
4. ***Firm** h charges $\overline{p}_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A$ with probability 1 and receives $R_t \in \{0,1\}$.*
5. ***Firm** l either charges a mixed-strategy equilibrium such that:*
   (a) It charges $\overline{p}_{\mathbb{H}_t} > 0$ with probability $\delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{-1, 0\}$, where $\delta^*_{\mathbb{H}_t} \in (\underline{\delta}_t, 1]$.
   (b) It charges $\underline{p}_{\mathbb{H}_t} = -q^A < 0$ with probability $1 - \delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{0,1\}$.

   Otherwise, if this mixed-strategy equilibrium does not exist, firm l sets $\delta^*_{\mathbb{H}_t} = 1$ such that $\overline{p}_{\mathbb{H}_t} < 0$ and receives either $R_t = 1$ or $R_t = 0$.

Towards proving this proposition, we will show a range of lemmas and corollaries that follow along the lines of the proof of Proposition 4. Recall that Lemmas 3 to 9 and Corollary 5 to 8 hold also for the three period model.

Lemma 18. *Given* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ *such that firm B sells if and only if $\mathbb{H} = \{\emptyset, \{1\}, \{1,1\}\}$:*

The period 2 beliefs if both types of newcomers enter in period 1 and 2 are

$$E[q_2^B|\mathbb{H}_1, p_2] =$$
$$\begin{cases} \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}(1-F(|\overline{p}_\emptyset|))} & \text{for } \mathbb{H}_1=\{0\},\ \forall p_2=\overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset \le 0 \\ \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}\left[\delta^*_\emptyset(1-F(\overline{p}_\emptyset))+(1-\delta^*_\emptyset)(1-F(q^A))\right]} & \text{for } \mathbb{H}_1=\{0\},\ \forall p_2=\overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)\delta^*_{\{1\}}F(|\overline{p}_\emptyset|)} & \text{for } \mathbb{H}_1=\{1\},\ \forall p_2=\overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset \le 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}}F(q^A)} & \text{for } \mathbb{H}_1=\{1\},\ \forall p_2=\overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset > 0 \\ 0 & \text{otherwise.} \end{cases}$$

And the period 3 beliefs, if both types of newcomers entered in period 1 and 2, are

$$E[q_3^B|\mathbb{H}_2, p_3] =$$
$$\begin{cases} \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)(1-F(|\overline{p}_{\{1\}}|))} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset, \overline{p}_{\{1\}} \le 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-F(|\overline{p}_{\{1\}}|))} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset > 0,\ \overline{p}_{\{1\}} \le 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset \le 0,\ \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset, \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset, \overline{p}_{\{1\}} \le 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset > 0,\ \overline{p}_{\{1\}} \le 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset \le 0,\ \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset, \overline{p}_{\{1\}} > 0 \\ 0 \qquad \text{for } \mathbb{H}_2=\{1,-1\},\ \forall p_3 \\ E[q_2^B|\mathbb{H}_1, p_2] \qquad \text{otherwise.} \end{cases}$$

where $\delta^*_\emptyset$ *is the mixed strategy in the first period,* $\delta^*_{\mathbb{H}_t}$ *is the mixed strategy in the period* t *following* *a history $\mathbb{H}_t$. And $\overline{p}_{\mathbb{H}_t}$ and $\underline{p}_{\mathbb{H}_t}$ are the prices following a history $\mathbb{H}_t$.*

After histories where not both types of newcomers enter, beliefs are $q^l = 0$.

Proof of Lemma 18.
We now explain how we apply Bayes Rule to construct the above beliefs.

Recall from Corollary 8 that $\underline{p}_{\mathbb{H}_t} = -q^A$.

Corollary 7 implies that for any history which includes -1, consumer beliefs are q^l. Such histories are $\mathbb{H} = \{\{-1\}, \{1, -1\}, \{0, -1\}, \{-1, 1\}, \{-1, 0\}, \{-1, -1\}\}$.

We focus on the histories where firm B does not sell following no rating. In other words, following no rating, any subsequent beliefs must be the same as the previous period. Hence, following $\mathbb{H}_1 = 0$, it must be that beliefs are unchanged (for $\mathbb{H}_2 = \{\{0, 1\}, \{0, 0\}, \{0, -1\}\}$).

In any equilibrium where $\overline{p}_{\mathbb{H}_t} \leq 0$, Lemma 7 shows that $\delta^*_{\mathbb{H}_t} = 1$.

We now construct beliefs for $\mathbb{H}_1 = \{\{0\}, \{1\}\}$. Suppose $\overline{p}_\emptyset \leq 0$, then $\delta^*_\emptyset = 1$ such that low-quality firms may only get no rating or a good rating. If the low-quality firm gets $R_1 = 1$, it does so with probability $F(|\overline{p}_\emptyset|)$. High-quality firms get a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firms only play $\overline{p}_{\{1\}}$ with some probability $\delta^*_{\{1\}}$, otherwise they play $\underline{p}_{\{1\}}$.

If, instead, the low-quality firm gets $R_1 = 0$, it does so with complementary probability $1 - F(|\overline{p}_\emptyset|)$. Likewise, the high-quality firm gets $R_1 = 0$ with complementary probability $1 - F(q^h - \overline{p}_\emptyset)$. Following $R_1 = 0$, low-quality firms only play $\overline{p}_{\{0\}}$ with some probability $\delta^*_{\{0\}}$, or they play $\underline{p}_{\{0\}}$ with probability $1 - \delta^*_{\{0\}}$.

Suppose instead that $\overline{p}_\emptyset > 0$, then $\delta^*_\emptyset \in (0, 1)$. From Lemmas 6 and 7, high-quality firm B plays $\overline{p}_\emptyset$ with probability 1, obtaining a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firm B mixes between $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$ with probabilities $\delta^*_\emptyset$ and $1 - \delta^*_\emptyset$, respectively. The low-quality firm may only obtain $R_1 = 1$ with probability $F(q^A)$ if it plays $\underline{p}_\emptyset$, which it does with probability $1 - \delta^*_\emptyset$.

Consider instead when firms get the rating $R_1 = 0$. The high-quality firm gets this rating with probability $(1 - F(q^h - \overline{p}_\emptyset))$, and l with probability $\left[\delta^*_\emptyset(1 - F(\overline{p}_\emptyset)) + (1 - \delta^*_\emptyset)(1 - F(q^A))\right]$.

This characterizes all period 2 beliefs.

We now characterize beliefs in period 3 following $\mathbb{H}_2 = \{1, 1\}$. If prices are $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$. Firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})$, and l if it charges a low price in period 1 an draws sufficiently low rating effort, i.e. $(1 - \delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in each period and obtains a sequence of good ratings with probability $F(|\overline{p}_\emptyset|) \cdot F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_\emptyset$ in the first period, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

Next, we characterize beliefs in the in period 3 following $\mathbb{H} = \{1, 0\}$. If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$, firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)(1 - F(q^h - \overline{p}_{\{1\}}))$. Firm l sets a low price in period 1, leading to $(1 - \delta^*_\emptyset)F(q^A)(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in both periods, obtaining a good rating in period 1 with probability $F(|\overline{p}_\emptyset|)$ and no rating in period 2 with probability $(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_\emptyset$ in the first period with probability 1, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$. In the second period, the low-quality firm sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A)$. Additionally, it sets the price $\overline{p}_{\{1\}}$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$, the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A))$. Additionally, it sets the price $\overline{p}_{\{1\}} > 0$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If not both newcomers enter either in period 1 or 2, these histories are off-equilibrium and we set beliefs to q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each subsequent period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in the period where they enter, a contradiction. We conclude that histories after which not both types enter are off-equilibrium.

This characterizes consumer beliefs in every period following every history $\mathbb{H}$ given firm B sells only after a history where both types of newcomers enter and sell only after positive ratings.

We now find the condition where firm B does not sell in any history following at least a single instance of no rating or a bad rating. To do so, we find the most optimistic history following at least a single instance of no rating or a bad rating. Then we find the condition for which that belief is worse than q^A—such that consumers prefer to purchase from firm A instead of firm B following this history.

To find the most optimistic history, note that any such history cannot include $R_t = -1$. Following any history with $R_t = -1$, beliefs are $q^l = 0$ which are the most pessimistic beliefs possible. Next, note that the histories $\mathbb{H}_t = \{\{0,0\},\{0,1\}\}$ are off-path as firm B does not sell following $\mathbb{H}_t = \{0\}$. We fix these beliefs equal $q^l = 0$. Moreover, since there are no sales on the path-of-play following $\{0\}$, this implies that $\overline{p}_{\{0\}} = \underline{p}_{\{0\}} = 0$, which implies $\delta^*_{\{0\}} = 1$. Given these statements, we know the most optimistic beliefs following a history which includes no rating or a bad rating must be either $\{0\}$ or $\{1,0\}$. Thus, $\max\{E[q^B_3|\{1,0\}], E[q^B_2|\{0\}]\} < q^A$ implies that if both newcomers enter, they sell only after histories with no ratings if $\max\{E[q^B_3|\{1,0\}], E[q^B_2|\{0\}]\} < q^A$ holds. □

Lemma 19. *Suppose* $\max\{E[q^B_3|\{1,0\}], E[q^B_2|\{0\}]\} < q^A$ *holds and* f *is sufficiently flat such that* (1), (2) *and* (3) *hold. Then for all* $\mathbb{H}_t$ *on the path of play, there exists a unique* $\delta^*_{\mathbb{H}_t}$ *such that in period 3, firms charge* $p_3 = \max\{E[q^B_3|\mathbb{H}_t] - q^A, 0\}$. *Additionally, for all* $\mathbb{H}_t$ *on the path of play.*

1. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, then* $\delta^*_\emptyset \in (0,1)$.
2. *If* $\overline{p}_\emptyset > 0$, (14), $\overline{p}_{\{1\}} > 0$ *and* (13) *hold, then* $\delta^*_{\{1\}} \in (0,1)$.
3. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$, *then* $\delta^*_{\{1\}} = 1$.
4. *If* $\overline{p}_\emptyset \leq 0$, *then* $\delta^*_\emptyset = 1$, *and the low-quality firm* B *obtains good ratings with some positive probability.*
5. *If* $\overline{p}_\emptyset \leq 0$, *then* $\overline{p}_{\{1\}} > 0$ *and* (13) *holds, then* $\delta^*_{\{1\}} \in (0,1)$.
6. *If* $\overline{p}_\emptyset \leq 0$, *and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$, *then* $\delta^*_{\{1\}} = 1$.

For all other histories, newcomers charge a price at marginal cost.

There exist parameters where entry does or does not occur. Indeed, if $q^A \to q^h$, *newcomers will not sell even after a positive rating, implying that newcomers never enter. However, if* q^h *is sufficiently large, reputation becomes increasingly valuable so that newcomers sell after a good rating and enter.*

Proof of Lemma 19.
We proceed as follows. We start by looking at the low-quality firm B's strategy in the third period, then in the second period, and finally in the first period.

In the third period, our Selection Assumption 2 implies that a low-quality firm B sets the highest possible positive price at which it would sell. This is because the third period is the terminal period and there is no continuation reputation effect that the firm needs to consider. Hence, to maximize profits it sets the highest price at which it sells with probability 1, given this price is positive.

In the second period, by Lemma 18, $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that a low-quality firm B only sells if it has a history of $\mathbb{H}_t = \{1\}$. By Lemma 7, when the low-quality firm B sells, it mixes between $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. If the low-quality firm sets the price $\overline{p}_{\{1\}} = E[q_2^B|\{1\}, \overline{p}_{\{1\}}] - q^A > 0$, it makes the expected profits of $\overline{p}_{\{1\}} + 0$. The firm makes no continuation profit since we focus on the scenario where the firm B sells if it only has a history of good ratings. If the low-quality firm B sets $\underline{p}_{\{1\}} = -q^A$, it makes the expected profits of $\underline{p}_{\{1\}} + F(q^A)(E[q_2^B|\{1,1\}] - q^A)$ as it only sells if it obtains a good rating in period 3 with probability $F(q^A)$. Hence, $\delta^*_{\{1\}}$ makes the firm indifferent between these two choices. To find the unique $\delta^*_{\{1\}}$, we first show that the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$ and the continuation profits after setting $\underline{p}_{\{1\}}$ is strictly increasing in $\delta^*_{\{1\}}$. Then we show that when $\delta^*_{\{1\}} = 0$, the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly greater than setting $\underline{p}_{\{1\}}$, which implies that $\delta^*_{\{1\}} > 0$. Finally, we fix $\delta^*_{\{1\}} = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_{\{1\}} = 1$.

To see that the continuation profits after $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$, note the derivative of the continuation profits is

$$-\frac{\gamma F(q^h - \overline{p}_\emptyset)(1-\gamma)(1-\delta^*_\emptyset)F(q^A)q^h}{(\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)\delta^*_{\{1\}})^2} < 0.$$

To see that the continuation profits after $\underline{p}_{\{1\}}$ are strictly increasing in $\delta^*_{\{1\}}$, note first that $\underline{p}_{\{1\}} = -q^A$, and second, for the case $\overline{p}_\emptyset > 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)}.$$

Since by the previous argument, $\overline{p}_{\{1\}}$ decreases in $\delta^*_{\{1\}}$, this expression strictly increases in $\delta^*_{\{1\}}$. Similarly, for the case $\overline{p}_\emptyset \leq 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)},$$

which is strictly increasing for the same reason.

Then let $\delta^*_{\{1\}} = 0$, the continuation profits after $\overline{p}_{\{1\}}$ becomes $q^h - q^A$ and, for $\overline{p}_\emptyset > 0$, the continuation profits after $\underline{p}_{\{1\}}$ becomes $-q^A + F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]$. This is strictly smaller than $q^h - q^A$, since

$$\begin{aligned} q^h - q^A &> \frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A \\ &> F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]. \end{aligned}$$

The argument for $\overline{p}_\emptyset \leq 0$ follows directly from the same steps.

Finally, let $\delta^*_{\{1\}} = 1$. Then the continuation profits after $\overline{p}_{\{1\}}$ becomes $\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset) F(q^A)} - q^A$ and the continuation profits after $\underline{p}_{\{1\}}$, for $\overline{p}_\emptyset > 0$, become $-q^A + F(q^A)\left[q^h - q^A\right]$. Then if the continuation profits after $\overline{p}_{\{1\}}$ is still larger than the continuation profits after $\underline{p}_{\{1\}}$, we have $\delta^*_{\{1\}} = 1$. The interior solution exists if and only if

$$\begin{aligned}\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset) F(q^A)} - q^A &< -q^A + F(q^A)\left[q^h - q^A\right] \\ \frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset) F(q^A)} &< +F(q^A)(q^h - q^A) \\ \frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset) F(q^A)} &< F(q^A)(q^h - q^A). \end{aligned} \tag{13}$$

Following the same argument for $\overline{p}_\emptyset \leq 0$ shows that the same condition applies to this case.

We now turn our attention to period 1.

In the first period the low-quality firm B sets $\overline{p}_\emptyset$ with strictly positive probability and obtains a continuation profit of $\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*_\emptyset} - q^A$. Otherwise, it may set $\underline{p}_\emptyset$ and make a continuation profit of

$$-q^A + F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset) F(q^A)\delta^*_{\{1\}}} - q^A\right).$$

Note that firm l only sets $\underline{p}_\emptyset$ with strictly positive probability if $\overline{p}_\emptyset > 0$.

Applying the same procedure, we show that the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$ and the continuation profits after $\underline{p}_\emptyset$ is strictly increasing, implying that any $\delta^*_\emptyset$ is unique. Then allowing $\delta^*_\emptyset = 0$ we show that the continuation profits after setting $\overline{p}_\emptyset$ is strictly greater than setting $\underline{p}_\emptyset$, which tells us $\delta^*_\emptyset > 0$. We then let $\delta^*_\emptyset = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_\emptyset = 1$.

To see the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$, note the derivative of the continuation profits is

$$-\frac{\gamma(1-\gamma)q^h}{(\gamma + (1-\gamma)\delta^*_\emptyset)^2} < 0.$$

To see the continuation profits after $\underline{p}_\emptyset$ is strictly increasing in $\delta^*_\emptyset$, note that by the previous argument, $\overline{p}_\emptyset$ decreases in $\delta^*_\emptyset$, implying that these continuation profits strictly increase in $\delta^*_\emptyset$.

Then for $\delta^*_\emptyset = 0$, the continuation profit from $\overline{p}_\emptyset$ becomes $q^h - q^A$, and the continuation profit from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma) F(q^A)\delta^*_{\{1\}}} - q^A\right)$. The continuation profits after a high price are strictly larger, since

$$q^h - q^A > \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A$$
$$> F(q^A)(\frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A).$$

Finally, for $\delta^*_\emptyset = 1$, the continuation profits after $\overline{p}_\emptyset$ is γq^h, and the continuation profits from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)(q^h - q^A)$. Then if the continuation profits after $\overline{p}_\emptyset$ is still larger than the continuation profits after $\underline{p}_\emptyset$, we have $\delta^*_\emptyset = 1$. The interior solution exists if and only if

$$\gamma q^h - q^A < -q^A + F(q^A)(q^h - q^A)$$
$$\gamma q^h < F(q^A)(q^h - q^A). \tag{14}$$

We now address each of the statements in Lemma 19 in turn.

Statement 1. If $\overline{p}_\emptyset > 0$ and (14) hold, then there exists a unique solution which is interior in period 1, $\delta^*_\emptyset \in (0,1)$.

Statement 2. If, in addition to Statement 1, $\overline{p}_{\{1\}} > 0$ and (13) hold, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 3. If, in addition to Statement 1 we have $\overline{p}_{\{1\}} \leq 0$, then Lemma 7 implies $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 1, (13) is violated, then $\delta^*_{\{1\}} = 1$.

Statement 4. If $\overline{p}_\emptyset \leq 0$, Lemma 7 implies $\delta^*_\emptyset = 1$. Additionally, because $\overline{p}_\emptyset \leq 0$, the low-quality firm B obtains good ratings with some positive probability.

Statement 5. If, in addition to Statement 4, $\overline{p}_{\{1\}} > 0$ and (13) holds, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 6. If, in addition to Statement 4, $\overline{p}_{\{1\}} \leq 0$ then by Lemma 7, we have $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 4, (13) is violated, then $\delta^*_{\{1\}} = 1$.

These statements describe $\delta^*_{\mathbb{H}_t}$ after histories that occur in equilibrium with strictly positive probability and shows they are unique up to off-path-beliefs.

All other histories occur with probability zero on the path of play, so we set newcomer prices to marginal cost.

This concludes the proof. □

Lemma 20. *After any history $\mathbb{H}_t$, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in any period $t+1$, it also sells in period t.*

Suppose (17) *and* (18) *hold. Then there exist unique values* $\underline{\delta}_\emptyset \in (0,1)$ *and* $\underline{\delta}_{\{1\}} \in (0,1)$ *such that the following holds. Both types of newcomer* B *enter in period 1 if and only if* $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$*; otherwise none enters. Both types of newcomer* B *enter in period 2 if and only if* $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ *and* $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ *hold; otherwise none enters in period 2.*

Proof of Lemma 20.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now show that if h enters, then also l enters. Towards a contradiction, suppose only firm h enters. Then $E[q^B_t|p_t] = q^h \forall p_t$. For any rating, newcomers are identified as h, which allows the firm to always get the profit of $q^h - q^A$ in each period. But then l has a profitable deviation to enter the market with positive prices $q^h - q^A$ in period t, contradicting that l does not enter. We conclude that after any history, h enters if and only if l enters.

Suppose instead the high-quality firm B chooses to enter the market in some period t but not the period $t-1$. Then it faces the same payoffs as entering in $t-1$ as it is unable to build its reputation. Thus if the high-quality firm chooses to enter the market in a period t rather than $t-1$, it forgoes the additional continuation profit it could make from a good reputation. Therefore, it must be that if the high-quality firm chooses to sell at all, it must sell in period 1.

Since, if the high-quality firm B sells at all it sells in period 1 and the low-quality firm B chooses to sell if the high-quality firm is selling, then it must be that if firm B enters the market it enters in the first period and this decision is independent of its quality realization.

We now check for the conditions which ensure firm B sells after a history of good ratings. In other words, firm B has to be able to sell following a good rating, $E[q^B_t|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Suppose to the contrary that $E[q^B_t|\mathbb{H}_t, p_t] < q^A$ for some $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Then it must be that firm B does not sell in period t. This means there is no incentive for the low-quality firm B to harvest ratings and set the low price in period $t-1$. Hence $\delta^*_{\mathbb{H}_t} = 1$ and firm B becomes inactive in period t. Therefore, firm B sells after a good rating if $E[q^B_t|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$.

We now derive more precise conditions for selling, conditional on entry, for these separate histories.

We start with $\mathbb{H}_t = \{\{1\}\}$ Rearranging $E[q^B_t|\{1\}, p_t] > q^A$, implies that

$$\begin{aligned} E[q^B_2|\{1\}, p_2] > q^A \Leftrightarrow \\ \gamma F(q^h - \overline{p}_\emptyset)(q^h - q^A) > (1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}} F(q^A) q^A \end{aligned} \tag{15}$$

where the left-hand side is strictly increasing in $\delta^*_\emptyset$ and, since by Lemma 19 $\delta^*_{\{1\}} > 0$, the right-hand side strictly decreasing in $\delta^*_\emptyset$. Therefore, if $\delta^*_\emptyset$ is sufficiently large, firm B sells after $\mathbb{H}_t = \{\{1\}\}$.

Next, consider $\mathbb{H}_t = \{1,1\}$. Firm B sells if

$$\begin{aligned} E[q_3^B|\{1,1\},p_3] > q^A \Leftrightarrow \\ \gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})(q^h - q^A) > (1-\gamma)(1-\delta_\emptyset^*)F(q^A)^2(1-\delta_{\{1\}}^*)q^A \end{aligned} \tag{16}$$

where the left-hand side is strictly increasing in $\delta_{\{1\}}^*$ and the right-hand side strictly decreasing if $\delta_\emptyset^* < 1$. If $\delta_\emptyset^* = 1$, the right had side is zero, and the condition always holds. Therefore, conditional on entry, firm B sells in period 3 if $\delta_{\{1\}}^*$ is sufficiently large.

We now characterize entry decisions in period 1 and 2. We start with period 1. Note that if $\overline{p}_\emptyset > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta_\emptyset^* > 0$, implying they earn strictly positive profits. If $\overline{p}_\emptyset \leq 0$, we know from Lemma 7 that $\delta_\emptyset^* = 1$ such that $\overline{p}_\emptyset < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$\begin{aligned} \gamma q^h - q^A + F(q^A - \gamma q^h)\Bigg[\frac{\gamma F(q^A + (1-\gamma)q^h)q^h}{\gamma F(q^A + (1-\gamma)q^h) + (1-\gamma)F(q^A - \gamma q^h)} - q^A + \\ F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right]\Bigg] > 0, \end{aligned} \tag{17}$$

This is the profits when both $\overline{p}_\emptyset < 0$ and $\overline{p}_{\{1\}} < 0$. Clearly, since h must earn weakly larger profits, this implies that also h sells with strictly positive probability. We conclude that if (15) holds, both types of firm B have strictly positive demand in period 1 and therefore enter. Since they clearly do not enter if this condition is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_\emptyset \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_\emptyset \in (0,1)$ such that all types of newcomers enter in period 1 if and only if $\delta_\emptyset^* \geq \underline{\delta}_\emptyset$.

We continue with period 2. Note that if $\overline{p}_{\{1\}} > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta_{\{1\}}^* > 0$, implying they earn strictly positive profits. If $\overline{p}_{\{1\}} \leq 0$, we know from Lemma 7 that $\delta_{\{1\}}^* = 1$ such that $\overline{p}_{\{1\}} < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$E[q_2^B|\{1\},p_2] - q^A + F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right] > 0, \tag{18}$$

Firm B can only sell if it obtained a positive rating in period 1. Then firm l sells despite a negative price in period 2 if this condition holds. Thus, since firm h earns weakly larger profits, also firm h sells.

We conclude that if (17) and (18) hold, both types of newcomers enter in period 2 if and only if (15) and (16) hold. Since they clearly do not enter in period 2 if (16) is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_{\{1\}} \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_{\{1\}} \in (0,1)$ such that all types of newcomers enter in period 2 if and only if $\delta^*_{\emptyset} \geq \underline{\delta}_{\emptyset}$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$.

Finally, by our Selection Assumption 1, the equilibria with entry are played whenever they exist. This concludes the proof. □

Proof of Proposition 6.

First, note that the claims on entry follow directly from Lemma 20.

Statement 1: Suppose $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$. Then Lemma 18 shows that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}] = q^l = 0$ and $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] \in (0, q^A)$. Next, from Lemma 20 we know that $E[q_t^B|\mathbb{H}_t, p_t] > q^A$ for $\mathbb{H}_t = \{\{1\},\{1,1\}\}$ whenever firm B enters. Therefore, we know that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > q^A > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}]$. This shows that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$ if the history $\mathbb{H}_{t-1}$ includes only positive ratings. Now recall that if the history includes negative or no rating, then there is no sales and no updating. Hence, obtaining a rating in the period t does not change consumer expectations and $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$.

Statement 2: $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that firm B does not sell if it receives either no or negative ratings. By Lemma 20, newcomers enter after a history of positive ratings. Then Lemma 19 characterized the prices in the statement.

Statement 3: Follows directly from Lemmas 19 and 20. Lemma 20 characterized which histories induce positive demand, and Lemma 19 characterizes the prices.

Statement 4: Comes directly from Corollary 6.

Statement 5: The strategies are characterized in Lemma 19. The price levels set by the firm B is follows from Corollary 8.

□

B.3 Comparative statics

As before, we focus on mixed-strategy equilibria for our comparative statics. Our arguments in Lemma 19 imply that such equilibria indeed exist for some parameters, i.e. if $\frac{F(q^A)(q^h - q^A)}{\gamma q^h}$ is sufficiently large.

Corollary 15. *Consider an increase in $\overline{e}$. In each period, the low-quality firms harvest ratings more, $\frac{\partial \delta^*_{\emptyset}}{\partial \overline{e}} > 0$ and $\frac{\partial \delta^*_{\{1\}}}{\partial \overline{e}} > 0$.*

Proof of Corollary 15.
In period 2, in equilibrium, the low-quality firm B is indifferent between the profit it obtains from

setting $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. This means

$$\frac{\gamma(q^h - \overline{p}_\emptyset)q^h}{\gamma(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}}q^A} = \frac{q^A}{\overline{e}}(\frac{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2} - q^A).$$

Then for a given $\delta^*_\emptyset$, it is immediate that setting a higher $\delta^*_{\{1\}}$ decreases the left side of this equation and changes in $\overline{e}$ only affects this equation through $\delta^*_{\{1\}}$.

On the right side, from Lemma 19 we know that the payoff $\frac{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2} - q^A$ is increasing in $\delta^*_{\{1\}}$. Further, fixing $\delta^*_{\{1\}}$, increases in $\overline{e}$ leads to a decrease in the probability of obtaining the period 3 payoff $\frac{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_\emptyset)(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2} - q^A$.

Taken together, this means that following an increase in $\overline{e}$, the direct effect is the right side of the equation decreases because the probability of obtaining a period 3 payoff decreases. Hence, to compensate for this decrease and restore indifference, an increase in $\delta^*_{\{1\}}$ is required as this simultaneously decreases the left side of the equation and increases the period 3 payoff in the right side of the equation. Therefore, it follows that $\frac{\partial \delta^*_{\{1\}}}{\partial \overline{e}} > 0$.

We now turn our attention to the first period. In equilibrium, the low-quality firm is indifferent between the profit it obtains from setting $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$. This means

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*_\emptyset} = \frac{q^A}{\overline{e}}(\frac{\gamma(q^h - \overline{p}_\emptyset)q^h}{\gamma(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)(q^A)\delta^*_{\{1\}}} - q^A).$$

Observe here that the left side of the equation is decreasing in $\delta^*_\emptyset$ and that changes in $\overline{e}$ affects the left side of the equation only through $\delta^*_\emptyset$. On the right side of the equation, increases in $\overline{e}$ have a direct effect of reducing the probability of obtaining a continuation payoff. Note that the size of the continuation payoff is increasing in $\delta^*_\emptyset$.

Hence, considering an increase in $\overline{e}$, the direct effect leads to a decrease to the right side of the equation. To restore indifference, an increase in $\delta^*_\emptyset$ decreases the left side of the equation and simultaneously increases the probability of the continuation payoff following a good rating (the terms in brackets on the right side of the equation). Therefore, to restore equilibrium, it must be that $\frac{\partial \delta^*_\emptyset}{\partial \overline{e}} > 0$. □