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The Need For Absorptive Capacity Alleviates the Free-Rider Problem in Knowledge Production*

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Knowledge sharing is central to strategy and organizational learning, yet its effect on knowledge production remains underexplored. When knowledge diffuses too easily, individuals may free-ride on others' costly knowledge production, creating a suboptimal equilibrium in which knowledge sharing persists but the average payoff is no greater than if everyone produced knowledge independently—a paradox first identified by Rogers (1988). We develop a game-theoretic model to reexamine this puzzle. In our baseline model, we reproduce Rogers' paradox: frictionless sharing does not increase performance beyond individual knowledge production alone. We then extend the model to incorporate absorptive capacity—the need for prior investment in one's own knowledge before learning from others. Absorptive capacity discourages pure free-riding. Counterintuitively, although it introduces frictions in knowledge sharing, absorptive capacity increases collective knowledge production beyond the level attainable through individual knowledge production alone. This explains why extensive knowledge sharing in domains such as academia does not erode incentives for knowledge production. It also contributes to knowledge-based theories of the firm by showing that while hierarchical control may be crucial in contexts where knowledge is simple and codifiable, it may not be necessary where knowledge is complex and tacit. More broadly, our analysis illustrates how formal modeling can uncover overlooked mechanisms and integrate insights across cultural evolution, organizational learning, and strategy.

Key words: Absorptive capacity, organizational learning, knowledge production, knowledge sharing

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Appendix

Social Optimum

If all n individuals spend a fraction equal to q on individual learning, the sum of the payoffs equals $\pi_s = nwq + n(1-q)\theta(n-1)q$. This is a strictly concave function (the second derivative is $-2\theta(n-1)n < 0$). There is thus a unique maximum that must satisfy

$$\frac{\partial \pi_s}{\partial q} = -2\theta(n-1)nq + \theta(n-1)n + nw = 0 \implies q^o = \frac{1}{2}\left(1 + \frac{w}{\theta(n-1)}\right),$$

which is less than one when $\theta > \frac{w}{n-1}$. The per individual payoff at $q = q^o$ equals $\frac{(\theta(n-1)+w)^2}{4\theta(n-1)}$, which is larger than w. To see this note that

$$\frac{(\theta(n-1)+w)^2}{4\theta(n-1)} > w \implies (\theta(n-1)+w)^2 > w4\theta(n-1)$$
$$\implies (\theta(n-1))^2 + 2w\theta(n-1) + w^2 > 4w\theta(n-1) \implies (\theta(n-1)-w)^2 > 0,$$

which holds since we assumed $\theta > \frac{w}{n-1}$.

If $\theta \leq \frac{w}{n-1}$, the unconstrained maximizer satisfies $q^o \geq 1$, so the constrained optimum on $[0,1]$ is 1.

Proof of Theorem 1

To calculate the equilibrium, consider how the payoff for individual i varies with q_i, keeping the choices of the other individuals constant. The payoff to individual i choosing q_i is $\pi_i = wq_i + (1-q_i)P$, where $P = \theta\sum_{j\neq i} q_j$. This can be written as $\pi_i = P + q_i(w-P)$.

Whenever $P > w$, individual i maximizes her payoff by setting $q_i = 0$. It follows that there cannot be an equilibrium in which $P > w$: if everyone sets $q = 0$, then $P = 0$, contradicting $P > w > 0$.

Whenever $P < w$, individual i maximizes her payoff by choosing $q_i = 1$. If all choose $q_i = 1$, then $P = \theta\sum_{j\neq i} q_j = \theta(n-1)$. However, since we assumed $\theta(n-1) > w$, this is inconsistent with $P < w$.

The only symmetric equilibrium occurs when q is at a level such that $P = w$, i.e., $P = \theta(n-1)q = w$, implying $q^e = \frac{w}{\theta(n-1)}$. In this case, no individual can increase their expected payoff by changing q_i. However, when $P = w$, the expected individual payoff is $\pi_i = wq_i + (1-q_i)P = w$, identical to the payoff when only individual learning is possible.

To show that $q^e = \frac{w}{\theta(n-1)}$ is an evolutionarily stable strategy (ESS), let $\pi(x \mid y)$ be the payoff to an individual who chooses $q = x$ when all other $n-1$ individuals choose $q = y$. A strategy $q^e \in [0,1]$ is an ESS if, for all alternative strategies $q \neq q^e$, 1) $\pi(q^e \mid q^e) \geq \pi(q \mid q^e)$, and 2) if $\pi(q^e \mid q^e) = \pi(q \mid q^e)$ then $\pi(q^e \mid q) > \pi(q \mid q)$. In our case, $\pi(q^e \mid q^e) = w$. Moreover, $\pi(q \mid q^e) = w$; thus a single mutant with $q \neq q^e$ will do equally well. However, $\pi(q^e \mid q) > \pi(q \mid q)$. To see this, note that

$$\pi(q \mid q) = qw + (1-q)\theta(n-1)q,$$
$$\pi(q^e \mid q) = q^e w + (1-q^e)\,\theta(n-1)q,$$

and thus

$$\begin{aligned}\pi(q^e \mid q) - \pi(q \mid q) &= \big[q^e w + (1-q^e)\theta(n-1)q\big] - \big[qw + (1-q)\theta(n-1)q\big] \\ &= (q^e - q)\big(w - \theta(n-1)q\big).\end{aligned}$$

Because $q^e = \frac{w}{\theta(n-1)}$, we have $w = \theta(n-1)q^e$. Using this, we get

$$\pi(q^e \mid q) - \pi(q \mid q) = (q^e - q)^2\,\theta(n-1) > 0,$$

when $q \neq q^e$. Hence, mutants do strictly worse against themselves than individuals who choose $q = q^e$ do against the mutant, $\pi(q \mid q) < \pi(q^e \mid q)$. Consequently, the interior symmetric equilibrium q^e is an ESS.

Asymmetric Equilibrium in the Game without Absorptive Capacity

Here we show that there exist asymmetric equilibria in which the payoffs to social learners exceed w. However, they become irrelevant when n is large.

Suppose exactly k individuals choose $q_i = 1$ (“individual learners”) and the remaining $n-k$ choose $q_i = 0$ (“social learners”). This profile is a Nash equilibrium if and only if $\theta(k-1) \le w \le \theta k$.

To prove this, let $S_i = \sum_{j \neq i} q_j$. For an individual learner we have $S_i = k-1$, while for a social learner $S_i = k$. Thus, payoffs are $\pi_i^{\text{ind}} = 1 \cdot w + (1-1)\theta S_i = w$ and $\pi_i^{\text{soc}} = 0 \cdot w + (1-0)\theta S_i = \theta k$. If one of the k individual learners deviates to $q_i = 0$, the number of other individual learners drops to $k-1$, so their payoff after deviating would be $\pi_i^{\text{deviate}} = \theta(k-1)$. Deviation is profitable iff $\theta(k-1) > w$. Hence to prevent deviation we require $w \ge \theta(k-1)$. If one of the $n-k$ social learners deviates to $q_i = 1$, their payoff after deviating would be $\pi_i^{\text{deviate}} = w$. Their current payoff is θk. Deviation is profitable iff $w > \theta k$. Hence to prevent deviation we require $w \le \theta k$. Combining both conditions we get $\theta(k-1) \le w \le \theta k$.

In an asymmetric equilibrium, social learners achieve a higher payoff compared to individual learners, except if $w = \theta k$ when payoffs are identical. This follows since $\pi_i^{\text{ind}} = w$, $\pi_i^{\text{soc}} = \theta k$ and $w \le \theta k$. Individual learners earn only w, but are still not tempted to deviate since they would get only $\theta(k-1) \le w$ if they switched to social learning because there would be one less individual learner. For example, suppose $n = 7$, $\theta = 0.4$, and $w = 0.5$. Consider an asymmetric equilibrium with $k = 2$ individual and 5 social learners. Then $\pi^{\text{ind}} = 0.5$ and $\pi^{\text{soc}} = \theta k = 0.4 \cdot 2 = 0.8 > 0.5$. If an individual switched to social learning, only one individual learner would remain and $\pi^{\text{soc}} = \theta(k-1) = 0.4 \cdot 1 = 0.4 < 0.5$.

However, the payoff difference between social and individual learners shrinks as θ becomes small. Specifically, the equilibrium conditions $\theta(k-1) \le w \le \theta k$ imply $0 \le \theta k - w \le \theta$, i.e., $|\theta k - w| \le \theta$.

Hence, if $\theta \to 0$, then $\theta k \to w$ along any sequence of such asymmetric equilibria. This scaling is natural if n (and hence potentially k) grows large while payoffs are assumed to remain bounded, since otherwise the term θk could diverge. Formally, if payoffs are bounded as $k \to \infty$, then necessarily $\theta \to 0$, and therefore $\theta k \to w$ in equilibrium.

Equilibrium in the Game with Absorptive Capacity

The equilibrium value of q is the value q^e such that if all others spend q^e on individual learning, individual i also wants to spend q^e on individual learning. To find the equilibrium, we first derive the best response function for individual i: the optimal value of q_i given $\sum_{j \neq i} q_j$. The payoff if individual i chooses q_i is $\pi_i = wq_i + (1-q_i)f(q_i)P$, where $P = \theta \sum_{j \neq i} q_j$. If each of the other individuals spends q on individual learning, then $P = \theta q(n-1)$.

Let $\Pi(q;P) = wq + (1-q)f(q)P$ and $g(P) = \text{argmax}_q \Pi(q;P)$; the optimal value of q given P. Because $\Pi(q;P)$ is continuous in q and $[0,1]$ is a closed and bounded set, Weierstrass's extreme value theorem implies that there is at least one maximizer (Sundaram 1996, p. 90) [13].

Recall that we assume (Assumption 3) that the absorptive capacity function, $f(q)$, is continuously twice differentiable on $[0,1]$, and that one of the following holds: i) f is linear or concave on $[0,1]$ (i.e., $f''(q) \leq 0$ for all q). ii) $\phi(q) := (1-q)f''(q) - 2f'(q) < 0$ for all $q \in [0,1]$. iii) There exists $c \in (0,1)$ such that $\phi(q) > 0$ for all $q \in (0,c)$ and $\phi(q) < 0$ for all $q \in (c,1)$. Moreover, $\Pi_q(0;P) > 0$ for all $P > 0$ or whenever $\Pi_q(0;P) \leq 0$ we have $\Pi_q(q;P) < 0$ for all $q \in (0,1]$ and all $P > 0$. This implies that $\Pi_{qq}(q;P) = P\phi(q)$ changes sign at most once, from positive to negative.

First, we show that i) choosing $q = 1$ is optimal only if P is small enough, ii) choosing $q = 0$ is only optimal if P is large enough, and iii) if choosing $q = 0$ is optimal for some value of P, it remains optimal for all larger values of P, and iv) there is an interval where there is an interior optimum and neither $q = 1$ or $q = 0$ is optimal.

Lemma 1. *i)* $g(P) = 1$ *if and only if* $P \leq \dfrac{w}{f(1)}$*. ii) If* $g(P) = 0$*, then necessarily* $f(0) > f'(0)$ *and* $P \geq \dfrac{w}{f(0) - f'(0)}$*. iii) If* $g(P_1) = 0$ *for some* $P_1 > 0$*, then* $g(P) = 0$ *for all* $P > P_1$*. iv) If* $f(0) > f'(0)$*, then* $g(P) \in (0,1)$ *whenever*

$$\frac{w}{f(1)} < P < \frac{w}{f(0) - f'(0)}.$$

If $f(0) \leq f'(0)$*, then* $g(P) \in (0,1)$ *for all* $P > \dfrac{w}{f(1)}$.

Proof: i) First, we show that if $P \leq w/f(1)$, then $g(P) = 1$. For any $q \in (0,1)$,

$$\Pi(q;P) = wq + (1-q)f(q)P \leq wq + (1-q)f(q)\frac{w}{f(1)}.$$

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[13] Sundaram, R. K. (1996). A First Course in Optimization Theory. Cambridge University Press: Cambridge, UK.

Since f is strictly increasing, $f(q) < f(1)$ for $q \in (0,1)$, hence $f(q)/f(1) < 1$ and therefore

$$\Pi(q;P) < wq + (1-q)w = w = \Pi(1;P).$$

Moreover, $\Pi(0;P) = f(0)P \leq f(0)\frac{w}{f(1)} < w = \Pi(1;P)$. Thus, $q = 1$ is the unique maximizer and $g(P) = 1$.

Next, suppose $P > w/f(1)$, so that $Pf(1) > w$. By continuity of f, there exists $q_b < 1$ such that $Pf(q_b) > w$. For this q_b we have $(1-q_b)\big[Pf(q_b) - w\big] > 0$, hence

$$\Pi(q_b;P) = wq_b + (1-q_b)f(q_b)P > w = \Pi(1;P),$$

so $q = 1$ cannot be optimal. Therefore, $g(P) = 1$ if and only if $P \leq w/f(1)$.

ii) A necessary condition for $q = 0$ to be optimal is $\Pi_q(0;P) \leq 0 \implies w + P\big(f'(0) - f(0)\big) \leq 0$, which requires $f(0) > f'(0)$ and $P \geq w/(f(0) - f'(0))$.

iii) The fact that choosing $q_i = 0$ is optimal for P_1 means that $\Pi(0;P_1) = f(0)P_1 > \Pi(q;P_1) = wq + (1-q)f(q)P_1$ for all $q \in (0,1]$. This can be written as $P_1\left(f(0) - (1-q)f(q)\right) > wq$ for all $q \in (0,1]$. Note that since $wq > 0$, and $P_1 > 0$, $f(0) - (1-q)f(q) > 0$. It follows that $P\left(f(0) - (1-q)f(q)\right) > wq$ for all $q \in (0,1]$ for any $P > P_1$.

iv) Suppose first that $f(0) > f'(0)$. By part (i), $g(P) \neq 1$ whenever $P > w/f(1)$. By part (ii), a necessary condition for $g(P) = 0$ is $P \geq w/(f(0) - f'(0))$; hence $g(P) \neq 0$ whenever $P < w/(f(0) - f'(0))$. Therefore, for $w/f(1) < P < w/(f(0) - f'(0))$ neither boundary choice $q = 0$ nor $q = 1$ is optimal. Since a maximizer exists on $[0,1]$, it follows that $g(P) \in (0,1)$ in this range.

Now suppose that $f(0) \leq f'(0)$. Then $\Pi_q(0;P) = w + P(f'(0) - f(0)) > 0$ for all $P > 0$, so $q = 0$ cannot be optimal. Combining this with part (i), we again have that for every $P > w/f(1)$ neither boundary is optimal, and hence $g(P) \in (0,1)$. □

Next, we establish a Lemma characterizing when a function f (not necessarily the absorptive capacity function) has a unique global maximum:

Lemma 2. *Let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and twice continuously differentiable on (a,b). Assume:*

(i) There exists $c \in (a,b)$ such that $f''(x) > 0$ for all $x \in (a,c)$ and $f''(x) < 0$ for all $x \in (c,b)$ (allowing $f''(c) = 0$).

(ii) either (a) $f'(a) > 0$, or (b) if $f'(a) \leq 0$ then $f'(x) < 0$ for all $x \in (a,b]$;

Then f has a unique global maximizer $x^* \in [a,b]$. Moreover, if the maximizer is interior, then it is the unique $x^* \in (a,b)$ satisfying $f'(x^*) = 0$ and it satisfies $f''(x^*) < 0$.

Proof. Condition (i) implies that $f'(x)$ is strictly increasing on (a,c) and strictly decreasing on (c,b). Hence f' has at most one zero, and if it has one then it changes sign from positive to negative.

Case 1: $f'(a)>0$. If $f'(x)>0$ for all $x\in(a,b]$, then f is strictly increasing on $[a,b]$ and therefore $x=b$ is the unique global maximizer. Otherwise, suppose there exists $x\in(a,b)$ with $f'(x)\le 0$. Then there exists $x^*\in(a,b)$ such that $f'(x^*)=0$. Because f' is strictly increasing on (a,c) and strictly decreasing on (c,b), f' cannot be identically zero on any interval; hence $f'(x)>0$ for $x<x^*$ close to x^* and $f'(x)<0$ for $x>x^*$ close to x^*. Therefore x^* is a strict local maximum. Moreover, $x^*\in(c,b)$ (it cannot lie in (a,c) since f' is increasing there), so $f''(x^*)<0$. Since f is increasing on $[a,x^*]$ and decreasing on $[x^*,b]$, x^* is the unique global maximizer.

Case 2: $f'(a)\le 0$ *and* $f'(x)<0$ *for all* $x\in(a,b]$. Then f is strictly decreasing on $[a,b]$, and hence $x=a$ is the unique global maximizer.

□

Next, we characterize the implications of the first-order condition that any interior optimum must satisfy:

Lemma 3. *Let* $P>0$. *If the best reply is interior,* $q(P)\in(0,1)$, *then:*

1. $w+(1-q)f'(q)\,P\;=\;f(q)\,P$ *(the FOC), and equivalently* $w=P\big[f(q)-(1-q)f'(q)\big]$.
2. $f(q)-(1-q)f'(q)\;>\;0$.

Proof: The derivative $\Pi_q(q;P)$ equals $w-f(q)P+(1-q)f'(q)P$ and the first-order condition is: $w+(1-q)f'(q)P-f(q)P=0$ which implies that $w+(1-q)f'(q)P=f(q)P$ or $w=P[f(q)-(1-q)f'(q)]$. Because $w>0$, the first-order condition also implies $f(q)-(1-q)f'(q)>0$ whenever $P>0$. □

We now demonstrate, using Lemma 2, that Assumptions 1-3 imply that the best response function is, in an interval with an interior optimum, a single valued, decreasing, and continuous function of P.

Lemma 4. *Suppose that* $g(P)\in(0,1)$ *for all* $P_1<P<P_2$. *Let* $\phi(q)=(1-q)f''(q)-2f'(q)$. *Under Assumption 3, a) the best response function,* $g(P)$, *is single-valued and b)* $g(P)$ *is a strictly decreasing continuous function of* P *in this region and c) there is a unique solution to* $\Pi_q(q,P)=0$ *for* $q\in(0,1)$.

Proof. Note that

$$\Pi_{qq}(q;P)=P\big((1-q)f''(q)-2f'(q)\big)=P\phi(q),$$

which is continuous.

Case A (Assumption 3(i)). If $f''(q)\le 0$ for all $q\in[0,1]$ and $f'(q)>0$, then $\phi(q)<0$ for all $q\in[0,1]$ and hence $\Pi(\cdot;P)$ is strictly concave. Therefore $g(P)$ is single-valued. Since $\Pi(q;P)$ is continuous in (q,P), Berge's maximum theorem implies that $g(P)$ is continuous in P on (P_1,P_2). If $g(P)\in(0,1)$, the implicit function theorem gives, for $q^*(P):=g(P)$,

$$\frac{dq^*(P)}{dP}=-\frac{\Pi_{qP}(q^*(P);P)}{\Pi_{qq}(q^*(P);P)}=\frac{f(q^*)-(1-q^*)f'(q^*)}{P\phi(q^*)}.$$

By Lemma 3, the numerator is positive, and since $\phi(q^*) < 0$ we obtain $dq^*/dP < 0$.

Case B (Assumption 3(ii)). If $\phi(q) < 0$ for all $q \in [0,1]$, the same argument as in Case A applies.

Case C (Assumption 3(iii)). Under Assumption 3(iii), Lemma 2 implies that $\Pi(\cdot;P)$ has a unique global maximizer on $[0,1]$, and if the maximizer is interior it is the unique $q^*(P) \in (0,1)$ solving $\Pi_q(q^*(P);P) = 0$ and it satisfies $\Pi_{qq}(q^*(P);P) < 0$. Hence $g(P)$ is single-valued on (P_1,P_2) and continuous by Berge's maximum theorem. Moreover, since $\Pi_{qq}(q^*(P);P) < 0$ and $\Pi_{qq}(q;P) = P\phi(q)$ with $P > 0$, we have $\phi(q^*(P)) < 0$, so the implicit-function calculation above again yields $dq^*(P)/dP < 0$. □

The above lemmas characterize the best response function $g(P)$:

Lemma 5. *Let $g(P) \in \arg\max_{q\in[0,1]} \Pi(q;P)$ where $\Pi(q;P) = wq + (1-q)f(q)P$.*

1. *For $P \le w/f(1)$, $g(P) = 1$.*
2. *Let $\delta := \dfrac{w}{f(0) - f'(0)}$, with the convention $\delta = \infty$ if $f(0) \le f'(0)$. For $\dfrac{w}{f(1)} < P < \delta$, the best reply $g(P)$ lies in $(0,1)$, is single-valued, continuous, and strictly decreasing in P, and is the unique $q \in (0,1)$ solving the first-order condition $\Pi_q(q;P) = 0$.*
3. *If $f(0) > f'(0)$ (so that $\delta < \infty$), then $g(P) = 0$ for all $P \ge \delta$. In particular, if $\delta \le \theta(n-1)$, then $g(P) = 0$ for all feasible $P \in [\delta, \theta(n-1)]$.*

Proof. Step 1. Lemma 1(i) gives $g(P) = 1$ if and only if $P \le w/f(1)$.

Step 2. Step 2. Consider P in the interval $(w/f(1), \delta)$. By Lemma 1(i), $g(P) \ne 1$. By Lemma 1(ii), $g(P) \ne 0$ for $P < \delta$ (and if $f(0) \le f'(0)$ then $q = 0$ is never optimal). Hence $g(P) \in (0,1)$ for all $P \in \big(w/f(1), \delta\big)$. Applying Lemma 4 on any interval (P_1,P_2) with $P_1 = w/f(1)$ and $P_2 < \delta$ yields that on (P_1,P_2), $g(P)$ is single-valued, continuous, and strictly decreasing, and that $g(P) \in (0,1)$ is the unique solution to the first-order condition $\Pi_q(g(P);P) = 0$.

Step 3. Assume $f(0) > f'(0)$, so that $\delta < \infty$. Recall that

$$\Pi(q;P) = wq + (1-q)f(q)P \quad \text{so} \quad \Pi_q(q;P) = w + P\big[(1-q)f'(q) - f(q)\big] = w - P\big[f(q) - (1-q)f'(q)\big].$$

In particular,

$$\Pi_q(0;P) = w + P\big(f'(0) - f(0)\big) = w - P\big(f(0) - f'(0)\big).$$

Hence $\Pi_q(0;\delta) = 0$, and for all $P > \delta$ we have $\Pi_q(0;P) < 0$.

Case A: Assumption 3(i) (with $f'(q) > 0$ on $(0,1]$) or Assumption 3(ii). Under Assumption 3(i), $f''(q) \le 0$ and $f'(q) > 0$ for $q \in (0,1]$ imply $\phi(q) = (1-q)f''(q) - 2f'(q) < 0$ for all $q \in (0,1]$. Under Assumption 3(ii), $\phi(q) < 0$ holds by assumption for all $q \in [0,1]$. Therefore, in either case,

$$\Pi_{qq}(q;P) = P\phi(q) < 0 \quad \text{for all } q \in (0,1),\ P > 0,$$

so $\Pi(\cdot;P)$ is strictly concave on $[0,1]$ and $\Pi_q(\cdot;P)$ is strictly decreasing in q.

At $P=\delta$, since $\Pi_q(0;\delta)=0$ and $\Pi_q(\cdot;\delta)$ is strictly decreasing, we obtain $\Pi_q(q;\delta)<0$ for all $q\in(0,1]$. Thus $\Pi(\cdot;\delta)$ is strictly decreasing on $[0,1]$ and its unique maximizer is $q=0$.

For $P>\delta$, we have $\Pi_q(0;P)<0$; strict monotonicity of $\Pi_q(\cdot;P)$ implies $\Pi_q(q;P)<\Pi_q(0;P)<0$ for all $q\in(0,1]$. Hence $\Pi(\cdot;P)$ is strictly decreasing on $[0,1]$ and the unique maximizer is again $q=0$.

Case B: Assumption 3(iii). By Assumption 3(iii), whenever $\Pi_q(0;P)\leq 0$ we have $\Pi_q(q;P)<0$ for all $q\in(0,1]$. Since $\Pi_q(0;\delta)=0$, it follows that $\Pi_q(q;\delta)<0$ for all $q\in(0,1]$, so $\Pi(\cdot;\delta)$ is strictly decreasing and its unique maximizer is $q=0$. Moreover, for $P>\delta$ we have $\Pi_q(0;P)<0$, hence again $\Pi_q(q;P)<0$ for all $q\in(0,1]$ and the unique maximizer is $q=0$. □

With knowledge of the best response function, we can demonstrate that a unique symmetric equilibrium exists.

Proposition 1. *Under Assumptions 1–3 the game with absorptive capacity has a unique symmetric interior pure-strategy equilibrium. Each individual chooses the same level* $q^e\in(0,1)$ *which is the unique solution to* $w=\theta q^e(n-1)[f(q^e)-(1-q^e)f'(q^e)]$.

Proof: If all others select q_o, then $P=\theta(n-1)q_o$ and the best reply is $g(\theta(n-1)q_o)$. A symmetric equilibrium exists if there is a value q such that $q=g(\theta(n-1)q)$. A value $q=0$ cannot be an equilibrium because then $P=0$ and Lemma 1 shows that $g(0)=1$. A value $q=1$ cannot be an equilibrium because then $P=\theta(n-1)$. According to Assumption 2, $f(1)>\frac{w}{\theta(n-1)}$ and thus $\theta(n-1)>\frac{w}{f(1)}$. Thus, if $P=\theta(n-1)$ then $P>\frac{w}{f(1)}$. However, according to Lemma 1, $g(P)=1$ only if $P\leq\frac{w}{f(1)}$. Hence, a symmetric equilibrium must be interior.

Next we show that an interior symmetric equilibrium exists and is unique. Let $r_1:=\frac{w}{f(1)\theta(n-1)}$. According to Assumption 2, $f(1)>\frac{w}{\theta(n-1)}$ and thus $r_1<1$. Then $\theta(n-1)r_1=\frac{w}{f(1)}$. By Lemma 5(1), $g(P)=1$ for $P\leq w/f(1)$, hence $g(\theta(n-1)q_o)=1$ for all $q_o\leq r_1$; in particular $g(\theta(n-1)r_1)=1$.

Now define $h(q):=g(\theta(n-1)q)-q$. Then $h(r_1)=g(\theta(n-1)r_1)-r_1=1-r_1>0$ since $r_1<1$.

Suppose that $f(0)>f'(0)$ and $\delta<\theta(n-1)$. Lemma 5(3) then shows that $g(P)=0$ for all $P\geq\delta$. It follows that $g(\theta(n-1)q_o)=0$ for all $q_o\geq r_2=\frac{\delta}{\theta(n-1)}>0$. We thus get $h(r_2)=g(\theta(n-1)r_2)-r_2=0-r_2<0$.

Suppose next that $f(0)>f'(0)$ but $\delta\geq\theta(n-1)$. In this case, the region $P\geq\delta$ where Lemma 5(3) would imply $g(P)=0$ is not feasible, since $P\leq\theta(n-1)$. We therefore set $r_2=1$ and evaluate h at the upper boundary. Note that $g(P)\in[0,1]$ for every P, since $g(P)\in\arg\max_{q\in[0,1]}\Pi(q;P)$. By Assumption 2, $f(1)>\frac{w}{\theta(n-1)}$, hence $\theta(n-1)>\frac{w}{f(1)}$. Lemma 5(1) implies that $g(P)=1$ iff $P\leq\frac{w}{f(1)}$, so $g(\theta(n-1))\neq 1$. Therefore $g(\theta(n-1))<1$, and $h(1)=g(\theta(n-1))-1<0$.

Finally, suppose that $f(0) \le f'(0)$ (so $\delta = \infty$). Then $q = 0$ is never optimal. Again set $r_2 = 1$. As in the previous case, Assumption 2 implies $\theta(n-1) > \frac{w}{f(1)}$, so by Lemma 5(1) we have $g(\theta(n-1)) \neq 1$ and hence $h(1) = g(\theta(n-1)) - 1 < 0$.

Thus, in all cases, h takes opposite signs at r_1 and r_2. Moreover, for $q \in (r_1, r_2)$ we have $P = \theta(n-1)q > w/f(1)$, and if $\delta < \infty$ then also $P < \delta$. Thus Lemma 5(2) implies that $g(\theta(n-1)q) \in (0,1)$ and is continuous on (r_1, r_2). Hence h is continuous on (r_1, r_2). Therefore, by the Intermediate Value Theorem, there exists $q^e \in (r_1, r_2)$ such that $h(q^e) = 0$, i.e., $q^e = g(\theta(n-1)q^e)$, so an interior symmetric equilibrium exists.

On (r_1, r_2), Lemma 5(2) implies that g is strictly decreasing in q_o (because g is strictly decreasing in P and $P = \theta(n-1)q_o$ is increasing in q_o). Hence $h(q) = g(\theta(n-1)q) - q$ is strictly decreasing on (r_1, r_2). Therefore, the fixed point q^e is unique.

Because $q^e \in (0,1)$, it satisfies the first-order condition for the maximization of $\Pi(q; P)$ at $P = \theta(n-1)q^e$, i.e.,

$$0 = \Pi_q(q^e; \theta(n-1)q^e) = w - \theta(n-1)q^e f(q^e) + \theta(n-1)q^e(1-q^e)f'(q^e),$$

which is equivalent to

$$w = \theta(n-1)q^e\left[f(q^e) - (1-q^e)f'(q^e)\right].$$

□

Computations for specific cases

Suppose that $f(q) = q$. In that case, $f(q) - (1-q)f'(q) = q - (1-q)$. Using Proposition 1, the equilibrium value of q is the solution to the equation:

$$w = \theta q^e(n-1)[q^e - (1-q^e)] = \theta q^e(n-1)(2q^e - 1).$$

The (non-negative) solution is

$$q^e = \frac{1}{4} + \frac{\sqrt{8w + \theta(n-1)}}{4\sqrt{\theta(n-1)}}. \tag{2}$$

We have $q^e \in (0,1)$ when $\theta > w/(n-1)$. The individual payoff in equilibrium is then $\pi^e = wq^e + (1-q^e)q^e\theta q^e(n-1)$. Using $w = \theta q^e(n-1)(2q^e-1)$, this can be written as

$$\pi^e = wq^e + (1-q^e)q^e\frac{w}{2q^e-1} = \frac{w(q^e)^2}{2q^e-1}. \tag{3}$$

Functions that satisfy Assumption 3

Assumption 3 states that the absorptive capacity function, $f(q)$, should be i) either concave or linear ($f''(q) \leq 0$ for all $q \in [0,1]$) or ii) $\phi(q) = (1-q)f''(q) - 2f'(q) < 0$ for all q iii) or such that $\Pi_{qq}(q,P) = P\phi(q)$ changes sign at most once, from positive to negative, for $q \in [0,1]$. The latter condition holds for several convex functions or functions with convex segments:

The power function: $f(q) = q^b$, $b > 1$.[14] Then $\phi(q)$ equals $-bq^{b-2}(b(q-1)+q+1)$, which is negative if $b(q-1)+q+1 > 0 \implies q > (b-1)/(b+1)$ and positive when $q < (b-1)/(b+1)$. Thus, $\Pi_{qq}(q,P)$ changes sign once, from positive to negative, for $q \in [0,1]$.

The Exponential function: $f(q) = Ae^{kq}$ with $A > 0$, $k > 0$, and $Ae^k \leq 1$. In this case $\phi(q)$ equals $Ake^{kq}[k(1-q)-2]$ which changes sign at most once, from positive to negative.

The Logistic function: $f(q) = 1/(1+e^{-s(q-t)})$ with $s > 0$. Then $f'(q) = sf(q)(1-f(q))$ and $f''(q) = sf'(q)(1-2f(q))$. Thus, $\phi(q)$ equals $f'(q)\,[-2+s(1-q)(1-2f(q))]$. Since $s > 0$, $f'(q) = sf(q)(1-f(q)) > 0$ for $q \in (0,1)$. Thus, the sign is determined by $\kappa(q) = -2+s(1-q)(1-2f(q))$. Consider first $\kappa(q)$ for $q \in [t,1]$. For $q \in [t,1]$, we have $f(q) \geq 1/2$, which implies $1-2f(q) \leq 0$. Thus, $\kappa(q) < 0$. Consider next $q \in [0,t)$. Then $f(q) < 1/2$, which implies $1-2f(q) > 0$ and $\kappa(q)$ can be positive, zero, or negative. To determine how $k(q)$ changes, we compute its derivative:

$$\kappa'(q) = s\,[-1+2f(q)-2(1-q)f'(q)]$$

Since $f(q) < 0.5$ when $q < t$, $-1+2f(q) < 0$. Moreover, $2(1-q)f'(q) \geq 0$. Thus, $\kappa'(q) < 0$ when $q < t$. Overall, $\kappa(q)$ is a strictly decreasing function on $q \in (0,t)$ and is negative for $q \geq t$. This implies that $\kappa(q)$ can cross the zero line from positive to negative at most once over the entire interval $[0,1]$. If $\kappa(q)$ is positive for some initial q, it must decrease and then eventually cross zero into negative territory and remain negative for $q \geq t$. It cannot become positive again.

Proof of Theorem 2

Theorem 2: Under Assumptions 1–3 the game possesses a unique symmetric interior pure-strategy equilibrium $q^e \in (0,1)$ with the following properties:

1. The equilibrium payoff for each player satisfies $\pi^e > w$, which is larger than in the equilibrium without absorptive capacity, when $f \equiv 1$.
2. The total knowledge available to each individual, $K(q^e)$, is larger than in the equilibrium without absorptive capacity, when $f \equiv 1$.

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[14] Formally, for $f(q) = q^b$, $f''(q) = b(b-1)q^{b-2}$. If $1 < b < 2$, then $f''(q)$ diverges as $q \downarrow 0$, so $f \notin C^2[0,1]$ which Assumption 3 requires.

Moreover, the equilibrium is an evolutionarily stable strategy (ESS).

Proof: Proposition 1 implies that under Assumptions 1–3 there is a unique symmetric equilibrium and that it is interior, $q^e \in (0,1)$. Hence q^e satisfies the first-order condition (Lemma 3):

$$w + (1-q^e)f'(q^e)\,\theta(n-1)q^e \;=\; f(q^e)\,\theta(n-1)q^e. \tag{4}$$

If $f'(q^e) > 0$ and $q^e > 0$, then (4) implies $f(q^e)\theta(n-1)q^e > w$, and therefore

$$\pi^e = wq^e + (1-q^e)f(q^e)\theta(n-1)q^e > wq^e + (1-q^e)w = w.$$

Consider next the available knowledge in a symmetric profile, $K_f(q) \;=\; q + (1-q)\,f(q)\,\theta(n-1)\,q$. In the benchmark case, $f \equiv 1$, the unique symmetric equilibrium is $q_1^* = w/T$ where $T := \theta(n-1)$, and thus

$$K_1(q_1^*) \;=\; q_1^* + (1-q_1^*)\,Tq_1^* \;=\; q_1^* + (1-q_1^*)\,w.$$

In the absorptive-capacity case, Lemma 3 gives

$$w \;=\; Tq^e\big[f(q^e) - (1-q^e)f'(q^e)\big].$$

Since $f'(q^e) > 0$ and $q^e \in (0,1)$, we have $(1-q^e)f'(q^e) > 0$ and therefore $f(q^e) - (1-q^e)f'(q^e) < f(q^e) \leq 1$. Hence

$$w = Tq^e\big[f(q^e) - (1-q^e)f'(q^e)\big] \leq Tq^e,$$

so $q^e \geq w/T = q_1^*$. Moreover, from $f(q^e)Tq^e > w$ (shown above) we obtain

$$K_f(q^e) = q^e + (1-q^e)\,f(q^e)\,Tq^e > q^e + (1-q^e)\,w. \tag{5}$$

Finally, since $w \in (0,1)$ and $q^e \geq q_1^*$, it follows that

$$q^e + (1-q^e)w \;\geq\; q_1^* + (1-q_1^*)w \;=\; K_1(q_1^*).$$

Combining this inequality with (5) yields $K_f(q^e) \;>\; K_1(q_1^*)$.

Finally, by Lemma 4 the best reply to $P = Tq^e$ is unique and equals q^e. Thus, q^e is a strict symmetric Nash equilibrium. A strict symmetric equilibrium is evolutionarily stable, so q^e is an ESS. $\square$

Asymmetric Equilibrium in the Game with Absorptive Capacity

Here we show that there exist asymmetric equilibria in which the introduction of absorptive capacity reduces the equilibrium payoff.

To illustrate this, suppose $\theta = 1$ and $w = 0.5$. Then without absorptive capacity, payoff equals $\pi_i = 0.5q_i + (1-q_i)\sum_{j\neq i} q_j$. Consider an asymmetric equilibrium in which one individual chooses $q_i = 1$ and the other $n-1$ individuals choose $q_i = 0$. The payoff for the individual learner is $\pi_i = 0.5$ and the payoff for social learners is $\pi_i = 1$. To see that this is an equilibrium, suppose that the individual learner chooses $q < 1$. Then, his payoff is $q0.5 < 0.5$. Suppose that social learners choose $q > 0$. Then, their payoff is $q0.5 + (1-q) < 1$.

Consider now the case with absorptive capacity with $f(q_i) = 0.9 + 0.1q_i$. The payoff is $\pi_i = q_i 0.5 + (1-q_i)(0.9+0.1q_i)\sum_{j\neq i} q_j$. There is again an asymmetric equilibrium in which one individual chooses $q_i = 1$ and the other $n-1$ individuals choose $q_i = 0$. The payoff for the individual learner is $\pi_i = 0.5$ and the payoff for social learners is $\pi_i = q_i 0.5 + (1-q_i)(0.9+0.1q_i)\sum_{j\neq i} q_j = 0.9$ since $q_i = 0$ and $\sum_{j\neq i} q_j = 1$. To see that this is an equilibrium, suppose that the individual learner chooses $q < 1$. Then his payoff is $0.5q < 0.5$. Suppose that social learners choose $q > 0$. Then their payoff is $q0.5 + (1-q)(0.9+0.1q) = 0.9 - 0.3q - 0.1q^2 < 0.9$.

In this case, absorptive capacity reduces the equilibrium payoff for social learners from 1 to 0.9. The reason is that no one changes their strategy (the amount of individual learning is not increased), and the introduction of absorptive capacity merely makes it more difficult for social learners to absorb the knowledge produced by the single individual learner.

Proof of Theorem 3

Let $\pi(x \mid y)$ be the payoff of an individual who chooses $q = x$ when the other $n-1$ players choose $q = y$. Then $\pi(x \mid y) = wx + (1-x)f(x)\,a\,y$, where $a = \theta(n-1)$. Recall that

$$H(q) = \frac{1}{1-q}\left[q^o - \frac{2w}{a+w}\,q\right] = \frac{w(q^o - q)}{(1-q)aq^o} + \frac{1-q^o}{1-q}.$$

where $q^o = \frac{1}{2}\left(1 + \frac{w}{a}\right)$.

Step 0 (properties of H). Let $\alpha := \frac{2w}{a+w}$. Then $H(q) = \frac{q^o - \alpha q}{1-q}$. A direct calculation gives

$$H'(q) = \frac{q^o - \alpha}{(1-q)^2}, \qquad H''(q) = \frac{2(q^o - \alpha)}{(1-q)^3}.$$

Moreover,

$$q^o - \alpha = \frac{a+w}{2a} - \frac{2w}{a+w} = \frac{(a-w)^2}{2a(a+w)} > 0 \quad \text{since } a = \theta(n-1) > w.$$

Hence $H'(q) > 0$ and $H''(q) > 0$ on $[0,1)$, so H is strictly increasing and strictly convex. Also, $H(0) = q^o$ and $H(q^o) = 1$ follow by direct substitution.

Step 1 ($q < q^o$ cannot be a symmetric equilibrium). Assume $f(q) < H(q)$ for all $q \in [0, q^o)$ and $f(q^o) = 1$. Fix $q \in (0, q^o)$. We have

$$\pi(q \mid q) = wq + (1-q)f(q)aq, \qquad \pi(q^o \mid q) = wq^o + (1-q^o) \cdot 1 \cdot aq.$$

Thus $\pi(q^o \mid q) > \pi(q \mid q)$ iff

$$f(q) < \frac{w(q^o - q)}{(1-q)aq} + \frac{1-q^o}{1-q} =: H_2(q).$$

But for $q \in (0, q^o)$,

$$H_2(q) - H(q) = \frac{w(q^o - q)}{(1-q)a}\Big(\frac{1}{q} - \frac{1}{q^o}\Big) > 0,$$

since $q^o > q > 0$. Hence $H_2(q) > H(q)$, and the assumption $f(q) < H(q)$ implies $f(q) < H_2(q)$, which yields $\pi(q^o \mid q) > \pi(q \mid q)$. Therefore, no $q \in (0, q^o)$ can be a symmetric equilibrium. (And $q = 0$ cannot be an equilibrium since then $\pi(q^o \mid 0) = wq^o > 0 = \pi(0 \mid 0)$.)

Step 2 ($q > q^o$ cannot be a symmetric equilibrium). Take $q > q^o$. Under the theorem's condition $f(t) = 1$ for all $t \geq q^o$, any deviator choosing $t \in [q^o, 1]$ gets

$$\pi(t \mid q) = wt + (1-t)\, aq = t(w - aq) + aq.$$

We have $w - aq < 0$ whenever $q > w/a$, which is satisfied when $q > q^o$ since $q^o = \frac{1}{2}\big(1 + \frac{w}{a}\big)$ and $0 < w/a < 1$ (by assumption). Hence $\pi(t \mid q)$ is strictly decreasing in t on $[q^o, 1]$. Since $q > q^o$ and $\pi(t \mid q)$ is decreasing in t on $[q^o, 1]$, we have $\pi(q^o \mid q) > \pi(q \mid q)$, so q cannot be a symmetric equilibrium.

Step 3 (q^o is a symmetric equilibrium). We show q^o is a best response to itself.

If a deviator chooses $q \geq q^o$, then $f(q) = 1$ and $\pi(q \mid q^o) = wq + (1-q)aq^o$. At $q = q^o$, the payoff is $\pi(q^o \mid q^o) = wq^o + (1-q^o)aq^o$. So q^o is a best response to itself iff $\pi(q^o \mid q^o) \geq \pi(q \mid q^o)$ for all $q \in [0, 1]$. Now consider the difference:

$$\pi(q^o \mid q^o) - \pi(q \mid q^o) = w(q^o - q) + aq^o\big[(1-q^o) - (1-q)\big] = (q^o - q)\big(w - aq^o\big).$$

Since $q \geq q^o$, we have $(q^o - q) \leq 0$. And because $q^o = \frac{1}{2}\left(1 + \frac{w}{a}\right)$, we have

$$aq^o = \frac{a+w}{2} \quad \Rightarrow \quad w - aq^o = \frac{w-a}{2} < 0 \;\; (\text{since } a > w).$$

So $(q^o - q)(w - aq^o) \geq 0$. Hence $\pi(q^o \mid q^o) \geq \pi(q \mid q^o)$ for all $q \in [q^o, 1]$ with strict inequality for $q > q^o$.

If instead $q \in [0, q^o)$, note

$$\pi(q \mid q^o) = wq + (1-q)f(q)aq^o, \qquad \pi(q^o \mid q^o) = wq^o + (1-q^o) \cdot 1 \cdot aq^o.$$

We have $\pi(q^o \mid q^o) > \pi(q \mid q^o)$ whenever

$$\begin{aligned} wq^o + (1-q^o)aq^o &> wq + (1-q)\,f(q)\,aq^o \\ \implies \frac{wq^o + (1-q^o)aq^o - wq}{(1-q)\,aq^o} &> f(q) \\ \implies H(q) = \frac{w(q^o-q)}{(1-q)aq^o} + \frac{1-q^o}{1-q} &> f(q), \end{aligned}$$

which holds by assumption for all $q \in [0, q^o)$. Finally, for $q = 1$, $\pi(1 \mid q^o) = w < \pi_o = \pi(q^o \mid q^o)$ since $a > w$.

Thus q^o is a best response to itself, hence a symmetric equilibrium. Uniqueness follows from Steps 1–2. Since $f(q^o) = 1$, the equilibrium payoff is $\pi(q^o \mid q^o) = wq^o + (1-q^o)aq^o = \pi_o$.

Step 4 (if the conditions fail, symmetric-equilibrium payoff is $< \pi_o$). If $f(q^o) < 1$, then at q^o the symmetric payoff is

$$wq^o + (1-q^o)f(q^o)aq^o < wq^o + (1-q^o)aq^o = \pi_o.$$

More generally, define $G(q) := wq + (1-q)aq = (w+a)q - aq^2$. This is strictly concave and maximized uniquely at $q^o = (w+a)/(2a)$. Since $f(q) \le 1$ for all q, for any $q \ne q^o$ we have

$$wq + (1-q)f(q)aq \le wq + (1-q)aq = G(q) < G(q^o) = \pi_o,$$

so any symmetric equilibrium payoff is $< \pi_o$.

If $f(\bar{q}) > H(\bar{q})$ for some $\bar{q} < q^o$, then $\pi(\bar{q} \mid q^o) > \pi(q^o \mid q^o)$, so q^o is not a best response to itself and cannot be a symmetric equilibrium. Any symmetric equilibrium must then have $q \ne q^o$, and by the same bound $wq + (1-q)f(q)aq \le G(q) < \pi_o$, its payoff is strictly below π_o.

The Cohen and Levinthal Model

The payoff to individual i is $\pi_i = h\big(q_i + f(q_i, b)\theta \sum_{j \ne i} q_j\big) - cq_i$, where $\theta > 0$, $h : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave with $h'(x) > 0$ and $h''(x) < 0$. Assume $h'(0) > c$ and $\lim_{x\to\infty} h'(x) = 0$. For absorptive capacity $f(q, b)$, assume $f_q > 0$, $f_{qq} < 0$, $f_b < 0$, and $f_{qb} > 0$. Finally, we assume an interior finite equilibrium $q > 0$.

Without absorptive capacity: Proof of Theorem 4

The payoff for individual i is then $\pi_i = h(q_i + \theta \sum_{j \ne i} q_j) - cq_i$. Let $Q_{-i} = \sum_{j \ne i} q_j$. Player i's payoff is $\pi_i(q_i, Q_{-i}) = h\big(q_i + \theta Q_{-i}\big) - c\,q_i$, with $h'(x) > 0$, $h''(x) < 0$ and $h'(0) > c$. Given Q_{-i} the first-order condition for an interior optimum is $h'\big(q_i + \theta Q_{-i}\big) = c$. Because $h'(\cdot)$ is strictly decreasing, h' is invertible, and $h'\big(q_i + \theta Q_{-i}\big) = c$ has the unique solution $q_i^{\mathrm{BR}}(Q_{-i}) = \max\{0,\ z^* - \theta Q_{-i}\}$, where $z^* := (h')^{-1}(c) > 0$.

Symmetric Nash equilibrium In a symmetric equilibrium $Q_{-i} = (n-1)q$, $h'\big(q^{*e} + \theta(n-1)q^{*e}\big) = c$, and thus

$$q^{*e} = \frac{\big(h'\big)^{-1}(c)}{1+\theta(n-1)} \implies q^{*e}(1+\theta(n-1)) = \big(h'\big)^{-1}(c). \tag{6}$$

The assumption $h'(0) > c$ implies $(h')^{-1}(c) > 0$; hence $q^{*e} > 0$ since $\theta > 0$.

Define the symmetric best-response mapping (on the interior region) by $BR(q) := (h')^{-1}(c) - \theta(n-1)q$. A symmetric equilibrium is a fixed point of this mapping, i.e., it solves $q = BR(q)$. But

$$\frac{d}{dq}\big(BR(q) - q\big) = -\theta(n-1) - 1 < 0,$$

so $BR(q) - q$ is strictly decreasing and can cross zero at most once. Hence, the symmetric equilibrium is unique.

The social optimum: The sum of the payoffs is

$$\sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} h(q_i + \theta \sum_{j \neq i} q_j) - c \sum_{i=1}^{n} q_i.$$

If we assume symmetry $q_i = q_j = q$, then $\sum_{i=1}^{n} \pi_i = nh(q(1+\theta(n-1))) - cnq$. Because $h(x)$ is strictly concave, there is a unique maximum that has to satisfy the first-order condition

$$nh'(q^s(1+\theta(n-1)))(1+\theta(n-1)) = cn$$
$$\implies h'(q^s(1+\theta(n-1)))(1+\theta(n-1)) = c.$$

In contrast, in the symmetric equilibrium we have $h'\big(q^{*e}(1+\theta(n-1))\big) = c$. Since $\theta(n-1) > 0$ for $n > 1$, the social optimum condition implies

$$h'\big(q^s(1+\theta(n-1))\big) = \frac{c}{1+\theta(n-1)} < c = h'\big(q^{*e}(1+\theta(n-1))\big).$$

Because h' is strictly decreasing, it follows that $q^s(1+\theta(n-1)) > q^{*e}(1+\theta(n-1))$, and hence $q^s > q^{*e}$. Thus, the equilibrium knowledge stock (and investment) is below the social optimum.

Only individual learning: The payoff to individual i is then $\pi_i = h(q_i) - cq_i$. The unique maximum must satisfy $h'(q^i) = c$, hence $q^i = \big(h'\big)^{-1}(c)$. However, equation 6 showed that $q^{*e}(1+\theta(n-1)) = \big(h'\big)^{-1}(c)$. Thus, $q^i = q^{*e}(1+\theta(n-1))$. That is, the knowledge stock individual i has access to when social learning is possible is *identical* to the knowledge stock when only individual learning is possible. Moreover, $q^{*e}(1+\theta(n-1)) = q^i$ implies $q^{*e} = \dfrac{q^i}{1+\theta(n-1)} < q^i$ when $n > 1$.

The payoff when only individual learning is possible is $h(q^i) - cq^i$. The payoff when social learning is possible is

$$h\big(q^{*e}(1+\theta(n-1))\big) - cq^{*e} = h(q^i) - \frac{cq^i}{1+\theta(n-1)} > h(q^i) - cq^i,$$

when $n > 1$. Thus, the payoff when social learning is possible is larger than when only individual learning is possible. The reason is that the cost per individual is lower (while the knowledge stock is identical).

With absorptive capacity

The payoff to individual i is

$$\pi_i \;=\; h\Bigg(q_i + f(q_i)\theta \sum_{j\neq i} q_j\Bigg) \;-\; cq_i,$$

where $h'(x) > 0,\ h''(x) < 0,$ and $f'(q) > 0,\ f''(q) < 0$. Let $Q_{-i} := \sum_{j\neq i} q_j$ and define $x_i := q_i + f(q_i)\theta Q_{-i}$. Then player i's payoff can be written as

$$\pi_i(q_i, Q_{-i}) \;=\; h(x_i) - cq_i.$$

Its first derivative is

$$\frac{\partial \pi_i}{\partial q_i} = h'(x_i)\Big(1 + \theta f'(q_i)Q_{-i}\Big) - c,$$

and the second derivative is

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = h''(x_i)\Big(1 + \theta f'(q_i)Q_{-i}\Big)^2 + h'(x_i)\,\theta f''(q_i)\,Q_{-i}.$$

Since $h''(x_i) < 0$, $h'(x_i) > 0$, $f''(q_i) < 0$, and $Q_{-i} \geq 0$, we have $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$. Hence $\pi_i(\cdot, Q_{-i})$ is strictly concave in q_i, so player i has a unique best response. An interior best response $q_i \in (0,1)$ satisfies the first-order condition:

$$h'(q_i + f(q_i)\theta Q_{-i})\,\Big(1 + \theta f'(q_i)Q_{-i}\Big) = c.$$

Consider a symmetric equilibrium with $q_i = q_a^e > 0$ for all i. Then $Q_{-i} = (n-1)q_a^e$, and q_a^e satisfies

$$h'(q_a^e + f(q_a^e)\theta(n-1)q_a^e)\,\Big(1 + \theta f'(q_a^e)(n-1)q_a^e\Big) = c.$$

Let q^e denote the symmetric equilibrium investment in the benchmark case $f \equiv 1$. Define the equilibrium knowledge stocks

$$X_a := q_a^e + f(q_a^e)\theta(n-1)q_a^e, \qquad X_1 := q^e + \theta(n-1)q^e.$$

In the absorptive-capacity game, the symmetric equilibrium satisfies

$$h'(X_a)\Big(1 + \theta f'(q_a^e)(n-1)q_a^e\Big) = c,$$

whereas when $f \equiv 1$ it satisfies

$$h'(X_1) = c.$$

If $n > 1$, $q_a^e > 0$, and $f'(q_a^e) > 0$, then $1 + \theta f'(q_a^e)(n-1)q_a^e > 1$, and therefore

$$h'(X_a) = \frac{c}{1 + \theta f'(q_a^e)(n-1)q_a^e} < c = h'(X_1).$$

Since h' is strictly decreasing, it follows that $X_a > X_1$.

The Cohen and Levinthal Model with costly social learning

Without absorptive capacity

The payoff to individual i is $\pi_i = h(q_{1,i} + q_{2,i}\theta \sum_{j\neq i} q_{1,j}) - c(q_{1,i} + q_{2,i})$, $q_{1,i} \geq 0$, $q_{2,i} \geq 0$, where $h(x)$ is strictly increasing and concave, $h'(x) > 0$ and $h''(x) < 0$ for all $x \geq 0$. We assume $h'(0) > c$. If $h'(0) < c$ then a single individual would set $q_1 = 0$ and not engage in individual learning. We also assume that $\lim_{x\to\infty} h'(x) = 0$, ensuring a finite optimal value of x.

Consider first the case with only individual learning when $\pi_i = h(q_{1,i}) - cq_{1,i}$. The optimal choice of q_1 then satisfies $h'(q_1^*) = c$. The payoff is $h(q_1^*) - q_1^* c$.

Next, consider the symmetric equilibrium when social learning is possible. To find the Nash equilibrium, we derive the first-order conditions for each player i with respect to their two decision variables, $q_{1,i}$ and $q_{2,i}$, taking the actions of other players as given. Due to the non-negativity constraints ($q_{1,i} \geq 0$, $q_{2,i} \geq 0$), we employ the Karush-Kuhn-Tucker (KKT) conditions. Let $K_i = q_{1,i} + q_{2,i}\theta \sum_{j\neq i} q_{1,j}$.

The KKT conditions are:

$$\frac{\partial \pi_i}{\partial q_{1,i}} = h'(K_i) - c \leq 0, \quad \text{with equality if } q_{1,i} > 0 \quad (1)$$

$$\frac{\partial \pi_i}{\partial q_{2,i}} = h'(K_i)\left(\theta \sum_{j\neq i} q_{1,j}\right) - c \leq 0, \quad \text{with equality if } q_{2,i} > 0 \quad (2)$$

Since the payoff is jointly concave in $(q_{1,i}, q_{2,i})$ these conditions are necessary and sufficient for a best response. We look for a symmetric Nash equilibrium where $q_{1,i} = q_1^e$ and $q_{2,i} = q_2^e$ for all i. In a symmetric equilibrium, $\sum_{j\neq i} q_{1,j} = (n-1)q_1^e$. Note that $q_1^e = 0$ cannot be an equilibrium since $h'(0) > c$.

Case 1: Interior Solution ($q_1^e > 0$ and $q_2^e > 0$) In this case, both KKT conditions hold with equality:

$$h'(q_1^e + q_2^e\theta(n-1)q_1^e) = c$$

$$h'(q_1^e + q_2^e\theta(n-1)q_1^e)(\theta(n-1)q_1^e) = c$$

This can only hold if $\theta(n-1)q_1^e = 1$ or $q_1^e = \frac{1}{\theta(n-1)}$. From the first equation we get, $q_1^e + q_2^e\theta(n-1)q_1^e = (h')^{-1}(c)$. Inserting the value of q_1^e, we get

$$\frac{1}{\theta(n-1)} + q_2^e = (h')^{-1}(c) \implies q_2^e = (h')^{-1}(c) - \frac{1}{\theta(n-1)}.$$

For this interior solution to be valid, we require $q_1^e > 0$ (which is ensured by $\theta > 0, n > 1$) and $q_2^e > 0$. The condition for $q_2^e > 0$ is: $(h')^{-1}(c) > \frac{1}{\theta(n-1)}$ or $\theta(n-1)(h')^{-1}(c) > 1$.

The total knowledge stock available to individual i, $q_1^e + q_2^e\theta(n-1)q_1^e$, satisfies $h'(q_1^e + q_2^e\theta(n-1)q_1^e) = c$, and thus $q_1^e + q_2^e\theta(n-1)q_1^e = q_1^*$, the knowledge stock when there is only individual learning. The payoff in a symmetric equilibrium is $h(q_1^e + q_2^e\theta(n-1)q_1^e) - c(q_1^e + q_2^e)$. Note that because $\theta(n-1)q_1^e = 1$, $q_1^e + q_2^e\theta(n-1)q_1^e = q_1^e + q_2^e$. Next,

$$q_1^e + q_2^e = \frac{1}{\theta(n-1)} + (h')^{-1}(c) - \frac{1}{\theta(n-1)} = (h')^{-1}(c) = q_1^*.$$

Thus, $q_1^e + q_2^e\theta(n-1)q_1^e = q_1^e + q_2^e = q_1^*$ and the payoff in a symmetric equilibrium is equal to $h(q_1^*) - q_1^* c$, the payoff when only individual learning is possible. Hence, Rogers' paradox holds.

Case 2: Boundary Solution ($q_2^e = 0$) If $q_2^e = 0$ then the KKT condition for q_1^e implies $h'(q_1^e) = c$ (since $q_1^e > 0$ as $h'(0) > c$). This uniquely determines q_1^e: $q_1^e = (h')^{-1}(c)$. Now, we must check that the KKT condition for q_2^e holds as an inequality: $h'(q_1^e)(\theta(n-1)q_1^e) - c \leq 0$. Using $h'(q_1^e) = c$ we get $c(\theta(n-1)q_1^e) - c \leq 0$ or $c[(\theta(n-1)q_1^e) - 1] \leq 0$ Since $c > 0$, this simplifies to: $\theta(n-1)q_1^e \leq 1$. Substituting $q_1^e = (h')^{-1}(c)$ we get $\theta(n-1)(h')^{-1}(c) \leq 1$. This condition specifies when $q_2^e = 0$ is optimal.

In this case, only individual learning is used and the knowledge stock and equilibrium payoff is identical to when only individual learning is possible.

With absorptive capacity

The payoff to individual i is now given by: $\pi_i = h(q_{1,i} + q_{2,i}f(q_{1,i})\theta\sum_{j\neq i} q_{1,j}) - c(q_{1,i} + q_{2,i})$, where $h(x)$ is strictly increasing ($h'(x) > 0$) and concave ($h''(x) < 0$). We assume $h'(0) > c$ and $\lim_{x\to\infty} h'(x) = 0$. The function $f(q)$ is strictly increasing and strictly concave ($f'(q) > 0, f''(q) < 0$). We assume $c > 0$, $\theta > 0$, and $n > 1$. Individual investments are non-negative ($q_{1,i} \geq 0, q_{2,i} \geq 0$).

To characterize the symmetric Nash equilibrium, we derive the Karush-Kuhn-Tucker (KKT) conditions for each player i, taking the actions of other players as given. These conditions are necessary (although not sufficient) for a best response. Let $K_i = q_{1,i} + q_{2,i}f(q_{1,i})\theta\sum_{j\neq i} q_{1,j}$.

The partial derivatives are:

$$\frac{\partial \pi_i}{\partial q_{1,i}} = h'(K_i)\left(1 + q_{2,i}f'(q_{1,i})\theta\sum_{j\neq i} q_{1,j}\right) - c$$
$$\frac{\partial \pi_i}{\partial q_{2,i}} = h'(K_i)\left(f(q_{1,i})\theta\sum_{j\neq i} q_{1,j}\right) - c$$

The KKT conditions are:

$$\frac{\partial \pi_i}{\partial q_{1,i}} \leq 0, \quad \text{with equality if } q_{1,i} > 0 \quad (1)$$
$$\frac{\partial \pi_i}{\partial q_{2,i}} \leq 0, \quad \text{with equality if } q_{2,i} > 0 \quad (2)$$

In a symmetric equilibrium, $q_{1,i} = q_1^e$ and $q_{2,i} = q_2^e$ for all i, so $\sum_{j\neq i} q_{1,j} = (n-1)q_1^e$. Note that $q_1^e = 0$ cannot be an equilibrium as $h'(0) > c$.

Case 1: Interior Solution ($q_1^e > 0$ and $q_2^e > 0$) In this case, both KKT conditions hold with equality. Let $K^e = q_1^e + q_2^e f(q_1^e)\theta(n-1)q_1^e$. KKT condition (1) (for $q_{1,i}$) states that $h'(K^e)\left(1 + q_2^e f'(q_1^e)\theta(n-1)q_1^e\right) = c$ and KKT condition (2) (for $q_{2,i}$) states $h'(K^e)\left(f(q_1^e)\theta(n-1)q_1^e\right) = c$. It follows that

$$1 + q_2^e f'(q_1^e)\theta(n-1)q_1^e = f(q_1^e)\theta(n-1)q_1^e.$$

Solving for q_2^e:

$$q_2^e = \frac{f(q_1^e)\theta(n-1)q_1^e - 1}{f'(q_1^e)\theta(n-1)q_1^e} \quad (A)$$

For $q_2^e > 0$, we must have $f(q_1^e)\theta(n-1)q_1^e > 1$. If this condition holds, we can find the equilibrium as follows. KKT condition (2) implies that

$$q_1^e + q_2^e f(q_1^e)\theta(n-1)q_1^e = (h')^{-1}\left(\frac{c}{f(q_1^e)\theta(n-1)q_1^e}\right)$$

Substitute the expression for q_2^e from (A) into this equation and simplifying we get:

$$q_1^e + \frac{f(q_1^e)}{f'(q_1^e)}\left(f(q_1^e)\theta(n-1)q_1^e - 1\right) = (h')^{-1}\left(\frac{c}{f(q_1^e)\theta(n-1)q_1^e}\right) \quad (B)$$

Equation (B) implicitly determines the equilibrium value of q_1^e. Once q_1^e is solved from (B), q_2^e can be directly computed using (A).

Case 2: Boundary Solution ($q_2^e = 0$) If individuals optimally choose $q_2^e = 0$, the KKT condition (1) for q_1^e becomes $h'(q_1^e) = c$. Since $h'(x)$ is strictly decreasing, this uniquely determines q_1^e: $q_1^e = (h')^{-1}(c)$. For this solution to be valid, the KKT condition (2) for q_2^e must hold as an inequality:

$$h'(q_1^e + 0 \cdot f(q_1^e)\theta(n-1)q_1^e)\left(f(q_1^e)\theta(n-1)q_1^e\right) - c \le 0 \implies h'(q_1^e)\left(f(q_1^e)\theta(n-1)q_1^e\right) - c \le 0$$

Substitute $h'(q_1^e) = c$ we get $c\left[f(q_1^e)\theta(n-1)q_1^e - 1\right] \le 0$. Since $c > 0$, we must have: $f(q_1^e)\theta(n-1)q_1^e \le 1$, which specifies the condition under which the symmetric equilibrium has $q_2^e = 0$. This condition holds if q_1^e is low enough, which occurs when c is large enough. If this condition holds, social learning is not sufficiently valuable. If c is high enough so that $q_1^e < \frac{1}{\theta(n-1)}$, then $f(q_1^e)\theta(n-1)q_1^e \le 1$ for all functions $f(q)$ for which $f(q) \le 1$. In this case, introducing absorptive capacity cannot increase the equilibrium payoff since individuals will select $q_2^e = 0$ and the payoff is always $h(q_1^e) - cq_1^e$.

7. Model with heterogeneity in θ

Proof of Theorem 5.

Fix a conjectured population mean $E \in [0,1]$. Given type $\theta_i \in [a,1]$, individual i's payoff from $q_i \in [0,1]$ is $\pi_i(q_i, \theta_i; E) = wq_i + (1-q_i)\theta_i rE$. The best reply is $q_i = 1$, for $w > \theta_i rE$, $q_i \in [0,1]$ for $w = \theta_i rE$, and $q_i = 0$ for $w < \theta_i rE$. If $E = 0$, then $\pi_i(q, \theta_i; 0) = wq$ and the unique best reply is $q_i = 1$ for all θ_i. Hence $E = 0$ cannot be an equilibrium.

Now take $E > 0$ and define the cutoff $\bar{\theta}(E) := \frac{w}{rE}$. Then, types with $\theta_i < \bar{\theta}(E)$ choose $q_i = 1$ and types with $\theta_i > \bar{\theta}(E)$ choose $q_i = 0$. Since $\theta \sim U[a,1]$ with cdf $H(\theta) = \frac{\theta - a}{1-a}$, the implied population mean is

$$\mathrm{BR}(E) := \mathbb{E}[q_i \mid E] = \begin{cases} 1, & \bar{\theta}(E) \geq 1 \iff E \leq \frac{w}{r}, \\ 0, & \bar{\theta}(E) \leq a \iff E \geq \frac{w}{ra}, \\ \dfrac{\bar{\theta}(E) - a}{1-a} = \dfrac{\frac{w}{rE} - a}{1-a}, & \text{if } \frac{w}{r} < E < \frac{w}{ra}. \end{cases}$$

A Nash equilibrium corresponds to a fixed point $E = \mathrm{BR}(E)$.

The cases $E \leq w/r$ and $E \geq w/(ra)$ cannot be fixed points. If $E \leq w/r$, then $\mathrm{BR}(E) = 1 \neq E$. If $E \geq w/(ra)$, then $\mathrm{BR}(E) = 0 \neq E$. Therefore, any equilibrium must satisfy $E \in (w/r,\, w/(ra))$ and solve

$$E = \frac{\frac{w}{rE} - a}{1-a}.$$

Equivalently $(1-a)E^2 + aE - \frac{w}{r} = 0$. This quadratic has exactly one positive root in $(0,1)$, namely

$$E^* = \frac{-a + \sqrt{a^2 + 4\frac{w}{r}(1-a)}}{2(1-a)}.$$

The corresponding cutoff is

$$\theta^* = \bar{\theta}(E^*) = \frac{w}{rE^*} = \frac{1}{2}\left(a + \sqrt{a^2 + 4\frac{w}{r}(1-a)}\right),$$

so in equilibrium $q_i^e = 1$ for $\theta_i < \theta^*$ and $q_i^e = 0$ for $\theta_i > \theta^*$. This equilibrium is unique (a.e.) because the best-response mean $\mathrm{BR}(E)$ is strictly decreasing in E on the interior region, so it can cross the 45° line at most once.

Finally, the expected equilibrium payoff is

$$\mathbb{E}[\pi_i^e] = \int_a^{\theta^*} w\, h(\theta)\, d\theta + \int_{\theta^*}^1 \theta r E^*\, h(\theta)\, d\theta.$$

Since $w = \theta^* r E^*$ by construction and $\theta r E^* > w$ for all $\theta > \theta^*$, we have

$$\mathbb{E}[\pi_i^e] - w = \int_{\theta^*}^1 \big(\theta r E^* - w\big) h(\theta)\, d\theta \;>\; 0.$$

$\square$

Equilibrium with Absorptive Capacity

We first characterize the individual's optimal choice q for a given population mean $E[q] = E$.

Lemma 6. *For a given population mean learning intensity* $E \in (0,1]$*, the individual's optimal learning intensity* $q^*(\theta, E) = \arg\max_{q \in [0,1]} \pi(q, \theta; E)$ *is uniquely determined by:*

$$q^*(\theta, E) = \min\left\{1, \max\left\{0, 1 - \frac{1}{2\beta} + \frac{w}{2\beta\theta r E}\right\}\right\}.$$

Furthermore, $q^*(\theta, E)$ *is continuous and non-increasing in* E *for all* $\theta \in [a,1]$.

Proof: The individual's payoff function is $\pi(q,\theta;E)=wq+(1-q)(1-\beta+\beta q)\theta rE$. Let $K=\theta rE$. Expanding the terms involving q, we have:

$$\pi(q,\theta;E)=wq+\left[(1-\beta)+q(2\beta-1)-\beta q^2\right]K.$$

The first-order condition (FOC) with respect to q is:

$$\frac{\partial\pi}{\partial q}=w+(2\beta-1)K-2\beta qK=0.$$

Solving for q yields the unconstrained interior solution:

$$q_{int}(\theta,E)=1-\frac{1}{2\beta}+\frac{w}{2\beta\theta rE}.$$

The second-order condition is $\frac{\partial^2\pi}{\partial q^2}=-2\beta K$. Since $\beta>0$, $\theta>0$, and $E>0$, we have $\frac{\partial^2\pi}{\partial q^2}<0$, ensuring strict concavity. Thus, the optimal q^* is the projection of q_{int} onto the interval $[0,1]$:

$$q^*(\theta,E)=\min\left\{1,\max\left\{0,q_{int}(\theta,E)\right\}\right\}.$$

To show monotonicity, observe that $\frac{\partial q_{int}}{\partial E}=-\frac{w}{2\beta\theta rE^2}<0$. Since q^* is a non-decreasing transformation (clamping) of a strictly decreasing function q_{int}, it follows that $q^*(\theta,E)$ is non-increasing in E. Continuity follows from the continuity of q_{int} and the min/max operators. □

We then show that there is a unique equilibrium population mean learning intensity E^*:

Proposition 2. *For any $w,a\in(0,1)$, $r\geq 1$, and $\beta\in(0,1]$, there exists a unique equilibrium population mean learning intensity $E^*\in(0,1]$.*

Proof: An equilibrium is defined as a fixed point $E^*=G(E^*)$, where $G(E)$ is the aggregate best-response function:

$$G(E)=\int_a^1 q^*(\theta,E)h(\theta)\,d\theta=\frac{1}{1-a}\int_a^1 q^*(\theta,E)\,d\theta.$$

We define $\Delta(E):=G(E)-E$ and seek its roots on the interval $(0,1]$.

Existence: Since $q^*(\theta,E)\in[0,1]$ for all $\theta\in[a,1]$, it follows that $G(E)\in[0,1]$. By Lemma 6, $q^*(\theta,E)$ is continuous in E, which ensures that $G(E)$ and $\Delta(E)$ are also continuous. As $E\to 0^+$, the interior solution q_{int} defined in Lemma 6 approaches infinity due to the term $\frac{w}{2\beta\theta rE}$. This implies $q^*(\theta,E)=1$ for all $\theta\in[a,1]$ as E becomes sufficiently small. Consequently, $\lim_{E\to 0^+}G(E)=1$, which implies $\Delta(E)>0$ for E near zero. Given that $G(1)\leq 1$, it follows that $\Delta(1)=G(1)-1\leq 0$. By the Intermediate Value Theorem, there exists at least one $E^*\in(0,1]$ such that $\Delta(E^*)=0$.

Uniqueness: From Lemma 6, $q^*(\theta,E)$ is non-increasing in E for all θ. Since the integral is a monotonic operator, the aggregate function $G(E)$ is also non-increasing in E. To demonstrate uniqueness, consider two values $E_H>E_L>0$. The difference in the function Δ is:

$$\Delta(E_H)-\Delta(E_L)=\underbrace{[G(E_H)-G(E_L)]}_{\leq 0}-\underbrace{(E_H-E_L)}_{>0}.$$

Since the first term is non-positive and the second term is strictly negative, $\Delta(E_H) - \Delta(E_L) < 0$. Thus, $\Delta(E)$ is strictly decreasing on $(0,1]$, which ensures that the root E^* is unique. □

To compute the equilibrium E^*, we solve the fixed-point problem $E = G(E)$. We must account for the fact that optimal learning intensity q^* varies with θ. The population can be partitioned into three regimes, defined by two thresholds $\theta_L(E)$ and $\theta_H(E)$ (found by setting $q_{\text{int}}(\theta, E)$ equal to 1 or 0):

- $\theta_L(E) := \frac{w}{rE}$. If $\theta \leq \theta_L(E)$, then $q^*(\theta, E) = 1$.
- If $\theta_L(E) < \theta < \theta_H(E)$, then $q^*(\theta, E) = q_{\text{int}}(\theta, E) \in (0,1)$.
- If $\beta < \frac{1}{2}$, define $\theta_H(E) := \frac{w}{(1-2\beta)rE}$. If $\theta \geq \theta_H(E)$, then $q^*(\theta, E) = 0$. (If $\beta \geq \frac{1}{2}$, then $\theta_H(E) = \infty$ and no type chooses $q^* = 0$.)

The aggregate function $G(E)$ is the weighted average of these three regimes. When the thresholds fall within the support $[a,1]$, we compute the integral as follows:

$$G(E) = \frac{1}{1-a}\left[\underbrace{\int_a^{\theta_L(E)} 1\, d\theta}_{q^*=1} + \underbrace{\int_{\theta_L(E)}^{\theta_H(E)} q_{int}(\theta, E)\, d\theta}_{q^* \in (0,1)} + \underbrace{\int_{\theta_H(E)}^{1} 0\, d\theta}_{q^*=0}\right]$$

The numerical computation of E^* involves using a root-finding algorithm to solve $G(E^*) - E^* = 0$, while dynamically updating the thresholds at each iteration.

Once the equilibrium E^* is determined, the average population payoff $\bar{\Pi}$ is computed by integrating the individual payoff function $\pi^*(\theta, E^*)$ across the same three regimes:

$$\bar{\Pi} = \frac{1}{1-a}\left[\int_a^{\theta_L} w\, d\theta + \int_{\theta_L}^{\theta_H} \pi(q_{int}, \theta; E^*)\, d\theta + \int_{\theta_H}^{1} (1-\beta)\theta r E^*\, d\theta\right]$$

For individuals who choose $q^* = 1$, the payoff is w. For individuals who choose $q^* = 0$, the payoff is the return to pure social learning $(1 - \beta + \beta \times 0)\theta r E^* = (1-\beta)\theta r E^*$. In the middle regime, the payoff is $\pi(q_{int}, \theta; E^*) = w q_{int} + (1 - q_{int})(1 - \beta + \beta q_{int})\theta r E^*$