Online Appendix for Information Asymmetry at Debt Rollover and Startup Loans

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Online Appendix

OA.1 A Joint Design Problem

In this appendix, we formulate the borrowing capacity maximization problem and the social welfare maximization problem as two closely related yet distinct joint design problems.

We begin with the borrowing capacity maximization problem. By definition, if V^σ is the entrepreneur's borrowing capacity under the signal structure σ, then there is an equilibrium $\mathcal{E}(V^\sigma;\sigma)$ in which the entrepreneur can borrow the amount $B = V^\sigma$. Therefore, to determine the entrepreneur's borrowing capacity, we must characterize such an equilibrium. In our model, this involves designing the initial contract at day 0 and the refinancing contracts for all possible scenarios at day 1, as well as deriving the agents' best responses to these contracts, such that the entrepreneur can indeed borrow the amount B at day 0.

The objective of the contract design problem is to maximize the insider's ex-ante return at day 0. This follows from the observation that competition among creditors at day 0 drives the insider's expected profit to zero, implying that, in equilibrium, the insider's ex-ante return from issuing the startup loan must equal his lending cost B. As a result, under any signal structure, the entrepreneur's borrowing capacity is the maximal ex-ante return to the insider.

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Given any monotone signal structure σ, a contract design problem for the entrepreneur's borrowing capacity is formulated as

$$V^{\sigma} = \max_{\mathcal{P}^j, m_1, m_2^O(s), m_2^I(p)} \int_{\mathcal{P}^I \cap \mathcal{P}^O} \left[(1-\lambda) p e(p) m_2^I(p) + \lambda m_1 \right] \mathrm{d}G(p)$$
$$+ \int_{\mathcal{P}^O \setminus \mathcal{P}^I} m_1 \mathrm{d}G(p) + \int_{\mathcal{P}^I \setminus \mathcal{P}^O} (1-\lambda) p e(p) m_2^I(p) \mathrm{d}G(p) \tag{OA.1.1}$$

subject to

$$e(p) = \mathbb{1}(p[v - \min\{m_2^O(s), m_2^I(p)\}] \geq c); \tag{OA.1.2}$$
$$pe(p) m_2^I(p) \geq m_1, \text{ and } m_2^I(p) \leq m_2^O(s), \forall p \in \mathcal{P}^I \cap \mathcal{P}^O; \tag{OA.1.3}$$
$$p m_2^O(s) < m_1, \forall p \in \mathcal{P}^O \setminus \mathcal{P}^I; \tag{OA.1.4}$$
$$pe(p) m_2^I(p) \geq 0, \forall p \in \mathcal{P}^I \setminus \mathcal{P}^O; \tag{OA.1.5}$$
$$m_2^O(s) \in \arg\max_{m_2} \mathbb{E}(pe(p)|s, \text{outsider winning}) m_2, \forall \sigma^{-1}(s) \subseteq \mathcal{P}^O; \tag{OA.1.6}$$
$$m_1 = \mathbb{E}(pe(p)|s, \text{outsider winning}) m_2^O(s), \forall \sigma^{-1}(s) \subseteq \mathcal{P}^O; \tag{OA.1.7}$$
$$m_1 > \max_{m_2} \mathbb{E}(pe(p)|s, \text{outsider winning}) m_2, \forall \sigma^{-1}(s) \nsubseteq \mathcal{P}^O. \tag{OA.1.8}$$

Here, $\mathcal{P}^I$ denotes the set of project qualities for which the insider is induced (or recommended) to bid at the debt rollover stage, and $\mathcal{P}^O$ represents the set of project qualities that generate signals inducing the outsiders to bid at debt rollover.[1] Importantly, $\mathcal{P}^j$ ($j = I, O$) forms part of the contract design. For any given project quality p, while the signals revealed to outsiders are determined by the signal structure σ, the creditors' bidding decisions at debt rollover are endogenously determined in equilibrium. Finally, m_1, $m_2^O(s)$, and $m_2^I(p)$ denote the insider's initial loan offer, the outsiders' refinancing offer conditional on observing signal s, and the insider's refinancing offer conditional on observing project quality p, respectively.

The seven constraints describe the agents' continuation plays in a perfect Bayesian equilibrium and are derived by backward induction. (OA.1.2) is the *entrepreneur's incentive compatibility* constraint at day 2, conditional on the project is refinanced at day 1. Given the refinancing offer $\min\{m_2^O(s), m_2^I(p)\}$ and the realized project quality p, the entrepreneur will work if and only if her expected payoff from working, which is $p[v - \min\{m_2^O(s), m_2^I(p)\}]$, can at least compensate her effort cost c.

(OA.1.3) is the *insider's incentive compatibility* constraint when both the insider and the outsiders are induced to bid at day 1. In this case, the insider wins to refinance the project in

[1] The insider can follow the prescribed bidding strategy at day 1 only if he does not experience a liquidity shock.

equilibrium when he does not face a liquidity shock. This requires, first, that the insider's refinancing offer $m_2^I(p)$ be weakly smaller than the outsiders' refinancing offer $m_2^O(s)$, and, second, that his expected return from refinancing the project, $pe(p)m_2^I(p)$, exceed the repayment m_1 he would obtain by exiting at debt rollover. (OA.1.4) is the *insider's incentive compatibility* constraint when only the outsiders are induced to bid at day 1. In this case, if $pm_2^O(s) < m_1$ holds, the insider optimally refrains from bidding at debt rollover, since no refinancing offer would both win the auction and yield an expected return exceeding m_1. Finally, (OA.1.5) is the *insider's incentive compatibility* constraint when only the insider is induced to bid at day 1. In this case, the insider's refinancing offer $m_2^I(p)$ only needs to generate a non-negative expected return, as his initial lending cost is already sunk.

(OA.1.6) is the *outsiders' incentive compatibility* constraint when they are induced to bid at day 1. It requires that, for any signal s generated by a project quality $p \in \mathcal{P}^O$, the refinancing offer $m_2^O(s)$ maximizes an outsider's expected return, conditional on the offer allowing him to win at debt rollover. (OA.1.7) is the *outsiders' individual rationality* constraint when they are induced to bid at day 1. In this case, given competition among the outsiders, the equilibrium refinancing offer $m_2^O(s)$ must yield zero expected profit to the winning outsider. Lastly, (OA.1.8) is the *outsiders' incentive compatibility* constraint when they are not induced to bid at day 1. For any signal s generated by a project quality $p \notin \mathcal{P}^O$, any refinancing offer that would enable an outsider to win at debt rollover would instead generate a loss. Consequently, no outsider is willing to refinance the project in this case.

Given these constraints, (OA.1.1) represents the objective function of the contract design—maximizing the insider's ex-ante return at day 0. It comprises three terms, corresponding to the insider's expected returns in three scenarios at day 1.[2]

On the right-hand side of (OA.1.1), the first term is the insider's expected return when both the insider and the outsiders are induced to bid at debt rollover. In this case, with probability $1-\lambda$, the insider does not face a liquidity shock and wins the refinancing with offer $m_2^I(p)$, earning an expected payment of $pe(p)m_2^I(p)$; with probability λ, he experiences a liquidity shock and receives the repayment m_1 specified in the startup loan contract. The second term captures the insider's expected return when only the outsiders bid at day 1. Here, the insider simply receives repayment m_1, as an outsider refinances the project. The third term is the insider's expected return when he is the only creditor induced to bid at day 1. In this case,

[2]There is a fourth scenario in which neither the insider nor the outsiders bid at day 1, occurring when $p \in [0,1]\backslash(\mathcal{P}^O \cup \mathcal{P}^I)$. However, in any optimal contract design, $[0,1]\backslash(\mathcal{P}^O \cup \mathcal{P}^I) \subseteq [0, \frac{c}{v})$, so the project can only yield zero expected return to the creditors. For simplicity, the creditors' incentive compatibility and individual rationality constraints under this scenario are also omitted.

when the insider does not encounter a liquidity shock, he offers $m_2^I(p)$ to refinance the project and obtains the expected payment $pe(p)m_2^I(p)$.

Once the entrepreneur's borrowing capacity has been determined under any monotone signal structure σ, we can characterize the signal structure that maximizes this capacity. Formally, the second step of the joint design problem is an information design problem:

$$\sup_{\sigma} V^{\sigma}, \tag{OA.1.9}$$

where σ is a monotone signal structure.

The joint design problem for maximizing social welfare follows a similar structure and can be decomposed into two steps: contract design followed by information design. In our model, social welfare coincides with the entrepreneur's ex-ante payoff in any equilibrium. Accordingly, given a monotone signal structure σ and the entrepreneur's borrowing need B, the contract design problem that maximizes the entrepreneur's ex-ante payoff $\Pi^{\sigma}(B)$ can be formulated as

$$\begin{aligned}\Pi^{\sigma}(B) = \max_{\mathcal{P}^j, m_1, m_2^O(s), m_2^I(p)} & \int_{\mathcal{P}^I \cap \mathcal{P}^O} e(p)\left[(1-\lambda)[p(v - m_2^I(p)) - c] + \lambda[p(v - m_2^O(s)) - c]\right] \mathrm{d}G(p) \\ & + \int_{\mathcal{P}^O \setminus \mathcal{P}^I} e(p)[p(v - m_2^O(s)) - c]\mathrm{d}G(p) \\ & + \int_{\mathcal{P}^I \setminus \mathcal{P}^O} e(p)(1-\lambda)[p(v - m_2^I(p)) - c]\mathrm{d}G(p)\end{aligned} \tag{OA.1.10}$$

subject to

$$\begin{aligned} m_1 \in \arg\max_{m_1'} \Big\{ & \int_{\mathcal{P}^I \cap \mathcal{P}^O} \left[(1-\lambda)pe(p)m_2^I(p) + \lambda m_1'\right] \mathrm{d}G(p) \\ & + \int_{\mathcal{P}^O \setminus \mathcal{P}^I} m_1' \mathrm{d}G(p) + \int_{\mathcal{P}^I \setminus \mathcal{P}^O} (1-\lambda)pe(p)m_2^I(p)\mathrm{d}G(p) \Big| \text{initial winning} \Big\}, \end{aligned} \tag{OA.1.11}$$

$$\begin{aligned} B = & \int_{\mathcal{P}^I \cap \mathcal{P}^O} \left[(1-\lambda)pe(p)m_2^I(p) + \lambda m_1\right] \mathrm{d}G(p) \\ & + \int_{\mathcal{P}^O \setminus \mathcal{P}^I} m_1 \mathrm{d}G(p) + \int_{\mathcal{P}^I \setminus \mathcal{P}^O} (1-\lambda)pe(p)m_2^I(p)\mathrm{d}G(p), \end{aligned} \tag{OA.1.12}$$

$$B \leq V^{\sigma}, \tag{OA.1.13}$$

and constraints (OA.1.2) to (OA.1.8).

(OA.1.11) is the *insider's incentive compatibility* constraint at day 0. This constraint requires that the designed contract m_1 at day 0 maximizes the insider's ex-ante return, conditional on enabling the insider to win to issue the startup loan. The insider's ex-ante return is given by the objective function in (OA.1.1). (OA.1.12) is the *insider's individual rationality* constraint at day 0, which ensures that, due to competition among creditors, the insider's ex-ante return is just sufficient to cover his lending cost B. (OA.1.13) is the *feasibility* constraint at day 0, stating that under the signal structure σ, the project can be financed only if the entrepreneur's borrowing need B does not exceed her borrowing capacity V^{σ}. In addition to these day-0 constraints, the continuation constraints (OA.1.2) to (OA.1.8) must also be satisfied.

Given all the constraints, (OA.1.10) represents the objective function of the contract design—maximizing the entrepreneur's ex-ante payoff at day 0. As before, it comprises three terms corresponding to the three possible competition scenarios at debt rollover.[3]

On the right-hand side of (OA.1.10), the first term is the entrepreneur's expected payoff when both the insider and the outsiders are induced to bid at debt rollover. In this case, when the insider does not encounter a liquidity shock and hence wins at debt rollover with the offer $m_2^I(p)$, the entrepreneur earns the expected return $e(p)[p(v - m_2^I(p)) - c]$ from the project; when the insider encounters a liquidity shock and therefore an outsider wins at debt rollover with the offer $m_2^O(s)$, the entrepreneur's expected return is $e(p)[p(v - m_2^O(s)) - c]$. The second term is the entrepreneur's expected payoff when only the outsiders are bidding at day 1. In this case, the outsiders offer $m_2^O(s)$ to refinance the project, and the entrepreneur's expected return is $e(p)[p(v - m_2^O(s)) - c]$. Lastly, the third term is the entrepreneur's expected payoff when only the insider is induced to bid at day 1. In this case, when the insider does not encounter a liquidity shock, he offers $m_2^I(p)$ to refinance the project, and the entrepreneur's expected return is $e(p)[p(v - m_2^I(p)) - c]$.

Once the entrepreneur's ex-ante payoff has been determined under any monotone signal structure σ, we can identify the signal structure that maximizes this payoff, which is equivalent to maximizing social welfare. The second step of the joint design problem is thus an information design problem, formally expressed as

$$\sup_{\sigma} \Pi^{\sigma}(B), \tag{OA.1.14}$$

where σ is a monotone signal structure.

[3]Analogous to the contract design problem for maximizing the entrepreneur's borrowing capacity, the fourth scenario, in which the project yields a zero expected payoff for any player, is omitted.

OA.2 Proofs of Lemmas, Propositions, and Corollaries in the Manuscript

We slightly reorder the proofs in this section, presenting those of Lemmas 1 and 2 before that of Proposition 3.

Proof of Proposition 1:

This result is a direct extension of Proposition 3 to the case without moral hazard, i.e., when $c = 0$. In particular, as shown in the proof of Corollary 1, when $c = 0$, the threshold p_T^* equals 0. Hence, the pass-or-fail signal structure $\sigma(p; p_T^*)$ that maximizes the entrepreneur's borrowing capacity, as described in Proposition 3, reduces to the non-revealing signal structure σ_N which induces outsider participation in debt rollover for all project qualities. Consequently, the entrepreneur's maximum borrowing capacity, $V(p_T^*)$ as defined in (2), equals $v\mathbb{E}(p)$. Furthermore, following the equilibrium construction in the proof of Proposition 4, when $c = 0$, the equilibrium refinancing offer given by m_2^* in (OA.2.14) equals v.

Q.E.D.

Proof of Proposition 2:

This result is a direct extension of Proposition 3 to the case without liquidity shocks, i.e., when $\lambda = 0$. Specifically, as shown in the proof of Corollary 1, when $\lambda = 0$, the threshold p_T^* is equal to 1. Accordingly, the pass-or-fail signal structure $\sigma(p; p_T^*)$ that maximizes the entrepreneur's borrowing capacity, as demonstrated in Proposition 3, degenerates into the no-revealing signal structure σ_N that induces no outsider participation in debt rollover for any project quality. Consequently, the entrepreneur's maximum borrowing capacity, $V(p_T^*)$ as defined in (2), is given by $\mathbb{E}\left(pv - c | p \geq \frac{c}{v}\right)$.

Q.E.D.

Proof of Lemma 1:

Given a monotone signal structure σ and a borrowing need B, suppose that the project is financed in equilibrium $\mathcal{E}(B;\sigma)$, with the insider's startup loan offer being m_1. In $\mathcal{E}(B;\sigma)$, for any signal s revealed at day 1, two cases arise. If $\Pr(\sigma^{-1}(s)) > 0$, the signal s conveys only coarse information to the outsiders, so competition at debt rollover features adverse selection. If instead $\Pr(\sigma^{-1}(s)) = 0$, the signal s perfectly reveals the project quality, thus eliminating information asymmetry among the creditors at debt rollover.

Case (1): $\Pr(\sigma^{-1}(s)) > 0$.

The result in Part 1 of Lemma 1 is straightforward. If upon observing the signal s no outsider participates in debt rollover, the insider remains the only potential creditor. When the insider does not face a liquidity shock, he offers refinancing terms as follows: for $p \in \sigma^{-1}(s) \cap [\frac{c}{v}, 1]$, he sets $m_2^I(p) = v - \frac{c}{p}$, thus extracting all net surplus $pv - c$ from the project while keeping the entrepreneur motivated to exert effort; for $p \in \sigma^{-1}(s) \cap [0, \frac{c}{v})$, he makes a zero-profit random offer, as the entrepreneur cannot be motivated to work. On the other hand, when the insider experiences a liquidity shock, the project is liquidated, and hence his payoff is zero. Thus, the insider's expected return conditional on the signal s is

$$(1-\lambda)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]}(pv-c)\mathrm{d}G(p).$$

The result in Part 2 of Lemma 1 illustrates the key interactions among creditors under asymmetric information.

First, consider the entrepreneur's decision at day 2. Upon observing the signal s, if the outsiders compete to make the offer $m_2^O(s)$, the project is surely refinanced at this offer. The entrepreneur then exerts effort at day 2 if and only if she observes $p \geq p_E(s)$, where

$$p_E(s) = \frac{c}{v - m_2^O(s)}. \tag{OA.2.1}$$

Since $m_2^O(s) \geq 0$, it follows that $p_E(s) \geq \frac{c}{v}$.

Second, consider the insider's decision at day 1. Given the outsiders' offer $m_2^O(s)$, the insider, if not facing a liquidity shock, chooses to match this offer if he observes $p \geq p_{I,E}(s) \equiv \max\{p_I(s), p_E(s)\}$ and chooses to exit otherwise, where

$$p_I(s) \equiv \frac{m_1}{m_2^O(s)}, \tag{OA.2.2}$$

and the subscript "I" refers to "information advantage". The reason is as follows. For $p \geq p_{I,E}(s)$, matching the outsiders' offer allows the insider to refinance the project and earn an expected return $pm_2^O(s)$, which exceeds the repayment m_1 he can obtain from exiting. In contrast, for $p < p_{I,E}(s)$, depending on whether the entrepreneur will exert effort, matching the outsiders' offer yields an expected return $pm_2^O(s)$ or 0 to the insider, which is strictly less than m_1.

Third, consider the outsiders' offer making decisions at day 1. Anticipating the insider's reaction in the refinancing competition, the outsiders update their beliefs according to Bayes' rule. In particular, conditional on one of them winning the competition and refinancing the project with an offer $m_2^O(s)$, they infer that, with probability

$$\frac{\lambda}{\lambda + (1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}},$$

the insider experiences a liquidity shock and therefore does not participate in the competition, in which case the project quality is drawn from the set $\sigma^{-1}(s)$ with density $\frac{g(p)}{\Pr(\sigma^{-1}(s))}$. Conversely, with probability

$$\frac{(1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}}{\lambda + (1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}},$$

the insider does not face a liquidity shock but chooses not to compete, in which case the project quality is drawn from the set $\sigma^{-1}(s)\cap[0,p_{I,E}(s))$ with density $\frac{g(p)}{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}$. In addition, the repayment $m_2^O(s)$ can be made at day 2 only when the entrepreneur exerts effort and the project's cash flow is v. Thus, the winning outsider's expected return from providing the refinancing loan is

$$\frac{\lambda\int_{\sigma^{-1}(s)\cap[p_E(s),1]}\frac{pm_2^O(s)g(p)}{\Pr(\sigma^{-1}(s))}\mathrm{d}p}{\lambda + (1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}} + \frac{(1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}\int_{\sigma^{-1}(s)\cap[p_E(s),p_{I,E}(s))}\frac{pm_2^O(s)g(p)}{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}\mathrm{d}p}{\lambda + (1-\lambda)\frac{\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}{\Pr(\sigma^{-1}(s))}}.$$

Due to perfect competition, the winning outsider should earn a zero expected profit. Simplifying the expected return above, the break-even condition for the winning outsider therefore is

$$m_1 = \frac{\lambda\int_{\sigma^{-1}(s)\cap[p_E(s),1]}pm_2^O(s)\mathrm{d}G(p) + (1-\lambda)\int_{\sigma^{-1}(s)\cap[p_E(s),p_{I,E}(s))}pm_2^O(s)\mathrm{d}G(p)}{\lambda\Pr(\sigma^{-1}(s)) + (1-\lambda)\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))}, \quad \text{(OA.2.3)}$$

where, because of competition, $m_2^O(s)$ is the smallest refinancing offer that satisfies this equation. Implicitly, in this equation we should have $p_E(s) < \sup(\sigma^{-1}(s))$, otherwise the entrepreneur will always shirk after the project is refinanced with the offer $m_2^O(s)$.

Given the players' decisions described above, the insider's total expected return conditional on the signal s is

$$\lambda\Pr(\sigma^{-1}(s))m_1 + (1-\lambda)\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))m_1 + (1-\lambda)\int_{\sigma^{-1}(s)\cap[p_{I,E}(s),1]}pm_2^O(s)\mathrm{d}G(p). \quad \text{(OA.2.4)}$$

The interpretation of this expression is as follows. With probability λ, the insider experiences a liquidity shock, in which case he receives the repayment m_1 and exits the competition. With probability $1-\lambda$, the insider does not face a liquidity shock. In this case, he receives the repayment m_1 and exits for $p < p_{I,E}(s)$, but matches the outsiders' offer $m_2^O(s)$ and hence refinances the project for $p \geq p_{I,E}(s)$. The three terms in (OA.2.4) represent the insider's expected returns in these three scenarios, respectively.

Note that the sum of the first two terms in (OA.2.4) equals the product of m_1 and the denominator in (OA.2.3), which in turn equals the numerator in (OA.2.3). Hence, by replacing the sum of these two terms with the numerator in (OA.2.3) and making some simplification, the insider's total expected return conditional on the signal s, as expressed in (OA.2.4), can be rewritten as

$$\int_{\sigma^{-1}(s)\cap[p_E(s),1]} p m_2^O(s) \mathrm{d}G(p). \tag{OA.2.5}$$

Substituting $m_2^O(s) = v - \frac{c}{p_E(s)}$ into (OA.2.5) then yields (5) in Lemma 1.

Case (2): $\Pr(\sigma^{-1}(s)) = 0$.

In this case, $\sigma^{-1}(s)$ is a singleton. So, the integrals below should be read as evaluations at that single posterior $p = \sigma^{-1}(s)$.

If upon observing the signal s no outsider participates in debt rollover, the insider, if not facing a liquidity shock, will refinance the project with an offer $v - \frac{c}{\sigma^{-1}(s)}$ when $\sigma^{-1}(s) \geq \frac{c}{v}$ and makes a zero-profit random offer otherwise. Abusing the integral notation slightly,the insider's expected return conditional on the signal s can be written as

$$(1-\lambda)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]} (pv - c)\mathrm{d}G(p).$$

On the other hand, if upon observing the signal s the outsiders participate in debt rollover, the competition between the insider and the outsiders is of complete information, and the outsiders' offer $m_2^O(s)$ satisfies $\sigma^{-1}(s)m_2^O(s) = m_1$. In addition, given this refinancing offer, the entrepreneur exerts effort if and only if $\sigma^{-1}(s)$ exceeds the threshold $p_E(s)$, where $p_E(s) = \frac{c}{v - m_2^O(s)}$. In this case, the insider's expected return is always equal to $\sigma^{-1}(s)m_2^O(s)$, regardless of whether he chooses to match the outsiders' offer. Abusing the integral notation again, his expected return conditional on the signal s can be written as

$$\int_{\sigma^{-1}(s)\cap[p_E(s),1]} p\Big(v - \frac{c}{p_E(s)}\Big)\mathrm{d}G(p).$$

Q.E.D.

Proof of Lemma 2:

Given a monotone signal structure σ and a borrowing need B, suppose that the project is financed in equilibrium $\mathcal{E}(B;\sigma)$.

In $\mathcal{E}(B;\sigma)$, let $\mathbb{S}_H(\sigma)$ denote the set of signals upon observing which the outsiders participate in debt rollover, and $\mathbb{S}_L(\sigma)$ denote the set of signals upon observing which no outsider participates. By Lemma 1, in equilibrium, the insider's expected return $U^\sigma(B)$ from financing the project with the startup loan offer m_1 is then given by

$$U^\sigma(B) = \sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[p_E(s),1]} p\Big(v - \frac{c}{p_E(s)}\Big)\mathrm{d}G(p) + \sum_{s\in\mathbb{S}_L(\sigma)} (1-\lambda)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]} (pv-c)\mathrm{d}G(p), \tag{OA.2.6}$$

where $p_E(s) = \frac{c}{v - m_2^O(s)}$, and $m_2^O(s)$ is the outsiders' equilibrium refinancing offer conditional on the signal s.

If $\mathbb{S}_H(\sigma)$ is empty, then $U^\sigma(B)$ in (OA.2.6) is

$$U^\sigma(B) = (1-\lambda)\int_{\frac{c}{v}}^{1} (pv-c)\mathrm{d}G(p). \tag{OA.2.7}$$

Hereafter, we focus on the case where $\mathbb{S}_H(\sigma)$ is non-empty. Let $\underline{s}$ denote the smallest signal in $\mathbb{S}_H(\sigma)$. We next show that if $s \in \mathbb{S}_H(\sigma)$ and $s' > s$, then $s' \in \mathbb{S}_H(\sigma)$, $m_2^O(s') < m_2^O(s)$ and $p_E(s') < p_E(s)$.

To see this, suppose that $s \in \mathbb{S}_H(\sigma)$ and $s' > s$. Consider the case where $\Pr(\sigma^{-1}(s)) > 0$ and $\Pr(\sigma^{-1}(s')) > 0$. In equilibrium, the following equation and inequalities must then hold:

$$\begin{aligned} m_1 &= \frac{\lambda \int_{\sigma^{-1}(s)\cap[p_E(s),1]} p m_2^O(s)\mathrm{d}G(p) + (1-\lambda)\int_{\sigma^{-1}(s)\cap[p_E(s),p_{I,E}(s))} p m_2^O(s)\mathrm{d}G(p)}{\lambda \Pr(\sigma^{-1}(s)) + (1-\lambda)\Pr(\sigma^{-1}(s)\cap[0,p_{I,E}(s)))} \\ &< \sup(\sigma^{-1}(s))\cdot m_2^O(s) \\ &\le \inf(\sigma^{-1}(s'))\cdot m_2^O(s) \\ &< \frac{\lambda \int_{\sigma^{-1}(s')\cap[p_E(s),1]} p m_2^O(s)\mathrm{d}G(p) + (1-\lambda)\int_{\sigma^{-1}(s')\cap[p_E(s),p_{I,E}(s))} p m_2^O(s)\mathrm{d}G(p)}{\lambda \Pr(\sigma^{-1}(s')) + (1-\lambda)\Pr(\sigma^{-1}(s')\cap[0,p_{I,E}(s)))}. \end{aligned} \tag{OA.2.8}$$

In (OA.2.8), the equation represents the outsider's break-even condition when, after observing the signal s, he wins the refinancing competition with an offer $m_2^O(s)$. The final inequality holds because, in equilibrium, we must have $p_E(s) < \sup(\sigma^{-1}(s)) \le \inf(\sigma^{-1}(s'))$, which implies that $\sigma^{-1}(s') \cap [p_E(s),1] = \sigma^{-1}(s')$. In other words, if an outsider can also win to refinance the project with offer $m_2^O(s)$ after observing the signal s', he knows that the entrepreneur will certainly exert effort for any $p \in \sigma^{-1}(s')$.

The conditions in (OA.2.8) imply that if, upon observing the signal s, an outsider breaks even from winning to refinance the project with an offer $m_2^O(s)$, then upon observing a higher signal s', the outsider would earn a positive expected profit if he wins to refinance the project with the same offer $m_2^O(s)$. Consequently, after observing the signal s', competition among the outsiders drives the equilibrium refinancing offer to $m_2^O(s')$, which is determined by

$$m_1 = \frac{\lambda \int_{\sigma^{-1}(s')\cap[p_E(s'),1]} p m_2^O(s')\mathrm{d}G(p) + (1-\lambda)\int_{\sigma^{-1}(s')\cap[p_E(s'),p_{I,E}(s'))} p m_2^O(s')\mathrm{d}G(p)}{\lambda \Pr(\sigma^{-1}(s')) + (1-\lambda)\Pr(\sigma^{-1}(s')\cap[0,p_{I,E}(s')))},$$

and satisfies $m_2^O(s') < m_2^O(s)$. Accordingly, $p_E(s') < p_E(s)$. Intuitively, after the realization of a more favorable signal, competition among the outsiders leads to a lower refinancing offer, which in turn reduces the entrepreneur's effort threshold.

By slightly modifying the arguments above, we can verify that the same results hold for the remaining cases: (i) $\Pr(\sigma^{-1}(s)) > 0$ and $\Pr(\sigma^{-1}(s')) = 0$, (ii) $\Pr(\sigma^{-1}(s)) = 0$ and $\Pr(\sigma^{-1}(s')) > 0$, and (iii) $\Pr(\sigma^{-1}(s)) = \Pr(\sigma^{-1}(s')) = 0$. In all these cases, we have $m_2^O(s') < m_2^O(s)$ and $p_E(s') < p_E(s)$. For example, in the case where $\Pr(\sigma^{-1}(s)) = 0$ and $\Pr(\sigma^{-1}(s')) > 0$, the result follows by replacing the equation in (OA.2.8) with $m_1 = \sup(\sigma^{-1}(s)) \cdot m_2^O(s)$.

In addition, note that $p_E(\underline{s}) < \sup(\sigma^{-1}(\underline{s}))$; otherwise, upon observing the signal $\underline{s}$, no outsider will participate in debt rollover. Consequently, for any $s > \underline{s}$, we have $s \in \mathbb{S}_H(\sigma)$ and

$$p_E(s) < p_E(\underline{s}) < \sup(\sigma^{-1}(\underline{s})) \leq \inf(\sigma^{-1}(s)).$$

Therefore, for any $s > \underline{s}$, if the project is refinanced with the offer $m_2^O(s)$, the entrepreneur will exert effort for any $p \in \sigma^{-1}(s)$. In contrast, for the signal $\underline{s}$, it is indeterminate whether $\inf(\sigma^{-1}(\underline{s}))$ is greater than $p_E(\underline{s})$ or not.

Thus, (OA.2.6) can be rewritten as

$$\begin{aligned} U^\sigma(B) \;=\; & \sum_{s\in\mathbb{S}_H(\sigma)\setminus\{\underline{s}\}} \int_{\sigma^{-1}(s)} p\Big(v - \frac{c}{p_E(s)}\Big)\mathrm{d}G(p) + \int_{\sigma^{-1}(\underline{s})\cap[p_E(\underline{s}),1]} p\Big(v - \frac{c}{p_E(\underline{s})}\Big)\mathrm{d}G(p) \\ & + (1-\lambda)\int_{\frac{c}{v}}^{\max\{\inf(\sigma^{-1}(\underline{s})),\frac{c}{v}\}} (pv-c)\mathrm{d}G(p). \end{aligned} \tag{OA.2.9}$$

Denote $\underline{p} = \inf(\sigma^{-1}(\underline{s}))$. (OA.2.9) can be further simplified as

$$U^\sigma(B) = \int_{\max\{\underline{p},p_E(\underline{s})\}}^1 p\Big(v - \frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}} (pv-c)\mathrm{d}G(p). \tag{OA.2.10}$$

Finally, since in equilibrium the insider must also break even from issuing the startup loan, it follows that $B = U^\sigma(B)$.

Q.E.D.

Proof of Proposition 3:

By Proposition 4, the entrepreneur can borrow an amount of $V(p_T^*)$ under the pass-or-fail signal structure $\sigma(p; p_T^*)$. In this proof, we show that under any monotone signal structure, the entrepreneur's borrowing capacity is bounded above by $V(p_T^*)$.

Consider any monotone signal structure σ, and suppose that the project is financed in equilibrium $\mathcal{E}(B;\sigma)$. Let $\mathbb{S}_H(\sigma)$ and $\mathbb{S}_L(\sigma)$ be defined as in the proof of Lemma 2.

In $\mathcal{E}(B;\sigma)$, when $\mathbb{S}_H(\sigma)$ is empty, the insider's expected return $U^\sigma(B)$ from issuing the startup loan is given by (OA.2.7) and hence satisfies

$$U^\sigma(B) = (1-\lambda)\int_{\frac{c}{v}}^{1}(pv-c)\mathrm{d}G(p) = V(1) < V(p_T^*). \tag{OA.2.11}$$

In $\mathcal{E}(B;\sigma)$, when $\mathbb{S}_H(\sigma)$ is non-empty, let $\underline{s}$ and $\underline{p}$ be defined as above and denote $p_T = \max\{\underline{p}, p_E(\underline{s})\}$. Note also that, in equilibrium, $p_E(\underline{s}) \geq \frac{c}{v}$. Then, the insider's ex-ante return $U^\sigma(B)$ from issuing the startup loan is given by (OA.2.10) and therefore satisfies

$$\begin{aligned}
U^\sigma(B) &= \int_{\max\{\underline{p},p_E(\underline{s})\}}^{1} p\Big(v-\frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p)\\
&\leq \int_{\max\{\underline{p},p_E(\underline{s})\}}^{1} p\Big(v-\frac{c}{p_E(\underline{s})}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p)\\
&\leq V(p_T) = \int_{p_T}^{1} p\Big(v-\frac{c}{p_T}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T}(pv-c)\mathrm{d}G(p)\\
&\leq V(p_T^*).
\end{aligned} \tag{OA.2.12}$$

In (OA.2.12), the three inequalities correspond to the three characterization steps discussed in the main text. Specifically, the first inequality holds because, for any $s > \underline{s}$, we have $s \in \mathbb{S}_H(\sigma)$ and $p_E(s) < p_E(\underline{s})$. Moreover, this inequality is strict if the set of project qualities generating signals $s > \underline{s}$ has a positive measure. The second inequality follows from the definition of p_T and is strict when $\underline{p} \neq p_E(\underline{s})$. Therefore, we have $U^\sigma(B) \leq V(p_T^*)$.

As a result, under any monotone signal structure σ, the insider's expected return from issuing the startup loan is bounded above by $V(p_T^*)$. Consequently, the entrepreneur's borrowing capacity is also bounded above by $V(p_T^*)$. Combined with Proposition 4, this establishes that the entrepreneur's maximum borrowing capacity is $V(p_T^*)$.

Q.E.D.

Proof of Proposition 4:

Recall that $p_T(B)$ denotes the smallest value of p_T satisfying $B = V(p_T)$. Let $\mathbf{P}_T$ denote the set of all such values $p_T(B)$. We show below that, for any borrowing need $B = V(p_T)$, the project can be financed under the binary signal structure $\sigma(p; p_T)$. In addition, if $p_T \in \mathbf{P}_T$, $\sigma(p; p_T)$ constitutes a pass-or-fail signal structure, thus establishing Proposition 4.

Suppose that the entrepreneur faces a borrowing need $B = V(p_T)$ for some $p_T \in [\frac{c}{v}, 1]$, and consider the binary signal structure $\sigma(p; p_T)$. We examine whether the following behavior of the agents, together with the outsiders' belief updating consistent with Bayes' rule, consists of an equilibrium. The agents behave as follows:

(i) At day 0, the creditors compete to finance the project by offering m_1^*, which is determined by

$$m_1^* = \frac{\lambda \int_{p_T}^{1} p\left(v - \frac{c}{p_T}\right) \mathrm{d}G(p) + (1-\lambda) \int_{p_T}^{\max\{\frac{m_1^*}{v - c/p_T}, p_T\}} p\left(v - \frac{c}{p_T}\right) \mathrm{d}G(p)}{\lambda[1 - G(p_T)] + (1-\lambda)[G(\max\{\frac{m_1^*}{v - c/p_T}, p_T\}) - G(p_T)]}. \tag{OA.2.13}$$

(ii) At day 1, upon observing the signal $s = 1$, the outsiders compete to refinance the project by offering

$$m_2^* = v - \frac{c}{p_T}. \tag{OA.2.14}$$

In contrast, upon observing the signal $s = 0$, no outsider competes to refinance the project.

(iii) At day 1, if the insider does not experience a liquidity shock, then upon observing the signal $s = 1$, he matches the outsiders' offer m_2^* when $p \geq \max\{\frac{m_1^*}{m_2^*}, p_T\}$, and exits the competition when $p < \max\{\frac{m_1^*}{m_2^*}, p_T\}$. In contrast, upon observing the signal $s = 0$, he refinances the project by offering $v - \frac{c}{p}$ when $p \geq \frac{c}{v}$ and making a zero-profit random offer when $p < \frac{c}{v}$.

(iv) At day 2, the entrepreneur exerts effort if and only if the refinancing offer m_2 made by the creditors satisfies $p(v - m_2) \geq c$.

Note that under the binary signal structure $\sigma(p; p_T)$, if the strategy profile (i)–(iv) is played in equilibrium, then $\sigma(p; p_T)$ is indeed a pass-or-fail signal structure. Furthermore, as established in the analysis following Lemma 1, given the behavior of the other agents in this profile, the insider's behavior at day 1 and the entrepreneur's behavior at day 2 are optimal. Therefore, it remains to verify the optimality of the creditors' behavior at day 0 and the outsiders' behavior at day 1.

To this end, we introduce a function $\phi(m_1; m_2)$ defined as

$$\begin{aligned}\phi(m_1; m_2) \;=\; & \Big\{\lambda \int_{\max\{\frac{c}{v-m_2}, p_T\}}^{1} p m_2 \mathrm{d}G(p) + (1-\lambda) \int_{\max\{\frac{c}{v-m_2}, p_T\}}^{\max\{\frac{m_1}{m_2}, \frac{c}{v-m_2}, p_T\}} p m_2 \mathrm{d}G(p)\Big\} \\ & - m_1 \Big\{\lambda[1 - G(p_T)] + (1-\lambda)[G(\max\{\frac{m_1}{m_2}, \frac{c}{v - m_2}, p_T\}) - G(p_T)]\Big\}\end{aligned} \tag{OA.2.15}$$

This function serves to characterize the outsiders' expected profit from refinancing the project. In particular, suppose that the insider's startup loan offer is m_1 and, upon observing the signal $s=1$, the lowest refinancing offer made by the outsiders is m_2. Then, following the argument in the proof of Lemma 1, the expression in the first large brace of (OA.2.15) represents the outsiders' total expected return, while the expression in the second large brace captures their total expected payment. When $\phi(m_1;m_2)=0$, equation (OA.2.15) collapses to the outsiders' break-even condition. Moreover, it follows directly from the construction that $\frac{m_1^*}{m_2^*} > p_T$ and $\phi(m_1^*;m_2^*)=0$.[4]

Claim 1. Given the other agents' behavior in the profile (i)–(iv), the outsiders' behavior described in (ii) is optimal for them.

Fix $m_1 = m_1^*$. Because $\frac{m_1^*}{m_2^*} > p_T$, for any $m_2 \in [m_1^*, m_2^*]$, we have

$$\frac{\mathrm{d}\phi(m_1^*;m_2)}{\mathrm{d}m_2} = \lambda \int_{p_T}^{1} p\mathrm{d}G(p) + (1-\lambda)\int_{p_T}^{\frac{m_1^*}{m_2}} p\mathrm{d}G(p) > 0.$$

Together with

$$\phi(m_1^*;m_1^*) < 0, \text{ and } \phi(m_1^*;m_2^*) = 0,$$

it follows that $\phi(m_1^*;m_2) < 0$ for any $m_2 \in [m_1^*, m_2^*)$. Thus, m_2^* is the smallest value of m_2 that satisfies $\phi(m_1^*;m_2)=0$. Consequently, if the insider's offer is m_1^*, upon observing the signal $s=1$, it is optimal for the outsiders to offer m_2^* to refinance the project.

Also note that $\frac{m_1^*}{m_2^*} > p_T$ implies

$$m_1^* > p_T v - c > \frac{\int_{\frac{c}{v}}^{p_T}(pv-c)\mathrm{d}G(p)}{G(p_T)},$$

where the last term represents an upper bound on the expected return that an outsider can obtain from refinancing the project upon observing the signal $s=0$. Hence, if the insider's offer is m_1^*, after observing $s=0$, no outsider has an incentive to roll over the loan.

Claim 2. If $p_T \in \mathbf{P}_T$, the creditors' behavior described in (i) is optimal for them. If $p_T \notin \mathbf{P}_T$, the behavior described in (i) may or may not be optimal for them, but the project can always be financed under the binary signal structure $\sigma(p;p_T)$.

[4] If $\frac{m_1^*}{m_2^*} \le p_T$, (OA.2.13) can be expressed as

$$m_1^* = \frac{\lambda\int_{p_T}^{1} p\left(v-\frac{c}{p_T}\right)\mathrm{d}G(p)}{\lambda[1-G(p_T)]} = \frac{\left(v-\frac{c}{p_T}\right)\int_{p_T}^{1} p\mathrm{d}G(p)}{1-G(p_T)} > \left(v-\frac{c}{p_T}\right)p_T = m_2^* p_T.$$

A contradiction.

Fix $m_2 = m_2^*$. We have

$$\phi(0; m_2^*) > 0, \text{ and } \phi(m_2^*; m_2^*) < 0.$$

Moreover, for any $m_1 \in [0, m_2^*]$,

$$\frac{\mathrm{d}\phi(m_1; m_2^*)}{\mathrm{d}m_1} = -\lambda[1 - G(p_T)] - (1-\lambda)[G(\max\{\frac{m_1}{m_2^*}, p_T\}) - G(p_T)] < 0.$$

Thus, for $m_1 \in [0, m_2^*]$, m_1^* is the unique value of m_1 that satisfies $\phi(m_1; m_2^*) = 0$.

Suppose a creditor wins to finance the project at day 0 with the offer m_1^*. Given the agents' behavior described in the profile (i)–(iv), and by Lemmas 1 and 2, the insider's expected return from issuing the startup loan is

$$V(p_T) = \int_{p_T}^{1} p\Big(v - \frac{c}{p_T}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T}(pv - c)\mathrm{d}G(p). \tag{OA.2.16}$$

Hence, the insider's break-even condition $B = V(p_T)$ holds.

Next, we examine whether any creditor has an incentive to make an offer lower than m_1^* to finance the project at day 0.

Suppose a creditor wins to finance the project at day 0 with an offer m_1', where $m_1' < m_1^*$. Note that for any $m_1' \in (0, m_1^*)$, if we take m_1' as given, then $\phi(m_1'; m_1') < 0$ and $\phi(m_1'; m_2^*) > 0$, which implies the existence of a value $m_2' \in (m_1', m_2^*)$ such that $\phi(m_1'; m_2') = 0$. Therefore, if the insider's offer is m_1', after observing the signal $s = 1$, the outsiders will compete to refinance the project. Consequently, two cases arise: (a) the outsiders compete to refinance the project if and only if the realized signal is $s = 1$; (b) the outsiders compete to refinance the project after the realization of both signals.

Consider case (a). Let $m_2^O(1)$ denote the lowest refinancing offer made by the outsiders after the realization of signal $s = 1$. By the argument in the previous paragraph, $m_2^O(1) < m_2^*$ and $\phi(m_1'; m_2^O(1)) = 0$. By Lemma 2, the insider's expected return at day 0 in this case is given by

$$\int_{\max\{p_T, p_E(1)\}}^{1} p\Big(v - \frac{c}{p_E(1)}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T}(pv - c)\mathrm{d}G(p), \tag{OA.2.17}$$

where $p_E(1) = \frac{c}{v - m_2^O(1)}$. Since $m_2^O(1) < m_2^*$, it follows that $p_E(1) < p_T$. Thus, the expected return in (OA.2.17) is strictly less than $V(p_T)$, indicating that in this case the offer m_1' would lead the insider to incur a loss.

Consider case (b). Let $m_2^O(1)$ and $m_2^O(0)$ denote the lowest refinancing offers made by the outsiders after the realization of signals $s = 1$ and $s = 0$, respectively. By Lemma 2, the insider's expected return at day 0 in this case is given by

$$\int_{\max\{p_T, p_E(1)\}}^{1} p\Big(v - \frac{c}{p_E(1)}\Big)\mathrm{d}G(p) + \int_{p_E(0)}^{p_T} p\Big(v - \frac{c}{p_E(0)}\Big)\mathrm{d}G(p) \tag{OA.2.18}$$

where $p_E(1) = \frac{c}{v - m_2^O(1)}$ and $p_E(0) = \frac{c}{v - m_2^O(0)}$. Note that $p_E(0) < p_T$; otherwise, upon observing the signal $s = 0$, no outsider would be willing to refinance the project.

Following the proof of Lemma 2, we have $m_2^O(1) < m_2^O(0)$ and consequently $p_E(1) < p_E(0)$. It then follows immediately that the expected return given in (OA.2.18) is strictly less than

$$\int_{p_E(0)}^{1} p\Big(v - \frac{c}{p_E(0)}\Big) \mathrm{d}G(p). \tag{OA.2.19}$$

If $p_T \in \mathbf{P}_T$, given that $p_E(0) < p_T$ as shown above, we have

$$\int_{p_E(0)}^{1} p\Big(v - \frac{c}{p_E(0)}\Big) \mathrm{d}G(p) \leq V(p_E(0)) < V(p_T) = B.$$

Therefore, in this case, the offer m_1' would lead the insider to incur a loss.

If $p_T \notin \mathbf{P}_T$, it is possible that the expected return given in (OA.2.18) exceeds $V(p_T)$. For example, when p_T and λ are close to 1, $V(p_T)$ approaches 0, while the expected return in (OA.2.18) can exceed $V(p_T)$ for an appropriate value of $p_E(0)$. In this case, the offer m_1' allows the insider to earn a positive profit. Consequently, competition among creditors at day 0 will further drive down their offers. However, when $p_T \notin \mathbf{P}_T$, it is also possible that the expected return in (OA.2.18) falls below $V(p_T)$, in which case the offer m_1' would result in a loss for the insider.

Hence, the analysis of the two cases above establishes the statement in Claim 2.

In summary, if the entrepreneur has a borrowing need $B = V(p_T)$, the project can always be financed under the binary signal structure $\sigma(p; p_T)$. Moreover, when $p_T \in \mathbf{P}_T$, $\sigma(p; p_T)$ is a pass-or-fail signal structure, and the agents' behavior follows the profile (i)–(iv). When $p_T \notin \mathbf{P}_T$, $\sigma(p; p_T)$ is either a pass-or-fail signal structure or a binary signal structure under which the outsiders compete to refinance the project after observing both signals.

Finally, by the definition of the set $\mathbf{P}_T$, we have $p_T^* \in \mathbf{P}_T$. Therefore, when the entrepreneur's borrowing need equals $V(p_T^*)$, the project is financed under the pass-or-fail signal structure $\sigma(p; p_T^*)$.

Q.E.D.

Proof of Corollary 1:

By assumption, $V(p_T)$ reaches a unique maximum at $p_T^* \in (\frac{c}{v}, 1)$. The first-order and second-order conditions are

$$V'(p_T^*) = \int_{p_T^*}^{1} \frac{pc}{(p_T^*)^2} \mathrm{d}G(p) - \lambda(p_T^* v - c) g(p_T^*) = 0, \text{ and,} \quad V''(p_T^*) < 0.$$

Totally differentiating the first-order condition yields

$$\frac{\mathrm{d}p_T^*}{\mathrm{d}\lambda} = -\frac{-(p_T^* v - c)g(p_T^*)}{V''(p_T^*)} < 0,$$

and

$$\frac{\mathrm{d}p_T^*}{\mathrm{d}c} = -\frac{\int_{p_T^*}^1 \frac{p}{(p_T^*)^2}\mathrm{d}G(p) + \lambda g(p_T^*)}{V''(p_T^*)} > 0.$$

Thus, the optimal threshold p_T^* decreases in λ, but increases in c.

By the envelope theorem,

$$\frac{\mathrm{d}V(p_T^*)}{\mathrm{d}\lambda} = -\int_{\frac{c}{v}}^{p_T^*}(pv - c)\mathrm{d}G(p) < 0,$$

and

$$\frac{\mathrm{d}V(p_T^*)}{\mathrm{d}c} = -\int_{p_T^*}^1 \frac{p}{p_T^*}\mathrm{d}G(p) - (1-\lambda)\int_{\frac{c}{v}}^{p_T^*} 1\mathrm{d}G(p) < 0.$$

Hence, the maximum borrowing capacity $V(p_T^*)$ decreases in both λ and c.

Finally, consider the limiting cases. As $\lambda \to 0$, if p_T^* were bounded away from 1, then $V'(p_T^*) > 0$, contradicting the first-order condition. Hence,

$$\lim_{\lambda \to 0} p_T^* = 1.$$

Similarly, As $c \to 0$, if p_T^* were bounded away from 0, then $V'(p_T^*) < 0$, which also contradicts the first-order condition. Thus,

$$\lim_{c \to 0} p_T^* = 0.$$

These limits imply

$$\lim_{\lambda \to 0} V(p_T^*) = \mathbb{E}(pv - c | p \geq \frac{c}{v}), \text{ and, } \lim_{c \to 0} V(p_T^*) = v\mathbb{E}(p).$$

So, as λ or c approaches 0, the results converge to the corresponding polar cases in Section 3.1.

Q.E.D.

Proof of Proposition 5:

Given a monotone signal structure σ and a borrowing need B, suppose the project is financed in equilibrium $\mathcal{E}(B;\sigma)$. Following the proof of Lemma 2, in $\mathcal{E}(B;\sigma)$ the entrepreneur's borrowing constraint, which is equivalent to the insider's break-even condition, takes the form

$$B = \sum_{s \in \mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s) \cap [p_E(s), 1]} p m_2^O(s) \mathrm{d}G(p) + \sum_{s \in \mathbb{S}_L(\sigma)} (1-\lambda) \int_{\sigma^{-1}(s) \cap [\frac{c}{v}, 1]} (pv - c)\mathrm{d}G(p). \quad \text{(OA.2.20)}$$

Meanwhile, the entrepreneur's ex-ante payoff, denoted by $\Pi^\sigma(B)$, is given by

$$\Pi^\sigma(B) = \sum_{s \in \mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s) \cap [p_E(s), 1]} [p(v - m_2^O(s)) - c] \mathrm{d}G(p). \tag{OA.2.21}$$

Note that because for any signal $s \in \mathbb{S}_L(\sigma)$ the project is either liquidated at debt rollover or refinanced by the insider who extracts all net surplus, the payoff $\Pi^\sigma(B)$ contains no term corresponding to signals $s \in \mathbb{S}_L(\sigma)$.

Consequently, maximizing the entrepreneur's ex-ante payoff from the project amounts to finding the signal structure σ that maximizes $\Pi^\sigma(B)$ subject to the borrowing constraint in (OA.2.20).

When $\mathbb{S}_H(\sigma)$ is empty, it is immediate that $\Pi^\sigma(B) = 0$. Henceforth, we focus on the case where $\mathbb{S}_H(\sigma)$ is non-empty.

Let $\Pi^\sigma(B) = \Pi^\sigma(B) + B - B$. By substituting the first occurrence of B with the expression on the right-hand side of (OA.2.20), the payoff $\Pi^\sigma(B)$ can be rewritten as

$$\begin{aligned} \Pi^\sigma(B) &= \underbrace{\int_{\frac{c}{v}}^{1} (pv - c) \mathrm{d}G(p) - B}_{\text{net project value}} \\ &\quad - \underbrace{\sum_{s \in \mathbb{S}_L(\sigma)} \lambda \int_{\sigma^{-1}(s) \cap [\frac{c}{v}, 1]} (pv - c) \mathrm{d}G(p)}_{\text{efficiency loss from liquidation}} - \underbrace{\sum_{s \in \mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s) \cap [\frac{c}{v}, p_E(s))} (pv - c) \mathrm{d}G(p)}_{\text{efficiency loss from shirking}}. \end{aligned} \tag{OA.2.22}$$

Intuitively, because creditors are competitive at day 0, they earn zero expected profit. Therefore, under the signal structure σ, the entrepreneur ultimately obtains all net surplus from the project, which is equal to the net project value minus the efficiency losses arising from early liquidation and shirking.

Let $\underline{s}$ and $\underline{p}$ be defined as in Lemma 2. We have shown previously that if $s > \underline{s}$, then $s \in \mathbb{S}_H(\sigma)$ and $p_E(s) < p_E(\underline{s}) \leq \inf(\sigma^{-1}(s))$. The payoff $\Pi^\sigma(B)$ therefore can be further rewritten as

$$\begin{aligned} \Pi^\sigma(B) &= \int_{\frac{c}{v}}^{1} (pv - c) \mathrm{d}G(p) - B \\ &\quad - \lambda \int_{\frac{c}{v}}^{\max\{\underline{p}, \frac{c}{v}\}} (pv - c) \mathrm{d}G(p) - \int_{\max\{\underline{p}, \frac{c}{v}\}}^{\max\{\underline{p}, p_E(\underline{s})\}} (pv - c) \mathrm{d}G(p). \end{aligned} \tag{OA.2.23}$$

We show next that the entrepreneur's ex-ante payoff is maximized under the pass-or-fail signal structure $\sigma(p; p_T(B))$. To see this, note that for a general signal structure σ, the payoff $\Pi^\sigma(B)$ takes the form given in (OA.2.23). We then consider three possible cases that may arise in the equilibrium $\mathcal{E}(B; \sigma)$ induced by σ:

- (a): $\sum_{s\in\mathbb{S}_H(\sigma)\setminus\{\underline{s}\}}\Pr(\sigma^{-1}(s))>0$;
- (b): $\sum_{s\in\mathbb{S}_H(\sigma)\setminus\{\underline{s}\}}\Pr(\sigma^{-1}(s))=0$, but $\underline{p}\neq p_E(\underline{s})$;
- (c): $\sum_{s\in\mathbb{S}_H(\sigma)\setminus\{\underline{s}\}}\Pr(\sigma^{-1}(s))=0$ and $\underline{p}=p_E(\underline{s})$, but $\underline{p}=p_E(\underline{s})\neq p_T(B)$.

Consider cases (a) and (b) first. As shown in (OA.2.12) in the proof of Proposition 3, under the signal structure σ, if cases (a) or (b) appear in equilibrium, then

$$V(\max\{\underline{p},p_E(\underline{s})\})>B=\int_{\max\{\underline{p},p_E(\underline{s})\}}^{1}pm_2^O(\sigma(p))\mathrm{d}G(p)+(1-\lambda)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p).$$

By definition, $p_T(B)$ is the smallest value of p_T that satisfies $V(p_T)=B$. Hence, we have

$$p_T(B)<\max\{\underline{p},p_E(\underline{s})\}.$$

Instead of an arbitrary signal structure σ, suppose now that the project is financed under the pass-or-fail signal structure $\sigma(p;p_T(B))$, which is feasible by Proposition 4. In this case, the entrepreneur's ex-ante payoff, denoted by $\Pi^*(B)$, takes the form

$$\Pi^*(B)=\int_{\frac{c}{v}}^{1}(pv-c)\mathrm{d}G(p)-B-\lambda\int_{\frac{c}{v}}^{p_T(B)}(pv-c)\mathrm{d}G(p),\tag{OA.2.24}$$

which is derived using the same arguments that lead to (OA.2.23).

Subtracting $\Pi^\sigma(B)$ in (OA.2.23) from $\Pi^*(B)$ in (OA.2.24) yields

$$\Pi^*(B)-\Pi^\sigma(B)=\lambda\int_{p_T(B)}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p)+(1-\lambda)\int_{\max\{\underline{p},\frac{c}{v}\}}^{\max\{\underline{p},p_E(\underline{s})\}}(pv-c)\mathrm{d}G(p).$$

Because $p_T(B)<\max\{\underline{p},p_E(\underline{s})\}$, it is immediate that $\Pi^*(B)>\Pi^\sigma(B)$. Therefore, if cases (a) or (b) arise under the signal structure σ, the entrepreneur's ex-ante payoff can be strictly increased by financing the project under the pass-or-fail signal structure $\sigma(p;p_T(B))$ instead of σ.

Now consider case (c). In this case, the signal structure σ is (or is equivalent to) a pass-or-fail signal structure $\sigma(p;p_T)$, where $p_T=\underline{p}=p_E(\underline{s})$, $p_T>p_T(B)$ and $V(p_T)=V(p_T(B))=B$. Substituting $\underline{p}$ and $p_E(\underline{s})$ in (OA.2.23) with p_T, the entrepreneur's ex-ante payoff under σ is

$$\Pi^\sigma(B)=\int_{\frac{c}{v}}^{1}(pv-c)\mathrm{d}G(p)-B-\lambda\int_{\frac{c}{v}}^{p_T}(pv-c)\mathrm{d}G(p).\tag{OA.2.25}$$

Since $p_T>p_T(B)$, it follows that $\Pi^*(B)>\Pi^\sigma(B)$. Therefore, among all pass-or-fail signal structures, the entrepreneur's ex-ante payoff is strictly maximized under the pass-or-fail signal structure $\sigma(p;p_T(B))$.

In conclusion, for any borrowing need $B \leq V(p_T^*)$, the entrepreneur's maximum ex-ante payoff is achieved under the pass-or-fail signal structure $\sigma(p; p_T(B))$, given by $\Pi^*(B)$ in (OA.2.24).

Q.E.D.

Proof of Corollary 2:

For any borrowing need $B \leq V(p_T^*)$, the entrepreneur's ex-ante payoff is maximized under the pass-or-fail signal structure $\sigma(p; p_T(B))$. Thus, in equilibrium, the entrepreneur's borrowing constraint is

$$B = \int_{p_T(B)}^{1} p\Big(v - \frac{c}{p_T(B)}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T(B)} (pv - c)\mathrm{d}G(p),$$

and her maximum ex-ante payoff is

$$\Pi^*(B) = \int_{\frac{c}{v}}^{1} (pv - c)\mathrm{d}G(p) - B - \lambda\int_{\frac{c}{v}}^{p_T(B)} (pv - c)\mathrm{d}G(p).$$

Note that the probability of a liquidity shock λ and the effort cost c affect the entrepreneur's maximum ex-ante payoff directly through the value $\Pi^*(B)$ and indirectly through their impact on the threshold $p_T(B)$.

From the borrowing constraint,

$$\frac{\mathrm{d}p_T(B)}{\mathrm{d}\lambda} = \frac{\int_{\frac{c}{v}}^{p_T(B)} (pv - c)\mathrm{d}G(p)}{\int_{p_T(B)}^{1} \frac{pc}{[p_T(B)]^2}\mathrm{d}G(p) - \lambda[p_T(B)v - c]g(p_T(B))} > 0,$$

because the denominator equals $V'(p_T(B))$ which, by the definition of the value $p_T(B)$, is positive (or zero when $V(p_T)$ reaches a local maximum). Similarly,

$$\frac{\mathrm{d}p_T(B)}{\mathrm{d}c} = \frac{\int_{p_T(B)}^{1} \frac{p}{p_T(B)}\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T(B)} 1\mathrm{d}G(p)}{\int_{p_T(B)}^{1} \frac{pc}{[p_T(B)]^2}\mathrm{d}G(p) - \lambda[p_T(B)v - c]g(p_T(B))} > 0.$$

Therefore, $p_T(B)$ increases in both λ and c.

On the other hand, for the direct effects of λ, c and $p_T(B)$ on $\Pi^*(B)$,

$$\frac{\partial \Pi^*(B)}{\partial \lambda} = -\int_{\frac{c}{v}}^{p_T(B)} (pv - c)\mathrm{d}G(p) < 0,$$

$$\frac{\partial \Pi^*(B)}{\partial c} = \lambda\int_{\frac{c}{v}}^{p_T(B)} 1\mathrm{d}G(p) - \int_{\frac{c}{v}}^{1} 1\mathrm{d}G(p) < 0,$$

and

$$\frac{\partial \Pi^*(B)}{\partial p_T(B)} = -\lambda [p_T(B)v - c] g(p_T(B)) < 0.$$

Hence, the total effects of λ and c on $\Pi^*(B)$ are

$$\frac{\mathrm{d}\Pi^*(B)}{\mathrm{d}\lambda} = \frac{\partial \Pi^*(B)}{\partial p_T(B)} \frac{\mathrm{d}p_T(B)}{\mathrm{d}\lambda} + \frac{\partial \Pi^*(B)}{\partial \lambda} < 0,$$

and

$$\frac{\mathrm{d}\Pi^*(B)}{\mathrm{d}c} = \frac{\partial \Pi^*(B)}{\partial p_T(B)} \frac{\mathrm{d}p_T(B)}{\mathrm{d}c} + \frac{\partial \Pi^*(B)}{\partial c} < 0.$$

Therefore, $\Pi^*(B)$ decreases in both λ and c.

Lastly, by definition $p_T(B)$ increases in B. Then,

$$\frac{\mathrm{d}\Pi^*(B)}{\mathrm{d}B} = -1 + \frac{\partial \Pi^*(B)}{\partial p_T(B)} \frac{\mathrm{d}p_T(B)}{\mathrm{d}B} < 0.$$

Hence, $\Pi^*(B)$ decreases in B.

Q.E.D.

OA.3 Extensions and Discussions

In this section, we relax several assumptions of the baseline model. Specifically, Section OA.3.1 examines financing with long-term loans, Section OA.3.2 analyzes debt renegotiation after rollover failures, and Section OA.3.3 allows for publicly observable creditor liquidity shocks. We characterize the optimal signal structures in these settings and compare them with those derived in the preceding sections. Section OA.3.4 provides some further discussions.

OA.3.1 Financing with Long-term Loans

Instead of restricting loans to be short-term, we now consider that creditors issue long-term loans at day 0 to finance the entrepreneur's project, maturing at day 2.[5] Thus, the entrepreneur faces no liquidation risk at day 1. Following studies such as Parlour and Plantin (2008), Dang, Gorton, Holmström, and Ordonez (2017), Monnet and Quintin (2017) and Goldstein and Leitner (2018), we assume a secondary market exists at day 1 where creditors can trade assets. In particular, let $m_{lt} \leq v$ denote the startup loan offer with which a creditor (who becomes the insider) wins to finance the project at day 0, where the subscript "lt" stands for "long-term". At day 1, based on the public signal s, the other creditors (the outsiders) bid the highest price $r_{lt}(s)$ to purchase the insider's loan claim. Given $r_{lt}(s)$, the insider decides whether to sell. The loan claim holder then receives repayment m_{lt} at day 2 if the project produces cash flow v. The information environment remains the same as in the baseline model.

Let $V^{lt}(p_{lt})$ be a function given by

$$V^{lt}(p_{lt}) = \int_{p_{lt}}^{1} p\Big(v - \frac{c}{p_{lt}}\Big)\mathrm{d}G(p), \quad \forall p_{lt} \in \Big[\frac{c}{v}, 1\Big].$$

Then, the function $V^{lt}(p_{lt})$ is non-monotonic and reaches its maximum at a value $p^*_{lt} \in (\frac{c}{v}, 1)$. We assume p^*_{lt} is unique.

Proposition OA.1 *When the project is financed with long-term loans, the entrepreneur's borrowing capacity is the same under all monotone signal structures and equals $V^{lt}(p^*_{lt})$. Moreover, $V^{lt}(p^*_{lt}) < V(p^*_T)$.*

Proofs of the results in this section are provided in Section OA.4. We focus on the intuition here. When the project is funded with a startup loan offer $m_{lt} \leq v - c$, the entrepreneur exerts

[5] Owing to the binary 0–v structure of cash flows, long-term debt is equivalent to equity in our model. As is standard in corporate finance theory, a binary cash flow structure with a positive low realization can distinguish long-term debt from equity. However, incorporating such a model would complicate the exposition without altering the qualitative results. We provide a detailed discussion of this in Section OA.5.4.

effort at day 2 if and only if $p \geq \frac{c}{v-m_{lt}}$, regardless of whether the insider sells the loan claim at day 1. In other words, with long-term loans, the secondary market trading of the loan claim does not affect the realization of the project's cash flow.

In the secondary market, information asymmetry about the project quality affects the insider's expected returns at day 1 in two opposing ways. On the one hand, it allows the insider to sell the loan claim even when the project quality is low. On the other hand, it depresses the outsiders' price offers, reducing the insider's return when a liquidity shock occurs. Since the project and the loan values depend only on the startup loan offer m_{lt}, for any degree of information asymmetry, these two effects offset each other in equilibrium. Consequently, information asymmetry in the secondary market does not affect the insider's ex-ante return from issuing the startup loan. As shown in the proof, given a startup loan offer $m_{lt} \leq v - c$, the insider's ex-ante return is $\int_{\frac{c}{v-m_{lt}}}^{1} pm_{lt}\mathrm{d}G(p)$, regardless of the signal structure.

Maximizing cash flow pledgeability or the entrepreneur's borrowing capacity thus reduces to finding a startup loan offer m_{lt}^* that optimally balances the entrepreneur's incentive to exert effort and the creditors' claim on the project's cash flow. We find that $m_{lt}^* = v - \frac{c}{p_{lt}^*}$, and the entrepreneur's maximum borrowing capacity is $V^{lt}(p_{lt}^*)$, which can be achieved under any monotone signal structure.

The second part of Proposition OA.1 follows from the inequalities

$$V(p_T^*) \geq V(p_{lt}^*) > V^{lt}(p_{lt}^*).$$

It illustrates that the entrepreneur's maximum borrowing capacity under long-term loan financing is lower than that under short-term loan financing. The intuition is that, relative to long-term loans, short-term financing has two opposing effects on the project value: it exposes the project to the risk of early liquidation, yet allows loan terms to be adjusted at rollover to better incentivize the entrepreneur to exert effort. When the signal structure is properly designed, the latter effect dominates the former, leading to a higher borrowing capacity.

This finding, by linking information design to the choice of debt maturity, adds our study to the literature on optimal debt maturity structures, e.g., Flannery (1986), Diamond (1991), Rajan (1992), and Wei and Zhou (2021). It also links our study to the accounting literature examining the interaction between disclosure quality and corporate debt maturity (García-Teruel, Martínez-Solano, and Sánchez-Ballesta 2010; Allaya, Derouiche, and Muessig 2022). While the prevailing view in this literature is that high-quality disclosure serves as a substitute for short-term monitoring—thereby enabling firms to lengthen their debt maturity—we provide a novel rationale for the opposite relationship. We demonstrate that for innovative firms facing moral hazard, disclosure is a strategic mechanism to facilitate short-term financing. Unlike the granular transparency often advocated for real efficiency (Kanodia and Sapra 2016), we show that coarse disclosure at the rollover stage is what makes short-term debt a more efficient funding vehicle than long-term debt.

Define $p_{lt}(B)$ as the smallest value of p_{lt} that satisfies $B = V^{lt}(p_{lt})$. Denote by $\Pi^{lt}(B)$ the entrepreneur's maximum ex-ante payoff when her borrowing need is $B \leq V^{lt}(p_{lt}^*)$ and the project is financed with long-term loans.

Corollary OA.1 *When the project is financed with long-term loans, for any borrowing need $B \leq V^{lt}(p_{lt}^*)$ and any monotone signal structure, the entrepreneur's maximum ex-ante payoff is*

$$\Pi^{lt}(B) = \int_{p_{lt}(B)}^{1} (pv - c)\,\mathrm{d}G(p) - B. \tag{OA.3.1}$$

Moreover, $\Pi^{lt}(B) < \Pi^*(B)$.

For any borrowing need $B \leq V^{lt}(p_{lt}^*)$, competition among creditors at day 0 drives them to make the lowest startup loan offer consistent with their participation condition. When the project is financed with long-term loans, for the reason discussed above, this lowest loan offer is identical across all monotone signal structures. Therefore, the entrepreneur's ex-ante payoff is also the same across all monotone signal structures, which is $\Pi^{lt}(B)$.

The result $\Pi^{lt}(B) < \Pi^*(B)$ indicates that, relative to long-term loans, the entrepreneur's maximum ex-ante payoff is higher when the project is financed with short-term loans. This is also because, under an optimally designed signal structure, the benefit of short-term loans in mitigating the entrepreneur's moral hazard outweighs the associated risk of early liquidation. As a result, for any borrowing need $B \leq V^{lt}(p_{lt}^*)$, the entrepreneur's ex-ante payoff is maximized when the project is financed with short-term loans and under the pass-or-fail signal structure $\sigma(p; p_T(B))$.

OA.3.2 Renegotiation

Debt rollover failures often lead to premature liquidation of projects, resulting in efficiency losses. A potential remedy is to restructure the initial loan contracts through renegotiation, inducing creditors to partially write down their claims or, equivalently, to "take a haircut." This section extends the baseline model by introducing a renegotiation stage following the debt rollover stage. Allowing renegotiation after rollover failure mitigates liquidation risk and thus increases both the entrepreneur's maximum borrowing capacity and maximum ex-ante payoff, which are now achieved under variants of pass-or-fail signal structures.

More specifically, based on the short-term loan financing structure, we now assume that if no creditor offers to refinance the project at the debt rollover stage, then with probability $\kappa \in [0,1]$ the game proceeds to a renegotiation stage, while with probability $1-\kappa$ the project is liquidated as in the baseline model.[6] At the renegotiation stage, given the previously observed signal s, each outsider j competes by proposing an offer $(m_{1j}^R(s), m_{2j}^R(s))$, where $m_{1j}^R(s)$ is the payment to the insider, $m_{2j}^R(s)$ is the amount claimed from the project's final cash flow, and the superscript "R" stands for "renegotiation."[7] Importantly, $m_{1j}^R(s)$ can be strictly lower than the insider's initial loan claim m_1, reflecting a renegotiation of the startup loan. For simplicity, we assume that the offer $(m_1^R(s), m_2^R(s))$ with $m_1^R(s) \equiv \max\{m_{1j}^R(s)\}$ is accepted at this stage, ensuring the insider receives the highest possible compensation.[8]

Let $V^R(p_R)$ be a function given by

$$V^R(p_R) = \int_{p_R}^{1} p(v - \frac{c}{p_R})\mathrm{d}G(p) + (1-\lambda+\kappa\lambda)\int_{\frac{c}{v}}^{p_R}(pv-c)\mathrm{d}G(p), \quad \forall p_R \in \left[\frac{c}{v}, 1\right].$$

Then, the function $V^R(p_R)$ is non-monotonic and reaches its maximum at a value $p_R^* \in (\frac{c}{v}, 1]$. We assume that p_R^* is unique.

We introduce a class of signal structures, termed *upper-censorship signal structures*. Denoted by $\sigma^{uc}(p; p_R)$, where the superscript "uc" refers to "upper-censorship", such a signal structure pools the project qualities above the threshold p_R while revealing those below it (see, e.g., Hu and Zhou (2022) and Kolotilin, Mylovanov, and Zapechelnyuk (2022)). Our main result for the model with renegotiation is as follows.

Proposition OA.2 *In the model with renegotiation, among all monotone signal structures, the entrepreneur's maximum borrowing capacity is achieved under the upper-censorship signal structure $\sigma^{uc}(p; p_R^*)$ and equals $V^R(p_R^*)$. Moreover, $V^R(p_R^*) \geq V(p_T^*)$.*

Proposition OA.2 is consistent with and further extends our baseline analysis. In the presence of debt renegotiation, the fundamental trade-off among moral hazard, liquidation risk,

[6]Alternatively, we may interpret that renegotiation is always viable, but it can break down with probability $1-\kappa$ for reasons outside the model.

[7]In equilibrium, the insider will never deliberately delay refinancing until the renegotiation stage—otherwise, he could have refinanced the project earlier to capture the entire refinancing surplus. Hence, for ease of exposition, we describe only the outsiders' offers at this stage.

[8]For notational simplicity, we assume that if $m_1^R(s)$ exceeds the startup loan claim m_1—though this situation never arises in any equilibrium that maximizes the entrepreneur's ex-ante payoff (see the discussion at the end of this section), the insider captures the difference. Moreover, we assume that if the creditors make refinancing offers at debt rollover, the entrepreneur cannot reject all of them; otherwise, he could strategically trigger renegotiation to benefit from a lower final payment.

and rent dissipation continues to induce outsider participation in the debt rollover stage only upon observing a pass signal. However, now fully revealing project qualities below the pass signal threshold eliminates information asymmetry between the entrepreneur and outsiders at the renegotiation stage, thus maximizing the outsiders' willingness to refinance. Consequently, under renegotiation, an upper-censorship signal structure $\sigma^{uc}(p; p_R^*)$, with the optimal pass signal threshold p_R^*, maximizes the entrepreneur's borrowing capacity. Importantly, compared to the baseline model, the possibility of renegotiation reduces the liquidation risk and thus improves maximum borrowing capacity, i.e., $V^R(p_R^*) \geq V(p_T^*)$.

In the model with renegotiation, the upper-censorship signal structure $\sigma^{uc}(p; p_R^*)$ in Proposition OA.2 is essentially a variant of the pass-or-fail signal structure $\sigma(p; p_R^*)$. Intuitively, under $\sigma^{uc}(p; p_R^*)$, signals that fully reveal project qualities below the threshold p_R^* still function as fail signals, discouraging outsider participation in debt rollover and affecting only the renegotiation stage. Moreover, this signal structure can be implemented through a two-step disclosure process: first, applying the pass-or-fail signal structure $\sigma(p; p_R^*)$ at the debt rollover stage, and second, if renegotiation occurs, fully revealing project qualities.[9]

It is straightforward that a higher probability of renegotiation raises both the pass signal threshold and the entrepreneur's maximum borrowing capacity; that is, both p_R^* and $V^R(p_R^*)$ increase in κ. Furthermore, when $\kappa = 0$, we have $p_R^* = p_T^*$ and $V^R(p_R^*) = V(p_T^*)$, so the model reduces to the baseline case. In contrast, when $\kappa = 1$, we have $p_R^* = 1$ and $V^R(p_R^*) = \mathbb{E}\left(pv - c | p \geq \frac{c}{v}\right)$, indicating the full pledgeability of the net project value.

Let $p_R(B)$ denote the smallest threshold p_R that satisfies $B = V^R(p_R)$, and let $\Pi^R(B)$ indicate the entrepreneur's maximum ex-ante payoff in the model with renegotiation.

Corollary OA.2 *Consider the model with renegotiation. For any borrowing need $B \leq V^R(p_R^*)$, among all monotone signal structures, the upper-censorship signal structure $\sigma^{uc}(p; p_R(B))$ maximizes the entrepreneur's ex-ante payoff and hence social welfare, which is*

$$\Pi^R(B) = \int_{\frac{c}{v}}^{1} (pv - c)\mathrm{d}G(p) - B - \lambda(1-\kappa) \int_{\frac{c}{v}}^{p_R(B)} (pv - c)\mathrm{d}G(p).$$

Moreover, $\Pi^R(B) \geq \Pi^*(B)$.

The interpretation of Corollary OA.2 parallels that of Proposition 5. With competition among creditors, the entrepreneur captures the net project value after accounting for efficiency

[9] In practice, renegotiation often coincides with project or firm restructuring, typically involving a shift of control from the entrepreneur to initial creditors. If full disclosure is mandatory at this stage, then the pass-or-fail signal structure $\sigma(p; p_R^*)$ alone suffices to maximize the entrepreneur's borrowing capacity.

losses from moral hazard and liquidation risk. For any borrowing need $B \leq V^R(p_R^*)$, the upper-censorship signal structure $\sigma^{uc}(p; p_R(B))$ optimally balances these forces, minimizing efficiency losses and thereby maximizing the entrepreneur's ex-ante payoff. Furthermore, the possibility of renegotiation further mitigates the inefficiency caused by liquidity shocks, leading to a higher maximum ex-ante payoff for the entrepreneur, i.e., $\Pi^R(B) \geq \Pi^*(B)$.

Lastly, for any borrowing need $B \leq V^R(p_R^*)$, we show in the proof that, in the equilibrium achieving the entrepreneur's maximum ex-ante payoff, the payment that the insider receives at the renegotiation stage is strictly less than his startup loan claim. Hence, the insider indeed "takes a haircut" under renegotiation.

OA.3.3 Publicly Observed Liquidity Shocks

We revisit the baseline model under short-term loan financing, maintaining all previous assumptions except for the observability of the liquidity shock. We now assume that whether the insider experiences a liquidity shock is publicly observable. Consequently, at $t = 1$, outsiders know with certainty whether they will face competition from the insider during the debt rollover process.

Let $V^{os}(p_{os})$ be a function given by

$$V^{os}(p_{os}) = \lambda \int_{p_{os}}^{1} p\Big(v - \frac{c}{p_{os}}\Big)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{1}(pv - c)\mathrm{d}G(p), \quad \forall p_{os} \in \Big[\frac{c}{v}, 1\Big].$$

We refer the subscript "os" to "observed shock". Then, the function $V^{os}(p_{os})$ is non-monotonic and reaches its maximum at a value $p_{os}^* \in (\frac{c}{v}, 1)$. We also assume p_{os}^* to be unique.

Proposition OA.3 *If liquidity shocks are publicly observed, then among all monotone signal structures, there exists a class of binary signal structures $\sigma(p; \tilde{p})$, with $\tilde{p} \in [0, p_{os}^*]$, which enables the entrepreneur to attain the maximum borrowing capacity given by $V^{os}(p_{os}^*)$. Moreover, $V^{os}(p_{os}^*) > V(p_T^*)$.*

The binary signal structures $\sigma(p; \tilde{p})$ characterized in Proposition OA.3 share a key feature: the signal $s = 1$ revealed under each of them is sufficiently coarse regarding the project quality, as indicated by a small threshold $\tilde{p} \leq p_{os}^*$. When liquidity shocks are publicly observed, this coarseness serves two purposes. First, if the insider faces a liquidity shock, it increases the likelihood of outsider participation in debt rollover and mitigates the risk of early liquidation. Second, if the insider does not face a liquidity shock, it discourages outsider participation in debt rollover, allowing the insider to capture the entire refinancing surplus. Consequently, such signal structures maximize the insider's expected return from offering the startup loan and generate the maximum borrowing capacity $V^{os}(p^*_{os})$ for the entrepreneur.[10]

Moreover, the optimal coarseness of the signal structures $\sigma(p; \tilde{p})$ with $\tilde{p} \in [0, p^*_{os}]$ yields that

$$V^{os}(p^*_{os}) - V(p^*_T) \geq V^{os}(p^*_T) - V(p^*_T) = (1-\lambda) \int_{p^*_T}^{1} \Big(\frac{p}{p^*_T} - 1\Big) c \mathrm{d}G(p) > 0.$$

Therefore, the entrepreneur's maximum borrowing capacity is strictly higher when liquidity shocks are publicly observed than when they are unobserved.

OA.3.4 Discussions

In this section, we discuss some other potential extensions of the model.

Monopolistic initial creditor. In our baseline model, creditors are competitive in the startup loan market. It is also instructive to consider the opposite scenario, in which an initial creditor possesses monopoly power in this market and, consequently, can design both the signal structure and a take-it-or-leave-it startup loan contract to the entrepreneur.[11]

Our analysis in Section 3 demonstrates that the entrepreneur's maximum borrowing capacity is equal to the highest expected return that the insider can obtain from issuing the startup loan, irrespective of who designs the signal structure. Therefore, if the initial creditor is monopolistic, then for any borrowing need $B \leq V(p^*_T)$ of the entrepreneur, he will always implement the borrowing-capacity-maximizing signal structure $\sigma(p; p^*_T)$—which is independent of B—and offer the corresponding startup loan contract. In doing so, the initial creditor earns an expected return $V(p^*_T)$ and hence an expected profit $V(p^*_T) - B$.

Comparing with Proposition 5 in the baseline model, the result above indicates that the pass-or-fail signal structure maximizing the initial creditor's payoff requires a higher signal threshold than the one maximizing the entrepreneur's payoff. As a result, when the initial creditor is monopolistic, the project faces a greater risk of debt rollover failure and incurs higher efficiency losses from premature liquidation, leading to lower overall social welfare.[12]

[10] As in the baseline model, Proposition OA.3 also indicates that if multiple signals induce outsider participation in debt rollover, pooling them into a single signal and adopting a binary structure can improve the entrepreneur's borrowing capacity.

[11] We maintain the assumption that outsiders are competitive in the refinancing market.

[12] By slightly modifying the analysis in Section 4, we can show that when the initial creditor's payoff is maximized, overall social welfare equals

$$\int_{\frac{c}{v}}^{1} (pv - c) \mathrm{d}G(p) - B - \lambda \int_{\frac{c}{v}}^{p^*_T} (pv - c) \mathrm{d}G(p),$$

Allocation of bargaining power. In the baseline model, when the insider is the only creditor offering to roll over the loan, he makes a take-it-or-leave-it offer to the entrepreneur. A natural extension is to introduce an allocation of bargaining power to the entrepreneur-insider bilateral monopoly situation and examine how this allocation affects the outcomes.

Specifically, suppose that in the bilateral monopoly, the insider and the entrepreneur can each make a take-it-or-leave-it offer with probabilities β and $1-\beta$, respectively.[13] The analysis of this extension closely parallels that in Sections 3 and 4, except that it now accounts for the insider's return being zero when the entrepreneur makes the offer in the bilateral monopoly.

The qualitative results of the baseline model remain unchanged. From a comparative statics perspective, as the insider's bargaining power in the bilateral monopoly increases (i.e., as β rises), the insider's expected return at debt rollover improves, which in turn strengthens his incentive to issue the startup loan. Consequently, the entrepreneur's maximum borrowing capacity increases, and—provided her borrowing need can be satisfied—her maximum ex-ante payoff also rises. An extended discussion is provided in Section OA.5.1.

Long-term loan financing with strong creditor control. Related to the discussion in Section OA.3.1, another interesting question is whether giving initial creditors stronger control can enhance the effectiveness of long-term loan financing in shaping the entrepreneur's borrowing capacity.

In particular, consider a long-term loan contract in which, at the interim stage, the initial creditor holds the control right to solicit a refinancing offer from outside creditors or from himself to replace his existing claim. A refinancing offer from creditor j is specified by a payment pair $(\hat{m}_{1j}(s), \hat{m}_{2j}(s))$, where $\hat{m}_{1j}(s)$ is the payment to the initial creditor and $\hat{m}_{2j}(s)$ is his claim on the project's final cash flow. The initial creditor selects the offer $(\hat{m}_1(s), \hat{m}_2(s))$ that yields the highest interim payment, with $\hat{m}_1(s) \equiv \max\{\hat{m}_{1j}(s)\}$, and then decides to continue or liquidate the project. In practice, such arrangements can be implemented through restrictive covenants that the initial creditor may selectively waive to permit project continuation.

The entrepreneur's maximum borrowing capacity under this long-term loan contract mirrors the renegotiation outcome with $\kappa = 1$ in Section OA.3.2. Under a full-revealing signal structure, the outsiders compete to make the offer $(pv - c, v - \frac{c}{p})$ for any $p \geq \frac{c}{v}$. Thus, whether or not the insider makes an offer himself, his interim expected return is $\mathbb{E}\left(pv - c | p \geq \frac{c}{v}\right)$, which equals the project's full net value. The entrepreneur's maximum borrowing capacity is there-

which is less than $\Pi^*(B)$ in (10) because $p_T(B) < p_T^*$ for any $B < V(p_T^*)$.

[13]This is equivalent to assume that, in the baseline model, after the insider makes the only offer at debt rollover, with probability $1-\beta$ the entrepreneur can make a counteroffer and threaten to liquidate the project if it is rejected.

fore the same. Intuitively, when the insider has the control right to solicit refinancing offers and determine project continuation, full disclosure maximizes outsiders' willingness to pay, allowing the insider to extract the entire surplus and thereby maximizing borrowing capacity.[14]

Reversing the timing of offer making at debt rollover. In practice, the sequence of offer making during the debt rollover stage may vary across contexts. For instance, unlike in our baseline model, the entrepreneur may first seek refinancing from the inside creditor and, only if no agreement is reached, turn to outside creditors for refinancing offers. In this case, beyond the information disclosed through the signal structure, the entrepreneur's attempt to obtain external funding may further signal the project's quality to outside creditors.

A specific *insider-rollover-first* model we consider assumes that whether the insider has made a refinancing offer to the entrepreneur is unobservable to the outsiders.[15] As a result, when approached, the outsiders cannot fully distinguish whether the insider is unwilling or simply unable to refinance the project due to a liquidity shock. In this model, the entrepreneur's maximum borrowing capacity is achieved under the pass-or-fail signal structure $\sigma(p; p_T^*)$ and equals $V(p_T^*)$, identical to the result in Proposition 3. Likewise, the entrepreneur's maximum ex-ante payoff is achieved under the pass-or-fail signal structure $\sigma(p; p_T(B))$ and equals $\Pi^*(B)$, consistent with Proposition 5.

Intuitively, although the timing of offer making in this model differs from that in the baseline model, the information environments that the outsiders face in two models are essentially identical. Consequently, the tension among liquidation risk, moral hazard, and rent dissipation leads to the same outcomes. A detailed discussion on this model is provided in Section OA.5.2.

Uncertainty about final cash flow. In the baseline model, we assume that the outsiders' uncertainty pertains to the probability that the project yields the positive cash flow v. An alternative setting of interest is one in which the outsiders are instead uncertain directly about the realization of the final cash flow.

To this end, we consider a *quality-of-cash-flow* model in which the project generates a cash flow $v \in [\underline{v}, \overline{v}]$ drawn from a continuous distribution $G(v)$ if the entrepreneur exerts effort, and zero if she shirks. Thus, in this context, project quality is captured by v.

[14] Under this long-term loan contract, the signal structure still matters. A proof parallel to that of Proposition OA.2 shows that a coarse signal structure, which pools project qualities over certain ranges, distorts outsiders' refinancing offers, causes moral hazard induced efficiency loss, and reduces the entrepreneur's borrowing capacity.

[15] In this model, we allow the insider to make any refinancing offer, rather than restricting him to either matching the outsiders' offer or exiting.

In this model, maximizing the entrepreneur's borrowing capacity induces outsider participation in debt rollover only if the project quality v exceeds an optimal threshold v_T^*, consistent with the baseline model. Notably, any monotone signal structure that separates project qualities above and below v_T^*, including the binary signal structure $\sigma(v; v_T^*)$, can achieve this maximum capacity. This is because, given the cash flow structure, the outsiders' refinancing loan is always safe in any equilibrium that maximizes borrowing capacity, rendering them insensitive to additional information disclosure once the project quality is known to exceed v_T^*. A similar result holds when maximizing the entrepreneur's ex-ante payoff. We elaborate more on this in Section OA.5.3.

Imperfect insider information. An interesting extension considers the case in which the insider's information about project quality is informative but imperfect, reflecting limited access to the firm's internal records. For instance, at debt rollover the insider's private information can be captured by a partition $\{z_n\}_{n=1}^N$ of $[0, 1]$, so that he only knows the project quality p lies in some set z_n. After the public signal s is realized, the insider further refines his information accordingly.

We conjecture that, provided the partition $\{z_n\}_{n=1}^N$ is sufficiently fine for the insider to retain an informational advantage at debt rollover, an optimal upper-censorship signal structure, as described in Section OA.3.2, maximizes the entrepreneur's borrowing capacity. The intuition is twofold. For project qualities that induce outsider participation, pooling them under a single signal softens creditor competition, mitigates rent dissipation, and improves borrowing capacity. In contrast, for project qualities that deter outsider participation, full revelation allows the insider to better tailor refinancing terms to the entrepreneur's effort incentives, thereby also enhancing borrowing capacity. By similar reasoning, a borrowing-need–dependent upper-censorship signal structure maximizes the entrepreneur's ex-ante payoff and social welfare.

Thus, our qualitative results does not rely on the insider having perfect information at debt rollover. However, due to the complexity of possible information partitions for the insider, fully characterizing the optimal signal thresholds and contracts can be technically involved.

OA.4 Proofs of Propositions and Corollaries in the Extensions

In this appendix, we present proofs of propositions and corollaries in Section OA.3.

Proof of Proposition OA.1:

Consider a monotone signal structure σ, and suppose that under σ there exists a signal s such that $\sigma^{-1}(s) = [p_0, p_1]$. In other words, σ pools all project qualities within the interval $[p_0, p_1]$ into a single signal s.[16]

Assume that the project is financed by a startup loan offer m_{lt}, where, without loss of generality, $m_{lt} \leq v - c$. Given this, all creditors in the secondary market know that, at day 2, the entrepreneur will exert effort if and only if $p \geq p_E(m_{lt}) = \frac{c}{v - m_{lt}}$. Hence, $p_E(m_{lt})$ represents the entrepreneur's effort threshold. We now proceed to analyze the following cases.

Case (a): $p_1 < p_E(m_{lt})$. In this case, since the entrepreneur will shirk for any $p \in [p_0, p_1]$, the loan claim has zero value to any creditor. Hence, conditional on observing signal s in the secondary market, the outsiders offer a price of $r_{lt}(s) = 0$. Consequently, regardless of whether the insider sells or holds the claim, his expected return (before learning whether he faces a liquidity shock) over the set $[p_0, p_1]$ is 0.

Now consider the full-revealing signal structure σ_F, under which project qualities in $[p_0, p_1]$ are perfectly disclosed to the outsiders. In this case, conditional on observing any $p \in [p_0, p_1]$, the outsiders again offer $r_{lt}(p) = 0$, and the insider's expected return over this set remains 0.

Therefore, when $p_1 < p_E(m_{lt})$, the insider's expected return over $[p_0, p_1]$ is independent of whether project qualities within this set are pooled or fully revealed.

Case (b): $p_0 < p_E(m_{lt}) \leq p_1$. In this case, suppose that upon observing signal s the outsiders offer a price $r_{lt}(s)$, where equilibrium requires $r_{lt}(s) \leq p_1 m_{lt}$. Then the insider who does not experience a liquidity shock will sell the loan claim if he observes $p \leq \max\{p_E(m_{lt}), \frac{r_{lt}(s)}{m_{lt}}\}$ and will otherwise hold it. By contrast, an insider facing a liquidity shock will sell the loan claim with certainty. Then, price $r_{lt}(s)$ is determined by the outsiders' break-even condition

$$r_{lt}(s) = \frac{\lambda \int_{p_E(m_{lt})}^{p_1} p m_{lt} \mathrm{d}G(p) + (1-\lambda) \int_{p_E(m_{lt})}^{\max\{p_E(m_{lt}), \frac{r_{lt}(s)}{m_{lt}}\}} p m_{lt} \mathrm{d}G(p)}{\lambda[G(p_1) - G(p_0)] + (1-\lambda)[G(\max\{p_E(m_{lt}), \frac{r_{lt}(s)}{m_{lt}}\}) - G(p_0)]}, \tag{OA.4.1}$$

where the term on the right-hand side is an outsider's expected return from buying the loan claim at price $r_{lt}(s)$.

[16] If the set takes the form $[p_0, p_1)$, $(p_0, p_1]$, or (p_0, p_1), the proof proceeds analogously.

The insider's total expected return over the set $[p_0, p_1]$ is

$$\lambda\Big(G(p_1) - G(p_0)\Big) r_{lt}(s) + (1-\lambda)\Big(G(\max\{p_E(m_{lt}), \frac{r_{lt}(s)}{m_{lt}}\}) - G(p_0)\Big) r_{lt}(s) + \\ (1-\lambda)\int_{\max\{p_E(m_{lt}), \frac{r_{lt}(s)}{m_{lt}}\}}^{p_1} p m_{lt} \mathrm{d}G(p). \tag{OA.4.2}$$

By substituting $r_{lt}(s)$ in (OA.4.2) with the expression on the right-hand side of (OA.4.1), the insider's expected return over the set $[p_0, p_1]$ can be written as

$$\int_{\frac{c}{v-m_{lt}}}^{p_1} p m_{lt} \mathrm{d}G(p), \text{ or equivalently, } \int_{p_E(m_{lt})}^{p_1} p\Big(v - \frac{c}{p_E(m_{lt})}\Big)\mathrm{d}G(p).$$

Again, consider the full-revealing signal structure σ_F. It is straightforward to see that, under σ_F, the outsiders will offer $r_{lt}(p) = pm_{lt}$ for $p \geq p_E(m_{lt})$ and offer $r_{lt}(p) = 0$ for $p < p_E(m_{lt})$. When the insider does not encounter a liquidity shock, he is indifferent between selling and holding the loan claim. Accordingly, the insider's expected return over the set $[p_0, p_1]$ is also given by

$$\int_{\frac{c}{v-m_{lt}}}^{p_1} p m_{lt} \mathrm{d}G(p), \text{ or equivalently, } \int_{p_E(m_{lt})}^{p_1} p\Big(v - \frac{c}{p_E(m_{lt})}\Big)\mathrm{d}G(p).$$

Therefore, when $p_0 < p_E(m_{lt}) \leq p_1$, the insider's expected return over $[p_0, p_1]$ is independent of whether project qualities within this set are pooled or fully revealed.

Case (c): $p_E(m_{lt}) \leq p_0$. In this case, by modifying the arguments in case (b) slightly, we can show that the insider's expected return over the set $[p_0, p_1]$ is

$$\int_{p_0}^{p_1} p\Big(v - \frac{c}{p_E(m_{lt})}\Big)\mathrm{d}G(p),$$

no matter whether project qualities in this set are pooled together or revealed fully.

Summing the insider's expected return over the full set of project qualities $[0, 1]$, we find that the insider's ex-ante return from financing the project with a startup loan m_{lt} is

$$\int_{\frac{c}{v-m_{lt}}}^{1} p m_{lt} \mathrm{d}G(p), \text{ or equivalently, } \int_{p_E(m_{lt})}^{1} p\Big(v - \frac{c}{p_E(m_{lt})}\Big)\mathrm{d}G(p), \tag{OA.4.3}$$

and this holds regardless of the signal structure employed.

Therefore, when the project is financed with long-term loans, all monotone signal structures are equivalent in terms of maximizing the entrepreneur's borrowing capacity. To determine this maximum capacity, it suffices to find the optimal startup loan m^*_{lt} that maximizes the value in (OA.4.3). It follows directly that $m_{lt}^* = v - \frac{c}{p_{lt}^*}$, and the entrepreneur's maximum borrowing capacity is

$$V^{lt}(p_{lt}^*) = \int_{p_{lt}^*}^{1} p\Big(v - \frac{c}{p_{lt}^*}\Big)\mathrm{d}G(p).$$

Lastly, we have

$$V(p_T^*) \geq V(p_{lt}^*) > V^{lt}(p_{lt}^*).$$

Q.E.D.

Proof of Corollary OA.1:

Suppose the entrepreneur has a borrowing need $B \leq V^{lt}(p_{lt}^*)$ and consider any monotone signal structure σ. Together with Proposition OA.1, we know that if the project is financed with a startup loan offer m_{lt}, the insider's break-even condition at day 0 should take the form

$$B = \int_{p_E(m_{lt})}^{1} p\Big(v - \frac{c}{p_E(m_{lt})}\Big)\mathrm{d}G(p), \tag{OA.4.4}$$

where $p_E(m_{lt}) = \frac{c}{v - m_{lt}}$ is the entrepreneur's effort threshold at day 2, which strictly increases with m_{lt}.

Competition at day 0 drives the loan offer m_{lt}, and equivalently the effort threshold $p_E(m_{lt})$, down to the lowest values consistent with this break-even condition. Recall that $p_{lt}(B)$ is the smallest value of p_{lt} that satisfies $B = V^{lt}(p_{lt})$. Then, in equilibrium, the insider's startup loan offer and the entrepreneur's effort threshold are

$$m_{lt}(B) = v - \frac{c}{p_{lt}(B)}, \text{ and } p_E(m_{lt}(B)) = p_{lt}(B).$$

Consequently, for any borrowing need $B \leq V^{lt}(p_{lt}^*)$ and any monotone signal structure σ, the entrepreneur's ex-ante payoff, denoted by $\Pi^{lt}(B)$, is given by

$$\Pi^{lt}(B) = \int_{p_E(m_{lt}(B))}^{1} [p(v - m_{lt}(B)) - c]\mathrm{d}G(p) = \int_{p_{lt}(B)}^{1} \left(\frac{p}{p_{lt}(B)} - 1\right) c\mathrm{d}G(p).$$

Employing $\Pi^{lt}(B) = \Pi^{lt}(B) + B - B$ and substituting the first term of B with the insider's break-even condition, we have

$$\Pi^{lt}(B) = \int_{p_{lt}(B)}^{1} (pv - c)\,\mathrm{d}G(p) - B.$$

Finally, note that $V(\hat{p}) > V^{lt}(\hat{p})$ for any threshold $\hat{p} \in (\frac{c}{v}, 1)$. Therefore, for $V(p_T(B)) = V^{lt}(p_{lt}(B)) = B$, we have $p_T(B) < p_{lt}(B)$. Then, it is straightforward that $\Pi^{lt}(B) < \Pi^*(B)$.

Q.E.D.

Proof of Proposition OA.2:

We begin by showing that if, in an equilibrium played under a signal structure, project qualities are not fully revealed at the renegotiation stage, then this equilibrium cannot achieve the entrepreneur's maximum borrowing capacity.

Consider a monotone signal structure σ, and suppose that there is a signal s such that $\sigma^{-1}(s) = [p_0, p_1]$, where $p_0 \geq \frac{c}{v}$.[17] Furthermore, assume that in the equilibrium achieving the entrepreneur's borrowing capacity under σ, denoted as $\mathcal{E}(\sigma)$ here, at the debt rollover stage no outsider competes to refinance the project upon observing signal s.

First note that, when facing no liquidity shock, the insider can capture the net project value, $\int_{p_0}^{p_1}(pv - c)\mathrm{d}G(p)$, from refinancing the project at the rollover stage, as shown previously. If instead he postpones refinancing and waits until renegotiation, his expected return cannot exceed this value, due to the outsiders' participation constraint and the entrepreneur's incentive compatibility. Therefore, an insider without a liquidity shock has no incentive to delay refinancing. In turn, at the renegotiation stage, only the outsiders compete.

In $\mathcal{E}(\sigma)$, conditional on signal s, if an outsider wins at the renegotiation stage with a repayment request $m_2^R(s) \leq v - c$, the entrepreneur will exert effort if and only if $p \geq p_E^R(s) \equiv \frac{c}{v - m_2^R(s)}$. Therefore, the winning outsider's break-even condition is

$$m_1^R(s) = \int_{\max\{p_0, p_E^R(s)\}}^{p_1} p\left(v - \frac{c}{p_E^R(s)}\right)\mathrm{d}G(p), \tag{OA.4.5}$$

indicating that the insider's expected return at the renegotiation stage is $m_1^R(s)$ as determined in (OA.4.5).

Now, by contrast, consider a signal structure $\hat{\sigma}$ that differs from σ only in that, under $\hat{\sigma}$, project qualities $p \in [p_0, p_1]$ are fully revealed. Suppose that in an equilibrium played under $\hat{\sigma}$, denoted as $\mathcal{E}(\hat{\sigma})$ here, the insider's startup loan offer is identical to that in $\mathcal{E}(\sigma)$. Then, it is straightforward that, upon observing signals indicating $p \notin [p_0, p_1]$, the players' behavior in $\mathcal{E}(\hat{\sigma})$ coincides with that in $\mathcal{E}(\sigma)$. In contrast, when $p \in [p_0, p_1]$ is revealed in $\mathcal{E}(\hat{\sigma})$ and the insider faces a liquidity shock, at the renegotiation stage the outsiders offer $m_1^R(p) = pv - c$ and $m_2^R(p) = v - \frac{c}{p}$ to refinance the project. Hence, over the range $[p_0, p_1]$, the insider's expected return under renegotiation is $\int_{p_0}^{p_1}(pv - c)\mathrm{d}G(p)$ in $\mathcal{E}(\hat{\sigma})$, which is strictly higher than $m_1^R(s)$ in $\mathcal{E}(\sigma)$ as shown above. This implies that the insider's ex-ante return in $\mathcal{E}(\hat{\sigma})$ is also strictly higher than that in $\mathcal{E}(\sigma)$. Consequently, compared to σ, signal structure $\hat{\sigma}$ strictly increases the entrepreneur's borrowing capacity.

[17] The case with $p_0 < \frac{c}{v}$ follows analogously.

The analysis above implies that, to characterize the entrepreneur's maximum borrowing capacity in the model with renegotiation, it is without loss of generality to focus on signal structures that fully reveal project qualities at the renegotiation stage.

Then, following the proof of Lemma 1, in any equilibrium under such a signal structure, if at the debt rollover stage no outsider offers to refinance the project upon observing a signal s, the insider's expected return conditional on this signal is

$$(1-\lambda+\lambda\kappa)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]}(pv-c)\mathrm{d}G(p).$$

Conversely, if at the debt rollover stage the outsiders compete to refinance the project with an offer $m_2^O(s)$ upon observing a signal s, the insider's expected return conditional on this signal remains

$$\int_{\sigma^{-1}(s)\cap[p_E(s),1]} p\left(v-\frac{c}{p_E(s)}\right)\mathrm{d}G(p),$$

where $p_E(s)=\frac{c}{v-m_2^O(s)}$.

Furthermore, following the proof of Lemma 2, in any equilibrium under such a signal structure, the insider's ex-ante return is

$$\int_{\max\{\underline{p},p_E(\underline{s})\}}^{1} p\Big(v-\frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p)+(1-\lambda+\lambda\kappa)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p),$$

which, by applying an argument analogous to the proof of Proposition 3, can be shown to be bounded above by $V^R(p_R^*)$. In particular, achieving the borrowing capacity $V^R(p_R^*)$ requires that the signal structure takes the form of the upper-censorship structure $\sigma^{uc}(p;p_R^*)$, which pools all project qualities above the threshold p_R^* while fully revealing those below.

By modifying the proof of Proposition 4 slightly, we can construct a class of equilibria and verify that, for any borrowing need $B\leq V^R(p_R^*)$, the project can be financed under the upper-censorship signal structure $\sigma^{uc}(p;p_R(B))$.

Finally, note that $V^R(p_R^*)\geq V^R(p_T^*)\geq V(p_T^*)$. Therefore, allowing renegotiation after debt rollover failure improves the entrepreneur's maximum borrowing capacity.

Q.E.D.

Proof of Corollary OA.2:

The proof closely follows that of Proposition 5, and we therefore only sketch the key steps.

Given a monotone signal structure σ and a borrowing need B, suppose that the project is financed in equilibrium $\mathcal{E}(B;\sigma)$.

In $\mathcal{E}(B;\sigma)$, define $\mathbb{S}_H(\sigma)$ and $\mathbb{S}_L(\sigma)$ as in the proof of Lemma 2. Moreover, for any $s \in \mathbb{S}_L(\sigma)$, if the game proceeds to the renegotiation stage, then for $\Pr(\sigma^{-1}(s)) > 0$, we have

$$m_1^R(s) = \int_{\sigma^{-1}(s)\cap[p_E^R(s),1]} p\left(v - \frac{c}{p_E^R(s)}\right) \mathrm{d}G(p),$$

where $p_E^R(s) = \frac{c}{v - m_2^R(s)}$; for $\Pr(\sigma^{-1}(s)) = 0$ and thereby $\sigma^{-1}(s) = p$, we have $m_1^R(p) = pv - c$ and $m_2^R(p) = v - \frac{c}{p}$ when $p \geq \frac{c}{v}$ and $m_1^R(p) = m_2^R(p) = 0$ otherwise.

Then, in $\mathcal{E}(B;\sigma)$, the entrepreneur's borrowing constraint is given by

$$\begin{aligned} B \;=\; & \sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[p_E(s),1]} p m_2^O(s)\mathrm{d}G(p) \\ & + \sum_{s\in\mathbb{S}_L(\sigma)} (1-\lambda) \int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]} (pv-c)\mathrm{d}G(p) + \sum_{s\in\mathbb{S}_L(\sigma)} \lambda\kappa m_1^R(s). \end{aligned} \tag{OA.4.6}$$

Meanwhile, the entrepreneur's ex-ante payoff, $\Pi^\sigma(B)$, is given by

$$\Pi^\sigma(B) = \sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[p_E(s),1]} [p(v - m_2^O(s)) - c]\mathrm{d}G(p). \tag{OA.4.7}$$

Thus, in the model with renegotiation, maximizing the entrepreneur's ex-ante payoff from the project amounts to finding the signal structure σ that maximizes $\Pi^\sigma(B)$ in (OA.4.7) subject to the borrowing constraint in (OA.4.6).

When $\mathbb{S}_H(\sigma)$ is empty, it is immediate that $\Pi^\sigma(B) = 0$. Henceforth, we focus on the case where $\mathbb{S}_H(\sigma)$ is non-empty.

Let $\Pi^\sigma(B) = \Pi^\sigma(B) + B - B$. By substituting the first occurrence of B with the expression on the right-hand side of (OA.4.6), the payoff $\Pi^\sigma(B)$ can be rewritten as

$$\begin{aligned} \Pi^\sigma(B) \;=\; & \underbrace{\int_{\frac{c}{v}}^{1} (pv-c)\mathrm{d}G(p) - B}_{\text{net project value}} - \underbrace{\sum_{s\in\mathbb{S}_L(\sigma)} \lambda(1-\kappa) \int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]} (pv-c)\mathrm{d}G(p)}_{\text{efficiency loss from liquidation}} \\ & - \underbrace{\sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[\frac{c}{v},p_E(s))} (pv-c)\mathrm{d}G(p)}_{\text{efficiency loss from shirking}} - \underbrace{\sum_{s\in\mathbb{S}_L(\sigma)} \lambda\kappa \int_{\sigma^{-1}(s)\cap[\frac{c}{v},p_E^R(s))} (pv-c)\mathrm{d}G(p)}_{\text{efficiency loss from shirking}}. \end{aligned}$$

Therefore, in $\mathcal{E}(B;\sigma)$, the entrepreneur obtains the net project value minus the efficiency losses arising from early liquidation and shirking. Importantly, for any $s \in \mathbb{S}_L(\sigma)$ with $\Pr(\sigma^{-1}(s)) > 0$, if the game reaches the renegotiation stage, there is an additional efficiency loss from shirking, captured by the last term above.

Let $\Pi^R(B)$ be a function defined as

$$\Pi^R(B) = \int_{\frac{c}{v}}^{1} (pv - c)\mathrm{d}G(p) - B - \lambda(1-\kappa)\int_{\frac{c}{v}}^{p_R(B)} (pv - c)\mathrm{d}G(p).$$

With a slight modification of the proof of Proposition 5, it follows that

$$\Pi^\sigma(B) = \Pi^R(B)$$

if σ is the upper-censorship signal structure $\sigma^{uc}(p; p_R(B))$, and

$$\Pi^\sigma(B) < \Pi^R(B)$$

otherwise. As a result, for any borrowing need $B \leq V^R(p_R^*)$, the upper-censorship signal structure $\sigma^{uc}(p; p_R(B))$ maximizes the entrepreneur's ex-ante payoff, which is $\Pi^R(B)$.

Note that $V^R(\hat{p}) \geq V(\hat{p})$ for any threshold $\hat{p} \in [\frac{c}{v}, 1]$. Hence, for $V^R(p_R(B)) = V(p_T(B)) = B$, we must have $p_R(B) \leq p_T(B)$. It then follows immediately that $\Pi^R(B) \geq \Pi^*(B)$. As a result, allowing for renegotiation following debt rollover failure improves the entrepreneur's maximum ex-ante payoff as well as social welfare.

Lastly, for any borrowing need $B \leq V^R(p_R^*)$, by slightly modifying the proof of Proposition 4, we can explicitly construct the equilibrium that achieves the entrepreneur's maximum ex-ante payoff, $\Pi^R(B)$, under the upper-censorship signal structure $\sigma^{uc}(p; p_R(B))$. Specifically, in this equilibrium, the insider's startup loan offer, m_1^*, satisfies

$$\begin{aligned} m_1^* &= \frac{\lambda \int_{p_R(B)}^{1} p\left(v - \frac{c}{p_R(B)}\right)\mathrm{d}G(p) + (1-\lambda)\int_{p_R(B)}^{\max\{\frac{m_1^*}{v-c/p_R(B)}, p_R(B)\}} p\left(v - \frac{c}{p_R(B)}\right)\mathrm{d}G(p)}{\lambda[1 - G(p_R(B))] + (1-\lambda)[G(\max\{\frac{m_1^*}{v-c/p_R(B)}, p_R(B)\}) - G(p_R(B))]} \\ &> p_R(B)v - c. \end{aligned}$$

In contrast, for $p \in [\frac{c}{v}, p_R(B))$, at the renegotiation stage the insider receives payment $m_1^R(p) = pv - c < p_R(B)v - c$. Therefore, in equilibrium, the insider indeed "takes a haircut".

Q.E.D.

Proof of Proposition OA.3:

The proof includes two main steps. First, we show that the insider's ex-ante return and equivalently the entrepreneur's borrowing capacity is bounded above by $V^{os}(p_{os}^*)$. Second, we show that there is a class of binary signal structures $\sigma(p; \tilde{p})$, where $\tilde{p} \in [0, p_{os}^*]$, that enables the entrepreneur to borrow the amount $V^{os}(p_{os}^*)$. Therefore, $V^{os}(p_{os}^*)$ is the entrepreneur's maximum borrowing capacity when liquidity shocks are publicly observable.

Step 1. (a) At day 1, the insider's expected return is bounded above by $\int_{p^*_{os}}^1 p(v - \frac{c}{p^*_{os}})\mathrm{d}G(p)$ in the case that he encounters a liquidity shock. (b) At day 1, the insider's expected return is bounded above by $\int_{\frac{c}{v}}^1 (pv - c)\mathrm{d}G(p)$ in the case that he does not encounter a liquidity shock. (c) At day 0, the insider's ex-ante return is bounded above by $V^{os}(p^*_{os})$.

To show part (a), the exercise is equivalent to assume $\lambda = 1$ in the baseline model and repeat the analysis and proofs of Lemma 1 and Lemma 2. This leads to the result that, under any signal structure σ, when it is publicly observed that the insider encounters a liquidity shock, his expected return at debt rollover is given by

$$\int_{\max\{\underline{p}, p_E(\underline{s})\}}^1 p\Big(v - \frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p).$$

In this term, as before, $\underline{s}$ is the smallest signal following which the outsiders compete to refinance the project, $\underline{p} = \inf(\sigma^{-1}(\underline{s}))$, and $p_E(\sigma(p))$ is the entrepreneur's effort threshold conditional on the outsiders' participation in debt rollover following the signal $\sigma(p) \geq \underline{s}$. Also in this term, $\sigma(p) > \underline{s}$ implies $p_E(\sigma(p)) < p_E(\underline{s})$.

Denote by $p_{os} \equiv \max\{\underline{p}, p_E(\underline{s})\}$ here, then

$$\int_{\max\{\underline{p}, p_E(\underline{s})\}}^1 p\Big(v - \frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p) \leq \int_{p_{os}}^1 p\Big(v - \frac{c}{p_{os}}\Big)\mathrm{d}G(p) \leq \int_{p^*_{os}}^1 p\Big(v - \frac{c}{p^*_{os}}\Big)\mathrm{d}G(p).$$

Thus, $\int_{p^*_{os}}^1 p(v - \frac{c}{p^*_{os}})\mathrm{d}G(p)$ is an upper bound to the insider's expected return at debt rollover in the case that he encounters a liquidity shock.

Part (b) is immediate. Under any signal structure σ, to motivate the entrepreneur to work at day 2, any creditor at most can offer $v - \frac{c}{p}$ to refinance the project, where $p \geq \frac{c}{v}$. Therefore, $\int_{\frac{c}{v}}^1 (pv - c)\mathrm{d}G(p)$ is an upper bound to any creditor's expected return at debt rollover.

Part (c) follows from the fact that the insider encounters a liquidity shock with probability λ. Thus,

$$V^{os}(p^*_{os}) = \lambda \int_{p^*_{os}}^1 p\Big(v - \frac{c}{p^*_{os}}\Big)\mathrm{d}G(p) + (1 - \lambda) \int_{\frac{c}{v}}^1 (pv - c)\mathrm{d}G(p)$$

forms an upper bound to the insider's ex-ante return at day 0.

Step 2. There is a class of binary signal structures $\sigma(p; \tilde{p})$, where $\tilde{p} \in [0, p^*_{os}]$, which enables the entrepreneur to borrow the amount $V^{os}(p^*_{os})$.

To see this, suppose now that there is a binary signal structure $\sigma(p; \tilde{p})$ with $\tilde{p} \leq p^*_{os}$ and the entrepreneur has a borrowing need $B = V^{os}(p^*_{os})$. We characterize an equilibrium and show that in this equilibrium the entrepreneur's borrowing need is fulfilled. The creditors' behavior in this equilibrium is constructed as follows.

(i) At day 0, the creditors compete to finance the project with an offer $m^*_{1,os}$ given by

$$m^*_{1,os} = \frac{\int_{p^*_{os}}^1 p\left(v - \frac{c}{p^*_{os}}\right) \mathrm{d}G(p)}{1 - G(\tilde{p})}.$$

(ii) At day 1, after learning that the insider encounters a liquidity shock, the outsiders compete to refinance the project with an offer $m^*_{2,os} = v - \frac{c}{p^*_{os}}$ upon seeing signal $s = 1$ but do not make refinancing offers upon seeing signal $s = 0$; after learning that the insider does not encounter a liquidity shock, the outsiders do not make refinancing offers upon seeing any signal.

(iii) At day 1, if the insider does not encounter a liquidity shock, he offers $v - \frac{c}{p}$ to refinance the project when $p \geq \frac{c}{v}$ but does not offer to refinance it when $p < \frac{c}{v}$.

(iv) At day 2, the entrepreneur works if and only if the refinancing offer m_2 made by the creditors satisfies $p(v - m_2) \geq c$.

By backward induction, it is straightforward that the entrepreneur's behavior described in (iv) and the insider's behavior described in (iii) are optimal.

For the outsiders' behavior described in (ii), there are two cases to consider: (a) the insider encounters a liquidity shock; (b) the insider does not encounter a liquidity shock.

Consider case (a). In this case, the outsiders are sure that the insider cannot compete at debt rollover. Upon observing signal $s = 1$, if an outsider offers $m^*_{2,os}$ and wins to refinance the project, then the entrepreneur will work at day 2 if and only if $p \geq p^*_{os}$. Thus, this outsider's expected return from winning at refinance is

$$\frac{\int_{p^*_{os}}^1 p\left(v - \frac{c}{p^*_{os}}\right) \mathrm{d}G(p)}{1 - G(\tilde{p})},$$

which equals $m^*_{1,os}$ and makes the outsider break-even.

We need to verify that no outsider has incentive to offer m_2, where $m_2 < m^*_{2,os}$, to win at debt rollover upon observing signal $s = 1$. To see this, note that if indeed an outsider offers $m_2 < m^*_{2,os}$ and wins at debt rollover upon observing signal $s = 1$, we have the relations

$$\frac{\int_{\max\{\tilde{p}, \frac{c}{v-m_2}\}}^1 p m_2 \mathrm{d}G(p)}{1 - G(\tilde{p})} \leq \frac{\int_{p_{os}}^1 p\left(v - \frac{c}{p_{os}}\right) \mathrm{d}G(p)}{1 - G(\tilde{p})} \leq \frac{\int_{p^*_{os}}^1 p\left(v - \frac{c}{p^*_{os}}\right) \mathrm{d}G(p)}{1 - G(\tilde{p})} = m^*_{1,os}, \quad \text{(OA.4.8)}$$

in which $p_{os} \equiv \max\{\tilde{p}, \frac{c}{v-m_2}\}$, and $\frac{c}{v-m_2}$ is the entrepreneur's effort threshold at day 2 which satisfies $\frac{c}{v-m_2} < p^*_{os}$. In (OA.4.8), the first term is the outsider's expected return from refinancing the project with the offer m_2. Moreover, in (OA.4.8), the first inequality is strict if $\tilde{p} > \frac{c}{v-m_2}$, and the second inequality is strict if $\tilde{p} \leq \frac{c}{v-m_2}$. Thus, at least one of the inequalities is strict.

As a result, offering $m_2 < m^*_{2,os}$ to win at debt rollover will cause the outsider to make a loss, which deters the outsiders from making such a deviation.

On the other hand, upon observing signal $s = 0$, each outsider knows that the expected return from winning to refinance the project is bounded above by $\tilde{p}v - c$. Because $m^*_{1,os} > p^*_{os}v - c \geq \tilde{p}v - c$, upon observing signal $s = 0$ it is optimal for the outsiders to refrain from refinancing the project.

Consider case (b). In this case, the outsiders are sure that the insider can compete at debt rollover. The key for this case is that, because the threshold $\tilde{p}$ of the binary signal structure $\sigma(p; \tilde{p})$ is sufficiently low (i.e., $\widetilde{p} \leq p^*_{os}$), even after the realization of a good signal ($s = 1$), adverse selection is so severe that it will lead an outsider who makes an offer at debt rollover to win with a positive probability and subsequently suffer a loss. As a result, the outsiders are deterred from making offers at debt rollover.

For signal $s = 1$, we proceed the proof with three sub-cases: (b1) the outsiders do not make any offer $m_2 \geq v - c$ at debt rollover; (b2) the outsiders do not make any offer $m_2 \in (m^*_{1,os}, v - c)$ at debt rollover; (b3) the outsiders do not make any offer $m_2 \leq m^*_{1,os}$ at debt rollover.

Consider sub-case (b1) and suppose that, in contrast, an outsider makes a refinancing offer $m_2 \geq v - c$. Let $\widetilde{\widetilde{p}}$ be determined by $m^*_{1,os} = \widetilde{\widetilde{p}}v - c$ in this case. So, $\widetilde{\widetilde{p}} > p^*_{os} \geq \widetilde{p}$. It is direct to see that the insider will take $m^*_{1,os}$ and exit the competition at debt rollover for $p < \widetilde{\widetilde{p}}$. As a result, the outsider at least wins to refinance the project when $p \in [\widetilde{p}, \widetilde{\widetilde{p}})$. Because an offer $m_2 \geq v - c$ will lead the entrepreneur to shirk for any project quality, the outsider's expected profit from winning to refinance the project will be negative. Therefore, the outsiders do not make any offer $m_2 \geq v - c$ at debt rollover.

Consider sub-case (b2) and suppose that, in contrast, an outsider makes a refinancing offer $m_2 \in (m^*_{1,os}, v - c)$. Similar to case (b1), we can show that the insider will offer the same m_2 to win at debt rollover for $p \geq \max\{\frac{c}{v-m_2}, \frac{m^*_{1,os}}{m_2}\}$, but will take $m^*_{1,os}$ and exit the competition for $p < \frac{m^*_{1,os}+c}{v}$. Note that $\frac{m^*_{1,os}+c}{v} > p^*_{os} \geq \widetilde{p}$, so in this sub-case the outsider offering m_2 also wins with a positive probability. Moreover, if the insider makes an offer to win at debt rollover at project quality p, in equilibrium he will win at debt rollover at any project quality higher than p. This implies that in this sub-case there exists a cutoff value $\widetilde{\widetilde{p}} \in (p^*_{os}, 1)$ such that the outsider offering m_2 wins at debt rollover when $p \in [\widetilde{p}, \widetilde{\widetilde{p}})$. We then have the relations

$$\frac{\int_{\min\{\max\{\widetilde{p}, \frac{c}{v-m_2}\}, \widetilde{\widetilde{p}}\}}^{\widetilde{\widetilde{p}}} pm_2 \mathrm{d}G(p)}{G(\widetilde{\widetilde{p}}) - G(\widetilde{p})} < \frac{\int_{\max\{\widetilde{p}, \frac{c}{v-m_2}\}}^{1} pm_2 \mathrm{d}G(p)}{1 - G(\widetilde{p})} \leq \frac{\int_{p_{os}}^{1} p\left(v - \frac{c}{p_{os}}\right)\mathrm{d}G(p)}{1 - G(\widetilde{p})} \leq m^*_{1,os}, \tag{OA.4.9}$$

in which $p_{os} \equiv \max\{\tilde{p}, \frac{c}{v-m_2}\}$, and $\frac{c}{v-m_2}$ is the entrepreneur's effort threshold at day 2 conditional on the outsider's refinancing of the project. In (OA.4.9), the first term is the outsider's expected return from refinancing the project with the offer m_2, and the first inequality is strict because $\widetilde{\widetilde{p}} < 1$. Hence, the outsiders do not make any offer $m_2 \in (m^*_{1,os}, v - c)$ at debt rollover.

Consider sub-case (b3) and suppose that, in contrast, an outsider makes a refinancing offer $m_2 \leq m^*_{1,os}$. It is direct to see that in this sub-case the insider will take $m^*_{1,os}$ and exit the competition at debt rollover for any project quality p, leading the outsider to win at debt rollover and suffer a loss. Thus, the outsiders do not make any offer $m_2 \leq m^*_{1,os}$ at debt rollover.

For signal $s = 0$, because $m^*_{1,os} > pv - c$ for any $p < \widetilde{p} \leq p^*_{os}$, if an outsider makes an offer m_2 at debt rollover, the insider will surely let the outsider win to refinance the project, leading the outsider to suffer a loss. Therefore, upon observing signal $s = 0$, no outsider will make an offer at debt rollover.

Lastly, for the creditors' behavior described in (i), note that any creditor who wins at day 0 with the offer $m^*_{1,os}$ has an ex-ante return

$$\lambda[1 - G(\widetilde{p})]m^*_{1,os} + (1 - \lambda)\int_{\frac{c}{v}}^{1}(pv - c)\mathrm{d}G(p),$$

which equals $V^{os}(p^*_{os})$. As a result, for the entrepreneur having a borrowing need $B = V^{os}(p^*_{os})$, at least the creditors can make a startup loan offer $m^*_{1,os}$ to finance the project, which will make the winner break-even.

We show that no creditor has incentive to make an offer m_1, where $m_1 < m^*_{1,os}$, to finance the project. To see this, suppose that a creditor wins the competition at day 0 with an offer $m_1 < m^*_{1,os}$ and thus becomes the insider. Note that the insider's expected return at day 1 in the case he does not encounter a liquidity shock is still bounded above by $\int_{\frac{c}{v}}^{1}(pv - c)\mathrm{d}G(p)$.

So we can focus on the insider's expected return at day 1 in the case he encounters a liquidity shock. If the startup loan offer m_1 only induces the outsiders to compete at debt rollover conditional on observing signal $s = 1$, then the insider's expected return before the realization of the signals is $[1 - G(\widetilde{p})]m_1$, which is strictly less than $[1 - G(\widetilde{p})]m^*_{1,os} = \int_{p^*_{os}}^{1} p(v - \frac{c}{p^*_{os}})\mathrm{d}G(p)$. On the other hand, if the startup loan offer m_1 induces the outsiders to compete at debt rollover upon observing any signal ($s = 1$ or $s = 0$), then following the analysis and proofs of Lemma 1 and Lemma 2, the insider's expected return before the realization of the signals has the form

$$\int_{\widetilde{p}}^{1} p\Big(v - \frac{c}{p_E(1)}\Big)\mathrm{d}G(p) + \int_{p_E(0)}^{\widetilde{p}} p\Big(v - \frac{c}{p_E(0)}\Big)\mathrm{d}G(p), \tag{OA.4.10}$$

in which $p_E(1)$ and $p_E(0)$ are the entrepreneur's effort thresholds conditional on the project being refinanced following signals $s = 1$ and $s = 0$, respectively. In addition, $\frac{c}{v} < p_E(1) <$

$p_E(0)$. It is direct to see that the expected return in (OA.4.10) is strictly less than $\int_{p_E(0)}^1 p(v - \frac{c}{p_E(0)})\mathrm{d}G(p)$, and therefore, less than $[1 - G(\widetilde{p})]m_{1,os}^* = \int_{p_{os}^*}^1 p(v - \frac{c}{p_{os}^*})\mathrm{d}G(p)$.

As a result, offering $m_1 < m_{1,os}^*$ will only lead the creditor to win at day 0 and obtain an ex-ante return strictly less than $V^{os}(p_{os}^*)$. Therefore, when the entrepreneur has a borrowing need $B = V^{os}(p_{os}^*)$ and the signal structure is $\sigma(p;\tilde{p})$ with $\tilde{p} \leq p_{os}^*$, in equilibrium the creditors offer $m_{1,os}^*$ at day 0 to finance the project.

Q.E.D.

OA.5 Extended Discussions

OA.5.1 Discussion: Allocation of bargaining power

In this section, we extend the baseline model by introducing an allocation of bargaining power to the entrepreneur-insider bilateral monopoly situation and examine its impacts on the outcomes.

One way to formalize this extension is to assume that if no outsider makes a refinancing offer at the debt rollover stage, then the insider (provided he faces no liquidity shock) and the entrepreneur can each make a take-it-or-leave-it offer with probabilities $\beta \in [0,1]$ and $1-\beta$, respectively. Apparently, when the insider makes such an offer, he requests repayment of $v - \frac{c}{p}$ for $p \geq \frac{c}{v}$ and makes a zero-profit random offer otherwise. Conversely, when the entrepreneur makes the offer, she proposes a payment of zero to the insider for any project quality.

Equivalently, one may assume that in the bilateral monopoly, the two parties divide the project's net surplus according to a Nash bargaining protocol, with shares β and $1-\beta$ representing the insider's and the entrepreneur's bargaining powers, respectively. This formulation implies that, in the bilateral monopoly, the insider's refinancing offer is $\beta\left(v - \frac{c}{p}\right)$ for $p \geq \frac{c}{v}$ and 0 otherwise.

Under either specification, the key modification to the baseline model is that, conditional on being the only creditor to refinance the project, the insider's expected return becomes $\beta(pv-c)$ for $p \geq \frac{c}{v}$, rather than $pv - c$ in the baseline case. The statements in Lemmas 1 and 2 should therefore be adjusted accordingly, with (4) in Lemma 1 being replaced by

$$\beta(1-\lambda)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]}(pv-c)\mathrm{d}G(p),$$

and (7) in Lemma 2 being replaced by

$$U^{\sigma}(B) = \int_{\max\{\underline{p},p_E(\underline{s})\}}^1 p\Big(v - \frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p) + \beta(1-\lambda)\int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}}(pv-c)\mathrm{d}G(p).$$

The characterization of the entrepreneur's maximum borrowing capacity or her maximum ex-ante payoff closely follows the analysis in Sections 3 and 4. Let $V^{bm}(p_{bm})$ be defined as

$$V^{bm}(p_{bm}) = \int_{p_{bm}}^{1} p(v - \frac{c}{p_{bm}})\mathrm{d}G(p) + \beta(1-\lambda)\int_{\frac{c}{v}}^{p_{bm}}(pv-c)\mathrm{d}G(p), \quad \forall p_{bm} \in \left[\frac{c}{v}, 1\right],$$

where "bm" denotes the bilateral monopoly setting. The function $V^{bm}(p_{bm})$ is non-monotonic and reaches its maximum at some $p^*_{bm} \in (\frac{c}{v}, 1)$, which is assumed to be unique.

Claim 1 *Consider the model with an allocation of bargaining power. Among all monotone signal structures, the maximum entrepreneur borrowing capacity is achieved under the pass-or-fail signal structure $\sigma(p; p^*_{bm})$ and is $V^{bm}(p^*_{bm})$. Moreover, $V^{bm}(p^*_{bm})$ increases with β.*

The first part of this claim closely parallels Proposition 3, and we therefore omit its proof. The second part characterizes how the allocation of bargaining power affects the entrepreneur's maximum borrowing capacity. By the envelope theorem,

$$\frac{\mathrm{d}V^{bm}(p^*_{bm})}{\mathrm{d}\beta} = (1-\lambda)\int_{\frac{c}{v}}^{p^*_{bm}}(pv-c)\mathrm{d}G(p) > 0.$$

Intuitively, as the insider's bargaining power in the bilateral monopoly increases (i.e., as β rises), the insider's expected return at debt rollover improves, which in turn strengthens his willingness to provide the startup loan. As a result, the entrepreneur's maximum borrowing capacity increases.

Let $p_{bm}(B)$ denote the smallest value of p_{bm} that satisfies $B = V^{bm}(p_{bm})$, and let $\Pi^{bm}(B)$ denote the entrepreneur's maximum ex-ante payoff in the model with an allocation of bargaining power.

Claim 2 *Consider the model with an allocation of bargaining power. For any borrowing need $B \leq V^{bm}(p^*_{bm})$, among all monotone signal structures, the pass-or-fail signal structure $\sigma(p; p_{bm}(B))$ maximizes the entrepreneur's ex-ante payoff and hence social welfare, which is*

$$\Pi^{bm}(B) = \int_{\frac{c}{v}}^{1}(pv-c)\mathrm{d}G(p) - B - \lambda\int_{\frac{c}{v}}^{p_{bm}(B)}(pv-c)\mathrm{d}G(p).$$

Moreover, $\Pi^{bm}(B)$ increases with β.

The first part of this claim closely parallels Proposition 5, and we also omit its proof. The second part characterizes how the allocation of bargaining power influences the entrepreneur's maximum ex-ante payoff. Note that β affects $\Pi^{bm}(B)$ through $p_{bm}(B)$. Given the entrepreneur's borrowing constraint

$$B = \int_{p_{bm}(B)}^{1} p\Big(v - \frac{c}{p_{bm}(B)}\Big)\mathrm{d}G(p) + \beta(1-\lambda)\int_{\frac{c}{v}}^{p_{bm}(B)} (pv - c)\mathrm{d}G(p),$$

we have

$$\frac{\mathrm{d}p_{bm}(B)}{\mathrm{d}\beta} = \frac{-(1-\lambda)\int_{\frac{c}{v}}^{p_{bm}(B)}(pv-c)\mathrm{d}G(p)}{\int_{p_{bm}(B)}^{1}\frac{pc}{[p_{bm}(B)]^2}\mathrm{d}G(p) - (1-\beta(1-\lambda))[p_{bm}(B)v - c]g(p_{bm}(B))} < 0.$$

Therefore,

$$\frac{\mathrm{d}\Pi^{bm}(B)}{\mathrm{d}\beta} = \frac{\partial \Pi^{bm}(B)}{\partial p_{bm}(B)}\frac{\mathrm{d}p_{bm}(B)}{\mathrm{d}\beta} > 0.$$

Intuitively, for any given borrowing need of the entrepreneur, granting the insider greater bargaining power in the bilateral monopoly strengthens the insider's incentive to provide the startup loan, lowers the optimal pass signal threshold that induces outsider participation at debt rollover, mitigates the efficiency loss from early liquidation, and thereby increases both the entrepreneur's ex-ante payoff and overall social welfare.

OA.5.2 Discussion: Reversing the timing of offer making at debt rollover

In this section, we examine the *insider-rollover-first* model in greater detail. Relative to the baseline model, this model reverses the timing of offers at debt rollover. Specifically, at the debt rollover stage, if facing no liquidity shock, the insider makes a refinancing offer first. The entrepreneur then chooses whether to accept it. If no agreement is reached, the entrepreneur seeks refinancing from the outsiders, accepting the lowest offer if any; otherwise, the project is liquidated. In particular, we assume that whether the insider has made a refinancing offer to the entrepreneur is unobservable to the outsiders. This assumption is reasonable and realistic, as internal corporate decisions are typically less transparent to external parties.

Note that in the insider-rollover-first model, the insider's refinancing offer should be unrestricted. In the subsequent discussion, when comparing results with the baseline model, we refer to an earlier circulated version of the paper in which the insider's refinancing offer is likewise not limited to either matching the outsiders' offer or exiting.

In the insider-rollover-first model, the entrepreneur's attempt to obtain external funding can also reveal information about the project quality to the outsiders. This feature introduces a new complication: given a signal structure σ, the offer game at the debt rollover stage may admit multiple equilibria, owing to a self-fulfilling loop between the insider's rollover decisions and the outsiders' inferences about them. Such equilibrium multiplicity, in turn, complicates the characterization of the entrepreneur's maximum borrowing capacity.

Nevertheless, our main result is that the entrepreneur's maximum borrowing capacity remains identical under the insider-rollover-first model and the baseline model.

Claim 3 *Consider the insider-rollover-first model. Among all monotone signal structures, the entrepreneur's maximum borrowing capacity is achieved under the pass-or-fail signal structure $\sigma(p; p_T^*)$, which is $V(p_T^*)$.*

Two observations help clarify this result. First, given a signal structure σ and a borrowing need B, there exists an equilibrium in the insider-rollover-first model that is outcome-equivalent to the (unique) equilibrium in the baseline model. This is because, in both models, if an outsider succeeds in rolling over the loan, they cannot fully distinguish between the insider's inability or unwillingness to refinance the project. As a result, the winner's curse operates similarly in both settings. This observation then implies that the entrepreneur's maximum borrowing capacity in the insider-rollover-first model is at least $V(p_T^*)$.

Second, in any equilibrium of the insider-rollover-first model, the insider's expected returns at the debt rollover stage and the initial financing stage are still characterized by the forms in Lemmas 1 and 2, respectively, from the baseline model. Consequently, in the insider-rollover-first model, the highest expected return the insider can obtain from issuing the startup loan and, equivalently, the entrepreneur's maximum borrowing capacity, is bounded above $V(p_T^*)$.

Building on the discussion above, we can also characterize the entrepreneur's maximum ex-ante payoff and social welfare in the insider-rollover-first model.

Claim 4 *Consider the insider-rollover-first model. For any borrowing need $B \leq V(p_T^*)$, among all monotone signal structures, the pass-or-fail signal structure $\sigma(p; p_T(B))$ maximizes the entrepreneur's ex-ante payoff and hence social welfare, which equals $\Pi^*(B)$.*

We provide the proofs of the results below.

Proof of Claim 3:

The proof proceeds in two parts, corresponding to showing that, in the insider-rollover-first model, the entrepreneur's maximum borrowing capacity is both at least and at most $V(p_T^*)$.

Part 1. We show that, for any given signal structure σ and borrowing need B, there exists an equilibrium in the insider-rollover-first model that is outcome-equivalent to the unique equilibrium in the baseline model.

Specifically, let $\mathcal{E}(B;\sigma)$ denote the equilibrium in the baseline model, and consider the case where, upon observing a signal s, the outsiders compete to roll over the loan. Denote by $\overline{m}_1$, $\overline{m}_2^O(s)$, $\overline{m}_2^I(p)$ the initial and refinancing offers made on the equilibrium path, and let $\overline{p}_E(s)$, $\overline{p}_A(s)$, $\overline{p}_M(s)$ be defined as in Lemma 1 of the previous version of the paper (the same applies hereafter). In this case, if the insider faces no liquidity shock, he offers $\overline{m}_2^I(p) = \overline{m}_2^O(s)$ for $p \geq \max\{\overline{p}_A(s), \overline{p}_E(s)\}$, offers $\overline{m}_2^I(p) = v - \frac{c}{p}$ for $\max\{\overline{p}_A(s), \overline{p}_M(s)\} \leq p < \max\{\overline{p}_A(s), \overline{p}_E(s)\}$, and exits the refinancing competition for $p < \max\{\overline{p}_A(s), \overline{p}_M(s)\}$. Moreover, whenever the insider makes a refinancing offer, the entrepreneur accepts his offer.

Now, suppose that in the insider-rollover-first model, with the same signal structure σ and borrowing need B, the offer-making decisions described above are also played following the realization of signal s. We outline their optimality in this model by backward induction.

First, if the entrepreneur accepts a refinancing offer m_2, she exerts effort if and only if $p(v - m_2) \geq c$, which is identical to the condition in $\mathcal{E}(B;\sigma)$.

Second, consider each outsider's decision regarding the refinancing offer. If an outsider wins the refinancing competition with an arbitrary offer $m_2^O(s)$, his expected payoff is

$$\frac{\lambda \int_{\sigma^{-1}(s)\cap[p_E(s),1]} p m_2^O(s) \mathrm{d}G(p) + (1-\lambda) \int_{\sigma^{-1}(s)\cap[p_E(s),1]\cap[0,\max\{\overline{p}_A(s),\overline{p}_M(s)\})} p m_2^O(s) \mathrm{d}G(p)}{\lambda \Pr(\sigma^{-1}(s)) + (1-\lambda)\Pr(\sigma^{-1}(s)\cap[0,\max\{\overline{p}_A(s),\overline{p}_M(s)\}))}.$$

Here, $\overline{p}_A(s)$ and $\overline{p}_M(s)$ are the values from $\mathcal{E}(B;\sigma)$, while $p_E(s) = \frac{c}{v - m_2^O(s)}$ depends on the outsider's offer $m_2^O(s)$ here. Importantly, this payoff expression reflects the outsiders' inference that, absent a liquidity shock, the insider would have refinanced the project himself for $p \geq \max\{\overline{p}_A(s), \overline{p}_M(s)\}$.

If there exist values of $m_2^O(s)$ that make this expression equal to $\overline{m}_1$, then $\overline{m}_2^O(s)$ is one such value. Hence,

$$\overline{m}_1 = \frac{\overline{m}_2^O(s)\left(\lambda \int_{\sigma^{-1}(s)\cap[\overline{p}_E(s),1]} p \mathrm{d}G(p) + (1-\lambda) \int_{\sigma^{-1}(s)\cap[\overline{p}_E(s),1]\cap[0,\max\{\overline{p}_A(s),\overline{p}_M(s)\})} p \mathrm{d}G(p)\right)}{\lambda \Pr(\sigma^{-1}(s)) + (1-\lambda)\Pr(\sigma^{-1}(s)\cap[0,\max\{\overline{p}_A(s),\overline{p}_M(s)\}))},$$

which takes the same form as equation (A.2) in the proof of Lemma 1. Consequently, it is optimal for each outsider to offer $\overline{m}_2^O(s)$ when approached by the entrepreneur.

Third, consider the entrepreneur's decision to accept or reject the insider's refinancing offer, if one is made. Given that the insider offers $\overline{m}_2^I(p)$ and the entrepreneur expects the outsiders to offer $\overline{m}_2^O(s)$ when approached, it remains optimal for her to accept the insider's offer. The key reason is that if the entrepreneur were to deviate from the equilibrium path by rejecting the insider's offer, this action would not alter the outsiders' inference about the project's quality and, consequently, would not affect their subsequent offer. Therefore, the entrepreneur cannot gain from such a deviation.

Fourth, consider the insider's decision at debt rollover, given that he does not face a liquidity shock. For $p \geq \max\{\overline{p}_A(s), \overline{p}_M(s)\}$, if making the offer $\overline{m}_2^I(p)$ induces the entrepreneur to accept, while any higher offer would induce her to reject, it is optimal for the insider to choose $\overline{m}_2^I(p)$. Conversely, for $p < \max\{\overline{p}_A(s), \overline{p}_M(s)\}$, it is optimal for the insider to exit and allow the outsiders to refinance the project. The reasoning is analogous to the previous case: any deviation by the insider would not alter the outsiders' inference about the project's quality, nor would it affect their refinancing offers. Hence, the insider cannot benefit from deviating.

By a similar argument, we can show that if, in $\mathcal{E}(B;\sigma)$, upon observing a signal s the outsiders do not compete to roll over the loan, then the players' equilibrium decisions in $\mathcal{E}(B;\sigma)$ would likewise be optimal in the insider-rollover-first model.

Lastly, given the anticipated equilibrium decisions at debt rollover and beyond, the offer $\overline{m}_1$ represents the lowest startup loan offer that allows a creditor to break even when financing the project. Therefore, offering this amount is optimal.

As a result, for any equilibrium $\mathcal{E}(B;\sigma)$ in the baseline model, there exists an outcome-equivalent equilibrium in the insider-rollover-first model. Moreover, since a borrowing need of $V(p_T^*)$ can be achieved under the pass-or-fail signal structure $\sigma(p;p_T^*)$ in the baseline model, the maximum borrowing capacity of the entrepreneur in the insider-rollover-first model is at least $V(p_T^*)$.

Part 2. We show that, in any equilibrium of the insider-rollover-first model, the insider's expected return from issuing the startup loan and, consequently, the entrepreneur's borrowing capacity, is bounded above by $V(p_T^*)$. The proof of this result closely follows the arguments in Lemmas 1 and 2, as well as Proposition 4.

To see this, fix a signal structure σ and consider any equilibrium in the insider-rollover-first model, denoted $\hat{\mathcal{E}}(B;\sigma)$. We then verify that, in this equilibrium, the borrowing need B must satisfy $B \leq V(p_T^*)$.

In $\hat{\mathcal{E}}(B;\sigma)$, consider the case where, given the realization of signal s, the outsiders do not make refinancing offers when approached on the equilibrium path. In this case, the insider who does not face a liquidity shock should offer $m_2^I(p) = v - \frac{c}{p}$ for $p \geq \frac{c}{v}$ and makes a zero-profit random offer otherwise, thereby capturing the entire net surplus. Consequently, the insider's expected return at debt rollover is

$$(1-\lambda)\int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]}(pv-c)\mathrm{d}G(p),$$

which takes the same form as Equation (10) in Lemma 1.

In $\hat{\mathcal{E}}(B;\sigma)$, consider the case where, given the realization of signal s, the outsiders make a refinancing offer $m_2^O(s)$ when approached on the equilibrium path. Then, in this case, the insider who does not face a liquidity shock should offer $m_2^I(p) = m_2^O(s)$ for $p \geq \max\{p_A(s), p_E(s)\}$, offer $m_2^I(p) = v - \frac{c}{p}$ for $\max\{p_A(s), p_M(s)\} \leq p < \max\{p_A(s), p_E(s)\}$, and exit the refinancing competition for $p < \max\{p_A(s), p_M(s)\}$, where $p_E(s)$, $p_A(s)$, and $p_M(s)$ are defined as in Equation (12) in Lemma 1. Moreover, whenever the insider makes a refinancing offer, the entrepreneur should accept his offer. Regardless of whether $m_2^O(s)$, and accordingly the other values, are unique, the insider's expected return at debt rollover can be expressed as

$$\int_{\sigma^{-1}(s)\cap[p_E(s),1]} p\left(v - \frac{c}{p_E(s)}\right)\mathrm{d}G(p) + (1-\lambda)\int_{\sigma^{-1}(s)\cap[p_M(s),p_E(s))} (pv - c)\mathrm{d}G(p),$$

which has the same form as Equation (11) in Lemma 1.

The key reasoning in both cases above is that, because the insider faces a liquidity shock with positive probability, in $\hat{\mathcal{E}}(B;\sigma)$ the entrepreneur also approaches the outsiders with positive probability upon the realization of any signal s. Consequently, any unobservable deviation by the insider or the entrepreneur cannot affect the outsiders' inference about the project's quality, and therefore does not influence their decision to roll over the loan.

Next, we can show that in $\hat{\mathcal{E}}(B;\sigma)$, for signals s and s' with $s < s'$, if the outsiders are willing to refinance the project upon observing signal s, they are also willing to do so upon observing signal s'. Moreover, on the equilibrium path of $\hat{\mathcal{E}}(B;\sigma)$, we have $m_2^O(s) > m_2^O(s')$, regardless of whether these values are unique. Let $\underline{s}$ denote the smallest signal for which the outsiders make refinancing offers, if such a signal exists, and define $\underline{p} = \inf(\sigma^{-1}(\underline{s}))$ if $\underline{s}$ exists, or $\underline{p} = 1$ otherwise. Then, in $\hat{\mathcal{E}}(B;\sigma)$, the insider's expected payoff from making the startup loan is

$$\begin{aligned} U^\sigma(B) \;=\; & \int_{\max\{\underline{p},p_E(\underline{s})\}}^{1} p\Big(v - \frac{c}{p_E(\sigma(p))}\Big)\mathrm{d}G(p) + \\ & (1-\lambda)\int_{\max\{\underline{p},p_M(\underline{s})\}}^{\max\{\underline{p},p_E(\underline{s})\}} (pv-c)\mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{\max\{\frac{c}{v},\underline{p}\}} (pv-c)\mathrm{d}G(p), \end{aligned}$$

which has the same form as Equation (14) in Lemma 2. In equilibrium, $B = U^\sigma(B)$, since the insider earns zero profit from the startup loan.

Finally, as shown in Equation (A.16) in the proof of Proposition 4, $U^\sigma(B) \leq V(p_T) \leq V(p_T^*)$, with $p_T = \max\{\underline{p}, p_E(\underline{s})\}$. Thus, in $\hat{\mathcal{E}}(B;\sigma)$, we have $B \leq V(p_T^*)$. This implies that in the insider-rollover-first model, the entrepreneur's borrowing capacity under any signal structure is bounded above by $V(p_T^*)$.

Combining both parts of the proof, we conclude that in the insider-rollover-first model, the entrepreneur's maximum borrowing capacity is $V(p_T^*)$, identical to that in the baseline model. Moreover, since the insider attains this highest expected return $V(p_T^*)$ only under the pass-or-fail signal structure $\sigma(p; p_T^*)$, the entrepreneur's maximum borrowing capacity is uniquely achieved under this signal structure.

Q.E.D.

Proof of Claim 4:

The proof closely follows that of Proposition 5 of the previous version of the paper (the same applies hereafter), so we outline only the key steps.

In the insider-rollover-first model, for any signal structure σ and borrowing need B, suppose that the project is financed in equilibrium $\mathcal{E}(B;\sigma)$. Let $\mathbb{S}_H(\sigma)$, $\mathbb{S}_L(\sigma)$, $\underline{s}$ and $\underline{p}$ be defined as before.

Then, in $\mathcal{E}(B;\sigma)$, the entrepreneur's borrowing constraint has the form

$$\begin{aligned} B &= \sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[p_E(s),1]} p m_2^O(s)\mathrm{d}G(p) + \\ &\quad \sum_{s\in\mathbb{S}_H(\sigma)} (1-\lambda) \int_{\sigma^{-1}(s)\cap[p_M(s),p_E(s))} (pv-c)\mathrm{d}G(p) + \\ &\quad \sum_{s\in\mathbb{S}_L(\sigma)} (1-\lambda) \int_{\sigma^{-1}(s)\cap[\frac{c}{v},1]} (pv-c)\mathrm{d}G(p). \end{aligned}$$

On the other hand, the entrepreneur's ex-ante payoff, $\Pi^\sigma(B)$, has the form

$$\Pi^\sigma(B) = \sum_{s\in\mathbb{S}_H(\sigma)} \int_{\sigma^{-1}(s)\cap[p_E(s),1]} [p(v-m_2^O(s))-c]\mathrm{d}G(p).$$

After some algebra, $\Pi^\sigma(B)$ can be re-expressed as

$$\begin{aligned} \Pi^\sigma(B) &= \int_{\frac{c}{v}}^{1} (pv-c)\mathrm{d}G(p) - B - \lambda \int_{\frac{c}{v}}^{\max\{\underline{p},\frac{c}{v}\}} (pv-c)\mathrm{d}G(p) - \\ &\quad \lambda \int_{\max\{\underline{p},\frac{c}{v}\}}^{\max\{\underline{p},p_E(\underline{s})\}} (pv-c)\mathrm{d}G(p) - (1-\lambda)\int_{\{\underline{p},\frac{c}{v}\}}^{\max\{\underline{p},p_M(\underline{s})\}} (pv-c)\mathrm{d}G(p). \end{aligned}$$

Following the proof of Proposition 5, we have that

$$\Pi^\sigma(B) = \Pi^*(B)$$

if σ is the pass-or-fail signal structure $\sigma(p;p_T(B))$, and

$$\Pi^\sigma(B) < \Pi^*(B)$$

otherwise. As a result, for any borrowing need $B \leq V(p_T^*)$, the pass-or-fail signal structure $\sigma(p;p_T(B))$ maximizes the entrepreneur's ex-ante payoff, which is $\Pi^*(B)$.

Q.E.D.

OA.5.3 Discussion: Uncertainty about final cash flow

In this section, we assume that the creditors' uncertainty is about the realization of the project's final cash flow, and examine its implications on the entrepreneur's borrowing capacity and her ex-ante payoff.

Suppose that, while keeping other aspects of the baseline model unchanged, if the entrepreneur exerts effort, the project yields a cash flow v drawn from the interval $[\underline{v}, \overline{v}]$ according to a continuous distribution $G(v)$. In contrast, if the entrepreneur shirks, the project generates a cash flow of zero with certainty. Accordingly, project quality in this setting is captured by the cash flow v. We refer to this formulation as the *quality-of-cash-flow* model. For ease of exposition, we assume that $\underline{v} \leq c$.

The characterization of the entrepreneur's maximum borrowing capacity or her maximum ex-ante payoff closely parallels the analysis in Sections 3 and 4. Define

$$V^{cf}(v_T) = \int_{v_T}^{\overline{v}} (v_T - c)\mathrm{d}G(v) + (1-\lambda)\int_{c}^{v_T} (v - c)\mathrm{d}G(v), \quad \forall v_T \in [c, \overline{v}],$$

where "cf" refers to "cash flow". Let v_T^* denote the value of v_T at which $V^{cf}(v_T)$ attains its maximum, and assume that this maximizer is unique.

Claim 5 *Consider the quality-of-cash-flow model. The entrepreneur's maximum borrowing capacity can be achieved under any monotone signal structure that separates projects with $v \geq v_T^*$ from those with $v < v_T^*$, which equals $V^{cf}(v_T^*)$.*

Following the proofs and arguments of Lemmas 1, 2 and Proposition 3, it is straightforward to show that, in the quality-of-cash-flow model, the entrepreneur's maximum borrowing capacity is bounded above by $V^{cf}(v_T^*)$. We hence omit this part of proof.

We now show that the entrepreneur's borrowing capacity can attain $V^{cf}(v_T^*)$. Consider any monotone signal structure $\sigma(v) = s_n$ for $v \in x_n$, satisfying $\cup_{n=0}^{N} x_n = [\underline{v}, \overline{v}]$, where the sets x_n are ordered increasingly and mutually disjoint, i.e., $x_n \cap x_{n+1} = \emptyset$. In particular, suppose there exists an index K such that $\sup(x_{K-1}) = \inf(x_K) = v_T^*$.[18] This signal structure thus separates projects with $v \geq v_T^*$ from those with $v < v_T^*$.

Then, following the proof of Proposition 4, we can show that under such a signal structure, if the entrepreneur's borrowing need equals $V^{cf}(v_T^*)$, there exists a unique equilibrium enabling her to meet this borrowing requirement. Importantly, in this equilibrium, the insider makes an initial offer $m_1 = v_T^* - c$, while the outsiders make a refinancing offer $m_2^O(s_n) = v_T^* - c$ upon

[18] A signal structure that fully reveals v within an interval can be viewed as a special case of this signal structure and its definition is therefore omitted.

observing any signal $s_n \geq s_K$, and make no refinancing offer when $s_n < s_K$. As a result, all monotone signal structures that separate project qualities above and below the threshold v_T^*, including the binary signal structure $\sigma(v; v_T^*)$, allow the entrepreneur to achieve the borrowing capacity $V^{cf}(v_T^*)$.

As shown above, the key feature of an equilibrium that maximizes the entrepreneur's borrowing capacity in the quality-of-cash-flow model is that, conditional on knowing the project quality exceeds v_T^*, the outsiders' refinancing loan is a safe loan, rendering them indifferent to additional information disclosure. This property implies that, in the present model, information asymmetry between the insider and the outsiders plays only a limited role, and the winner's curse disappears once the outsiders possess even a minimal level of information. Consequently, design of information disclosure is less critical.

Let $v_T(B)$ denote the smallest value of v_T that satisfies $B = V^{cf}(v_T)$, and let $\Pi^{cf}(B)$ denote the entrepreneur's maximum ex-ante payoff in the quality-of-cash-flow model.

Claim 6 *Consider the quality-of-cash-flow model. For a borrowing need $B \leq V^{cf}(v_T^*)$, any monotone signal structure that separates projects with $v \geq v_T(B)$ from those with $v < v_T(B)$ can maximize the entrepreneur's ex-ante payoff and hence social welfare, which is*

$$\Pi^{cf}(B) = \int_c^{\overline{v}} (v - c)\mathrm{d}G(v) - B - \lambda \int_c^{v_T(B)} (v - c)\mathrm{d}G(v).$$

The proof of this claim closely parallels that of Proposition 5, and is therefore omitted. The key insight is that, for a borrowing need $B \leq V^{cf}(v_T^*)$, under any monotone signal structure that maximizes the entrepreneur's ex-ante payoff, the outsiders' refinancing loan is safe in equilibrium. Specifically, they make a refinancing offer $v_T(B) - c$ when inferring that $v \geq v_T(B)$, and make no offer when inferring that $v < v_T(B)$. Therefore, conditional on knowing that the project quality is above $v_T(B)$, the outsiders are insensitive to any further information disclosure, consistent with the earlier interpretation.

As a result, any monotone signal structure that separates project qualities above and below the threshold $v_T(B)$, including the binary signal structure $\sigma(v; v_T(B))$, allows the entrepreneur to achieve her maximum ex-ante payoff.

OA.5.4 Discussion: A binary cash flow structure with a positive low realization

We provide a discussion on a binary cash flow structure with a positive low realization in this section. Specifically, keeping all other aspects of the baseline model unchanged, we assume that if the entrepreneur exerts effort, the project generates a cash flow of $\overline{v}$ with probability p and $\underline{v}$ with probability $1 - p$, where $\overline{v} > \underline{v} > 0$. In contrast, if the entrepreneur shirks, the project yields $\underline{v}$ with certainty. We focus on the case in which the entrepreneur has a borrowing need $B > \underline{v} - c$.[19] Define $\hat{p} \equiv \frac{c-\underline{v}}{\overline{v}-\underline{v}}$ if $\underline{v} < c$, and $\hat{p} \equiv 0$ otherwise. Then, exerting effort is socially efficient if and only if $p \geq \hat{p}$.

Under this cash flow structure, the characterization of the entrepreneur's maximum borrowing capacity and her maximum ex-ante payoff closely follows the analysis in Sections 3 and 4, with corresponding adjustments to several expressions. In particular, the statement in Lemma 1 should be modified, replacing (4) with

$$(1-\lambda)\int_{\sigma^{-1}(s)\cap[\hat{p},1]}[p\overline{v}+(1-p)\underline{v}-c]\mathrm{d}G(p),$$

and replacing (5) with

$$\int_{\sigma^{-1}(s)\cap[p_E(s),1]}[pm_2^O(s)+(1-p)\min\{\underline{v},m_2^O(s)\}]\mathrm{d}G(p),$$

where

$$p_E(s) \equiv \frac{c-[\underline{v}-\min\{\underline{v},m_2^O(s)\}]}{[\overline{v}-m_2^O(s)]-[\underline{v}-\min\{\underline{v},m_2^O(s)\}]}.$$

Moreover, the statement in Lemma 2 should be revised, replacing (7) with

$$\begin{aligned} U^\sigma(B) &= \int_{\max\{\underline{p},p_E(\underline{s})\}}^1 [pm_2^O(s)+(1-p)\min\{\underline{v},m_2^O(s)\}]\mathrm{d}G(p) \\ &\quad +(1-\lambda)\int_{\hat{p}}^{\max\{\underline{p},\hat{p}\}}[p\overline{v}+(1-p)\underline{v}-c]\mathrm{d}G(p). \end{aligned} \tag{OA.5.1}$$

As evident from the expressions above, the presence of the term $\min\{\underline{v},m_2^O(s)\}$—which represents the creditor's repayment when he refinances the project with the offer $m_2^O(s)$ and the realized cash flow is $\underline{v}$—adds complexity to the analysis. The key feature under the binary cash flow structure is that the creditors' refinancing loan may or may not be safe. Specifically, in any equilibrium $\mathcal{E}(B;\sigma)$ under it, if $m_2^O(s) \leq \underline{v}$, then $m_2^O(s') = m_2^O(s)$ holds for any $s' > s$. This property implies that if the refinancing loan is safe upon observing a signal s, it remains the same upon observing any higher signal $s' > s$. In contrast, in any equilibrium of the baseline model, $m_2^O(s') < m_2^O(s)$ for any $s' > s$.

In the baseline model, the function $V(p_T)$ defined in (2), which captures the upper bound of the insider's expected return from issuing the startup loan, is obtained by setting $m_2^O(s) = m_2^O(\underline{s})$ and, accordingly, $p_E(s) = p_E(\underline{s})$ for $s \geq \underline{s}$, and then setting $\underline{p} = p_E(\underline{s}) = p_T$ in (7),

[19] If $B \leq \underline{v} - c$, then under any signal structure, a set of safe loan contracts, i.e., $m_1 = B$ and $m_2^O(s) = B$ for any signal s, enables the entrepreneur to borrow the amount B. Hence, information design in this case is irrelevant.

as shown in (OA.2.12) in the proof of Proposition 3. This step captures the idea that pooling signals above $\underline{s}$ enhances the entrepreneur's borrowing capacity.

Under the binary cash flow structure with a positive low realization, the function $V(p_T)$ should be modified but can be derived in a similar way, that is, by setting $m_2^O(s) = m_2^O(\underline{s})$ and $p_E(s) = p_E(\underline{s})$ for $s \geq \underline{s}$, and then setting $\underline{p} = p_E(\underline{s}) = p_T$ in (OA.5.1). Crucially, during this process, depending on whether $m_2^O(\underline{s}) \leq \underline{v}$, or equivalently whether $p_E(\underline{s}) = p_T \leq \frac{c}{\overline{v}-\underline{v}}$, the function $V(p_T)$ takes the form:

$$V(p_T) = \begin{cases} \int_{p_T}^1 [p_T\overline{v} + (1-p_T)\underline{v} - c]\mathrm{d}G(p) + (1-\lambda)\int_{\hat{p}}^{p_T}[p\overline{v} + (1-p)\underline{v} - c]\mathrm{d}G(p), \text{if } \; p_T \leq \frac{c}{\overline{v}-\underline{v}}; \\ \int_{p_T}^1 [p(\overline{v} - \frac{c}{p_T}) + (1-p)\underline{v}]\mathrm{d}G(p) + (1-\lambda)\int_{\hat{p}}^{p_T}[p\overline{v} + (1-p)\underline{v} - c]\mathrm{d}G(p), \text{if } \; p_T > \frac{c}{\overline{v}-\underline{v}}. \end{cases}$$

Hence, the function $V(p_T)$ exhibits a kink at $\frac{c}{\overline{v}-\underline{v}}$. This is because that, in any equilibrium that maximizes the entrepreneur's borrowing capacity or her ex-ante payoff, the outsiders' refinancing offer can be either risky (exceeding $\underline{v}$) or safe (below $\underline{v}$).

To see this, suppose $p_T^* \in (\hat{p}, 1)$ is the unique maximizer of the function $V(p_T)$ above. Then, following the proofs of Lemmas 1, 2 and Proposition 3, it is straightforward that the entrepreneur's maximum borrowing capacity is bounded above by $V(p_T^*)$.

Following the proof of Proposition 4, if $p_T^* > \frac{c}{\overline{v}-\underline{v}}$, the entrepreneur's borrowing need $V(p_T^*)$ can be attained under the pass-or-fail signal structure $\sigma(p; p_T^*)$, where the outsiders' refinancing offer in equilibrium is $\overline{v} - \frac{c}{p_T^*} > \underline{v}$. Moreover, consistent with the analysis in Section 3, pooling project qualities above p_T^* is necessary to maximize the entrepreneur's borrowing capacity.

In contrast, if $p_T^* \leq \frac{c}{\overline{v}-\underline{v}}$, the borrowing need $V(p_T^*)$ is attainable under any monotone signal structure that separates project qualities above p_T^* from those below it, including the pass-or-fail signal structure $\sigma(p; p_T^*)$. In this case, the outsiders' refinancing offer in equilibrium is $p_T^*\overline{v} + (1-p_T^*)\underline{v} - c \leq \underline{v}$. The intuition mirrors that in Section OA.5.3: conditional on knowing that the project quality exceeds p_T^*, the outsiders' refinancing loan is safe in any equilibrium that maximizes the entrepreneur's borrowing capacity, making them indifferent to further information disclosure.

Following the proof of Proposition 5, we can further establish that, under the binary cash flow structure with a positive low realization, for any borrowing need $B \leq V(p_T^*)$, the entrepreneur's maximum ex-ante payoff $\Pi^*(B)$ is given by

$$\Pi^*(B) = \int_{\hat{p}}^1 [p\overline{v} + (1-p)\underline{v} - c]\mathrm{d}G(p) - B - \lambda\int_{\hat{p}}^{p_T(B)}[p\overline{v} + (1-p)\underline{v} - c]\mathrm{d}G(p),$$

where $p_T(B)$ denotes the smallest threshold p_T that satisfies $B = V(p_T)$. Moreover, similar to the preceding analysis, if $p_T(B) > \frac{c}{\overline{v}-\underline{v}}$, pooling project qualities above $p_T(B)$ is necessary to maximize the entrepreneur's ex-ante payoff. Conversely, if $p_T(B) \leq \frac{c}{\overline{v}-\underline{v}}$, any monotone signal structure that separates project qualities above $p_T(B)$ from those below it can achieve the entrepreneur's maximum ex-ante payoff, indicating that in this case the outsiders are insensitive to further information disclosure once the project quality is known to exceed $p_T(B)$.

OA.5.5 Discussion: Non-monotone signal structures

For tractability, our baseline model restricts attention to monotone signal structures. However, it is both interesting and important to consider non-monotone signal structures and to examine whether pass-or-fail signal structures remain optimal for maximizing borrowing capacity or social welfare. We have explored this extension and made some progress; in particular, we can construct an example in which non-monotone signal structures improve borrowing capacity, though the improvement is very limited. In this section, we briefly discuss some of our findings on non-monotone signal structures.

In our baseline model, among all monotone signal structures, the entrepreneur's borrowing capacity is maximized under the pass-or-fail signal structure $\sigma(p; p_T^*)$, yielding

$$V(p_T^*) = \underbrace{\int_{p_T^*}^{1} p\left(v - \frac{c}{p_T^*}\right) \mathrm{d}G(p)}_{\text{cash flow pledged after the pass signal}} + \underbrace{(1-\lambda)\int_{\frac{c}{v}}^{p_T^*} (pv - c)\mathrm{d}G(p)}_{\text{cash flow pledged after the fail signal}} .$$

Accordingly, $V(p_T^*)$ decomposes into cash flows pledged following the pass and fail signals. The first term is below the corresponding net project value because of rent dissipation (interacting with moral hazard), while the second term is below the net project value because of liquidation risk.

Building on the results in the baseline model, a natural non-monotone signal structure to consider is a *two-threshold signal structure* $\sigma(p; p_T^*, \overline{p})$, under which project qualities in $[0, p_T^*) \cup (\overline{p}, 1]$ are pooled to generate a fail signal that deters outsider participation, while project qualities in $[p_T^*, \overline{p}]$ are pooled to generate a pass signal that induces outsider participation. Analogous to the characterization in the main content, we can verify that the entrepreneur's borrowing capacity under this signal structure, denoted by $W(\overline{p})$, is

$$W(\overline{p}) = (1-\lambda)\int_{\overline{p}}^{1} (pv - c)\mathrm{d}G(p) + \int_{p_T^*}^{\overline{p}} p\left(v - \frac{c}{p_T^*}\right) \mathrm{d}G(p) + (1-\lambda)\int_{\frac{c}{v}}^{p_T^*} (pv - c)\mathrm{d}G(p),$$

where $\overline{p} \in [p_T^*, 1]$. Note that for achieving the borrowing capacity $W(\overline{p})$, the threshold $\overline{p}$ must be chosen to ensure that the fail signal effectively deters outsider participation in debt rollover.

Relative to the pass-or-fail signal structure $\sigma(p;p_T^*)$, the signal structure $\sigma(p;p_T^*,\overline{p})$ increases pledgeable cash flow for $p\in(\overline{p},1]$ when the insider does not face a liquidity shock, but reduces it when the insider does. Specifically, let

$$\Delta(\overline{p})\equiv W(\overline{p})-V(p_T^*)=\int_{\overline{p}}^{1}\left((\frac{p}{p_T^*}-1)c-\lambda(pv-c)\right)\mathrm{d}G(p).$$

We are interested in the sign of $\Delta(\overline{p})$. Note that $\Delta(1)=0$, and

$$\Delta'(\overline{p})=\left[(\lambda v-\frac{c}{p_T^*})\overline{p}+(1-\lambda)c\right]g(\overline{p}).$$

Hence, if $\Delta'(1)<0$, which is equivalent to the condition

$$p_T^*<\frac{c}{\lambda v+(1-\lambda)c}, \tag{C}$$

then $\Delta(\overline{p})>0$ for $\overline{p}$ close enough to 1.

The intuition behind Condition (C) is as follows. A small p_T^* amounts to the situation that, under the pass-or-fail signal structure $\sigma(p;p_T^*)$, creditor competition following the pass signal significantly limits the cash flow pledgeability. In this case, truncating and pooling a proper high-quality range $(\overline{p},1]$ into the fail signal allows the insider to capture all net surplus over this range when no liquidity shock occurs, thereby potentially increasing cash flow pledgeability and borrowing capacity. However, whether Condition (C) holds is not immediate, because, as discussed in the manuscript, p_T^* is determined by the parameters c, λ, v, and the distribution $G(p)$.

Below, we present the results for two exercises.

Exercise I. Consider a numerical example with $v=1$, $c=\lambda=\frac{1}{8}$, and $G(p)$ following a Beta distribution $Beta(2,5)$. In this case, Condition (C) holds and there are non-monotone signal structures of the form $\sigma(p;p_T^*,\overline{p})$ that yield a borrowing capacity higher than $V(p_T^*)$.

More specifically, with these parameter values and distribution, simulations yield

$$p_T^*\approx 0.4623,\ \text{and}\ V(p_T^*)\approx 0.15137.$$

Let $\overline{p}^*$ denote the maximizer of $W(\overline{p})$. Then

$$\overline{p}^*\approx 0.7514,\ \text{and}\ W(\overline{p}^*)\approx 0.15140.$$

In addition, since $\Delta'(\overline{p}^*)=0$, it follows that $\Delta'(\overline{p})<0$ for all $\overline{p}>\overline{p}^*$. Hence, $\Delta(\overline{p})>0$ for $\overline{p}\in(\overline{p}^*,1)$. These outcomes are illustrated in the following figure.

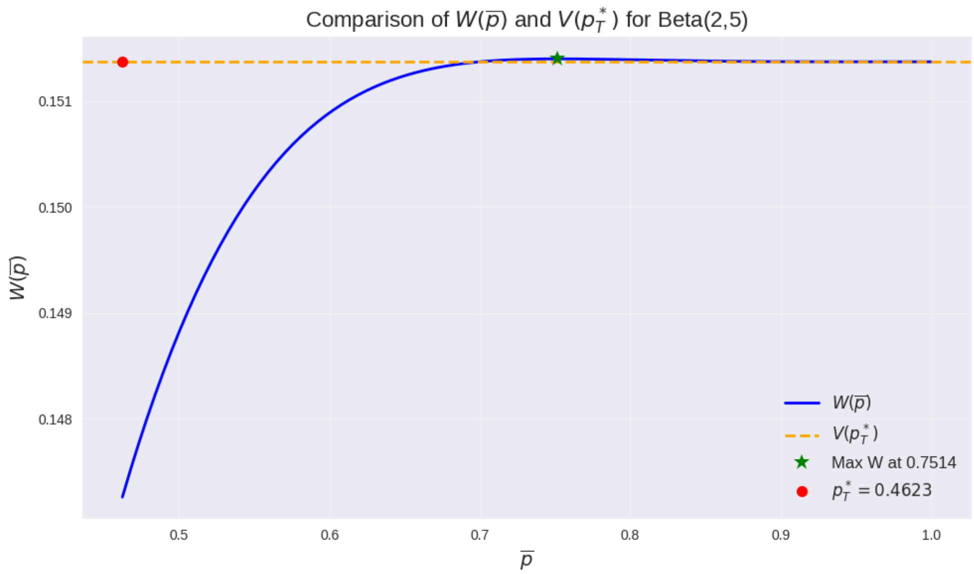

Figure 1. Comparison of $W(\overline{p})$ and $V(p_T^*)$

Therefore, when the distribution $G(p)$ is sufficiently right-skewed—leading to a small value of p_T^*, as in the case of $Beta(2,5)$—it is possible for a two-threshold signal structure $\sigma(p; p_T^*, \overline{p})$ to generate a borrowing capacity higher than $V(p_T^*)$ obtained in the baseline model.

However, the improvement of borrowing capacity is expected to be very small. In the present numerical example, for instance, this improvement is bounded above by

$$\frac{W(\overline{p}^*) - V(p_T^*)}{V(p_T^*)} \approx 0.02\%.$$

This may provide some justification for the use of simple binary signal structures in practice.

Exercise II. Consider the example in which $G(p)$ is uniformly distributed over $[0,1]$. In this case, Condition (C) never holds, and thereby no non-monotone signal structure of the form $\sigma(p; p_T^*, \overline{p})$ can generate a borrowing capacity higher than $V(p_T^*)$.

To see this, suppose that $G(p)$ is the uniform distribution. Define

$$f(x) = \frac{c}{2x^2} - \lambda v x + (\lambda - \frac{1}{2})c,$$

where $x \in [c, 1]$. Indeed, $f(x)$ is constructed so that $f(p_T) = V'(p_T)$.

Let

$$\widehat{x} \equiv \frac{c}{\lambda v + (1-\lambda)c}.$$

After some algebra, one can verify that $f(\widehat{x}) > 0$. Since $f'(x) < 0$ and $f(p_T^*) = 0$, it follows that $p_T^* > \widehat{x}$. Hence, Condition (C) never holds in this case.

We then have $\Delta'(\overline{p}) > 0$ for any $\overline{p} \in [p_T^*, 1]$, so $\Delta(\overline{p})$ increases in $\overline{p}$ until reaching 0 at $\overline{p} = 1$. Equivalently, $\Delta(\overline{p}) < 0$ for all $\overline{p} \in [\overline{p}^*, 1)$. As a result, in this case, there is no two-threshold signal structure $\sigma(p; p_T^*, \overline{p})$ that can yield a borrowing capacity exceeding $V(p_T^*)$. Roughly, the reason for this result is that the uniform distribution $G(p)$ is not sufficiently right-skewed.

Lastly, we shall acknowledge that the exercises above remain quite restrictive, as they only consider two-threshold signal structures with the lower threshold fixed at p_T^*. In principle, even the two-threshold signal structures can take a more general form $\sigma(p; \underline{p}, \overline{p})$, allowing both thresholds $\underline{p}$ and $\overline{p}$ to vary. This added flexibility may further increase borrowing capacity relative to the pass–or-fail signal structure $\sigma(p; p_T^*)$ derived in the baseline model. Technically, characterizing the optimal two-threshold signal structure requires solving a constrained optimization problem over $\underline{p}$ and $\overline{p}$, subject to constraints on inducing or deterring outsider participation in debt rollover. This remains highly challenging in our model, not to mention more complex non-monotone signal structures.

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