Online Appendix A

The Effects of Personal Data Management on Competition and Welfare

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This online appendix provides proofs for the formal results in the paper. The proofs for Propositions 1 and all Corollaries are omitted from this appendix, as they can be readily derived through basic calculations. The detailed analysis of the extensions in Sections 5.5 and 6 are included in the Online Appendix B, which also contains additional extensions.

1 Proof of Lemma 1

Let us consider a consumer of firm B_1. If she erases her data, she receives utility $v_A - tx - \alpha_1^a - \varepsilon$ or utility $v_A - t(1-x) - \alpha_2^a - \varepsilon$ in market A. Otherwise, she receives utility $v_A - tx - p_1^a(x)$ or utility $v_A - t(1-x) - \alpha_2^a$. When her location x is smaller than $\bar{x}_1 = 1/2 + \alpha_2^a/(2t)$, firm A_1 charges the optimal personalized price $p_1^a(x) = \alpha_2^a + t(1-2x)$, equalizing $v_A - tx - p_1^a(x)$ and $v_A - t(1-x) - \alpha_2^a$, and wins over the consumer. Therefore, a consumer $x \in [0, \bar{x}_1]$ does not erase her data if and only if $v_A - tx - p_1^a(x) \geq \max\{v_A - tx - \alpha_1^a, v_A - t(1-x) - \alpha_2^a\} - \varepsilon$, which is equivalent to $x > \tilde{x}_1$. When the consumer's location $x > \bar{x}_1$, she always purchases from firm A_2 under price α_2, irrespective of data erasure. She refrains from data erasure because erasing data does not generate any benefit but brings costs ε. In summary, a consumer of firm B_1 chooses to erase data if and only if her location x in market A satisfies $x < \tilde{x}_1$. The data erasure strategy of firm B_2's consumers can be derived using a similar approach.

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2 Proof of Proposition 2

We have $\beta_i^n - \beta_i^* = (t-\varepsilon)(t+15\varepsilon)/(16t) > 0$. Consequently, it follows that $\pi_{B_i}^n - \pi_{B_i}^* = (\beta_i^n - \beta_i^*)/2 > 0$. In market A, we find that $\pi_{A_i}^n - \pi_{A_i}^* = (t-\varepsilon)(3\varepsilon - t)/(32t) > 0$. Thus, we conclude that $\pi_{A_i}^n + \pi_{B_i}^n > \pi_{A_i}^* + \pi_{B_i}^*$.

3 Proof of Proposition 3

Consumer surplus in market A under no data management and data management is $CS_A^n = E[CS_{B_1}]/2 + E[CS_{B_2}]/2 = v_A - t$ and $CS_A^* = E[CS_{B_1}]/2 + E[CS_{B_2}]/2 = v_A - \frac{5t^2-\varepsilon^2}{4t}$, respectively. Consumer surplus in market B under no data management and data management is

$$CS_B^n = \int_0^{1/2} (v_B - ky - \beta_1^n)dy + \int_{1/2}^1 (v_B - k(1-y) - \beta_2^n)dy = v_B - \frac{20k - 7t}{16},$$
$$CS_B^* = \int_0^{1/2} (v_B - ky - \beta_1^*)dy + \int_{1/2}^1 (v_B - k(1-y) - \beta_2^*)dy = v_B - \frac{20tk - 8t^2 - 14t\varepsilon + 15\varepsilon^2}{16t}.$$

With these expressions, verifying Proposition 3 is straightforward.

4 Proof of Proposition 4

■ **Collection opt-out** Firm A_1's profit from the uniform price is $\alpha_1[(1-r)(1-\delta_h)\bar{x}_2 + r\hat{x}]$, where $\bar{x}_2 = \frac{1}{2} - \frac{\alpha_1}{2t}$ and $\hat{x} = \frac{1}{2} + \frac{\alpha_2 - \alpha_1}{2t}$. Similarly, firm A_2's profit from the uniform price is $\alpha_2[(1-r)\delta_h(1-\bar{x}_1) + r(1-\hat{x})]$. The equilibrium uniform prices are $\alpha_1^\circ = \frac{t((\delta_h-1)r-\delta_h)(-2\delta_h+2\delta_h r+r+2)}{4(\delta_h-1)\delta_h+(1-2\delta_h)^2r^2+\left(-8\delta_h^2+8\delta_h-4\right)r}$ and $\alpha_2^\circ = \frac{t\left(2(\delta_h-1)\delta_h+\delta_h(2\delta_h-3)r^2+\left(-4\delta_h^2+5\delta_h-3\right)r\right)}{4(\delta_h-1)\delta_h+(1-2\delta_h)^2r^2+\left(-8\delta_h^2+8\delta_h-4\right)r}$, in which the superscript $\circ$ indicates opting out of data collection. In the competition of market A, firms' optimal personalized prices $p_1^\circ(x)$ and $p_2^\circ(x)$ are determined in the same manner as in the baseline model. The profits of firm A_1 and firm A_2 are

$$\pi_{A_1} = \alpha_1^\circ[(1-r)(1-\delta_h)\bar{x}_2^\circ + r\hat{x}^\circ] + (1-r)\delta_h \int_0^{\bar{x}_1^\circ} p_1^\circ(x)dx,$$
$$\pi_{A_2} = \alpha_2^\circ[(1-r)\delta_h(1-\bar{x}_1^\circ) + r(1-\hat{x}^\circ)] + (1-r)(1-\delta_h) \int_{\bar{x}_2^\circ}^1 p_2^\circ(x)dx.$$

The expected utility of B_1's opt-in consumers, B_2's opt-in consumers, and opt-out consumers are

$$E[CS^h_{B_1}] = \int_0^{\bar{x}_1^\circ} (v_A - p_1^\circ(x) - tx)dx + \int_{\bar{x}_1^\circ}^1 (v_A - \alpha_2^\circ - t(1-x))dx = \int_0^1 (v_A - \alpha_2^\circ - t(1-x))dx,$$
$$E[CS^h_{B_2}] = \int_0^{\bar{x}_2^\circ} (v_A - \alpha_1^\circ - tx)dx + \int_{\bar{x}_2^\circ}^1 (v_A - p_2^\circ(x) - t(1-x))dx = \int_0^1 (v_A - \alpha_1^\circ - tx)dx,$$
$$E[CS^l_{B_1}] = E[CS^l_{B_2}] = \int_0^{\hat{x}^\circ} (v_A - \alpha_1^\circ - tx)dx + \int_{\hat{x}^\circ}^1 (v_A - \alpha_2^\circ - t(1-x))dx.$$

The indifferent consumer δ_l in market B is determined by $v_B - \beta_1 - t\delta_l + E[CS^l_{B_1}] = v_B - \beta_2 - t(1-\delta_l) + E[CS^l_{B_2}]$, resulting in $\delta_l = \frac{t+\beta_2-\beta_1}{2t}$. This expression is the same as the Hotelling indifferent consumer. The indifferent consumer δ_h in market B is determined by $v_B - \beta_1 - t\delta_h + E[CS^h_{B_1}] = v_B - \beta_2 - t(1-\delta_h) + E[CS^h_{B_2}]$. Since $E[CS^h_{B_1}] > E[CS^h_{B_2}]$ holds if and only if $\delta_h > 1/2$, firms' price competition in market B intensifies compared to the competition analyzed in Section 4.1.

The profit of firm B_1 is $\pi_{B_1} = \beta_1[r\delta_l + (1-r)\delta_h]$ and the profit of firm B_2 is $\pi_{B_2} = \beta_2[r(1-\delta_l) + (1-r)(1-\delta_h)]$. Firm i chooses β_i to maximize its two-market profit $\pi_{A_i} + \pi_{B_i}$, resulting in equilibrium prices expressed as $\beta_i^\circ = \frac{t(9+9r+15r^2+11r^3+4r^4)}{16(1+2r-r^2+r^3)}$ and $\delta^\circ = 1/2$. We observe that β° exhibits a U-shaped pattern with respect to $r \in [0,1]$, attaining its minimum value at $r = 0.16$. It is straightforward to verify that $\beta_i^\circ < \beta_i^H$ consistently holds, where $\beta_i^H = t$ represents the Hotelling price, and that $\beta_i^\circ > \beta_i^n$ holds whenever $r > 0.36$. Additionally, we can confirm that $\pi^\circ_{A_i} + \pi^\circ_{B_i} > \pi^n_{A_i} + \pi^n_{B_i}$ whenever $r > 0.19$; $CS_A^\circ < CS_A^n$ always holds; $CS_B^\circ < CS_B^n$ whenever $r > 0.36$; and $CS_A^\circ + CS_B^\circ < CS_A^n + CS_B^n$ whenever $r > 0.16$.

■ **Full control** Firm A_1's profit from the uniform price is $\alpha_1[(1-r)(1-\delta_h)\bar{x}_2 + r\hat{x} + (1-r)\delta_h\tilde{x}_1]$. Similarly, firm A_2's profit from the uniform price is $\alpha_2[(1-r)\delta_h(1-\bar{x}_1) + r(1-\hat{x}) + (1-r)(1-\delta_h)(1-\tilde{x}_2)]$. The equilibrium uniform prices are $\alpha_1^\circ = \frac{t(2\delta_h - 2(\delta_h-1)r+1)-(r-1)\varepsilon((\delta_h-1)r-2\delta_h)}{r+2}$ and $\alpha_2^\circ = \frac{(r-1)\varepsilon(\delta_h(r-2)+2)+t(2\delta_h(r-1)+3)}{r+2}$. In the competition of market A, firms' optimal personalized prices $p_1^\circ(x)$ and $p_2^\circ(x)$ are determined in the same manner as in the baseline model. The profits of firm A_1 and firm A_2 are

$$\pi_{A_1} = \alpha_1^\circ[(1-r)(1-\delta_h)\bar{x}_2^\circ + r\hat{x}^\circ + +(1-r)\delta_h\tilde{x}_1^\circ] + (1-r)\delta_h \int_{\tilde{x}_1^\circ}^{\bar{x}_1^\circ} p_1^\circ(x)dx,$$
$$\pi_{A_2} = \alpha_2^\circ[(1-r)\delta_h(1-\bar{x}_1^\circ) + r(1-\hat{x}^\circ) + (1-r)(1-\delta_h)(1-\tilde{x}_2^\circ)] + (1-r)(1-\delta_h) \int_{\bar{x}_2^\circ}^{\tilde{x}_2^\circ} p_2^\circ(x)dx.$$

The expected utility of B_1's opt-in consumers, B_2's opt-in consumers, and opt-out consumers are

$$E[CS^h_{B_1}] = \int_0^{\tilde{x}_1^\circ} (v_A - \alpha_1^\circ - tx - \varepsilon)dx + \int_{\tilde{x}_1^\circ}^{\bar{x}_1^\circ} (v_A - p_1^\circ(x) - tx)dx + \int_{\bar{x}_1^\circ}^1 (v_A - \alpha_2^\circ - t(1-x))dx,$$

$$E[CS^h_{B_2}] = \int_0^{\bar{x}_2^\circ} (v_A - \alpha_1^\circ - tx)dx + \int_{\bar{x}_2^\circ}^{\tilde{x}_2^\circ} (v_A - p_2^\circ(x) - t(1-x))dx + + \int_{\tilde{x}_2^\circ}^1 (v_A - \alpha_2^\circ - t(1-x) - \varepsilon)dx,$$

$$E[CS^l_{B_1}] = E[CS^l_{B_2}] = \int_0^{\hat{x}^\circ} (v_A - \alpha_1^\circ - tx)dx + \int_{\hat{x}^\circ}^1 (v_A - \alpha_2^\circ - t(1-x))dx.$$

The indifferent consumer δ_l in market B is determined by $v_B - \beta_1 - t\delta_l + E[CS^l_{B_1}] = v_B - \beta_2 - t(1-\delta_l) + E[CS^l_{B_2}]$, resulting in $\delta_l = \frac{t+\beta_2-\beta_1}{2t}$. This expression is same as the Hotelling indifferent consumer, which is less sensitive to firms' prices in market B compared to the indifferent consumer in Section 4.2. The indifferent consumer δ_h in market B is determined by $v_B - \beta_1 - t\delta_h + E[CS^h_{B_1}] = v_B - \beta_2 - t(1-\delta_h) + E[CS^h_{B_2}]$. Since $E[CS^h_{B_1}] > E[CS^h_{B_2}]$ holds if and only if $\delta_h > (2t(r-4) + \varepsilon(6-5r+2r^2))/(4(r-1)(2t+(r-2)\varepsilon))$, firms compete more aggressively in market B to attract consumers.

The profit of firm B_1 is $\pi_{B_1} = \beta_1[r\delta_l + (1-r)\delta_h]$ and the profit of firm B_2 is $\pi_{B_2} = \beta_2[r(1-\delta_l) + (1-r)(1-\delta_h)]$. Firm i chooses β_i to maximize its two-market profit $\pi_{A_i} + \pi_{B_i}$, resulting in the equilibrium prices as

$$\beta_i^\circ = \frac{t\left(4\left(r^2+7r+4\right)t^2 + 4\left(r^3+r^2+5r-7\right)t\varepsilon + \left(r^4-r^3+25r^2-55r+30\right)\varepsilon^2\right)}{16\left(r\left(r^2-3r+2\right)\varepsilon^2 + (r+2)t^2 + 2(r-1)rt\varepsilon\right)}$$

and $\delta^\circ = 1/2$. Given ε, we observe that β° exhibits a U-shaped pattern with respect to $r \in [0,1]$. It is straightforward to verify that $\beta_i^\circ < \beta_i^H$ consistently holds, where $\beta_i^H = t$ represents the Hotelling price, and that $\beta_i^\circ > \beta_i^*$ holds whenever $r > \hat{r}(t,\varepsilon)$. Additionally, we can confirm that $\pi^\circ_{A_i} + \pi^\circ_{B_i} > \pi^*_{A_i} + \pi^*_{B_i}$ whenever $r > \hat{r}_\pi(t,\varepsilon)$; $CS^\circ_A < CS^*_A$ always holds; $CS^\circ_B < CS^*_B$ whenever $r > \hat{r}_B(t,\varepsilon)$; and $CS^\circ_A + CS^\circ_B < CS^*_A + CS^*_B$ whenever $r > \hat{r}_{AB}(t,\varepsilon)$.

5 Proof for Proposition 5

Firm A_i's uniform price is used solely to poach the rival's targeted consumers, as in the benchmark of no data management in Section 4.1. Consequently, the equilibrium uniform prices in market A are $\alpha_1^{**} = \alpha_2^{**} = t/2$, implying $\bar{x}_1^{**} = 3/4$ and $\bar{x}_2^{**} = 1/4$. It is straightforward to check that no firm has incentive to deviate from α_1^{**} and α_2^{**}. Two firms' equilibrium profits in market

A are

$$\pi_{A_1}^{**} = \alpha_1^{**}(1-\delta)\bar{x}_2^{**} + \delta\int_0^{\frac{1}{2}} t(1-2x)dx + (1-\delta)\int_{\bar{x}_2^{**}}^{\frac{1}{2}} t(1-2x)dx = \frac{t}{16}(3+\delta),$$
$$\pi_{A_2}^{**} = \alpha_2^{**}\delta(1-\bar{x}_1^{**}) + \delta\int_{\frac{1}{2}}^{\bar{x}_1^{**}} t(2x-1)dx + (1-\delta)\int_{\frac{1}{2}}^{1} t(2x-1)dx = \frac{t}{16}(4-\delta).$$

Notice that $E[CS_{B_1}] = E[CS_{B_2}]$ always holds, implying the indifferent consumer in market B is $\delta = (t+\beta_2-\beta_1)/(2t)$. Firm B_i chooses β_i to maximize its two-market profit: $\Pi_i = \pi_{A_i}^{**} + \pi_{B_i}$. The equilibrium uniform prices are $\beta_1^{**} = \beta_2^{**} = 15t/16$, implying $\delta^{**} = 1/2$.

We consider two benchmarks for comparison. In benchmark one (with superscript n), consumers cannot manage their data (Section 4.1). In the other benchmark (with superscript $*$), consumers can only use data erasure for data management (Section 4.2). The ranking of firms' profits is: $\pi_{A_i}^n > \pi_{A_i}^* > \pi_{A_i}^{**}$, $\pi_{B_i}^{**} > \pi_{B_i}^n > \pi_{B_i}^*$, and $\Pi_i^{**} > \Pi_i^n > \Pi_i^*$.

In market A, consumer surplus without data management is $CS_A^n = v_A - t$; consumer surplus under data erasure is $CS_A^* = v_A - \frac{5t^2-\varepsilon^2}{4t}$; consumer surplus under data erasure and data portability is

$$\begin{aligned}
CS_A^{**} =& \frac{1}{2}\left[\int_0^{\frac{1}{2}}(v_A - tx - t(1-2x))dx + \int_{\frac{1}{2}}^{\bar{x}_1^{**}}(v_A - t(1-x) - t(2x-1))dx + \int_{\bar{x}_1^{**}}^{1}(v_A - t(1-x) - \alpha_2^{**})dx\right] \\
&+ \frac{1}{2}\left[\int_0^{\bar{x}_2^{**}}(v_A - tx - \alpha_1^{**})dx + \int_{\bar{x}_2^{**}}^{\frac{1}{2}}(v_A - tx - t(1-2x))dx + \int_{\frac{1}{2}}^{1}(v_A - t(1-x) - t(2x-1))dx\right] \\
=& v_A - \frac{11}{16}t.
\end{aligned}$$

It is straightforward to check that $CS_A^* < CS_A^n < CS_A^{**}$ holds.

In market B, consumer surplus without data management is $CS_B^n = v_B - \frac{13}{16}t$; consumer surplus under data erasure is $CS_B^* = v_B - \frac{12t^2+2t\varepsilon-\varepsilon^2}{16t}$; consumer surplus under data erasure and data portability is

$$CS_B^{**} = \int_0^{1/2}(v_B - tx - \beta_1^{**})dx + \int_{1/2}^{1}(v_B - t(1-x) - \beta_2^{**})dx = v_B - \frac{19}{16}t.$$

We can check that $CS_B^* > CS_B^n > CS_B^{**}$ holds. Moreover, $CS_A^n + CS_B^n > CS_A^{**} + CS_B^{**}$ always holds.

6 Proof of Proposition 6

■ **Benchmark: no data management** Our analysis begins with the observation that the indifferent consumer in market B, δ, is determined by (3), which is independent of the servicing cost c_B.

Suppose firm B_1 acquires consumers from $[0,\delta]\times[0,\bar{c}_B]$, while firm B_2 obtains consumers from $[\delta,1]\times[0,\bar{c}_B]$ in market B. In market A, firm A_1's optimal personalized prices $p_1(x,c_A)$ are determined by: $v_A - tx - p_1(x,c_A) = v_A - t(1-x) - \alpha_2$, leading to $p_1(x,c_A) = \alpha_2 + t(1-2x)$. Similarly, firm A_2's optimal personalized prices are defined by $p_2(x,c_A) = \alpha_1 + t(2x-1)$. With these personalized prices, firm A_1 can successfully poach firm A_2's targeted consumers whose x lies in $[0,\bar{x}_2]$, where $\bar{x}_2 = (t + c_A - \alpha_1)/(2t)$ and increases with c_A (see Figure 6). Firm A_2 can poach firm A_1's targeted consumers whose x lies in $[\bar{x}_1,1]$, where $\bar{x}_1 = (\alpha_2 + t - c_A)/(2t)$ and decreases with c_A (see Figure 6). Firm A_1 selects α_1 to maximize its profit from the uniform price: $\int_0^{\bar{c}_A}\int_0^{\bar{x}_2}(\alpha_1 - c_A)\frac{1-\delta}{\bar{c}_A}dxdc_A$, where $(1-\delta)/\bar{c}_A$ is the density function of firm A_2's targeted consumers. Similarly, firm A_2 selects α_2 to maximize its profit from the uniform price: $\int_0^{\bar{c}_A}\int_{\bar{x}_1}^{1}(\alpha_2 - c_A)\frac{\delta}{\bar{c}_A}dxdc_A$. The equilibrium uniform prices and the cutoffs are

$$\alpha_1^n = \alpha_2^n = \frac{t+\bar{c}_A}{2}, \quad \bar{x}_1^n = \frac{3}{4} + \frac{\bar{c}_A - 2c_A}{4t}, \quad \bar{x}_2^n = \frac{1}{4} - \frac{\bar{c}_A - 2c_A}{4t}.$$

Both firms' optimal personalized prices, $p_i^n(x,c_A)$, can be calculated accordingly. Their profits from personalized prices are:

$$\int_0^{\bar{c}_A}\int_0^{\bar{x}_1^n}(p_1^n(x,c_A) - c_A)\frac{\delta}{\bar{c}_A}dxdc_A \quad \text{and} \quad \int_0^{\bar{c}_A}\int_{\bar{x}_2^n}^{1}(p_2^n(x,c_A) - c_A)\frac{1-\delta}{\bar{c}_A}dxdc_A.$$

As a result, the equilibrium profits of firm A_1 and firm A_2 are

$$\pi_{A_1}^n = \frac{3(2+7\delta)t^2 + (3\delta-2)(\bar{c}_A)^2}{48t}, \quad \pi_{A_2}^n = \frac{3(9-7\delta)t^2 + (1-3\delta)(\bar{c}_A)^2}{48t}. \tag{1}$$

Firm A_1's profit increases with $\bar{c}_A$ whenever $\delta > 2/3$, while firm A_2's profit increases with $\bar{c}_A$ whenever $\delta < 1/3$.

In market B, a consumer indexed by (y,c_B) receives the same expected utilities $E[CS_{B_1}]$ and $E[CS_{B_2}]$. Consequently, the indifferent consumer is given by $\delta = (\beta_2 - \beta_1 + t)/(2t)$. The profits of firms in market B are

$$\pi^n_{B_1} = \int_0^{\bar{c}_B}\int_0^{\delta}(\beta_1 - c_B)\frac{1}{\bar{c}_B}dydc_B = (\beta_1 - \frac{\bar{c}_B}{2})\delta,\ \ \pi^n_{B_2} = \int_0^{\bar{c}_B}\int_{\delta}^{1}(\beta_2 - c_B)\frac{1}{\bar{c}_B}dydc_B = (\beta_2 - \frac{\bar{c}_B}{2})(1-\delta).$$

The equilibrium prices in market B are $\beta^n_1 = \beta^n_2 = (9t^2 + 8t\bar{c}_B - (\bar{c}_A)^2)/(16t)$, resulting in $\delta^n = 1/2$. Note that as $\bar{c}_A$ increases, the equilibrium price β^n_i decreases. Firm B_i's equilibrium profits are $\pi^n_{B_1} = \pi^n_{B_2} = (9t^2 - (\bar{c}_A)^2)/(32t)$.

■ **Under data management** Suppose firm B_1 obtains consumers from $[0,\delta]\times[0,\bar{c}_B]$ and firm B_2 obtains consumers from $[\delta,1]\times[0,\bar{c}_B]$ in market B. In market A, the consumers who choose to erase data are identical to those in Lemma 1, indicating that firm A_i's opt-out consumers are independent of their servicing costs c_A. Each firm in market A selects its uniform price to maximize the profits from poached consumers and opt-out consumers. Specifically, firm A_1 selects α_1 to maximize its profit from the uniform price:

$$\int_0^{\bar{c}_A}\int_0^{\bar{x}_2}(\alpha_1 - c_A)\frac{1-\delta}{\bar{c}_A}dxdc_A + \int_0^{\bar{c}_A}\int_0^{\tilde{x}_1}(\alpha_1 - c_A)\frac{\delta}{\bar{c}_A}dxdc_A.$$

Similarly, firm A_2 selects α_2 to maximize its profit from the uniform price:

$$\int_0^{\bar{c}_A}\int_{\bar{x}_1}^{1}(\alpha_2 - c_A)\frac{\delta}{\bar{c}_A}dxdc_A + \int_0^{\bar{c}_A}\int_{\tilde{x}_2}^{1}(\alpha_2 - c_A)\frac{1-\delta}{\bar{c}_A}dxdc_A.$$

The equilibrium uniform prices in market A are

$$\alpha^*_1 = \frac{t+\bar{c}_A}{2} + \delta(t-\varepsilon),\quad \alpha^*_2 = \frac{t+\bar{c}_A}{2} + (1-\delta)(t-\varepsilon).$$

Consequently, the equilibrium cutoffs are

$$\tilde{x}^*_1 = \frac{(1-\delta)(t-\varepsilon)}{t},\quad \tilde{x}^*_2 = 1 - \frac{\delta(t-\varepsilon)}{t},$$
$$\bar{x}^*_1(c_A) = \frac{3t + 2(1-\delta)(t-\varepsilon) + \bar{c}_A - 2c}{4t},\quad \bar{x}^*_2(c_A) = \frac{t - 2\delta(t-\varepsilon) - \bar{c}_A + 2c}{4t}.$$

Both firms' optimal personalized prices, $p^*_i(x, c_A)$, can be calculated accordingly. The profits from personalized prices for the two firms are

$$\int_0^{\bar{c}_A}\int_{\tilde{x}^*_1}^{\bar{x}^*_1}(p^*_1(x,c_A) - c_A)\frac{\delta}{\bar{c}_A}dxdc_A \quad \text{and} \quad \int_0^{\bar{c}_A}\int_{\bar{x}^*_2}^{\tilde{x}^*_2}(p^*_2(x,c_A) - c_A)\frac{1-\delta}{\bar{c}_A}dxdc_A.$$

We can verify that firm A_1's profit increases with $\bar{c}_A$ whenever $\delta > 2/3$, while firm A_2's profit increases with $\bar{c}_A$ whenever $\delta < 1/3$.

In market B, each consumer can obtain the expected utilities $E[CS_{B_1}]$ and $E[CS_{B_2}]$ from market A, calculated in the same manner as in the baseline model. We find

$$E[CS_{B_1}] - E[CS_{B_2}] = \frac{(\delta - (1-\delta))(t-\varepsilon)\varepsilon}{t} > 0 \text{ if and only if } \delta > 1-\delta.$$

The indifferent consumer is $\delta = \frac{1}{2} + \frac{t(\beta_2-\beta_1)}{2(\varepsilon^2 - t\varepsilon + t^2)}$, which is independent of c_B. The profits of firms in market B are

$$\pi_{B_1} = \int_0^{\bar{c}_B} \int_0^{\delta} (\beta_1 - c_B)\frac{1}{\bar{c}_B} dy dc_B = (\beta_1 - \frac{\bar{c}_B}{2})\delta, \quad \pi_{B_2} = \int_0^{\bar{c}_B} \int_{\delta}^{1} (\beta_2 - c_B)\frac{1}{\bar{c}_B} dy dc_B = (\beta_2 - \frac{\bar{c}_B}{2})(1-\delta).$$

The equilibrium prices in market B are

$$\beta_1^* = \beta_2^* = \frac{8t^2 + 15\varepsilon^2 - 14t\varepsilon - (\bar{c}_A)^2 + 8t\bar{c}_B}{16t},$$

implying $\delta^* = 1/2$. Note that as $\bar{c}_A$ increases, the equilibrium price β_i^* decreases. Firm B_i's equilibrium profits are $\pi_{B_1}^* = \pi_{B_2}^* = (8t^2 + 15\varepsilon^2 - 14t\varepsilon - (\bar{c}_A)^2)/(32t)$.

7 Proof for Proposition 7

Given consumers' expectation of prices in market A, a consumer of firm A_1 chooses to opt in if and only if her location in market A is larger than $\tilde{x}_1 = \frac{1}{2} + \frac{\alpha_2^a - \alpha_1^a}{2t} - \frac{b}{2t}$. Conversely, a consumer of firm A_2 opts in if and only if her location in market A is smaller than $\tilde{x}_2 = \frac{1}{2} + \frac{\alpha_2^a - \alpha_1^a}{2t} + \frac{b}{2t}$. Because consumers rationally anticipate equilibrium prices, inequality $\tilde{x}_1 < \hat{x} < \tilde{x}_2$ should hold. Moreover, we can define $\bar{x}_1 = \frac{1}{2} + \frac{\alpha_2}{2t}$ and $\bar{x}_2 = \frac{1}{2} - \frac{\alpha_1}{2t}$. After consumers' opt-in choices, the promised benefits paid to opt-in consumers become sunk. Firm A_1 chooses α_1 to maximize $\alpha_1 [(1-\delta)\bar{x}_2 + \delta\tilde{x}_1]$ and firm A_2 chooses α_2 to maximize $\alpha_2 [\delta(1-\bar{x}_1) + (1-\delta)(1-\tilde{x}_2)]$. The subsequent analysis mirrors that of the baseline model. We find:

$$\alpha_1^\star = \frac{t}{2} + \delta(t-b), \quad \alpha_2^\star = \frac{t}{2} + (1-\delta)(t-b), \quad \tilde{x}_1^\star = \frac{(1-\delta)(t-b)}{t}, \quad \tilde{x}_2^\star = 1 - \frac{\delta(t-b)}{t}.$$

The equilibrium personalized prices are $p_1^\star(x) = \alpha_2^\star + t(1-2x)$ and $p_2^\star(x) = \alpha_1^\star + t(2x-1)$. It is important to note that the opt-in benefit b is already promised to all opting-in consumers, and therefore does not influence the personalized prices.

In market B, a consumer's expected market-A surplus if she purchases from firm B_i is:

$$E[CS_{B_1}] = \int_0^{\tilde{x}_1^\star} (v_A - \alpha_1^\star - tx)dx + \int_{\tilde{x}_1^\star}^{\bar{x}_1^\star} (v_A - p_1^\star(x) - tx + b)dx + \int_{\bar{x}_1^\star}^{1} (v_A - \alpha_2^\star - t(1-x) + b)dx,$$
$$E[CS_{B_2}] = \int_0^{\bar{x}_2^\star} (v_A - \alpha_1^\star - tx + b)dx + \int_{\bar{x}_2^\star}^{\tilde{x}_2^\star} (v_A - p_2^\star(x) - t(1-x) + b)dx + \int_{\tilde{x}_2^\star}^{1} (v_A - \alpha_2^\star - t(1-x))dx.$$

The indifferent consumer δ is determined in the same manner as in equation (3), resulting in $\delta = \frac{1}{2} + \frac{t(\beta_2 - \beta_1)}{2(b^2 - tb + t^2)}$. The equilibrium uniform prices in market B are detailed in Section 5.4, which implies that $\delta^\star = 1/2$.

Consumer surplus in the two market is

$$CS_A^\star = v_A - \frac{5t^2 - b^2 - 4tb}{4t}, \quad CS_B^\star = \int_0^{1/2} (v_B - ty - \beta_1^\star)dy + \int_{1/2}^{1} (v_B - t(1-y) - \beta_2^\star)dy.$$

The comparison to the benchmark equilibrium in Section 4.2 under the condition $b = \varepsilon$ is straightforward and will be omitted here.