Balancing Power in Decentralized Governance: Quadratic Voting and Information Aggregation

Online Appendix

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A Technical Details

A.1 The Poisson Binomial

We provide the definition of a Poisson Binomial below for reference:

Definition 9 (Poisson Binomial). *When X is distributed as a Poisson Binomial, i.e., $X \sim \mathrm{PoiBin}(p_1, \ldots, p_n)$ then*

$$\Pr[X = k] = \sum_{A \in \binom{[n]}{k}} \prod_{i \in A} p_i \prod_{j \in A^c} (1 - p_j), \tag{19}$$

where $\binom{[n]}{k}$ is the set of all subsets of k integers that can be selected from $[n]$.

A.2 Partitioning the strategy space

In many situations, the success probability does not change when a voter changes their vote. For example, in the two-voter setting, if Voter 1 votes $v_1 = 1$, then the success probability will be the same whether the Voter 2 votes $v_2 = 2$ or $v_2 = 3$, or $v_2 = k$ for any $k \geq 2$. We formalize this idea as the equivalence of voting strategies.

Definition 10 (Equivalence of strategies). *We say that voting strategies, $\vec{v}$ and $\vec{w}$ are equivalent (written $\vec{v} \equiv \vec{w}$) if*

$$F_{\vec{v}} = F_{\vec{w}}. \tag{20}$$

Let $\mathcal{S}_{\vec{v}}$ denote the set of voting strategies that are equivalent to $\vec{v}$, i.e.,

$$\mathcal{S}_{\vec{v}} = \{\vec{w} \mid \vec{v} \equiv \vec{w}\}. \tag{21}$$

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We use the word "dominate" to reflect the fact that if two voting strategies lead to the same final outcome, voters will prefer the one with the smallest weights (because voting is costly).

Lemma 3 shows that the relation in Definition 10 is in fact a mathematical "equivalence relation."

Lemma 3. *Definition 10 partitions the space of strategies into a* finite *number of equivalence classes, i.e., there are a finite number of voting strategies* $(\vec{v}_1^*, \ldots, \vec{v}_t^*)$ *with the property that for every* $\vec{v} \in (\mathbb{Z}_{\geq 0})^n$*, there exists an* i *such that*

$$\vec{v}_i^* \equiv \vec{v}. \tag{22}$$

Alternatively,

$$\bigcup_{i=1}^{t} \mathcal{S}_{\vec{v}_i^*} = (\mathbb{Z}_{\geq 0})^n \tag{23}$$

and

$$\mathcal{S}_{\vec{v}_i^*} \cap \mathcal{S}_{\vec{v}_j^*} = \emptyset \quad \textit{for all } 1 \leq i < j \leq t. \tag{24}$$

Proof of Lemma 3. To see that the relation in Definition 10 is indeed an equivalence relation, we need to check Reflexivity, Symmetry and Transitivity.

Since $\vec{v} \equiv \vec{w}$ if and only if $F_{\vec{v}} = F_{\vec{w}}$, the reflexivity, symmetry and transitivity of this relation follow immediately from the reflexivity, symmetry and transitivity of the equality relation on sets.

Since this is an equivalence relation, it partitions the set of strategies $\vec{v} \in \mathbb{Z}_{\geq 0}^n$.

Now, by Definition 10, $\vec{v} \equiv \vec{w}$ if and only if $F_{\vec{v}} = F_{\vec{w}}$, but when there are n voters, there are at most 2^n different sets $A \subseteq [n]$, so there are at most 2^n "canonical" strategies. In practice, we will find that there are often many fewer. □

Lemma 3 establishes that even though each voter has an infinite strategy space ($v_i \in \mathbb{Z}_{\geq 0}$), there are only a *finite* set of outcomes of interest. This is because since voting is costly, if two vote vectors lead to the same outcome, voters will always prefer the one in which they place lower weights on their vote.

Since voting is costly, for a fixed success probability, voters will always prefer the strategy that has them voting the *least*. For example, in the two voter setting, if Voter 1 votes $v_1 = 1$, then Voter 2 will always prefer $v_2 = 2$ to $v_2 = 3$ since both strategies have the same success probability (p_2), but voting $v_2 = 2$ costs less. This is formalized in Definition 11.

Definition 11 (Dominating strategies)**.** *We say that* $\vec{v}$ dominates $\vec{w}$ *if* $\vec{w} \in \mathcal{S}_{\vec{v}}$ *and* $v_i \leq w_i$ *for all*

$i \in [n]$. *We say that* $\vec{v}$ strictly dominates $\vec{w}$ *if* $\vec{v}$ *dominates* $\vec{w}$ *and there exists an* $i \in [n]$ *such that* $v_i < w_i$.

Another way to view Definition 11 is that the success probability, $p(\vec{v})$ is piecewise constant in each v_i, so voter i is always incentivized to choose the lowest v_i that guarantees some level of $p(\vec{v})$.

Example 4. *Assume we are in a game with 3 voters, and consider two possible voting strategies:* $\vec{v} = (1, 1, 4)$ *and* $\vec{w} = (1, 1, 5)$. *Using our definitions, the two strategies are "equivalent," and* $\vec{v}$ *"dominates"* $\vec{w}$. *The two strategies equivalent because in either case, voter 3 unilaterally decides the vote. Furthermore, strategy* $\vec{v}$ *dominates* $\vec{w}$ *because there is no need for voter 3 to add additional weight (from 4 to 5) to their own vote – doing so costs more and will not change the outcome.*

In this case

$$S_{(1,1,3)} = S_{(1,1,4)} = S_{(1,1,5)} = \{(v_1, v_2, v_3) \mid v_1 + v_2 < v_3\}. \tag{25}$$

Because Definition 10 defines an equivalence relation over strategies, we can partition the strategy space into a set of equivalence classes. Critically, however, although the strategy space is infinite ($\mathbb{Z}^n_{\geq 0}$), for any parameter vector $(\vec{u}, \vec{p}) \in \mathcal{P}$, there are only a *finite* number of equivalence classes of strategies.

Example 5 (Winning coalitions in 1p1v voting)**.** *In the 1p1v voting game,* $v_i = 1$ *for all* i, *so* $F_{\vec{v}} = F_{\vec{1}} = \left\{A \subset [n] \mid |A| > \left\lfloor \frac{n}{2} \right\rfloor \right\}$. *In other words,* $F_{\vec{1}}$ *is all subsets that have at least* $\lfloor n/2 \rfloor$ *people, that is, all simple majorities.*

B Analysis of 1p1v Mechanism

In this section, we compare the strategic games (e.g. LV, QV) to the 1p1v game, where every voter purchases exactly one vote.

B.1 3-voter setting

Proposition 2 shows that there are only 5 possible equilibrium success probabilities in the 3-voter game, $0, p_3, p_2, p_1, P_3$, under any mechanism. In order to characterize which voting mechanism is "better," we need to identify which success probabilities each allows and how they compare to each other. In other words, as a first step, we need to order them.

By definition $p_1 \geq p_2 \geq p_3$, but where does P_3 fit? Lemma 4 shows that P_3 (the probability that the majority is correct) is always higher than the two lower-precision voters.

Lemma 4. *The majority vote is always more accurate than voter 2 or voter 3, i.e.,*

$$P_3 > p_2 > p_3. \tag{26}$$

On the other hand, p_1, the success probability of the most accurate voter, can be larger or smaller than the majority success probability, P_3. Lemma 5 gives a simple criteria for identifying when dictatorship by voter 1 yields better outcomes than democracy (i.e., success probability P_3).

Lemma 5 (When is dictatorship by voter 1 optimal?). *Dictatorship by the most informed voter is optimal, i.e., $p_1 > P_3$, when*

$$\ln\left(\frac{p_1}{1-p_1}\right) > \sum_{i=2}^{n} \ln\left(\frac{p_i}{1-p_i}\right). \tag{27}$$

In light of Lemma 4, the only way LV or QV can do better than 1p1v is when $p_1 > P_3$ (i.e., voter 1 is more accurate than the majority). To see this, note that Proposition 2 shows that the only possible success probabilities in the strategic game are $\{0, p_1, p_2, p_3, P_3\}$. In light of Lemma 4, we have $0 \leq p_3 \leq p_2 \leq P_3$. So the only uncertainty is where p_1 falls relative to P_3. If $p_1 \leq P_3$, then the maximum possible success probability under LV or QV is P_3 (which is the success probability under 1p1v), so in this case, LV and QV can do no better than 1p1v.

Corollary 5. *If $p_1 < P_3$, then both LV and QV are no better than 1p1v.*

Corollary 5 raises the question: when is $p_1 > P_3$? One criterion is given in Lemma 5, but an important special case is when all voters are equally well informed, i.e., $p_1 = p_2 = p_3$. In this case, the Condorcet Jury Theorem tells us that $p_1 < P_3$, so we have

Corollary 6. *In the 3-voter setting, if the voters are homogeneous (i.e., all p_i are equal), then LV and QV are no better than 1p1v.*

Another implication of Proposition 2 is that LV and QV differ from the 1p1v mechanism because they allow "dictatorship" equilibria (i.e., $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$). Then, the partial ordering of success probabilities in Lemma 4 allows us to conclude that the dictatorship by *uninformed* voters (voter 2 or voter 3) is always worse than democracy.

B.2 Asymptotic analysis of 1p1v

Plugging $v_{\max} = 1$ into Proposition 6, we recover the 1p1v setting, and obtain the following corollary.

Corollary 7 (The Condorcet Jury Theorem for the 1p1v Mechanism)**.** *Suppose the precisions* $\{p_i\}$ *are bounded away from* $\frac{1}{2}$*, then under the 1p1v mechanism (where all voters are forced to vote* $v_i = 1$*) we have*

$$\lim_{n\to\infty} p\left(\vec{1}_n\right) = 1. \tag{28}$$

In other words, 1p1v without abstention effectively aggregates information asymptotically.

C Robustness to Alternative Tie-Breaking Rules

The main paper adopts an adversarial tie-breaking rule, a well-established method in the cryptography literature on security proofs and Byzantine fault-tolerant protocols (Canetti 2001, Benhaim et al. 2023). The idea is that, in the event of a tie, a hypothetical adversary could nudge the outcome toward the worst-case option at minimal cost. This abstraction also has the added benefit of simplifying the exposition and analysis, as it bypasses the need to specify voter beliefs on whether a tie leads to a "good" or "bad" outcome. It simply sets the probability of a bad outcome in case of a tie, to 100%.

While this abstraction is well established in the theoretical literature, it may not directly translate to smart contract implementation in the context of Web3 protocols. In the following two sections, we establish that a key result of the paper—that QV can underperform LV under uncertainty—continues to hold under more practical tie-breaking mechanisms.

C.1 Breaking Ties Towards the Status Quo

We re-visit the 2-voter game from Section 4.1, but now assume that ties are broken towards the status quo of the platform. For example, take the DAI stablecoin platform mentioned in the introduction and imagine the current status quo is that "WBTC tokens are not allowed as a collateral type for minting DAI tokens". Users are asked to vote on whether to reverse this rule or not (binary yes/no vote). If the election results in a tie, the status quo is automatically maintained.

To be able to compute voter payoffs, we need to specify the probability that the status quo is the "good" option *conditioned* on the fact that there is a tie. Following Bayes' Rule, we have

$$\Pr[\text{SQ is good} \mid \text{Tie}] = \frac{\Pr[\text{Tie} \mid \text{SQ is good}] \cdot \Pr[\text{SQ is good }]}{\Pr[\text{Tie}]} \tag{29}$$

$$= \frac{\Pr[\text{Tie} \mid \text{SQ is good}] \cdot \Pr[\text{SQ is good }]}{\Pr[\text{Tie} \mid \text{SQ is good}] \cdot \Pr[\text{SQ is good }] + \Pr[\text{Tie} \mid \text{SQ is bad}] \cdot \Pr[\text{SQ is bad }]} \tag{30}$$

Now,

$$\Pr[\text{Tie} \mid \text{SQ is good}] = \Pr[\text{Tie} \mid \text{SQ is bad}] \tag{31}$$

$$= p_1 \cdot (1 - p_2) + (1 - p_2) \cdot p_1, \tag{32}$$

where Eq. (32) comes from the fact that there can only be a tie if both voters choose the same vote weight, v, and one votes for the "good" option and one votes for the "bad" option.

Thus we can factor a $\Pr[\text{Tie} \mid \text{SQ is good}]$ out of the numerator and denominator of Eq. (30), and we obtain

$$\Pr[\text{SQ is good} \mid \text{Tie}] = \Pr[\text{SQ is good}]\,. \tag{33}$$

In other words, the probability that the status quo is the "good" option conditioned on a tied vote is just the *a priori* probability that the status quo is the good option. For simplicity, assume this probability to be $\frac{1}{2}$.[2]

Under this type of tie-break rule, if votes are chosen to be $(0, 0)$, the payout to voter i is $\frac{1}{2} \cdot u_i$. In other words, even if no one votes, voters may still obtain a non-zero payout.

When both voters vote $v > 0$, the probability of obtaining the "good" outcome in case of a tie is:

$$\underbrace{\frac{1}{2}}_{\text{Prob SQ is good}} \cdot \underbrace{\left(\underbrace{p_1 \cdot (1 - p_2)}_{\text{1 right 2 wrong}} + \underbrace{p_2 \cdot (1 - p_1)}_{\text{2 right 1 wrong}} \right)}_{\text{tied vote}} + \underbrace{p_1 \cdot p_2}_{\text{both right}} \tag{34}$$

$$= \frac{1}{2}\left(p_1 - p_1 \cdot p_2 + p_2 - p_1 \cdot p_2\right) + p_1 \cdot p_2 \tag{35}$$

$$= \frac{p_1 + p_2}{2} \tag{36}$$

Lemma 6. *When ties are broken towards the status quo (with prior $\frac{1}{2}$), then the only possible*

[2]Note that if we assume that ties are broken towards the status quo, then we *must* make an assumption about the prior probability that the status quo is the good option. By contrast, if we assume that ties are always broken towards the "bad" option (as in the main body of this paper, or if we assumed that ties were broken by a fair coin-flip), then we would *not* need to make any assumptions about the prior probability that the status quo is the good option.

equilibria are of the form:

$$(0,0) \tag{37}$$

$$(1,0) \tag{38}$$

$$(0,1) \tag{39}$$

The only equilibria in this game are of the form $v_1 = v_2$ *or* $v_1 = 1$*,* $v_2 = 0$*.*

Proof of Lemma 6. If $v_{q_1} > v_{q_2} > 0$, then voter q_2 can profitably deviate by $v_{q_2} \to v_{q_2} - 1$. The payoff is the same $(u_{q_2} \cdot p_{q_1})$, but the voting cost is lower. If $v_{q_1} > v_{q_2} + 1$, then voter q_1 can deviate to $v_{q_1} - 1$, which reducing their voting cost, but leaves their payoff $(u_{q_1} \cdot p_{q_1})$ unchanged.

This only leaves the cases where $v_{q_1} = v_{q_2}$, or $v_{q_1} = v_{q_2} + 1$, and $v_2 = 0$. □

Lemma 7. *Vote vector* $(0,0)$ *is an equilibrium if and only if the following conditions hold:*

$$p_1 \leq \frac{1}{2} + \frac{1}{u_1} \tag{40}$$

$$p_2 \leq \frac{1}{2} + \frac{1}{u_2} \tag{41}$$

Proof of Lemma 7. First, we show that if $(0,0)$ is an equilibrium, we consider the possible deviations for each player. At $(0,0)$ voter i's payoff is $\frac{u_i}{2}$. If voter 1 votes 1, then the state is $(1,0)$, then voter 1's payoff becomes $u_1 \cdot p_1$, so voter 1 will deviate if (and only if) $\frac{u_1}{2} < u_1 \cdot p_1 - 1$. And similarly for voter 2. We can summarize this as:

Deviation	**Condition**
$(1,0)$	$u_1 \cdot \left(p_1 - \frac{1}{2}\right) > 1$
$(0,1)$	$u_2 \cdot \left(p_2 - \frac{1}{2}\right) > 1$

□

Lemma 8. *Votes* $v_1 = v_2 = v > 0$ *is* never *an equilibrium.*

Proof of Lemma 8. In this game, the success probability at (v,v) is $\frac{p_1+p_2}{2}$.

But voter 2 can profitably deviate to $(v,0)$, by setting $v_2 = 0$, which increases the success probability from $\frac{p_1+p_2}{2}$ to p_1, and decreases voter 2's cost from v_2^m to 0.

So (v,v) is never an equilibrium with $v > 0$. □

Lemma 9. *Vote vector* $(1,0)$ *is an equilibrium if and only if the following conditions hold:*

$$p_1 \geq \frac{1}{2} + \frac{1}{u_1} \tag{42}$$

$$\tag{43}$$

and $(0,1)$ *is an equilibrium if and only if:*

$$p_2 \geq \frac{1}{2} + \frac{1}{u_2} \tag{44}$$

$$p_2 \geq p_1 - \frac{2}{u_1} \tag{45}$$

$$p_2 \geq p_1 - \frac{2^m}{u_1} \tag{46}$$

Proof of Lemma 9. From $(1,0)$ the possible deviations are, voter 1 can profitably deviate to $(0,0)$ if $u_1 \cdot \frac{1}{2} > u_1 \cdot p_1 - 1$, i.e., $p_1 < \frac{1}{2} + \frac{1}{u_1}$. Voter 2 cannot profitably deviate since setting $v_2 = 1$, will *decrease* the success probability from p_1 to $\frac{p_1+p_2}{2}$. Setting $v_2 = v > 1$, will *decrease* the success probability from p_1 to p_2, and increase the voting cost from 0 to v^m.

From $(0,1)$, voter 2 can profitably deviate to $(0,0)$ if $u_2 \cdot \frac{1}{2} > u_2 \cdot p_2 - 1$, i.e., $p_2 < \frac{1}{2} + \frac{1}{u_2}$. From $(0,1)$, voter 1 can profitably deviate to $(1,1)$ if $u_1 \cdot \frac{p_1+p_2}{2} - 1 > u_1 \cdot p_2$, i.e., $p_2 < p_1 - \frac{2}{u_1}$. From $(0,1)$, voter 1 can profitably deviate to $(2,1)$ if $u_1 \cdot p_1 - 2^m > u_1 \cdot p_2$, i.e., $p_2 < p_1 - \frac{2^m}{u_1}$. □

Lemma 10. *If*

$$\frac{1}{2} + \frac{1}{u_2} \leq p_2 \tag{47}$$

$$\frac{1}{2} + \frac{1}{u_1} \leq p_1 \tag{48}$$

$$\frac{2}{u_1} \leq p_1 - p_2 \leq \frac{4}{u_1}, \tag{49}$$

then the equilibria are:

QV	***LV***
$(1,0)$	$(1,0)$
$(0,1)$	

Since $p_2 < p_1$, $(0,1)$ *is a strictly* worse *equilibrium (in the sense of Definition 7).*

Proof. From Lemma 7, $(0,0)$ is not an equilibrium under QV or LV. From Lemma 8, (v,v) is an equilibrium under QV or LV for any $v > 0$. From Lemma 9, $(0,1)$ is not an equilibrium under

LV, because $p_2 \leq p_1 - \frac{2}{u_1}$. On the other hand, by Lemma 9, $(0, 1)$ is an equilibrium under QV because $p_2 \geq p_1 - \frac{4}{u_1}$. □

Corollary 8. *When ties are broken towards the status quo (with prior $\frac{1}{2}$), there exists a non-empty set of parameters where QV is worse than LV. Figure 4 illustrates this set.*

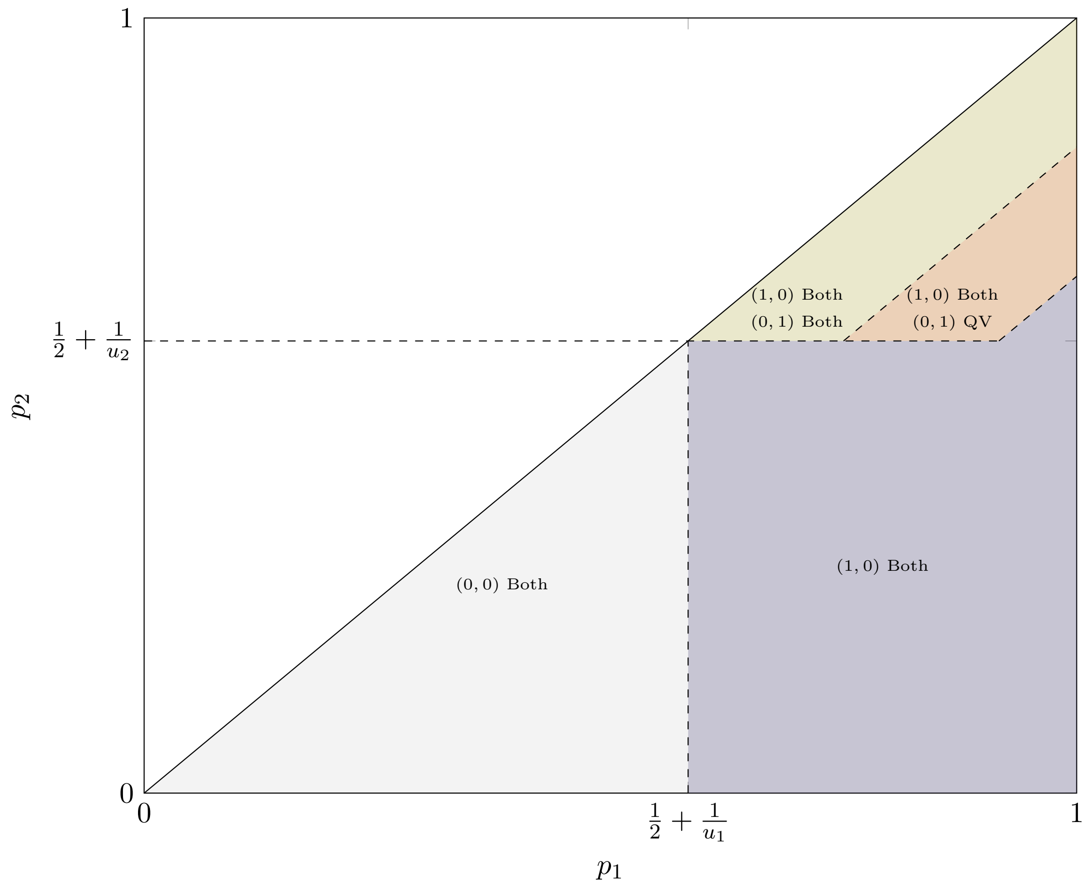

Figure 4: 2-Player game equilibrium votes (v_1, v_2) for each mechanism under a "default to status-quo" type of tie-breaking rule, with prior $\frac{1}{2}$ (c.f. Fig. 2).

C.2 Breaking Ties Randomly

Consider the more general case of the two-voter setting where ties are broken towards the "good" option with probability τ.

Now, at $(0, 0)$, the payoffs are $u_i \cdot \tau$. When both voters vote $v > 0$, the probability of the correct outcome is

$$\tau \cdot \left(\underbrace{p_1 \cdot (1-p_2)}_{\text{1 right 2 wrong}} + \underbrace{p_2 \cdot (1-p_1)}_{\text{2 right 1 wrong}} \right) + \underbrace{p_1 \cdot p_2}_{\text{both right}} = \tau \left(p_1 - p_1 \cdot p_2 + p_2 - p_1 \cdot p_2\right) + p_1 \cdot p_2 \tag{50}$$

$$= \tau \left(p_1 + p_2\right) + (1 - 2 \cdot \tau) p_1 \cdot p_2 \tag{51}$$

In the special case $\tau = \frac{1}{2}$, where ties are broken by a coin-flip, then the success probability at (v, v) (for $v > 0$) given in Eq. (51) reduces to $\frac{p_1+p_2}{2}$ as in Eq. (36) in Section C.1. When $\tau = 0$, we have the setting analyzed in the main text (Section 4.1)

Lemma 11. *In the two voter game, when ties are broken towards the "good" option with probability τ, then the only possible equilibria are of the form:*

$$(0, 0) \tag{52}$$

$$(v, v) \quad \textit{(for } v > 0\textit{)} \tag{53}$$

$$(1, 0) \tag{54}$$

$$(0, 1) \tag{55}$$

The only equilibria in this game are of the form $v_1 = v_2$ or $v_1 = 1$, $v_2 = 0$.

Proof of Lemma 11. By renaming the players, we can assume that $v_1 \geq v_2 \geq 0$. If $v_1 > v_2 > 0$, then voter 2 can profitably deviate by $v_2 \to v_2 - 1$. The payoff is the same $(u_2 \cdot p_1)$, but the voting cost is lower. If $v_1 > v_2 + 1$, then voter 1 can deviate to $v_1 - 1$, which reducing their voting cost, but leaves their payoff $(u_1 \cdot p_1)$ unchanged.

This only leaves the cases where $v_1 = v_2$, or $v_1 = v_2 + 1$, and $v_2 = 0$. □

Lemma 12. *In the two voter game, when ties are broken towards the "good" option with probability τ, then $(0, 0)$ is an equilibrium if and only if the following conditions hold:*

$$p_1 \leq \tau + \frac{1}{u_1} \tag{56}$$

$$p_2 \leq \tau + \frac{1}{u_2} \tag{57}$$

Proof of Lemma 12. First, we show that if $(0, 0)$ is an equilibrium, we consider the possible deviations for each player. At $(0, 0)$ voter i's payoff is $u_i \cdot \tau$. If voter 1 votes 1, and we deviate to

$(1, 0)$, then voter 1's payoff becomes $u_1 \cdot p_1$, so voter 1 will deviate if (and only if) $u_1 \cdot \tau < u_1 \cdot p_1 - 1$. And similarly for voter 2. We can summarize this as:

Deviation	**Condition**
$(1, 0)$	$u_1 \cdot (p_1 - \tau) > 1$
$(0, 1)$	$u_2 \cdot (p_2 - \tau) > 1$

□

Lemma 13. *In the two voter game, when ties are broken towards the "good" option with probability τ, then $v_1 = v_2 = v > 0$ is an equilibrium if and only if the following conditions hold:*

$$p_1 \leq \tau\,(p_1 + p_2) + (1 - 2 \cdot \tau)\,p_1 \cdot p_2 - \frac{v^m}{u_2} \tag{58}$$

$$p_2 \leq \tau\,(p_1 + p_2) + (1 - 2 \cdot \tau)\,p_1 \cdot p_2 - \frac{v^m}{u_1} \tag{59}$$

when $\tau \leq \frac{1}{2}$, these equations are never satisfied and (v, v) is not *an equilibrium. (In Lemma 8 we showed that (v, v) is not an equilibrium when $\tau = \frac{1}{2}$)*

Proof of Lemma 13. From Eq. (51), we have the success probability at (v, v) (with $v > 0$) is

$$p_{\text{tie}} \stackrel{\text{def}}{=} \tau\,(p_1 + p_2) + (1 - 2 \cdot \tau)\,p_1 \cdot p_2 \tag{60}$$

The possible deviations are:

Deviation	**Condition**
$(v + 1, v)$	$p_1 - p_{\text{tie}} > \frac{(v+1)^m - v^m}{u_1}$
$(0, v)$	$p_2 - p_{\text{tie}} > \frac{-v^m}{u_1}$
$(v, v + 1)$	$p_2 - p_{\text{tie}} > \frac{(v+1)^m - v^m}{u_2}$
$(v, 0)$	$p_1 - p_{\text{tie}} > \frac{-v^m}{u_2}$

Now, notice that since $v > 0$,

$$p_1 - p_{\text{tie}} > \frac{(v + 1)^m - v^m}{u_1} \Rightarrow p_1 - p_{\text{tie}} > 0 \tag{61}$$

$$\Rightarrow p_1 - p_{\text{tie}} > -\frac{v^m}{u_2} \tag{62}$$

So if the deviation $(v, v) \rightarrow (v+1, v)$ is profitable, then the deviation $(v, v) \rightarrow (v, 0)$ is also profitable.

Similarly,

$$p_2 - p_{\text{tie}} > \frac{(v+1)^m - v^m}{u_2} \Rightarrow p_2 - p_{\text{tie}} > 0 \tag{63}$$

$$\Rightarrow p_2 - p_{\text{tie}} > -\frac{v^m}{u_1} \tag{64}$$

So if the deviation $(v, v) \to (v, v+1)$ is profitable, then the deviation $(v, v) \to (0, v)$ is also profitable. Thus, (v, v) is an equilibrium if and only if $(v, v) \to (v, 0)$ and $(v, v) \to (0, v)$ are both *not* profitable. Now, note that when $\tau \leq \frac{1}{2}$, $p_{\text{tie}} \leq \frac{p_1+p_2}{2} \leq \max(p_1, p_2)$, so in this case either $p_1 - p_{\text{tie}}$ or $p_2 - p_{\text{tie}}$ must be positive, and (v, v) is not an equilibrium. □

Lemma 14. *In the two voter game, when ties are broken towards the "good" option with probability* τ*, then* $(1, 0)$ *is an equilibrium if and only if the following conditions hold:*

$$p_1 \geq \tau + \frac{1}{u_1} \tag{65}$$

$$p_1 \geq p_{tie} - \frac{1}{u_2} \tag{66}$$

$$p_1 \geq p_2 - \frac{2^m}{u_2} \tag{67}$$

and $(0, 1)$ *is an equilibrium if and only if:*

$$p_2 \geq \tau + \frac{1}{u_2} \tag{68}$$

$$p_2 \geq p_{tie} - \frac{1}{u_1} \tag{69}$$

$$p_2 \geq p_1 - \frac{2^m}{u_1} \tag{70}$$

Proof of Lemma 14. From $(1, 0)$ the possible deviations are:

Deviation	**Condition**
$(0, 0)$	$u_1 \cdot (\tau - p_1) > -1$
$(1, 1)$	$u_2 \cdot (p_{\text{tie}} - p_1) > 1$
$(1, 2)$	$u_2 \cdot (p_2 - p_1) > 2^m$

The case is symmetric for $(0, 1)$. □

Lemma 15. *If*

$$\tau + \frac{1}{u_2} \leq p_2 \tag{71}$$

$$\tau + \frac{1}{u_1} \leq p_1 \tag{72}$$

$$p_{tie} - \frac{1}{u_2} \leq p_1 \tag{73}$$

$$p_{tie} - \frac{1}{u_1} \leq p_2 \tag{74}$$

$$p_1 - \frac{4}{u_1} \leq p_2 \leq p_1 - \frac{2}{u_1} \tag{75}$$

Then the equilibria are:

QV	***LV***
$(1, 0)$	$(1, 0)$
$(0, 1)$	

Since $p_2 < p_1$, $(0, 1)$ *is a strictly* worse *equilibrium.*

Proof. From Lemma 12, $(0, 0)$ is not an equilibrium under QV or LV. From Lemma 13, (v, v) is an equilibrium under QV or LV for any $v > 0$. From Lemma 14, $(0, 1)$ is not an equilibrium under LV, because $p_2 \leq p_1 - \frac{2}{u_1}$. On the other hand, by Lemma 14, $(0, 1)$ is an equilibrium under QV because $p_2 \geq p_1 - \frac{4}{u_1}$. □

Corollary 9. *In the two-voter game, when ties are broken towards the "good" option with probability* τ*, there exists a non-empty set of parameters where QV is worse than LV. Figure 5 illustrates this set.*

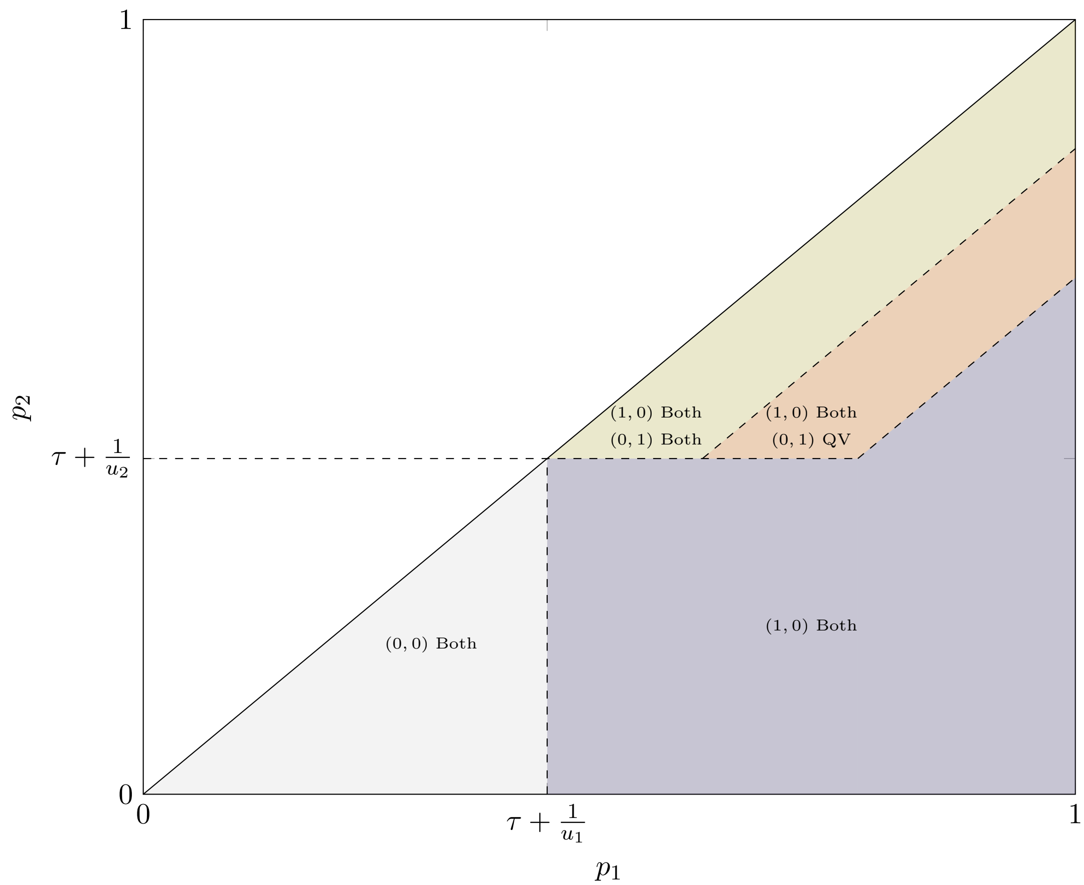

Figure 5: 2-Player game equilibrium votes (v_1, v_2) for each mechanism under a general randomized tie-breaking rule τ (c.f. Fig. 2).

D Alternative characterizations of the model

D.1 "Signals"

In Nitzan and Paroush (1982), voters have a precision but do not explicitly receive a "signal" telling them which option to vote for. It is common, however, in the voting literature to imagine that each voter receives a signal, and in this section we show how this is equivalent to our model.

Let $q \in \{\pm 1\}$ be the random variable denoting which option is the "good" option. Initially, we assume voters are uninformed, i.e., their prior is that

$$\Pr[q = 1] = \Pr[q = -1] = \frac{1}{2} \tag{76}$$

Voter i receives a "signal" $s_i' \in \{\pm 1\}$, with

$$p_i \stackrel{\text{def}}{=} \Pr[s_i' = q] \tag{77}$$

With this terminology, each voter receives a signal, $s'_i \in \{\pm 1\}$, and decides how many votes to purchase as a function of this signal.

Now, let

$$s_i \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } s'_i = q \\ -1 & \text{otherwise} \end{cases} \tag{78}$$

and we obtain the "correctness" value used in our model.

We call this a change-of-variables because

$$s_i \stackrel{\text{def}}{=} 1 - \left|q - s'_i\right| \tag{79}$$

As it is more convenient to work directly with the s_i (rather than the signals $\{s'_i\}$), this change-of-variables from s' to s, makes our analysis significantly simpler, and so we do not include the underlying signals $\{s'_i\}$ in the definition of our model in the main body of the text (Note that this similar to Nitzan and Paroush (1982), where they model precisions directly but never discuss underlying signals).

D.2 Linear strategies

As in Tsoukalas and Falk (2020), Vives (1988), Myatt and Wallace (2012), we assume voters follow a *linear strategy*, i.e., voter i's strategy is given by

$$w(s_i) = v_i \cdot s'_i \tag{80}$$

for some integer $v_i \in \mathbb{Z}_{\geq 0}$. This assumption was also used in Nitzan and Paroush (1982) (although there, they assumed $v_i \in \mathbb{R}^+$).

Note that this assumption of linear strategies is a restriction on the strategy space, since, in principle, voters could follow a more general (non-linear) strategy. In our setting, where the signal is dichotomous, the most general strategy would be parameterized by *two* values

$$w_i(s'_i) = \begin{cases} v_i^+ \text{ if } s'_i = 1 \\ v_i^- \text{ if } s'_i = -1 \end{cases} \tag{81}$$

As in other scenarios, this assumption is justified by the fact that if all other voters follow a linear strategy, then any voter's best response is also linear.

Lemma 16. *If all voters $j \neq i$ follow a linear strategy ($w_j(s'_j) \stackrel{\text{def}}{=} v_j \cdot s'_j$, for some $v_j \in \mathbb{Z}_{\geq 0}$), then* *voter i's best-response is also linear.*

Proof. By assumption, all voters other than voter i follow a linear strategy. So for all $j \neq i$, there is a v_j such that $w_j(s_i') = v_j \cdot s_i'$.

Now, suppose Voter i's best response is a strategy (v_i^+, v_i^-). Let U^+ (resp. U^-) denote Voter i's expected payoff on receiving $s_i = 1$ (resp. $s_i = -1$). So

$$U^+ \stackrel{\text{def}}{=} u_i \cdot \Pr\left[v_i^+ \cdot s_i + \sum_{j \neq i} v_j \cdot s_j > 0 \;\middle|\; s_i' = 1\right] \tag{82}$$

$$U^- \stackrel{\text{def}}{=} u_i \cdot \Pr\left[v_i^- \cdot s_i + \sum_{j \neq i} v_j \cdot s_j > 0 \;\middle|\; s_i' = -1\right] \tag{83}$$

Although the underlying *signals*, $\left\{s_j'\right\}$, are dependent, (i.e., s_j' is dependent on s_i'), the *correctness* of each voter j is *independent* of voter i's signal, i.e., s_j is *independent* of s_i'. In fact, because the prior is balanced, even s_i is independent of s_i'

$$p_i = \Pr[s_i = 1] = \Pr\left[s_i = 1 \mid s_i' = 1\right] = \Pr\left[s_i = 1 \mid s_i' = -1\right] \tag{84}$$

This means

$$\Pr\left[\sum_{j \neq i} v_j \cdot s_j = X \;\middle|\; s_i = 1\right] = \Pr\left[\sum_{j \neq i} v_j \cdot s_j = X \;\middle|\; s_i = -1\right] = \Pr\left[\sum_{j \neq i} v_j \cdot s_j = X\right] \tag{85}$$

Plugging this back into Eqs. (82) and (83), we have

$$U^+ = u_i \cdot \Pr\left[v_i^+ \cdot s_i + \sum_{j \neq i} v_j \cdot s_j > 0\right] = u_i \cdot \Pr\left[\sum_{j \neq i} v_j \cdot s_j > -v_i^+ \cdot s_i\right] \tag{86}$$

$$U^- = u_i \cdot \Pr\left[-v_i^- \cdot s_i + \sum_{j \neq i} v_j \cdot s_j > 0\right] = u_i \cdot \Pr\left[\sum_{j \neq i} v_j \cdot s_j > v_i^- \cdot s_i\right] \tag{87}$$

Note that with a linear strategy $U^+ = U^-$ because $v_i^+ = -v_i^-$. Now, we have that Voter i's payoff is

$$U^+ \cdot p_i + U^- \cdot (1 - p_i) \tag{88}$$

Now, if $U^+ > U^-$, then Voter i can set $v_i^- = -v_i^+$, and achieve $U^+ > U^+ \cdot p_i + U^-(1 - p_i)$, with a linear strategy. Similarly, if $U^+ < U^-$, then Voter i can set $v_i^+ = -v_i^-$, and achieve $U^- > U^+ \cdot p_i + U^-(1-p_i)$, with a linear strategy.

On the other hand, if $U^+ = U^-$, Voter i can set $v_i^+ = -v_i^-$ (or vice-versa) and achieve the same payoff with a linear strategy.

□

Note that the parallel to Lemma 16 in the continuous setting was argued in Myatt and Wallace (2012)[Appendix A].

Lemma 17. *If voters follow a linear strategy $w_i(s_i') = v_i \cdot s_i'$, then in equilibrium, $v_i \geq 0$, so it suffices to consider strategies where voters follow their signals (rather than go against them).*

Proof. Suppose the strategies for Voter j for $j \neq i$ are fixed as $w_j(s_j') = v_j \cdot s_j'$.

Now, suppose Voter i follows the strategy $w_i(s_i') = v_i \cdot s_i'$. Then if we define $V_{-i} \stackrel{\text{def}}{=} \sum_{j \neq i} v_j \cdot s_j$, Voter i's payoff will be

$$u_i \cdot \Pr\left[v_i \cdot s_i + \sum_{j \neq i} v_j \cdot s_j > 0\right] = u_i \cdot \Pr\left[v_i \cdot s_i > V_{-i}\right] \tag{89}$$

$$= u_i \left(p_i \cdot \Pr\left[v_i > -V_{-i}\right] + (1-p_i) \cdot \Pr\left[-v_i > -V_{-i}\right]\right) \tag{90}$$

Now if $v_i > 0$,

$$\Pr\left[v_i > -V_{-i}\right] > \Pr\left[-v_i > -V_{-i}\right] \tag{91}$$

Since, by assumption, $p_i > \frac{1}{2}$, choosing $v_i > 0$, yields a higher payoff than choosing $v_i < 0$. □

Note that Lemma 17 seems obvious, but there are models where voters are incentivized to disregard their signals. For example, in the Swing Voter's Curse Feddersen and Pesendorfer (1996), uninformed voters may vote contrary to their priors in order to "cancel out" the votes of non-strategic "partisan" voters. Now, in Feddersen and Pesendorfer (1996), uninformed voters are *completely uninformed*, i.e., they receive no signal, but it is likely that the "curse" would still appear if they had very weak signals as well.

This type of behavior, where voters vote *against* their best-guess of the truth occurs in Feddersen and Pesendorfer (1996) because there are non-strategic, partisan voters, and voting is costless. Lemma 17 shows that this does *not* occur in our model, where all voters are strategic, and voting is costly.

Lemma 18 (Order-irrelevance of signals)**.** *When voters follow linear strategies (i.e., $w_i(s_i') = v_i \cdot s_i'$) then the two models lead to equivalent equilibria*

- ***Model 1:***
  - *Voters receive their signals,* s'_i
  - *Voters decide how many votes to buy and cast,* $|w_i(s'_i)|$
- ***Model 2:***
  - *Voters purchase a number of voting "shares,"* V_i
  - *Voters receive their signals,* s'_i
  - *Voters decide how many votes to cast* $w_i(s'_i)$*, with* $|w_i(s'_i)| \leq V_i$

Proof. When voters follow a *linear* strategy, then v_i is the same for both realizations of s'_i, i.e., the voter will want to cast v_i votes whether $s'_i = 1$ or $s'_i = -1$ (the voter uses the signal to determine which option to vote for, but not to define the strength of their signal).

In Model 2, if voters knows they will follow a linear strategy, then this strategy is defined by a single value, v_i (since $w_i(s'_i) = v_i \cdot s_i$, so in round 1, voters will purchase $V_i = v_i$ votes, and in round 2 voters will cast them for option s'_i. □

E Proofs for the Main Paper

Proof of Lemma 1. We have:

$$W(\vec{v}) = \sum_i u_i \cdot p(\vec{v}) - f(v_i) + r_i(v_i) \tag{92}$$

$$= \sum_i u_i \cdot p(\vec{v}) \text{ (redistribution is budget-balanced)} \tag{93}$$

$$= p(\vec{v}) \sum_i u_i. \tag{94}$$

Since the u_i are fixed (and thus independent of $\vec{v}$),

$$\arg\max_{\vec{v}} W(\vec{v}) = \arg\max_{\vec{v}} p(\vec{v}). \tag{95}$$

□

Proof of Lemma 2.

- **Irreflexivity:** It is clearly irreflexive, i.e., $m \not\prec_{\vec{u},\vec{p}} m$.

- **Antisymmetry:** If $m_1 \prec_{\vec{u},\vec{p}} m_2$, then if m_1 has an additional worse equilibrium, $\mathcal{E}_1$, then $W(\mathcal{E}_1)$ is less than the minimum utility of all of m_2's equilibria, so m_2 cannot have an additional worse equilibrium. On the other hand, if $\mathcal{E}_1 \in \mathcal{N}_{m_1} \setminus \mathcal{N}_{m_2}$, so m_2 cannot be missing a better equilibrium. Thus it cannot be that $m_2 \prec_{\vec{u},\vec{p}} m_1$. The other possibility is that m_1 is missing better equilibria. In this case, there is an equilibrium $\mathcal{E}_2 \in \mathcal{N}_{m_2} \setminus \mathcal{N}_{m_1}$, and this $W(\mathcal{E}_2)$ is greater than the maximum of all of m_1's equilibria. In this case, m_2 cannot have additional worse equilibria or be missing better equilibria.

- **Transitivity:** Suppose $m_1 \prec_{\vec{u},\vec{p}} m_2$ and $m_2 \prec_{\vec{u},\vec{p}} m_3$. We consider 4 cases depending on which of the two conditions holds for $m_1 \prec_{\vec{u},\vec{p}} m_2$, and which of the two conditions holds for $m_2 \prec_{\vec{u},\vec{p}} m_3$. We use the terms "worse" and "better" to denote two possible conditions in Definition 6.

  - m_1 is worse than m_2, and m_2 is worse than m_3. Then m_1 is worse than m_3.
  - m_1 is worse than m_2 and m_3 is better than m_2. Then m_3 is better than m_1.
  - m_2 is better than m_1 and m_2 is worse than m_3. Then m_1 is worse than m_3.
  - m_2 is better than m_1 and m_3 is better than m_2. Then m_3 is better than m_1.

□

Proof of Proposition 1. Let v_1, v_2 be the voters' votes in equilibrium *ordered by vote weight* (so $v_1 \geq v_2$). Then $v_2 = 0$, because if $v_2 > 0$, then v_2 could decrease their vote to $v_2 - 1$, and this would decrease their voting cost, and would *increase* the success probability if $v_1 = v_2$ and leave it unchanged if $v_1 > v_2$. Now, if $v_2 = 0$, v_1 must equal 0 or 1 because if $v_1 > 1$, voter 1 could decrease their success probability, and leave the success probability unchanged by setting $v_1 = 1$.

So the only possible equilibria are $(0, 0)$, $(1, 0)$ and $(0, 1)$. For $(0, 0)$ to be an equilibrium it must be both voters' best-response, so it must beat the utility of voting $(1, 0)$ or $(0, 1)$ so $\max_{i \in \{1,2\}} u_i \cdot p_i - 1 \leq 0$.

For $(1, 0)$ to be an equilibrium, voter 1 must not be better off by voting 0, $u_1 \cdot p_1 \geq 1$. For $(0, 1)$ to be an equilibrium, voter 1 must not be willing to vote 2, (so $u_1 \cdot p_1 - 2^m \leq u_1 \cdot p_2$) and voter 2 must not be willing to vote 0, so $u_2 p_2 - 1 \geq 0$. □

Proof of Corollary 1. Looking at Table 3, we can see that the conditions for the $(0, 0)$ and $(1, 0)$ equilibria are independent of m, so they are equilibria under QV if and only if they are also equilibria under LV.

Table 3 shows that the strategy $(0, 1)$ is an equilibrium if and only if $u_1 \cdot p_1 - 2^m \leq u_1 \cdot p_2$ and $u_2 \cdot p_2 \geq 1$.

Clearly if this holds for $m = 1$, it also holds for $m = 2$. So if $(0, 1)$ is an equilibrium under LV it is also an equilibrium under QV. □

Proof of Corollary 2. Looking at Table 3, $(0, 0)$ and $(1, 0)$ are equilibria under LV if and only if they are equilibria under QV (since the equilibrium criteria are independent of m).

Now, note that if $(0, 1)$ is an equilibrium under LV, it is also an equilibrium under QV (because $u_1 \cdot p_1 - 2^2 < u_1 \cdot p_1 - 2$).

So, the only way QV can be worse is if $(0, 1)$ is an equilibrium under QV, but not under LV, and LV has a better equilibrium (i.e., $(1, 0)$).

Looking at Table 3, if $(0, 1)$ is an equilibrium under QV but *not* LV, we have

$$u_1 \cdot p_1 - 4 \leq u_1 \cdot p_2 \leq u_1 \cdot p_1 - 2 \tag{96}$$

Now, if $u_2 \cdot p_1 \geq 1$, and $u_2 \cdot p_2 \leq u_1 \cdot p_1 - 2$, we have $u_1 \cdot p_1 \geq u_1 \cdot p_2 + 2 \geq 3$, so by Table 3, we have that $(1, 0)$ is an equilibrium.

Thus, for this range of parameters the equilibria under QV are $(1, 0)$, $(0, 1)$, but the only equilibrium under LV is $(1, 0)$, so QV has an additional worse equilibrium, and so by Definition 6, QV is worse. □

Note that if utilities are low enough, all voters will abstain.

Lemma 19. *In the 3-voter game,* $(0, 0, 0)$ *is an equilibrium if and only if* $u_i \cdot p_i < 1$ *for all* i.

Proof of Lemma 19. If every voter votes 0, their payoff is 0. Voter i can earn $u_i \cdot p_i - v_i^m$ by voting $v_i > 0$, so Voter i's maximum achievable profit by deviating is $u_i \cdot p_i - 1$. So $(0, 0, 0)$ is an equilibrium if and only if $u_i \cdot p_i < 1$ for all i. □

Proof of Proposition 2. Lemma 19 shows that $(0, 0, 0)$ is an equilibrium if and only if $\max_i u_i \cdot p_i < 1$. Lemma 20 shows that all nonzero equilibria are in Region I or Region III.

Lemma 22 shows that the only possible equilibria in Region III are dictatorships, i.e., $(1, 0, 0)$, $(0, 1, 0)$ or $(0, 0, 1)$.

Lemma 23 shows that the only possible equilibria in Region I are of the form $(v, v, 1)$, $(v, 1, v)$ or $(1, v, v)$ for some $v \geq 1$. □

Proof of Lemma 4. Plugging $x = p_1, y = p_3$ into Lemma 25, we get

$$\begin{aligned} P_3 &= p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3 \\ &> p_1p_2 + p_2p_3 + p_2p_3 - 2p_1p_2p_3 \\ &= p_2(p_1 + 2p_3 - 2p_1p_3) \\ &> p_2 \\ &> p_3 \end{aligned}$$

□

Proof of Lemma 5. Lemma 5 is pure question of probabilities and does not depend on voter strategies or the cost of votes. It simply asks, for probabilities $p_1 \geq p_2 \geq p_3$, when is $p_1 > p_1 \cdot p_2 + p_2 \cdot p_3 + p_1 \cdot p_3 - 2 \cdot p_1 \cdot p_2 \cdot p_3$.

The bound is given in Nitzan and Paroush (1982)[Corollary 2]. □

Proof of Corollary 5. Proposition 2 shows that the only possible equilibria in the 3-voter game lead to success probabilities $0, p_3, p_2, p_1, P_3$. The net utilities for these equilibria are $0, U \cdot p_3, U \cdot p_2, U \cdot p_1, U \cdot P_3$. Now, Lemma 4 shows that $0 \leq p_3 \leq p_2 \leq P_3$, so if $p_1 < P_3$, there is no equilibrium under LV or QV that leads to higher net utility than $U \cdot P_3$, which is the utility achieved in the 1p1v game. □

Proof of Corollary 6. When $p_1 = p_2 = p_3$ the Condorcet Jury Theorem tells us that $P_3 > p_1$, so by Corollary 5, LV and QV are no better than 1p1v. □

Proof of Proposition 3. First, consider $(1, 0, 0)$. If Voter 1 increases their vote, the success probability remains the same, but their voting cost increases, so Voter 1 will not increase their vote. If Voter 1 *decreases* their vote to zero, their net profit drops from $u_1 \cdot p_1 - 1$ to 0, so Voter 1 will drop their vote to 0 if and only if $u_1 \cdot p_1 < 1$. Voter i (for $i \in \{1, 2\}$) will *decrease* the success probability and *increase* their voting cost by voting $v_i > 0$, so $(1, 0, 0)$ will be an equilibrium if and only if $u_1 \cdot p_1 > 1$.

Next, consider $(0, 1, 0)$. Voter 2 will not increase their vote (it increases voting cost, with no increase in success probability), but Voter 2 will *decrease* their vote to 0 if $u_2p_2 < 1$. Now, Voter i (for $i \in inset 1, 3$) will not increase their vote to 1 (this increases voting cost while decreasing success probability), but Voter i will vote 2 if $u_i \cdot p_i - 2^m > u_i \cdot p_2$. Note, however, that $p_3 \leq p_2$, so $u_3 \cdot p_3 - 2^m < u_3 \cdot p_2$, so Voter 3, will never increase their vote to 2. So $(0, 1, 0)$ will be an equilibrium if $u_1 \cdot p_1 - 2^m < u_1 \cdot p_2$ and $u_2 \cdot p_2 > 1$.

Finally, consider $(0, 0, 1)$. Voter 3 will decrease their vote to 0 if $u_3 \cdot p_3 < 1$. Voter i for $i \in \{1, 2\}$

will increase their vote to 2 if $u_i \cdot p_i - 2^m > u_i \cdot p_3$ $\square$

Proof of Corollary 3. From the table in Proposition 3, we can check the following.

For $(1,0,0)$, the criterion is independent of m, so $(1,0,0)$ is an equilibrium under LV if and only if it is an equilibrium under QV.

For $(0,1,0)$, if $u_1 \cdot (p_1 - p_2) < 2$, then $u_1 \cdot (p_1 - p_2) < 2^2$, so if $(0,1,0)$ is an equilibrium under LV ($m = 1$), it is also an equilibrium under QV ($m = 2$).

For $(0,0,1)$, if $\max_{i\in\{1,2\}} u_i \cdot (p_i - p_3) < 2$, then $\max_{i\in\{1,2\}} u_i \cdot (p_i - p_3) < 2^2$, so if $(0,0,1)$ is an equilibrium under LV ($m = 1$), it is also an equilibrium under QV ($m = 2$). $\square$

Proof of Proposition 4. First, consider $(1,1,1)$. $(1,1,1)$, the strategy $(v,1,1)$ for $v > 3$ is always dominated by $(3,1,1)$ (and similarly for voters 2 and 3. Similarly, the strategy $(2,1,1)$ is always dominated by $(1,1,1)$, since the success probability is lower ($\{2,3\}$ is not longer a winning set), but the voter cost is higher (and similarly for voters 2 and 3). So $(1,1,1)$ will be an equilibrium if and only if Voter i cannot profitably deviate by setting $v_i = 0$ or $v_i = 3$. Voter i's payoff at $(1,1,1)$ is $u_i P_3$. Voter i's payoff at $v_i = 3$ is $u_i \cdot p_i - 3^m$. Voter i's payoff at $v_i = 0$ is $u_i \cdot p_{i+1} \cdot p_{i+2}$. Thus $(1,1,1)$ will be an equilibrium if and only if

$$\max\left(u_i \cdot p_i - 3^m, u_i \cdot p_{i+1} p_{i+2}\right) \leq u_i \cdot P_3 - 1 \text{ for all } i \tag{97}$$

Consider the strategy, $\vec{v}^*$ where $v_i = 1, v_{i+1} = v, v_{i+2} = v$. In this case, any two-out-of-three voters is enough to win the vote. Now, let $\vec{v}_{j\to w}$ denote the strategy where voter j changes their vote to w (and voters $j+1$ and $j+2$ vote as in $\vec{v}^*$). Then

$\vec{v}$	$F_{\vec{v}}$
$\vec{v}_{i\to 0}$	$\{\{i+1, i+2\}, \{1,2,3\}\}$
$\vec{v}_{i\to 1}$	$F_{\vec{v}^*} = \{\{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$
$\vec{v}_{i\to w}$ $(1 \leq w < 2v-1)$	$\{\{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$
$\vec{v}_{i\to 2v}$	$\{\{i, i+1\}, \{i, i+2\}, \{1,2,3\}\}$
$\vec{v}_{i\to w}$ $(w \geq 2v+1)$	$\{\{i\}, \{i, i+1\}, \{i, i+2\}, \{1,2,3\}\}$

From this table it is easy to see that $\vec{v}_{i\to w}$ for $w = \{2, \ldots, 2v\}$ is strictly dominated by $w = 1$, and $\vec{v}_{i\to w}$ for $w > 2v+1$ is strictly dominated by $w = 2v+1$. So to check whether 1 is player i's best response to voters $i+1$, $i+2$ voting v, we just need to check $\vec{v}_{i\to 0}$ and $\vec{v}_{i\to 2v+1}$.

Now, $p(\vec{v}_{i\to 1}) = p(\vec{v}^*) = P_3$, $p(\vec{v}_{i\to 0}) = p_{i+1} \cdot p_{i+2}$ and $p(\vec{v}_{i\to 2v+1}) = p_i$.

So 1 will be Voter i's best response if and only if

$$\max\left(u_i \cdot p_{i+1} \cdot p_{i+2}, u_i \cdot p_i - (2v+1)^m\right) \le u_i P_3 - 1 \tag{98}$$

Similarly, for Voter j, with $j \neq i$, we can calculate (note that $\{-i-j\} = \{1,2,3\} \setminus \{i,j\}$)

$\vec{v}$	$F_{\vec{v}}$
$\vec{v}_{j\to w}$ $(w < v-1)$	$\{\{-i-j\}, \{-i-j, i\}, \{-i-j, j\}, \{1,2,3\}\}$
$\vec{v}_{j\to v-1}$	$\{\{-i-j, i\}, \{-i-j, j\}, \{1,2,3\}\}$
$\vec{v}_{j\to v} = \vec{v}^*$	$F_{\vec{v}^*} = \{\{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$
$\vec{v}_{j\to v+1}$	$\{\{-i-j, j\}, \{i,j\}, \{1,2,3\}\}$
$\vec{v}_{j\to w}$ $(w > v+1)$	$\{\{j\}, \{-i-j, i\}, \{i,j\}, \{1,2,3\}\}$

From this, we see that $\vec{v}_{j\to 0}$ strictly dominates $\vec{v}_{j\to w}$ for $w \in \{1,\ldots,v-1\}$. Similarly, $\vec{v}^*$ dominates $\vec{v}_{j\to v+1}$. So the only possible deviating strategies for Voter j that we need to consider are $\vec{v}_{j\to 0}$ and $\vec{v}_{j\to v+2}$.

Now $p(\vec{v}_{j\to 0}) = p_{-i-j}$, $p(\vec{v}^*) = P_3$ and $p(\vec{v}_{j\to v+2}) = p_j$. So v will be voter j's best response if and only if

$$\max\left(u_j \cdot p_{-i-j}, u_j \cdot p_j - (v+2)^m\right) \le u_j \cdot P_3 - v^m \tag{99}$$

□

Proof of Corollary 4. From Proposition 4, we see that $(1,1,1)$ is an equilibrium if and only if $\max\left(u_i \cdot p_i - 3^m, u_i \cdot p_{i+1}p_{i+2}\right) \le u_i \cdot P_3 - 1$ for all i. If this criterion holds for $m = 1$, then it certainly holds for $m = 2$ as well. □

Proof of Proposition 5. Consider the strategy where voter i votes $v_i = 1$, and $v_j = 0$ for all $j \neq i$.

This will be an equilibrium if and only if player i cannot profitably deviate by reducing their vote, i.e., $u_i \cdot p_i - 1 \ge 0$, and player j cannot profitably deviate by increasing their vote i.e., $u_j \cdot p_j - 2^m \le u_j \cdot p_i$.

If $u_j \cdot p_j - 2^2 \le u_j \cdot p_i < u_j \cdot p_j - 2$, then this dictatorship will be an equilibrium under QV but *not* under LV.

Now, it is straightforward to check that there are values $\vec{p}$ and $\vec{u}$ such that both these criteria hold. In addition, if these criteria hold for LV $(m = 1)$ then they will also hold for QV $(m = 2)$.

Note also that this is the *only* equilibrium where voter i is the dictator. In other words, if $v_i > \sum_{j\neq i} v_j$, then $v_i = 1$. To see this, note that in an equilibrium with $v_i > \sum_{j\neq i} v_j$, if $v_j > 0$, then voter j can profitably deviate by setting their vote to 0 (thus reducing their voting cost, but leaving the payoff unchanged). Thus in an equilibrium with $v_i > \sum_{j\neq i} v_j$, $v_j = 0$ for all $j \neq i$. Then we must have $v_i = 1$, since if $v_i > 1$, voter i could always profitably deviate by reducing their vote to $v_i = 1$. □

Proof of Proposition 6. Let $X_i \stackrel{\text{def}}{=} \frac{1}{2} + \frac{s_i v_i}{2v_{\max}}$. Let $X \stackrel{\text{def}}{=} \sum_{i=1}^{n} X_i$. With these definition, the "good" outcome will be selected if $X > \frac{n}{2}$

A basic Chernoff bound (Dubhashi and Panconesi (2009)[Theorem 1.1]) that for any $t > 0$

$$\Pr\left[X < E[X] - t\right] \leq e^{-2t^2/n} \tag{100}$$

$$E[X] = \frac{n}{2} + \sum_{i=1}^{n} p_i \frac{v_i}{2v_{\max}} \tag{101}$$

$$> \frac{n}{2} + \frac{\epsilon}{4v_{\max}} \sum_{i=1}^{n} v_i \tag{102}$$

$$\geq \frac{n}{2} + \frac{n\epsilon}{4v_{\max}} \tag{103}$$

Setting $t = \frac{n\epsilon}{4v_{\max}}$

$$\Pr\left[X < \frac{n}{2}\right] \leq e^{-\frac{\epsilon^2}{8v_{\max}^2} n} \tag{104}$$

which gives the result. Note that we need the assumption that $v_{\max}$ is bounded. To see this, consider the situation where $v_i = 2^i$. Then for any n, voter n is the dictator, and the success probability is p_n, which may not tend towards 1. □

Proof of Corollary 7. Under 1p1v, we have $v_{\max} = 1$, since the p_i are bounded away from $\frac{1}{2}$, there is an $\epsilon > 0$ such that $\inf_i p_i > \frac{1}{2} + \epsilon$. Thus we can apply Proposition 6 to conclude the success probability is bounded below by

$$1 - e^{-\frac{\epsilon^2}{8} n}. \tag{105}$$

which tends to 1 as the number of voters, n, goes to infinity. □

Proof of Proposition 7. Consider a set of votes $\vec{v}$. Let $v_{\text{tot}} \stackrel{\text{def}}{=} \sum v_j$ be the total number of votes cast. Let X_{-j} be the random variable denoting the number of successful votes cast by all voters *except* voter j. So

$$X_{-j} \stackrel{\text{def}}{=} \sum_{i \neq j} v_i \cdot \text{Ber}(p_i) \tag{106}$$

where $\text{Ber}(p_i)$ is a Bernoulli random variable with probability p_i.

Now, the probability that voter j is *pivotal* is

$$q_{\text{piv}}^{(j)} \stackrel{\text{def}}{=} \Pr[\, j \text{ is pivotal }] \tag{107}$$
$$= \Pr\left[\frac{v_{\text{tot}}}{2} - v_j < X_{-j} \leq \frac{v_{\text{tot}} - v_j}{2}\right] \tag{108}$$
$$\leq \Pr\left[X_{-j} \leq \frac{v_{\text{tot}} - v_j}{2}\right] \tag{109}$$

Now, if $\min_i p_i \geq \frac{1}{2} + \epsilon$

$$E\left[X_{-j}\right] = \sum_{i \neq j} v_i \cdot p_i \geq (v_{\text{tot}} - v_j) \cdot \min_i (p_i) \geq (v_{\text{tot}} - v_j) \cdot \left(\frac{1}{2} + \epsilon\right) \tag{110}$$

Then

$$q_{\text{piv}}^{(j)} = \Pr[\, j \text{ is pivotal }] \tag{111}$$
$$\leq \Pr\left[X_{-j} \leq \frac{v_{\text{tot}} - v_j}{2}\right] \tag{112}$$
$$\leq \Pr\left[X_{-j} \leq E\left[X_{-j}\right] - (v_{\text{tot}} - v_j) \cdot \epsilon\right] \tag{113}$$

A Chernoff bound (Dubhashi and Panconesi (2009)[Problem 1.15]) gives

$$q_{\text{piv}}^{(j)} = \Pr[\, j \text{ is pivotal }] \tag{114}$$
$$\leq \Pr\left[X_{-j} \leq E\left[X_{-j}\right] - (v_{\text{tot}} - v_j) \cdot \epsilon\right] \tag{115}$$
$$\leq 2e^{-2\frac{\left((n - v_j)\cdot\epsilon\right)^2}{n \cdot v_{\max}}} \tag{116}$$
$$\leq 2e^{-2\epsilon^2 \cdot \frac{v_{\text{tot}}^2 - 2v_{\text{tot}} \cdot v_j + v_j^2}{v_{\text{tot}} \cdot v_{\max}}} \tag{117}$$
$$\leq 2e^{-2\epsilon^2 \left(\frac{v_{\text{tot}}}{v_{\max}} - 2\right)} \tag{118}$$
$$\tag{119}$$

Now, if $\vec{v}$ is an equilibrium, then voter j cannot profitably deviate by setting $v_j = 0$. If voter j does set $v_j = 0$, then the success probability drops by at most $p_j \cdot q_{\text{piv}}^{(j)}$, so voter j's utility drops by at most $u_j \cdot p_j \cdot q_{\text{piv}}^{(j)}$. On the other hand, voter j's voting cost drops by v_j^m. So if $\vec{v}$ is an equilibrium, we have

$$u_j \cdot p_j \cdot q_{\text{piv}}^{(j)} \geq v_j^m \tag{120}$$

Plugging in Eq. (118), we have

$$v_j^m \leq 2 \cdot u_j \cdot p_j \cdot e^{-2\epsilon^2\left(\frac{v_{\text{tot}}}{v_{\max}} - 2\right)} \tag{121}$$

So if

$$\frac{v_{\text{tot}}}{v_{\max}} > \frac{\ln(2 \cdot u_j \cdot p_j)}{2\epsilon^2} + 2 \tag{122}$$

then $v_j^m < 1$, so $v_j = 0$. If there are k nonzero voters, then $v_{\text{tot}} \geq k$, and the result follows. □

F Additional Technical Lemmas Relating to the 3-Voter Setting

Remark 1 (Notation)**.** *As discussed in the model section, we order voters by their precision, implying*

$$p_1 \geq p_2 \geq p_3. \tag{123}$$

In this section, it is also sometimes useful to order voters by their vote weight instead. For a given voting strategy, $\vec{v}$, let q_i be the permutation of $[n]$ that takes the i-th vote weight to the i-th voter, so that

$$v_{q_1} \geq v_{q_2} \geq v_{q_3}. \tag{124}$$

As an example, suppose the votes are $(3, 1, 2)$

Voter	1	2	3
Precision	p_1	p_2	p_3
Vote weight	*3*	*1*	*2*

Then, the voter with the highest vote weight (equal to 3) is also the one with the highest precision, so $q_1 = 1$; the voter with the second highest vote weight (equal to 2) has the 3rd highest precision, so $q_2 = 3$; the voter with the third highest vote weight (equal to 1) has the second highest precision, so $q_3 = 2$.

We use P_3 to denote the probability of success in the 1p1v game.

$$P_3 \stackrel{\text{def}}{=} p_1p_2 + p_2p_3 + p_1p_3 - 2p_1p_2p_3 \quad \text{(prob. that majority is correct)} \tag{125}$$

With this notation, Table 4 becomes

Criteria	$p(\vec{v})$	Label
$v_{q_1} < v_{q_2} + v_{q_3}$	P_3	Region I
$v_{q_1} = v_{q_2} + v_{q_3}$	$P_3 - P_{\bar{1}23}$	Region II
$v_{q_1} > v_{q_2} + v_{q_3}$	p_{q_1}	Region III

Table 5: We give these three regions names that will be useful in the following analysis. P_3 is the probability that the majority (i.e., 2-out-of-3) is correct, p_{q_1} is the probability voter q_1 is correct, and $P_{\bar{1}23}$ is the probability voter q_1 is incorrect, while voters q_2 and q_3 are correct.

Remark 2 (Ordering voters)**.** *Note that in Table 5, the payoff in Region* III*, voter q_1 is the dictator, so the success probability is p_{q_1}. The success probability is* not p_1*, because p_1 is the voter with the highest precision, and v_{q_1} is the voter who purchases the most votes.*

Lemma 20. *In the 3-voter game, all possible nonzero equilibria are in Region* I *and Region* III *(as defined in Table 4).*

Proof of Lemma 20. To see this, note that if we are in Region II, and $v_{q_1} > 0$, then voter q_1 can profitably deviate. To see this, note that voter q_1 can *reduce* their vote to move from Region II to Region I. This *increases* their success probability, and *decreases* their voting cost, so it strictly *increases* their payoff.

Thus the only possible equilibria are in Region I (where the payoff is the same as 1p1v) and Region III (where a single voter is the dictator). □

Lemma 21. *In the 3-voter game, every nonzero equilibrium is of the form*

$$v_{q_1} = v_{q_2} + v_{q_3} + 1 \tag{126}$$

$$\text{or} \tag{127}$$

$$v_{q_1} = v_{q_2} + v_{q_3} - 1 \tag{128}$$

Proof of Lemma 21. From Lemma 20, every equilibrium is of the form $v_{q_1} < v_{q_2} + v_{q_3}$ (Region I), or $v_{q_1} > v_{q_2} + v_{q_3}$ (Region III). If $v_{q_1} < v_{q_2} + v_{q_3} + 1$, then voters 2 and 3 can strictly increase their payoff by reducing their vote by 1, since this keeps the probability of success the same, but reduces their voting cost. Thus every equilibrium in Region I is of the form $v_{q_1} = v_{q_2} + v_{q_3} + 1$. Similarly, if $v_{q_1} > v_{q_2} + v_{q_3}$ in equilibrium, if $v_{q_1} > v_{q_2} + v_{q_3} + 1$, then voter 1 could strictly increase their payoff by reducing their vote by 1. Thus every equilibrium in Region III is of the form $v_{q_1} = v_{q_2} + v_{q_3} + 1$. □

Lemma 22. *In the 3-voter game, the only possible equilibria in Region* III *are* $(1, 0, 0)$*,* $(0, 1, 0)$ *and* $(0, 0, 1)$*.*

Proof of Lemma 22. If we have an equilibrium in Region III, then $v_{q_1} > v_{q_2} + v_{q_3}$. In this case, voter q_2 or q_3 can always achieve a strictly greater payoff by reducing their vote to the minimum. So, if voters and 3 are playing their best-response, it must be that $v_{q_2} = v_{q_3} = 0$ Then by Lemma 21, this means that $v_{q_1} = 1$ (or $v_{q_1} = 3$ if abstention is not allowed). □

Lemma 23. *The only possible equilibria in Region* I *are*

$$(v, v, 1) \tag{129}$$

$$(v, 1, v) \tag{130}$$

$$(1, v, v) \tag{131}$$

for some $v > 1$

Proof of Lemma 23. First, note that Lemma 24 implies that all possible equilibria in Region I have $v_{q_3} = 1$. To see this, note that by Lemma 24, either the equilibrium is $(1, 1, 1)$, or $v_{q_2} > v_{q_3}$. Now, by Lemma 21 $v_{q_1} = v_{q_2} + v_{q_3} - 1$, so if $v_{q_3} = 0$, we have $v_{q_1} < v_{q_2}$, which is not possible since $v_{q_1} \geq v_{q_2}$. On the other hand, if $v_{q_1} \geq v_{q_2} > v_{q_3} > 1$, then it's not possible for $v_{q_1} = v_{q_2} + v_{q_3} - 1$.

In an equilibrium in Region I, with $v_{q_3} = 1$, Lemma 21 tells us that $v_{q_1} = v_{q_2}$. Thus every possible equilibrium in Region I is of the form $v_{q_1} = v_{q_2} = v$ and $v_{q_3} = 1$ for some $v \in \mathbb{Z}^+$. □

Lemma 24. *In the 3-voter game, the only possible equilibrium in Region* I *with* $v_{q_2} = v_{q_3}$ *is* $(1, 1, 1)$*.*

Proof of Lemma 24. Consider a set of votes in Region I with $v_{q_2} = v_{q_3} > 1$. Then $v_{q_1} = v_{q_2} + v_{q_3} - 1 > v_{q_2}$. In this case, voter 1 could reduce their vote to v_{q_2}, which would make $v_{q_1} = v_{q_2} = v_{q_3}$. In this case, the probability of winning would stay the same at P_3 (See Table 4), but Voter 1's cost would strictly decrease. □

Lemma 25. *Let* $x, y \in \mathbb{R}$*. If* $1 > x > y > 0.5$ *then* $x + 2y - 2xy > 1$*.*

Proof of Lemma 25.

$$\begin{aligned}1 &= x + (1 - x)\\ &< x + 2y \cdot (1 - x)\\ &= x + 2y - 2xy\end{aligned}$$

□

Lemma 26. *If QV has a "worse" equilibrium than LV, then it must be a "democratic" equilibrium.*

Proof of Lemma 26.

From Corollary 3, we know that if dictatorship is an equilibrium under LV, then it is an equilibrium under QV. Similarly, from Lemma 19, if $(0, 0, 0)$ is an equilibrium under LV it is an equilibrium under QV.

So if LV has an equilibrium that is worse than QV, it cannot be a dictatorship equilibrium, so it must be a "democratic" equilibrium (since Proposition 2 shows that the only possible equilibria are "democratic," "dictatorship" or $(0, 0, 0)$).

□

Lemma 27.

$$P_3 - p_3 \geq P_3 - p_2 \geq |p_1 - P_3| \tag{132}$$

Proof of Lemma 27. First, note that $P_3 - p_3 \geq P_3 - p_2 \geq 0$ by Lemma 4.

Now

$$\begin{aligned}(p_1 - P_3) - (P_3 - p_2) &= p_1 + p_2 - 2P_3 && (133)\\ &= p_1 + p_2 - 2\left(p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3\right) && (134)\\ &= p_1\left(1 - 2p_2 - 2p_3 + 4p_2p_3\right) + p_2(1 - 2p_3) && (135)\\ &= p_1(1 - 2p_2)(1 - 2p_3) + p_2(1 - 2p_3) && (136)\\ &= (1 - 2p_3)\left(p_1(1 - 2p_2) + p_2\right) && (137)\\ &= (1 - 2p_3)\left(p_1 + p_2 - 2p_1p_2\right) && (138)\\ &= (1 - 2p_3)\left(p_1(1 - p_2) + p_2(1 - p_1)\right) && (139)\end{aligned}$$

Now, since $\frac{1}{2} \leq p_3 \leq p_2 \leq p_1 \leq 1$,

$$1 - 2p_3 \leq 0 \tag{140}$$

$$1 - p_2 \geq 0 \tag{141}$$

$$1 - p_3 \geq 0 \tag{142}$$

So

$$(p_1 - P_3) - (P_3 - p_2) \leq 0 \Rightarrow P_3 - p_2 \geq p_1 - P_3 \tag{143}$$

Since $p_1 \geq p_2$, we have $P_3 - p_2 \geq P_3 - p_1$ as well, so $P_3 - p_2 \geq |p_1 - P_3|$ □

G Uniform Utilities in the 3-Voter Setting

Lemma 28 (Proposition 4 with uniform utilities)**.** *If $u_1 = u_2 = u_3 = u$, then Proposition 4 becomes*

Equilibrium	***Criteria***
$(1,1,1)$	$u(p_1 - P_3) \leq 3^m - 1$ *and* $u(p_1 p_2 - P_3) \leq 1$
$(v,v,1)$	$u \cdot (p_1 - P_3) \leq -v^m$
$(v,1,v)$	$u \cdot (p_1 - P_3) \leq -v^m$
$(1,v,v)$	$u\,(p_2 - P_3) \leq -v^m$ $u\,(p_1 - P_3) \leq (2v+1)^m - 1$

for $v > 1$.

Lemma 29. *If utilities are uniform, i.e., $u_1 = u_2 = u_3$, then LV is no worse than QV.*

Proof of Lemma 29.

For LV to be worse than QV, we need either (1) a worse equilibrium under LV or (2) a better equilibrium under QV.

Consider case (1) – LV has a worse equilibrium. Lemma 26 shows that this equilibrium must be a democratic equilibrium. If a democratic equilibrium yields a lower probability of success than a dictatorship equilibrium, then by Lemma 4 we must have $p_1 > P_3$.

Now, Corollary 4 shows that if $(1,1,1)$ is an equilibrium under LV, it is also an equilibrium under QV. So this "worse" equilibrium must be a democratic equilibrium of the form $(v,v,1), (v,1,v)$ or $(v,v,1)$ for some $v > 1$.

Now, looking at Lemma 28, if the u_i are all equal, then if $(v,v,1)$ or $(v,1,v)$ is an equilibrium, then $(1,1,1)$ is also an equilibrium since $u(p_1 p_2 - P_3) \leq u(p_1 - P_3)$.

On the other hand, suppose $(1, v, v)$ is an equilibrium under LV for some $v > 1$. This means $u(p_2 - P_3) \leq -v^m$, which implies that $u(p_1 p_2 - P_3) \leq 1$. So if $(1, 1, 1)$ is *not* an equilibrium under QV, we will show that $(1, v', v')$ *is* an equilibrium under QV for some v'. (If $(1, 1, 1)$ *were* an equilibrium under QV, LV could not be worse). First, note that if $(1, 1, 1)$ is *not* an equilibrium under QV, $u(p_1 - P_3) > 3^2 - 1$. Now, we have

$$u(p_1 - P_3) \geq 8 \tag{144}$$

$$\Downarrow$$

$$u(P_3 - p_2) \geq 8 \quad \text{(Lemma 27)} \tag{145}$$

$$\Downarrow$$

$$u(p_2 - P_3) \leq -8 \tag{146}$$

$$\Downarrow$$

$$u(p_2 - P_3) \leq -2^2 \tag{147}$$

So if $u(p_1 - P_3) \leq (2v + 1)^m - 1$ for $v = 2$, then we are done because $(1, 2, 2)$ is an equilibrium under QV.

Now, we can proceed by induction. Note that if

$$u(p_1 - P_3) > (2v' + 1)^m - 1 \tag{148}$$

for some v, then by Lemma 27,

$$u(p_2 - P_3) \leq -(2v')^m \tag{149}$$

So if Condition 2 ($u(p_1 - P_3) \leq (2v' + 1)^2 - 1$) fails for some v', then Condition 1 ($u(p_2 - P_3) \leq -w^m$) *holds* for all $w \leq 2v'$.

So if Condition 2 fails for $v' = 2$, then Condition 1 holds for $v' \leq 4$. If Condition 2 fails for $v' = 4$, then Condition 1 holds for $v' \leq 8$ etc. Since we know Condition 2 holds for $v' = v$, this process must terminate with some v' for which Conditions (1) and (2) hold, thus $(1, v', v')$ is an equilibrium under QV.

Thus LV *cannot* be worse than QV when the utilities are uniform.

□

Example 6 (QV can be worse than LV when utilities are uniform.)**.** *When the utilities are uniform, i.e., $u_1 = u_2 = u_3$, Lemma 29 shows that LV* cannot *be worse than QV. On the other hand, there* *are parameters where QV is worse than LV. This is the same phenomenon we see in Example 2, where QV allows a "bad" dictatorship* $(0, 1, 0)$.

Strategy	*Mechanism*	*Net Utility*
$(1, 2, 2)$	*Both*	370.97
$(1, 1, 1)$	*Both*	370.97
$(1, 4, 4)$	*LV*	370.97
$(2, 1, 2)$	*LV*	370.97
$(1, 3, 3)$	*LV*	370.97
$(2, 2, 1)$	*LV*	370.97
$(1, 0, 0)$	*Both*	363.75
$(0, 1, 0)$	*QV*	356.25

$p_1 = 0.97, p_2 = 0.95, p_3 = 0.88$ $u_1 = 125, u_2 = 125, u_3 = 125$