Online Appendix to "Information Design with Supplemental Investment"*

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This Online Appendix presents three extensions of the main model. Section A allows the supplemental investment to have negative NPV that is unobservable to both parties. Sections B.1 and B.2 introduce uncertainty regarding the nature of the supplemental investment, respectively through non-mutually-exclusive outcomes (B.1) and noisy private information (B.2). Across these settings, our main qualitative results continue to hold whenever the second dimension of information asymmetry remains economically significant. When this dimension becomes negligible, the problem collapses to a one-dimensional setting in which a perfectly informative system achieves the first best.

A Supplemental Investment May Have Negative NPV and is Unobservable to Both Parties

In Section 5.1, we consider the possibility that the supplemental investment may have negative net present value and the realization of the supplemental return $r \in \{r_h, r_l\}$ is privately observed by the firm. In this section, we examine the case in which neither the

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firm nor the investor observes the realization of r, and study the role of negative NPV supplemental investments without introducing additional information asymmetry.

Specifically, we consider the case in which $r \in \{r_h, r_l\}$ is not observed by either the firm or the investor, but the prior belief is that $r = r_h > I$ occurs with probability q, while $r = r_l < I$ occurs with probability $1-q$. As neither the firm nor the investor observes the realization of r, there is no additional information asymmetry compared with our main setting. We further assume the prior expected net return remains positive ($E[r] \equiv qr_h + (1-q)r_l > I$) so that the firm has an incentive to invest. Since the realization of r is unknown, the first-best is still achieved when all supplemental investments are financed (full-investment equilibrium). For notational simplicity, we set $r_l = 0$ in this section.

When negative NPV is possible, the downside-improving supplemental investment is no longer like a "full insurance" that protects against the project's failure. Instead, the project failure is only partially protected and the investor still faces downside risk if the major project fails and the supplemental investment return is negative. In other words, there is no risk-free loan even with the downside-improving investment. As a result, the nature of supplemental investment (downside- or upside-improving) is not so important to the investor as in the main setting. We summarize our findings in the following proposition.

Proposition OA.1 *If neither the firm nor the investor knows the realization of $r \in \{r_h, r_l\}$:*

(i) When q is low, $q \leq \frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$, the first-best full-investment equilibrium is always achieved by a perfectly informative system.

(ii) When q is high, $q > \frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$, a perfectly informative system cannot achieve the first-best investment, and increasing the informativeness of the system may reduce the efficiency. In particular,

- ***(Low credit risk)*** *When $\delta \leq \bar{\delta}'(\theta, q) \equiv \frac{[p_\theta+(1-p_\theta)q]-\frac{I}{E[r]}[p_G+(1-p_G)q]}{(1-p_\theta)q}$, an uninformative system that reveals nothing about the firm's fundamental achieves the first-best full-investment equilibrium.*

– ***(High credit risk)*** *When $\delta > \bar{\delta}'(\theta, q)$, it is impossible to achieve the first best and a biased information system or a perfectly informative system may attain the second best.*

When q is low, the supplemental investment is very likely to have a negative NPV, and thus even a downside-improving supplemental investment offers little protection against the project's failure. As a result, the nature of the supplemental investment (whether downside- or upside-improving) has little impact on debt contracting; only information about the fundamental type matters. In other words, only one dimension of information asymmetry is relevant for the debt contract. In such a situation, we find that a perfectly informative system achieves the first-best full-investment equilibrium, similar to that in our benchmark case.

When q is high, a downside-improving investment still offers substantial protection against project failure. The nature of the supplemental investment thus plays a more significant role in debt contracting. Consequently, information about both the fundamental project type and the supplemental investment type becomes important. In other words, both dimensions of information asymmetry are relevant for the debt contract. We show that our main conclusion still holds: greater informativeness may not improve the efficiency. Similar to our main setting, when q is high, we find that a completely uninformative system can achieve the first-best outcome when credit risk δ is low. With high credit risk, a biased information system or a perfectly informative system may attain the second-best outcome.

B Uncertainties about the Supplemental Investment

In the main model, we assume that the supplemental investment is either upside-improving or downside-improving with certainty, and that the firm learns the supplemental investment type perfectly. In this section, we relax this assumption and allow uncertainty regarding the supplemental investment. In Section B.1, the supplemental investment outcomes are not mutually exclusive, that is, the supplemental investment can be both upside-improving and

downside-improving. In Section B.2, the firm only observes a noisy private signal about the supplemental investment type.

B.1 Supplemental Investment Outcomes are not Mutually Exclusive

In this subsection, we introduce uncertainty regarding the outcome of the supplemental investment and allow both improvements to occur simultaneously with some probability. Specifically, the supplemental investment yields an upside improvement with probability q_u and a downside improvement with probability q_d, independently; thus, both improvements can occur with probability $q_u \cdot q_d$. Note that only a downside improvement helps reduce the credit risk from the investor's perspective; therefore, the lower the q_d (i.e., the higher the $1 - q_d$), the higher the expected credit risk level. To keep a similar information asymmetry regarding supplemental investment to that in the main setting, we assume there are two types of supplemental investment, denoted by $\{H, L\}$, and the probability of the H-type is $\delta \in (0, 1)$. The H-type supplemental investment yields an upside improvement with probability q_u^H and a downside improvement with probability q_d^H, while the L-type yields an upside improvement and a downside improvement with probabilities q_u^L and q_d^L respectively. We assume $q_d^H < q_d^L$; that is, the H-type has a higher level of expected credit risk than the L-type. We further assume $q_u^H + q_d^H = q_u^L + q_d^L = q_{ud}$ to ensure that both types have the same expected NPV, and without loss of generality, we normalize $q_{ud} = 1$. We summarize the result of this extension below:

Proposition OA.2 *In the presence of uncertainty regarding investment outcomes,*

(i) If the H-type and the L-type have similar levels of expected credit risk (that is, q_d^H and q_d^L are close enough, $q_d^L - q_d^H \leq \frac{1-\frac{I}{r}}{1-p_B}\frac{p_B+(1-p_B)q_d^L}{\delta}$), the first-best full-investment equilibrium is always achieved by a perfectly informative system.

(ii) If the H-type and the L-type have sufficiently different levels of expected credit risk (that is, q_d^H and q_d^L are sufficiently different, $q_d^L - q_d^H > \frac{1-\frac{I}{r}}{1-p_B}\frac{p_B+(1-p_B)q_d^L}{\delta}$), a perfectly

informative system cannot achieve the first-best full-investment equilibrium. In this case, increasing the informativeness of the system may reduce efficiency.

Due to the uncertainty in investment outcomes, even the L-type supplemental investment cannot guarantee a downside improvement and does not provide a full insurance to investors. Proposition OA.2 demonstrates that, as long as the H-type and the L-type supplemental investments have sufficiently distinct levels of expected credit risk, our main results remain qualitatively unchanged in that a perfectly informative system cannot achieve the first-best full-investment equilibrium, while a biased information system can achieve a second-best outcome. However, when the two types' expected credit risk levels become too similar, the second dimension of information asymmetry regarding investment types becomes trivial, and the scenario reduces to that with one-dimensional information asymmetry similar to our benchmark case. As a result, a perfectly informative system that reveals firms' fundamental types is sufficient to implement the first-best full-investment equilibrium. To build intuition, consider two extremes. When the model reduces to $q_d^H = 0$ and $q_d^L = 1$, it is effectively the same as our main setting. When we reach the opposite extreme, where $q_d^H = q_d^L$, there is effectively only one type of supplemental investment. In this case, the manager no longer possesses private information about investment type, leaving only one dimension of information asymmetry concerning the firm's fundamentals. A perfectly informative system fully resolves this asymmetry, thereby achieving the first-best full-investment equilibrium.

B.2 Noisy Information on Supplemental Investment Type

In this subsection, we introduce noise in the information regarding the supplemental investment type. Specifically, we assume the firm does not observe the supplemental investment type but observes privately a noisy signal, $m \in \{u, d\}$, that is informative about the investment type, $\tau \in \{U, D\}$, with $\Pr[m = u|\tau = U] = \Pr[m = d|\tau = D] = 1 - \varphi$ and $\varphi \in [0, \frac{1}{2}]$. Contingent on observing a signal m, the firm's posterior of the supplemental investment being upside-improving is determined by $\mu_u \equiv \Pr[\tau = U|m = u] = \frac{\delta(1-\varphi)}{\delta(1-\varphi)+(1-\delta)\varphi}$

and $\mu_d \equiv \Pr[\tau = U | m = d] = \frac{\delta\varphi}{\delta\varphi+(1-\delta)(1-\varphi)}$. That is, a signal $m = d$ no longer guarantees a downside improvement, and upon signal $m = d$, the supplemental investment is no longer a "full insurance" to the investor. Conversely, while the signal $m = u$ still indicates a higher probability of an upside improvement, it is not guaranteed to be upside-improving.

Proposition OA.3 *With noisy information regarding the supplemental investment type,*

(i) If the firm's private information regarding supplemental investment type is sufficiently noisy (i.e., $\varphi \geq \frac{(\delta-\frac{1-\frac{I}{r}}{1-p_B})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_B})}$), the first-best full-investment equilibrium is always achieved by a perfectly informative system.

(ii) If the firm's private information regarding supplemental investment type is sufficiently precise (i.e., $\varphi < \frac{(\delta-\frac{1-\frac{I}{r}}{1-p_B})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_B})}$), a perfectly informative system cannot achieve the first-best full-investment equilibrium. In this case, increasing the informativeness of the system may reduce efficiency.

Proposition OA.3 demonstrates that as long as the firm's private information regarding the supplemental investment type is sufficiently precise, our main results remain qualitatively unchanged: a perfectly informative system cannot achieve the first best, and improving informativeness of the system may reduce efficiency. However, when the firm's private information regarding the supplemental investment type becomes sufficiently noisy, the information asymmetry concerning the investment type diminishes in significance. Therefore the two-dimensional information asymmetry problem reduces to one-dimensional asymmetry only on the fundamental type. As a result, a perfectly informative system that reveals the firm's fundamental type can achieve the first-best full-investment equilibrium.

Proofs of Online Appendix

Proof of Proposition OA.1. Given a perfectly informative system, upon a good signal, $\theta_g = 1$, the debt repayment is $F(\theta_g = 1) = \frac{I}{p_G+(1-p_G)q(1-\delta)}$. A good-type firm with a downside-improving investment will invest if $E[r] \geq F(\theta_g = 1)[p_G + (1-p_G)q]$, which is $q \leq \frac{p_G}{(1-p_G)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$. Similarly upon a bad signal, a bad type with downside improving will invest if $q \leq \frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$. Since $\frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)} < \frac{p_G}{(1-p_G)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$, the binding constraint is $q \leq \frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$ and a full-investment equilibrium is achieved under a perfectly informative system.

If $q > \frac{p_B}{(1-p_B)(\frac{\delta}{1-\frac{I}{E[r]}}-1)}$, similarly to the main setting, we define $\bar{\delta}'(\theta, q) \equiv \frac{[p_\theta+(1-p_\theta)q]-\frac{I}{E[r]}[p_G+(1-p_G)q]}{(1-p_\theta)q}$. When $\delta \leq \bar{\delta}'(\theta, q)$, a good type with downside improving is willing to invest with the market's belief of θ. With an uninformative system, the market holds the prior belief of θ, all types of firm will invest and full-investment equilibrium is achieved.

We define $\underline{\delta}(\theta, q) \equiv \frac{[p_B+(1-p_B)q](1-\theta)[1-\frac{I}{E[r]}]}{[p_B+(1-p_B)q][1-\theta+\theta\frac{I}{E[r]}]-p_\theta}$. When $\bar{\delta}'(\theta, q) < \delta \leq \underline{\delta}(\theta, q)$, a good-type firm with downside-improving forgoes the supplemental investment, but a bad-type firm with downside-improving is still willing to invest by pooling with upside-improving firms. When $\delta \in (\underline{\delta}(\theta, q), \bar{\delta}'(1, q)]$, an optimistic biased system is the optimal. That is, even with the highest possible θ_b^* (which is θ), the bad type downside improving firms choose not to pool with upside firms and invest. Under this condition, an optimistically biased system is optimal since the bad-type firms with downside-improving investments may obtain a good signal and choose to invest.

When $\delta > \bar{\delta}'(1, q)$, a perfectly informative system could be optimal if $q > \frac{p_G\frac{E[r]}{I}-p_B}{1-p_B}$. The intuition is that the bad type with downside-improving still do not want to invest even by pooling with good upside-improving types. Since no downside-improving investment is undertaken, a perfectly informative system achieves the second best. ■

Proof of Proposition OA.2. With a perfectly informative system, the fundamental firm type $\omega \in \{G, B\}$ is revealed, and the investor conjectures the plausible supplemental investment types that seek financing. If the first-best full investment is achieved in equilibrium, the investor's break-even condition is

$$F_\omega p_\omega + (\delta q_d^H + (1-\delta)q_d^L)\min\{(1-p_\omega)F_\omega, r\} = I,$$

which is equivalent to

$$F_\omega = \begin{cases} \frac{I}{(1-p_\omega)(\delta q_d^H+(1-\delta)q_d^L)+p_\omega}, & \text{if } (\delta q_d^H + (1-\delta)q_d^L) \geq \frac{p_\omega}{1-p_\omega} + \frac{I}{r}, \\ \frac{I-(\delta q_d^H+(1-\delta)q_d^L)r}{p_\omega}, & \text{if } (\delta q_d^H + (1-\delta)q_d^L) < \frac{p_\omega}{1-p_\omega} + \frac{I}{r}. \end{cases} \quad \text{(OA.1)}$$

The firm chooses to invest if and only if the expected payoff with the supplemental investment outweighs the cost of financing. Specifically, for firms with investment type j,

$$p_\omega X + r - p_\omega F_\omega - q_d^j \min\{(1-p_\omega)F_\omega, r\} \geq p_\omega X,\ \omega \in \{G,B\}, j \in \{H,L\}, \tag{OA.2}$$

which can be rewritten to be $r \geq p_\omega F_\omega + q_d^j \min\{(1-p_\omega)F_\omega, r\}$. For the same fundamental type, the condition for low-risk investment implies that for high-risk investment, given $r > I$ and $q_d^H < q_d^L$. The condition for low-risk investment holds if and only if $r \geq (p_\omega + q_d^L(1-p_\omega))F_\omega$, which is equivalent to $\delta \leq \frac{1-\frac{I}{r}}{1-p_\omega}\frac{p_\omega+(1-p_\omega)q_d^L}{q_d^L-q_d^H}$. Since $p_G > p_B$, the binding condition is determined by $\delta \leq \frac{1-\frac{I}{r}}{1-p_B}\frac{p_B+(1-p_B)q_d^L}{q_d^L-q_d^H}$, which is equivalent to $q_d^L - q_d^H \leq \frac{1-\frac{I}{r}}{1-p_B}\frac{p_B+(1-p_B)q_d^L}{\delta}$. Considering a special case with $q_d^L = 1-\sigma \geq q_d^H = \sigma$, where $\sigma \in \left[0, \frac{1}{2}\right]$, we obtain $\sigma \geq \frac{\delta - \frac{1-\frac{I}{r}}{1-p_B}}{2\delta-(1-\frac{I}{r})}$, or equivalently, $q_d^L - q_d^H \leq \frac{1+p_B}{1-p_B}\frac{1-\frac{I}{r}}{2\delta-(1-\frac{I}{r})}$.

If $q_d^L - q_d^H > \frac{1-\frac{I}{r}}{1-p_B}\frac{p_B+(1-p_B)q_d^L}{\delta}$, it is no longer possible to achieve first-best full investment with a perfectly informative system. Contingent on the posterior belief θ_s ($s \in \{g,b\}$) upon the realized signal, if all types invest in equilibrium, the investor's break even condition is

$$F_s^A p_s(\theta_s) + (\delta q_d^H + (1-\delta)q_d^L)[\theta_s(1-p_G)F_s^A + (1-\theta_s)(1-p_B)F_s^A] = I,$$

which can be rewritten as

$$F_s^A = \frac{I}{(1-p_s(\theta_s))(\delta q_d^H + (1-\delta)q_d^L) + p_s(\theta_s)}. \tag{OA.3}$$

The firm chooses to invest if and only if the expected payoff of the investment outweighs the cost of financing. That is, for firms with fundamental type i and risk type j investment,

$$p_\omega X + r - p_\omega F_s^A - q_d^j(1-p_\omega)F_s^A > p_\omega X,\ \omega \in \{G,B\}, j \in \{H,L\},\ \text{and } s \in \{g,b\}, \tag{OA.4}$$

which can be rewritten as

$$r > p_\omega F_s^A + q_d^j(1-p_\omega)F_s^A. \tag{OA.5}$$

Since $q_d^L > q_d^H$ and $p_G > p_B$, the low-risk and good type's incentive compatibility constraint implies that of other types.

$$r > p_G F_s^A + q_d^L \min\{(1-p_G)F_s^A, r\} = (q_d^L(1-p_G) + p_G)F_s^A. \tag{OA.6}$$

Considering a special case with $q_d^L = 1-\sigma \geq q_d^H = \sigma$, where $\sigma \in \left[0, \frac{1}{2}\right]$, the condition is equivalent to

$$\theta_s > \frac{1}{p_G - p_B}\left(1 - \frac{1-(1-\sigma(1-p_G))\frac{I}{r}}{(1-2\sigma)\delta + \sigma} - p_B\right) \equiv \theta_s(\sigma), \tag{OA.7}$$

and $\theta_s(\sigma) \in (0,1)$ for $\sigma \in (\frac{\delta-\frac{1-\frac{I}{r}}{1-p_G}}{2\delta-(1-\frac{I}{r})}, \frac{\delta-\frac{1-\frac{I}{r}}{1-p_B}}{2\delta-(1-\frac{I}{r})})$, in which case if $\theta_s(\sigma) \leq \theta$, an uninformative system achieves the first-best; otherwise if $\theta_s(\sigma) > \theta$, sometimes a biased system with $\theta_g = \theta_s(\sigma) > \theta$ and $\theta_b = 0$ achieves the second best outcome. ■

Proof of Proposition OA.3. With a perfectly informative system, the fundamental firm type $\omega \in \{G, B\}$ is revealed, and the investor conjectures the plausible signals regarding supplemental investment types received by the manager that seek financing. If the first-best full investment is achieved in equilibrium, the investor's break-even condition is the same as the main setting, leading to

$$F_\omega = \begin{cases} \frac{I}{1-(1-p_\omega)\delta}, & \text{if } \delta \leq \frac{1}{1-p_\omega} - \frac{I}{r}, \\ \frac{I-(1-\delta)r}{p_\omega}, & \text{if } \delta > \frac{1}{1-p_\omega} - \frac{I}{r}. \end{cases} \tag{OA.8}$$

The firm chooses to invest if and only if the expected payoff with the supplemental investment outweighs the cost of financing. Specifically, for firms with private signal m,

$$p_\omega X + r - p_\omega F_\omega - (1-\mu_m)\min\{(1-p_\omega)F_\omega, r\} \geq p_\omega X, \ \omega \in \{G,B\}, m \in \{u,d\}, \tag{OA.9}$$

which can be rewritten to be $r \geq p_\omega F_\omega + (1-\mu_m)\min\{(1-p_\omega)F_\omega, r\}$. For the same fundamental type, the condition for signal d implies that for signal u, given $r > I$ and $\mu_u > \mu_d$. The condition for signal d holds if and only if $r \geq (p_\omega + (1-\mu_d)(1-p_\omega))F_\omega$, which is equivalent to $\delta \leq \frac{1-\frac{I}{r}}{1-p_\omega} + \frac{I}{r}\mu_d$. Plugging in $\mu_d = \frac{\delta\varphi}{\delta\varphi+(1-\delta)(1-\varphi)}$, we obtain $\varphi \geq \frac{(\delta-\frac{1-\frac{I}{r}}{1-p_B})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_B})}$ since $p_G > p_B$.

If $\varphi < \frac{(\delta-\frac{1-\frac{I}{r}}{1-p_B})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_B})}$, it is no longer possible to achieve first-best full investment with a perfectly informative system. Contingent on the posterior belief θ_s ($s \in \{g,b\}$) upon the realized signal, if all types invest in equilibrium, the investor's break-even condition is the same as the main setting, leading to

$$F_s^A = \frac{I}{1-\delta+\delta p_s(\theta_s)}. \tag{OA.10}$$

The firm chooses to invest if and only if the expected payoff of the investment outweighs the cost of financing. That is, for firms with fundamental type ω, public signal s and private signal m,

$$p_\omega X + r - p_\omega F_s^A - (1-\mu_m)\min\{(1-p_\omega)F_s^A, r\} > p_\omega X, \ \omega \in \{G,B\}, m \in \{u,d\}, \text{ and } s \in \{g,b\}, \tag{OA.11}$$

which can be rewritten as

$$r > p_\omega F_s^A + (1-\mu_m)(1-p_\omega)F_s^A. \tag{OA.12}$$

Since $\mu_d < \mu_u$ and $p_G > p_B$, the incentive compatibility constraint for a good-type firm with signal $m = d$ implies that of other types.

$$r > p_G F_s^A + (1 - \mu_d) \min\{(1 - p_G)F_s^A, r\} = (1 - \mu_d(1 - p_G))F_s^A, \tag{OA.13}$$

which after plugging in $\mu_d \equiv \Pr[\tau = U|m = d] = \frac{\delta\varphi}{\delta\varphi+(1-\delta)(1-\varphi)}$ is equivalent to

$$\theta_s > \frac{1}{\delta(p_G - p_B)} \left((1 - p_B)\delta - 1 + \frac{I}{r} \frac{p_G\delta\varphi + (1 - \delta)(1 - \varphi)}{\delta\varphi + (1 - \delta)(1 - \varphi)} \right) \equiv \theta_s(\varphi), \tag{OA.14}$$

and $\theta_s(\varphi) \in (0, 1)$ for $\varphi \in (\frac{(\delta-\frac{1-\frac{I}{r}}{1-p_G})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_G})}, \frac{(\delta-\frac{1-\frac{I}{r}}{1-p_B})(1-\delta)}{\frac{I}{r}\delta-(1-2\delta)(\delta-\frac{1-\frac{I}{r}}{1-p_B})})$, in which case if $\theta_s(\varphi) \leq \theta$, an uninformative system achieves the first-best; otherwise if $\theta_s(\varphi) > \theta$, a suitably biased system with $\theta_g = \theta_s(\varphi) > \theta$ and $\theta_b = 0$ achieves the second best outcome. ■