Online Appendices to

"Left-Digit Bias, Quality Choice, and Channel Coordination"

Xuefeng Peng, Zemin (Zachary) Zhong

Appendix A Proof of All Results

A.1 Summary of Notations

Before proving the main results, we summarize the notations used in the paper. We focus on four cases: centralized quality-exogenous (CX), centralized quality-endogenous (CN), decentralized quality-exogenous (DX), and decentralized quality-endogenous (DN). Uppercase (e.g., CX) and lowercase (e.g., cx) superscripts represent the bias scenario ($\lambda > 0$) and the rational benchmark ($\lambda = 0$), respectively, and the superscript * denotes equilibrium outcomes. We use the subscript 9 for *99-ending pricing* ($\Delta_9^* = 1^-$) and the subscript o for *interior pricing* ($0 \leq \Delta_{n,o}^* < 1$). The subscripts m, r, and c denote the manufacturer, retailer, and channel parties, respectively. For example, $\pi_{9r}^{DX^*}$ is the retailer's equilibrium profit under 99-ending pricing in the decentralized quality-exogenous case. Furthermore, for simplicity, we use $\Delta_9^* = \lim_{\epsilon \to 0^+}(1-\epsilon)$ or $\Delta_9^* = 1^-$ to denote 99-ending pricing interchangeably.

A.2 Proof of Lemma 1

Given the consumer utility function $u = \theta q - \tilde{p}(\lambda, n, \Delta)$, where $\theta \sim U[0,1]$ and $\tilde{p}(\lambda, n, \Delta) = n + (1-\lambda)\Delta$, with $n \equiv \left\lfloor \frac{p}{10^{\lfloor \log_{10} p \rfloor}} \right\rfloor \in \mathbb{N}^+$ and $\Delta = p - n \cdot 10^{\lfloor \log_{10} p \rfloor}$, consumers purchase the product if their perceived utility is non-negative, i.e., $u = \theta q - \tilde{p} \geq 0$. This implies the market demand $D = 1 - \frac{\tilde{p}}{q} = 1 - \frac{n+(1-\lambda)\Delta}{q}$. The firm's profit-maximization problem in the centralized-exogenous scenario (denoted by CX) is:

$$\max_{n \in \mathbb{N}^+,\, \Delta \in [0,1)} \pi(q, n, \Delta) = (n + \Delta - cq^2 - f)\left(1 - \frac{\tilde{p}(n, \Delta, \lambda)}{q}\right).$$

Without loss of generality, we assume $f = 0$. The firm's actual price $p = n + \Delta$ differs from the consumers' perceived price $\tilde{p}$, due to left-digit bias, allowing the firm to charge $p \geq \tilde{p}$. Demand depends on the perceived price, while profit per unit depends on the actual price. The equilibrium pricing strategy comprises two regimes: interior pricing (continuous in the right digits) and 99-ending pricing (discontinuous at the leftmost digit threshold). We analyze each regime separately and then determine the global optimum by comparison.

Interior pricing regime: The firm optimizes Δ for a fixed n, subject to $0 \leq \Delta < 1$. The second-order condition is $\frac{\partial^2 \pi^{CX}}{\partial \Delta^2} = -\frac{2(1-\lambda)}{q} \leq 0$ (since $\lambda \leq 1$), indicating concavity. The first-order condition is

$\frac{\partial \pi^{CX}}{\partial \Delta} = 1 - \frac{\tilde{p}}{q} - \frac{(n+\Delta-cq^2)(1-\lambda)}{q} = 0$, which simplifies to $\frac{cq(q-\lambda q)+(\lambda-2)n+2(\lambda-1)\Delta+q}{q} = 0$. Solving yields $\Delta_o^* = \frac{q(c(\lambda-1)q-1)-(\lambda-2)n}{2(\lambda-1)}$ and $p_o^* = n+\Delta_o^* = \frac{q(c(\lambda-1)q-1)+\lambda n}{2(\lambda-1)}$. For $\Delta_o^* \in [0,1)$, the quality must satisfy:

$$\frac{\sqrt{4c(\lambda^2-3\lambda+2)n+1}-1}{2c(1-\lambda)} \leq q < \frac{\sqrt{4c((1-\lambda)^2(n+2)+n(1-\lambda))+1}-1}{2c(1-\lambda)}. \tag{A.1}$$

Under this condition, the demand is $D_{n,o}^{CX^*} = \frac{c(\lambda-1)q^2-\lambda n+q}{2q}$, profit is $\pi_{n,o}^{CX^*} = \frac{[c(\lambda-1)q^2-\lambda n+q]^2}{4(1-\lambda)q}$, true consumer surplus (based on actual p) is $CS_{n,o}^{CX^*} = \int_{1-D_{n,o}^{CX^*}}^{1} (\theta q - n^* - \Delta^*)\, d\theta = \frac{[c(\lambda-1)q^2-\lambda n+q][c(\lambda^2-1)q^2-(\lambda-3)\lambda n-3\lambda q+q]}{8(1-\lambda)q}$, and social welfare is $SW_{n,o}^{CX^*} = \pi^{CX^*} + CS^{CX^*} = \frac{[c(\lambda-1)q^2-\lambda n+q][q(c(\lambda+3)q-3)-\lambda n]}{8(1-\lambda)q}$. For notational convenience, define $q(\lambda)_{n,1}^{CX}$ and $q(\lambda)_{n,2}^{CX}$ as lower and upper bound of constraint A.1, respectively.

99-ending pricing: Under this pricing scheme, the firm solves a discrete pricing problem given consumer demand, optimizing n given $\Delta_9^{CX^*} = \lim_{\epsilon \to o^+}(1-\epsilon)$. With $\epsilon \to 0^+$, the optimal price $p_9^{CX^*} = n+1-\epsilon$, market demand $D_9^{CX^*} = 1-\frac{n+(1-\lambda)(1-\epsilon)}{q}$, firm profit $\pi_9^{CX^*} = \frac{(n+1-cq^2-\epsilon-1)(\lambda-n+q-\lambda\epsilon+\epsilon-1)}{q}$, consumers' true surplus $CS_9^{CX^*} = \frac{(-n+q+\epsilon-1)^2-\lambda^2(\epsilon-1)^2}{2q}$, and social welfare $SW_9^{CX^*} = \frac{(q-n-(1-\lambda)(1-\epsilon))(-2cq^2+n+q+(\lambda-1)(\epsilon-1))}{2q}$.

Optimal pricing: Based on the solutions derived for the two regimes, we determine the globally optimal pricing strategy through comparative analysis. The optimal leftmost digit n primarily depends on quality q, while the marginal production cost c influences the parameter ranges but not the overall qualitative patterns. Accordingly, we focus on q and the bias intensity λ to characterize the equilibrium pricing outcomes. The interior pricing regime is conditional on constraint (A.1), whereas the 99-ending pricing regime is unconditional. For a given q, comparisons between the interior profit $\pi_{n,o}^{CX^*}$ and the 99-ending profits $\pi_{n,9}^{CX^*}$ and $\pi_{n-1,9}^{CX^*}$ follow a systematic logic under the constraint (A.1). Outside this constraint, we compare $\pi_{n,9}^{CX^*}$ with $\pi_{n-1,9}^{CX^*}$ and $\pi_{n+1,9}^{CX^*}$. Under constraint (A.1), the concavity of the profit function in Δ and the interior location of Δ_o^* imply that $\pi_{n,o}^{CX^*} > \pi_{n,9}^{CX^*}$. Furthermore, the difference $\pi_{n,o}^{CX^*} - \pi_{n-1,9}^{CX^*} = \frac{4(cq^2-n)(\lambda-n+q)-\frac{(c(\lambda-1)q^2-\lambda n+q)^2}{\lambda-1}}{4q}$ may be positive or negative. The derivatives are $\frac{\partial \pi_{n,o}^{CX^*}}{\partial q} = \frac{(c(\lambda-1)q^2-\lambda n+q)(3c(\lambda-1)q^2+\lambda n+q)}{4(1-\lambda)q^2}$ and $\frac{\partial \pi_{n-1,9}^{CX^*}}{\partial q} = \frac{n(cq^2-\lambda)-cq^2(\lambda+2q)+n^2}{q^2}$, satisfying $\frac{\partial \pi_{n,o}^{CX^*}}{\partial q} > \frac{\partial \pi_{n-1,9}^{CX^*}}{\partial q} > 0$. Evaluating at the bounds of constraint (A.1), we have $(\pi_{n,o}^{CX^*} - \pi_{n-1,9}^{CX^*})|_{q(\lambda)_{n,1}^{CX}} < 0$ and $(\pi_{n,o}^{CX^*} - \pi_{n-1,9}^{CX^*})|_{q(\lambda)_{n,2}^{CX}} > 0$. Therefore, by the single-crossing property and the intermediate value theorem, there exists a unique $\underline{q}(\lambda)_n^{CX} \in (q(\lambda)_{n,1}^{CX}, q(\lambda)_{n,2}^{CX})$ such that $\pi_{n,o}^{CX^*} < \pi_{n-1,9}^{CX^*}$ for $q(\lambda)_{n,1}^{CX} < q < \underline{q}(\lambda)_n^{CX}$, and $\pi_{n,o}^{CX^*} > \pi_{n-1,9}^{CX^*}$ for $\underline{q}(\lambda)_n^{CX} < q < q(\lambda)_{n,2}^{CX}$. To complete the characterization, the interior pricing regime is optimal when $\underline{q}(\lambda)_n^{CX} < q < q(\lambda)_{n,2}^{CX}$ and $0 < \lambda < \lambda_n^{CX}$, where λ_n^{CX} is the unique solution to $\underline{q}(\lambda)_n^{CX} = q(\lambda)_{n,2}^{CX}$. For conciseness, we omit the closed-form expression for $\underline{q}(\lambda)_n^{CX}$, available upon request. For the remaining ranges where $\lambda_n^{CX} < \lambda < 1$, we determine the optimal left digit under 99-ending pricing. We have $\frac{\partial \pi_{n,9}^{CX^*}}{\partial q} = \frac{cnq^2-cq^2(\lambda+2q-1)+n^2}{q^2} > 0$, implying steeper slopes for larger n, i.e., $\frac{\partial \pi_{n,9}^{CX^*}}{\partial q} > \frac{\partial \pi_{n-1,9}^{CX^*}}{\partial q} > 0$. Thus, by the single-crossing property, $\pi_{n-1,9}^{CX^*} > \pi_{n,9}^{CX^*}$ for $q < q(\lambda)_n^{CX}$ and $\pi_{n-1,9}^{CX^*} < \pi_{n,9}^{CX^*}$ for $q > q(\lambda)_n^{CX}$, where $q(\lambda)_n^{CX} \equiv \frac{\sqrt{4c(-\lambda+2n+1)+1}-1}{2c}$. In summary, the equilibrium pricing strategies are: Interior pricing is optimal when $\underline{q}(\lambda)_n^{CX} < q < q(\lambda)_{n,2}^{CX}$ and $0 \leq \lambda < \lambda_n^{CX}$. 99-ending pricing is optimal when (1) $q(\lambda)_{n,2}^{CX} < q < \underline{q}(\lambda)_{n+1}^{CX}$

and $0 < \lambda < \lambda_{n+1}^{CX}$, (2) $q(\lambda)_{n,2}^{CX} < q < q(\lambda)_{n+1}^{CX}$ and $\lambda_{n+1}^{CX} < \lambda < \lambda_n^{CX}$, and (3) $q(\lambda)_n^{CX} < q < q(\lambda)_{n+1}^{CX}$ and $\lambda_n^{CX} < \lambda < 1$. For expositional ease in subsequent proofs, we denote the ranges for interior (99-ending) pricing by CON_o^{CX} (CON_9^{CX}).

A.3 Proof of Corollary 1

Given the equilibrium outcomes and parameter ranges characterized in Lemma **??**, we first confirm that left-digit bias raises firm profitability in both regimes: $\frac{\partial \pi_{n,o}^{CX^*}}{\partial \lambda} = \frac{\frac{(n-q)^2}{(1-\lambda)^2} - (n-cq^2)^2}{4q} > 0$ under CON_o^{CX} and $\frac{\partial \pi_{n,9}^{CX^*}}{\partial \lambda} = \frac{(1-\epsilon)(n-cq^2+\epsilon)}{q} > 0$ under CON_9^{CX}. We next show that this profitability gain does not always come at the expense of consumer surplus. In the rational benchmark ($\lambda = 0$) we have $CS^{cx^*} = \frac{q(1-cq)^2}{8}$. Under 99-ending pricing the difference is $CS_{n,9}^{CX^*} - CS^{cx^*} = \frac{(-n+q+\epsilon-1)^2 - \lambda^2(\epsilon-1)^2}{2q} - \frac{q(1-cq)^2}{8}$, positive or negative. Differentiating with respect to q yields $\frac{\partial (CS_{n,9}^{CX^*} - CS^{cx^*})}{\partial q} = \frac{q^2\left(cq(4-3cq)+3\right) - 4n^2 + 8n(\epsilon-1) + 4(\lambda^2-1)(\epsilon-1)^2}{8q^2} > 0$ under CON_9^{CX}. Moreover, $(CS_{n,9}^{CX^*} - CS^{cx^*})\big|_{\overline{q}(\lambda)_n^{CX}} < 0$, $(CS_{n,9}^{CX^*} - CS^{cx^*})\big|_{\underline{q}(\lambda)_n^{CX}} > 0$, $(CS_{n,9}^{CX^*} - CS^{cx^*})\big|_{q(\lambda)_n^{CX}} > 0$. Since the difference is continuous and strictly increasing in q within each 99-ending interval, the Zero Theorem and the Single-Crossing Condition imply the existence of a unique $q(\lambda)_{n,a}^{CX}$ solving $CS_{n,9}^{CX^*} = CS^{cx^*}$. Thus $CS_{n,9}^{CX^*} > CS^{cx^*}$ for all $q(\lambda)_{n,a}^{CX} < q < \underline{q}(\lambda)_{n+1}^{CX}$ and $0 < \lambda < \lambda_{n,a}^{CX}$, where $\lambda_{n,a}^{CX}$ is the largest bias level such that the 99-ending regime still contains qualities above $q(\lambda)_{n,a}^{CX}$. Under interior pricing, the difference is $CS_{n,o}^{CX^*} - CS^{cx^*} = \frac{\lambda\left[2nq\left(c(\lambda-1)^2q-\lambda-1\right)+q^2\left(2-c(\lambda-1)q(c\lambda q-2)\right)-(\lambda-3)\lambda n^2\right]}{8(\lambda-1)q} < 0$ under CON_o^{CX}. Hence consumer surplus strictly declines under interior pricing.

A.4 Proof of Lemma 2 and Corollary 2

We now consider the endogenous-quality scenario. The centralized firm's profit-maximization problem (denoted by superscript CN) is

$$\max_{q,\, n\in\mathbb{N}^+,\, \Delta\in[0,1)} \pi(q,n,\Delta) = (n+\Delta-cq^2-f)\cdot(1-\frac{\tilde{p}(n,\Delta,\lambda)}{q})$$

We solve by backward induction, first deriving the optimal quality conditional on the pricing regime characterized in the proof of Lemma 1 and then characterizing the globally optimal pricing.

Interior pricing. The second-order condition (SOC) of the profit function with respect to the rightmost digit of the price is given by $\frac{\partial^2 \pi^{CN}}{\partial \Delta^2} = -\frac{2(1-\lambda)}{q} < 0$, indicating that the profit function is concave and unimodal. Additionally, we derive the first-order condition (FOC) of the profit function and set it to zero, i.e., $\frac{\partial \pi^{CX}}{\partial \Delta} = \frac{\lambda\left(-cq^2-f+n+2\Delta\right)+cq^2+f-2(n+\Delta)+q}{q} = 0$, implying $\Delta_o^{CN} = \frac{cq(q-\lambda q)+f(-\lambda)+f+(\lambda-2)n+q}{2-2\lambda}$ and thus $p_o^{CN} = \frac{c(\lambda-1)q^2+f(\lambda-1)+\lambda n-q}{2(\lambda-1)}$. Substituting it into the retailer's profit function, we have $\pi_r^{CN} = \frac{\left(c(\lambda-1)q^2+f(\lambda-1)-\lambda n+q\right)^2}{4(1-\lambda)q}$, whose SOC and FOC are $\frac{\partial^2 \pi^{CN}}{\partial q^2} = \frac{1}{2}\left(-3c^2(\lambda-1)q - 2c - \frac{(f(-\lambda)+f+\lambda n)^2}{(\lambda-1)q^3}\right) < 0$

and $\frac{\partial \pi^{CN}}{\partial q} = \frac{\left(c(\lambda-1)q^2+f(\lambda-1)-\lambda n+q\right)\left(3c(\lambda-1)q^2-f\lambda+f+\lambda n+q\right)}{4(1-\lambda)q^2} = 0$, implying $q_o^{CN^*} = \frac{\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+1}{6c(1-\lambda)}$. Substituting $q_o^{CN^*}$ into optimal price, we have $\Delta_o^{CN^*} = \frac{3c(\lambda-1)(2f(\lambda-1)+(3-2\lambda)n)+\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+1}{9c(\lambda-1)^2}$. The requirement $0 \leq \Delta_o^{CN^*} < 1$ holds if and only if

$$\frac{2(n+1-\lambda)}{(1-\lambda)(2f(\lambda-1)-3\lambda+(3-2\lambda)n+3)^2} < c \leq \frac{2n}{(1-\lambda)(2f(\lambda-1)+(3-2\lambda)n)^2} \tag{A.2}$$

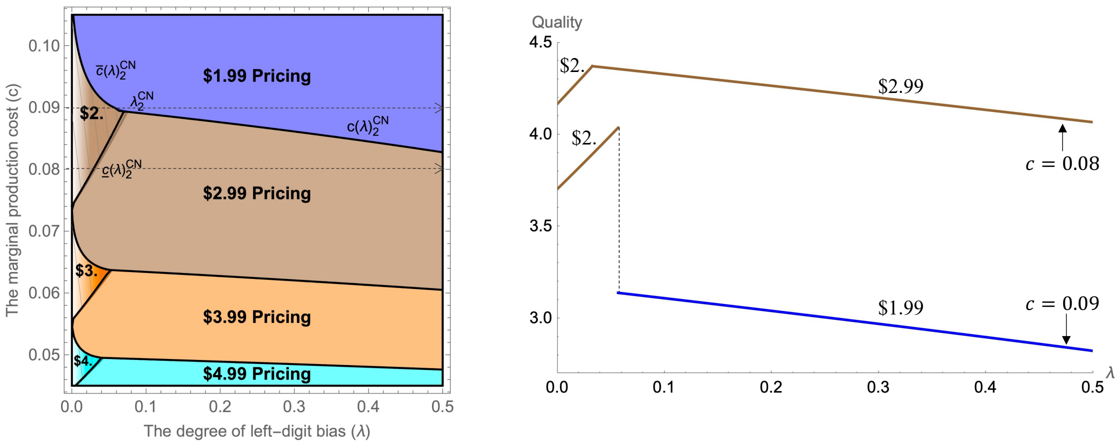
Note that, similar to Lemma 1, a lower c represents higher quality q in equilibrium. Under condition (A.2), we have $p_{n,o}^{CN^*} = \frac{3c(\lambda-1)(2f(\lambda-1)+\lambda n)+A_1+1}{9c(\lambda-1)^2}$, $D_{n,o}^{CN^*} = \frac{2-A_1}{3}$, $\pi_{n,o}^{CN^*} = \frac{(-12c(\lambda-1)(f(\lambda-1)-\lambda n)+A_1+1)^2}{54c(\lambda-1)^2(A_1+1)}$, $CS_{n,o}^{CN^*} = \frac{1}{54}\left(\frac{72c(f(\lambda-1)-\lambda n)\left(f\left(\lambda^2-1\right)-(\lambda-2)\lambda n\right)}{\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+1} - \frac{(5\lambda-1)(A_1-1)}{2c(\lambda-1)^2} + \frac{1-5\lambda}{c(\lambda-1)^2} + 12f(2\lambda-1) - \frac{6\lambda(4\lambda+1)n}{\lambda-1}\right)$, and welfare $SW_{n,o}^{CN^*} = \frac{(12c(1-\lambda)(f(\lambda-1)-\lambda n)+A_1+1)\left(12c(\lambda-1)\left(f(\lambda-1)(\lambda+3)-\lambda^2 n\right)+(5\lambda-3)(A_1+1)\right)}{-108c(\lambda-1)^2\left(\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+1\right)}$ where $A_1 = \sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}$.

99-ending pricing. Under the 99-ending pricing scheme, the firm solves a discrete pricing problem in anticipation of consumers' behaviors, optimizing n given c, f, and λ where $\Delta = \lim_{\epsilon\to 0^+} 1-\epsilon$. Thus, the optimal price is $p_9^{CN^*} = n+1-\epsilon$, we thus have $\pi_9^{CN} = \frac{(\lambda-n+q-\lambda\epsilon+\epsilon-1)\left(cq^2+f-n+\epsilon-1\right)}{-q}$, and, following FOC and SOC conditions, we derive the equilibrium quality $q^{CN^*} = \frac{c^2(n+(\lambda-1)(\epsilon-1))^2+(A_2+c(n+(\lambda-1)(\epsilon-1)))A_2}{6cA_2}$ where $A_2 = \sqrt[3]{6\sqrt{3}\sqrt{B_1^2\left(-c^4\right)(f-n+\epsilon-1)\left(B_1^2c-27(f-n+\epsilon-1)\right)}+B_1^3c^3-54B_1c^2(f-n+\epsilon-1)}$ and $B_1 = n+(1-\lambda)(1-\epsilon)$, profit $\pi^{CN^*} = \left(1-\frac{6B_1c}{\frac{B_1^2c^2}{A_2}+A_2+B_1c}\right)\left(n+1-\frac{\left(\frac{B_1^2c^2}{A_2}+A_2+B_1c\right)^2}{36c}-f-\epsilon\right)$, consumers' true collective surplus $CS^{CN^*} = \frac{\left(-5A_2B_1c+A_2^2+B_1^2c^2\right)\left(A_2c(7B_1-12(n-\epsilon+1))+A_2^2+B_1^2c^2\right)}{12A_2c\left(A_2B_1c+A_2^2+B_1^2c^2\right)}$, and social welfare $SW^{CN^*} = \frac{\left(5A_2B_1c-A_2^2-B_1^2c^2\right)\left(A_2B_1^2c^2(2B_1c-3)+3A_2^2c(B_1(B_1c-7)+12f)+A_2^3(2B_1c-3)+A_2^4+B_1^4c^4\right)}{36A_2^2c\left(A_2B_1c+A_2^2+B_1^2c^2\right)}$.

Optimal pricing. The interior-pricing solution exists only subject to constraint (A.2), whereas the 99-ending solution for any given n is unconditional. An increase in c raises the marginal cost of quality, so higher c corresponds to lower equilibrium quality (exactly as q plays the role of the state variable in Lemma **??**). We therefore treat c as the primary market parameter and compare candidate profits across regimes and adjacent n, following the same logic as in the proof of Lemma **??**. Additionally, f is inherently irrelevant to quality, and we thus treat c as the main market condition to derive the firm's decisions on both quality and pricing. Given certain c, the comparison between interior ($\pi_o^{CN^*}$) and 99-ending ($\pi_9^{CN^*}$) pricing follows a similar logic; that is, we compare $\pi_{n,9}^{CN^*}$ and $\pi_{n-1,9}^{CN^*}$ with $\pi_{n,o}^{CN^*}$ under constraint (A.2), then compare $\pi_{n,9}^{CN^*}$ with $\pi_{n-1,9}^{CN^*}$ and $\pi_{n+1,9}^{CN^*}$ under other conditions. For conciseness, we show the following equilibrium results without the detailed process, interior pricing is optimal when $\underline{c}(\lambda)_n^{CN} < c < \overline{c}(\lambda)_n^{CN}$ and $0 < \lambda < \lambda_n^{CN}$, wherein $\underline{c}(\lambda)_n^{CN}$ is the lower bound of constraint (A.2) (i.e., $\pi_{n,o}^{CN^*} > \pi_{n,9}^{CN^*}$), $\overline{c}(\lambda)_n^{CN}$ is the unique solution of equation $\pi_{n,o}^{CN^*} = \pi_{n-1,9}^{CN^*}$, and λ_n^{CN} is the unique solution of $\underline{c}(\lambda)_n^{CN} = \overline{c}(\lambda)_n^{CN}$. 99-ending pricing is optimal when otherwise, specifically (1) $c(\lambda)_{n+1}^{CN} < c < \underline{c}(\lambda)_n^{CN}$ and $0 < \lambda < \lambda_{n+1}^{CN}$, (2) $c(\lambda)_{n+1}^{CN} < c < \underline{c}(\lambda)_n^{CN}$ and $\lambda_{n+1}^{CN} \leq \lambda \leq \lambda_n^{CN}$, and (3) $c(\lambda)_{n+1}^{CN} < c < c(\lambda)_n^{CN}$ and $\lambda_n^{CN} \leq \lambda \leq 1$. In summary, the thresholds (i.e., $\overline{c}(\lambda)_n^{CN}$, $\underline{c}(\lambda)_n^{CN}$, $c(\lambda)_n^{CN}$, and λ_n^{CN}) can be derived as following method: $\overline{c}(\lambda)_n^{CN}$ is unique solution of $\pi_{n,o}^{CN^*} = \pi_{n-1,9}^{CN^*}$ due to $\frac{\partial \pi_{n,o}^{CN^*}}{\partial c} < 0$ and $\frac{\partial \pi_{n-1,9}^{CN^*}}{\partial c} < 0$, and $\overline{c}(\lambda)_n^{CN} \geq \underline{c}(\lambda)_n^{CN}$, $c(\lambda)_n^{CN}$ is unique solution of $\pi_{n,9}^{CN^*} = \pi_{n-1,9}^{CN^*}$,

λ_n^{CN} is unique solution of $\underline{c}(\lambda)_n^{CN} = \overline{c}(\lambda)_n^{CN}$ due to $\frac{\partial \underline{c}(\lambda)_n^{CN}}{\partial c} > 0$ $\frac{\partial \overline{c}(\lambda)_n^{CN}}{\partial c} > 0$, and $\overline{c}(\lambda)_n^{CN} \geq \underline{c}(\lambda)_n^{CN}$. We further verify $\frac{\partial q_o^{CN*}}{\partial \lambda} > 0$ and $\frac{\partial q_9^{CN*}}{\partial \lambda} < 0$, and the optimal quality exhibits a discontinuous downward jump from $(n, n+1)$ to $(n-1, n)$ with a 99-ending price approaching n^-, as is shown in Figure A.1 right panel. The global equilibrium pricing is shown in Figure A.1 by parameter space (c, λ) for illustration.

Figure A.1: Optimal Quality-Pricing in Quality-Endogenous Scenario



Note. The left panel show the equilibrium prices for all levels of c with $f = 0$. The right panel draws comparative static of optimal quality regarding consumer bias across two levels of c, i.e., $c = 0.08$ and $c = 0.09$. For simplicity, we here use $\$n.$ to indicate interior pricing.

A.5 Proof of Proposition 1

In this proof, we first rationalize the shrinkflation phenomenon under 99-ending pricing and then explore how consumer left-digit bias affects a firm's strategic responses to cost shocks under interior pricing.

Cost shocks under 99-ending pricing. Given the equilibrium results under 99-ending pricing, we have $\frac{\partial p_9^{CN*}}{\partial c} = 0$ and $\frac{\partial p_9^{CN*}}{\partial f} = 0$, implying the rigid price under cost shocks. For firm quality responses under various cost shocks, we have shrinkflation $\frac{\partial q_9^{CN*}}{\partial c} = -\frac{\left(B_1^2c^2 - A_2^2\right)\left(B_1c^2\left(A_3 + B_1^2c + 36(n+1-f-\epsilon)\right) - A_2^3\right)}{6A_2^4c^2} < 0$, and $\frac{\partial q_9^{CN*}}{\partial f} = -\frac{(18 - A_4)B_1c\left(A_2^2 - B_1^2c^2\right)}{6A_2^4} < 0$, where $A_3 = \frac{\sqrt{3}c^2(f-n+\epsilon-1)(n+(\lambda-1)(\epsilon-1))\left(108(f-n+\epsilon-1) - 5c(n+(\lambda-1)(\epsilon-1))^2\right)}{\sqrt{-c^4(f-n+\epsilon-1)(n+(\lambda-1)(\epsilon-1))^2(c(n+(\lambda-1)(\epsilon-1))^2 - 27(f-n+\epsilon-1))}}$ and $A_4 = \frac{\sqrt{3}c^2(n+(\lambda-1)(\epsilon-1))\left(54(f-n+\epsilon-1) - c(n+(\lambda-1)(\epsilon-1))^2\right)}{\sqrt{-c^4(f-n+\epsilon-1)(n+(\lambda-1)(\epsilon-1))^2(c(n+(\lambda-1)(\epsilon-1))^2 - 27(f-n+\epsilon-1))}}$. Further, we can derive $\frac{\partial^2 q_9^{CN*}}{\partial c \partial \lambda} > 0$ and $\frac{\partial^2 q_9^{CN*}}{\partial f \partial \lambda} > 0$. Thus, cost shocks trigger quality reductions while prices remain fixed (shrinkflation), as is shown in right panel of Figure A.1.

Cost shocks under interior pricing. From Lemma 2, direct differentiation of the closed-form equilibrium price and quality gives, we have $\frac{\partial p_o^{CN*}}{\partial c} = -\frac{6c(1-\lambda)(f(1-\lambda)+\lambda n) + \sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1} + 1}{9c^2(\lambda-1)^2\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}} < 0$ and quality sensitivity $\frac{\partial q_o^{CN*}}{\partial c} = \frac{6c(\lambda-1)(f(\lambda-1)-\lambda n) + \sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1} + 1}{6c^2(\lambda-1)\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}} < 0$ due to $6c(\lambda - 1)(f(\lambda -$

$1) - \lambda n) > 0$. This shows that, under cost shocks depending on quality (i.e., c), firm has incentives to decrease quality with corresponding lower price, and consumer bias will boost such degradation, i.e., $\frac{\partial^2 q^*}{\partial c \partial \lambda} = \frac{-\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+6c(\lambda-1)(f(\lambda-1)-\lambda n)\left(-2\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}+6c(\lambda-1)n-3\right)-1}{6c^2(\lambda-1)^2(12c(\lambda-1)(f(\lambda-1)-\lambda n)+1)^{3/2}} < 0$ and $\frac{\partial^2 p^*}{\partial c \partial \lambda} < 0$. For f cost shocks, we have quality sensitivity regarding such cost shocks $\frac{\partial q_o^{CN^*}}{\partial f} = \frac{1-\lambda}{\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}} > 0$ and $\frac{\partial p_o^{CN^*}}{\partial f} = \frac{2}{3}\left(\frac{1}{\sqrt{12c(\lambda-1)(f(\lambda-1)-\lambda n)+1}} + 1\right) > 0$.

A.6 Proof of Corollary 3

Under 99-ending pricing, for particular bias degree λ, the length of the cost interval that triggers shrinkflation is $C(\lambda)_n^{CN} = \min\{\underline{c}(\lambda)_n^{CN}, c(\lambda)_n^{CN}\} - \max\{\overline{c}(\lambda)_{n+1}^{CN}, c(\lambda)_{n+1}^{CN}\}$. Treating n as continuous for a moment, direct differentiation of the threshold expressions yields $\frac{\partial\left(C(\lambda)_n^{CN} - C(\lambda)_{n+1}^{CN}\right)}{\partial n} > 0$. In the discrete case the inequality $C(\lambda)_n^{CN} > C(\lambda)_{n+1}^{CN}$ holds for all $n \in \mathbb{N}^+$. Thus the range of cost shocks that induce shrinkflation is strictly larger when the leftmost digit n is smaller. Moreover, differentiating the equilibrium quality $q_9^{CN^*}$ twice with respect to the bias parameter gives $\frac{\partial^2 q_9^{CN^*}}{\partial c\, \partial \lambda} > 0$ and $\frac{\partial^2 q_9^{CN^*}}{\partial f\, \partial \lambda} > 0$. Stronger left-digit bias therefore reduces the magnitude of quality adjustments required to absorb a given cost shock.

A.7 Proof of Corollary 4

Shrinkflation reduces consumer surplus because quality falls while the nominal price remains fixed. Formally, $\frac{\partial CS_9^{CN^*}}{\partial c} < 0$ and $\frac{\partial CS_9^{CN^*}}{\partial f} < 0$, which follows immediately from $\partial u/\partial q > 0$ and the fact that price is invariant to cost shocks in this regime, i.e., $\frac{\partial u(q,p)}{\partial q} < 0$. We further consider consumer surplus changes in certain points (i.e., $\overline{c}(\lambda)_n^{CN}$ and $c(\lambda)_n^{CN}$). At the regime-switching thresholds the welfare effect is more subtle. Consider first the upper boundary of the interior-pricing region, $\overline{c}(\lambda)_n^{CN}$. Crossing this threshold discontinuously shifts the optimal price downward (e.g., from an interior price near \$$(n+1)$ to the 99-ending price \$$n.99$). Substituting the common value $c = \overline{c}(\lambda)_n^{CN}$ into the two surplus expressions shows $(CS_{n,9}^{CN^*} - CS_{n,o}^{CN^*})|_{c=\overline{c}_n^{CN}} > 0$ when $0 < \lambda < \lambda_n^{CN}$. Although we choose not to directly show the whole comparative results given its complicated forms, we derive some numerical results here; for example, consider $f = 0.1$, $\lambda = 0.025$, and $n = 2$, we can easily have $\overline{c}_2^{CN^*} = 0.0996$ and $(CS_{1,9}^{CN^*} - CS_{2,o}^{CN^*})|_{c=\overline{c}_2^{CN}} = 0.1764 - 0.1634 = 0.013 > 0$.

Further consider the point $c(\lambda)_n^{CN}$, at which optimal price shifts from \$$n.99$ to \$$(n.99 - 1)$. Similarly, we are able to derive $(CS_{n-1,9}^{CN^*} - CS_{n,9}^{CN^*})|_{c=c_n^{CN}} > 0$ only when $0 < \lambda < \dot{\lambda}(c)_n$. Consider $f = 1/10$ and $n = 2$, we have $\dot{\lambda}(c)_n = 0.45$, and we further suppose $\lambda = 0.25$ and derive $c_2 = 0.092$, thereby showing $(CS_{1,9}^{CN^*} - CS_{2,9}^{CN^*})|_{c=c_2^{CN}} = 0.143 - 0.126 = 0.017 > 0$. It is noteworthy that because consumer surplus decreases with λ, there is the unique solution of $\dot{\lambda}(c)_n^{CN}$ via $(CS_{n-1,9}^{CN^*} = CS_{n,9}^{CN^*})|_{c=c_n^{CN}}$ according to the single crossing condition and zero theorem. We further consider the consumer surplus dynamics regarding costs f. When λ is high ($\ddot{\lambda}(c)_n < \lambda < 1$), it increases as f increases at $f(\lambda)_n^{CN}$, where optimal price shifts from \$$n$.99 to \$(n.99 + 1). The proving logic is quite similar to the above comparative analyses.

A.8 Proof of Lemma 3

Now consider the decentralized channel (denoted by superscript DX) wherein the manufacturer sets its wholesale price (w^{DX}), and subsequently, the retailer chooses its retail price (p^{DX}) in anticipation of left-digit biased consumers' behaviors, assuming $f = 0$ here for analytical comparison. We respectively solve two cases (i.e., interior and 99-ending pricing) and then compare them to derive the globally optimal solutions.

$$\max_{n\in\mathbb{N}^+,\ \Delta\in[0,1)} \pi(n,\Delta)_r^{DX} = (n+\Delta-w)\cdot(1-\frac{n+(1-\lambda)\Delta}{q})$$

$$\max_{w} \pi(q,w)_m^{DX} = (w-cq^2)\cdot(1-\frac{n+(1-\lambda)\Delta}{q})$$

Interior pricing: We first consider the retailer's profit-maximization problem and thus solve its best response function. The FOC and SOC of the profit function are $\frac{\partial\pi_{or}^{DX}}{\partial\Delta} = \frac{(\lambda-2)n+q-(\lambda-1)(w-2\Delta)}{q}$ and $\frac{\partial^2\pi_{or}^{DX}}{\partial\Delta^2} = \frac{2(\lambda-1)}{q} < 0$, which imply $\Delta_o^* = -\frac{-\lambda n+2n-q+\lambda w-w}{2(\lambda-1)}$ by setting $\frac{\partial\pi_{or}^{DX}}{\partial\Delta} = 0$. Substituting Δ_o^* into the manufacturer's profit function, we can get $\pi_{om}^{DX} = -\frac{\left(cq^2-w\right)\left(-\lambda n+q+(\lambda-1)w\right)}{2q}$. Similarly, the FOC and SOC of the profit function are $\frac{\partial\pi_{om}^{DX}}{\partial w} = \frac{q^2(c-c\lambda)-\lambda n+q+2(\lambda-1)w}{2q}$ and $\frac{\partial^2\pi_{om}^{DX}}{\partial w^2} = \frac{\lambda-1}{q} \le 0$, which imply $w_o^{DX} = \frac{c(\lambda-1)q^2+\lambda n-q}{2(\lambda-1)}$ by setting $\frac{\partial\pi_{om}^{DX}}{\partial w} = 0$, implying $\Delta_o^{DX^*} = \frac{q(c(\lambda-1)q-3)-(\lambda-4)n}{4(\lambda-1)}$. To satisfy the constraint (i.e., $0 \le \Delta^{DX^*} < 1$), we must have the following feasible condition (A.3) to enable interior pricing:

$$\frac{\sqrt{4c\left(\lambda^2-5\lambda+4\right)n+9}-3}{2c(1-\lambda)} \le q < \frac{\sqrt{4c(1-\lambda)((1-\lambda)(n+4)+3n)+9}-3}{2c(1-\lambda)} \tag{A.3}$$

Under the condition (A.3), we can derive the following market outcomes: Demand $D_o^{DX^*} = \frac{c(\lambda-1)q^2-\lambda n+q}{4q}$, retailer profit $\pi_{n,or}^{DX^*} = \frac{\left(c(\lambda-1)q^2-\lambda n+q\right)^2}{16(1-\lambda)q}$, manufacturer profit $\pi_{n,om}^{DX^*} = \frac{\left(c(\lambda-1)q^2-\lambda n+q\right)^2}{8(1-\lambda)q}$, channel profit $\pi_{n,oc}^{DX^*} = \frac{3\left(c(\lambda-1)q^2-\lambda n+q\right)^2}{16(1-\lambda)q}$, consumer surplus $CS_{n,o}^{DX^*} = \frac{\left(c(\lambda-1)q^2-\lambda n+q\right)\left(c\left(\lambda^2-1\right)q^2-(\lambda-7)\lambda n-7\lambda q+q\right)}{32(1-\lambda)q}$, and social welfare $SW_{n,o}^{DX^*} = \frac{\left(c(\lambda-1)q^2-\lambda n+q\right)(7-q(c(\lambda+7)q)+\lambda n)}{32q}$.

99-ending pricing: The manufacturer and retailer sequentially determine the wholesale and retail prices in anticipation of left-digit biased consumers' choices. In the 99-ending pricing scheme, the rightmost digits are fixed at the corner solution $\Delta = \lim_{\epsilon\to 0^+}(1-\epsilon)$, so the retail price is $p = n + 1^-$ in the limit. The retailer only chooses the integer leftmost digit $n \in \mathbb{N}^+$. Since n is a discrete number (i.e., $n \in \mathbb{N}^+$), we first treat it as a continuous number $\tilde{n}$ and find the optimal $\tilde{n}^*$, which could be any non-negative numbers. Note that $\mathbb{N}^+$ is a set of non-negative round numbers, and n is a certain non-negative round number. We can thus derive $\tilde{n}^* = \lim_{\epsilon\to 0^+}\frac{1}{2}(q+w-(\lambda-2)(\epsilon-1))$ due to $\frac{\partial\pi^{DX}}{\partial\tilde{n}} = \frac{\lambda-2\tilde{n}+q+w-\lambda\epsilon+2\epsilon-2}{q}$ and $\frac{\partial^2\pi^{DX}}{\partial\tilde{n}^2} = \frac{-q}{2} < 0$ by setting $\frac{\partial\pi^{DX}}{\partial\tilde{n}} = 0$. Suppose $n-1 \le \tilde{n}^* \le n$, the optimal n^* must be either

$n-1$ or n due to its concavity and unimodality, i.e., $\frac{\partial^2 \pi^{DX}}{\partial \tilde{n}^2} < 0$. This also represents that $n^* = n$ when $\pi_{n-1,r} < \pi_{n,r}$, $\pi_{n,r} > \pi_{n+1,r}$ and $\pi_{n,r} > 0$. Thus, given $\Delta_9^* = 1-\epsilon$, we can derive $p_9^* = n+1-\epsilon$ when $2n-q-2\epsilon-\lambda+\lambda\epsilon < w \leq \min\{2n+2-q-2\epsilon-\lambda+\lambda\epsilon, n+1-\epsilon\}$. Thus, the manufacturer's profit is $\pi_m^{DX} = \frac{(w-cq^2)(\lambda-n+q-\lambda\epsilon+\epsilon-1)}{q}$, which means the manufacturer's profit is increasing in its wholesale price under the constraint conditions, and the manufacturer chooses w as high as possible subject to the constraint that prevents the retailer from profitably deviating to the next left digit. It is noteworthy that the wholesale price must be lower than the retail price, i.e., $w^* \leq n+p$. Also, for clarification, we focus on the parameter range wherein $n^* = n$. Thus, given retailer response function, the manufacturer's optimal wholesale price is:

$$w^{DX^*} = \begin{cases} n+1-\epsilon, & p^* = n+1-\epsilon, \quad 2n-q-2\epsilon-\lambda+\lambda\epsilon < w \leq n+1-\epsilon \\ 2n+2-q-2\epsilon-\lambda+\lambda\epsilon, & p^* = n+1-\epsilon, \quad w \leq 2n+2-q-2\epsilon-\lambda+\lambda\epsilon \end{cases};$$

Substituting the optimal wholesale and retail prices into the profit function to specify the conditions:

$$\pi_m^{DX^*} = \begin{cases} \frac{(n-\epsilon+1-cq^2)(\lambda-n+q-\lambda\epsilon+\epsilon-1)}{q} & \{w^{DX^*} = n+1-\epsilon,\ p^{DX^*} = n+1-\epsilon\}, \\ \frac{(n+1-\lambda-q+\lambda\epsilon-\epsilon)(\lambda-2n+q-\lambda\epsilon+2\epsilon-2-cq^2)}{q} & \{w^{DX^*} = 2n+2-q-2\epsilon-\lambda+\lambda\epsilon,\ p^{DX^*} = n+1-\epsilon\}; \end{cases}$$

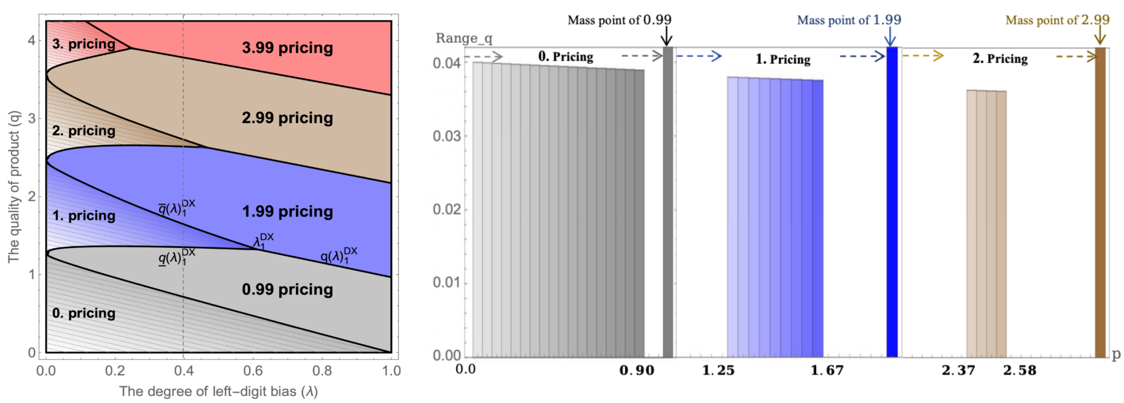
Thus, the manufacturer setting the wholesale price at $n+1-\epsilon$ derives strictly lower profits than pricing at $2n+2-q-2\epsilon-\lambda+\lambda\epsilon$. Thus, we have the optimal pricing: $w^* = 2(n+1-\epsilon)-q--\lambda+\lambda\epsilon$ and $p^* = n+1-\epsilon$.

Thus, the other wholesale prices under different n can be derived similarly. We use the subscript $9r$ and $9m$ denote the 99-ending pricing for the retailer and manufacturer, respectively. Given q and the optimal Δ_9^* (i.e., $\Delta_9^* = 1-\epsilon$ where $\epsilon \to 0^+$), optimal n and wholesale price are settled, we summarize the equilibrium market outcomes: demand $D_{n,9}^{DX^*} = \frac{\lambda-n+q-\lambda\epsilon+\epsilon-1}{q}$, retailer profit $\pi_{n,9r}^{DX^*} = \frac{(\lambda-n+q-\lambda\epsilon+\epsilon-1)^2}{q}$, manufacturer profit $\pi_{n,9m}^{DX^*} = \frac{(\lambda-n+q-\lambda\epsilon+\epsilon-1)\left(cq^2-2n+q-(\lambda-2)(\epsilon-1)\right)}{-q}$, channel profit $\pi_{n,9c}^{DX^*} = \frac{\left(cq^2-n+\epsilon-1\right)(\lambda-n+q-\lambda\epsilon+\epsilon-1)}{-q}$, consumer surplus $CS_{n,9}^{DX^*} = \frac{(q-n+\epsilon-1)^2-\lambda^2(\epsilon-1)^2}{2q}$, and social welfare $SW_{n,9}^{DX^*} = \frac{(q-n-(\lambda-1)(\epsilon-1))\left(-2cq^2+n+q+(\lambda-1)(\epsilon-1)\right)}{2q}$. It is noteworthy that the manufacturer has to decrease its price without enforcing the retailer to increase its left-digit number significantly as left-digit bias or quality increases (i.e., quantity contract): $\frac{w_9^{DX^*}}{\partial\lambda} = \epsilon-1 < 0$ and $\frac{w_9^{DX^*}}{\partial q} = -1 < 0$. We further derive the globally optimal pricing.

Optimal pricing: Thus far, we have derived the respective conditions for interior and 99-ending pricing for further comparison. In our setting, the manufacturer can determine its wholesale price given the retailer's best response function, i.e., first-mover advantage; that is, it is the manufacturer who actually decides the ultimate pricing scheme, either interior or 99-ending pricing. Thus, the equilibrium pricing scheme is derived through a comparison of the manufacturer's profit. Note that the interior pricing equilibrium is conditional on the constraint (A.3), whereas the 99-ending pricing is unconditional. Similar to the proof of Lemma **??**, we can derive the following equilibrium pricing decisions without repeating the comparison process. (i) Interior pricing is optimal when $\underline{q}(\lambda)_n^{DX^+} < q < \bar{q}(\lambda)_n^{DX}$ and $0 < \lambda < \min\{\lambda_n^{DX}, 1\}$. (ii) 99-ending pricing is optimal when (1) $\bar{q}(\lambda)_n^{DX^+} < q < min\{\underline{q}(\lambda)_{n+1}^{DX}, q(\lambda)_{n+1}^{DX}\}$ and $0 < \lambda < \min\{\lambda_n^{DX}, 1\}$ or (2) $\max\{\bar{q}(\lambda)_n^{DX}, q(\lambda)_n^{DX}\} <$

$q < q(\lambda)_{n+1}^{DX}$ and $\lambda_n^{DX} \leq \lambda \leq 1$. We further clarify the existence and uniqueness of these thresholds, i.e., $\underline{q}(\lambda)_n^{DX}$, $\overline{q}(\lambda)_n^{DX}$, $q(\lambda)_n^{DX}$, and λ_n^{DX}. Let's consider the comparative static of the manufacturer's profit in terms of quality, we can derive $\frac{\partial \pi_{9m}^{DX^*}}{\partial q} = \frac{(-\lambda+n+(\lambda-1)\epsilon+2)\left(-cq^2-\lambda+2n-q+\lambda\epsilon-2\epsilon+3\right)-q(2cq+1)(\lambda-n+q-\lambda\epsilon+\epsilon-2)}{q^2} > 0$ under 99-ending pricing, $\frac{\partial \pi_{om}^{DX^*}}{\partial q} = \frac{\left(c(\lambda-1)q^2-\lambda n+q\right)\left(3c(\lambda-1)q^2+\lambda n+q\right)}{8(1-\lambda)q^2} > 0$ under interior pricing. Thus, $q(\lambda)_n^{DX}$ is the unique feasible solution of $\pi_{n,9m}^{DX^*} = \pi_{n-1,9m}^{DX^*}$ according to single crossing condition, deriving $q(\lambda)_n^{DX} = \frac{\sqrt{-12c\lambda+16cn+12c\lambda\epsilon-16c\epsilon+20c+9}-3}{2c}$. Similarly, $\underline{q}(\lambda)_n^{DX}$ is the unique solution of $\pi_{n-1,9m}^{DX^*} = \pi_{n,om}^{DX^*}$, $\underline{q}(\lambda)_n^{DX} = \frac{-8\lambda^2+19\lambda+2\sqrt{2}\sqrt{(\lambda-1)\lambda(4(\lambda-1)+(\lambda-1)n^2+n(4\lambda-2(\lambda-1)\epsilon-5)+(\lambda-1)\epsilon^2+(5-4\lambda)\epsilon)}+11\lambda n-12n+8\lambda^2\epsilon-20\lambda\epsilon+12\epsilon-12}{8\lambda-9}$ in the special case of $c = 0$, and the form of $c > 0$ is complicated and available. Also, $\overline{q}(\lambda)_n^{DX}$ is the upper bound of q, which is $\overline{q}(\lambda)_n^{DX} = \frac{\sqrt{4c(1-\lambda)((1-\lambda)(n+4)+3n)+9}-3}{2c(1-\lambda)}$ since $\pi_{n,om}^{DX^*} > \pi_{n,9m}^{DX^*}$. Thus, λ_n^{DX} is the unique solution of $\underline{q}(\lambda)_n^{DX} = \overline{q}(\lambda)_n^{DX}$ due to $\frac{\partial \underline{q}(\lambda)_n^{DX}}{\partial \lambda} > 0$ and $\frac{\partial \overline{q}(\lambda)_n^{DX}}{\partial \lambda} < 0$, getting $\lambda_n^{DX}|_{c=0} = 2 + \frac{14\epsilon+13}{(2\epsilon+3)(\epsilon-n)} - \frac{8}{(n-\epsilon)^2} - \frac{4(n(2n-2\epsilon+3)-\epsilon+3)\sqrt{(-3n+\epsilon-3)(n(2n-2\epsilon-9)+\epsilon-12)}}{(2n+3)^2(n-\epsilon)^2} - \frac{20\epsilon+46}{4n\epsilon+6n+6\epsilon+9} + \frac{16}{(2n+3)^2}$. Note that the value of marginal production cost does not essentially affect the whole equilibrium pattern. For the expositional ease, we denote the ranges for interior and 99-ending pricing by CON_o^{DX} and CON_9^{DX}, respectively. We show the above equilibrium pricing patterns in Figure A.2 for illustration.

Figure A.2: Equilibrium Pricing in Decentralized Channels ($c = 0.1$)



A.9 Proof of Proposition 2

Firstly, the equilibrium results without left-digit bias have been extensively explored in the previous studies, we show the main results for completeness. Under decentralized channels without consumer left-digit bias (i.e., $\lambda = 0$, denoted by superscript ^{dx}), the equilibrium outcomes are given as: $w^{dx^*} = \frac{q(cq+1)}{2}$, $p^{dx^*} = \frac{q(cq+3)}{4}$, $D^{dx^*} = \frac{(1-cq)}{4}$, $\pi_m^{dx^*} = \frac{q(1-cq)^2}{8}$, $\pi_r^{dx^*} = \frac{q(1-cq)^2}{16}$, and the total channel profit $\pi^{dx^*} = \frac{3q(1-cq)^2}{16}$. Notably, $\frac{\partial w^{dx^*}}{\partial q} > 0$, $\frac{\partial p^{dx^*}}{\partial q} > 0$, $\frac{\partial (D^{cx^*}-D^{dx^*})}{\partial q} < 0$, and $\frac{\partial (\pi^{cx^*}-\pi^{dx^*})}{\partial q} > 0$. Given the building block results, we further analyze how consumer left-digit bias affects the profitability of channel members.

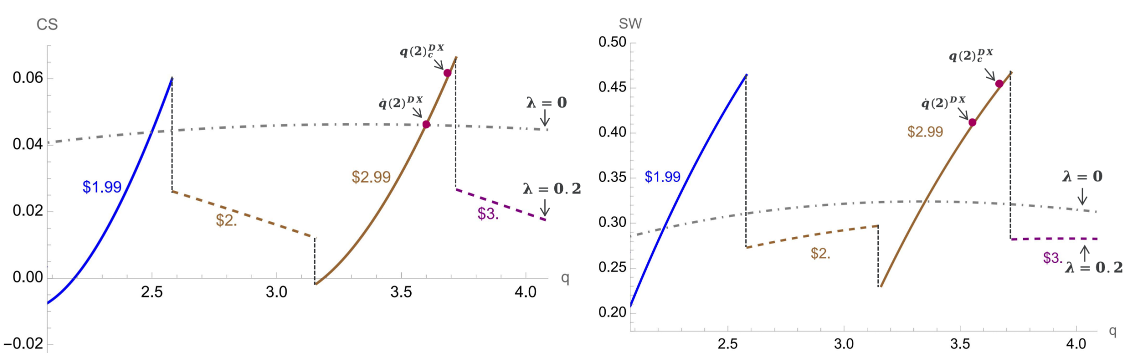
Interior pricing. Note that, regardless of consumer left-digit bias, the retailer always gets half profit relative to the manufacturer, and the comparative pattern regarding two parties can be analyzed by one party(e.g., manufacturer). For comparative statics, we have $\pi_{om}^{DX^*} - \pi_m^{dx^*} = -\frac{\left(c(\lambda-1)q^2-\lambda n+q\right)^2}{8(\lambda-1)q} - \frac{1}{8}q(cq-1)^2$, implying $q(\lambda)_{n,a}^{DX} = \frac{1-\sqrt{1-4n\left(\sqrt{-c^2(\lambda-1)}+c(\lambda-1)\right)}}{2c\sqrt{1-\lambda}}$ via equation $\pi_{om}^{DX^*} - \pi_m^{dx^*} = 0$. We have $\frac{\partial \pi_{om}^{DX^*}}{\partial q} = -\frac{\left(c(\lambda-1)q^2-\lambda n+q\right)\left(3c(\lambda-1)q^2+\lambda n+q\right)}{8(\lambda-1)q^2}$, and find $\frac{\partial \pi_{om}^{DX^*}}{\partial q} > 0$ ($\frac{\partial \pi_{om}^{DX^*}}{\partial q} < 0$) when $q < q(\lambda)_{n,o}^{DN^*}$ ($q > q_{n,o}^{DN^*}$) where $q_{n,o}^{DN^*} = \frac{\sqrt{1-12c(\lambda-1)\lambda n}+1}{6c(1-\lambda)}$ due to $\frac{\partial^2 \pi_{om}^{DX}}{\partial q^2} = \frac{1}{4}\left(-3c^2(\lambda-1)q - 2c - \frac{\lambda^2 n^2}{(\lambda-1)q^3}\right) < 0$ only if c is not small. Note that $q_{n,o}^{DN^*}$ is the optimal quality level when we consider the decentralized channel with endogenous quality. Further, we have $q_{n,o}^{DN^*} - q(\lambda)_{n,a}^{DX} = (\frac{\sqrt{1-12c(\lambda-1)\lambda n}+1}{6c(1-\lambda)}) - (\frac{1-\sqrt{1-4n\left(\sqrt{-c^2(\lambda-1)}+c(\lambda-1)\right)}}{2c\sqrt{1-\lambda}}) > 0$. This implies that when $\underline{q}(\lambda)_n^{DX} < q < q(\lambda)_{n,o}^{DN^*}$, manufacturer profit is monotonically increasing in quality, implying that $q(\lambda)_{n,a}^{DX}$ is the unique solution via single crossing condition. We further prove both Region I and Region II; that is, $\overline{q}(\lambda)_n^{DX} - q(\lambda)_{n,a}^{DX} = \frac{\sqrt{1-\lambda}\left(\sqrt{1-4n\left(\sqrt{-c^2(\lambda-1)}+c(\lambda-1)\right)}-1\right)+\sqrt{4c(\lambda-1)(\lambda(n+4)-4(n+1))+9}-3}{2(c-c\lambda)} > 0$ and $\pi_{om}^{DX^*} - \pi_m^{dx^*} = \frac{64(1-\lambda)\left(\sqrt{A_5+9}+2c(\lambda-1)(-\lambda+n+1)-3\right)^2+\left(\sqrt{A_5+9}-3\right)^2\left(\sqrt{A_5+9}+2\lambda-5\right)^2}{64\left(\sqrt{A_5+9}-3\right)c(1-\lambda)^3} > 0$ when $q = \overline{q}(\lambda)_n^{DX}$, where $A_5 = 4c(1-\lambda)(4(n+1)-\lambda(n+4)) > 0$. In summary, under interior pricing, we have formally proved the key results: $\underline{q}(\lambda)_n^{DX} < q(\lambda)_{n,a}^{DX} < q(\lambda)_{n,o}^{DN^*}$ and $\underline{q}(\lambda)_n^{DX} < q(\lambda)_{n,a}^{DX} < \overline{q}(\lambda)_n^{DX}$, enabling Regions I and II. Moreover, we can further check how consumer left-digit bias intensity affects the region scope; that is, $\frac{\partial \underline{q}(\lambda)_n^{DX}}{\partial \lambda} > 0$, $\frac{\partial \overline{q}(\lambda)_n^{DX}}{\partial \lambda} < 0$, $\frac{\partial q(\lambda)_{n,a}^{DX}}{\partial \lambda} = \frac{2n\sqrt{-c^2(\lambda-1)}+\sqrt{1-4n\left(\sqrt{-c^2(\lambda-1)}+c(\lambda-1)\right)}-1}{4c(1-\lambda)^{3/2}\sqrt{1-4n\left(\sqrt{-c^2(\lambda-1)}+c(\lambda-1)\right)}} > 0$, and $\frac{\partial \pi_{om}^{DX^*}}{\partial \lambda} = \frac{\frac{(n-q)^2}{(\lambda-1)^2}-\left(n-cq^2\right)^2}{8q} < 0$. This implies that a higher bias intensifies double marginalization, hurting both.

99-ending pricing. For Region III, the manufacturer obtains higher profits, whereas the retailer has lower profits. We substitutes $\overline{q}(\lambda)_n^{DX}$ into the profit function, deriving $(\pi_{9r}^{DX^*} - \pi_r^{dx^*})|_{q=\overline{q}(\lambda)_n^{DX}} = \left(1 - \frac{2c(1-\lambda)(n+(1-\lambda)(1-\epsilon))}{\sqrt{A_5+9}-3}\right)\left(\frac{\sqrt{A_5+9}-3}{2c(1-\lambda)} + \lambda - n - \lambda\epsilon + \epsilon\right) - \frac{\left(\sqrt{A_5+9}-3\right)\left(-\sqrt{A_5+9}-2\lambda+5\right)^2}{128c(1-\lambda)^3} < 0$ and also $(\pi_{9m}^{DX^*} - \pi_m^{dx^*})|_{q=\overline{q}(\lambda)_n^{DX}} = \frac{B_2\left(\left(\sqrt{A_5+9}-3\right)(\lambda+2)+2c(\lambda-1)((\lambda+2)n+(\lambda-1)(\lambda(\epsilon-1)-2(\epsilon+1)))\right)+\left(-3+\sqrt{A_5+9}\right)^2\left(\sqrt{A_5+9}+2\lambda-5\right)^2}{64\left(\sqrt{A_5+9}-3\right)c(\lambda-1)^3} > 0$ where $B_2 = 32(\lambda-1)\left(\sqrt{A_5+9}+2c(\lambda-1)(n+(\lambda-1)(\epsilon-1))-3\right)$. As quality q increases, we have $\frac{\partial \pi_{n,9r}^{DX^*}}{\partial q} = \frac{(q^2-n^2)+n(2\lambda-2(\lambda-1)\epsilon-1)+(1-\lambda)(1-\epsilon)(\lambda(1-\epsilon)+\epsilon)}{q^2} > 0$ due to $q > n$, whereas manufacturer's profit sensitivity analysis is more nuanced, $\frac{\partial \pi_{n,9m}^{DX^*}}{\partial q} = \frac{n\left(cq^2+(3\lambda-4)(\epsilon-1)\right)-2cq^3+q^2(c(\lambda-1)(\epsilon-1)-1)+2n^2+(\lambda-2)(\lambda-1)(\epsilon-1)^2}{q^2}$ can be positive or negative. Further, we have $\frac{\partial^2 \pi_{n,9m}^{DX^*}}{\partial q^2} = -\frac{2\left(cq^3+2n^2+(4-3\lambda)n(1-\epsilon)+\left(2+\lambda^2-3\lambda\right)(\epsilon-1)^2\right)}{q^3} < 0$, implying the non-monotonic pattern. We further prove that the manufacturer's profit first increases and then decreases with quality, which will help prove the existence of Regions IV and V. Thus, we have $q_{n,9}^{DN^*} = \frac{\sqrt[3]{B_4+B_5}(c\lambda+c(-n)-c\lambda\epsilon+c\epsilon-c+1)+(B_4+B_5)^{2/3}+(c(n+(1-\lambda)(1-\epsilon))-1)^2}{-6\sqrt[3]{B_4+B_5}c}$, which is the unique solution of the first-order condition $\frac{\partial \pi_{n,9m}^{DX^*}}{\partial q} = 0$, where $B_3 = 17n^2 + (25\lambda-34)n(\epsilon-1) + \left(8\lambda^2-25\lambda+17\right)(\epsilon-1)^2$, $B_4 = -c^3(n+(\lambda-1)(\epsilon-1))^3 - 3c^2\left(35n^2+2(26\lambda-35)n(\epsilon-1)+\left(17\lambda^2-52\lambda+35\right)(\epsilon-1)^2\right) - 3c(n+(\lambda-1)(\epsilon-1)) + 1$, $B_5 = 6\sqrt{3}\sqrt{c^2\left(B_6^3c^3+3B_3c^2+3B_6c-1\right)\left(2n^2+(3\lambda-4)n(\epsilon-1)+\left(\lambda^2-3\lambda+2\right)(\epsilon-1)^2\right)}$, and $B_6 = n+(\lambda-1)(\epsilon-1)$. Further, we can derive $q(\lambda)_{n,b}^{DX} = \frac{\sqrt{16c(n+(\lambda-1)(\epsilon-1))+9}-3}{2c}$, which is the unique feasible solution of equation $\pi_{9r}^{DX^*} = \pi_r^{dx^*}$ due to $\frac{\partial \pi_{9r}^{DX^*}}{\partial q} > 0$, and we can also verify that $\max\{\overline{q}(\lambda)_n^{DX}, q(\lambda)_{n-1}^{DX}\} < q(\lambda)_{n,b}^{DX} < q_{n,9}^{DN^*}$. We then substitute $\overline{q}(\lambda)_n^{DX}$ into manufacturer's profit, deriving $(\pi_{n,9m}^{DX^*} - \pi_m^{dx^*})|_{q=\overline{q}(\lambda)_n^{DX}} = \frac{32(\lambda-1)\left(\sqrt{A_5+9}+2B_6c(\lambda-1)-3\right)\left(\left(-3+\sqrt{A_5+9}\right)(\lambda+2)+2c(\lambda-1)((\lambda+2)n+(\lambda-1)(\lambda(\epsilon-1)-2(\epsilon+1)))\right)+\left(-3+\sqrt{A_5+9}\right)^2\left(-5+\sqrt{A_5+9}+2\lambda\right)^2}{64\left(-3+\sqrt{A_5+9}\right)c(\lambda-1)^3} < 0$. Thus, there is a unique solution (i.e., $q(\lambda)_{n,c}^{DX}$) of $\pi_{n,9m}^{DX^*} = \pi_m^{dx^*}$ according to single crossing condition and zero theorem. This completes our proof of the existence of the regions. We here also consider channel profit (subscript c) changes with left-digit bias, deriving $\frac{\partial \pi_o^{CX^*}}{\partial \lambda} \equiv \frac{\frac{(n-q)^2}{(\lambda-1)^2}-\left(n-cq^2\right)^2}{4q} > \frac{\partial \pi_{oc}^{DX^*}}{\partial \lambda} \equiv \frac{3\left(\frac{(n-q)^2}{(\lambda-1)^2}-\left(n-cq^2\right)^2\right)}{16q} > 0$. Under 99-ending pricing, $\frac{\partial \pi_{9c}^{DX^*}}{\partial \lambda} \equiv \frac{(1-\epsilon)\left(n+2-cq^2-\epsilon\right)}{q} > \frac{\partial \pi_9^{CX^*}}{\partial \lambda} \equiv \frac{(1-\epsilon)\left(n+1-cq^2-\epsilon\right)}{q} > 0$.

A.10 Proof of Corollary 5

Given the equilibrium outcomes characterized in Lemma 3, we compare consumer surplus with and without left-digit bias, complementing the profitability comparison in Proposition 2. Under interior pricing, we have $CS_o^{DX^*} - CS^{dx^*} = \frac{\lambda\left(2nq(c(\lambda-3)(\lambda-1)q-3(\lambda+1))+q^2(6-c(\lambda-1)q(c\lambda q-6))-(\lambda-7)\lambda n^2\right)}{32(\lambda-1)q} < 0$ only if $\lambda > 0$ under CON_o^{DX}, and we further have $\frac{\partial(CS_o^{DX^*}-CS^{dx^*})}{\partial \lambda} = -\frac{\lambda\left(n-cq^2\right)^2+3(q-n)\left(n-cq^2\right)+\frac{3(n-q)^2}{(\lambda-1)^2}}{16q} < 0$ where the negativity follows from $q > n$ throughout the feasible region. Thus consumer surplus is strictly lower under bias case than in the rational benchmark, and stronger bias further reduces surplus. Combined with Proposition **??** , this produces a "triple-lose" outcome (manufacturer profit, retailer profit, and consumer surplus all decline relative to the rational benchmark) when $q < q(\lambda)_{n,a}^{DX}$. Under 99-ending pricing, we have $\frac{\partial CS_9^{DX^*}}{\partial q} = \frac{-n^2+2n(\epsilon-1)+q^2+\left(\lambda^2-1\right)(\epsilon-1)^2}{2q^2} < 0$, $\frac{\partial(CS_9^{DX^*}-CS^{dx^*})}{\partial q} = \frac{-q^2(cq-3)(3cq+5)-16n^2+32n(\epsilon-1)+16\left(\lambda^2-1\right)(\epsilon-1)^2}{32q^2} < 0$, and $CS_9^{DX^*} - CS^{dx^*} = \frac{(-n+q+\epsilon-1)^2-\lambda^2(\epsilon-1)^2}{2q} - \frac{1}{32}q(cq-1)^2$, which could be positive or negative. Because the difference is continuous in q and strictly decreasing, by the intermediate-value theorem there exists a unique crossing point $\dot{q}(\lambda)_n^{DX}$ that solves $CS_9^{DX^*} = CS^{dx^*}$. The results are illustrated in Figure A.3.

Figure A.3: Comparative Analysis of Left-Digit Bias on Consumer Surplus



Note. The left panel plots consumer surplus (CS) across quality levels q, contrasting the biased case ($\lambda = 0.2$)—with interior pricing (colored dashed line) and 99-ending pricing (colored solid line)—against the rational benchmark ($\lambda = 0$, gray dot-dashed line). The right panel compares social welfare (SW) under the same scenarios.

Therefore, consumer surplus under 99-ending pricing exceeds the rational benchmark precisely when $\dot{q}(\lambda)_n^{DX} < q < q(\lambda)_{n,c}^{DX}$, where $q(\lambda)_{n,c}^{DX}$ is the upper boundary of Region IV defined in the proof of Proposition **??**. In this interval the bias produces a "triple-win" outcome.

A.11 Proof of Lemma 4 and Corollary 6

Further, we characterize the manufacturer's quality–choice problem under interior and 99-ending pricing, incorporating the retailer's equilibrium pricing response derived in Lemma 3. Under interior pricing, the manufacturer's optimal quality is $q_{n,o}^{DN^*} = \frac{\sqrt{1-12c(\lambda-1)\lambda n}+1}{6c(1-\lambda)}$. Substituting $q_o^{DN^*}$ into the corresponding profit functions yields $\pi_{n,om}^{DN^*} = \frac{\left(12c(\lambda-1)\lambda n+\sqrt{1-12c(\lambda-1)\lambda n}+1\right)^2}{108c(\lambda-1)^2\left(\sqrt{1-12c(\lambda-1)\lambda n}+1\right)}$ and $\pi_{or}^{DN^*} = \frac{1}{2}\pi_{om}^{DN^*}$. Under 99-ending pricing, the optimal quality $q_{n,9}^{DN^*}$ is obtained from the first-order condition of the manufacturer's problem, yielding the closed-form expression reported above. Substituting $q_{n,9}^{DN^*}$ into the profit functions gives the corresponding equilibrium profits under 99-ending pricing. Comparative statics with respect to λ show that $\frac{\partial q_{n,o}^{DN^*}}{\partial \lambda} = \frac{q-n}{2(\lambda-1)^2} > 0$ and $\frac{\partial q_{n,9}^{DN^*}}{\partial \lambda} < 0$. Hence, stronger left-digit bias increases equilibrium quality under interior pricing but reduces it under 99-ending pricing, highlighting the opposing effects across regimes.

A.12 Proof of Proposition 3

Under interior case, comparing with the no-bias benchmark $\pi_m^{dn^*} = 1/(54c)$ and $\pi_r^{dn^*} = 1/(108c)$, we obtain $\pi_{om}^{DN^*} - \pi_m^{dn^*}$ and $\pi_{or}^{DN^*} - \pi_r^{dn^*} = \frac{1}{2}\left(\pi_{om}^{DN^*} - \pi_m^{dn^*}\right)$. Differentiating with respect to c shows that $\frac{\partial \pi_{om}^{DN^*}}{\partial c} - \frac{\partial \pi_m^{dn^*}}{\partial c} < 0$, for all (n, c) implying a single-crossing property. It is noteworthy that parameter c will directly maps into the optimally discrete choice of left digit n. Evaluating profits at the upper boundary of the interior-pricing region, $c = \bar{c}(\lambda)_n^{DN}$, confirms $\pi_{om}^{DN^*}\big|_{c=\bar{c}(\lambda)_n^{DN}} > \pi_m^{dn^*}\big|_{c=\bar{c}(\lambda)_n^{DN}}$, and hence there exists a unique threshold $\tilde{c}(\lambda)_n^{DN}$ such that $\pi_{om}^{DN^*} > \pi_m^{dn^*}$ for $c > \max\{\underline{c}(\lambda)_n^{DN}, \tilde{c}(\lambda)_n^{DN}\}$. Under 99-ending pricing, we similarly compare profits under bias with those under the rational benchmark. In this regime, both $\pi_m^{DN^*}$ and $\pi_r^{DN^*}$ are decreasing in c. Although the closed-form expressions are algebraically cumbersome, all sign comparisons are verified computationally (e.g., via Mathematica and MATLAB). Consequently, the equilibrium profit regions are fully characterized by the parameter pair (c, λ), as illustrated in Figure 3. The corresponding comparisons of wholesale price, quality, and retail price between biased and rational benchmark under both interior and 99-ending pricing follow from analogous comparative methods.

A.13 Proof of Corollary 7

Consider the decentralized–endogenous benchmark without consumer bias. The equilibrium quality, wholesale price, and retail price are given by $q^{dn^*} = \frac{1}{3c}$, $w^{dn^*} = \frac{2}{9c}$, and $p^{dn^*} = \frac{5}{18c}$, respectively. We decompose the equilibrium retail price as $p^{dn^*} = n + (p^{dn^*} - n)$, where n denotes the left digit. Under this "new" benchmark, consumer utility is $\tilde{u} = \theta q - n - (1-\lambda)(p^{dn^*} - n)$. Setting $\tilde{u} = 0$ yields the marginal consumer $\tilde{\theta} = \frac{5}{6}\big((18c-1)\lambda+1\big)$, which implies demand $\tilde{D} = \frac{1}{6}\big((5-90c)\lambda+1\big)$. Accordingly, manufacturer and retailer profits are $\pi_{m,n}^{dn^*} = \frac{1+5\lambda-90c\lambda}{54c}$, and $\pi_{r,n}^{dn^*} = \frac{1+5\lambda-90c\lambda}{108c}$, and total channel profit is $\Pi_n^{dn^*} = \frac{1+5\lambda-90c\lambda}{36c}$.

We then compare these benchmark profits with equilibrium channel profits under endogenous pricing and quality choices in the presence of bias, characterized in Lemma 4. Specifically, we compare $\{\Pi_{n,o}^{DN^*}, \Pi_{n,9}^{DN^*}, \Pi_{n-1,9}^{DN^*}\}$ with $\Pi_n^{dn^*}$ across the parameter space. These comparisons follow the same logic as in the proof of Proposition 3 and are therefore omitted for brevity. Importantly, because all equilibrium outcomes depend only on (c, λ), we conduct global comparisons over the feasible parameter space and summarize the results in Figure 3. The threshold $c_n^{dn} = \frac{5}{18(n+1)}$ defines the boundary at which the no-bias retail price transitions from $n.xx$ to $(n+1).xx$. These comparisons establish when bias-induced price distortions mitigate (exacerbate) double marginalization when $n.xx \to n^-$ ($n.xx \to n.$ or $n+1^-$).

A.14 Proof of Corollary 8

Based on the optimal quality choices characterized above, we examine the manufacturer's response to marginal cost shocks. Under interior pricing, differentiating the optimal quality with respect to c yields $\frac{\partial q_o^{DN^*}}{\partial c} = \frac{\sqrt{1-12c(\lambda-1)\lambda n}+1}{6c^2(\lambda-1)} + \frac{\lambda n}{c\sqrt{1-12c(\lambda-1)\lambda n}} < 0$. Hence, an increase in marginal cost reduces equilibrium quality. Under 99-ending pricing, we similarly obtain $\frac{\partial q_9^{DN^*}}{\partial c} < 0$. Although the closed-form expression is algebraically cumbersome, the sign is unambiguous. Therefore, shrinkflation—quality reduction in response to higher marginal production cost—arises under both interior and 99-ending pricing. In addition, for cost shocks to the fixed component f, we obtain $\frac{\partial q_9^{DN^*}}{\partial f} < 0$, implying that quality also decreases when fixed production costs increase. Furthermore, following the equilibrium results in Lemma 4, the manufacturer's optimal wholesale price response to cost shocks satisfies $\frac{\partial w_9^{DN^*}}{\partial c} > 0$ and $\frac{\partial w_9^{DN^*}}{\partial f} > 0$.

A.15 Proof of Corollary 9

Comparing consumer surplus, we have $CS^{dn^*} = \frac{1}{216c}$, $CS_{n,o}^{DN^*}$, and $CS_{n,9}^{DN^*}$. Since $\frac{\partial CS_{n,o}^{DN^*}}{\partial \lambda} < 0$, implying $CS_{n,o}^{DN^*} < CS^{dn^*}$. Moreover, $\frac{\partial CS_{n,9}^{DN^*}}{\partial \lambda} < 0$. Let $\underline{\lambda}_{cs}$ denote the unique solution to $CS^{dn^*} = CS_{n,9}^{DN^*}$. Then $CS^{dn^*} < CS_{n,9}^{DN^*}$ when $\underline{c}^{-1}(\lambda)_{n+1}^{DN} < \lambda < \underline{\lambda}_{cs}$; $CS_{n,9}^{DN^*} < CS^{dn^*}$ holds under other (c, λ) regions.

A.16 Proof of Proposition 4

When consumers have misperception for quality changes, we define perceived quality as $\tilde{q}(q_{\text{old}}, q_{\text{new}}, \beta) = q_{\text{old}} + (1-\beta)(q_{\text{new}} - q_{\text{old}})$, with q_{old} and q_{new} representing the original and new optimal quality, respectively. The consumer with WTP θ derives utility $\tilde{u} = \theta[q_{\text{old}}+(1-\beta)(q_{\text{new}}-q_{\text{old}})]-(n+(1-\lambda)p)$. Consider the no-bias benchmark case of $\lambda = 0$, we have $q_{\text{old}}|_{\lambda=0} = \frac{\sqrt{12cf+1}+1}{6c}$, deriving $D^{QP} = 1 - \frac{6cp}{\beta+\beta\sqrt{12cf+1}-6\beta c q_{\text{new}}+6c c q_{\text{new}}}$ and $\pi^{QP} = \left(1 - \frac{6cp}{\beta+\beta\sqrt{12cf+1}-6\beta c q_{\text{new}}+6c q_{\text{new}}}\right)\left(-(\Delta_c + c)\left(\frac{\sqrt{12cf+1}+1}{6c} + q_{\text{new}}\right)^2 - \Delta_c - f + p\right)$ where $\Delta_c > 0$ and $\Delta_f > 0$ represent cost shocks. Further, we are able to have the optimal pricing $p^{QP^*}|_{\lambda=0} = \frac{a\left(6c\left(q_{\text{new}}\left(\sqrt{12cf+1}+3cq_{\text{new}}+1\right)+f\right)+\sqrt{12cf+1}+1\right)+c\left(18bc+18c^2 q_{\text{new}}^2+(3\beta+1)\left(\sqrt{12cf+1}+1\right)+6c\left(q_{\text{new}}\left(\sqrt{12cf+1}+4-3\beta\right)+4f\right)\right)}{36c^2}$, and thus $q_{new}^{QP^*}|_{\lambda=0}$ is unique feasible solution of equation $\frac{\partial \pi^{QP^*}}{\partial q_{\text{new}}} = 0$. While the solution is quite complicated, we use numerical studies to examine the firm's product design under cost shocks without consumer left-digit bias. For numerical illustration, by setting $\beta = 0.5$, $c = 0.1$, and $f = 0.1$ without loss of generality, we have $q_{\text{new}}^{QP^*} = \frac{-\sqrt{-3600\Delta_c\Delta_f-360\Delta_c-360\Delta_f+189}-40\sqrt{7}\Delta_c-100\Delta_c-4\sqrt{7}+5}{6(10\Delta_c+1)}$ and $p^{QP^*} = \frac{5-\sqrt{-40\Delta_c(10\Delta_f+1)-40\Delta_f+21}}{40\Delta_c+4}$. From this simple yet general case, we can verify that $\frac{\partial q_{\text{new}}^{QP^*}}{\partial \Delta_c} < 0$, $\frac{\partial q_{\text{new}}^{QP^*}}{\partial \Delta_f} > 0$, $\frac{\partial p^{QP^*}}{\partial \Delta_c} < 0$, and $\frac{\partial p^{QP^*}}{\partial \Delta_f} > 0$. Thus, under quality misperception and absent left-digit bias, marginal production cost increases induce quality reductions (shrinkflation), while fixed-cost shocks mitigate this incentive.

A.17 Proof of Corollary 10

Following the same approach as in Section A.16, we substitute the pre-shock optimal quality under 99-ending pricing, q_9^{CN}, into the profit function as q_{old}^{QP}. We then solve the post-shock problem under quality misperception and characterize the optimal adjustment numerically due to the analytical complexity. We use numerical studies to demonstrate the main insights, i.e., $c = 0.07$, $f = 0.1$, and $\lambda = 0.5$. Thus, we are able to derive $\frac{\partial q_9^{CN^*}}{\partial \Delta_c} < 0$, $\frac{\partial q_9^{CN^*}}{\partial \Delta_f} < 0$, $\frac{\partial^2 q_9^{CN^*}}{\partial \Delta_c \partial \beta} < 0$, and $\frac{\partial^2 q_9^{CN^*}}{\partial \Delta_f \partial \beta} < 0$. Due to the complexity of analytical expressions, we have verified the results across all feasible parameter zones, showing that higher degree of consumers inattention to quality changes intensifies the shrinkflation behavior.

A.18 Proof of More General Formulation of Left-Digit Bias

We generalize formulation (**??**) by allowing greater flexibility in how consumers perceive interior prices, while preserving the key behavioral discontinuity at 99-ending pricing. Consumers perceive a price $p = n + \Delta$ as

$$\underbrace{\widetilde{p}(n, \Delta, \lambda)}_{\text{Perceived price}} = \underbrace{n}_{\text{Leftmost digit}} + \underbrace{F(\lambda, \Delta)}_{\text{Perceived right digits}} .$$

Consumer demand is therefore given by $D_G = 1 - \frac{n+F(\lambda,\Delta)}{q}$. Substituting this demand into the firm's profit function yields $\pi_G = (n + \Delta - cq^2)\left(1 - \frac{n+F(\lambda,\Delta)}{q}\right)$.

We first examine the centralized channel with endogenous quality choice (CN), focusing on the firm's incentive to engage in shrinkflation under 99-ending pricing. In this case, the optimal pricing satisfies $\Delta = 1^-$, so that perceived price converges to $n + 1 - \lambda$. The resulting profit function becomes

$$\lim_{\Delta\to 1^-} \pi_G^{CN} = (n + \Delta - cq^2)\left(1 - \frac{n + 1 - \lambda}{q}\right).$$

Taking the first-order condition with respect to quality q yields

$$\frac{\partial \lim_{\Delta\to 1^-} \pi_G^{CN}}{\partial q} = \frac{(1-\lambda)\left(cq^2 + 1^- + n\right) + n\left(cq^2 + 1^-\right) - 2cq^3 + n^2}{q^2}.$$

Setting this derivative equal to zero and rearranging implies

$$c = \frac{(1^- + n)\big(1 - \lambda + n\big)}{q^{*2}\big(2q^* - n - 1 + \lambda\big)}, \qquad \text{as } \Delta \to 1^-.$$

By the Envelope Theorem, the above expression immediately implies that an increase in the marginal cost parameter c strictly reduces the optimal quality q^*, confirming that shrinkflation persists under the generalized formulation. Moreover, since $F(\lambda, \Delta)$ is strictly decreasing in λ, an increase in λ lowers the numerator of the expression, and the denominator must decrease, which requires a reduction in q^*.

We next consider the decentralized channel. Retailer and manufacturer profits are given by $\pi_{rG} = (n+\Delta-w)D_G$ and $\pi_{mG} = (w - cq^2)D_G$ respectively. Under 99-ending pricing ($\Delta^* \to 1^-$), the manufacturer optimally sets the wholesale price $w_G^* = 2n + \Delta - q + F(\lambda, \Delta)$, following the same steps as in the proof of Lemma 3. Differentiating yields

$$\frac{\partial w_G^*}{\partial \lambda} = \frac{\partial F(\lambda,\Delta)}{\partial \lambda} < 0, \qquad \frac{\partial w_G^*}{\partial q} = \frac{\partial w_G^*}{\partial D} \cdot \frac{\partial D}{\partial q} = -1 < 0,$$

indicating that stronger left-digit bias induces the manufacturer to lower wholesale prices in order to preserve retailer demand—i.e., a *quantity-contract* (or coordination) effect.

Substituting the optimal prices into the profit functions yields

$$\pi_{mG} = \left(1 - \frac{F(\lambda,\Delta) + n}{q}\right)\big(F(\lambda,\Delta) - cq^2 + \Delta + 2n - q\big), \quad \pi_{rG} = \frac{(F(\lambda,\Delta) + n - q)^2}{q}.$$

Differentiating with respect to λ gives

$$\frac{\partial \pi_{mG}}{\partial \lambda} = \left(-\frac{\partial F(\lambda,\Delta)}{\partial \lambda}\right)\frac{(n + \Delta - cq^2) + 2(n + F(\lambda,\Delta) - q)}{q} > 0,$$

and

$$\frac{\partial \pi_{rG}}{\partial \lambda} = \left(-\frac{\partial F(\lambda,\Delta)}{\partial \lambda}\right)\frac{2(q + \lambda - n - 1)}{q} > 0,$$

where the inequalities follow from $1 > D_G > 0$ as $\Delta \to 1^-$.

Relative to the no-bias benchmark, the manufacturer's profit difference is

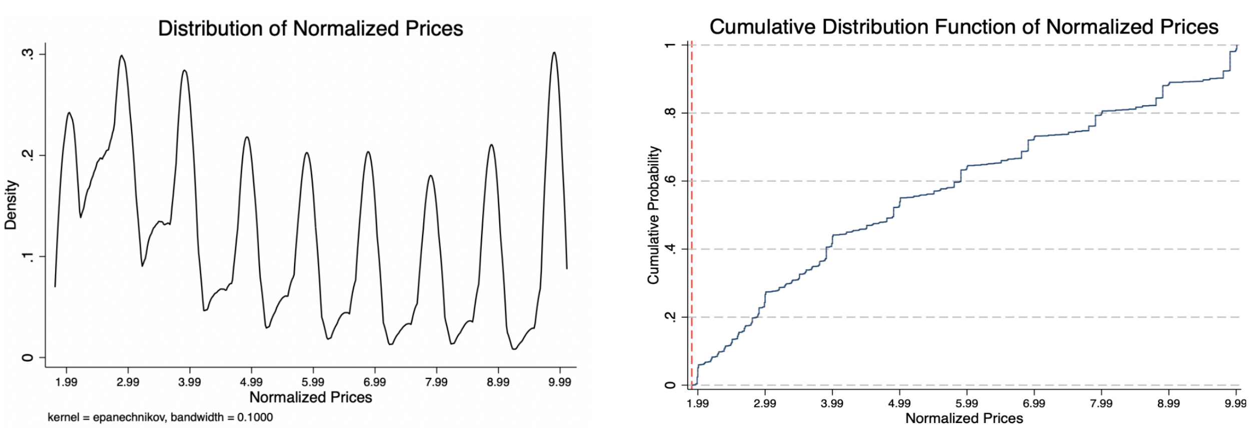
$$\lim_{\Delta \to 1^-} \left(\pi_{mG}^{DX^*} - \pi_m^{dx^*} \right) = \left(1 - \frac{F(\lambda, \Delta) + n}{q} \right) \left(F(\lambda, \Delta) - cq^2 + \Delta + 2n - q \right) - \frac{1}{8} q (cq - 1)^2$$

Evaluating at the extreme $\lambda = 1$ yields $\lim_{\Delta \to 1^-} \left(\pi_{mG}^{DX^*} - \pi_m^{dx^*} \right) \big|_{\lambda=1} = -\frac{(q(cq+3)-4n)^2 + 8\Delta(n-q)}{8q} > 0$, implying the existence of a threshold $\underline{\lambda}_m$ such that the manufacturer strictly benefits from bias whenever $\lambda > \underline{\lambda}_m$. Similarly, the retailer's profit difference is $\pi_{rG}^{DX^*} - \pi_r^{dx^*} = \frac{16(-\lambda+n-q+1)^2 - q^2(cq-1)^2}{16q}$. At $\lambda = 1$, we obtain $\lim_{\Delta \to 1^-} \left(\pi_{rG}^{DX^*} - \pi_r^{dx^*} \right) \big|_{\lambda=1} = \frac{16(n-q)^2 - q^2(cq-1)^2}{16q} > 0$, which implies the existence of a threshold $\underline{\lambda}_r$ above which the retailer also strictly benefits from consumer bias. Therefore, when $\lambda > \max\{\underline{\lambda}_m, \underline{\lambda}_r\}$, both manufacturer and retailer earn higher profits relative to the no-bias benchmark.

Appendix B Empirical Observations

To empirically validate the predictions proposed in Lemma **??**, we analyze 142,133 retail price observations from a major e-commerce platform, with prices ranging from 0 to 736,000. We extract the leading three digits from each price and normalize them into the form of (first digit).(right digits), ensuring all values are in the range of [0,10]. We further restrict the analysis to normalized values within [1.9,10], as in Strulov-Shlain (2023). This refinement yields 103,613 observations, which are used to generate the empirical distribution and C.D.F., shown in Figures B.1 and B.2.

Figure B.1: Distribution and CDF of Normalized Prices

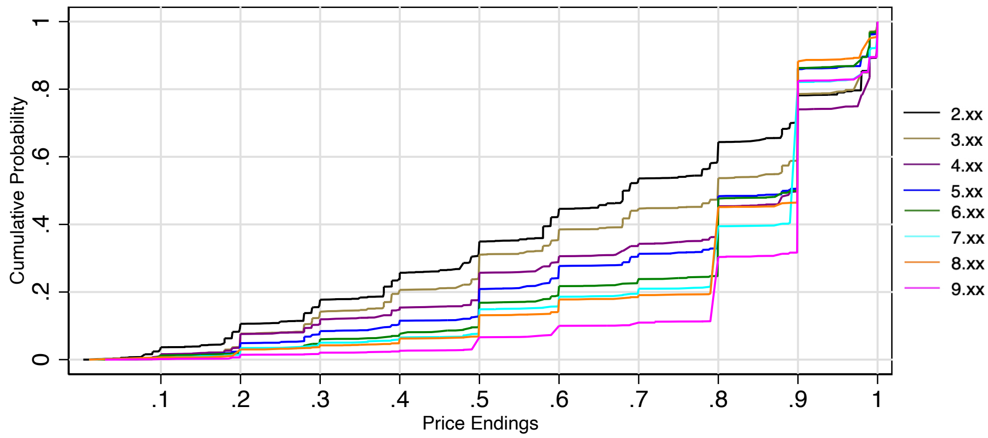


(a) Distribution of Normalized Prices (b) CDF of Normalized Prices

Note. The left and right panels respectively show the distribution and CDF of normalized prices (within 1.9 to 10).

Figure B.1 highlights two patterns. First, 99-ending pricing is notably prevalent, with a clear mass point at 99-ending values, as shown in Figure B.1(a). Second, Figure B.1(b) demonstrates that as the leftmost digit increases, the slope of the interior price distribution declines. This pattern is further confirmed by

Figure B.2: CDF of Price Endings by Left Digit



Note. This figure illustrates the empirical cumulative distributions of price endings by dollar digit. Different colors represent different left digits of the prices, as specified in the right-hand column. The figure reveals that as the left digit of the price increases, firms tend to use fewer lower price endings.

Figure B.2, which shows that, as the left-most digit increases, firms use fewer lower price endings. Together, these observations indicate that interior prices are more common when the leftmost digit is smaller, while 99-ending prices become more dominant as the leftmost digit rises. These findings align with the theoretical predictions presented in the main paper.

Appendix C Numerical Illustrations

To complement our analytical results and highlight their economic significance, we conduct numerical studies across the examined scenarios. These numerical studies quantify the magnitude of left-digit effects on firm pricing, quality choice, channel coordination, consumer surplus, and profitability. Specifically, we report percentage differences relative to the no-bias benchmark, thereby providing a direct measure of the economic significance of left-digit bias. We consider three bias scenarios: (i) no bias ($\lambda = 0$); (ii) low bias ($\lambda = 0.1$), corresponding to the lower bound of inattention estimates in the literature, e.g., Strulov-Shlain (2023); and (iii) high bias ($\lambda = 0.5$), consistent with the estimate in List et al. (2023). The numerical analysis is organized to mirror the main scenarios examined in the model.

- In Section C.1, we examine the *centralized–exogenous quality* (CX) case, focusing on how left-digit bias affects firm pricing, consumer surplus, and firm profit.
- In Section C.2, we analyze the *centralized–endogenous quality* (CN) case to illustrate quality choice.
- In Section C.3, we consider the *decentralized–exogenous quality* (DX) case, highlighting how left-digit bias influences wholesale pricing, retail pricing, and channel coordination.
- In Section C.4, we turn to the *decentralized–endogenous quality* (DN) case to show channel coordination.

C.1 Centralized-Exogenous Case

Table C.1 presents the economic significance of consumer left-digit bias by comparing outcomes under the benchmark without bias ($\lambda = 0$) to cases with varying degrees of bias ($\lambda = 0.1, 0.5$). To illustrate how the effects vary across different regions, we examine two exogenous quality levels ($q = 2.9$ and $q = 3.2$).

Table C.1: The Centralized Exogenous (CX) Quality Case: $c = 0.1$

Quality	Bias	Price	Margin	Quantity	Profit	Consumer Surplus	Social Welfare
$q = 2.9$	$\lambda = 0$	1.87	1.03	0.355	0.365	0.183	0.548
	$\lambda = 0.1$	1.97	1.14	0.352	0.400	0.146	0.545
		+ 5.7%	+ 10.3%	− 0.9%	+ 9.4%	− 20.4%	− 0.5%
	$\lambda = 0.5$	1.99	1.15	0.485	0.557	0.101	0.657
		+ 6.4%	+ 11.6%	+ 36.5%	+ 52.4%	− 45.0%	+ 19.9%
$q = 3.2$	$\lambda = 0$	2.11	1.09	0.34	0.37	0.185	0.555
	$\lambda = 0.1$	1.99	0.97	0.41	0.40	0.227	0.622
		− 5.8%	− 11.2%	+ 20.3%	+ 6.8%	+ 22.9%	+ 12.2%
	$\lambda = 0.5$	1.99	0.97	0.53	0.52	0.191	0.705
		− 5.8%	− 11.2%	+ 56.7%	+ 39.1%	+ 3.0%	+ 27.1%

Note. All percentage changes are rounded to one decimal place, while other values are rounded according to the scale of the corresponding benchmark model. Because percentage changes are calculated using the underlying unrounded values rather than the rounded table entries, trivial discrepancies across entries may occur due to rounding.

The table reports the percentage changes in firm pricing, profitability, and consumer welfare, highlighting how left-digit bias distorts market outcomes. Overall, firm profit increases with stronger bias. For instance, relative to the no-bias benchmark ($\lambda = 0$), firm profit rises by as much as +55% when $\lambda = 0.5$ and $q = 2.9$. The impact on consumer surplus, however, depends critically on the quality region. When the quality level induces an upward shift in price from $n.xx$ to $n.99$ (e.g., $q = 2.9$), stronger bias reduces consumer surplus, reflecting that more consumers purchase despite experiencing negative surplus. In contrast, when the quality level induces a downward shift from $n.xx$ to $(n.99 - 1)$ (e.g., $q = 3.2$), consumer surplus increases, as more consumers purchase with positive surplus. Notably, relative to the benchmark, consumer surplus improves by up to +22.9% and +3.0%, alongside profit increases of +6.8% and +39.1%, respectively.

C.2 Centralized-Endogenous Case

Table C.2 reports the economic significance of consumer left-digit bias on quality choice and firm profit in the centralized-endogenous case, under two marginal cost levels ($c = 0.09$ and $c = 0.1$).

Table C.2: The Centralized Endogenous (CN) Quality Case: $f = 0$

Cost	Bias	Price	Quality	margin	Quantity	Profit	Consumer Surplus	Social Welfare
$c = 0.09$	$\lambda = 0$	2.47	3.70	1.235	0.333	0.412	0.205	0.617
	$\lambda = 0.1$	1.99	3.11	1.120	0.392	0.439	0.200	0.639
		− 19.4%	− 16.1%	− 9.3%	+ 17.5%	+ 6.6%	− 2.9%	+ 3.4%
	$\lambda = 0.5$	1.99	2.80	1.270	0.470	0.600	0.080	0.680
		− 19.4%	− 23.8%	+ 3.1%	+ 41.1%	+ 45.5%	− 61.4%	+ 9.9%
$c = 0.1$	$\lambda = 0$	2.22	3.33	1.11	0.33	0.37	0.185	0.55
	$\lambda = 0.1$	1.99	3.02	1.08	0.37	0.40	0.173	0.58
		− 10.5%	− 9.5%	− 2.7%	+ 11.9%	+ 8.8%	− 6.8%	+ 3.7%
	$\lambda = 0.5$	1.99	2.74	1.24	0.45	0.56	0.057	0.62
		− 10.5%	− 17.9%	+ 11.8%	+ 36.0%	+ 52.2%	− 69.3%	+ 11.7%

The overall pattern mirrors the centralized-exogenous case: stronger bias primarily benefits the firm at the expense of consumers (e.g., at $\lambda = 0.5$, profit increases by +52.2% but consumer surplus drops by −69.3%). What distinguishes the endogenous-quality setting is the additional lever of firm adjustment through quality. As λ rises, firms strategically reduce quality—by −16.1% at $\lambda = 0.1$ and by −23.8% at $\lambda = 0.5$ when $c = 0.09$. More importantly, when marginal production costs increase (from $c = 0.09$ to $c = 0.1$) under the same 1.99 pricing, the bias-driven quality downgrading is also significant: quality decreases by an additional −2.9% and −3.1% at $\lambda = 0.1$ and $\lambda = 0.5$, respectively. As cost increases from 0.09 to 0.1, these quality reductions are accompanied by profit losses (−8%, −6.6%) and substantial consumer surplus declines (−13.4%, −28.8%), underscoring how cost shocks amplify shrinkflation under left-digit bias.

C.3 Decentralized-Exogenous Case

Table C.3 reports the economic significance of consumer left-digit bias on manufacturer-retailer decisions and profit in the decentralized-exogenous case, under two quality levels ($q = 2.9$ and $q = 3.6$).

While retail prices broadly follow the centralized pattern, the key insights emerge from how wholesale pricing and profit allocation adjust significantly under bias.

In the interior pricing case ($q = 2.9$), both the manufacturer and retailer raise their markups as bias intensifies. Numerically, at $\lambda = 0.1$ and $q = 2.9$, wholesale and retail prices increase by +2.7% and +3.1%, respectively. However, these higher markups reduce demand by −5.6%, leading to profit declines of −1.1% for both firms and a significant consumer surplus loss of −27.8%. Here, the distortion effect dominates the demand-expansion effect, creating a prisoner's dilemma (also illustrated in Table **??**) for channel members.

Table C.3: The Decentralized Exogenous (DX) Quality Case: $c = 0.1$

Quality	Bias	Wholesale Price	Retail Price	Manufacturer Margin	Retailer Margin	Quantity	Manufacturer Profit	Retailer Profit	Channel Profit	Consumer Surplus
$q = 2.9$	$\lambda = 0$	1.87	2.39	1.03	0.51	0.178	0.183	0.091	0.274	0.0457
	$\lambda = 0.1$	1.92	2.46	1.08	0.54	0.167	0.181	0.090	0.271	0.0330
		+ 2.7%	+3.1%	+ 4.9%	+ 4.9%	− 5.6%	− 1.1%	− 1.1%	− 1.1%	− 27.8%
	$\lambda = 0.5$	2.59	2.99	1.74	0.41	0.140	0.240	0.056	0.300	−0.0400
		+ 38.2%	+ 25.4%	+ 69.4%	− 21.3%	− 21.3%	+ 33.3%	− 38.1%	+ 9.5%	− 189.4%
$q = 3.6$	$\lambda = 0$	2.45	3.02	1.15	0.58	0.16	0.184	0.092	0.276	0.046
	$\lambda = 0.1$	2.28	2.99	0.99	0.71	0.20	0.194	0.140	0.333	0.050
		− 6.8%	− 1.1%	− 14.5%	+ 23.1%	+ 23.1%	+ 5.3%	+ 51.5%	+ 20.7%	+ 9.2%
	$\lambda = 0.5$	1.89	2.99	0.59	1.10	0.31	0.180	0.340	0.520	0.018
		− 23.0%	− 1.1%	− 48.9%	+ 91.8%	+ 91.8%	− 2.0%	+ 268.0%	+ 88.0%	− 161.7%

In the 99-ending pricing case ($q = 3.6$), the retailer strategically adjusts its original price of 3.02 to 2.99 and maintains this left-digit position as bias grows. To support this price, the manufacturer lowers its wholesale markup—for instance, at $\lambda = 0.1$, wholesale price falls by −6.8% and markup by −14.5%. This adjustment shifts surplus back into the channel: manufacturer profit rises by +5.3%, retailer profit surges by +51.5%, and consumer surplus also benefits, increasing by +9.2% at $\lambda = 0.1$. Thus, unlike the interior case, strong consumer left-digit bias under 99-ending pricing enhance channel coordination gains and channel members' profitability.

C.4 Decentralized-Endogenous Case

Table C.4 reports the economic significance of consumer left-digit bias on channel quality choice and firm profit in the decentralized-endogenous case, under two quality levels ($c = 0.09$ and $c = 0.1$).

Two important insights emerge: the persistence of shrinkflation and the role of bias in improving channel coordination when bias is higher. For shrinkflation, as the marginal cost c rises from 0.09 to 0.1, the manufacturer responds by lowering optimal quality even under 99-ending pricing. Numerically, when $\lambda = 0.1$ and 0.5, quality falls by −1.2% and −1.3%, respectively. With $c = 0.09$, channel profit rises by +8.4%, and +45.8% as λ increases from 0.1 to 0.5. This indicates that bias-induced demand expansion and pricing rigidity can align incentives, mitigating double marginalization and strengthening channel coordination.

Table C.4: The Decentralized Endogenous (DN) Quality Case

Cost	Bias	Wholesale Price	Retail Price	Quality	Manufacturer Margin	Retailer Margin	Quantity	Manufacturer Profit	Retailer Profit	Channel Profit	Consumer Surplus
$c =$ 0.09	$\lambda = 0$	2.47	3.09	3.70	1.23	0.62	0.167	0.206	0.10	0.31	0.051
	$\lambda = 0.1$	2.36	2.99	3.52	1.25	0.63	0.178	0.222	0.11	0.33	0.038
		− 4.3%	− 3.1%	− 5.0%	+ 1.1%	+ 1.7%	+ 7.0%	+ 8.2%	+ 8.9%	+ 8.4%	− 25.5%
	$\lambda = 0.5$	2.30	2.99	3.19	1.39	0.69	0.220	0.300	0.15	0.45	−0.030
		− 6.9%	− 3.1%	− 14.0%	+ 12.3%	+ 11.8%	+ 30.3%	+ 46.0%	+ 45.4%	+ 45.8%	− 163.1%
$c =$ 0.10	$\lambda = 0$	2.22	2.78	3.33	1.11	0.56	0.167	0.185	0.0925	0.28	0.0463
	$\lambda = 0.1$	2.40	2.99	3.48	1.20	0.59	0.169	0.201	0.0987	0.30	0.0327
		+ 8.2%	+ 7.6%	+ 4.3%	+ 7.6%	+ 5.5%	+ 1.1%	+ 8.8%	+ 6.7%	+ 8.1%	− 29.4%
	$\lambda = 0.5$	2.33	2.99	3.15	1.34	0.65	0.207	0.280	0.1360	0.42	−0.0350
		+ 5.1%	+ 7.6%	− 5.5%	+ 21.0%	+ 17.7%	+ 24.6%	+ 50.8%	+ 46.6%	+ 49.4%	− 175.4%

Appendix D Extension of Shrinkflation under Menu Costs

In the main model, pricing rigidity arises solely from consumer left-digit bias. In practice, however, firms also face menu costs when adjusting posted prices (e.g., re-tagging, coordination, and communication costs). We therefore introduce a supply-side friction and examine how it interacts with left-digit bias in shaping quality adjustment under cost shocks. Suppose a marginal cost increase from (c, f) to $(c + \Delta_c, f + \Delta_f)$, and let p^* and q^* denote the optimal price and quality before the cost shock. Changing the posted price triggers a menu cost $m_c > 0$, proportional to the size of price adjustment: Menu cost $= m_c \cdot |p' - p^*|$. Our main results are robust to a fixed menu cost specification (i.e., a lump-sum cost m_u), as the underlying trade-off between repricing and quality adjustment remains unchanged. The firm compares two strategies: (i) jointly adjusting price and quality to (p', q'), or (ii) adjusting quality only to (p^*, q') while keeping the price unchanged. The corresponding profit-maximization problems are given by Equations (D.1) and (D.2):

$$\max_{p',q'} \ \pi(p', q') = \Big(p' - (c + \Delta_c)q'^2 - (f + \Delta_f)\Big) \cdot D(p', q', \lambda) - m_c \cdot |p' - p^*|. \tag{D.1}$$

$$\max_{q'} \ \pi(p^*, q') = \Big(p^* - (c + \Delta_c)q'^2 - (f + \Delta_f)\Big) \cdot D(p^*, q', \lambda). \tag{D.2}$$

Closed-form solutions for the optimal choices (q'^*, p'^*) and (q'^*_{na}, p^*) are derived using the similar methods.

The firm's choice between these two strategies is governed by the relative magnitude of the menu cost. We define $\underline{m_c} > 0$ as the critical menu cost threshold satisfying the indifference condition: $\pi(q'^*_{na}, p^*) = \pi(q'^*, p'^*)$. If the menu cost m_c exceeds this threshold ($m_c > \underline{m_c}$), the firm optimally engages in shrinkflation via quality-only adjustment. Conversely, if $m_c < \underline{m_c}$, the firm adjusts both quality and price. Introducing a non-trivial menu cost therefore enables or expands the parameter region in which shrinkflation arises, even for interior prices. Because price adjustment is costly regardless of whether the initial price is interior or 99-ending, menu costs uniformly increase the likelihood of shrinkflation, independent of the bias parameter λ. In particular, when menu costs are sufficiently high, shrinkflation emerges purely from supply-side frictions—even in the absence of strong behavioral rigidity.

Proposition D.1: *Non-trivial menu costs make shrinkflation optimal even at interior prices, and left-digit bias at 99-ending thresholds further enlarges the shrinkflation region.*

Proof. Consider two regimes: (i) joint adjustment with profit $\pi^*_{q,p}$ and (ii) quality-only adjustment with profit π^*_{na}. We have $\frac{\partial \pi^*_{q,p}}{\partial m_c} < 0$ and $\frac{\partial \pi^*_{na}}{\partial m_c} = 0$. Thus, menu costs strictly reduce the profitability of joint adjustment but leave quality-only adjustment unaffected. When $m_c = 0$, joint adjustment strictly dominates, i.e., $\pi^*_{q,p} > \pi^*_{na}$. For sufficiently large m_c, this ranking reverses. By continuity, there exists a threshold $\underline{m}_c$ such that shrinkflation is optimal if and only if $m_c > \underline{m}_c$. For illustration, under $c = 0.08$, $\lambda = 0.02$, $f = 0$, $\Delta_f = 0$, and $\Delta_c = 0.01$, the threshold is $\underline{m}_c = 0.005$. This example shows how even sufficiently small menu costs can induce quality under-provision under cost shocks, e.g., shrinkflation. □

Menu costs provide a baseline incentive to avoid repricing across the entire price spectrum. Left-digit bias introduces an additional behavioral friction that is particularly strong at 99-ending thresholds, where crossing a left-digit boundary triggers a discrete perceived demand loss. Consequently, while menu costs alone can generate shrinkflation at interior prices, left-digit bias amplifies this incentive at 99-endings. Holding other factors constant, shrinkflation is therefore more likely when the product is priced at a 99-ending than at an interior price. The interaction between supply-side menu costs and demand-side left-digit bias provides a unified explanation for why shrinkflation is especially prevalent at psychologically salient price points.