Political Polarization and Nonmarket Strategy over the Policy Life Cycle

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A Appendix

A.1 Introduction

In this appendix we present modeling details omitted from the main text. These include the details of the belief formation process, the details of the stage game, and the proofs of our propositions. We bring together three influential strands of literature to create our model, the belief formation model of Golub and Jackson (2012), the private politics model of Baron (2012), and the legislative model of Grossman and Helpman (2000). In order to aid the reader in connecting the text to the appropriate parts of the Appendix, we have numbered the parts in parallel fashion. Where a proposition has been demonstrated in the text already, we present it here for continuity but do not reiterate the proof.

A.2 Overview of the Game

A Two-Phase Model of the Policy Life Cycle

As described in section 2 of the main text, we use a specific formal model of belief formation and evolution as the underpinning of our two-phase model of the policy life cycle. To capture the evolution of beliefs, we use a simple version of the model developed by Golub and Jackson (2012). There are two groups in the population, a majority with fraction $M \in (1/2, 1)$ of the population and a minority with fraction $1 - M$. We assume the overall size of the population is large enough that we can focus on the beliefs of representative agents in each group (Golub and Jackson, 2012). There is a true state of nature $\omega \in \{0, 1\}$, and the agents hold beliefs at time t about the probability that $\omega = 1$, which are represented by the vector

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$$b_t \equiv \begin{pmatrix} b_t^{Maj} \\ b_t^{Min} \end{pmatrix}.$$

At time $t = 0$, agents in the majority receive a signal that with probability $\mu < 1/2$ indicates the state of the world is $\omega = 1$ and with probability $1 - \mu$ indicates the state of the world is $\omega = 0$. Similarly, at time $t = 0$, agents in the minority receive a signal that with probability $\nu > 1/2$ indicates the state of the world is $\omega = 1$ and with probability $1 - \nu$ indicates the state of the world is $\omega = 0$. We assume the minority gets a stronger signal, in that $\nu > \mu$, as otherwise the majority would either vote for the policy immediately or would never vote for the policy, neither of which is interesting for analytical purposes.[1]

The overall fraction with a signal that $\omega = 1$ is $p = M\mu + (1 - M)\nu$. In each period, each individual interacts with another member of the population, after which each holds beliefs equal to the average of the pair's prior beliefs. The group level matrix of linking densities describes the probability each member of the population interacts with a member of their own group or a member outside their own group and is given by

$$G(z, M) = \begin{pmatrix} 1 - z & z \\ zQ & 1 - zQ \end{pmatrix},$$

where z is the probability that an individual in the majority encounters a member of the minority. The term $Q = M/(1 - M)$ is necessary to account for reciprocal communication and to ensure that probabilities add up to unity. We assume $1 - M > z$, which implies homophily, i.e., a bias towards encountering one's own type.[2]

Initial beliefs are $b_0^{Maj} = \mu$ for the majority and $b_0^{Min} = \nu$ for the minority. At time t, after encounters with other members of the population, the beliefs of the representative agent in each group evolve to

$$b_t = G(z, M) b_{t-1},$$

and Golub and Jackson (2012) show that the evolution of beliefs over time is represented by

$$\begin{pmatrix} b_t^{Maj} \\ b_t^{Min} \end{pmatrix} = \begin{pmatrix} p - \left(1 - \frac{z}{1-M}\right)^t (p - \mu) \\ p + Q\left(1 - \frac{z}{1-M}\right)^t (p - \mu) \end{pmatrix}. \tag{17}$$

Thus the beliefs of the representative agent in each group gradually converge to the common

[1] The belief updating process in our model is virtually identical to that of Golub and Jackson (2012), but our interpretation of signals is somewhat different. They assume majority agents receive a signal that is correct with probability μ, and they assume that the distribution of signals is sufficient to ensure that society will eventually converge to making the "correct" decision. We, instead, assume majority agents receive a signal that indicates $\omega = 1$ (which would favor legislative action) with probability μ, whether or not this is actually the true state. We are focused on a positivistic depiction of a world in which legislators may sometimes make mistakes, and how firms develop nonmarket strategy suites in response to that world.

[2] With this assumption, $1 - z > M$ and $1 - zQ > 1 - M$, so each group is disproportionately likely to encounter its own members.

belief p via a process whose convergence speed is affected by the degree of homophily $1 - z$ in the society, as illustrated in Figure 1. The figure depicts the evolution of beliefs for the minority and the majority for two different levels of homophily. Note that because $\nu > \mu$, $b_t^{Min} \geq b_t^{Maj}$ for all t. We show that the evolution of beliefs may result in two distinct phases of the game, one in which only private politics occurs and a second phase in which both private and public politics occur. The time period at which the second phase is reached is denoted by $t = T^{Public}$; it is the first period in which the majority finds the alternative practice desirable given its beliefs about the state of the world. (Of course, this may not happen at all if beliefs never reach a level high enough to justify legislation.) Equation (20) below implies that T^{Public} is the first period when $b_t^{Maj} \geq \widehat{b}^{Maj} = \frac{1+A}{2} + \delta^{Maj}$.[3]

Using (17) and (20) we can derive a closed-form solution

$$T^{Public} = \frac{\ln\left(\frac{p - \frac{(1+A)}{2} - \delta^{Maj}}{p - \mu}\right)}{\ln\left(1 - \frac{z}{1-M}\right)}. \tag{18}$$

We discuss the comparative statics of this expression with respect to parameters capturing polarization in section 5 of the main text.

Repertoire of Nonmarket Strategies

The description below fleshes out the details of the stage game played in each period.

Stage Game

In each period, the firm may face threats of activism and legislation, and decides under what conditions it will adopt the alternative practice. We focus on a setting in which the firm must make a binary choice between the existing practice and the alternative practice. By doing so, we capture the notion that in many cases the firm has incentives to resist the activist's demands. If instead we assumed that the firm could make a continuous choice of the extent to which it will adopt the alternative practice, this would tend to lead to an equilibrium in which the firm always invests in just enough of the alternative practice to preempt activist campaigning. Given the beliefs b_t, in period t the activist and firm choose strategies $s_t = \{e_t; h_t^R, L_t\}$ as follows.

Stage 1: The Activist's Campaign Strategy

As in Baron (2012), the activist chooses a level of campaign effort $e_t \geq 0$, which will generate a level of reputational harm h_t to the firm if the campaign is launched, according to a cumulative distribution $F(h_t|e_t)$ over the range $[0, \overline{h}]$, with $F(0|0) = 1$, $F_e(h_t|e_t) < 0$

[3]Although the model of Golub and Jackson (2012) is highly influential and offers numerous insights, other models of the evolution of beliefs are obviously possible and could merit exploration in future work. For example, Acemoglu et al. (2010) allow for "forceful" agents who have outsize influence on others and can cause misinformation to persist, although convergence ultimately occurs.

and $F_{ee}(h_t|e_t) > 0$. This structure captures the unpredictability of campaign success, and also ensures that campaigns occur in equilibrium. We assume $\overline{h}$ is large enough to induce the firm to concede, so that even when public politics is infeasible the activist campaign may still induce adoption. We assume the monotone likelihood ratio property (MLRP) holds, so that f_e/f is increasing in h_t.

The activist has a cost αe_t of launching a campaign, and obtains per-period payoff U^1 once the firm adopts the alternative practice.

Stage 2: The Firm's Resistance Strategy

Once the activist launches its campaign, its intensity and, therefore, its potential harm h_t are apparent to the firm. The firm will optimally resist the campaign as long as the potential harm is below h_t^R. That is, the firm's resistance strategy is to resist for all $h_t \leq h_t^R$. Concession is sometimes a desirable choice because if the firm resists, it suffers the full harm of the campaign and earns payoff $\pi^0 - h_t - \rho_t(L_t)A$ (where the last term reflects the risk that subsequent legislation in Stage 3 will force adoption if firm lobbying is not sufficient to block it). If the firm concedes then it avoids a portion $(1-\beta)$ of the reputational harm of the campaign, and earns $\pi^1 - \beta h_t$.

Stage 3: Legislation

We build on the influential model of legislation presented in Grossman and Helpman (2000). It is a good fit for purposes of exploring the implications of polarization because it includes both the ideological bias of legislators as well as their beliefs about the state of the world. These two factors combine naturally into legislative preferences about policy.

There are two groups of legislators who mirror the distribution of beliefs in society, a majority with fraction of seats M and a minority with fraction of seats $1-M$. Each legislator holds beliefs identical to those of the representative member of their group, as given by b_t in (17). Given the state of the world $\omega \in \{0,1\}$, a legislator in group $i \in \{Maj, Min\}$ has per-period utility of

$$\mathcal{L}^i(x) = -(x - \omega + \delta^i)^2 - Ax,$$

where $x \in \{0,1\}$ is a policy choice, δ^i is the "bias" of group i. Thus, the ideal point of a legislator of type i is

$$x = \omega - \delta^i - \frac{A}{2}$$

and a legislator with a larger δ prefers a weaker policy.

Given beliefs b_t^i, and policy x, a legislator of type i obtains expected payoff from policy x of

$$E_t\mathcal{L}^i(x) = -b_t^i(x - 1 + \delta^i)^2 - (1 - b_t^i)(x + \delta^i)^2 - Ax.$$

The net benefit received by a legislator of type i from voting for $x = 1$ instead of $x = 0$ is

$$B_t^i \equiv E_t \mathcal{L}^i(1) - E_t \mathcal{L}^i(0) = 2(b_t^i - \delta^i) - 1 - A. \tag{19}$$

If $B_t^i \geq 0$ then legislators representing group i will vote for the policy, and if $B_t^i < 0$ they will vote against the policy. This implies that a legislator of type i will vote for the policy if $b_t^i \geq \widehat{b^i}$, where

$$\widehat{b^i} = \frac{1+A}{2} + \delta^i. \tag{20}$$

In order to ensure that public politics is not meaningless, we assume $-1+2-2\delta^i - A > 0$ for each group, which ensures that legislation will be desired if $b^i = 1$. This implies that $\delta^i < (1-A)/2$ for both groups. For simplicity, since $b_t^{Min} > b_t^{Maj}$ for all t, we assume $\delta^{Maj} > \delta^{Min}$, which ensures that $B_t^{Min} > B_t^{Maj}$ for all t. Thus, taking both beliefs and ideological bias into account, the minority consistently is more strongly in favor of government action than is the majority.[4]

If social consensus is achieved, both groups agree that the state of the world is $\omega = 1$ with probability p, i.e. $b^{Maj} = b^{Min} = p$. Legislation, if proposed, will pass if the majority supports it. More precisely, after convergence the majority will vote for the policy if $-1 + 2p - 2\delta^{Maj} - A > 0$ or

$$\delta^{Maj} < \frac{2p - 1 - A}{2} = p - \frac{(1+A)}{2}.$$

We recognize that any legislature always faces competing priorities, and that it can be difficult to predict what will be most urgent at any given time. Thus, even if legislation would command the support of a majority, it is not necessarily brought to the floor for a vote, but instead is only proposed with probability $\lambda \in (0,1)$. The notion that even once a legislative majority exists there is still no certainty that a bill will be introduced is a familiar one in the political science literature. One paper that offers a formal underpinning for this view is Glazer and McMillan (1992), which captures the notion that introducing legislation is costly for any given legislator, so that there is a free-riding problem in the introduction of a bill. This leads to a probability of legislation strictly less than unity even when a majority would support it if introduced, which motivates our assumption that $\rho = \lambda < 1$ when the majority supports legislation. Absent lobbying, the probability that legislation is proposed, $\rho_t(0)$, take on one of two possible values,

$$\rho_t(0) = \begin{cases} 0 \text{ if } B_t^{Maj} < 0 \\ \lambda \text{ if } B_t^{Maj} \geq 0 \end{cases}. \tag{21}$$

[4] Recall that once the majority is in favor of legislation, it will occur. As long as legislation does not occur immediately, the model will feature two phases.

If $B_t^{Maj} < 0$ then there is no threat of legislation and $\rho_t(0) = 0$. If $B_t^{Maj} \geq 0$ then the majority will support legislation if it is proposed, which, absent lobbying, occurs with probability $\rho_t(0) = \lambda$. Of course, the firm always has the option of lobbying, and if it chooses to lobby, it will select a lobbying expenditure $L_t = B_t^{Maj}/2$ which is sufficient to block legislation, that is, it will set $\rho_t(B_t^{Maj}/2) = 0$.

The Dynamic Game and Player's Value Functions

With the structure of the stage game defined, we turn to the players' behavior in the dynamic game. The problems of the activist and the firm are presented in the text, so we jump immediately to the first proposition.

Proposition 1 *For any time $t < T^{Public}$, the closer is t to T^{Public} the lower is the firm's valuation, and the higher is the activist's valuation.*

Formally, for any $x > t$, $v^F(\mathcal{H}_t|T^{Public} = x) > v^F(\mathcal{H}_t|T^{Public} = x+1)$ and $v^A(\mathcal{H}_t|T^{Public} = x) < v^A(\mathcal{H}_t|T^{Public} = x+1)$. The text provides a verbal proof of the proposition so we do not repeat it here. The proposition has an important and closely related implication, namely

Corollary 1 *In Phase I, as T^{Public} approaches, the activist's value function increases and the firm's value function decreases.*

Formally, $v^A(\mathcal{H}_t) < v^A(\mathcal{H}_{t+1})$ and $v^F(\mathcal{H}_t) > v^F(\mathcal{H}_{t+1})$. That is, the firm's value is decreasing in t during Phase I as it approaches T^{Public}, and the activist's value is increasing in t during Phase I.

A.3 Equilibria in Phase I: Private Politics

Proposition 2 *During Phase I, the firm's resistance to activist campaigns and the activist's campaign effort decline as Phase II approaches. That is, for any time $t < T^{Public}$, $h_{t-1}^{R*} > h_t^{R*}$; in addition, $e_{t-1}^* > e_t^*$ as long as effort is a strategic complement to firm toughness (as measured by its resistance threshold) or the cost of effort is sufficiently low.*

Proof of Proposition 2: Recall from Corollary 1 that $v^A(\mathcal{H}_{t-1}) < v^A(\mathcal{H}_t)$ for $t < T^{Public}$. Because we are in the private politics phase, $\rho_t(L_t) = 0$, and if it were the case that $h_{t-1}^R = h_t^R \equiv h^R$ then the FOC (5) would imply

$$F_e(h^R|e_{t-1}) = \frac{\alpha}{\left[\frac{v^A(\mathcal{H}_t)}{1+r} - U^1\frac{(1+r)}{r}\right]} < \frac{\alpha}{\left[\frac{v^A(\mathcal{H}_{t+1})}{1+r} - U^1\frac{(1+r)}{r}\right]} = F_e(h^R|e_t),$$

and the direct effect would imply that $e_{t-1}^* > e_t^*$. However, we must also consider the indirect effect on effort of any change in h^R. Applying Corollary 1 to equation (7) implies that $\phi_{t-1} > \phi_t$, and using this observation, equation (8) implies $h^{R*}_{t-1} > h^{R*}_t$, so the indirect effect is indeed relevant. Still, as long as this indirect effect is sufficiently small, or reinforces the activist's decision to reduce e_t, then it will remain true that $e^*_{t-1} > e^*_t$. The indirect effect will reinforce the activist's reduction in effort if $de_t/dh^R_t > 0$. Totally differentiating the activist's FOC shows that at the equilibrium point, $de_t/dh^R_t = -F_{eh}/F_{ee}$. Thus a sufficient condition for the indirect effect to reinforce the direct effect is that $F_{eh} < 0$ at the equilibrium, which ensures that effort is a strategic complement to firm toughness. An alternative sufficient condition is that α is sufficiently small, as can be seen from (10), which implies that $de^*_t/dt < 0$ and $e^*_{t-1} > e^*_t$. Q.E.D.

A.4 Equilibria in Phase II: Private and Public Politics

Here we formally characterize the firm's joint resistance and lobbying strategies, as presented in the following Proposition, whereas we presented a verbal version in the main text.

Proposition 3 (Technical Statement) *For each $t > T^{Public}$, there exists a value $\tilde{\gamma}_t$ such that for $\gamma = \tilde{\gamma}_t$, $h^L_t = h^{R|L}_t = h^{R|N}_t$. For $\gamma \leq \tilde{\gamma}_t$ the firm's optimal strategy is as follows: if $h_t \geq h^L_t$, the firm concedes (and would not lobby if it resisted); if $h_t \in (h^{R|L}_t, h^L_t)$ the firm concedes (and would lobby if it resisted); and if $h_t \leq h^{R|L}_t$, the firm resists and lobbies. For $\gamma > \tilde{\gamma}_t$ the firm's optimal strategy is as follows: if $h_t \geq h^{R|N}_t$, the firm concedes (and would not lobby if it resisted); if $h_t \in (h^L_t, h^{R|N}_t)$ the firm resists and does not lobby; and if $h_t \leq h^L_t$, the firm resists and lobbies.*

Proof of Proposition 3: By referring to (13), (14) and (15), algebraic manipulations show that $h^L_t = h^{R|L}_t = h^{R|N}_t$ when

$$\tilde{\gamma}_t \equiv (1-\beta)\frac{\rho_t\phi_t - B^{Maj}_t/2}{\phi_t(1-\rho_t)B^{Maj}_t/2},$$

which is strictly positive since $\beta < 1$. For the case where $\gamma \leq \tilde{\gamma}_t$, $h^L_t > h^{R|L}_t > h^{R|N}_t$. Making use of the definition of each threshold, for $h > h^L_t$, the firm concedes (and would not lobby even if it resisted). If $h \in (h^{R|L}_t, h^L_t)$ the firm concedes (and would lobby if it resisted); and if $h < h^{R|L}_t$, the firm resists and lobbies. Note that in this case $h^{R|N}_t$ is irrelevant for the firm's ultimate lobbying and resistance decisions because it only applies in the situation where the firm would not lobby; however, because we are considering the region where $h < h^{R|L}_t$, the firm will lobby, rendering $h^{R|N}_t$ moot. For the case where $\gamma > \tilde{\gamma}_t$, $h^{R|N}_t > h^{R|L}_t > h^L_t$. For $h > h^{R|N}_t$, the firm concedes (and would not lobby if it resisted); if $h \in (h^{R|L}_t, h^{R|N}_t)$ the firm resists (and would not lobby if it resisted); and if $h < h^L_t$, the firm resists and lobbies. Note that in this case $h_t^{R|L}$ is irrelevant because it is only relevant in the case where the firm would lobby; however, because we are considering the region where $h > h_t^L$, the firm will not lobby, rendering $h_t^{R|L}$ moot. Q.E.D.

Proposition 4 *Lobbying and resistance strategies are complements for the firm: when the firm is able to lobby in Phase II, its resistance strategy is tougher in both Phase I and Phase II.*

The proof of the proposition is presented in the main text.

Phase II Strategy Dynamics The following proposition characterizes how the firm's incentives change over time.

Proposition 5 (Technical Statement) *Lobbying does not occur if $t < T^{Public}$ or $h^L_{T^{Public}} < 0$. Otherwise, for $t \geq T^{Public}$, and for any given level of e, the likelihood of lobbying is strictly positive at $t = T^{Public}$ and declines over time. A sufficient condition for the firm to eventually stop lobbying is*

$$p > (1 + A)/2 + \delta^{Maj} + \lambda\bar{\phi}.$$

Moreover, for any given level of activist effort, the firm's likelihood of resisting decreases over time.

Proof of Proposition 5: The firm is willing to lobby if and only if $h < h_t^L$. Recall from our explication of equation (13) that h_t^L is strictly decreasing over time for any value of γ. Hence for any value of e, and corresponding distribution of harm $F(h|e)$, the probability that the firm lobbies is decreasing over time. A sufficient condition for the firm to eventually cease lobbying altogether is that $h_t^L = 0$ when majority beliefs have converged to $b_t^{Maj} = p$. In this case, the firm will not lobby for any possible realized level of harm. As $b_t^{Maj} \to p$,

$$\lim_{t\to\infty} B_t^{Maj} = -1 + 2(p - \delta^{Maj}) - A,$$

and the firm will find lobbying too expensive if $B_t^{Maj}/2 > \lambda\bar{\phi}$, or equivalently, $p > (1 + A)/2 + \delta^{Maj} + \lambda\bar{\phi}$.

Thus, lobbying will eventually become too costly if the condition in the proposition (equation (16) in the text) holds. In this case, there is eventually a date $t > T^{Public}$ after which there is a stationary equilibrium in which legislation is introduced with probability λ each period. If (16) does not hold as $b_t^{Maj} \to p$, then Stage II features a permanent lobbying equilibrium for all $t > T^{Public}$ and legislation is never introduced.

Finally, equations (14) and (15) imply that the boundaries $h_t^{R|L}$ and $h_t^{R|N}$ are decreasing over time, as discussed in the text. Thus, for any level of activist effort e, the firm's likelihood of resistance is decreasing over time. Q.E.D.

A.5 Strategic Implications of Polarization

Proposition 6 *The time at which public politics becomes feasible is increasing in both the bias of the majority (ideological polarization) and the degree of homophily (affective polarization).*

Proof of Proposition 6 Differentiating equation (18) with respect to δ^{Maj} and z yields the result immediately. Q.E.D.

Proposition 7 *During Phase I, greater ideological or affective polarization intensifies both activism and corporate resistance.*

Proof of Proposition 7 Proposition 6 shows that T^{Public} is increasing in both ideological and affective polarization. In addition, Proposition 1 shows that $v^A(\mathcal{H}_t)$ is decreasing in T^{Public} and $v^F(\mathcal{H}_t)$ is increasing in T^{Public}. Then equation (5) implies directly that activist effort increases with polarization. Similarly, equations (7) and (8) imply that firm resistance increases with polarization. Q.E.D.

Proposition 8 *During Phase II, greater ideological or affective polarization (a) increases the likelihood of lobbying in periods before the firm ceases lobbying, and (b) extends the date at which the firm ceases lobbying. In addition, greater ideological polarization increases the likelihood that the firm never ceases lobbying, resulting in permanent legislative gridlock.*

Proof of Proposition 8 (a) Equation (11) shows that the value to a majority legislator of passing legislation is decreasing in δ^{Maj} and increasing in majority beliefs b_t^{Maj}, the latter of which are decreasing in both forms of polarization. Thus, the cost of lobbying, $B_t^{Maj}/2$, is decreasing in polarization; this increases the likelihood of lobbying in all periods before the firm ceases lobbying. (b) As shown in the proof of Proposition 7, affective polarization increases $\bar{\phi}$. Then Proposition 5 implies that both ideological and affective polarization increase the threshold level of beliefs, and hence the date, at which the firm stops lobbying. In addition, equation (16) shows that the likelihood of permanent lobbying increases directly in δ^{Maj}. Q.E.D.