Optimal Incentive Design for Decentralized Dynamic Matching Markets

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EC1 Proofs in Sections 3 and 4

EC1.1 Proof of Lemma 3.1

Recall that

$$V^* = \max_{\pi\in\Pi} \liminf_{t\to\infty} \frac{1}{t} V^\pi(t), \qquad \text{where } V^\pi(t) = \mathbb{E}\left[\sum_{m\in[K]} r_m N_m^\pi(t)\right],$$

and Π denotes the set of all feasible policies. Note that there is no submission term in $V^\pi(t)$ because there is only one player: the central planner.

For any feasible policy $\pi \in \Pi$, the number of type-j jobs that have been matched by time t is $\sum_{m\in[K]} M_{jm} N_m^\pi(t)$, which must be less than $X_j(0) + A_j(t)$, the number of type-j jobs that have arrived by time t. Therefore, for any sample path, the total matching rewards by time t under policy π is bounded from above by the following:

$$\begin{aligned} V(t)[\mathbf{X}(0) + \mathbf{A}(t)] \triangleq \max_{\mathbf{x}\in\mathbb{N}^K} \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} \le \mathbf{X}(0) + \mathbf{A}(t). \end{aligned} \tag{EC-1}$$

Let $\mathbf{x}[\mathbf{X}(0) + \mathbf{A}(t)]$ denote an optimal solution to (EC-1); it holds that

$$\mathbf{M}\,\mathbb{E}\big[\mathbf{x}[\mathbf{X}(0) + \mathbf{A}(t)]\big] \le \mathbf{X}(0) + \mathbb{E}\left[\mathbf{A}(t)\right] = \mathbf{X}(0) + N\boldsymbol{\lambda} t.$$

Therefore, $\mathbb{E}\left[\mathbf{x}[\mathbf{X}(0) + \mathbf{A}(t)]\right]$ is feasible to the problem of $V^{\mathrm{F}}(\mathbf{X}(0) + N\boldsymbol{\lambda}t)$, and it follows that

$$V^\pi(t) \le \mathbb{E}\left[V(t)[\mathbf{X}(0) + \mathbf{A}(t)]\right] \le V^{\mathrm{F}}(\mathbf{X}(0) + N\boldsymbol{\lambda}t).$$

According to the definition of $V^{\mathrm{F}}(\boldsymbol{\lambda})$ and its dual problem (2), the function $V^{\mathrm{F}}(\boldsymbol{\lambda})$ is concave and piecewise linear, and thus is continuous. As a result, we have

$$\lim_{t\to\infty} \frac{1}{t} V^{\mathrm{F}}(\mathbf{X}(0) + N\boldsymbol{\lambda}t) = \lim_{t\to\infty} V^{\mathrm{F}}\left(N\boldsymbol{\lambda} + \frac{1}{t}\mathbf{X}(0)\right) = V^{\mathrm{F}}(N\boldsymbol{\lambda}) = NV^{\mathrm{F}}(\boldsymbol{\lambda}).$$

In conclusion, we have $V^* \le V^{\mathrm{F}}(N\boldsymbol{\lambda}) = NV^{\mathrm{F}}(\boldsymbol{\lambda})$.

EC1.2 Proof of Lemma 3.2

EC1.2.1 Preparations

To start, we provide some useful lemmas as preparation for the proof of Lemma 3.2. First, we bound the optimal dual variable $\mathbf{p}^*$ from above in Lemma EC1.1.

Lemma EC1.1. *Let $\mathbf{p}^* = (p_j^*)_{j\in[J]}$ be an optimal solution to (2). It follows that $p_j^* \le r_{\max}$ for all $j \in [J]$.*

Proof. We prove by contradiction. Suppose, instead, that $p_j^* > r_{\max}$ for some index $j \in [J]$. If we set $p_j^* = r_{\max}$ while keeping all other components of $\mathbf{p}^*$ unchanged, $\mathbf{p}^*$ remains feasible to (2), and this adjustment strictly improves the objective value, contradicting the optimality of $\mathbf{p}^*$. □

Second, Lemma EC1.2 shows that the function $V^{\mathrm{F}}(\boldsymbol{\lambda})$ is Lipschitz continuous in $\boldsymbol{\lambda}$.

Lemma EC1.2. *For any two arrival rate vectors $\boldsymbol{\lambda}, \boldsymbol{\lambda}' \geq 0$, we have:*

$$V^{\mathrm{F}}(\boldsymbol{\lambda}') - V^{\mathrm{F}}(\boldsymbol{\lambda}) \leq r_{\max} \cdot \sum_{j \in [J]} \Big(\lambda'_j - \lambda_j\Big)^+.$$

Proof. By the strong duality, $V^{\mathrm{F}}(\boldsymbol{\lambda})$ equals the optimal value of (2). Let $\mathbf{p}^* = (p_j^*)_{j\in[J]}$ denote an optimal solution of (2). Therefore, we have $V^{\mathrm{F}}(\boldsymbol{\lambda}) = \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{p}^*$. Moreover, since $\mathbf{p}^*$ satisfies $\mathbf{M}^{\mathrm{T}}\mathbf{p}^* \geq \mathbf{r}$, it holds that $V^{\mathrm{F}}(\boldsymbol{\lambda}') \leq \boldsymbol{\lambda}'^{\mathrm{T}}\mathbf{p}^*$ by weak duality. As a result, we have:

$$V^{\mathrm{F}}(\boldsymbol{\lambda}') - V^{\mathrm{F}}(\boldsymbol{\lambda}) \leq \sum_{j\in[J]} p_j^*(\lambda'_j - \lambda_j) \leq \sum_{j\in[J]} p_j^* \left(\lambda'_j - \lambda_j\right)^+ \leq r_{\max} \cdot \sum_{j\in[J]} \left(\lambda'_j - \lambda_j\right)^+,$$

where the last inequality follows from Lemma EC1.1.

□

Finally, Lemma EC1.3 shows that the number of jobs that arrive within a time interval and remain till the end of the interval follows a specific Poisson distribution.

Lemma EC1.3. *For any time interval of length Δ, let Z_j denote the number of type-j jobs that arrive within the time interval and remain till the end of the interval. Then, Z_j follows a Poisson distribution with a mean value of $\frac{N\lambda_j}{\theta_j}\left(1 - \exp\left(-\theta_j\Delta\right)\right)$.*

Proof. Without loss of generality, we consider the time interval $(0, \Delta]$. We first compute the ex-ante probability $\mathbb{P}_j(\Delta)$ that a type-j job which arrived during $(0, \Delta]$ is still in the system at time Δ. Note that a type-j job will still be in the system if and only if its arrival time + waiting time $> \Delta$. Therefore, it holds that

$$\begin{aligned}
\mathbb{P}_j(\Delta) &= \mathbb{P}\left(\text{arrival time} + \text{waiting time} > \Delta\right) = \mathbb{P}\left(\text{waiting time} > \Delta - \text{arrival time}\right) \\
&= \frac{1}{\Delta}\int_0^\Delta \mathbb{P}\left(\text{waiting time} > x\right) dx = \frac{1}{\Delta}\int_0^\Delta \exp\left(-\theta_j x\right) dx \\
&= \frac{1}{\Delta\theta_j}\left(1 - \exp\left(-\theta_j\Delta\right)\right),
\end{aligned}$$

where the third equality follows from the fact that the arrival time is uniformly distributed over $(0, \Delta]$, conditional on the job arriving within this time interval (see Section 2.3 of Ross 1995).

Let $A_j(\Delta)$ denote the number of type-j jobs that have arrived by time Δ, which follows a Poisson distribution with a mean value of $\Delta N\lambda_j$. Since each of the $A_j(\Delta)$ jobs remains in the system at time Δ with a probability of $\mathbb{P}_j(\Delta)$, Z_j follows a Poisson distribution with a mean value of

$$\Delta N\lambda_j \cdot \mathbb{P}_j(\Delta) = \frac{N\lambda_j}{\theta_j}\Big(1 - \exp\left(-\theta_j\Delta\right)\Big).$$

□

EC1.2.2 Proof of Lemma 3.2

Consider a time interval of length Δ. Let Z_j denote the number of type-j jobs that arrive within the time interval and remain till the end of the interval. According to Lemma EC1.3, we have that $Z_j \sim \text{Poisson}\left(\frac{N\lambda_j}{\theta_j}\left(1 - \exp\left(-\theta_j\Delta\right)\right)\right)$.

Let $\mathbf{Z} = (Z_j)_{j\in[J]} \in \mathbb{R}_+^J$ be the concatenation of job arrivals, $V^{\mathrm{P}}(\Delta)$ the expected payoff of the periodic matching policy π^{P} in a period of length Δ, and $\mathbf{X} \in \mathbb{N}^J$ the number of jobs in the system

by the end of the time interval and before matches are conducted. It holds that $\mathbf{X} \geq \mathbf{Z}$ for any sample path, because there could be jobs that arrived before the time interval and still remain in the system. By the definition of policy π^{P}, we solve the fluid relaxation $V^{\mathrm{F}}(\mathbf{X})$, obtain an optimal solution $\mathbf{x}^* \in \mathbb{R}_+^K$, and perform $\lfloor x_m^* \rfloor$ type-m matchings for every $m \in \mathcal{M}_+$.

We first bound the rounding error. Note that:

$$\sum_{m \in \mathcal{M}_+} r_m \lfloor x_m^* \rfloor \geq \sum_{m \in \mathcal{M}_+} r_m x_m^* - \min\left\{K, J\right\} r_{\max}\,,$$

where the inequality follows from the fact that the fluid relaxation (1) has at most $\min\{K, J\}$ basic variables. Therefore, the following holds:

$$V^{\mathrm{P}}(\Delta) \geq V^{\mathrm{F}}(\mathbf{X}) - \min\left\{K, J\right\} \cdot r_{\max} \geq V^{\mathrm{F}}(\mathbf{Z}) - \min\left\{K, J\right\} \cdot r_{\max}\,, \tag{EC-2}$$

where the second inequality follows from the fact that $\mathbf{X} \geq \mathbf{Z}$ for every sample path. Consequently,

$$\begin{aligned}
V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}[V^{\mathrm{P}}(\Delta)] &\leq V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}[V^{\mathrm{F}}(\mathbf{Z})] + \min\left\{K, J\right\} \cdot r_{\max} \\
&\leq r_{\max} \cdot \sum_{j \in [J]} \mathbb{E}\Big(\Delta N\lambda_j - Z_j\Big)^+ + \min\left\{K, J\right\} \cdot r_{\max} \\
&\leq r_{\max} \cdot \sum_{j \in [J]} \left\{\mathbb{E}\Big(\Delta N\lambda_j - \mathbb{E}\left[Z_j\right]\Big)^+ + \mathbb{E}\Big(\mathbb{E}\left[Z_j\right] - Z_j\Big)^+\right\} + \min\left\{K, J\right\} \cdot r_{\max} \\
&\leq r_{\max} \cdot \sum_{j \in [J]} \left\{\frac{1}{2}\theta_j N\lambda_j \Delta^2 + \sqrt{N\lambda_j \Delta}\right\} + \min\left\{K, J\right\} \cdot r_{\max}.
\end{aligned}$$

where the first inequality follows from (EC-2), the second inequality from Lemma EC1.2, and the fourth inequality from the facts that

$$\mathbb{E}\left[Z_j\right] = N\lambda_j \int_0^\Delta \exp\left(-\theta_j x\right) dx \geq N\lambda_j \int_0^\Delta \left(1 - \theta_j x\right) dx = N\lambda_j \Delta \left(1 - \frac{1}{2}\theta_j \Delta\right), \tag{EC-3}$$

and that

$$\begin{aligned}
\mathbb{E}\Big(\mathbb{E}\left[Z_j\right] - Z_j\Big)^+ \leq \mathbb{E}\Big|\mathbb{E}\left[Z_j\right] - Z_j\Big| &\leq \left[\mathbb{E}\Big(\mathbb{E}\left[Z_j\right] - Z_j\Big)^2\right]^{\frac{1}{2}} = \sqrt{\mathbb{V}\mathrm{ar}\big(Z_j\big)} \\
&= \sqrt{\frac{N\lambda_j}{\theta_j}\left(1 - \exp\left(-\theta_j \Delta\right)\right)} \leq \sqrt{N\lambda_j \Delta}.
\end{aligned}$$

By taking $\Delta = N^{-\frac{1}{3}}$, we have

$$\begin{aligned}
\frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{NV^{\mathrm{F}}} &= \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}[V^{\mathrm{P}}(\Delta)]}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})} \\
&\leq r_{\max} \cdot \frac{\sum_{j\in[J]} \left\{ \sqrt{N\lambda_j\Delta} + \frac{1}{2}\theta_j N\lambda_j\Delta^2 \right\} + \min\{K, J\}}{\Delta N \cdot V^{\mathrm{F}}} \\
&\leq \frac{r_{\max}}{V^{\mathrm{F}}} \cdot \sum_{j\in[J]} \left(\sqrt{\lambda_j} \cdot \frac{1}{\sqrt{\Delta N}} + \frac{\lambda_j\theta_j}{2} \cdot \Delta + \frac{1}{\Delta N} \right) \\
&\leq \frac{r_{\max}}{V^{\mathrm{F}}} \cdot \sum_{j\in[J]} \left(\sqrt{\lambda_j} + \frac{\lambda_j\theta_j}{2} + 1 \right) \cdot N^{-\frac{1}{3}}.
\end{aligned}$$

Remark EC1.1 (Incorporating Enrollment Deferral)**.** We note that the proofs of Lemma 3.2 and Lemma 3.3 (provided in Section EC1.4) continue to hold when the enrollment deferral (Section 3.3) is incorporated. The proofs are literally the same, with the main difference being that the random variable Z_j, defined in Lemma EC1.3 as the number of newly arriving type-j jobs joining a specific matching epoch, now follows a Poisson distribution with a mean value of $\exp(-\theta_j\Delta) \cdot \Delta N\lambda_j$, which is greater than $(1 - \theta_j\Delta)\,\Delta N\lambda_j$.

EC1.3 Details of Remark 3.1

Proof of Bullet One We note that function $V^{\mathrm{F}}(\boldsymbol{\lambda})$ is piecewise linear and concave in $\boldsymbol{\lambda}$, and that $\mathbf{p}^* \in \mathbb{R}_+^J$ is an optimal dual variable of $V^{\mathrm{F}}(\boldsymbol{\lambda})$ if and only if it is a subgradient of $V^{\mathrm{F}}(\cdot)$ at the point $\boldsymbol{\lambda}$ (see Sections 5.2 and 5.3 of Bertsimas and Tsitsiklis 1997). Therefore, $\mathbf{p}^*$ is the unique optimal dual solution if and only if $\boldsymbol{\lambda}$ is not a kink of the function $V^{\mathrm{F}}(\cdot)$, and since $V^{\mathrm{F}}(\boldsymbol{\lambda})$ is piecewise linear, this condition holds if and only if $\mathbf{p}^*$ is the unique optimal dual solution in the neighborhood of $\boldsymbol{\lambda}$.

Proof of Bullet Two Let (EC-4) be the standard form of the fluid relaxation (1) by introducing the slack variable $\mathbf{s} \in \mathbb{R}_+^J$.

$$\begin{aligned}
V^{\mathrm{F}}(\boldsymbol{\lambda}) = \max_{\mathbf{x}\in\mathbb{R}_+^K,\, \mathbf{s}\in\mathbb{R}_+^J} \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\
\text{s.t.} \quad & \mathbf{M}\mathbf{x} + \mathbf{s} = \boldsymbol{\lambda}.
\end{aligned} \tag{EC-4}$$

We show that $\mathbf{x}$ is a non-degenerate basic feasible solution of (1) if and only if $\mathbf{x}$ together with $\mathbf{s} = \boldsymbol{\lambda} - \mathbf{M}\mathbf{x}$ is a non-degenerate basic feasible solution of (EC-4). For a matrix $\mathbf{M} \in \mathbb{R}^{J\times K}$ and subsets $A \subseteq [J]$ and $B \subseteq [K]$, let $\mathbf{M}_{A,B}$ denote the submatrix of $\mathbf{M}$ consisting of the rows indexed by A and the columns indexed by B. Furthermore, for a feasible solution $\mathbf{x}$ to (1), define $I \triangleq \{m \in [K] : x_m = 0\}$ and $H \triangleq \{j \in [J] : \mathbf{m}_j^{\mathrm{T}}\mathbf{x} = \lambda_j\}$, where $\mathbf{m}_j^{\mathrm{T}}$ represents the j-th row of the matching matrix $\mathbf{M}$. On one hand, $\mathbf{x}$ is a non-degenerate basic feasible solution of (1) if and only if $\mathbf{x}$ is feasible to (1) and the following conditions hold:

$$\begin{aligned}
&|I| + |H| = K, \\
&\text{The sub-matrix } \mathbf{M}_{H,[K]\setminus I} \text{ has full rank.}
\end{aligned} \tag{EC-5}$$

On the other hand, $\mathbf{x}$ together with $\mathbf{s} = \boldsymbol{\lambda} - \mathbf{M}\mathbf{x}$ is a non-degenerate basic solution of (EC-4) if and only if $\mathbf{x}$ is feasible to (1) and the following conditions hold:

$$\begin{aligned} &|I| + |H| = K, \\ &\text{The matrix } \left(\mathbf{M}_{[J],\,[K]\setminus I}\,, \mathbf{I}_{[J],\,[J]\setminus[H]}\right) \text{ has full rank,} \end{aligned} \tag{EC-6}$$

where $\mathbf{I}_{[J],\,[J]\setminus[H]}$ is a sub-matrix of the $J \times J$ identity matrix. Since $\left(\mathbf{M}_{[J],\,[K]\setminus I}\,, \mathbf{I}_{[J],\,[J]\setminus[H]}\right)$ has full rank if and only if $\mathbf{M}_{H,\,[K]\setminus I}$ has full rank, (EC-5) and (EC-6) are equivalent.

Note that (2) is also the dual of (EC-4). Therefore, if (1) has a non-degenerate optimal basic feasible solution, so does (EC-4), and this implies that (2) has a unique optimal solution according to Bertsimas and Tsitsiklis (1997) (see, for example, the discussion following Example 4.6 or preceding Theorem 5.1).

EC1.4 Proof of Lemma 3.3

Let $\mathbf{p}^*$ be the unique optimal dual variable of $V^{\mathrm{F}}(\hat{\boldsymbol{\lambda}})$ for any $\hat{\boldsymbol{\lambda}}$ such that $||\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda}||_\infty \le \delta$. We have:

$$V^{\mathrm{F}}(\hat{\boldsymbol{\lambda}}) = \hat{\boldsymbol{\lambda}}^{\mathrm{T}}\mathbf{p}^*,\ \forall\ \hat{\boldsymbol{\lambda}} \text{ such that } ||\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda}||_\infty \le \delta. \tag{EC-7}$$

Analogous to Section EC1.2, we focus on a time interval of length Δ and define random variable Z_j to be the number of type-j jobs that arrive within the time interval and remain till the end of the interval. Z_j follows a Poisson distribution with a mean value of $\frac{N\lambda_j}{\theta_j}\left(1 - \exp\left(-\theta_j\Delta\right)\right)$ by Lemma EC1.3. Note that the following holds:

$$\left(1 - \theta_j\Delta\right)\Delta N\lambda_j \le \left(1 - \frac{1}{2}\theta_j\Delta\right)\Delta N\lambda_j \le \frac{N\lambda_j}{\theta_j}\left(1 - \exp\left(-\theta_j\Delta\right)\right) \le \Delta N\lambda_j\,,$$

where the second inequality follows from (EC-3) and the third inequality follows from the fact that $e^x \ge 1 + x$ for any $x \in \mathbb{R}$. We can construct two Poisson random variables $\bar{Z}_j \sim \mathrm{Poisson}\left(\Delta N\lambda_j\right)$ and $\underline{Z}_j \sim \mathrm{Poisson}\left(\left(1 - \theta_j\Delta\right)\Delta N\lambda_j\right)$, such that with proper coupling,

$$\underline{Z}_j \le Z_j \le \bar{Z}_j$$

holds for every sample path.

EC1.4.1 Proof of Bullet One

Within a time interval of length Δ, the expected payoff of the periodic matching policy $V^{\mathrm{P}}(\Delta)$ is at least $V^{\mathrm{F}}(\mathbf{Z}) - \min\left\{K, J\right\} \cdot r_{\max}$ according to (EC-2), and the payoff of the fluid relaxation is $V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})$.

Take the interval length to be $\Delta = N^{-\frac{1}{2}}$, and define the following events:

$$\begin{aligned} &\bar{B}_j = \left\{Z_j > \lfloor \Delta N(\lambda_j + \delta)\rfloor\right\},\ \forall\, j \in [J], \\ &\underline{B}_j = \left\{Z_j < \lceil \Delta N(\lambda_j - \delta)\rceil\right\},\ \forall\, j \in [J], \\ &A = \left(\cup_{j\in[J]}\left(\bar{B}_j \cup \underline{B}_j\right)\right)^c \subseteq \left\{||\mathbf{Z} - \Delta N\boldsymbol{\lambda}||_\infty \le \Delta N\delta\right\}. \end{aligned}$$

By the concentration inequality for Poisson random variables (Lemma EC1.6), there exists positive

constants $c_1, c_2 > 0$, such that the following hold:

$$\begin{aligned}\mathbb{P}\left(\bar{B}_j\right) &\leq \mathbb{P}\big(\bar{Z}_j \geq \lfloor \Delta N(\lambda_j + \delta)\rfloor\big) \leq c_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right), \\ \mathbb{P}\left(\underline{B}_j\right) &\leq \mathbb{P}\big(\underline{Z}_j \leq \lceil \Delta N(\lambda_j - \delta)\rceil\big) \leq c_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right).\end{aligned} \tag{EC-8}$$

In the first line, the first inequality follows from the fact that $Z_j \leq \bar{Z}_j$, and the second inequality from taking $\Delta = N^{-\frac{1}{2}}$ and Lemma EC1.6. The second line can be justified analogously.

Using the union bound, we obtain:

$$\mathbb{P}\left(A^c\right) \leq \sum_{j\in[J]} \left(\mathbb{P}\left(\bar{B}_j\right) + \mathbb{P}\left(\underline{B}_j\right)\right) \leq 2Jc_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right). \tag{EC-9}$$

In addition, we have $||\frac{\mathbf{Z}}{\Delta N} - \boldsymbol{\lambda}||_\infty \leq \delta$ conditioning on event A. Therefore, from (EC-7) we have:

$$V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - V^{\mathrm{F}}(\mathbf{Z}) \leq \mathbb{1}[A] \cdot \mathbf{p}^{*\mathrm{T}} \cdot (\Delta N\boldsymbol{\lambda} - \mathbf{Z}) + \mathbb{1}[A^c] \cdot V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}).$$

Consequently,

$$\begin{aligned}\frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{NV^{\mathrm{F}}} &= \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}[V^{\mathrm{P}}(\Delta)]}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})} \\ &\leq \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}\big[V^{\mathrm{F}}(\mathbf{Z})\big] + \min\{K, J\} \cdot r_{\max}}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})} \\ &\leq \frac{\mathbb{E}\left[\mathbb{1}[A] \cdot \mathbf{p}^{*\mathrm{T}} \cdot (\Delta N\boldsymbol{\lambda} - \mathbf{Z})\right] + \min\{K, J\} \cdot r_{\max}}{\Delta N \cdot V^{\mathrm{F}}} + \mathbb{P}\left(A^c\right),\end{aligned} \tag{EC-10}$$

where the first inequality follows from (EC-2). Lemma EC1.4 serves as preparation to bound the first term in (EC-10) from above.

Lemma EC1.4. *For any job type $j \in [J]$, the following holds:*

$$\mathbb{E}[\mathbb{1}[A] \cdot Z_j] \geq \Delta N\lambda_j \cdot (1 - \theta_j\Delta) - \Delta N \cdot \big(\lambda_j + 2J(\lambda_j + \delta)\big) \cdot c_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right).$$

Proof. First, note that:

$$\mathbb{E}[\mathbb{1}[A] \cdot Z_j] = \mathbb{E}[Z_j] - \mathbb{E}[\mathbb{1}[A^c] \cdot Z_j] \geq \Delta N\lambda_j \cdot \big(1 - \theta_j\Delta\big) - \mathbb{E}[\mathbb{1}[A^c] \cdot Z_j],$$

where the inequality follows from $Z_j \geq \underline{Z}_j$, and hence $\mathbb{E}[Z_j] \geq \mathbb{E}\big[\underline{Z}_j\big] = \Delta N\lambda_j \cdot (1 - \theta_j\Delta)$.

We now bound the second term from above, as follows:

$$\begin{aligned}\mathbb{E}[\mathbb{1}[A^c] \cdot Z_j] &= \mathbb{E}\big[\mathbb{1}[A^c \cap \bar{B}_j] \cdot Z_j\big] + \mathbb{E}\big[\mathbb{1}[A^c \cap \bar{B}_j^c] \cdot Z_j\big] \\ &\leq \mathbb{E}[\mathbb{1}[\bar{B}_j] \cdot Z_j] + \mathbb{P}[A^c \cap \bar{B}_j^c] \cdot \Delta N\big(\lambda_j + \delta\big) \\ &\leq \mathbb{E}\left[\mathbb{1}\left[\bar{Z}_j > \lfloor \Delta N(\lambda_j + \delta)\rfloor\right] \cdot \bar{Z}_j\right] + \mathbb{P}[A^c] \cdot \Delta N\big(\lambda_j + \delta\big) \\ &\leq \Delta N\lambda_j \cdot \mathbb{P}\left(\bar{Z}_j \geq \lfloor \Delta N(\lambda_j + \delta)\rfloor\right) + 2J \cdot \Delta N\big(\lambda_j + \delta\big) \cdot c_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right) \\ &\leq \Delta N \cdot \big(\lambda_j + 2J(\lambda_j + \delta)\big) \cdot c_1 \cdot \exp\left(-c_2 \cdot N^{\frac{1}{2}}\right).\end{aligned}$$

In the above, the first inequality follows from the facts that $\bar{B}_j \subseteq A^c$ and that $Z_j \leq \Delta N(\lambda_j + \delta)$

conditional on event $\bar{B}_j^c$, the second inequality follows from $Z_j \le \bar{Z}_j$, the third inequality from (EC-9) and Lemma EC1.7, and the fourth inequality from (EC-8). □

From Lemma EC1.4, the following holds:

$$\begin{aligned}
&\mathbb{E}\big[\mathbb{1}[A]\cdot \mathbf{p}^{*\mathrm{T}}\cdot(\Delta N\boldsymbol{\lambda}-\mathbf{Z})\big] \le \sum_{j\in[J]} p_j^*\cdot\Big(\Delta N\lambda_j - \mathbb{E}\big[\mathbb{1}[A]\cdot Z_j\big]\Big)\\
&\le r_{\max}\cdot\sum_{j\in[J]}\Big(\Delta N\lambda_j\cdot\theta_j\Delta + \Delta N\cdot\big(\lambda_j+2J(\lambda_j+\delta)\big)\cdot c_1\cdot\exp\Big(-c_2\cdot N^{\frac{1}{2}}\Big)\Big),
\end{aligned} \tag{EC-11}$$

where the second inequality follows from Lemma EC1.4 and the fact that $p_j^* \le r_{\max}$ for any $j\in[J]$ (Lemma EC1.1). Combining (EC-9) – (EC-11), we obtain:

$$\begin{aligned}
\frac{NV^{\mathrm{F}}-V^{\mathrm{P}}}{NV^{\mathrm{F}}} &\le \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}\big[V^{\mathrm{F}}(\mathbf{Z})\big] + \min\{K,J\}\cdot r_{\max}}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})}\\
&\le \frac{r_{\max}}{V^{\mathrm{F}}}\cdot\sum_{j\in[J]}\left(\lambda_j\theta_j\Delta + \frac{1}{\Delta N} + \big(\lambda_j+2J(\lambda_j+\delta)\big)\cdot c_1\cdot\exp\Big(-c_2\cdot N^{\frac{1}{2}}\Big)\right) + 2Jc_1\cdot\exp\Big(-c_2\cdot N^{\frac{1}{2}}\Big)\\
&= O\Big(N^{-\frac{1}{2}}\Big)
\end{aligned}$$

by using $\Delta = N^{-\frac{1}{2}}$.

EC1.4.2 Proof of Bullet Two

Assume that the matching matrix $\mathbf{M}$ is totally unimodular. Then, the optimal solution to the matching problem solved by policy π^{P} at any time point $k\Delta$ is always integral. Therefore, rounding is not needed to perform the matching, and we have the following:

$$V^{\mathrm{P}}(\Delta) = V^{\mathrm{F}}(\mathbf{X}) \ge V^{\mathrm{F}}(\mathbf{Z}), \tag{EC-12}$$

following the same notation as in (EC-2).

Define a constant $c \triangleq \max_{j\in[J]}\big(\frac{1}{\lambda_j g(\delta/\lambda_j)} \vee \frac{2}{\lambda_j g(-\delta/\lambda_j)}\big)$, where $g(u)\triangleq(1+u)\ln(1+u)-u\ge 0$ is defined in Lemma EC1.6. Note that $g(0)=0$ and $g(u)>0$ for any $u\neq 0$. Take the interval length to be $\Delta = 2c\cdot\frac{\ln N}{N}$. Moreover, assume that N is sufficiently large so that: (i) $\Delta \le \min_{j\in[J]}\frac{\delta}{\lambda_j\theta_j}$, and (ii) $\frac{1}{2}\cdot g(-\delta/\lambda_j) \le (1-\theta_j\Delta)\cdot g\left(\frac{\lambda_j\theta_j\Delta-\delta}{\lambda_j(1-\theta_j\Delta)}\right)$ for all $j\in[J]$. Finally, define the following events:

$$\begin{aligned}
&\bar{B}_j = \big\{Z_j > \Delta N(\lambda_j+\delta)\big\},\ \forall\, j\in[J],\\
&\underline{B}_j = \big\{Z_j < \Delta N(\lambda_j-\delta)\big\},\ \forall\, j\in[J],\\
&A = \big(\cup_{j\in[J]}\big(\bar{B}_j\cup\underline{B}_j\big)\big)^c = \big\{||\mathbf{Z}-\Delta N\boldsymbol{\lambda}||_\infty \le \Delta N\delta\big\}.
\end{aligned}$$

By the concentration inequality for Poisson random variables (Lemma EC1.6), we have:

$$\begin{aligned}
&\mathbb{P}\left(\bar{B}_j\right) \le \mathbb{P}\big(\bar{Z}_j \ge \Delta N(\lambda_j+\delta)\big) \le \exp\big(-\Delta N\lambda_j\cdot g\big(\delta/\lambda_j\big)\big) \le \frac{1}{N^2},\\
&\mathbb{P}\left(\underline{B}_j\right) \le \mathbb{P}\big(\underline{Z}_j \le \Delta N(\lambda_j-\delta)\big) \le \exp\left(-\Delta N\lambda_j\cdot(1-\theta_j\Delta)\cdot g\left(\frac{\lambda_j\theta_j\Delta-\delta}{\lambda_j(1-\theta_j\Delta)}\right)\right) \le \frac{1}{N^2}.
\end{aligned}$$

In the first line, the first inequality follows from the fact that $Z_j \le \bar{Z}_j$, the second inequality from

Lemma EC1.6, and the third inequality from the definitions of the interval length Δ and constant c. Analogously, in the second line, the first inequality follows from the fact that $\underline{Z}_j \leq Z_j$, the second inequality from Lemma EC1.6, and the third inequality from the definitions of the interval length Δ and constant c and the aforementioned properties of Δ given that N is sufficiently large.

Using the union bound, we obtain:

$$\mathbb{P}\left(A^c\right) \leq \sum_{j\in[J]} \left(\mathbb{P}\left(\bar{B}_j\right) + \mathbb{P}\left(\underline{B}_j\right)\right) \leq \frac{2J}{N^2}\,. \tag{EC-13}$$

In addition, we have $||\frac{\mathbf{Z}}{\Delta N} - \boldsymbol{\lambda}||_\infty \leq \delta$ conditioning on event A. Therefore, from (EC-7) we get:

$$V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - V^{\mathrm{F}}(\mathbf{Z}) \leq \mathbb{1}[A]\cdot \mathbf{p}^{*\mathrm{T}}\cdot(\Delta N\boldsymbol{\lambda} - \mathbf{Z}) + \mathbb{1}[A^c]\cdot V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}).$$

Consequently,

$$\begin{aligned}
\frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{NV^{\mathrm{F}}} &= \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}[V^{\mathrm{P}}(\Delta)]}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})} \\
&\leq \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}\big[V^{\mathrm{F}}(\mathbf{Z})\big]}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})} \\
&\leq \frac{\mathbb{E}\left[\mathbb{1}[A]\cdot \mathbf{p}^{*\mathrm{T}}\cdot(\Delta N\boldsymbol{\lambda} - \mathbf{Z})\right]}{\Delta N\cdot V^{\mathrm{F}}} + \mathbb{P}\left(A^c\right),
\end{aligned} \tag{EC-14}$$

where the first inequality follows from (EC-12). Analogous to Lemma EC1.4, Lemma EC1.5 helps bound the first term in (EC-14) from above.

Lemma EC1.5. *For any job type $j \in [J]$, the following holds:*

$$\mathbb{E}[\mathbb{1}[A]\cdot Z_j] \geq \Delta N\lambda_j\cdot(1-\theta_j\Delta) - \Delta\lambda_j - 2J\cdot(\lambda_j+\delta)\cdot\frac{\Delta}{N}$$

when N is larger than a certain constant.

Proof. First, note that:

$$\mathbb{E}[\mathbb{1}[A]\cdot Z_j] = \mathbb{E}[Z_j] - \mathbb{E}[\mathbb{1}[A^c]\cdot Z_j] \geq \Delta N\lambda_j\cdot\big(1-\theta_j\Delta\big) - \mathbb{E}[\mathbb{1}[A^c]\cdot Z_j],$$

where the inequality follows from $Z_j \geq \underline{Z}_j$, and hence $\mathbb{E}[Z_j] \geq \mathbb{E}\big[\underline{Z}_j\big] = \Delta N\lambda_j\cdot(1-\theta_j\Delta)$.

We now bound the second term from above, as follows:

$$\begin{aligned}
\mathbb{E}[\mathbb{1}[A^c]\cdot Z_j] &= \mathbb{E}\big[\mathbb{1}[A^c\cap\bar{B}_j]\cdot Z_j\big] + \mathbb{E}\big[\mathbb{1}[A^c\cap\bar{B}_j^c]\cdot Z_j\big] \\
&\leq \mathbb{E}[\mathbb{1}[\bar{B}_j]\cdot Z_j] + \mathbb{P}[A^c\cap\bar{B}_j^c]\cdot\Delta N\big(\lambda_j+\delta\big) \\
&\leq \mathbb{E}\left[\mathbb{1}[\bar{Z}_j > \Delta N(\lambda_j+\delta)]\cdot\bar{Z}_j\right] + \mathbb{P}[A^c]\cdot\Delta N\big(\lambda_j+\delta\big) \\
&\leq \Delta N\lambda_j\cdot\mathbb{P}\left[\bar{Z}_j \geq \lfloor\Delta N(\lambda_j+\delta)\rfloor\right] + 2J\cdot\big(\lambda_j+\delta\big)\cdot\frac{\Delta}{N} \\
&\leq \Delta\lambda_j + 2J\cdot\big(\lambda_j+\delta\big)\cdot\frac{\Delta}{N}.
\end{aligned}$$

In the above, the first inequality follows from $\bar{B}_j \subseteq A^c$ and $Z_j \leq \Delta N(\lambda_j+\delta)$ conditional on event $\bar{B}_j^c$. The second inequality follows from $Z_j \leq \bar{Z}_j$. The third inequality follows from (EC-13) and Lemma EC1.7. Finally, let $\tilde{\delta}$ be a positive constant satisfying $\tilde{\delta} < \delta$ and $c \geq \max_{j\in[J]}\frac{1}{2\cdot\lambda_j g(\tilde{\delta}/\lambda_j)}$.

The constant $\tilde{\delta}$ exists by the definition of c. The fourth inequality follows from the facts that $\Delta N\big(\lambda_j+\tilde{\delta}\big) \le \lfloor \Delta N(\lambda_j+\delta)\rfloor$ when N is sufficiently large (specifically, when $\Delta N \ge 1/(\delta-\tilde{\delta})$) and that

$$\mathbb{P}\left(\bar{Z}_j \ge \Delta N(\lambda_j+\tilde{\delta})\right) \le \exp\left(-\Delta N\lambda_j \cdot g\big(\tilde{\delta}/\lambda_j\big)\right) \le \frac{1}{N}$$

by the definition of $\tilde{\delta}$ and Lemma EC1.6. □

From Lemma EC1.5, the following holds:

$$\begin{aligned}\mathbb{E}\big[\mathbb{1}[A]\cdot \mathbf{p}^{*\mathrm{T}}\cdot(\Delta N\boldsymbol{\lambda}-\mathbf{Z})\big] &\le \sum_{j\in[J]} p_j^*\cdot\Big(\Delta N\lambda_j - \mathbb{E}\big[\mathbb{1}[A]\cdot Z_j\big]\Big)\\ &\le r_{\max}\cdot\sum_{j\in[J]}\left(\Delta N\lambda_j\cdot\theta_j\Delta + \Delta\lambda_j + 2J\cdot(\lambda_j+\delta)\cdot\frac{\Delta}{N}\right),\end{aligned} \tag{EC-15}$$

where the second inequality follows from Lemma EC1.5 and the fact that $p_j^* \le r_{\max}$ for any $j\in[J]$ (Lemma EC1.1). Combining (EC-13) - (EC-15), we obtain:

$$\begin{aligned}\frac{NV^{\mathrm{F}}-V^{\mathrm{P}}}{NV^{\mathrm{F}}} &\le \frac{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda}) - \mathbb{E}\big[V^{\mathrm{F}}(\mathbf{Z})\big]}{V^{\mathrm{F}}(\Delta N\boldsymbol{\lambda})}\\ &\le \frac{r_{\max}}{V^{\mathrm{F}}}\cdot\sum_{j\in[J]}\left(\lambda_j\theta_j\Delta + \frac{\lambda_j}{N} + 2J\cdot(\lambda_j+\delta)\cdot\frac{1}{N^2}\right) + \frac{2J}{N^2}\\ &= O\left(\frac{\ln N}{N}\right)\end{aligned}$$

by using $\Delta = \Theta\left(\frac{\ln N}{N}\right)$.

Lemma EC1.6 (Concentration Inequality for Poisson Random Variables). *Let $X \sim Poisson(\lambda)$ be a Poisson random variable with a mean value of $\lambda > 0$. We have*

$$\mathbb{P}(X \ge \lambda(1+u)) \le \exp(-\lambda g(u))$$

for any $u \ge 0$, and

$$\mathbb{P}(X \le \lambda(1-u)) \le \exp(-\lambda g(-u))$$

for any $0 \le u < 1$, where $g(u) = (1+u)\ln(1+u) - u$.

Proof. We first prove the first inequality. For any $t \ge 0$, we have

$$\begin{aligned}\mathbb{P}(X \ge \lambda(1+u)) &= \mathbb{P}\Big[\exp(tX) \ge \exp(t\lambda(1+u))\Big]\\ &\le \mathbb{E}\Big[\exp(tX)\Big]\exp(-t\lambda(1+u))\\ &= \exp\big(\lambda(e^t-1) - t\lambda(1+u)\big)\end{aligned}$$

where the inequality follows from Markov's inequality and the second equality from the moment generating function of the Poisson distribution. It turns out that the right-hand side of the second equality is minimized by setting $t = \ln(1+u)$, in which case the right-hand side simplifies to $\exp(-\lambda g(u))$. The second inequality can be proven similarly. □

Lemma EC1.7. *Let $X \sim Poisson(\lambda)$ be a Poisson random variable with a mean value of $\lambda > 0$ and $z \in \mathbb{N}$ be a nonnegative integer. The following holds:*

$$\mathbb{E}\left[X \cdot \mathbb{1}[X > z]\right] = \sum_{k=z+1}^{\infty} k \cdot \frac{\lambda^k e^{-\lambda}}{k!} = \lambda \cdot \sum_{k=z}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = \lambda \cdot \mathbb{P}(X \geq z).$$

EC1.5 Proof of Lemma 3.4

Let x_m^P represent the long-run average matching rate of match m under policy π^{P}, and let $\mathbf{x}^P = (x_m^P)_{m\in[K]} \in \mathbb{R}_+^K$. It follows that $\mathbf{M}\mathbf{x}^P \leq N\boldsymbol{\lambda}$ because the rate of jobs joining matches must be less than the rate of job arrivals. Therefore, $\frac{\mathbf{x}^P}{N}$ is an optimal solution to the following LP (EC-16):

$$\begin{aligned} \max_{\mathbf{x}\in\mathbb{R}_+^K} \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} \leq \boldsymbol{\lambda}, \\ & \mathbf{r}^{\mathrm{T}}\mathbf{x} \leq \frac{\mathbf{r}^{\mathrm{T}}\mathbf{x}^P}{N}, \end{aligned} \tag{EC-16}$$

because $\frac{\mathbf{x}^P}{N}$ is feasible and obtains the highest possible objective value. On the other hand, since the optimal value of the fluid relaxation (1) is V^{F}, a solution is optimal to (1) if and only if it is optimal to the following LP (EC-17):

$$\begin{aligned} \max_{\mathbf{x}\in\mathbb{R}_+^K} \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} \leq \boldsymbol{\lambda}, \\ & \mathbf{r}^{\mathrm{T}}\mathbf{x} \leq V^{\mathrm{F}}. \end{aligned} \tag{EC-17}$$

By Lipschitz continuity of optimal solutions of LPs with respect to constraints' right-hand side (i.e., Lemma EC1.8 below), there exists an optimal solution $\mathbf{x}^*$ to the fluid relaxation (1) such that

$$\left\| \frac{\mathbf{x}^P}{N} - \mathbf{x}^* \right\|_\infty \leq C_4 \left| V^{\mathrm{F}} - \frac{\mathbf{r}^{\mathrm{T}}\mathbf{x}^P}{N} \right| = C_4 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}$$

for some constant $C_4 > 0$. The rest of Lemma 3.4 follows from the fact that policy π^{P} is asymptotically optimal (i.e., Lemmas 3.2 and 3.3 hold).

Lemma EC1.8 (Theorem 2.4 of Mangasarian and Shiau 1987)**.** *Consider the following LP:*

$$P(\mathbf{b}) = \max\left\{\mathbf{r}^{\mathrm{T}}\mathbf{x} : \mathbf{A}\mathbf{x} \leq \mathbf{b}\right\}.$$

There exists a constant κ such that, for any vectors $\mathbf{b}, \hat{\mathbf{b}} \in \mathbb{R}_+^n$ and any optimal solution $\mathbf{x}$ to $P(\mathbf{b})$, there exists an optimal solution $\hat{\mathbf{x}}$ to $P(\hat{\mathbf{b}})$ such that $\|\mathbf{x} - \hat{\mathbf{x}}\|_\infty \leq \kappa\|\mathbf{b} - \hat{\mathbf{b}}\|_\infty$.

EC1.6 Proof of Corollary 3.5

Since $0 \leq A_{jm}(t) \leq A_j(t)$, the bounded convergence theorem implies that

$$q_{jm} \triangleq \lim_{t\to\infty} \mathbb{E}\left[\frac{A_{jm}(t)}{A_j(t)}\right] = \mathbb{E}\left[\lim_{t\to\infty} \frac{A_{jm}(t)}{A_j(t)}\right].$$

We now characterize this limit. Let $N_m(t)$ denote the number of type-m matches completed by time t; we have $A_{jm}(t) = M_{jm}N_m(t)$. Therefore, the following holds:

$$\mathbb{E}\left[\lim_{t\to\infty}\frac{A_{jm}(t)}{A_j(t)}\right] = \mathbb{E}\left[\lim_{t\to\infty}\frac{A_{jm}(t)}{t}\frac{t}{A_j(t)}\right] = \frac{M_{jm}}{N\lambda_j}\cdot\mathbb{E}\left[\lim_{t\to\infty}\frac{N_m(t)}{t}\right] = \frac{M_{jm}}{N\lambda_j}\cdot\lim_{t\to\infty}\mathbb{E}\left[\frac{N_m(t)}{t}\right].$$

Specifically, the second equality follows from $A_{jm}(t) = M_{jm}N_m(t)$ and the fact that $\lim_{t\to\infty} A_j(t)/t = N\lambda_j$ almost surely by the strong law for renewal processes (Proposition 6.2 in Ross and Peköz 2023). The third equality follows from the generalized dominated convergence theorem (Theorem 19 in Royden and Fitzpatrick 2010) because $N_m(t) \le A_{j'}(t)$ and $\lim_{t\to\infty}\mathbb{E}\Big[\frac{A_{j'}(t)}{t}\Big] = \mathbb{E}\Big[\lim_{t\to\infty}\frac{A_{j'}(t)}{t}\Big] = N\lambda_{j'} < \infty$, where j' denotes a job type required by match m.

The above result, together with Lemma 3.4, yields Corollary 3.5 by setting $C_6 \triangleq C_4\cdot\max_{j\in[J],m\in[K]}\frac{M_{jm}}{\lambda_j}$.

EC1.7 Proof of Lemma 4.3

Let x_m denote the long-run average rate at which agent i performs match m locally, and s_j the long-run average rate at which type-j jobs submitted by agent i are successfully matched at the shared pool. The vectors $\mathbf{x} = (x_m)$ and $\mathbf{s} = (s_j)$ are feasible to (3) because, for each job type, the combined rates of jobs used for local matching and jobs matched at the shared pool cannot exceed the job arrival rate. Moreover, since every type-j job matched at the shared pool yields agent i an expected payoff of p_j^*, irrespective of the match type, the agent's long-run average payoff is $\mathbf{r}^{\mathrm{T}}\mathbf{x} + \mathbf{p}^{*\mathrm{T}}\mathbf{s}$, which is no larger than the optimal value of (3).

EC1.8 Proof of Lemma 4.4

Suppose all agents fully submit their jobs to the shared pool. Without loss of generality, assume that neither the shared pool nor any agent holds any job at time zero. Let $N_m(t)$ denote the number of type-m matches formed by the shared pool by time t, $x_m^P = \lim_{t\to\infty}\mathbb{E}[N_m(t)]/t$ the long-run average matching rate of match m at the shared pool, and $S_{ijm}(t)$ the number of type-j jobs submitted by agent i that participate in match m by time t. We have the following equation:

$$\mathbb{E}\big[S_{ijm}(t)\big] = \frac{\lambda_{ij}}{N\lambda_j}\cdot M_{jm}\cdot\mathbb{E}\big[N_m(t)\big], \tag{EC-18}$$

where $\lambda_j = \sum_{i\in[N]}\lambda_{ij}/N$ denote the normalized aggregate arrival rate of type-j jobs. To interpret (EC-18), note that a total of $M_{jm}N_m(t)$ type-j jobs participate in match m by time t, and each originates from agent i with probability $\frac{\lambda_{ij}}{N\lambda_j}$.

Let $\mathbf{x}^* = \big(x_m^*\big)_{m\in[K]}$ be an optimal solution to (1). The long-run average payoff of agent i,

denoted by V_i, satisfies the following:

$$\begin{aligned}
V_i &= \lim_{t\to\infty} \mathbb{E}\left[\frac{\sum_{j\in[J]} p_j^* \cdot \sum_{m\in[K]} S_{ijm}(t)}{t}\right] \\
&= \sum_{j\in[J]} p_j^* \cdot \frac{\lambda_{ij}}{\lambda_j} \sum_{m\in[K]} M_{jm} \cdot \frac{x_m^P}{N} \\
&\geq \sum_{j\in[J]} p_j^* \cdot \frac{\lambda_{ij}}{\lambda_j} \sum_{m\in[K]} M_{jm} \cdot \left(x_m^* - C_4 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}\right) \\
&\geq \sum_{j\in[J]} p_j^* \cdot \frac{\lambda_{ij}}{\lambda_j} \sum_{m\in[K]} M_{jm} \cdot x_m^* - C_7 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N} \\
&= \boldsymbol{\lambda}_i^{\mathrm{T}} \mathbf{p}^* - C_7 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}.
\end{aligned}$$

In the above, the first equality follows from the fact that every matched type-j job yields an expected payoff of p_j^*, the second equality follows from (EC-18) and the definition of x_m^P, the first inequality from Lemma 3.4, and the second inequality from the definition of the constant C_7, Lemma EC1.1, and Assumption 2.1. Finally, the third equality follows from the complementary slackness condition of (1), that is, $p_j^* \cdot (\mathbf{m}_j^{\mathrm{T}} \mathbf{x}^* - \lambda_j) = 0$ for all $j \in [J]$, where $\mathbf{m}_j^{\mathrm{T}}$ denotes the j-th row of matrix $\mathbf{M}$.

EC2 Proof in Section 5

EC2.1 Proof of Proposition 5.1

The proof is identical to the proof of Lemma 5.2 in Section EC2.3 with $\epsilon = 0$.

EC2.2 Further Details of Example 5.1

EC2.2.1 Expected Payoff of the Alternative Policy under the $\texttt{MV}(\epsilon)$ Mechanism

In this section, we derive agent one's long-run average expected payoff under the alternative policy within the $\texttt{MV}(\epsilon)$ mechanism. Under this policy, agent one possesses either zero or one type-two jobs, and the dynamics of the type-two jobs follow a birth-death process. Let μ_i be the stationary probability of having $i \in \{0, 1\}$ type-two jobs; these probabilities satisfy the following balance equation:

$$\lambda \cdot \mu_0 = (1 + \theta) \cdot \mu_1.$$

Since $\mu_0 + \mu_1 = 1$, we have:

$$\mu_0 = \frac{1+\theta}{1+\lambda+\theta}, \quad \mu_1 = \frac{\lambda}{1+\lambda+\theta}.$$

The long-run average payoff under the alternative policy is given by:

$$\begin{aligned}
V^{\mathrm{alt}} &= 1 \cdot \big(\mu_0 \cdot w \cdot (1-\epsilon) + \mu_1\big) + \lambda \cdot \mu_1 \cdot \epsilon \cdot \frac{w}{\lambda} \\
&= w + (1-w) \cdot \mu_1 + \epsilon \cdot w \cdot (\mu_1 - \mu_0) \\
&= w + (1-w) \cdot \frac{\lambda}{1+\lambda+\theta} + \epsilon \cdot w \cdot \frac{\lambda - 1 - \theta}{1+\lambda+\theta}.
\end{aligned}$$

For the first equality, note that an arriving type-one job is either matched internally with probability μ_1, yielding a reward of one, or submitted with probability μ_0, generating an expected payoff of $w \cdot (1-\epsilon)$. Conversely, an arriving type-two job is submitted with probability μ_1, generating an expected payoff of $\epsilon \cdot w/\lambda$.

EC2.2.2 Oblivious Equilibrium under the MV Mechanism

In this section, we derive the OE under the MV mechanism, i.e., the MV(ϵ) mechanism with $\epsilon = 0$. We demonstrate that the unique OE involves no job submission; that is, each agent performs only local matching and does not collaborate.

To see this, we show in Lemma EC2.1 that, in Example 5.1, an agent solving her mean-field problem never submits type-two jobs (under-demanded), because doing so yields zero value. Note that in Example 5.1, there is only one match type, which requires one type-one job and one type-two job. Since agents do not submit type-two jobs, type-one jobs cannot be matched in the shared pool. This renders $w = 0$, and thus, in an OE, agents do not submit over-demanded jobs either.

Lemma EC2.1. *Suppose the MV mechanism is used. For an agent solving her mean-field problem in Example 5.1, submitting an (under-demanded) type-two job is strictly suboptimal.*

Proof. Let $w < 1$ denote the steady-state matching probability of a type-one job in the shared pool. In the mean-field problem, submitting a type-one job yields payoff w, whereas matching it internally yields payoff 1. Hence, internal matching generates a strictly higher payoff, with incremental gain $1 - w > 0$.

Now consider an arriving type-two job. If the agent retains this job, then with positive probability it can later be matched internally with a future arriving type-one job. Whenever such a match occurs, the agent obtains the incremental gain $1 - w > 0$. Consequently, the expected payoff from retaining a type-two job is strictly positive. By contrast, under the MV mechanism, submitting a type-two job yields payoff 0. Therefore, it is strictly suboptimal to submit a type-two job. □

EC2.3 Proof of Lemma 5.2

EC2.3.1 Setting the Values of $C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$

Let $p^*_{\min} \triangleq \min_{j \in \mathcal{N}_+} p^*_j > 0$ denote the minimum marginal value of over-demanded jobs, $M_{\max} \triangleq \max_{m \in [K]} \sum_{j \in \mathcal{N}_+} M_{jm}$ the maximum number of over-demanded jobs required in a match, and $\Delta r_{\min} \triangleq \min_{m \in \mathcal{M}_0} \left\{ \sum_{j \in [J]} M_{jm}\, p^*_j - r_m \right\} > 0$ the smallest gap for suboptimal matchings. Define the constant $\nu \triangleq \min_{j \in [J]} \frac{\theta_j}{\theta_j + \sum_{\ell \in [J]} C_\ell \lambda_\ell} \in (0,1)$, where the constants $\{C_\ell\}_{\ell \in [J]}$ are specified in Assumption 2.1. We define $C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ as follows:

$$C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) = 1 - \frac{1}{8\, r_{\max}} \min \left\{ 2\Delta r_{\min},\, \nu p^*_{\min} \right\}$$
$$C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) = \frac{1}{8 M_{\max}} \min \left\{ 2\Delta r_{\min},\, \nu p^*_{\min} \right\}.$$

For simplicity, we denote $C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ as C_8 and C_9, respectively. In Lemma EC2.2, we establish basic properties of these two constants.

Lemma EC2.2. *We have that $C_8 \geq 7/8$ and $C_9 \leq p^*_{\min}/8$.*

Proof. The result follows directly from the observations that $p^*_{\min} \le r_{\max}$ by Lemma EC1.1, $\nu < 1$ by definition, and $M_{\max} \ge 1$ as each match must contain at least one over-demanded job under the constraints of (2). □

EC2.3.2 Proof of Lemma 5.2

We consider a representative agent i in the mean-field problem. Since every agent solves a Markov decision problem in the mean-field problem, it is without loss of optimality to assume that agent i takes actions only upon job arrivals or departures. Suppose agent i implements a policy π under which, in the long-run average, she retains a positive fraction $\rho > 0$ of type-j jobs, where type j is over-demanded (that is, $p^*_j > 0$). We use a coupling argument to show that policy π is strictly suboptimal.

An Alternative Policy $\tilde{\pi}$ Consider an alternative policy $\tilde{\pi}$ that mimics policy π but always submits type-j jobs upon their arrival. We refer to the type-j jobs on which π and $\tilde{\pi}$ differ as *marked jobs*. This difference may affect decisions at subsequent time points. Specifically, when policy π matches an incoming job, if one of the matched participants is a marked job, policy $\tilde{\pi}$ cannot replicate this match. We refer to such matches as *affected matches*. In other words, a match is affected if and only if it involves a marked job. In these cases, policy $\tilde{\pi}$ submits the incoming job and all other participant jobs at the moment when π executes an affected match. For jobs not involved in affected matches, policies π and $\tilde{\pi}$ make identical decisions. Therefore, any payoff difference between the two policies arises solely from the marked jobs and the affected matches.

Step One: Absence of Incentives for Forming Suboptimal Matches Let m be an affected match. We first show that $m \in \mathcal{M}_+$, that is, $\sum_{j\in\mathcal{N}_+} M_{jm}\, p^*_j = r_m$. Note that executing match m yields a reward of r_m, whereas submitting all participating jobs to the shared pool generates a payoff of at least $w_{\min} \sum_{j\in\mathcal{N}_+} M_{jm} \left(p^*_j - \epsilon\right)$. Suppose instead that $m \in \mathcal{M}_0$. Then,

$$\begin{aligned}
w_{\min} \sum_{j\in\mathcal{N}_+} M_{jm} \left(p^*_j - \epsilon\right) - r_m &= w_{\min} \sum_{j\in\mathcal{N}_+} M_{jm}\, p^*_j - \epsilon\, w_{\min} \sum_{j\in\mathcal{N}_+} M_{jm} - r_m \\
&\ge w_{\min} \left(r_m + \Delta r_{\min}\right) - \epsilon\, M_{\max} - r_m \\
&\ge w_{\min}\, \Delta r_{\min} - \epsilon\, M_{\max} - (1 - w_{\min}) \cdot r_{\max} \\
&> \Delta r_{\min} / 2 - \Delta r_{\min} / 4 - \Delta r_{\min} / 4 \\
&= 0,
\end{aligned}$$

where the first inequality follows from the definitions of $\Delta r_{\min}$ and $M_{\max}$, and the third inequality follows from the facts that $w_{\min} \ge C_8 > \frac{1}{2}$ (with second inequality due to Lemma EC2.2) and $\epsilon \le C_9$, and the definitions of C_8 and C_9. Thus, if $m \in \mathcal{M}_0$, agent i is strictly better off submitting all participating jobs and would never internally execute match m.

Step Two: Comparing Payoffs under the Two Policies Consider a representative marked job of type $j \in \mathcal{N}_+$ and, if the job is matched under policy π, its affected match. We compare the expected payoffs under the two policies by analyzing two scenarios:

1. With probability

$$\frac{\theta_j}{\theta_j + \sum_{\ell\in[J]} \lambda_{i\ell}} \;\ge\; \nu,$$

the marked job expires before a new job arrives under policy π. The inequality follows from Assumption 2.1 and the definition of ν. In this scenario, the job expires unused and generates zero reward under policy π. In contrast, under policy $\tilde{\pi}$, the same job yields a reward of at least $w_{\min} \cdot \big(p_j^* - \epsilon\big)$ through submission.

2. With probability at most $1-\nu$, the job is matched under policy π and generates a reward of r_m, where m represents the corresponding affected match. The policy $\tilde{\pi}$ instead submits all participating jobs of match m and earns at least $w_{\min} \sum_{j \in \mathcal{N}_+} M_{jm} \cdot \big(p_j^* - \epsilon\big)$.

Let m denote the affected match if the marked job is matched under policy π. Note that $m \in \mathcal{M}_+$ by Step One. Let $V^{\pi}(j)$ and $V^{\tilde{\pi}}(j)$ denote the expected payoffs from a type-j marked job and its affected match under policies π and $\tilde{\pi}$, respectively. The payoff difference between the two policies satisfies:

$$\begin{aligned} V^{\tilde{\pi}}(j) - V^{\pi}(j) &\geq \nu \cdot w_{\min} \cdot \big(p_j^* - \epsilon\big) + (1-\nu) \cdot \left(w_{\min} \sum_{j \in \mathcal{N}_+} M_{jm} \cdot \big(p_j^* - \epsilon\big) - r_m \right) \\ &\geq \nu \cdot w_{\min} \cdot \big(p_{\min}^* - \epsilon\big) - \epsilon \cdot M_{\max} - r_{\max} \cdot (1 - w_{\min}) \\ &> \nu p_{\min}^* \,/\, 4 - \nu p_{\min}^* \,/\, 8 - \nu p_{\min}^* \,/\, 8 \\ &= 0, \end{aligned}$$

where the second inequality follows from definitions of $p_{\min}^*$ and $M_{\max}$ and the facts that $m \in \mathcal{M}_+$ and that $r_m \leq r_{\max}$. The third inequality follows from $w_{\min} \geq C_8 > \frac{1}{2}$ (with the second inequality by Lemma EC2.2), $\epsilon \leq C_9 < p_{\min}^* \,/\, 2$ (with the second inequality by Lemma EC2.2), and the definitions of C_8 and C_9. Therefore, policy $\tilde{\pi}$ yields strictly higher rewards than π for marked jobs and affected matches, which implies that policy π is strictly suboptimal.

EC2.4 Proof of Lemma 5.3

EC2.4.1 Setting the Values of $C_{10}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$

We first define the following constants:

$$\begin{aligned} \nu_0 &= \prod_{j \in \mathcal{N}_0} \exp\big(-C_j \lambda_j / \theta_j\big) \\ \rho_1 &= \frac{\nu_0}{4} \cdot \frac{\sum_{j \in \mathcal{N}_+} \lambda_j}{\sum_{j \in \mathcal{N}_0} \lambda_j} \\ \rho_2 &= \frac{\min_{j \in \mathcal{N}_0} \theta_j}{\max_{j \in \mathcal{N}_+} C_j \lambda_j} \end{aligned}$$

where the constants $\{C_j\}$ are specified in Assumption 2.1. Using these parameters, we define $C_{10}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ as follows:

$$\begin{aligned} C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) &= \frac{r_{\max}}{\rho_1 \, \rho_2} \\ C_{10}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) &= \max\left\{ C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}), 1 - \frac{C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})}{C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})} \right\} \end{aligned}$$

where $C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ and $C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ are constants defined in Lemma 5.2. For simplicity, we denote $C_8(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$, $C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$, $C_{10}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$, and $C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$ as C_8, C_9, C_{10}, and C_{11},

respectively. Note that by the definition of C_{10} and C_{11}, we have:

$$C_8 \le C_{10} < 1$$
$$(1 - C_{10}) \cdot C_{11} \le C_9 \,.$$

Therefore, there always exist values of $w_{\min}$ and ϵ satisfying $w_{\min} \ge C_{10}$ and $(1-w_{\min})\, C_{11} \le \epsilon \le C_9$.

EC2.4.2 Proof of Lemma 5.3

We consider a representative agent i in the mean-field problem. Since every agent solves a Markov decision problem in the mean-field problem, it is without loss of optimality to assume that agent i takes actions only upon job arrivals or departures.

Furthermore, since $w_{\min} \ge C_{10} \ge C_8$ and $\epsilon \le C_9$ by assumption, Lemma 5.2 implies that no agent retains over-demanded jobs for internal matching. In other words, all agents either submit or internally match over-demanded jobs immediately upon their arrival. Since each match requires at least two jobs (that is, $\sum_{j\in[J]} M_{jm} \ge 2$ for all $m \in [K]$), it suffices to show that agent i never retains under-demanded jobs in anticipation of future arrivals of over-demanded jobs. This is because the full submission of under-demanded jobs, together with Lemma 5.2, ensures the full submission of over-demanded jobs, and hence of all jobs.

Suppose that agent i implements a policy π under which she performs internal matching at a positive long-run average rate $c > 0$. We will show that policy π is strictly suboptimal. By Lemma 5.2, every internal match performed by agent i is a linking match, involving exactly one over-demanded job and one or more under-demanded jobs. Moreover, each such match is performed immediately upon the arrival of the over-demanded job.

An Alternative Policy $\tilde{\pi}$ To prove that policy π is strictly suboptimal, we consider an alternative policy $\tilde{\pi}$ that submits all jobs immediately upon their arrival but otherwise closely follows policy π. We couple the system dynamics of the two policies so that jobs in the two systems arrive and depart at the same time, even after being converted into credits by submission. This coupling is feasible because under-demanded jobs and their corresponding credits expire at identical rates.

When policy π internally executes a linking matching m involving an arriving over-demanded job and M_{jm} under-demanded jobs of type j for each $j \in \mathcal{N}_0$, policy $\tilde{\pi}$ submits the arriving over-demanded job and pairs it with M_{jm} credits of type j for each $j \in \mathcal{N}_0$. This action is feasible under the coupling described above.

Note that the difference in rewards between policies π and $\tilde{\pi}$ arises from two sources:

- Relative Benefit: For every internal linking match m performed by policy π, policy π obtains a reward of r_m. In contrast, policy $\tilde{\pi}$ submits the over-demanded job and pairs it with the corresponding credits. This yields an expected payoff of at least $w_{\min}\, r_m$, because the over-demanded job is matched in the shared pool with probability at least $w_{\min}$. Therefore, policy π generates higher rewards from internal matches as jobs submitted to the shared pool might expire before being matched.

- Relative Loss: Since policy π executes linking matches immediately upon the arrival of participating over-demanded jobs, some under-demanded jobs must wait for subsequent arrivals of over-demanded jobs and hence will be inevitably wasted. In contrast, these under-demanded jobs are submitted to the shared pool and become lotteries once they expire under policy $\tilde{\pi}$, each generating a positive reward.

In the following, we provide a lower bound on the loss of policy π relative to $\tilde{\pi}$ (Step Two) and an upper bound on the relative benefit (Step Three). Ultimately, we demonstrate that policy $\tilde{\pi}$ achieves a strictly larger overall payoff (Step Four).

Step One: Value of Lotteries As preparation, we first bound the value of a lottery from below. In the mean-field problem, since the shared pool remains in steady state, each lottery receives the same expected value from the collected taxes. Lemma EC2.3 shows that this value is at least $\rho_1\epsilon$ when the number of agents N exceeds a constant depending only on the problem primitives. The proof is provided in Section EC2.4.3.

Lemma EC2.3. *In the mean-field problem, each lottery receives an expected value of at least $\rho_1\epsilon$ from the collected taxes, provided that the number of agents N exceeds a constant depending only on the problem primitives. Here, ρ_1 is defined in Section EC2.4.1.*

Step Two: Cost of Wasted Under-Demanded Jobs under Policy π Recall that agent i internally performs linking matches at rate $c > 0$ under policy π. We apply the approach of Aouad and Sarıtaç (2022) to derive a lower bound on the rate at which under-demanded jobs are wasted while waiting to be matched. Since policy π performs an internal linking match upon the arrival of some over-demanded job (referred to as a passive job), some under-demanded job must actively wait to be matched (referred to as an active job). Following the proof of Lemma 1 in Aouad and Sarıtaç (2022), the rate at which under-demanded jobs are wasted due to waiting for future arrivals of over-demanded jobs to perform linking matches, which we denote by ρ_e, satisfies:

$$\rho_e \geq c \cdot \frac{\min_{j\in\mathcal{N}_0}\theta_j}{\max_{j\in\mathcal{N}_+}\lambda_{ij}} \geq c \cdot \frac{\min_{j\in\mathcal{N}_0}\theta_j}{\max_{j\in\mathcal{N}_+}C_j\lambda_j} = c\cdot\rho_2.$$

Consequently, under-demanded jobs expire and are wasted at a positive rate ρ_e under policy π. In contrast, these under-demanded jobs are submitted and become lotteries under policy $\tilde{\pi}$, which, by Step One, yield the agent a reward of at least

$$(c\cdot\rho_2)\cdot(\rho_1\cdot\epsilon) = c\,\epsilon\cdot\frac{r_{\max}}{C_{11}}$$

per unit of time.

Step Three: Benefit of Internal Matching under Policy π Whenever policy π internally performs a linking match m and obtains a reward $r_m = p_j^*$—where j denotes the type of the over-demanded job in match m—policy $\tilde{\pi}$ instead submits the newly arriving over-demanded type-j job and pairs it with credits corresponding to the under-demanded jobs involved in match m, which yields an expected payoff of at least $w_{\min}\cdot p_j^*$. Therefore, the benefit of internal matching for policy π relative to $\tilde{\pi}$ is at most:

$$c\cdot(1-w_{\min})\cdot\max_{j\in\mathcal{N}_+}p_j^* \leq c\cdot(1-w_{\min})\cdot r_{\max},$$

where the inequality follows from Lemma EC1.1.

Step Four: Suboptimality of Policy π Since

$$c\,\epsilon\cdot\frac{r_{\max}}{C_{11}} - c\cdot(1-w_{\min})\cdot r_{\max} \geq 0,$$

where the inequality follows from $\epsilon \geq (1-w_{\min})\cdot C_{11}$, policy $\tilde{\pi}$ has a strictly higher long-run average payoff than π.

EC2.4.3 Proof of Lemma EC2.3

From Lemma 5.2, agents submit over-demanded jobs in isolation when they do not hold any under-demanded jobs or credits. We consider an alternative system in which each agent:

1. Submits all under-demanded jobs to the shared pool and never uses credits.
2. Submits all over-demanded jobs. However, a submitted over-demanded job is taxed by ϵ upon being matched only if the agent holds no credits of any job type at the time of submission.

It is straightforward to verify that, regardless of the agents' strategies in the original system, at any time t, both the number of credits and the number of lotteries in the alternative system are larger than those in the original system, and the number of the collected ϵ-values in the alternative system is smaller. Therefore, it suffices to show that each lottery in the alternative system yields an expected value of at least $\rho_1\epsilon$, which we do now.

Let $Y(\infty)$ and $Z(\infty)$ denote, respectively, the number of collected ϵ-values and the number of lotteries in the steady state of the shared pool in the alternative system. We first establish in Lemma EC2.4 that both $Y(\infty)$ and $Z(\infty)$ follow Poisson distributions with mean values in the order of $\Theta(\Delta N)$. We provide the proof at the end of this section.

Lemma EC2.4. *In the steady state of the alternative system, the following holds:*

1. *The number of collected ϵ-values, $Y(\infty)$, follows a Poisson distribution with mean value $a\,\Delta N$, where $\frac{\nu_0}{2}\sum_{j\in\mathcal{N}_+}\lambda_j = \underline{a} \leq a \leq \bar{a} = \sum_{j\in\mathcal{N}_+}\lambda_j$.*
2. *The number of lotteries, $Z(\infty)$, follows a Poisson distribution with mean value $b\,\Delta N$, where $b = \sum_{j\in\mathcal{N}_0}\lambda_j$.*

In the mean-field problem, the shared pool remains in steady state at any matching epoch. Moreover, the taxes collected in a matching epoch are fully allocated to lotteries in that epoch whenever a lottery exists. Therefore, the expected number of ϵ-values each lottery receives is given by the following expression

$$
\begin{aligned}
&\mathbb{E}\left(\frac{Y(\infty)}{Z(\infty)}\mathbb{1}[Z(\infty)\geq 1]\right)\\
\geq&\,\mathbb{E}\left(\frac{Y(\infty)}{\mathbb{E}[Z(\infty)]}\mathbb{1}[Z(\infty)\geq 1] - \frac{Y(\infty)}{\mathbb{E}[Z(\infty)]^2}\left(Z(\infty)-\mathbb{E}[Z(\infty)]\right)\mathbb{1}[Z(\infty)\geq 1]\right)\\
=&\,\frac{\mathbb{E}[Y(\infty)]}{\mathbb{E}[Z(\infty)]} - \mathbb{E}\left(\frac{Y(\infty)}{\mathbb{E}[Z(\infty)]}\mathbb{1}[Z(\infty)=0]\right) - \mathbb{E}\left(\frac{Y(\infty)}{\mathbb{E}[Z(\infty)]^2}\left(Z(\infty)-\mathbb{E}[Z(\infty)]\right)\right)\\
&\qquad + \mathbb{E}\left(\frac{Y(\infty)}{\mathbb{E}[Z(\infty)]^2}\left(Z(\infty)-\mathbb{E}[Z(\infty)]\right)\mathbb{1}[Z(\infty)=0]\right)\\
=&\,\frac{\mathbb{E}[Y(\infty)]}{\mathbb{E}[Z(\infty)]} - \frac{\mathbb{C}\mathrm{ov}[Y(\infty),Z(\infty)]}{\mathbb{E}[Z(\infty)]^2} - 2\,\frac{\mathbb{E}\big[Y(\infty)\,\mathbb{1}[Z(\infty)=0]\big]}{\mathbb{E}[Z(\infty)]}\\
\geq&\,\frac{\underline{a}}{b} - \underbrace{\frac{\mathbb{C}\mathrm{ov}[Y(\infty),Z(\infty)]}{\mathbb{E}[Z(\infty)]^2}}_{(a)} - 2\,\underbrace{\frac{\mathbb{E}\big[Y(\infty)\,\mathbb{1}[Z(\infty)=0]\big]}{\mathbb{E}[Z(\infty)]}}_{(b)},
\end{aligned}
$$

where the first inequality follows from Lemma EC2.5, the second equality follows from the fact that $Z(\infty)\mathbb{1}[Z(\infty)=0]=0$, and the second inequality from Lemma EC2.4.

We next bound the terms (a) and (b) from above using Cauchy–Schwarz inequality. For (a), we have

$$\frac{\mathbb{C}\mathrm{ov}[Y(\infty),Z(\infty)]}{\mathbb{E}[Z(\infty)]^2} \le \frac{\sqrt{\mathbb{V}\mathrm{ar}(Y(\infty))\mathbb{V}\mathrm{ar}(Z(\infty))}}{\mathbb{E}[Z(\infty)]^2} \le \sqrt{\frac{\bar{a}}{b^3}}\cdot\frac{1}{\Delta N},$$

where the second inequality follows from Lemma EC2.4. For (b), we have

$$\frac{\mathbb{E}\big[Y(\infty)\,\mathbb{1}[Z(\infty)=0]\big]}{\mathbb{E}[Z(\infty)]} \le \frac{\sqrt{\mathbb{E}\left[Y(\infty)^2\right]\,\mathbb{P}\left[Z(\infty)=0\right]}}{\mathbb{E}[Z(\infty)]} \le \frac{\sqrt{2\bar{a}}}{b}\cdot\frac{e^{-\frac{b}{2}\Delta N}}{\sqrt{\Delta N}},$$

where the second inequality also follows from Lemma EC2.4.

Therefore, each lottery yields an expected value of at least

$$\begin{aligned}
&\mathbb{E}\left(\frac{Y(\infty)}{Z(\infty)}\mathbb{1}[Z(\infty)\ge 1]\right)\cdot\epsilon\\
&\ge\left(\frac{\underline{a}}{\bar{b}}-\sqrt{\frac{\bar{a}}{b^3}}\cdot\frac{1}{\Delta N}-\frac{2\sqrt{2\bar{a}}}{b}\cdot\frac{e^{-\frac{b}{2}\Delta N}}{\sqrt{\Delta N}}\right)\cdot\epsilon\\
&\ge\frac{\underline{a}}{2b}\cdot\epsilon\\
&=\rho_1\cdot\epsilon,
\end{aligned}$$

where the second inequality holds when N exceeds a certain constant N_0, which depends only on the problem primitives, as ΔN increases with N by the setup in Lemmas 3.2 and 3.3.

Lemma EC2.5. *Let $Z>0$ be a positive random variable. Then, almost surely,*

$$\frac{1}{Z}\ge\frac{1}{\mathbb{E}[Z]}-\frac{Z-\mathbb{E}[Z]}{\mathbb{E}[Z]^2}.$$

Proof of Lemma EC2.5. Let $f(x)=\frac{1}{x}$. Since $f(x)$ is convex for $x>0$ with derivative $f'(x)=-\frac{1}{x^2}$, it follows that

$$f(x)\ge f(a)+f'(a)\,(x-a)\qquad\text{for all } x,a>0.$$

By setting $x=Z$ and $a=\mathbb{E}[Z]$, we obtain the desired result. □

Proof of Lemma EC2.4 Bullet 1. We first show that the number of collected ϵ-values in a steady state, $Y(\infty)$, follows a Poisson distribution. To do so, let $Y(k)$ denote the number of ϵ-values collected at the matching epoch $t=k\Delta$. We show that $Y(k)$ is Poisson distributed for each k.

Specifically, consider an over-demanded job that arrives at time $s\le k\Delta$. Let $p(s)$ denote the probability that this job contributes an ϵ to the collected taxes at epoch $k\Delta$. In the alternative system, all agents submit jobs fully. Define

$$\rho_{ij}=\frac{\lambda_{ij}}{\sum_{j\in\mathcal{N}_+}N\lambda_j}$$

as the probability that an over-demanded job is of type $j \in \mathcal{N}_+$ and arrives at agent $i \in [N]$. Then,

$$p(s) = \sum_{i \in [N]} \sum_{j \in \mathcal{N}_+} \rho_{ij} \, \mathbb{P}\Big(A(s) \cap B(s) \cap C(s)\Big),$$

where

$$\begin{aligned} A(s) &:= \big\{\text{agent } i \text{ holds no credits at time } s\big\}, \\ B(s) &:= \big\{\text{the job has not expired by } k\Delta\big\}, \\ C(s) &:= \big\{\text{the job is matched at epoch } k\Delta, \text{but not at earlier epochs}\big\}. \end{aligned}$$

Since over-demanded jobs arrive according to a Poisson process with rate $N \sum_{j \in \mathcal{N}_+} \lambda_j$, from Proposition 2.3.2 of Ross (1995), $Y(k)$ follows a Poisson distribution with mean value

$$\mathbb{E}[Y(k)] = \left(N \sum_{j \in \mathcal{N}_+} \lambda_j \right) \cdot \int_0^{k\Delta} p(s) \, ds.$$

Finally, since $Y(k)$ converges in distribution to $Y(\infty)$, $Y(\infty)$ also follows a Poisson distribution.

We next derive upper and lower bounds on the mean value of $Y(\infty)$. For an upper bound, note that for any $k \in \mathbb{N}$, the total number of ϵ-values collected in the first k matching epochs is at most

$$k \, \Delta \sum_{j \in \mathcal{N}_+} N \lambda_j$$

in expectation, because over-demanded jobs join the shared pool at rate $\sum_{j \in \mathcal{N}_+} N\lambda_j$, and each contributes at most ϵ to the collection. This implies that

$$\mathbb{E}[Y(\infty)] \leq \Delta N \sum_{j \in \mathcal{N}_+} \lambda_j \, .$$

We next derive a lower bound. Since all under-demanded jobs are submitted and remain unused until expiration, the number of credits of type $j \in \mathcal{N}_0$ held by agent i follows an $M/M/\infty$ queue process with arrival rate λ_{ij} and service rate θ_j. The stationary distribution is Poisson with mean λ_{ij}/θ_j. Therefore, when an over-demanded job arrives at agent i, the probability that agent i holds no credits of any job type is

$$\prod_{j \in \mathcal{N}_0} \exp\left(-\frac{\lambda_{ij}}{\theta_j}\right) \geq \prod_{j \in \mathcal{N}_0} \exp\left(-\frac{C_j \, \lambda_j}{\theta_j}\right) = \nu_0.$$

Each such job is matched in the shared pool with probability at least $w_{\min}$, and once matched, contributes a value of ϵ to the tax collection. Consequently, taxes are collected at a rate of at least

$$\nu_0 \cdot w_{\min} \sum_{j \in \mathcal{N}_+} N\lambda_j \geq \frac{\nu_0}{2} \sum_{j \in \mathcal{N}_+} N\lambda_j \, ,$$

where the inequality follows from $w_{\min} \geq C_{10} \geq C_8 \geq 1/2$ (the last inequality is due to Lemma EC2.2). This implies that

$$\mathbb{E}[Y(\infty)] \geq \Delta N \cdot \frac{\nu_0}{2} \sum_{j \in \mathcal{N}_+} \lambda_j \, . \qquad \square$$

Proof of Lemma EC2.4 Bullet 2. Let $Z(k)$ denote the number of lotteries at matching epoch $t = k\Delta$. We first derive its distribution. To do so, consider a type-j credit generated s units of time before epoch $k\Delta$. Let T denote its sojourn time, which follows an exponential distribution with rate parameter θ_j. We consider two cases:

1. If $s < \Delta$, for the credit to convert into a lottery that participates in the matching epoch $k\Delta$, the credit must expire before that epoch, which occurs with probability
$$\mathbb{P}[T \le s] = 1 - \exp(-\theta_j s) .$$
2. If $s \ge \Delta$, for the credit to convert into a lottery that participates in the matching epoch $k\Delta$, the credit must expire after epoch $(k-1)\Delta$ and before epoch $k\Delta$, which occurs with probability
$$\mathbb{P}[s - \Delta < T \le s] = \exp(-\theta_j(s-\Delta)) - \exp(-\theta_j s) .$$

Since agents submit all under-demanded jobs to the shared pool and never use credits, type-j credits are created according to a Poisson process with rate $N\lambda_j$. Let $Z_j(k)$ denote the number of lotteries at epoch $k\Delta$ that arise from expired type-j credits. By Proposition 2.3.2 of Ross (1995), $Z_j(k)$ follows a Poisson distribution with mean value

$$\begin{aligned} E[Z_j(k)] &= N\lambda_j \cdot \left\{ \int_0^{\Delta} \Big(1 - \exp(-\theta_j s)\Big) ds + \int_{\Delta}^{k\Delta} \Big(\exp(-\theta_j(s-\Delta)) - \exp(-\theta_j s)\Big) ds \right\} \\ &= N\lambda_j \left\{ \Delta - e^{-k\theta_j\Delta} \cdot \frac{e^{\theta_j\Delta} - 1}{\theta_j} \right\}. \end{aligned}$$

Note that for a fixed k, the lottery inventories $\{Z_j(k)\}_{j\in\mathcal{N}_0}$ are independent because they originate from jobs of different types that arrive independently. Since $Z(k) = \sum_{j\in\mathcal{N}_0} Z_j(k)$, it follows a Poisson distribution with mean value

$$\mathbb{E}\left[Z(k)\right] = \sum_{j\in\mathcal{N}_0} N\lambda_j \left\{ \Delta - e^{-k\theta_j\Delta} \cdot \frac{e^{\theta_j\Delta} - 1}{\theta_j} \right\} .$$

Letting k increase to infinity yields the desired result. □

EC2.5 Proof of Theorem 5.4

According to Corollary 3.5, for any over-demanded job type $j \in \mathcal{N}_+$, we have:

$$1 - w_j = \sum_{m\in[K]} \left(\frac{M_{jm} x_m^*}{\lambda_j} - q_{jm} \right) \le K \cdot C_6 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N} . \tag{EC-19}$$

The equality follows from the fact that $\sum_{m\in[K]} M_{jm} x_m^* = \lambda_j$ for an over-demanded job type j (by complementary slackness between (1) and (2)), and that $w_j = \sum_m q_{jm}$, where the probabilities q_{jm} are defined in Corollary 3.5. The inequality follows from Corollary 3.5.

From (EC-19), for any number of agents N, we have:

$$1 - w_{\min} = \max_{j\in\mathcal{N}_+} \left\{1 - w_j\right\} \le K \cdot C_6 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}.$$

Hence, there exists an integer $N_0 \in \mathbb{N}$ such that, for all $N \geq N_0$:

1. $w_{\min} \geq C_{10}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$,
2. $(1 - w_{\min})\, C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) \leq \epsilon = K \cdot C_6 \cdot C_{11}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M}) \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N} \leq C_9(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$.
3. N is greater than the constant specified in Lemma 5.3.

By Lemma 5.3, full job submission by all agents constitutes an OE under the `MVC` mechanism with parameter ϵ.

EC2.6 Proof of Proposition 5.5

Step One Suppose that all agents fully submit their jobs to the shared pool. Since every over-demanded job of type j yields a reward of at least $(p_j^* - \epsilon)$ if matched, by the same proof of Lemma 4.4, the long-run average payoff of an agent i from full submission is at least:

$$\boldsymbol{\lambda}_i^{\mathrm{T}} \mathbf{p}^* - \epsilon \cdot \sum_{j \in \mathcal{N}_+} \lambda_{ij} - C_7 \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N} \geq \boldsymbol{\lambda}_i^{\mathrm{T}} \mathbf{p}^* - \left(K \cdot C_6 \cdot C_{11} \cdot \sum_{j \in \mathcal{N}_+} C_j \lambda_j + C_7 \right) \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}$$

where C_7 is a positive constant specified in Lemma 4.4, and the term $\frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}$ diminishes to zero at a rate characterized by Lemmas 3.2 and 3.3.

Step Two Suppose that agents two to N fully submit their jobs to the shared pool and, whenever jobs are over-demanded, pair them with credits in an arbitrary way. We first show in Lemma EC2.6 that, the long-run average reward agent one can obtain from tax collection is at most a constant multiple of ϵ regardless of the strategy she uses.

Lemma EC2.6. *Suppose that:*

1. *Agents two to N submit all their jobs to the shared pool and, whenever jobs are over-demanded, pair them with credits in an arbitrary way;*
2. *The number of agents N is larger than some constant to ensure $\Delta \sum_{j \in \mathcal{N}_+} C_j \lambda_j \leq 1$, where Δ is the matching interval length used by the periodic matching policy π^{P} in the shared pool, as specified in Lemmas 3.2 and 3.3.*

Then, the long-run average reward agent one can obtain from tax collection is at most $c\,\epsilon$, regardless of the strategy agent one uses, where c is a positive constant depending only on the problem primitives $(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$.

Recall that the shared pool performs matching at every time epoch $t = k\Delta$, where $k \in \mathbb{N}_+$. In the proof of Lemma EC2.6, we show that the number of lotteries associated with agent one at time $t = k\Delta$ is only a fraction $\Theta\left(\frac{1}{N}\right)$ of the total number of lotteries at that time when k is large. This ensures that agent one obtains only a fraction $\Theta\left(\frac{1}{N}\right)$ of the collected taxes and thus leads to Lemma EC2.6. We provide further details in Section EC2.6.1.

Since submitting an over-demanded job of type j yields a reward of at most p_j^*, by Lemma EC2.6, the long-run average payoff of agent one is at most the optimal value of the following optimization problem:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}_+^K,\, \mathbf{s} \in \mathbb{R}_+^J} \quad & \mathbf{r}^{\mathrm{T}} \mathbf{x} + \mathbf{p}^{*\mathrm{T}} \mathbf{s} + c\,\epsilon \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} + \mathbf{s} \leq \boldsymbol{\lambda}_i \end{aligned}$$

whose optimal value is

$$\boldsymbol{\lambda}_i^{\mathrm{T}}\mathbf{p}^* + c\,\epsilon = \boldsymbol{\lambda}_i^{\mathrm{T}}\mathbf{p}^* + c \cdot K \cdot C_6 \cdot C_{11} \cdot \frac{NV^{\mathrm{F}} - V^{\mathrm{P}}}{N}\,.$$

Wrap-Up From the above two steps, the benefit of unilaterally deviating from full submission is only a negligible term of order $O\left(\frac{NV^{\mathrm{F}}-V^{\mathrm{P}}}{N}\right)$. Consequently, full job submission by all agents constitutes an approximate Nash equilibrium under the `MVC` mechanism, regardless of how agents pair credits with over-demanded jobs.

EC2.6.1 Proof of Lemma EC2.6

Without loss of generality, assume that at time zero, neither the shared pool nor any agent holds jobs or credits. In addition, assume that agents two to N submit every job to the shared pool.

Recall that the shared pool performs matching at every epoch $t = k\Delta$, with $k \in \mathbb{N}_+$. Fix such an epoch $t = k\Delta$. In the following, we first bound the number of lotteries held by agent one at that epoch from above (Step One), and the number of lotteries held by agents two to N at that epoch from below (Step Two). We then show that the number of lotteries associated with agent one at time $t = k\Delta$ is only a fraction $\Theta\left(\frac{1}{N}\right)$ of the total lotteries at that time when k is large. Consequently, agent one obtains only a fraction $\Theta\left(\frac{1}{N}\right)$ of the collected taxes, thereby establishing the desired result.

Step One: Bounding Agent One's Lotteries from Above For any integer $k \in \mathbb{N}_+$, let $Z_1(k)$ denote the number of lotteries held by agent one that participate in the matching epoch $k\Delta$. Since, for any under-demanded job type $j \in \mathcal{N}_0$, the sojourn time of type-j jobs and their associated credits follow the same exponential distribution, $Z_1(k)$ attains its maximum (in the sample path sense) when agent one submits all under-demanded jobs to the shared pool and never pairs the resulting credits with over-demanded jobs. In this case, Lemma EC2.7 shows that $Z_1(k)$ follows a Poisson distribution with mean value

$$\mathbb{E}[Z_1(k)] \leq \sum_{j\in\mathcal{N}_0} C_j\lambda_j\Delta.$$

Lemma EC2.7. *Suppose agent one submits all under-demanded jobs to the shared pool and never pairs credits with over-demanded jobs. This strategy maximizes the number of lotteries $Z_1(k)$ agent one holds at any matching epoch $k\Delta$. Furthermore, under this strategy, $Z_1(k)$ follows a Poisson distribution with mean value*

$$\mathbb{E}\left[Z_1(k)\right] = \sum_{j\in\mathcal{N}_0} \lambda_{1j}\left\{\Delta - e^{-k\theta_j\Delta}\cdot\frac{e^{\theta_j\Delta}-1}{\theta_j}\right\} \leq \sum_{j\in\mathcal{N}_0} C_j\lambda_j\Delta\,.$$

Proof of Lemma EC2.7. Consider a type-j credit generated by agent one s units of time prior to the matching epoch $k\Delta$. Let T denote its sojourn time, which follows an exponential distribution with rate parameter θ_j. We consider two cases:

1. If $s < \Delta$, for the credit to convert into a lottery that participates in the matching epoch $k\Delta$, the credit must expire before that epoch, which occurs with probability

$$\mathbb{P}[T \leq s] = 1 - \exp\left(-\theta_j s\right)\,.$$

2. If $s \geq \Delta$, for the credit to convert into a lottery that participates in the matching epoch $k\Delta$, the credit must expire after epoch $(k-1)\Delta$ and before epoch $k\Delta$, which occurs with probability

$$\mathbb{P}[s - \Delta < T \leq s] = \exp(-\theta_j(s-\Delta)) - \exp(-\theta_j s)\,.$$

Since agent one submits all under-demanded jobs to the shared pool and never uses credits, type-j credits are created following a Poisson process with rate λ_{1j}. Let $Z_{1j}(k)$ denote the number of lotteries at epoch $k\Delta$ that arise from expired type-j credits of agent one. By Proposition 2.3.2 of Ross (1995), $Z_{1j}(k)$ follows a Poisson distribution with mean value

$$\begin{aligned} E[Z_{1j}(k)] &= \lambda_{1j} \cdot \left\{ \int_0^{\Delta} \Big(1 - \exp(-\theta_j s)\Big) ds + \int_{\Delta}^{k\Delta} \Big(\exp(-\theta_j(s-\Delta)) - \exp(-\theta_j s)\Big) ds \right\} \\ &= \lambda_{1j} \left\{ \Delta - e^{-k\theta_j \Delta} \cdot \frac{e^{\theta_j \Delta} - 1}{\theta_j} \right\}. \end{aligned}$$

Note that $Z_1(k) = \sum_{j \in \mathcal{N}_0} Z_{1j}(k)$. Moreover, these lottery inventories $\{Z_{1j}(k)\}_{j \in \mathcal{N}_0}$ are independent because they originate from jobs of different types that arrive independently. Therefore, $Z_1(k)$ also follows a Poisson distribution with mean value

$$\mathbb{E}\left[Z_1(k)\right] = \sum_{j \in \mathcal{N}_0} \lambda_{1j} \left\{ \Delta - e^{-k\theta_j \Delta} \cdot \frac{e^{\theta_j \Delta} - 1}{\theta_j} \right\} \leq \sum_{j \in \mathcal{N}_0} C_j \lambda_j \Delta\,. \qquad \square$$

Step Two: Bounding Agents Two to N's Lotteries from Below For any integer $k \in \mathbb{N}_+$, let $Z_{-1}(k)$ denote the number of lotteries held by agents two to N that enter the matching epoch $k\Delta$. Lemma EC2.8 shows that when k is large, $Z_{-1}(k)$ is sample-path-wise no smaller than a Poisson random variable $\tilde{Z}_{-1}(k)$ with mean value $\mathbb{E}[\tilde{Z}_{-1}(k)] = \Delta \cdot \Theta(N)$.

Lemma EC2.8. *There exists a constant $c_1(N)$, which depends on the number of agents N and problem primitives $(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$, such that for any $k \geq c_1(N)$, regardless of how agents two to N pair credits with over-demanded jobs, the number of lotteries $Z_{-1}(k)$ held by agents two to N is, sample-path-wise, no smaller than a Poisson random variable $\tilde{Z}_{-1}(k)$ with mean value*

$$\mathbb{E}[\tilde{Z}_{-1}(k)] \geq c_2 \cdot \Delta\, N,$$

where c_2 is a positive constant depending only on the problem primitives $(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$.

Proof of Lemma EC2.8. Define

$$\phi_i \triangleq \sum_{j \in \mathcal{N}_+} \lambda_{ij} \leq \sum_{j \in \mathcal{N}_+} C_j \lambda_j$$

as the aggregate arrival rate of over-demanded jobs at agent i. Consider a type-j credit generated by agent i s units of time before the matching epoch $k\Delta$; let T denote its sojourn time, which follows an exponential distribution with rate parameter θ_j. We consider two cases:

1. If $s < \Delta$, for the credit to convert into a lottery that enters the matching epoch $k\Delta$, it must expire before that epoch and cannot be paired with an over-demanded job prior to expiration. A sufficient condition is that the credit expires before the epoch $k\Delta$ and no over-demanded jobs arrive at agent i before its expiration, which occurs with probability

$$\exp(-\phi_i s) \cdot \mathbb{P}[T \leq s] = \exp(-\phi_i s) \cdot \left(1 - \exp\left(-\theta_j s\right)\right)\,.$$

2. If $s \geq \Delta$, for the credit to convert into a lottery that enters the matching epoch $k\Delta$, it must expire after epoch $(k-1)\Delta$ and before epoch $k\Delta$, and it must not be paired with an over-demanded job before expiration. A sufficient condition is that the credit expires after epoch $(k-1)\Delta$ and before epoch $k\Delta$, and that no over-demanded jobs arrive at agent i before its expiration. This occurs with probability

$$\exp(-\phi_i s) \cdot \mathbb{P}[s-\Delta < T \leq s] = \exp(-\phi_i s) \cdot (\exp(-\theta_j(s-\Delta)) - \exp(-\theta_j s)) \ .$$

We consider an alternative system in which, for any agent $i \in [2\!:\!N]$, each of her credits perishes without converting into a lottery if an over-demanded job arrives before the credit's expiration. Let $\tilde{Z}_{-1}(k)$ denote the number of lotteries held by agents two to N in this alternative system that participate in epoch $k\Delta$. Clearly, $\tilde{Z}_{-1}(k)$ is no larger than $Z_{-1}(k)$, regardless of how agents two to N use credits. We now show that $\tilde{Z}_{-1}(k)$ satisfies the property stated in Lemma EC2.8.

Recall that agents two through N submit all jobs upon arrival. For a credit generated by these agents, define

$$\rho_{ij} = \frac{\lambda_{ij}}{\sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\lambda_{ij}}$$

as the probability that it is of type $j \in \mathcal{N}_0$ and generated by agent $i \in [2\!:\!N]$. By Proposition 2.3.2 of Ross (1995), $\tilde{Z}_{-1}(k)$ follows a Poisson distribution with mean value

$$\begin{aligned}
E\left[\tilde{Z}_{-1}(k)\right] =& \left(\sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\lambda_{ij}\right)\cdot\left\{\int_0^{\Delta}\sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\rho_{ij}\cdot\exp(-\phi_i s)\left(1-\exp(-\theta_j s)\right)ds\right. \\
&\left.+\int_{\Delta}^{k\Delta}\sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\rho_{ij}\cdot\exp(-\phi_i s)\left(\exp(-\theta_j(s-\Delta))-\exp(-\theta_j s)\right)ds\right\} \\
=& \sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\lambda_{ij}\left\{\int_0^{\Delta}\exp(-\phi_i s)\left(1-\exp(-\theta_j s)\right)ds\right. \\
&\left.+\int_{\Delta}^{k\Delta}\exp(-\phi_i s)\left(\exp(-\theta_j(s-\Delta))-\exp(-\theta_j s)\right)ds\right\} \\
=& \sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\lambda_{ij}\cdot\frac{\theta_j - e^{-\Delta\phi_i}\theta_j - e^{-k\Delta(\theta_j+\phi_i)}\left(-1+e^{\Delta\theta_j}\right)\phi_i}{\phi_i(\theta_j+\phi_i)} \\
\geq& \sum_{j\in\mathcal{N}_0}\sum_{i=2}^{N}\lambda_{ij}\cdot\frac{\frac{\Delta\theta_j}{2} - e^{-k\Delta(\theta_j+\phi_i)}\left(-1+e^{\Delta\theta_j}\right)}{\theta_j+\phi_i}
\end{aligned}$$

where the inequality follows from the facts that $1-e^{-x} \geq x/2$ for all $x \in [0,1]$ and that $\Delta\phi_i \leq \Delta\sum_{j\in\mathcal{N}_+} C_j\lambda_j \leq 1$ (with second inequality due to the assumption in Lemma EC2.6).

There obviously exists a constant $c_1(N)$, depending on the number of agents N, such that for any $k \geq c_1(N)$,

$$e^{-k\Delta\theta_j}\left(-1+e^{\Delta\theta_j}\right) \leq \frac{\Delta\theta_j}{4},\ \forall\, j \in \mathcal{N}_0\,.$$

Therefore, when $k \geq c_1(N)$, we have

$$\begin{aligned} E\left[\tilde{Z}_{-1}(k)\right] &\geq \Delta \sum_{j\in\mathcal{N}_0} \sum_{i=2}^{N} \frac{\lambda_{ij}}{4} \cdot \frac{\theta_j}{\theta_j + \phi_i} \\ &\geq \frac{\Delta}{4} \sum_{j\in\mathcal{N}_0} \frac{\theta_j}{\theta_j + \sum_{j\in\mathcal{N}_+} C_j\lambda_j} \cdot (N - C_j)\,\lambda_j \\ &\geq c_2\,\Delta\,N \end{aligned}$$

for some positive constant c_2 depending only on the problem primitives $(\boldsymbol{\lambda}, \boldsymbol{\theta}, \mathbf{r}, \mathbf{M})$. The second inequality follows from the facts that $\sum_{i\in[N]} \lambda_{ij} = N\lambda_j$, $\lambda_{1j} \leq C_j\lambda_j$, and $\phi_i \leq \sum_{j\in\mathcal{N}_+} C_j\lambda_j$ for all $i \in [N]$. □

Step Three: Bounding Agent One's Payoff from Collected Taxes Since the upper bound $Z_1(k)$ in Lemma EC2.7 depends only on the job arrivals at agent one, and the lower bound $\tilde{Z}_{-1}(k)$ in Lemma EC2.8 depends only on the job arrivals at agents two to N, $Z_1(k)$ and $\tilde{Z}_{-1}(k)$ are independent. Therefore, for $k \geq c_1(N)$, the fraction of rewards that agent one obtains from collected taxes is at most

$$\begin{aligned} \mathbb{E}\left[\frac{Z_1(k)}{Z_1(k) + \tilde{Z}_{-1}(k)} \cdot \mathbb{1}[Z_1(k) \geq 1]\right] &\leq \mathbb{E}\left[\frac{Z_1(k)}{1 + \tilde{Z}_{-1}(k)} \cdot \mathbb{1}[Z_1(k) \geq 1]\right] \\ &= \mathbb{E}\left[Z_1(k) \cdot \mathbb{1}[Z_1(k) \geq 1]\right] \cdot \mathbb{E}\left[\frac{1}{1 + \tilde{Z}_{-1}(k)}\right] \\ &\leq E[Z_1(k)] \cdot \frac{1}{E[\tilde{Z}_{-1}(k)]} \\ &\leq \frac{c_3}{N}, \end{aligned}$$

where the equality follows from the independence of $Z_1(k)$ and $\tilde{Z}_{-1}(k)$, the second inequality from Lemma EC2.9, and the third inequality from Lemmas EC2.7 and EC2.8, where $c_3 \triangleq \sum_{j\in\mathcal{N}_0} C_j\lambda_j/c_2$ is a constant that depends only on the problem primitives.

We note that, for any $k \in \mathbb{N}_+$, the total taxes collected in the first k matching epochs are at most

$$\epsilon \cdot k\Delta \sum_{j\in\mathcal{N}_+} N\lambda_j$$

in expectation, because over-demanded jobs join the shared pool at a rate no larger than $\sum_{j\in\mathcal{N}_+} N\lambda_j$, and each over-demanded job contributes at most ϵ to the collection. Therefore, the long-run average payoff that agent one receives from tax collection is bounded above by

$$\lim_{k\to\infty} \frac{1}{k\Delta}\,\epsilon \left(c_1(N)\,\Delta \sum_{j\in\mathcal{N}_+} N\lambda_j + \frac{c_3}{N} \cdot k\Delta \sum_{j\in\mathcal{N}_+} N\lambda_j \right) = \epsilon\, c_3 \sum_{j\in\mathcal{N}_+} \lambda_j\,,$$

because agent one receives at most all the taxes in the first $c_1(N)$ matching epochs and a fraction $\frac{c_3}{N}$ of the collected taxes in the remaining epochs. We thus complete the proof by letting $c \triangleq c_3 \sum_{j\in\mathcal{N}_+} \lambda_j$.

Lemma EC2.9. *Let $X \sim Poisson(\lambda)$ be a Poisson random variable with mean value $\lambda > 0$. Then*

$$\mathbb{E}\left[\frac{1}{1+X}\right] = \frac{1-e^{-\lambda}}{\lambda} < \frac{1}{\lambda}.$$

Proof. Note that

$$\mathbb{E}\left[\frac{1}{1+X}\right] = \sum_{k=0}^{\infty} \frac{1}{k+1}\frac{\lambda^k e^{-\lambda}}{k!} = \frac{e^{-\lambda}}{\lambda}\sum_{k=0}^{\infty}\frac{\lambda^{k+1}}{(k+1)!} = \frac{e^{-\lambda}}{\lambda}\left(e^{\lambda}-1\right) = \frac{1-e^{-\lambda}}{\lambda}. \qquad \square$$

EC2.7 Both Pathways are Essential to Incentivize Full Submission of Under-Demanded Jobs

In this section, we show that both pathways for deriving value from submitting under-demanded jobs—namely, tax exemption and lottery entry—are essential to sustain full job submission as an OE. To illustrate this, we use Example 5.1 to demonstrate that relying on only one pathway is insufficient to guarantee full submission as an OE.

EC2.7.1 Tax Exemption Alone Does Not Guarantee Full Submission as an OE

Consider a modification of the `MVC` mechanism that incentivizes the submission of under-demanded jobs only through tax exemption for over-demanded jobs. We show that, under this mechanism, withholding all type-two jobs strictly benefits an agent solving the mean-field problem in Example 5.1, regardless of the tax parameter $\epsilon > 0$ or the steady-state matching probability $w \in (0,1)$ of type-one jobs in the shared pool. Therefore, full job submission by all agents cannot constitute an OE.

To illustrate this, let π^{full} denote the policy that submits all jobs while optimally pairing submitted (over-demanded) type-one jobs with type-two credits. To establish that policy π^{full} is suboptimal, we consider an alternative policy $\tilde{\pi}$, which withholds all type-two jobs while mimicking the actions of policy π^{full} for type-one jobs in the following manner: when a type-one job arrives,

1. If policy π^{full} pairs the job with a type-two credit upon submission, then policy $\tilde{\pi}$ internally matches it with a type-two job;
2. If policy π^{full} submits the job in isolation (i.e., without pairing it with a credit), then policy $\tilde{\pi}$ also submits it in isolation.

We couple the system dynamics under the two policies so that jobs in both systems arrive and depart at the same time, even when type-two jobs are converted into credits upon submission.[EC1] Consequently, policy $\tilde{\pi}$ can mimic the actions of policy π^{full} as described.

For type-one jobs submitted in isolation under both policies, the resulting rewards are identical. Thus, the payoff difference between the two policies arises only from type-one jobs that policy π^{full} pairs with a credit upon submission. For each such job, policy π^{full} yields an expected payoff of $w < 1$. In contrast, since policy $\tilde{\pi}$ internally matches the job, it obtains a higher payoff of 1. Therefore, policy $\tilde{\pi}$ strictly outperforms π^{full} for every type-one job that π^{full} submits with a pair. This implies that full submission is strictly suboptimal, and consequently, full job submission by all agents cannot sustain an OE when only tax exemption is applied.

[EC1] Recall that type-two jobs and their credits expire according to exponential clocks with identical rates.

EC2.7.2 Lottery Entry Alone Does Not Guarantee Full Submission as an OE

We now consider a modification of the MVC mechanism that uses only lottery entry to incentivize submission of under-demanded jobs. We show that in the corresponding mean-field problem of Example 5.1, full job submission is strictly suboptimal for any tax parameter $\epsilon > 0$ and any steady-state matching probability $w \in (0, 1)$ of type-one jobs in the shared pool. Thus, full job submission by all agents cannot constitute an OE under the mechanism.

Note that without tax exemption, each submitted type-one job yields an expected payoff of $1-\epsilon$ when matched. Meanwhile, submitted type-two jobs receive value from the redistribution of taxes collected from type-one jobs. Suppose all agents fully submit their jobs to the shared pool. Each type-two job receives an expected payoff of at most $w\epsilon/\lambda$ upon submission.[EC2] Since a type-two job is matched in the shared pool with probability w/λ,[EC3] the reward structure is equivalent if a type-two job instead receives an expected payoff of at most ϵ upon being matched.

Consequently, the payoffs from submitting type-one and type-two jobs under the mechanism are analogous to those under the MV(ϵ) mechanism defined in Definition 5.3. As shown in Section 5.2.1, when $\lambda > 1 + \theta$, an alternative policy that retains one type-two job for local matching strictly outperforms full job submission. Therefore, full job submission by all agents cannot constitute an OE when lottery entry is the sole incentive for submitting type-two jobs.

EC3 Extension to Heterogeneous Matching Rewards

In this section, we consider a more general setting in which agents have heterogeneous matching rewards. Specifically, we model the dynamics of each agent as in Section 2. However, agent i generates a reward $r_{im} > 0$ from performing a type-m match, where these rewards $\{r_{im}\}$ may vary across agents. We let $\mathbf{r}_i = (r_{im})_{m\in[K]} \in \mathbb{R}_+^K$ denote the vector of matching rewards for agent i.

In the following, we first examine the centralized setting in Section EC3.1. We then show in Section EC3.2 that, if monetary transfers are allowed, the monetary mechanism proposed in Section 4.1 continues to make full submission a dominant strategy, thereby achieving the first-best performance, and remains approximately budget-balanced in large markets. Finally, in Section EC3.3, we show that no non-monetary mechanism can achieve the first-best performance when matching rewards are heterogeneous.

EC3.1 The Centralized Setting

We first consider a centralized setting in which a central planner has full control over all agents and aims to maximize the expected long-run average total matching value of the system. For each match type $m \in [K]$, let $r_m \triangleq \max_{i\in[N]} r_{im}$ denote the highest value that an agent can generate from a type-m match, and $S(m) \triangleq \{i \in [N] : r_{im} = r_m\}$ the set of agents achieving this maximum value.

When the central planner schedules a type-m match, she would assign it to an agent in $S(m)$ to perform it, so as to obtain the maximum matching reward. Accordingly, we define $\mathbf{r} = \max_{i\in[N]} \mathbf{r}_i =$

[EC2] This is because type-one jobs enter the shared pool at rate N, each being matched and taxed ϵ with probability w. Meanwhile, type-two jobs enter the shared pool at rate $N\lambda$, each converting into a lottery upon expiration. The result then follows from the following two observations: (i) under the mean-field approximation, every lottery receives the same expected value from the collected taxes, because the shared pool remains in steady state under this approximation; and (ii) the rate at which taxes are allocated cannot exceed the rate at which they are collected.

[EC3] This is because the shared pool matches an equal number of type-one and type-two jobs, and type-one jobs are matched with probability w.

$(r_m)_{m\in[K]} \in \mathbb{R}_+^K$ as the vector of matching rewards for the planner, where the maximization is taken componentwise.

Let V^* denote the optimal long-run average payoff achievable under centralized matching. Problem (EC-20) provides a fluid relaxation to the centralized matching problem, which is identical to (1) with the new definition of the matching value $\mathbf{r}$, and has the same interpretation as (1).

$$\begin{aligned} V^{\mathrm{F}}(\boldsymbol{\lambda}) = \max_{\mathbf{x}\in\mathbb{R}_+^K} \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} \le \boldsymbol{\lambda}. \end{aligned} \tag{EC-20}$$

The dual problem of (EC-20) is given by the following LP, which is identical to (2):

$$\begin{aligned} \min_{\mathbf{p}\in\mathbb{R}_+^J} \quad & \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{p} \\ \text{s.t.} \quad & \mathbf{M}^{\mathrm{T}}\mathbf{p} \ge \mathbf{r}. \end{aligned} \tag{EC-21}$$

We let $\mathbf{p}^* = (p_j^*)_{j\in[J]} \in \mathbb{R}_+^J$ denote an optimal solution to (EC-21), and we can interpret the value of p_j^* as the marginal value of a type-j job to the centralized problem.

EC3.2 Mechanism with Monetary Transfer

We consider the monetary mechanism proposed in Section 4.1, which rewards an agent with a value of p_j^* whenever she submits a job of type j to the shared pool. The mechanism then performs an asymptotically optimal matching in the shared pool (e.g., by implementing the periodic matching policy π^{P} described in Section 3.1), and lets agents in set $S(m)$ execute type-m matches, charging r_m per match. In what follows, we show that the properties established in Section 4.1 continue to hold. In particular, this mechanism incentivizes all agents to fully submit their jobs and remains approximately budget-balanced.

To do so, we analyze the decision problem faced by an individual agent i under this mechanism. The fluid relaxation to the decision problem of agent i is given in (EC-22):

$$\begin{aligned} \max_{\mathbf{x}\in\mathbb{R}_+^K,\, \mathbf{s}\in\mathbb{R}_+^J} \quad & \mathbf{r}_i^{\mathrm{T}}\mathbf{x} + \mathbf{p}^{*\mathrm{T}}\mathbf{s} \\ \text{s.t.} \quad & \mathbf{M}\mathbf{x} + \mathbf{s} \le \boldsymbol{\lambda}_i. \end{aligned} \tag{EC-22}$$

Note that (EC-22) is identical to (3), except that the matching rewards $\mathbf{r}_i$ reflect agent i's individual values. The dual problem of (EC-22) is given by (EC-23):

$$\begin{aligned} \min_{\mathbf{p}\in\mathbb{R}_+^J} \quad & \boldsymbol{\lambda}_i^{\mathrm{T}}\mathbf{p} \\ \text{s.t.} \quad & \mathbf{M}^{\mathrm{T}}\mathbf{p} \ge \mathbf{r}_i, \\ & \mathbf{p} \ge \mathbf{p}^*. \end{aligned} \tag{EC-23}$$

Proposition EC3.1 parallels Proposition 4.1 and provides optimal solutions to the primal and dual problems.

Proposition EC3.1. *$\mathbf{x} = \mathbf{0}$ and $\mathbf{s} = \boldsymbol{\lambda}_i$ is an optimal primal solution and $\mathbf{p} = \mathbf{p}^*$ is an optimal dual solution to (EC-22), with the optimal value being $\boldsymbol{\lambda}_i^{\mathrm{T}}\mathbf{p}^*$.*

Proof. Note that $\mathbf{x} = \mathbf{0}$ and $\mathbf{s} = \boldsymbol{\lambda}_i$ is feasible to (EC-22), and $\mathbf{p} = \mathbf{p}^*$ is feasible to (EC-23) because $\mathbf{p} = \mathbf{p}^*$ is feasible to (2) and $\mathbf{r} = \max_{i\in[N]} \mathbf{r}_i \ge \mathbf{r}_i$. Moreover, these solutions achieve the same

objective value. By weak duality, they must be optimal solutions to the primal and dual problems, respectively. □

From Proposition EC3.1, submitting all arriving jobs to the shared pool is optimal to an agent's fluid relaxation problem (EC-22). Moreover, under the mechanism, the marginal value of a type-j job to an agent's problem equals the job's marginal value in the centralized problem (EC-20), which is p_j^*. Finally, Proposition EC3.1 also implies that it is a dominant strategy for an agent to submit all jobs to the shared pool in the original problem, as stated in Proposition EC3.2.

Proposition EC3.2. *For any agent $i \in [N]$, it is a dominant strategy to submit all jobs to the shared pool upon their arrival, and this yields an expected long-run average payoff of $\boldsymbol{\lambda}_i^{\mathrm{T}} \mathbf{p}^*$.*

Proof. This is because full submission achieves the fluid relaxation upper bound (EC-22). □

Finally, we remark that the monetary mechanism remains approximately budget-balanced in the large market regime, by the same reasoning as in Remark 4.1.

EC3.3 Mechanism without Monetary Transfer

In this section, we show that when agents have heterogeneous matching rewards, no non-monetary mechanism can achieve the first-best performance. This contrasts with the case of homogeneous matching rewards. To illustrate this, we consider a simple example with two job types and one match type, as defined in Example EC3.1.

Example EC3.1. Consider a problem instance with two job types and one matching type. Each match requires one job from each type. Jobs of both types arrive at each agent at rates 1 and expire at rates 1. There are N agents; the matching value is 1 for the first $N/2$ agents and 2 for the remaining agents.

Suppose the designer cannot use monetary transfer and can only reimburse job submissions through matching allocations. In order to incentivize the first $N/2$ agents to fully submit their jobs, the mechanism must allocate matches to each of these agents at a constant rate that compensates them more than what they could achieve individually. However, since these first $N/2$ agents generate lower matching rewards than the remaining ones, a central controller would never assign them any jobs, and they would consequently receive zero matching rewards. This implies that a non-monetary mechanism cannot achieve the first-best performance for any number of agents N.

EC4 More on the Kidney Exchange Example in Section 6.2

Figure EC-1 (provided in Ashlagi and Roth 2021) illustrates the ABO compatibility structure.

Table EC-1 (reproduced from Table A.1 of Ashlagi and Roth 2021) reports statistics on the historical pool composition of APKD (a major U.S. kidney exchange program) from 2010 to 2019. The second column presents the empirical distribution of the patient-donor ABO pairs, and columns three to nine present the frequencies of PRA intervals, conditional on each ABO pair.

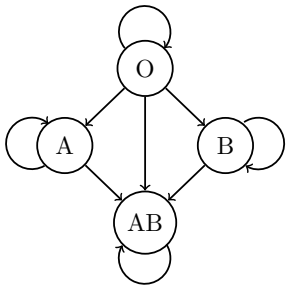


Figure EC-1: Illustration of ABO compatibility (Figure 1 of Ashlagi and Roth 2021). A directed arc from X to Y means that a donor with blood type X is compatible with a recipient with blood type Y.

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		Marginal frequencies (PRA intervals)						
Patient–donor ABO	% of pairs	$0-1$	$1-10$	$10-50$	$50-80$	$80-95$	$95-99$	$99-100$
AB–AB	0.2	0.0	0.0	0.0	50.0	0.0	25.0	25.0
AB–B	0.4	0.0	0.0	0.0	16.7	16.7	0.0	66.7
AB–A	0.7	0.0	8.3	0.0	8.3	50.0	0.0	33.3
AB–O	0.6	10.0	0.0	20.0	10.0	0.0	20.0	40.0
B–AB	0.9	37.5	6.2	18.8	6.2	12.5	0.0	18.8
B–B	2.4	0.0	4.9	12.2	12.2	31.7	9.8	29.3
B–A	5.8	46.5	8.1	13.1	9.1	12.1	1.0	10.1
B–O	4.2	9.9	1.4	4.2	16.9	19.7	15.5	32.4
A–AB	1.0	41.2	5.9	5.9	11.8	17.6	0.0	17.6
A–B	3.6	30.6	9.7	6.5	14.5	9.7	1.6	27.4
A–A	9.7	4.2	1.8	16.9	19.3	18.1	10.8	28.9
A–O	8.8	12.7	4.7	9.3	19.3	15.3	18.0	20.7
O–AB	2.3	46.2	10.3	23.1	5.1	12.8	0.0	2.6
O–B	9.2	47.1	10.8	14.0	7.6	8.3	4.5	7.6
O–A	29.4	49.9	10.0	12.8	8.8	6.4	3.6	8.6
O–O	20.7	4.5	2.8	13.9	17.3	23.9	16.2	21.3

Table EC-1: APKD pool composition (2010-2019) (Table A.1 of Ashlagi and Roth 2021).