Online Appendix for "Platform Disintermediation with Repeated Transactions" by Andreea Enache and Andrew Rhodes

A Omitted Proof for Section 5

Proof of Proposition 6. Since disintermediation reduces platform profit, from earlier work we must have $\phi \in (0,1)$. First, suppose all buyers have the same convenience benefit b. Using Proposition 2, the percentage change in platform profit due to disintermediation is

$$\frac{\max\{2(1-\phi)(v+b-c), v-c+(2-\phi)b\} - (2-\phi)(v+b-c)}{(2-\phi)(v+b-c)}$$
$$= -\max\left\{\frac{\phi}{2-\phi}, \frac{(1-\phi)(v-c)}{(2-\phi)(v+b-c)}\right\}, \tag{21}$$

which increases in b. Hence starting from $b=0$, a platform that invests in $b>0$ suffers less from disintermediation.

Second, suppose that buyers have heterogeneous b. Using the proof of Proposition 5, absent disintermediation the platform earns $(2-\phi)(v+\tilde{b}-c)[1-F(\tilde{b})]$ where $\tilde{b} = \arg\max_b (v+b-c)[1-F(b)]$. Now suppose disintermediation is possible. One thing the platform could do is set $p_{B,1} = 2v+\tilde{b}-c$ and $p_{B,2} = \tilde{b}+c$: using (11) it would earn $2(v+\tilde{b}-c)[1-F(\tilde{b})]$ from two-time buyers, and so its total profit would weakly exceed $2(1-\phi)(v+\tilde{b}-c)[1-F(\tilde{b})]$ (given that it may also earn profit from one-time buyers). Another thing the platform could do is set $p_{B,1} = v+\tilde{b}$ and $p_{B,2} = \arg\max(p-c)[1-F(p-c)]$: using (1) all one-time buyers with $b \geq \tilde{b}$ will purchase, and using (5) all two-time buyers with $b \geq \tilde{b}$ will do the first transaction (plus some others, if $\tilde{b} > \underline{b}$), and from (4) some of these two-time buyers will also do the second transaction on the platform, and so the platform would earn strictly more than $(v+\tilde{b}-c)[1-F(\tilde{b})]$. Of course the platform could also choose other prices and earn even higher profit. Thus the percentage change in its profit due to disintermediation is (weakly) larger than

$$\frac{\max\{2(1-\phi)(v+\tilde{b}-c)[1-F(\tilde{b})], (v+\tilde{b}-c)[1-F(\tilde{b})]\} - (2-\phi)(v+\tilde{b}-c)[1-F(\tilde{b})]}{(2-\phi)(v+\tilde{b}-c)[1-F(\tilde{b})]}$$
$$= -\max\{\frac{\phi}{2-\phi}, \frac{1-\phi}{2-\phi}\} \tag{22}$$

which equals (21) evaluated at $b=0$. Hence the change in platform profit due to disintermediation is (weakly) larger compared to that of a platform which offers $b=0$ to each buyer. □

B Omitted Proofs for Sections 6.1-6.3

Here we provide omitted proofs for the three extensions described in detail in the main text.

B.1 The Timing of Disintermediation

We first derive participation constraints *without* referral fees. When disintermediation is impossible, condition (1) remains valid for one-time buyers, and conditions (2) and (3) remain valid for two-time buyers. When disintermediation is possible, for the $1-\chi$ buyers who can only disintermediate after the first transaction, condition (1) remains valid for one-time buyers, and conditions (4) and (5) remain valid for two-time buyers. When disintermediation is possible, the χ one-time buyers buy on the platform if and only if

$$v + b - p_{B,1} \geq v - c \iff b \geq p_{B,1} - c. \tag{23}$$

Meanwhile, continuing with the case where disintermediation is possible, the χ two-time buyers do both transactions on the platform if

$$2(v+b) - p_{B,1} - p_{B,2} \geq \max\{v + b - p_{B,1} + v - c, 2(v-c)\}, \tag{24}$$

do the first transaction on the platform and the second transaction off the platform if

$$v + b - p_{B,1} + v - c \geq \max\{2(v+b) - p_{B,1} - p_{B,2}, 2(v-c)\}, \tag{25}$$

and otherwise do both transactions off the platform. In the case where a buyer is indifferent between two or more options, we assume she takes the option which entails performing the highest number of transactions on the platform.

Using the above we can now prove Proposition 7.

Proof of Proposition 7. First, suppose there is no buyer heterogeneity, i.e., all buyers wish to transact twice and have the same b. The proof of Proposition 1 showed that, when disintermediation is impossible, the platform can extract the maximum possible surplus $2(v+b-c)$ from each buyer. We now argue that, when disintermediation is possible, the platform is unable to do this. In particular, consider the $\chi > 0$ buyers that can disintermediate before the first transaction. As usual, the maximum possible surplus is attained when they do both transactions on the platform. However, the maximum profit the platform could extract from them when they use it for both transactions is $p_{B,1} + p_{B,2} - 2c \leq 2b$, where the inequality uses (24). This is clearly strictly less than $2(v+b-c)$. Hence platform profit is strictly lower when disintermediation is possible.

Second, suppose buyers have the same b but are heterogeneous in how many times they wish to transact. The proof of Proposition 2 showed that, when disintermediation is impossible, the platform can extract the maximal possible surplus—namely $v+b-c$ from one-time buyers and $2(v+b-c)$ from two-time buyers. However, when disintermediation is possible, the previous paragraph already shows that the platform can extract strictly less than $2(v+b-c)$ from the $\chi > 0$ two-time buyers. Hence platform profit is strictly lower with disintermediation.

Third, suppose buyers all wish to transact twice but have heterogeneous b.

We start by proving that when $\chi = 1$ platform profit is strictly lower with disintermediation. Using (24) and (25) the platform's profit under disintermediation equals

$$\pi^D = \begin{cases} (p_{B,1} - c)[1 - F(p_{B,1} - c)] + (p_{B,2} - c)[1 - F(p_{B,2} - c)] & \text{if } p_{B,1} \leq p_{B,2}, \\ (p_{B,1} + p_{B,2} - 2c)\left[1 - F\left(\frac{p_{B,1}+p_{B,2}}{2} - c\right)\right] & \text{otherwise.} \end{cases} \quad (26)$$

(Specifically, when $p_{B,1} \leq p_{B,2}$, buyers with $b < p_{B,1} - c$ do both transactions off the platform, buyers with $b \geq p_{B,2} - c$ do both transactions on the platform, and other buyers do only the first transaction on the platform. When instead $p_{B,1} > p_{B,2}$, buyers with $b < \frac{p_{B,1}+p_{B,2}}{2} - c$ do both transactions off the platform, and other buyers do both transactions on the platform.) Maximizing π^D in equation (26), and following the same steps as in the proof of Proposition 3, it is straightforward to show that the platform optimally sets prices satisfying $p_{B,1} + p_{B,2} = 2(b^D + c)$ and $p_{B,2} \leq p_{B,1}$, where $b^D = \underline{b}$ if $\underline{b}f(\underline{b}) \geq 1$ and otherwise $b^D > \underline{b}$ is the unique solution to $1 - F(b^D) - b^D f(b^D) = 0$. This yields a profit $2b^D[1 - F(b^D)]$. However, if disintermediation were impossible and the platform charged, for example, $p_{B,1} = p_{B,2} = v + b^D$, one can check using the π^{ND} expression in (8) that the platform would earn $2(v + b^D - c)[1 - F(b^D)] > 2b^D[1 - F(b^D)]$. Hence for $\chi = 1$ platform profit is strictly lower when disintermediation is possible.

Next, consider a general $\chi \in [0, 1]$. Let $\pi^{D,1}(p_{B,1}, p_{B,2})$ denote platform profit on a buyer who can disintermediate *before* the first transaction; this is given by (26). Let $\pi^{D,2}(p_{B,1}, p_{B,2})$ denote platform profit on a buyer who can only disintermediate *after* the first transaction; this is given by (11). Total platform profit is then given by $\Pi(\chi, p_{B,1}, p_{B,2}) \equiv \chi\pi^{D,1}(p_{B,1}, p_{B,2}) + (1 - \chi)\pi^{D,2}(p_{B,1}, p_{B,2})$. Introduce the following notation:

$$\{p^*_{B,1}(\chi), p^*_{B,2}(\chi)\} \equiv \arg\max_{p_{B,1},p_{B,2}} \Pi(\chi, p_{B,1}, p_{B,2}) \quad \text{and} \quad \Pi^*(\chi) \equiv \max_{p_{B,1},p_{B,2}} \Pi(\chi, p_{B,1}, p_{B,2}).$$

We know from earlier in this proof that $\max \pi^{D,1} < \max \pi^{ND}$, where $\max \pi^{ND}$ is the highest attainable profit when disintermediation is impossible. There are then two cases to consider. (1) Suppose that $\underline{b}f(\underline{b}) \geq 1$. We know from Proposition 3 that $\max \pi^{D,2} = \max \pi^{ND}$. It then follows immediately that $\Pi^*(\chi) < \max \pi^{ND}$ for all $\chi > 0$. (2) Suppose that $\underline{b}f(\underline{b}) < 1$. We know from Proposition 3 that $\max \pi^{D,2} > \max \pi^{ND}$. It then follows that $\Pi^*(0) > \max \pi^{ND} > \Pi^*(1)$. By continuity there exists a $\tilde{\chi} \in (0, 1)$ such that $\Pi^*(\tilde{\chi}) = \max \pi^{ND}$. However, notice that for all $\chi < \tilde{\chi}$ we have

$$\Pi^*(\tilde{\chi}) \equiv \Pi(\tilde{\chi}, p^*_{B,1}(\tilde{\chi}), p^*_{B,2}(\tilde{\chi})) < \Pi(\chi, p^*_{B,1}(\tilde{\chi}), p^*_{B,2}(\tilde{\chi})) \leq \Pi^*(\chi),$$

where the first inequality exploits the fact that $\max \pi^{D,1} < \max \pi^{ND}$ and $\Pi^*(\tilde{\chi}) = \max \pi^{ND}$ imply that $\pi^{D,1}(p^*_{B,1}(\tilde{\chi}), p^*_{B,2}(\tilde{\chi})) < \pi^{D,2}(p^*_{B,1}(\tilde{\chi}), p^*_{B,2}(\tilde{\chi}))$, and the second inequality uses the fact that for $\chi < \tilde{\chi}$ the platform may do better to choose $(p_{B,1}, p_{B,2})$ not equal to $(p^*_{B,1}(\tilde{\chi}), p^*_{B,2}(\tilde{\chi}))$. This then establishes uniqueness of a $\widehat{\chi}$ such that $\Pi^*(\widehat{\chi}) = \max \pi^{ND}$, as well as the fact that $\Pi^*(\chi) > \max \pi^{ND}$ if and only if $\chi < \widehat{\chi}$. □

We now turn to the case where the platform can charge a referral fee r as well as per-transaction fees $p_{B,1}$ and $p_{B,2}$ when disintermediation is possible.

We begin by deriving buyer participation constraints. First, consider one-time buyers. The χ such buyers who can disintermediate immediately will pay r and transact on the platform if

$$v + b - r - p_{B,1} \geq \max\{v - c - r, 0\}, \tag{27}$$

will pay r but then transact off the platform if

$$v - c - r \geq \max\{v + b - r - p_{B,1}, 0\}, \tag{28}$$

and otherwise do not pay r and hence do no transaction. Meanwhile the $1 - \chi$ one-time buyers who cannot disintermediate immediately will pay r and transact on the platform if

$$v + b - r - p_{B,1} \geq 0, \tag{29}$$

and otherwise do not pay r and hence do no transaction. Second, consider two-time buyers. The χ such buyers who can disintermediate immediately after paying r, will pay r and do both transactions on the platform if

$$2(v + b) - p_{B,1} - p_{B,2} - r \geq \max\{v + b - p_{B,1} + v - c - r, 2(v - c) - r, 0\}, \tag{30}$$

will pay r and do one transaction on the platform and the other transaction off the platform if

$$v + b - p_{B,1} + v - c - r \geq \max\{2(v + b) - p_{B,1} - p_{B,2} - r, 2(v - c) - r, 0\}, \tag{31}$$

will pay r but then do both transactions off the platform if

$$2(v - c) - r \geq \max\{2(v + b) - p_{B,1} - p_{B,2} - r, v + b - p_{B,1} + v - c - r, 0\}, \tag{32}$$

and otherwise will not pay r and hence will do no transactions. Meanwhile the $1 - \chi$ two-time buyers who can only disintermediate after performing the first transaction on the platform, will pay r and do both transactions on the platform if

$$2(v + b) - p_{B,1} - p_{B,2} - r \geq \max\{v + b - p_{B,1} + v - c - r, 0\}, \tag{33}$$

will pay r and do one transaction on the platform and the other transaction off the platform if

$$v + b - p_{B,1} + v - c - r \geq \max\{2(v + b) - p_{B,1} - p_{B,2} - r, 0\}, \tag{34}$$

and otherwise will not pay r and hence will do no transactions. In the case where a buyer is indifferent between two or more options, we assume she takes the option which entails performing the highest number of transactions on the platform; if a buyer is indifferent between paying r and subsequently transacting off the platform, or not doing anything, she takes the former option.

Using the above we can now prove Proposition 8.

Proof of Proposition 8. First, suppose there is no buyer heterogeneity (i.e., all buyers wish to transact twice and have the same benefit b). The proof of Proposition 1 showed that, when disintermediation is impossible, the platform can extract the maximum possible surplus $2(v+b-c)$ from each buyer. We now argue that the platform can do the same even when disintermediation is possible. For example, suppose the platform sets $r=2(v-c)$ and $p_{B,1}=p_{B,2}=b+c$. Using conditions (30)-(32) the χ buyers are willing to pay r and do both transactions on the platform. Using conditions (33) and (34) the same is true for the $1-\chi$ buyers as well. Hence the platform would sell to all buyers and earn $r+p_{B,1}+p_{B,2}-2c=2(v+b-c)$. Thus disintermediation is neutral for the platform.

Second, suppose buyers all have the same b but are heterogeneous in how many times they wish to transact. The proof of Proposition 2 showed that, when disintermediation is impossible, the platform can extract the maximal possible surplus—namely $v+b-c$ from one-time buyers and $2(v+b-c)$ from two-time buyers. We now argue that, even when it can use a referral fee, the platform is unable to do this when disintermediation is possible. It is sufficient to prove this for the χ buyers that can disintermediate after paying r. In order for the platform to fully extract the maximum surplus generated by one-time buyers, they must transact on the platform and pay the platform in total $r+p_{B,1}=v+b$. In order for the platform to fully extract the maximum surplus generated by two-time buyers, they must transact twice on the platform and pay the platform in total $r+p_{B,1}+p_{B,2}=2(v+b)$; however, from (30), a necessary condition for buyers to then do both transactions on the platform is $r+p_{B,1}\geq 2v+b-c$. However since $2v+b-c>v+b$ it is impossible to fully extract both the one-time and the two-time χ buyers. Hence platform profit must be strictly lower with disintermediation.

Third, suppose buyers all wish to transact twice but have heterogeneous b.

We start by proving that platform profit is at least as high with disintermediation as it is without disintermediation. To do this, recall from Proposition 3 that, when disintermediation is impossible, the platform chooses a marginal buyer b^{ND} and chooses prices $p_{B,1}\geq p_{B,2}$ and $p_{B,1}+p_{B,2}=2(v+b^{ND})$ such that buyers with $b\geq b^{ND}$ do both transactions on the platform, and other buyers do no transactions. This generates platform profit $2(v+b^{ND}-c)[1-F(b^{ND})]$. Now suppose that disintermediation is possible, and that the platform sets $r=2(v-c)$ and $p_{B,1}=p_{B,2}=b^{ND}+c$. Using (30)-(32), of the χ buyers, those with $b\geq b^{ND}$ pay r and transact twice on the platform, while those with $b<b^{ND}$ pay r but do both transactions off the platform. Using (33) and (34), of the $1-\chi$ buyers, those with $b\geq b^{ND}$ pay r and transact twice on the platform, while those with $b<b^{ND}$ do not pay r and thus do not transact at all. Hence the platform earns

$$\chi rF(b^{ND})+(r+p_{B,1}+p_{B,2}-2c)[1-F(b^{ND})],$$

which equals $2(v+b^{ND}-c)[1-F(b^{ND})]$ when $b^{ND}=\underline{b}$ and is otherwise strictly greater than

$2(v + b^{ND} - c)[1 - F(b^{ND})]$.

We now argue that when $\underline{b}f(\underline{b}) < 1$ the platform is strictly better off when disintermediation is possible. The previous paragraph already establishes this claim when $b^{ND} > \underline{b}$. Therefore, to complete this part of the proof, suppose $b^{ND} = \underline{b}$. Suppose the platform sets $r = 2(v - c)$ and $p_{B,1} = \underline{b} + c$ and $p_{B,2} \geq \underline{b} + c$. Using (30)-(32), of the χ buyers, those with $b \geq p_{B,2} - c$ do both transactions on the platform, while the others do only the first transaction on the platform. Using (33) and (34), the $1 - \chi$ buyers optimally behave exactly as the χ buyers. Hence we can write platform profit as

$$r + p_{B,1} - c + (p_{B,2} - c)[1 - F(p_{B,2} - c)] = 2(v - c) + \underline{b} + (p_{B,2} - c)[1 - F(p_{B,2} - c)].$$

As shown already, when $p_{B,2} = \underline{b} + c$ this equals $2(v + \underline{b} - c)$, i.e., platform profit absent disintermediation. Moreover, by the usual argument, the above profit is strictly increasing in $p_{B,2}$ around the point $p_{B,2} = \underline{b} + c$ given the assumption that $\underline{b}f(\underline{b}) < 1$. Hence the platform does strictly better than it would do when disintermediation is impossible. □

Finally we prove Proposition 9

Proof of Proposition 9. First, notice that if the platform follows a pure referral service, it will either charge $r = v - c$ and all buyers will pay it, or it will charge $r = 2(v - c)$ and only two-time buyers will pay it; it is straightforward to show that the former is optimal if and only if $\phi \geq 1/2$. Second, suppose the platform charges for referrals and for transactions. Suppose the platform keeps r the same, but (i) in the case where b is homogeneous, it charges $p_{B,1} = p_{B,2} = b + c$, and (ii) in the case where b is heterogeneous, it charges $p_{B,1} = p_{B,2} = \arg\max(p - c)[1 - F(p - c)]$. Using (27) one can check that any one-time buyer that participated under the pure referral model will still pay r, and will pay $p_{B,1}$ with positive probability. Using (30) one can check that all two-time buyers will pay r, and will pay $p_{B,1}$ and $p_{B,2}$ with positive probability. Hence platform profit is strictly higher than under the pure referral service mode. □

B.2 Uncertain Transaction Frequency

We begin by deriving buyer participation constraints. Start by considering a buyer who did the first transaction on the platform and then learned that she wishes to use the service again. Exactly as in (2) and (4), when disintermediation is impossible the buyer does the second transaction if and only if

$$v + b - p_{B,2} \geq 0, \tag{35}$$

whereas if disintermediation is possible she does the second transaction on the platform if

$$b \geq p_{B,2} - c, \tag{36}$$

and otherwise she does the second transaction off the platform. Next, consider the first transaction. When disintermediation is impossible, a buyer does the first transaction if and only if

$$v + b - p_{B,1} + (1-\phi)\max\{v + b - p_{B,2}, 0\} \geq 0, \tag{37}$$

because the buyer knows there is a probability $1-\phi$ she will wish to transact again later. Meanwhile, when disintermediation is possible, a buyer does the first transaction if and only if

$$v + b - p_{B,1} + (1-\phi)\max\{v + b - p_{B,2}, v - c\} \geq 0. \tag{38}$$

We can now use the above results to prove Proposition 10.

Proof of Proposition 10. First, suppose buyers are homogeneous in b. Notice that the maximum possible surplus is $(2-\phi)(v+b-c)$, and it is achieved when each buyer does all the transactions that she wants to do on the platform. We now argue that the platform can achieve this, irrespective of whether disintermediation is possible. For example, suppose the platform sets $p_{B,1} = v+b+(1-\phi)(v-c)$ and $p_{B,2} = b+c$. Using (35)-(38) one can check that a buyer will do the first transaction and then, conditional on needing the service a second time, will also do the second transaction on the platform. Hence the platform earns profit $p_{B,1} - c + (1-\phi)(p_{B,2} - c) = (2-\phi)(v+b-c)$, which as argued above is the maximum possible. Hence disintermediation has no effect on platform profit.

Second, suppose buyers have heterogeneous b. When disintermediation is impossible we can write platform profit as

$$\pi^{ND} = \begin{cases} (p_{B,1} - c)[1 - F(p_{B,1} - v)] + (1-\phi)(p_{B,2} - c)[1 - F(p_{B,2} - v)] & \text{if } p_{B,1} \leq p_{B,2}, \\ [p_{B,1} - c + (1-\phi)(p_{B,2} - c)]\left[1 - F\left(\frac{p_{B,1} + (1-\phi)p_{B,2}}{2-\phi} - v\right)\right] & \text{otherwise.} \end{cases} \tag{39}$$

(First, consider $p_{B,1} \leq p_{B,2}$. Using (35) and (37), one can check that buyers with $b < p_{B,1} - v$ do not do even the first transaction, buyers with $b \geq p_{B,2} - v$ do the first transaction and then also the second if they need the service a second time, while remaining buyers do only the first transaction irrespective of whether they end up needing the service twice. Second, consider $p_{B,1} > p_{B,2}$. Using (35) and (37), one can check that buyers with $b < \frac{p_{B,1} + (1-\phi)p_{B,2}}{2-\phi} - v$ do no transactions, while all remaining buyers do the first transaction and then also the second transaction if they need the service a second time.) Using the same approach as in the proof of Proposition 3, maximizing (39), we find that the platform optimally sets $p_{B,1} \geq p_{B,2}$ where $p_{B,1} + (1-\phi)p_{B,2} = (2-\phi)(v + b^{ND})$ and where $b^{ND} = \underline{b}$ if $(v + \underline{b} - c)f(\underline{b}) \geq 1$ and otherwise b^{ND} is the unique solution to $1 - F(b^{ND}) - (v + b^{ND} - c)f(b^{ND}) = 0$. This leads to platform profit $(2-\phi)(v + b^{ND} - c)[1 - F(b^{ND})]$.

When instead disintermediation is possible we can write platform profit as

$$\pi^D = \begin{cases} (p_{B,1} - c)[1 - F(\zeta)] + (1-\phi)(p_{B,2} - c)[1 - F(p_{B,2} - c)] & \text{if } p_{B,1} \leq p_{B,2} + (2-\phi)(v-c), \\ [p_{B,1} - c + (1-\phi)(p_{B,2} - c)] \left[1 - F\left(\frac{p_{B,1}+(1-\phi)p_{B,2}}{2-\phi} - v\right)\right] & \text{otherwise,} \end{cases} \tag{40}$$

where for brevity we use $\zeta \equiv p_{B,1} - v - (1-\phi)(v-c)$. (First, consider $p_{B,1} \leq p_{B,2} + (2-\phi)(v-c)$. Using (36) and (38), one can check that buyers with $b < \zeta$ do not do even the first transaction, buyers with $b \geq p_{B,2} - c$ do the first transaction and then if they need the service a second time they do the transaction on the platform, while the remaining buyers do the first transaction and then if they need the service a second time they do this second transaction off the platform. Second, consider $p_{B,1} > p_{B,2} + (2-\phi)(v-c)$. Using (36) and (38), one can check that buyers with $b < \frac{p_{B,1}+(1-\phi)p_{B,2}}{2-\phi} - v$ do no transactions, while all remaining buyers do the first transaction and then if they need the service a second time they do this second transaction on the platform as well.) We now use (40) to argue that the platform can always earn at least as much profit as it did absent disintermediation. For example, if the platform sets $p_{B,1} = v(2-\phi) + b^{ND} - (1-\phi)c$ and $p_{B,2} = b^{ND} + c$ one can check that it earns $(2-\phi)(v + b^{ND} - c)[1 - F(b^{ND})]$, i.e., the same as it did absent disintermediation. To complete the proof, we now show that when $\underline{b}f(\underline{b}) < 1$ the platform earns strictly more profit than when disintermediation is impossible. In particular, suppose the platform continues to set $p_{B,1} = v(2-\phi) + b^{ND} - (1-\phi)c$ but now charges $p_{B,2} > b^{ND} + c$. Using the first line of (40) the platform's profit is

$$[(2-\phi)(v-c) + b^{ND}][1 - F(b^{ND})] + (1-\phi)(p_{B,2} - c)[1 - F(p_{B,2} - c)].$$

We have already shown that this equals $(2-\phi)(v + b^{ND} - c)[1 - F(b^{ND})]$ when evaluated at $p_{B,2} = b^{ND} + c$. Moreover, its derivative with respect to $p_{B,2}$ evaluated at $p_{B,2} = b^{ND} + c$ is $(1-\phi)[1 - F(b^{ND}) - b^{ND} f(b^{ND})]$. If $b^{ND} = \underline{b}$ this is strictly positive because by assumption $\underline{b}f(\underline{b}) < 1$; if instead $b^{ND} > \underline{b}$ then it equals $(1-\phi)f(b^{ND})(v-c) > 0$. Hence profit is strictly higher than when disintermediation is impossible. □

B.3 Seller Pricing

Proof of Proposition 11. As a preliminary step, note that the buyer participation constraints in equations (1) to (5) remain valid in this extension.

Now consider part (i). As usual it is efficient for both transactions to occur on the platform, generating total surplus of $2(v + b - c)$; this puts a bound on the total profit that can be earned by the platform and seller. Notice that if the platform sets $\tau_{S,1} = 2(v + b - c)$ and $\tau_{S,2} = 0$ then the seller can never make strictly positive profit. However, by charging, e.g., $p_{B,1} = 2(v + b)$ and $p_{B,2} = 0$, the seller can induce the buyer to do both transactions on the platform—irrespective of whether disintermediation is possible—and so generate zero profit. Since $\tau_{S,1} + \tau_{S,2} = 2(v + b - c)$ the platform extracts the full surplus regardless of whether disintermediation is possible.

Now consider part (ii). As usual it is efficient for all transactions to occur on the platform, generating total surplus of $(2 - \phi)(v + b - c)$; this again puts a bound on the total profit that can be earned. Suppose disintermediation is impossible. Notice that if the platform sets $\tau_{S,1} = (2 - \phi)(v + b - c)$ and $\tau_{S,2} = 0$ then the seller can never make strictly positive profit. However, by charging $p_{B,1} = p_{B,2} = v + b$, the seller can induce the buyer to do all desired transactions on the platform—earning the seller a zero profit, and allowing the platform to extract the maximum possible surplus. Next, suppose disintermediation is possible. Recall from Proposition 2 that the amount which can be extracted from the buyer is strictly less than $(2 - \phi)(v + b - c)$, and so since the seller's profit cannot be negative, the platform must earn strictly less than $(2 - \phi)(v + b - c)$.

Now consider part (iii). Recall the definition $I(b) = \frac{1-F(b)}{f(b)} - b$.

We first prove that, when disintermediation is impossible, the platform chooses a marginal buyer type b^{ND}, where $b^{ND} = \underline{b}$ if $I'(\underline{b}) - 1 + (v + \underline{b} - c)f(\underline{b}) \geq 0$, and otherwise $b^{ND} > \underline{b}$ is the unique solution to

$$I'(b^{ND})\frac{1 - F(b^{ND})}{f(b^{ND})} + v - c - I(b^{ND}) = 0. \tag{41}$$

The platform then charges fees satisfying $\tau_{S,1} \geq \tau_{S,2}$ and $\tau_{S,1} + \tau_{S,2} = 2[v - c - I(b^{ND})]$, such that the seller charges prices satisfying $p_{B,1} \geq p_{B,2}$ and $p_{B,1} + p_{B,2} = 2(v + b^{ND})$. Given these seller prices, buyers with $b < b^{ND}$ do no transactions, and buyers with $b \geq b^{ND}$ do both transactions on the platform.

To prove the above result, first write the seller's profit as

$$\begin{cases} (p_{B,1} - c - \tau_{S,1})[1 - F(p_{B,1} - v)] + (p_{B,2} - c - \tau_{S,2})[1 - F(p_{B,2} - v)] & \text{if } p_{B,1} \leq p_{B,2}, \\ (p_{B,1} + p_{B,2} - 2c - \tau_{S,1} - \tau_{S,2}) \left[1 - F\left(\frac{p_{B,1}+p_{B,2}}{2} - v\right)\right] & \text{otherwise.} \end{cases} \tag{42}$$

Consider the first line of (42). Log-concavity of $1 - F$ implies that each term is quasiconcave in $p_{B,1}$ and $p_{B,2}$ respectively. The "unconstrained" optimum (i.e., ignoring the constraint $p_{B,1} \leq p_{B,2}$) is as follows: for $i = 1, 2$, $p_{B,i} = v + \underline{b}$ if $1 - (v + \underline{b} - c - \tau_{S,i})f(\underline{b}) \leq 0$, and otherwise $p_{B,i}$ is the unique solution to $I(p_{B,i} - v) = v - c - \tau_{S,i}$. Next, consider the second line of (42). Let $\overline{p_B} \equiv (p_{B,1} + p_{B,2})/2$ and $\overline{\tau_S} \equiv (\tau_{S,1} + \tau_{S,2})/2$. Then $\overline{p_B} = v + \underline{b}$ if $1 - (v + \underline{b} - c - \overline{\tau_S})\, f(\underline{b}) < 0$, and otherwise $\overline{p_B}$ is the unique solution to $I(\overline{p_B} - v) = v - c - \overline{\tau_S}$. Next, notice that any prices satisfying $p_{B,1} \geq p_{B,2}$ which have the same total price $p_{B,1} + p_{B,2}$ give the same seller profit. We can then conclude that if the unconstrained solution to the first line of (42) satisfies $p_{B,1} < p_{B,2}$ then it is the unique solution to the seller's optimization problem, and otherwise any $p_{B,1} \geq p_{B,2}$ that maximize the second line of (42) solve the seller's optimization problem.

Now consider the platform's optimization problem. We will argue that the platform optimally sets $\tau_{S,1} \geq \tau_{S,2}$. On the way to a contradiction, suppose the platform sets $\tau_{S,1} < \tau_{S,2}$. Define

$\hat{\tau} \equiv v + \underline{b} - c - [1/f(\underline{b})]$ (a) One possibility is that $\tau_{S,1} < \tau_{S,2} \leq \hat{\tau}$. Earlier work implies that the unconstrained $p_{B,1}$ and $p_{B,2}$ that maximize the first line of (42) are equal. Hence the seller chooses total price $p_{B,1} + p_{B,2}$ (satisfying $p_{B,1} \geq p_{B,2}$) to maximize the second line of (42). However, since $\overline{\tau_S} < \hat{\tau}$, the platform can profitably deviate by raising $\tau_{S,1}$ slightly: the seller will not change its $p_{B,1} + p_{B,2}$ so the platform will host the same transactions but for a higher fee. Hence we cannot have $\tau_{S,1} < \tau_{S,2} \leq \hat{\tau}$. (b) Another possibility is that $\tau_{S,1} < \hat{\tau} < \tau_{S,2}$. Earlier work implies that the unconstrained seller prices satisfy $p_{B,1} < p_{B,2}$. Hence the seller chooses prices to maximize the first line of (42). However, since $\tau_{S,1} < \hat{\tau}$, the platform can profitably deviate by raising $\tau_{S,1}$ slightly: the seller will not change its prices, so the platform will host the same transactions but at higher fees. Hence we cannot have $\tau_{S,1} < \hat{\tau} < \tau_{S,2}$. (c) The final possibility is that $\hat{\tau} \leq \tau_{S,1} < \tau_{S,2}$. Earlier work again implies that the unconstrained seller prices satisfy $p_{B,1} < p_{B,2}$.[25] Hence the seller chooses prices to maximize the first line of (42), and platform profit is $\tau_{S,1}[1 - F(p_{B,1} - v)] + \tau_{S,2}[1 - F(p_{B,2} - v)]$. Note that because (from earlier) $p_{B,i}$ satisfies $I(p_{B,i} - v) = v - c - \tau_{S,i}$ for $i = 1, 2$, we have that $dp_{B,i}/d\tau_{S,i} = -1/I'(p_{B,i} - v)$. The derivative of platform profit with respect to $\tau_{S,i}$ is therefore proportional to

$$\frac{1 - F(p_{B,i} - v)}{f(p_{B,i} - v)} + \frac{\tau_{S,i}}{I'(p_{B,i} - v)}.$$

This is strictly decreasing in $\tau_{S,i} \geq 0$ given that $I''(b) \geq 0$. Hence the platform will not choose $\tau_{S,1} < \tau_{S,2}$: it can move one fee closer to the other and strictly increase its profit.

We conclude from the above that the platform optimally chooses $\tau_{S,1} \geq \tau_{S,2}$. From earlier work, the unconstrained seller prices do not satisfy $p_{B,1} < p_{B,2}$ and so the seller chooses an average price to maximize the second line of (42). We can immediately rule out the platform choosing $\overline{\tau_S} < \hat{\tau}$ because it could slightly raise $\overline{\tau_S}$, the seller would continue to charge $\overline{p_B} = v + \underline{b}$, and the platform would earn higher profit. Hence, the platform sets $\overline{\tau_S} \geq \hat{\tau}$ and consequently, from earlier work, the seller sets an average price $\overline{p_B}$ which satisfies $I(\overline{p_B} - v) = v - c - \overline{\tau_S}$. Since the platform's profit is $2\overline{\tau_S}[1 - F(\overline{p_B} - v)]$, the derivative of its profit with respect to $\overline{\tau_S}$ is proportional to

$$\frac{1 - F(\overline{p_B} - v)}{f(\overline{p_B} - v)} + \frac{\overline{\tau_S}}{I'(\overline{p_B} - v)} = \frac{1}{I'(b^{ND})}\left[I'(b^{ND})\frac{1 - F(b^{ND})}{f(b^{ND})} + v - c - I(b^{ND})\right],$$

where in the second expression we have substituted in for $\overline{\tau_S}$ from the seller's first order condition $I(\overline{p_B} - v) = v - c - \overline{\tau_S}$, and substituted in $b^{ND} = \overline{p_B} - v$. Notice that the left-hand side is strictly decreasing in $\overline{\tau_S} \geq 0$ given our regularity condition $I''(b) \geq 0$, and hence platform profit is quasiconcave in $\overline{\tau_S}$. Notice also that the square-bracketed term on the right-hand side is strictly increasing in b^{ND} for the same reason. Therefore, following the usual logic, if the square-bracketed term is weakly positive when evaluated at $\underline{b}$ then the platform chooses $b^{ND} = \underline{b}$, and otherwise

[25] The reason is as follows. From earlier, in this case $p_{B,i}$ for $i = 1, 2$ satisfies $I(p_{B,i} - v) = v - c - \tau_{S,i}$. Since $I'(b) < 0$ due to $1 - F$ being log-concave, $p_{B,i}$ is strictly increasing in $\tau_{S,i}$.

it chooses the unique b^{ND} which sets the square-bracketed term to zero. The claimed threshold, fees and prices described at the start of this part of the proof then follow. This generates platform profit $2[v - c - I(b^{ND})][1 - F(b^{ND})]$.

Now suppose that disintermediation is possible. First write out the seller's profit:

$$\begin{cases} (p_{B,1} - c - \tau_{S,1})[1 - F(p_{B,1} + c - 2v)] + (p_{B,2} - c - \tau_{S,2})[1 - F(p_{B,2} - c)] & \text{if } p_{B,1} \leq p_{B,2} + 2(v - c), \\ (p_{B,1} + p_{B,2} - 2c - \tau_{S,1} - \tau_{S,2}) \left[1 - F\left(\frac{p_{B,1}+p_{B,2}}{2} - v\right)\right] & \text{otherwise.} \end{cases} \tag{43}$$

We begin by proving that the platform earns the same profit as it did when disintermediation was impossible, provided it chooses fees

$$\tilde{\tau}_{S,1} = 2(v - c) - I(b^{ND}) \quad \text{and} \quad \tilde{\tau}_{S,2} = -I(b^{ND}).$$

Notice that $(\tilde{\tau}_{S,1} + \tilde{\tau}_{S,2})/2$ equals the average fee chosen by the platform absent disintermediation. Consider the seller's pricing decision. Consider the first line of (43), and note that if we temporarily ignore the constraint $p_{B,1} \leq p_{B,2} + 2(v - c)$ on prices, the derivatives of seller profit with respect to $p_{B,1}$ and $p_{B,2}$ are, respectively

$$I(p_{B,1} + c - 2v) - I(b^{ND}) \quad \text{and} \quad I(p_{B,2} - c) - I(b^{ND}).$$

These equal zero if and only if the seller charges prices $\tilde{p}_{B,1} = b^{ND} + 2v - c$ and $\tilde{p}_{B,2} = b^{ND} + c$. These prices satisfy the pricing constraint, and moreover they ensure that buyers with $b < b^{ND}$ do no transaction, and buyers with $b \geq b^{ND}$ still do both transactions on the platform. Hence platform profit is exactly the same as before. (One can also verify that if the seller chooses prices to maximize the second line of (43), the same outcome arises.)

Finally, we prove that disintermediation enables the platform to earn strictly higher profit than before, provided that $1 - \underline{b}f(\underline{b}) > I'(\underline{b})$. Suppose the platform charges $\tau_{S,1} = \tilde{\tau}_{S,1}$ but slightly increases $\tau_{S,2}$ above $\tilde{\tau}_{S,2}$. The pricing constraint in the first line of (43) is satisfied strictly, from the seller's first order condition we have $I(p_{B,2} - c) = -\tau_{S,2}$, and platform profit equals

$$\tau_{S,1}[1 - F(p_{B,1} + c - 2v)] + \tau_{S,2}[1 - F(p_{B,2} - c)]. \tag{44}$$

The derivative of the platform's profit with respect to $\tau_{S,2}$ around $\tau_{S,2} = \tilde{\tau}_{S,2}$ is therefore proportional to

$$\frac{1 - F(b^{ND})}{f(b^{ND})} - \frac{I(b^{ND})}{I'(b^{ND})}.$$

One can check that if $b^{ND} = \underline{b}$ this is strictly positive since by assumption $1 - \underline{b}f(\underline{b}) > I'(\underline{b})$. One can check that it is also strictly positive if $b^{ND} > \underline{b}$, because in that case we have (from earlier in the proof) that $\frac{1-F(b^{ND})}{f(b^{ND})} = -\frac{v-c-I(b^{ND})}{I'(b^{ND})}$. But this means that starting from $\tilde{\tau}_{S,2}$ and $\tilde{\tau}_{S,2}$, where the platform makes the same profit as it did absent disintermediation, it can do even better—and so earn strictly more than when disintermediation was impossible—by slightly raising $\tau_{S,2}$. □

C Omitted Details for Section 6.4

Here we provide fuller details and explanations of the extensions that were briefly described in Section 6.4. At the end of this section we then collect together the proofs for each of these extensions.

C.1 Seller Benefits

In our baseline model the buyer receives a benefit from transacting on the platform and, when disintermediation is possible, chooses where to do the second transaction. Here we assume that the seller receives the platform benefit and decides where to transact.

Consider the following variant on our baseline model. The buyer wishes to buy twice, and each time her valuation for the product—irrespective of where the transaction occurs—is v. However, the seller's marginal cost now depends on where she transacts: her cost is c if she transacts off the platform, and $c - r$ if she transacts on the platform. Hence r is the reduction in cost (i.e., seller benefit) from using the platform. We assume that r has a distribution $G(r)$ with support $[\underline{r}, \overline{r}] \subseteq \mathbb{R}_+$, where $0 < \overline{r} < c$. Moreover, with probability $\phi < 1$ the seller can only serve the buyer once, and with probability $1 - \phi$ the seller can serve the buyer twice.[26] The seller is privately informed about her platform benefit r and whether she can serve the buyer once or twice. Mirroring our baseline analysis, the platform charges the buyer v for each transaction, and offers the seller $p_{S,1}$ and $p_{S,2}$ to complete, respectively, the first and second transactions on the platform. As usual, the buyer and seller can only meet with the help of the platform. However, after the first transaction, the seller can choose (when appropriate) whether to do the second transaction on or off the platform; if the second transaction occurs off the platform, the seller makes the buyer a take-it-or-leave-it offer.

Proposition 12. *Suppose the seller benefits from transacting on the platform. The ability of sellers to disintermediate: (i) is neutral for platform profit when sellers are homogeneous, (ii) strictly reduces platform profit when sellers are heterogeneous in how many times they can transact, and (iii) weakly benefits the platform when sellers are heterogeneous in their platform benefit r (and strictly so when $\underline{r} g(\underline{r}) < 1$).*

When buyers are homogeneous, the platform reacts to the threat of disintermediation by offering sellers a higher $p_{S,2}$ so as to prevent disintermediation, but a lower $p_{S,1}$ so as to reduce the overall compensation it pays out. As in Proposition 1, this rebalancing ensures the platform earns the same profit as when disintermediation was impossible. However, if sellers differ in how many times they can transact, the platform is worse off with disintermediation—because if it offers a lower $p_{S,1}$ to extract more surplus from two-time sellers, it loses revenues from one-time sellers who are no

[26] We allow this dimension of heterogeneity for completeness, and to remain symmetric with respect to the baseline model, even though in practice it is more natural that the seller can always do both transactions if required (and hence r would be the only dimension of seller heterogeneity).

longer willing to transact, similar to the trade-off it faced in Proposition 2. Finally, if instead sellers differ (enough) in their platform benefit r, the platform can use disintermediation to screen them and hence earn higher profit, just like in Proposition 3 where it used disintermediation to screen buyers.

C.2 Bargaining Between Buyers and Sellers

In our baseline analysis a buyer chooses where to transact and on what terms. Here we allow buyers and sellers to bargain over where to trade and at what price.

Consider our baseline model, but now suppose that the platform charges buyers and sellers $p_{B,1}$ and $p_{S,1}$ respectively to do the first transaction, and $p_{B,2}$ and $p_{S,2}$ respectively to do a second transaction. Buyers and sellers still need the platform to meet and perform the first transaction. However, after doing this first transaction, a buyer and seller can communicate and bargain with each other. To capture this in a simple way, suppose that with probability $\alpha \in [0, 1]$ the buyer chooses where a second transaction occurs, and at what price, and makes a take-it-or-leave-it offer to the seller; with probability $1 - \alpha$ the seller chooses where a second transaction occurs, and at what price, and makes a take-it-or-leave-it offer to the buyer. (Hence a higher α means the buyer has more bargaining power.) For simplicity, we assume that the buyer and seller match only with each other, and hence have zero outside option if they fail to make an agreement; we also assume that if the seller is the one making the offer, she learns the buyer's on-platform b before making an offer.[27] In addition to the usual tie-break rules, we assume that if the seller has the bargaining power and is indifferent about where to do the second transaction, she does it on the platform.

The platform must now satisfy not only buyer participation constraints like those in equations (1) to (5), but also seller participation constraints. In the proof we show that when disintermediation is possible, a second transaction occurs on the platform if and only only if $p_{B,2}+p_{S,2} \leq b$, i.e., provided the total margin taken by the platform is less than the convenience benefit it brings. Hence only the total margin matters, not its individual components $p_{B,2}$ and $p_{S,2}$.[28] Because the seller may earn positive surplus from bargaining at the time of the second transaction, the platform may charge it a positive fee for the first transaction.

Proposition 13. *Suppose there is bargaining. The ability of agents to disintermediate: (i) is neutral for platform profit when buyers are homogeneous, (ii) weakly reduces platform profit when buyers are heterogeneous in how many times they wish to transact, and strictly so for $\alpha > 0$, and*

[27] These assumptions avoid the well-known problems and technical complications associated with, respectively, multi-player bargaining, and bargaining under incomplete information.

[28] In the baseline model we assumed the platform compensates the seller c for each transaction, i.e., using our notation here, we set $p_{S,1} = p_{S,2} = -c$. As the above discussion shows, $p_{S,2} = -c$ is purely a normalization given that only $p_{B,2} + p_{S,2}$ matters. Moreover, since the seller has no bargaining power in the baseline model, one can show that it is indeed optimal for the platform to set $p_{S,1} = -c$.

(iii) weakly benefits the platform when buyers are heterogeneous in b (and strictly so when $\underline{b}f(\underline{b}) < 1$ and α exceeds a threshold $\breve{\alpha} \in (0,1)$).

When buyers are homogeneous, the platform reacts as usual to the threat of disintermediation by reducing its margin $p_{B,2}+p_{S,2}$ on the second transaction, so as to ensure that disintermediation does not occur. This leaves surplus to both buyers and sellers (depending on their bargaining powers), which the platform then extracts by raising $p_{B,1}$ and $p_{S,1}$. Just as in Proposition 1, this enables it to earn the same profit as when disintermediation was impossible. However, when buyers differ in their purchase frequency and $\alpha > 0$, the platform faces the usual dilemma from Proposition 2 between keeping $p_{B,1}$ low to host one-time buyers but give up surplus on two-time buyers, or raising $p_{B,1}$ to fully extract two-time buyers but lose one-time buyers. (The reason why this dilemma does not arise when $\alpha = 0$—and hence the platform is not harmed by disintermediation—is that in this case two-time buyers have no bargaining power over the second transaction, so their willingness-to-pay for the first transaction is the same as that of a one-time buyer.) Finally, if buyers differ in their convenience benefit b, the platform (weakly) benefits from disintermediation, and strictly so whenever $\underline{b}f(\underline{b}) < 1$ and α is sufficiently large. Intuitively, α must be relatively large because in that case buyers have a lot of bargaining power, and so buyers' payoffs are more sensitive to their b, implying more scope for the platform to screen them, just as in Proposition 3.

C.3 General Number of Transactions

In our baseline analysis a buyer wishes to transact either once or twice. Here we allow for a general number of transactions.

Consider the same set-up as our baseline model, except that now a buyer wishes to transact $n = 1, \dots, N$ times. Let ϕ_k be the probability a buyer wishes to transact k times, where $\phi_1 < 1$ and $\sum_{k=1}^{N} \phi_k = 1$. The platform compensates the seller c for each transaction, and charges the buyer $p_{B,k}$ to complete a k^{th} transaction on the platform. As usual the buyer and seller can only meet with the help of the platform, but after the first transaction they can disintermediate. We break ties as follows: if a buyer is indifferent about whether to do an additional transaction she does it, and if she is indifferent about doing a transaction on or off the platform she does it on the platform.

Proposition 14. *Consider a general number of transactions. The ability of buyers to disintermediate: (i) is neutral for the platform when buyers are homogeneous, (ii) strictly reduces platform profit when buyers are heterogeneous in n, and (iii) weakly benefits the platform when buyers are heterogeneous in b (and strictly so if $\underline{b}f(\underline{b}) < 1$).*

When buyers all have the same b and n, the platform reacts to the threat of disintermediation by reducing $\{p_{B,2}, \dots, p_{B,n}\}$ so as to ensure the last $n-1$ transactions occur on the platform. This leaves surplus to buyers, which as in Proposition 1 the platform extracts by raising $p_{B,1}$, such that it earns the same profit as when disintermediation was impossible. However, when different buyers have different desired purchase frequencies, buyers with larger n would benefit more from the lower $\{p_{B,2}, \ldots, p_{B,n}\}$, and so would need to be charged a higher $p_{B,1}$ to extract all their surplus. As in Proposition 2 the platform faces a dilemma: as it raises $p_{B,1}$ it extracts more surplus from high-n buyers, but forces low-n buyers off the platform completely, and so is always worse off compared to when disintermediation was impossible. Finally, if buyers differ only in their platform benefit b, the platform can use disintermediation to screen them and earn more profit, just like in Proposition 3.

Finally, recall that in Section 6.2 we extended our baseline model to the case where buyers are ex ante uncertain of how many times they will want to transact. We showed that disintermediation is weakly beneficial for the platform, and strictly so when b is heterogeneous and $\underline{b}f(\underline{b}) < 1$. We now show how that result generalizes to the above setting with a general number of transactions. Assume again that a buyer will need the service exactly n times with probability ϕ_n; as in Section 6.2 assume buyers have a correct prior ex ante about how many times they will need the service, but only learn they need the service again when they actually need it.[29] To make a buyer's purchase problem more tractable, we focus on the situation where the platform charges $p_{B,1}$ for the initial transaction and then charges the same $p_{B,>1}$ for each subsequent transaction that a buyer performs.

Proposition 15. *Consider a general number of transactions, and suppose buyers are uninformed ex ante about how many times they will want to transact. Suppose the platform charges $p_{B,1}$ for the first transaction and $p_{B,>1}$ for each subsequent transaction. The ability of buyers to disintermediate: (i) is neutral for platform profit when buyers have homogeneous b, and (ii) weakly benefits the platform when buyers have heterogeneous b, and strictly so whenever $\underline{b}f(\underline{b}) < 1$.*

Thus, as in Section 6.2, when buyers are initially uninformed about how many times they will ultimately wish to transact, disintermediation is always (weakly) beneficial for the platform.

C.4 Costly Off-Platform Transactions

In our baseline model buyers receive a convenience benefit when they transact *on* the platform. Here we allow buyers to incur a cost when they transact *off* the platform.

Only Costs We start with the case where there are only costs of going off the platform, i.e., there are no convenience benefits from transacting on the platform.

Consider the same set-up as in our baseline model, except that now a buyer gets value v from transacting on the platform, and value $v - \tau$ from transacting off the platform. Thus τ represents

[29] As pointed out by a referee, a very natural special case would be one in which the probability of a buyer needing the good exactly n times decreases geometrically in n; if there were N periods, at the start of each period the buyer would learn with some constant probability if she still needed the good.

an off-platform friction. Suppose that it has a cumulative distribution function $H(\tau)$ on $[\underline{\tau}, \overline{\tau}]$ where $\underline{\tau} \geq 0$ and $\overline{\tau} < v - c$.

A natural intuition would be that benefits of transacting on the platform are equivalent to costs of transacting off the platform, and hence the effect of disintermediation should be the same as in our baseline analysis. However, our next result shows this is incorrect. The reason is that, while on-platform benefits arise irrespective of whether disintermediation is possible, off-platform costs only play a role when disintermediation is possible. Thus the two are not equivalent. In particular:

Proposition 16. *Suppose transacting off the platform incurs a cost for the buyer. The ability of buyers to disintermediate: (i) is neutral for the platform when buyers are homogeneous, or are heterogeneous only in their off-platform cost τ, but (ii) strictly reduces platform profit when buyers are heterogeneous in how often they wish to transact.*

Different from our baseline analysis, disintermediation can *never* strictly benefit the platform when buyers have heterogeneous costs of transacting off the platform. Intuitively, because buyers all obtain the same value from transacting *on* the platform, absent disintermediation the platform is able to fully extract all their surplus. As a result, there is no scope for the platform to extract further surplus—and thus no scope for it to use disintermediation as a way to better screen buyers.[30] Proposition 16 aligns with Hagiu and Wright (2024), who consider a model where transacting off a platform is costly, and find that disintermediation harms a platform.

Costs and Benefits We now assume that buyers obtain both benefits from transacting on the platform and incur costs from transacting off the platform. We show that the platform can again strictly benefit from disintermediation.

Consider again our baseline model, but now suppose that a buyer gets value $v + \psi b$ from transacting on the platform, and gets value $v - (1 - \psi)b$ from transacting off the platform, where $\psi \in (0, 1)$. Hence $(v + \psi b) - [v - (1 - \psi)b] = b$ now denotes a buyer's *net* benefit from transacting on the platform compared to off it.[31] We assume that $v - c > (1 - \psi)b$ for all buyers: since a buyer who wishes to transact off the platform needs to compensate the seller c, this condition ensures that each buyer gets strictly positive surplus when transacting off the platform.

Proposition 17. *Suppose buyers enjoy both benefits from transacting on the platform and incur costs of transacting off it; let $\psi \in (0, 1)$ denote the weight on benefits and $1 - \psi$ the weight on costs.*

[30] Specifically, suppose the off-platform cost τ is heterogeneous. In the proof we show that the platform is no worse off due to disintermediation, because it rebalances its prices as in our baseline analysis. However, unlike in Proposition 3, the platform cannot strictly gain from disintermediation.

[31] We therefore capture off-platform costs and on-platform benefits using a one-dimensional variable. This makes the analysis much more tractable than, say, a setting where a buyer's cost and benefit are not perfectly correlated and instead are drawn from a joint distribution.

The ability of buyers to disintermediate: (i) is neutral for the platform when buyers are homogeneous, (ii) strictly reduces platform profit when buyers are heterogeneous in purchase frequency, and (iii) weakly benefits the platform when buyers are heterogeneous in b *(and strictly so if* $\psi \geq 1/2$ *and* $\underline{b}f(\underline{b}) < 1$*).*

The proposition again shows that disintermediation can strictly benefit the platform—provided that buyers' net benefit b of using the platform is sufficiently heterogeneous (i.e., $\underline{b}f(\underline{b}) < 1$, as usual) and provided benefits provided by the platform are relatively large (i.e., $\psi \geq 1/2$).[32] As usual, the intuition is that disintermediation enables the platform to better screen buyers' private valuation of the service it provides to them.

C.5 Omitted Proofs

C.5.1 Seller Benefits

We first derive a seller's participation constraints, which are the analogue of the buyers' participation constraints (1) to (5) from the baseline analysis. A "one-time seller" completes her first (and only) transaction provided

$$p_{S,1} \geq c - r. \tag{45}$$

When disintermediation is impossible, a "two-time seller" does the first transaction if and only if

$$p_{S,1} - (c - r) + \max\{p_{S,2} - (c - r), 0\} \geq 0, \tag{46}$$

and then conditional on doing the first transaction, does the second one if and only if

$$p_{S,2} \geq c - r. \tag{47}$$

When disintermediation is possible, she does the first transaction if and only if

$$p_{S,1} - (c - r) + \max\{p_{S,2} - (c - r), v - c\} \geq 0, \tag{48}$$

because this gives her the option to then either do the second transaction on the platform and get $p_{S,2} - (c - r)$, or do it off the platform and forego the platform benefit r but charge the buyer v and hence get $v - c$. Conditional on doing the first transaction, the seller therefore does the second transaction on the platform if and only if

$$p_{S,2} \geq v - r, \tag{49}$$

and otherwise does it off the platform. Using the above we can prove Proposition 12.

[32] In the proof of Proposition 17 we show that these are only sufficient conditions, and indeed platform profit can strictly increase with disintermediation even when $\psi < 1/2$.

Proof of Proposition 12. Consider part (i). Notice that it is efficient for both transactions to occur on the platform, and that this generates total surplus of $2(v+r-c)$ (which is therefore the maximum possible profit the platform can achieve). Notice also that $p_{S,1} = 2c - r - v$ and $p_{S,2} = v - r$ satisfy (46) and (47), as well as (48) and (49), and hence induce the seller to do both transactions on the platform, and generate platform profit $2(v+r-c)$, irrespective of whether disintermediation is possible.

Now consider part (ii). First, suppose disintermediation is impossible. It is easy to see that by charging $p_{S,1} = p_{S,2} = c - r$ the platform hosts all $(2 - \phi)$ transactions, and extracts the maximum possible surplus $v + r - c$ on each one, giving a total platform profit of $(2 - \phi)(v + r - c)$. Since this is the maximal total surplus, the platform cannot do better. Next, suppose disintermediation is possible. Adapting the proof of Lemma 1, the optimum must have $p_{S,2} \geq v - r$ such that the platform hosts all second transactions. We can then proceed as in the proof of Proposition 2. (a) If $p_{S,1} < 2(c - r) - p_{S,2}$ no seller participates, and the platform earns zero profit. (b) If $2(c-r) - p_{S,2} \leq p_{S,1} < c - r$ then only sellers interested in transacting twice participate. Platform profit is then $(1 - \phi)(2v - p_{S,1} - p_{s,2})$ which is maximized at $p_{S,1} = 2(c - r) - p_{S,2}$, for a total profit of $2(1 - \phi)(v + r - c)$. (c) If $p_{S,1} \geq c - r$ then all sellers participate. Platform profit is then $v - p_{S,1} + (1 - \phi)(v - p_{S,2})$ which is maximized at $p_{S,1} = c - r$ and $p_{S,2} = v - r$, for a total profit of $v + r - c + (1 - \phi)r$. In all cases profit is strictly lower than $(2 - \phi)(v + r - c)$.

Now consider part (iii). Using (46) and (47), platform profit when disintermediation is impossible is

$$\pi^{ND} = \begin{cases} (v - p_{S,1})[1 - G(c - p_{S,1})] + (v - p_{S,2})[1 - G(c - p_{S,2})] & \text{if } p_{S,1} \geq p_{S,2}, \\ (2v - p_{S,1} - p_{S,2}) \left[1 - G\left(c - \frac{p_{S,1}+p_{S,2}}{2}\right)\right] & \text{otherwise.} \end{cases}$$

Using (48) and (49), platform profit when disintermediation is possible is

$$\pi^{D} = \begin{cases} (v - p_{S,1})[1 - G(2c - v - p_{S,1})] + (v - p_{S,2})[1 - G(v - p_{S,2})] & \text{if } p_{S,1} + 2(v - c) \geq p_{S,2}, \\ (2v - p_{S,1} - p_{S,2}) \left[1 - G\left(c - \frac{p_{S,1}+p_{S,2}}{2}\right)\right] & \text{otherwise.} \end{cases}$$

Using a change of variables $q_i = v + c - p_{S,i}$ for $i = 1, 2$ we can rewrite these as

$$\pi^{ND} = \begin{cases} (q_1 - c)[1 - G(q_1 - v)] + (q_2 - c)[1 - G(q_2 - v)] & \text{if } q_1 \leq q_2, \\ (q_1 + q_2 - 2c) \left[1 - G\left(\frac{q_1+q_2}{2} - v\right)\right] & \text{otherwise.} \end{cases}$$

$$\pi^{D} = \begin{cases} (q_1 - c)[1 - G(q_1 + c - 2v)] + (q_2 - c)[1 - G(q_2 - c)] & \text{if } q_1 \leq q_2 + 2(v - c), \\ (q_1 + q_2 - 2c) \left[1 - G\left(\frac{q_1+q_2}{2} - v\right)\right] & \text{otherwise.} \end{cases}$$

However, notice that if we replace q_i with $p_{B,i}$ for $i = 1, 2$, and replace G with F, the last expressions for π^{ND} and π^{D} coincide with equations (8) and (11) from the baseline model. Hence the maximized profit earned by the platform with and without disintermediation is the same in this extension as in the baseline model; the profit comparison in the proposition then follows. □

C.5.2 Bargaining Between Buyers and Sellers

We first derive buyer and seller participation decisions, starting with the buyers.

A one-time buyer completes her first (and only) transaction if and only if (1) holds. Now turn to a two-time buyer. Begin by considering the *second* transaction. If the buyer has the bargaining power, she can: propose no transaction, and obtain a zero payoff; propose to transact on the platform, drive the seller to her outside option by offering her $c + p_{S,2}$, and hence obtain $v+b-c-(p_{B,2}+p_{S,2})$; when disintermediation is possible, propose to transact off the platform, drive the seller to her outside option by offering her c, and hence obtain $v-c$. If the buyer does not have the bargaining power, the seller fully extracts her so she gets zero payoff on the second transaction, irrespective of whether disintermediation is possible. Therefore, when disintermediation is *not* possible, the buyer is willing to do the first transaction if and only if

$$U_B^{ND}(b) \equiv v + b - p_{B,1} + \alpha \max\{v + b - c - (p_{B,2} + p_{S,2}), 0\} \geq 0. \tag{50}$$

Conditional on the first transaction occurring, the second transaction occurs when the buyer has the bargaining power if and only if

$$p_{B,2} + p_{S,2} \leq v + b - c. \tag{51}$$

When disintermediation *is* possible, the buyer is willing to do the first transaction if and only if

$$U_B^{D}(b) \equiv v + b - p_{B,1} + \alpha \max\{v + b - c - (p_{B,2} + p_{S,2}), v - c\} \geq 0. \tag{52}$$

Conditional on the first transaction occurring, the second transaction occurs on the platform when the buyer has the bargaining power if

$$p_{B,2} + p_{S,2} \leq b, \tag{53}$$

and otherwise occurs off the platform.

Now consider the seller's participation decision. Recall that at the time of the first transaction the seller does not know whether the buyer is a one- or two-time buyer, nor does she know the buyer's b. Again begin by considering the second transaction (for a seller dealing with a two-time buyer). If the seller has the bargaining power, she can: propose no transaction, and obtain a zero payoff; propose to transact on the platform, drive the buyer to her outside option by charging her a price of $v+b-p_{B,2}$, and hence obtain $v+b-c-(p_{B,2}+p_{S,2})$; when disintermediation is possible, propose to transact off the platform, drive the buyer to her outside option by charging her v, and hence obtain $v-c$. If the seller does not have the bargaining power, the buyer fully extracts her so she gets zero payoff on the second transaction irrespective of whether disintermediation is possible. Therefore, when disintermediation is *not* possible, the seller is willing to do the first transaction if and only if

$$-p_{S,1} - c + (1-\alpha)\mathbb{E}_b \left[\Pr(TT) \max\{v + b - c - (p_{B,2} + p_{S,2}), 0\}\right] \geq 0, \tag{54}$$

where $\Pr(TT)$ is the conditional probability (given platform prices) that a buyer who does the first transaction is a two-time buyer. (If no buyer does the first transaction, this seller constraint is moot so we can set $\Pr(TT) = 0$ without loss.) Conditional on the first transaction occurring, the second transaction occurs when the seller has the bargaining power if and only if (51) holds. When disintermediation *is* possible, the seller is willing to do the first transaction if and only if

$$-p_{S,1} - c + (1-\alpha)\mathbb{E}_b \left[\Pr(TT) \max\{v + b - c - (p_{B,2} + p_{S,2}), v - c\}\right] \geq 0, \tag{55}$$

where again where $\Pr(TT)$ is the conditional probability of the seller encountering a two-time buyer. Conditional on the first transaction occurring, the second transaction occurs on the platform when the seller has the bargaining power if (53) holds, and otherwise occurs off the platform. We can now prove Proposition 13.

Proof of Proposition 13. Consider part (i). Notice that it is efficient for both transactions to occur on the platform, and that this generates total surplus of $2(v + b - c)$. Notice also that $p_{B,1} = v + b + \alpha(v - c)$, $p_{S,1} = (1-\alpha)v - (2-\alpha)c$, and $p_{B,2} + p_{S,2} = b$ satisfy (50) to (55), and hence induce the buyer and seller to do both transactions on the platform, and moreover generate platform profit $2(v + b - c)$, irrespective of whether disintermediation is possible.[33]

Now consider part (ii). It is socially efficient for all transactions to occur on the platform, leading to total surplus of $(2-\phi)(v + b - c)$. First, suppose disintermediation is impossible. It is easy to check that $p_{B,1} = v + b$, $p_{S,1} = -c$, and $p_{B,2} + p_{S,2} = v + b - c$ satisfy (1), (50), (51), and (54). Hence these prices induce the buyer and seller to do all transactions on the platform, and moreover they enable the platform to extract the maximal possible surplus $(2-\phi)(v + b - c)$. Next, suppose disintermediation is possible. Using the same argument as in Lemma 1, the optimum must have $p_{B,2} + p_{S,2} \leq b$: if not, (53) would be violated, so the platform would make zero profit from second transactions, whereas if it deviated and set $p_{B,2} + p_{S,2}$ equal to b the first-transaction participation constraints (52) and (55) would be unchanged, but the platform would host (and hence profit from) second transactions. We now derive platform profit for different values of $p_{B,1}$ and $p_{S,1}$. First, if $p_{B,1} > v + b + \alpha[v + b - c - (p_{B,2} + p_{S,2})]$ then no buyer does the first transaction and the platform therefore earns zero profit. Second, if $v + b < p_{B,1} \leq v + b + \alpha[v + b - c - (p_{B,2} + p_{S,2})]$ then two-time but not one-time buyers do the first transaction. If the platform does not satisfy the

[33] Note that if v and c are such that $p_{S,1} < 0$, the platform pays the seller to do the first transaction.

seller's participation constraint (55) it earns zero profit. Otherwise it seeks to

$$\begin{aligned} &\max_{\{p_{B,1},p_{B,2},p_{S,1},p_{S,2}\}} (1-\phi)(p_{B,1}+p_{B,2}+p_{S,1}+p_{S,2}) \\ \text{s.t.} \quad &\text{(i)} \quad p_{S,1} \leq -c+(1-\alpha)[v+b-c-(p_{B,2}+p_{S,2})] \\ &\text{(ii)} \quad p_{B,2}+p_{S,2} \leq b \\ &\text{(iii)} \quad p_{B,1} \leq v+b+\alpha[v+b-c-(p_{B,2}+p_{S,2})]. \end{aligned}$$

However, adding constraints (i) and (iii) together gives $p_{B,1}+p_{B,2}+p_{S,1}+p_{S,2} \leq 2(v+b-c)$ and so given any $p_{B,2}$ and $p_{S,2}$ that satisfy (ii), the platform sets $p_{S,1}$ and $p_{B,1}$ as high as possible to make (i) and (iii) bind, and consequently earns $2(1-\phi)(v+b-c)$. Third, if $p_{B,1} \leq v+b$ then all buyers do the first transaction. If the platform does not satisfy the seller's participation constraint (55) it again earns zero profit. Otherwise it seeks to

$$\begin{aligned} &\max_{\{p_{B,1},p_{B,2},p_{S,1},p_{S,2}\}} p_{B,1}+p_{S,1}+(1-\phi)(p_{B,2}+p_{S,2}) \\ \text{s.t.} \quad &\text{(i)} \quad p_{S,1} \leq -c+(1-\alpha)(1-\phi)[v+b-c-(p_{B,2}+p_{S,2})] \\ &\text{(ii)} \quad p_{B,2}+p_{S,2} \leq b \\ &\text{(iii)} \quad p_{B,1} \leq v+b. \end{aligned}$$

Given any choice of $p_{B,2}+p_{S,2}$ constraints (i) and (iii) must bind for profit to be maximized; making them bind and substituting them, the platform's problem becomes

$$\max_{\{p_{B,2},p_{S,2}\}} (v+b-c)[1+(1-\alpha)(1-\phi)]+\alpha(1-\phi)(p_{B,2}+p_{S,2}) \quad \text{s.t.} \quad p_{B,2}+p_{S,2} \leq b.$$

Hence the platform sets $p_{B,2}+p_{S,2}=b$, for a total profit of $(2-\phi)(v+b-c)-\alpha(1-\phi)(v-c)$. Summing up, when disintermediation is possible platform profit is

$$\max\{2(1-\phi)(v+b-c),(2-\phi)(v+b-c)-\alpha(1-\phi)(v-c)\}$$

which is strictly less than the profit $(2-\phi)(v+b-c)$ earned when disintermediation is impossible, except at $\alpha=0$ where the two are equal.

Now consider part (iii).

First, suppose disintermediation is impossible. Notice that given prices $p_{B,1}$ and $p_{B,2}$ buyers follow a threshold strategy. Therefore define thresholds b_1^{ND} and b_2^{ND} satisfying $b_1^{ND} \leq b_2^{ND}$ such that buyers do the first transaction if and only if $b \geq b_1^{ND}$ and do the second transaction if and only if $b \geq b_2^{ND}$. Then, assuming $b_1^{ND} < \bar{b}$, the seller's participation constraint (54) becomes

$$p_{S,1} \leq -c+(1-\alpha)\frac{\int_{b_2^{ND}}^{\bar{b}}[v+b-c-(p_{B,2}+p_{S,2})]dF(b)}{1-F(b_1^{ND})}, \tag{56}$$

and conditional on this holding, platform profit is

$$(p_{B,1}+p_{S,1})[1-F(b_1^{ND})]+(p_{B,2}+p_{S,2})[1-F(b_2^{ND})]. \tag{57}$$

Since profit is increasing in $p_{S,1}$, the platform should make the seller's participation constraint bind by setting $p_{S,1}$ as high as possible. Hence platform profit equals

$$(p_{B,1}-c)[1-F(b_1^{ND})]+\int_{b_2^{ND}}^{\bar{b}}(1-\alpha)[v+b-c]dF(b)+\alpha(p_{B,2}+p_{S,2})[1-F(b_2^{ND})]. \tag{58}$$

There are then two subcases to consider. (a) Consider $p_{B,1}\le p_{B,2}+p_{S,2}+c$. It is easy to see from (50) and (51) that $b_1^{ND}=p_{B,1}-v$ and $b_2^{ND}=p_{B,2}+p_{S,2}+c-v$. Hence we can rewrite platform profit as

$$\begin{aligned}&(p_{B,1}-c)[1-F(p_{B,1}-v)]+(1-\alpha)\int_{p_{B,2}+p_{S,2}+c-v}^{\bar{b}}[v+b-c]dF(b)\\&+\alpha(p_{B,2}+p_{S,2})[1-F(p_{B,2}+p_{S,2}+c-v)].\end{aligned} \tag{59}$$

One can check that, if we ignore the constraint $p_{B,1}\le p_{B,2}+p_{S,2}+c$, the above expression is quasiconcave in $p_{B,1}$ and in $p_{B,2}+p_{S,2}$ given the log-concavity of $1-F$, and that its derivative with respect to $p_{B,1}$ is weakly larger than its derivative with respect to $p_{B,2}+p_{S,2}$ when evaluated at $p_{B,1}=p_{B,2}+p_{S,2}+c$. However, this implies that the constraint must bind (and so $b_1^{ND}=b_2^{ND}$). Hence, substituting in $p_{B,1}=p_{B,2}+p_{S,2}+c$, we can write platform profit as

$$(1+\alpha)(p_{B,1}-c)[1-F(p_{B,1}-v)]+(1-\alpha)\int_{p_{B,1}-v}^{\bar{b}}[v+b-c]dF(b). \tag{60}$$

(b) Consider $p_{B,1}>p_{B,2}+p_{S,2}+c$. It is easy to see from (50) and (51) that $b_1^{ND}=b_2^{ND}=\frac{p_{B,1}+\alpha[p_{B,2}+p_{S,2}+c]}{1+\alpha}-v$. Hence we can rewrite platform profit as

$$\begin{aligned}&(1+\alpha)\left(\frac{p_{B,1}+\alpha[p_{B,2}+p_{S,2}+c]}{1+\alpha}-c\right)\left[1-F\left(\frac{p_{B,1}+\alpha[p_{B,2}+p_{S,2}+c]}{1+\alpha}-v\right)\right]\\&+(1-\alpha)\int_{\frac{p_{B,1}+\alpha[p_{B,2}+p_{S,2}+c]}{1+\alpha}-v}^{\bar{b}}[v+b-c]dF(b).\end{aligned} \tag{61}$$

However, notice that (60) and (61) take the same form. Hence, following the same approach as in the proof of Lemma 2, there is a continuum of optimal prices which lead to profit

$$(1+\alpha)(v+b^{ND}-c)[1-F(b^{ND})]+(1-\alpha)\int_{b^{ND}}^{\bar{b}}[v+b-c]dF(b), \tag{62}$$

where $b_1^{ND}=b_2^{ND}=b^{ND}$, and where $b^{ND}=\underline{b}$ if $1+\alpha-2f(\underline{b})(v+\underline{b}-c)\le 0$ and otherwise b^{ND} is the unique solution to $[1-F(b^{ND})](1+\alpha)-2(v+b^{ND}-c)f(b^{ND})=0$.

Next, suppose disintermediation is possible. Again note that buyers follow a threshold strategy, and define b_1^D and b_2^D satisfying $b_1^D\le b_2^D$ such that buyers do the first transaction if and only if

$b \geq b_1^D$ and do the second transaction on the platform if and only if $b \geq b_2^D$. Then, assuming that $b_1^D < \bar{b}$, the seller's participation constraint (55) becomes

$$p_{S,1} \leq -c + (1-\alpha)\frac{(v-c)[F(b_2^D) - F(b_1^D)] + \int_{b_2^D}^{\bar{b}}[v+b-c-(p_{B,2}+p_{S,2})]dF(b)}{1-F(b_1^D)}. \tag{63}$$

Conditional on this holding, platform profit is still given by (57) after replacing b_1^{ND} and b_2^{ND} with b_1^D and b_2^D respectively. Since this is increasing in $p_{S,1}$ the platform should make the seller's participation constraint bind, which leads to platform profit

$$\begin{aligned}&(p_{B,1}-c)[1-F(b_1^D)] + (1-\alpha)(v-c)[F(b_2^D) - F(b_1^D)]\\ &+ \int_{b_2^D}^{\bar{b}}(1-\alpha)[v+b-c]dF(b) + \alpha(p_{B,2}+p_{S,2})[1-F(b_2^D)].\end{aligned} \tag{64}$$

There are again two subcases to consider. (a) Consider $p_{B,1} > p_{B,2} + p_{S,2} + v + \alpha(v-c)$. It is easy to see from (52) and (53) that $b_1^D = b_2^D$ and that profit is exactly the same as (61). (b) Consider $p_{B,1} \leq p_{B,2} + p_{S,2} + v + \alpha(v-c)$. It is easy to see from (52) and (53) that $b_1^D = p_{B,1} - v - \alpha(v-c)$ and $b_2^D = p_{B,2} + p_{S,2}$ and hence platform profit is

$$\begin{aligned}&(p_{B,1}-c)[1-F(p_{B,1}-v-\alpha(v-c))] + (1-\alpha)(v-c)[F(p_{B,2}+p_{S,2}) - F(p_{B,1}-v-\alpha(v-c))]\\ &+ \int_{p_{B,2}+p_{S,2}}^{\bar{b}}(1-\alpha)[v+b-c]dF(b) + \alpha(p_{B,2}+p_{S,2})[1-F(p_{B,2}+p_{S,2})].\end{aligned} \tag{65}$$

Notice that if the platform sets $p_{B,1} = v + b^{ND} + \alpha(v-c)$ and $p_{B,2}+p_{S,2} = b^{ND}$ it makes the pricing constraint just bind (i.e., $p_{B,1} = p_{B,2} + p_{S,2} + v + \alpha(v-c)$) and it earns the same profit (62) as it did when disintermedition was impossible. One can also check that, starting from these prices, the derivative of profit with respect to $p_{B,2} + p_{S,2}$ is $\alpha[1 - F(b^{ND})] - b^{ND} f(b^{ND})$. If $\underline{b} f(\underline{b}) \geq 1$ this is negative for all α and all $b^{ND} \geq \underline{b}$ given log-concavity of $1-F$. If $\underline{b} f(\underline{b}) < 1$ and $\alpha = 1$, the derivative is strictly positive: for $b^{ND} = \underline{b}$ this is immediate, and for $b^{ND} > \underline{b}$ this follows from the equation which determines b^{ND}. By continuity, if $\underline{b} f(\underline{b}) < 1$ and α is above a threshold, the derivative remains strictly positive. (The threshold must be strictly positive, since the derivative is weakly negative at $\alpha = 0$.) But then the platform does strictly better than when disintermediation is impossible.[34] □

C.5.3 General Number of Transactions

We first derive the optimal behavior of a buyer who wishes to transact n times in total. When disintermediation is impossible, let $V^{ND}(k)$ be the buyer's payoff from transacting k times on the

[34] Unfortunately, for a general distribution it is not possible to establish uniqueness of the cutoff $\breve{\alpha}$ in the proposition. Nevertheless, in some cases we can establish uniqueness, and also show that $\breve{\alpha}$ is small. For example, suppose b is uniformly distributed on $[0, 1]$. One can check that for $v - c \geq 1$ we have $b^{ND} = 0$, and that $\breve{\alpha}$ is unique and equal to 0: disintermediation strictly benefits the platform whenever buyers have some positive bargaining power.

platform. Note that $V^{ND}(0) = 0$ and $V^{ND}(k) = \sum_{j=1}^{k}(v + b - p_{B,j})$ for $1 \leq k \leq n$. When disintermediation is possible, let $V^D(k)$ be the buyer's payoff from transacting k times on the platform and $n - k$ times off the platform. Note that $V^D(0) = n(v - c)$ and $V^D(k) = \sum_{j=1}^{k}(v + b - p_{B,j}) + (n - k)(v - c)$ for $1 \leq k \leq n$. Given our tie-break rule, the buyer chooses the largest $k \in \{0, 1, \ldots, n\}$ that maximizes $V^{ND}(k)$ or $V^D(k)$, depending on whether disintermediation is possible.

Proof of Proposition 14. Consider part (i). Let n be the (common) number of times that buyers wish to transact, and b be the (common) platform benefit. Notice that it is efficient for all n transactions to occur on the platform, and that this generates total surplus of $n(v + b - c)$ (which is therefore the maximum possible profit the platform can achieve). Notice also that prices $p_{B,1} = v + b + (n - 1)(v - c)$ and $p_{B,2} = \cdots = p_{B,n} = b + c$ induce buyers to do all n transactions on the platform, and generate profit of $n(v + b - c)$, irrespective of whether disintermediation is possible.

Now consider part (ii). Let b be buyers' (common) platform benefit. First, suppose disintermediation is impossible. It is easy to see that by charging $p_{B,1} = \cdots = p_{B,N} = v + b$ a buyer who wishes to do $n = 1, \ldots, N$ total transactions does all n of them on the platform, and so the platform earns $\sum_{j=1}^{N}(\phi_j j)(v + b - c)$. Since this is the maximal total surplus, the platform cannot do better. Next, suppose disintermediation is possible. We argue that platform profit is strictly below $\sum_{j=1}^{N}(\phi_j j)(v + b - c)$. On the way to a contradiction, suppose the platform can earn $\sum_{j=1}^{N}(\phi_j j)(v + b - c)$. Since buyers have heterogeneous n, there must exist j' and $j'' > j'$ such that $\phi_{j'} > 0$ and $\phi_{j''} > 0$. Moreover buyers with $n = j'$ and $n = j''$ must do all their transactions on the platform, and in addition they must be fully extracted. The latter requires that

$$\sum_{i=1}^{j'} p_{B,i} = j'(v + b) \quad \text{and} \quad \sum_{i=1}^{j''} p_{B,i} = j''(v + b).$$

However, in that case a buyer with $n = j''$ will not do all j'' transactions on the platform: $V^D(j'') = 0 < (j'' - j')(v - c) = V^D(j')$. This yields a contradiction, and hence platform profit is strictly less than $\sum_{j=1}^{N}(\phi_j j)(v + b - c)$.

Now consider part (iii). Let n be the (common) number of desired transactions.

First, suppose disintermediation is impossible. Notice that for given $(p_{B,1}, \ldots, p_{B,n})$, if a buyer with benefit b prefers to do j'' transactions rather than $j' < j''$ transactions, so do all buyers with benefit above b. Hence buyers follow a threshold rule. Introduce thresholds $b_1^{ND} \leq b_2^{ND} \leq \cdots \leq b_n^{ND}$ satisfying $b_1^{ND} \geq \underline{b}$ and $b_n^{ND} \leq \bar{b}$, such that a buyer transacts zero times if $b < b_1^{ND}$, transacts $k = 1, \ldots, n - 1$ times if $b \in [b_k^{ND}, b_{k+1}^{ND})$, and transacts n times if $b \geq b_n^{ND}$. We claim that if $b_1^{ND} = \cdots = b_n^{ND} = \tilde{b}$ then prices satisfy

$$\sum_{i=1}^{n} p_{B,i} = n(v + \tilde{b}), \quad \text{and} \quad \sum_{i=j+1}^{n} p_{B,i} \leq (n - j)(v + \tilde{b}) \ \text{ for all } j = 1, \ldots, n - 1. \tag{66}$$

The equality in (66) says that the marginal buyer type $\tilde{b}$ is fully extracted across the n transactions: we must have $\sum_{i=1}^{n} p_{B,i} \leq n(v+\tilde{b})$, otherwise a buyer with $b=\tilde{b}$ will not do all n transactions, but if the inequality is strict then either $\tilde{b}=\underline{b}$ and the platform could do better by raising some prices and still host all buyers, or $\tilde{b}>\underline{b}$ and types slightly below $\tilde{b}$ would also wish to do the n transactions which contradicts $\tilde{b}$ being the marginal type. The inequality in (66) says that having completed $j=1,\ldots,n-1$ transactions, it is worthwhile for all buyers weakly above $\tilde{b}$ to do the remaining $n-j$ transactions. Hence if $b_1^{ND}=\cdots=b_n^{ND}=\tilde{b}$ platform profit equals

$$\left[\sum_{i=1}^{n}(p_{B,i}-c)\right][1-F(\tilde{b})]=\left(\sum_{i=1}^{n}p_{B,i}-nc\right)\left[1-F\left(\frac{\sum_{i=1}^{n}p_{B,i}}{n}-v\right)\right],$$

and using the same approach as in the proof of Lemma 2, its maximized value is $n(v+b^{ND}-c)[1-F(b^{ND})]$ where b^{ND} is the same as the one defined in Lemma 2. Next, we claim that the platform optimum must have $b_1^{ND}=\cdots=b_n^{ND}$. On the way to a contradiction, suppose not all the thresholds are identical. Then, we can partition them into $m\geq 2$ sets, $(\mathcal{B}_1,\ldots,\mathcal{B}_m)$, where all $i\in\mathcal{B}_j$ have the same threshold, but different partitions are associated with different thresholds. Using the same argument as above, in partition $\mathcal{B}_j$ we must have $\sum_{i\in\mathcal{B}_j}p_{B,i}=|\mathcal{B}_j|(v+b_{i\in\mathcal{B}_j})$ where $|\mathcal{B}_j|$ is the cardinality of that partition and $b_{i\in\mathcal{B}_j}$ is the value of the threshold in that partition. (Also, if the partition is not a singleton, prices associated with the thresholds in the partition must satisfy an inequality like the one in (66).) Hence we can write platform profit as

$$\sum_{j=1}^{m}\left[\sum_{i\in\mathcal{B}_j}(p_{B,i}-c)[1-F(b_{i\in\mathcal{B}_j})]\right]=\sum_{j=1}^{m}|\mathcal{B}_j|\left[\left(\frac{\sum_{i\in\mathcal{B}_j}p_{B,i}}{|\mathcal{B}_j|}-c\right)\left[1-F\left(\frac{\sum_{i\in\mathcal{B}_j}p_{B,i}}{|\mathcal{B}_j|}-v\right)\right]\right].$$

However, notice that each term in the (outer) summation takes the same form, namely $(X-c)[1-F(X-v)]$. Hence the expression for platform profit is maximized when $\sum_{i\in\mathcal{B}_j}p_{B,i}/|\mathcal{B}_j|$ is the same for each j. But this means that $b_{i\in\mathcal{B}_j}$ is the same for each j, which is a contradiction. Hence the optimum must have $b_1^{ND}=\cdots=b_n^{ND}$.

Summing up, when disintermediation is impossible, the platform earns $n(v+b^{ND}-c)[1-F(b^{ND})]$ where b^{ND} is the same as the one defined in Lemma 2.

Next, suppose disintermediation is possible. To show that the platform earns (weakly) higher profit than above, it is sufficient to allow for two prices: $p_{B,1}$ for the first transaction and, abusing notation, the same $p_{B,>1}$ for each subsequent transaction. If $p_{B,1}\leq p_{B,>1}+n(v-c)$ then one can check that platform profit equals

$$(p_{B,1}-c)[1-F(p_{B,1}-nv+(n-1)c)]+(n-1)(p_{B,>1}-c)[1-F(p_{B,>1}-c)]. \tag{67}$$

Hence, if the platform charges $p_{B,1}=b^{ND}+nv-(n-1)c$ and $p_{B,>1}=b^{ND}+c$ it earns $n(v+b^{ND}-c)[1-F(b^{ND})]$, which is the same profit as it earned absent disintermediation. Hence the platform is not harmed by disintermediation. Moreover, the derivative of the above profit expression with respect to $p_{B,>1}$, evaluated at $p_{B,>1} = b^{ND} + c$, is $n-1$ multiplied by

$$1 - F(b^{ND}) - b^{ND} f(b^{ND}). \tag{68}$$

We claim this is strictly positive if $\underline{b} f(\underline{b}) < 1$. The claim is immediate if $b^{ND} = \underline{b}$. If instead $b^{ND} > \underline{b}$ then, using equation (10), the expression simplifies to $f(b^{ND})(v-c) > 0$. Hence, starting from prices $p_{B,1} = b^{ND} + nv - (n-1)c$ and $p_{B,>1} = b^{ND} + c$, if $\underline{b} f(\underline{b}) < 1$ the platform can earn strictly higher profit by raising $p_{B,>1}$. □

Finally, we turn to the case where, in addition to having a general number of transactions n, buyers are also ex ante uncertain about how many times they will ultimately need to transact. We first derive a buyer's participation constraints. When disintermediation is impossible, conditional on doing the first transaction, the buyer does each subsequent transaction that she wishes to do if and only if

$$v + b \geq p_{B,>1}. \tag{69}$$

Moreover, the buyer does the first transaction if and only if

$$v + b - p_{B,1} + \sum_{n=2}^{N} \phi_n (n-1) \max\{v + b - p_{B,>1}, 0\} \geq 0, \tag{70}$$

where the final term on the left-hand side is expected surplus on transactions beyond the first one. When instead disintermediation is possible, conditional on doing the first transaction, the buyer does each subsequent transaction that she wishes to do on the platform if

$$b \geq p_{B,>1} - c, \tag{71}$$

and otherwise does it off the platform. Moreover, the buyer does the first transaction if and only if

$$v + b - p_{B,1} + \sum_{n=2}^{N} \phi_n (n-1) \max\{v + b - p_{B,>1}, v - c\} \geq 0. \tag{72}$$

Using the above we can now prove Proposition 15.

Proof of Proposition 15. First, suppose there is no heterogeneity in b. Notice that total surplus is maximized when all transactions occur on the platform, leading to surplus $\sum_{n=1}^{N} \phi_n n(v+b-c)$. We will prove that, irrespective of whether disintermediation is possible, the platform can extract this full surplus. For example, suppose it sets $p_{B,1} = v + b + \sum_{n=2}^{N} \phi_n (n-1)(v-c)$ and $p_{B,>1} = b + c$. Using (69)-(72) one can check that buyers do the first transaction, and then each subsequent time they need the service they again transact on the platform. Hence the platform earns profit

$$p_{B,1} - c + \sum_{n=2}^{N} \phi_n (n-1)(p_{B,>1} - c) = \sum_{n=1}^{N} \phi_n n(v + b - c),$$

which as noted above is the maximum possible profit it could ever achieve.

Second, suppose buyers have heterogeneous b. When disintermediation is impossible platform profit equals

$$\pi^{ND} = \begin{cases} (p_{B,1}-c)[1-F(p_{B,1}-v)] + \sum_{n=2}^{N} \phi_n (n-1)(p_{B,>1}-c)[1-F(p_{B,>1}-v)] & \text{if } p_{B,1} \leq p_{B,>1}, \\ [p_{B,1}-c+\sum_{n=2}^{N} \phi_n(n-1)(p_{B,>1}-c)]\left[1-F\left(\frac{p_{B,1}+\sum_{n=2}^{N}\phi_n(n-1)p_{B,>1}}{\sum_{n=1}^{N}\phi_n n}-v\right)\right] & \text{otherwise.} \end{cases} \tag{73}$$

(First, consider $p_{B,1} \leq p_{B,>1}$. Using (69) and (70), buyers with $b < p_{B,1} - v$ do not transact at all, buyers with $b \geq p_{B,>1} - v$ do all desired transactions, and the remaining buyers do only the first transaction. Second, consider $p_{B,1} > p_{B,>1}$. Using (69) and (70), buyers with $b < \frac{p_{B,1}+\sum_{n=2}^{N}\phi_n(n-1)p_{B,>1}}{\sum_{n=1}^{N}\phi_n n} - v$ do no transactions and all remaining buyers do all desired transactions.) Following the same approach as in the proof of Proposition 3, we can show that this is maximized by prices satisfying $p_{B,1} \geq p_{B,>1}$ and $p_{B,1}+\sum_{n=2}^{N}\phi_n(n-1)p_{B,>1} = \sum_{n=1}^{N}\phi_n n(v+b^{ND})$, where $b^{ND} = \underline{b}$ if $(v+\underline{b}-c)f(\underline{b}) \geq 1$ and otherwise b^{ND} is the unique solution to $1-F(b^{ND}-(v+b^{ND}-c)f(b^{ND}) = 0$. This generates a profit $\sum_{n=1}^{N}\phi_n n(v+b^{ND}-c)[1-F(b^{ND})]$.

When instead disintermediation is possible platform profit equals

$$\pi^{D} = \begin{cases} (p_{B,1}-c)[1-F(\zeta)] + \sum_{n=2}^{N} \phi_n (n-1)(p_{B,>1}-c)[1-F(p_{B,>1}-c)] & \text{if } p_{B,1} \leq \eta, \\ \pi^{ND} & \text{otherwise,} \end{cases} \tag{74}$$

where in the first line $\zeta = p_{B,1} - v - \sum_{n=2}^{N}\phi_n(n-1)(v-c)$ and $\eta = p_{B,>1} + \sum_{n=1}^{N}\phi_n n(v-c)$, and in the second line we have written π^{ND} for brevity. (First, consider $p_{B,1} \leq \eta$. Using (71) and (72), buyers with $b < \zeta$ do no transactions, buyers with $b \geq p_{B,>1} - c$ do all desired transactions on the platform, and all other buyers do the first transaction and then disintermediate for any subsequent transaction they wish to make. Second, consider $p_{B,1} > \eta$. Using (71) and (72), buyers with $b < \frac{p_{B,1}+\sum_{n=2}^{N}\phi_n(n-1)p_{B,>1}}{\sum_{n=1}^{N}\phi_n n} - v$ do no transactions and all remaining buyers do all desired transactions on the platform.)

We first show that platform profit is at least as high with disintermediation as it is without it. For example, suppose the platform sets $p_{B,1} = \sum_{n=1}^{N}\phi_n nv - \sum_{n=2}^{N}\phi_n(n-1)c + b^{ND}$ and $p_{B,>1} = b^{ND}+c$. Using (74) one can check that the platform earns $\sum_{n=1}^{N}\phi_n n(v+b^{ND}-c)[1-F(b^{ND})]$, i.e., the same as it did absent disintermediation. To complete the proof, we show that when $\underline{b}f(\underline{b}) < 1$ the platform is strictly better off with disintermediation. In particular, suppose the platform again sets $p_{B,1} = \sum_{n=1}^{N}\phi_n nv - \sum_{n=2}^{N}\phi_n(n-1)c + b^{ND}$ but now sets $p_{B,>1} \geq b^{ND} + c$. Following the same approach as in, e.g., the proof of Proposition 3, profit is strictly increasing in $p_{B,>1}$ around $p_{B,>1} = b^{ND} + c$. □

C.5.4 Costly Off-Platform Transactions

We start with the case where buyers only incur costs of transacting off the platform. As a first step we derive a buyer's optimal purchase behavior. If the buyer wishes to transact only once she transacts if and only if $v \geq p_{B,1}$. If the buyer wishes to transact twice and disintermediation is impossible, she does the first transaction if and only if

$$v - p_{B,1} + \max\{v - p_{B,2}, 0\} \geq 0, \tag{75}$$

and conditional on doing the first transaction, she then does the second one if and only if $v \geq p_{B,2}$. If instead the buyer wishes to transact twice and disintermediation is possible, she does the first transaction if and only if

$$v - p_{B,1} + \max\{v - p_{B,2}, v - \tau - c\}. \tag{76}$$

Conditional on doing the first transaction, she does the second transaction on the platform if $\tau \geq p_{B,2} - c$ and otherwise does it off the platform.

We can then use the above prove Proposition 16.

Proof of Proposition 16. First, note that when disintermediation is impossible the platform can set $p_{B,1} = p_{B,2} = v$, induce buyers to do all their transactions on the platform, and hence earn profit $(2 - \phi)(v - c)$. (This is true for all $\phi \in [0, 1)$ and irrespective of whether τ is heterogeneous.) Since it is efficient to do all transactions on the platform, the platform cannot do better than this. Second, suppose now that disintermediation is possible. Again, notice that because it is efficient to do all transactions on the platform, the platform cannot earn strictly more than $(2 - \phi)(v - c)$. (i) Consider the case where buyers all wish to transact twice. The platform can set $p_{B,1} = 2v - \underline{\tau} - c$ and $p_{B,2} = \underline{\tau} + c$, fully extract buyers on the first transaction and induce them to do the second transaction on the platform, giving platform profit of $2(v - c)$. Notice that this holds both when buyers are completely homogeneous (such that $\underline{\tau} = \overline{\tau}$) or when they differ in their off-platform cost (such that $\underline{\tau} < \overline{\tau}$). Hence in these cases disintermediation is neutral for the platform. (ii) Consider the case where $\phi \in (0, 1)$ buyers wish to transact once and $1 - \phi$ buyers wish to transact twice. In order to fully extract one-time buyers the platform must charge $p_{B,1} = v$, and in order to fully extract two-time buyers the platform must charge $p_{B,1} = 2v - p_{B,2}$ and $p_{B,2} \leq \tau + c$. But notice that $p_{B,2} \leq \tau + c$ implies that $2v - p_{B,2} \geq 2v - \tau - c > v$ so it is impossible to fully extract both types of buyer. Hence the platform is strictly worse off with disintermediation in this case. □

We now turn to the case where buyers obtain both benefits of transacting on the platform and incur costs of transacting off it. Again, as a first step, we begin by deriving buyers' participation decisions. A one-time buyer will do the transaction if and only if

$$v + \psi b \geq p_{B,1}. \tag{77}$$

Meanwhile, when disintermediation is impossible, conditional on having done the first transaction, a two-time buyer will also do the second transaction if

$$v + \psi b \geq p_{B,2}, \tag{78}$$

and will do the first transaction if and only if

$$v + \psi b - p_{B,1} + \max\{v + \psi b - p_{B,2}, 0\} \geq 0. \tag{79}$$

When instead disintermediation is possible, conditional on having done the first transaction, a two-time buyer will do the second transaction on the platform if

$$v + \psi b - p_{B,2} \geq v - c - (1 - \psi)b \iff b \geq p_{B,2} - c, \tag{80}$$

and otherwise will do it off the platform; she will do the first transaction if and only if

$$v + \psi b - p_{B,1} + \max\{v + \psi b - p_{B,2}, v - c - (1 - \psi)b\} \geq 0. \tag{81}$$

Using the above results we can now prove Proposition 17.

Proof of Proposition 17. First, suppose buyers are homogenegeous, i.e., they all wish to transact twice and have the same b. Notice that it is efficient for all buyers to do both transactions on the platform, resulting in total surplus $2(v + \psi b - c)$. We now argue that the platform can achieve this irrespective of whether disintermediation is possible. For example, suppose the platform sets $p_{B,1} = 2v + (2\psi - 1)b - c$ and $p_{B,2} = b + c$. Using (78)-(81) one can check that buyers do both transactions on the platform, leading to platform profit $p_{B,1} + p_{B,2} - 2c = 2(v + \psi b - c)$. Hence disintermediation is neutral for the platform.

Second, suppose buyers all have the same b but differ in how many times they wish to purchase the service. Notice that it is again efficient for all buyers to do all their transactions on the platform, resulting in total surplus $(2 - \phi)(v + \psi b - c)$. It is easy to check from (77)-(79) that, when disintermediation is impossible, by charging $p_{B,1} = p_{B,2} = v + \psi b$ the platform induces all buyers to do all transactions, and it earns profit $(2 - \phi)(v + \psi b - c)$. We now argue that when disintermediation is possible the platform earns strictly less, and hence is strictly worse off. Notice that in order to fully extract all surplus from one-time buyers they must transact on the platform and pay $p_{B,1} = v + \psi b$. Notice also that in order to fully transact all surplus from two-time buyers they must do both transactions on the platform and pay $p_{B,1} + p_{B,2} = 2(v + \psi b)$. However these are incompatible—they require $p_{B,2} = v + \psi b$ but then from (80) a two-time buyer will only do the second transaction if $v - c \leq b(1 - \psi)$, which by assumption does not hold. Hence the platform cannot fully extract the full surplus from each buyer type, and thus earns strictly lower profit.

Third, suppose buyers all wish to transact twice but have heterogeneous b. When disintermediation is impossible, we can write platform profit as

$$\pi^{ND} = \begin{cases} (p_{B,1} - c)\left[1 - F\left(\frac{p_{B,1}-v)}{\psi}\right)\right] + (p_{B,2} - c)\left[1 - F\left(\frac{p_{B,2}-v}{\psi}\right)\right] & \text{if } p_{B,1} \leq p_{B,2}, \\ (p_{B,1} + p_{B,2} - 2c)\left[1 - F\left(\frac{p_{B,1}+p_{B,2}-2v}{2\psi}\right)\right] & \text{otherwise.} \end{cases} \tag{82}$$

(First, consider $p_{B,1} \leq p_{B,2}$. Using (78) and (79), buyers with $\psi b < p_{B,1} - v$ do no transactions, buyers with $\psi b \geq p_{B,2} - v$ do both transactions, and other buyers do just the first transaction. Second, consider $p_{B,1} > p_{B,2}$. Using (78) and (79), buyers with $b \geq \frac{p_{B,1}+p_{B,2}-2v}{2\psi}$ do both transactions on the platform and all other buyers do no transactions.) Using the same procedure as in the proof of Proposition 3, one can check that this is maximized by setting prices that satisfy $p_{B,1} \geq p_{B,2}$ and $p_{B,1} + p_{B,2} = 2(v + \psi b^{ND})$ where $b^{ND} = \underline{b}$ if $f(\underline{b})(v - c + \psi\underline{b}) \geq \psi$ and otherwise b^{ND} is the unique solution to $\psi[1 - F(b^{ND})] - (v + \psi b^{ND} - c)f(b^{ND}) = 0$. Buyers with $b \geq b^{ND}$ then do both transactions on the platform while other buyers do no transactions; the platform earns $2(v + \psi b^{ND} - c)[1 - F(b^{ND})]$.

Next, we show that when disintermediation is possible the platform can at least earn as much profit as it did when disintermediation was impossible. For example, suppose the platform sets $p_{B,1} = 2v - (1 - 2\psi)b^{ND} - c$ and $p_{B,2} = b^{ND} + c$. Using (80) and (81) one can check that all buyers with $b \geq b^{ND}$ will do both transactions on the platform, paying the platform a total price $p_{B,1} + p_{B,2} = 2(v + \psi b^{ND})$; hence from these buyers the platform again earns $2(v + \psi b^{ND} - c)[1 - F(b^{ND})]$. One can also check that, if $\psi \leq 1/2$, buyers with $b < b^{ND}$ will also do the first transaction and then disintermediate; since $p_{B,1} \geq p_{B,2} \geq 0$ this generates additional profit for the platform. Hence platform profit is at least as high as it was when disintermediation was impossible.

To complete the proof, we demonstrate that under the conditions in the proposition the platform is strictly better off with disintermediation than without it. Suppose the platform again sets $p_{B,1} = 2v - (1 - 2\psi)b^{ND} - c$ but now sets $p_{B,2} \geq b^{ND} + c$. Using (80) and (81) one can check that all buyers with $b \geq b^{ND}$ do the first transaction, and all buyers with $b \geq p_{B,2} - c$ then also do the second transaction on the platform while those with $b \in [b^{ND}, p_{B,2} + c)$ disintermediate, leading to platform profit of at least (since, when $\psi = 1/2$ and $b^{ND} > \underline{b}$, buyers with $b < b^{ND}$ will also do the first transaction)

$$[2v - (1 - 2\psi)b^{ND} - 2c][1 - F(b^{ND})] + (p_{B,2} - c)[1 - F(p_{B,2} - c)].$$

As already shown above, when $p_{B,2} = b^{ND} + c$ this equals $2(v + \psi b^{ND} - c)[1 - F(b^{ND})]$. Following the usual steps, when $\underline{b}f(\underline{b}) < 1$ this is also strictly increasing in $p_{B,2}$ around $p_{B,2} = b^{ND} + c$. $\square$