Online Appendix for "Reducing Risk through Trade Credit: Theory and Evidence from Dual-Channel Suppliers"

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Online Appendix A. Proof of Lemma 1

Assume that the random market size A follows an arbitrary distribution and use μ_A and σ_A to denote its mean and standard deviation. The supplier's random share, X_s, of the overlapping market can have any distribution on support $[0,1]$. Let μ_s and σ_s denote the mean and standard deviation of X_s. We note that the distribution of X_s, and thus (μ_s, σ_s), may depend on firms' (strategic) decisions, such as marketing, that precede the production and ordering decisions. To keep our discussion general, we do not explicitly model these strategic decisions and the distribution of X_s. The key to our model is that the outcome of these strategic decisions is uncertain, i.e., X_s is a random variable.

It is important to note that self-consistency of the product market imposes a relationship between a firm's advantage in the loyal market and in the overlapping market. Specifically, $\mu_s \geq \frac{1}{2}$ means that the supplier has an advantage over the buyer in the overlapping market. Thus, it is natural to expect that the supplier also has an advantage in the loyal market, i.e., $\alpha_s - \alpha_b \geq 0$, implying that $(\mu_s - \frac{1}{2})(\alpha_s - \alpha_b) \geq 0$. The opposite is also true. If $\mu_s \leq \frac{1}{2}$, implying that the buyer has an advantage in the overlapping market, then it is likely that this advantage would extend to the loyal market as well, i.e., $\alpha_s - \alpha_b \leq 0$. Again, the condition $(\mu_s - \frac{1}{2})(\alpha_s - \alpha_b) \geq 0$ holds. The following assumption reflects this intuition.

Assumption 1 *A firm's advantage in the overlapping market implies an advantage in the loyal market, and vice versa:* $(\mu_s - \frac{1}{2})(\alpha_s - \alpha_b) \geq 0$.

Let R_s and R_b denote the sales revenue of the supplier's direct channel and the buyer channel, respectively, i.e., $R_s = p_s q_d$ and $R_b = p_b q_b$. Next we consider the effect of overlapping market size γ on the correlation coefficient $\mathrm{Corr}(R_s, R_b)$. For simplicity, we focus on the case where the quantities are not too high so that the market clearing prices in equ. (1) and (2) are positive.

The following lemmas state that the sales covariance $\mathrm{Cov}(R_s, R_b)$ decreases in the overlapping market size γ and becomes negative at $\gamma = 1$, and that the sales variance $\mathrm{Var}(R_k)$ increases in γ $(k = b, s)$.

Lemma A.1 *Under the condition* $p_s, p_b > 0$,

1. $Cov(R_s, R_b)$ *decreases in the size of the overlapping market* γ.
2. $Cov(R_s, R_b) \geq 0$ *in the absence of the overlapping market, i.e., when* $\gamma = 0$.
3. $Cov(R_s, R_b) < 0$ *when the overlapping market has the maximum size* $\gamma = 1$*, if the average market size is sufficiently higher than its standard deviation* $\frac{\mu_A}{\sigma_A} > \frac{1}{2\sigma_s}$
4. $Cov(R_s, R_b) < 0$ *for all* $\gamma > 0$ *in the absence of market uncertainty, i.e., when* $\sigma_A = 0$.

Lemma A.2 *Under the condition* $p_s, p_b > 0$*, the variance of either firm's sales,* $Var(R_k)$ *(*$k = b, s$*), increases in the size of the overlapping market.*

Lemma 1 follows immediately from Lemmas A.1 and A.2. The sales correlation decreases in the size of the overlapping market γ. Absent of the overlapping market, the firms' sales are positively correlated, confirming the intuition of their common market trend. If the average market size μ_A is sufficiently larger than the standard deviation σ_A, which is usually the case, a sufficiently large overlapping market makes the two sales R_d and R_b negatively correlated. Importantly, we note that these results are true for *any* distribution of the overlapping market split X_s and the market size A. In particular, incorporating firms' strategic competition and endogenizing the distribution of X_s would *not* affect this result, as long as Assumption 1 holds.

Proofs of Lemmas A.1 and A.2

Proof of Lemma A.1. By definition, for given (q_d, q_b), the covariance between the two sales revenues R_s and R_b is:

$$\begin{aligned}
\mathrm{Cov}(R_s, R_b) &= \mathrm{Cov}[((\alpha_s + X_s\gamma)A - q_d)q_d,\ ((\alpha_b + (1-X_s)\gamma)A - q_b)q_b] \\
&= \mathrm{Cov}[(\alpha_s + X_s\gamma)Aq_d,\ (\alpha_b + (1-X_s)\gamma)Aq_b] \\
&= E[(\alpha_s + X_s\gamma)(\alpha_b + (1-X_s)\gamma)A^2]q_bq_d - E[(\alpha_s + X_s\gamma)A]E[(\alpha_b + (1-X_s)\gamma)A]q_bq_d \\
&= q_bq_d\Big\{\alpha_s\alpha_bE[A^2] + E[\alpha_s(1-X_s)\gamma A^2] + E[\alpha_bX_s\gamma A^2] + E[X_s(1-X_s)\gamma^2A^2]\Big\} \\
&\quad - q_bq_d\Big\{E[\alpha_sA]E[\alpha_bA] + E[\alpha_sA]E[(1-X_s)\gamma A] + E[\alpha_bA]E[X_s\gamma A] + E[X_sA]E[(1-X_s)A]\gamma^2\Big\} \\
&= q_bq_d\Big\{\alpha_s\alpha_b\sigma_A^2 + \alpha_s\gamma Cov(A,(1-X_s)A) + \alpha_b\gamma Cov(A,X_sA) \\
&\quad + \gamma^2Cov(X_s,1-X_s)\mu_A^2 + \gamma^2E[X_s(1-X_s)]\sigma_A^2\Big\} \\
&= q_bq_d\Big\{\alpha_s\alpha_b\sigma_A^2 + \alpha_s\gamma E[1-X_s]\sigma_A^2 + \alpha_b\gamma\mu_s\sigma_A^2 - \gamma^2\sigma_s^2\mu_A^2 + \gamma^2E[X_s(1-X_s)]\sigma_A^2\Big\} \\
&= q_bq_d\Big(\Big[\alpha_s\alpha_b + \alpha_s\gamma(1-\mu_s) + \alpha_b\gamma\mu_s + \gamma^2E[X_s(1-X_s)]\Big]\sigma_A^2 - \gamma^2\sigma_s^2\mu_A^2\Big), \qquad \text{(A.1)}
\end{aligned}$$

where the last few lines use the independence assumption $\mathrm{Cov}(X_s, A) = 0$.

Because $\alpha_s + \alpha_b = 1-\gamma$, let $\alpha_s = \frac{1-\gamma}{2} + \delta$ and $\alpha_b = \frac{1-\gamma}{2} - \delta$, where $\delta \in [-\frac{1-\gamma}{2}, \frac{1-\gamma}{2}]$. Then (A.1) becomes

$$\begin{aligned}
\frac{Cov(R_s,R_b)}{q_bq_d} &= \left[\frac{(1-\gamma)^2}{4} - \delta^2 + (\frac{1-\gamma}{2} + \delta)\gamma - 2\delta\gamma\mu_s + \gamma^2E[X_s(1-X_s)]\right]\sigma_A^2 - \gamma^2\sigma_s^2\mu_A^2 \\
&= \left[\frac{1-\gamma^2}{4} - \delta^2 + \delta\gamma - 2\delta\gamma\mu_s + E[X_s(1-X_s)]\gamma^2\right]\sigma_A^2 - \gamma^2\sigma_s^2\mu_A^2 \\
&= \left[-\left(\frac{1}{4} - E[X_s(1-X_s)]\right)\gamma^2 - (2\mu_s - 1)\delta\gamma + \frac{1}{4} - \delta^2\right]\sigma_A^2 - \gamma^2\sigma_s^2\mu_A^2 \qquad \text{(A.2)}
\end{aligned}$$

Clearly, the second term in (A.2) decreases in γ. Now we focus on the first term. Denote the coefficient of σ_A^2 as a function of γ:

$$g(\gamma) \equiv -\left(\frac{1}{4} - E[X_s(1-X_s)]\right)\gamma^2 - (2\mu_s - 1)\delta\gamma + \frac{1}{4} - \delta^2.$$

Function $g(\gamma)$ is quadratic in γ. The coefficient $\frac{1}{4} - E[X_s(1-X_s)]$ is positive because

$$E[X_s(1-X_s)] = EX_s - E[X_s^2] < EX_s - (EX_s)^2 \leq \frac{1}{4}$$

for all $E[X_s] \in [0,1]$. The coefficient $(2\mu_s - 1)\delta$ can be expressed as $(\mu_s - \frac{1}{2})(\alpha_s - \alpha_b)$, which is nonnegative by Assumption 1. Thus, function $g(\gamma)$ decreases in γ.

Therefore, $\text{Cov}(R_s, R_b)$ decreases in the overlapping market size γ.

At zero overlapping market $\gamma = 0$, equ. (A.2) states that $\text{Cov}(R_d, R_b) = (\frac{1}{4} - \delta^2)\sigma_A^2 q_d q_b$, where $\delta \in [-\frac{1}{2}, \frac{1}{2}]$. Thus, $\text{Cov}(R_d, R_b) \geq 0$ at $\gamma = 0$.

At maximum overlapping market $\gamma = 1$ and $\delta = 0$, equ. (A.2) states that

$$\text{Cov}(R_s, R_b) = \left(E[X_s(1-X_s)]\sigma_A^2 - \sigma_s^2\mu_A^2\right) q_b q_d \leq (\frac{\sigma_A^2}{4} - \sigma_s^2\mu_A^2) q_b q_d.$$

Thus, $\text{Cov}(R_s, R_b) < 0$ at $\gamma = 1$ if $\frac{\mu_A}{\sigma_A} > \frac{1}{2\sigma_s}$.

Finally, if $\sigma_A = 0$, i.e., the market size is constant, then $\text{Cov}(R_s, R_b) < 0$ for all $\gamma > 0$, completing the proof. □

Proof of Lemma A.2. Consider $Var(R_s)$. By definition,

$$\begin{aligned}
\frac{Var(R_s)}{q_d^2} &= E[((\alpha_s + X_s\gamma)A - q_d)^2] - \Big(E[(\alpha_s + X_s\gamma)A - q_d]\Big)^2 \\
&= E[(\alpha_s + X_s\gamma)^2 A^2] - [(\alpha_s + \mu_s\gamma)\mu_A]^2 \\
&= Var(\alpha_s + X_s\gamma)E[A^2] + (\alpha_s + \mu_s\gamma)^2\sigma_A^2 \\
&= \sigma_s^2 E[A^2]\gamma^2 + (\alpha_s + \mu_s\gamma^2)\sigma_A^2.
\end{aligned}$$

Therefore, $Var(R_s)$ increases in γ. Similar argument yields that $Var(R_b)$ increases in γ, completing the proof. □

Online Appendix B. Proofs of Results in Section 4

Proofs in this section follow from the equilibrium outcome of the dynamic games under trade credit and COD. Online Appendix C presents these equilibrium results and their proofs.

Proof of Proposition 1. Under $w < \frac{1+\gamma}{2}$, the buyer's order quantity under trade credit $Q_T(w;\gamma) > 0$ from Proposition C.1. Under COD, the buyer's order quantity $Q_C(w;\gamma) = 0$ on $w \geq \frac{\theta}{2}$. Thus, $Q_T(w;\gamma) > Q_C(w;\gamma)$ on $w \in [\frac{\theta}{2}, \frac{1+\gamma}{2})$. (Note that $\gamma \in (0,1]$ by definition.)

When $w < \frac{\theta}{2}$, the conditions for the buyer's order quantities in Proposition C.1 can be rewritten as:

$$Q_C(w;\gamma) = \begin{cases} \frac{1-2w/\theta}{4} & \text{if } 0 \leq \gamma \leq \gamma_C \\ \frac{1+\gamma-4w/\theta}{4} & \text{if } \gamma_C < \gamma \leq 1 \end{cases}, \quad Q_T(w;\gamma) = \begin{cases} \frac{1-2w}{4} & \text{if } 0 \leq \gamma < \gamma_0 \\ \frac{1+\gamma-2w}{4} & \text{if } \gamma_0 \leq \gamma \leq 1 \end{cases}$$

where $\gamma_C = \sqrt{2} - 1 + \frac{2(2-\sqrt{2})w}{\theta}$ and $\gamma_0 = (\sqrt{2}-1)(1-2w)$. Since $\gamma_C > \gamma_0$, we have $Q_T(w;\gamma) \geq Q_C(w;\gamma)$ when $0 \leq \gamma \leq \gamma_0$, with equality holds when $\theta = 1$. In this case, $Q_T(w;\gamma) = Q_l(w)$ and $Q_C(w;\gamma) = Q_l(w)$. When $\gamma_0 < \gamma \leq \gamma_C$, $Q_T(w;\gamma) - Q_C(w;\gamma) = \frac{2w(1-\theta)+\gamma\theta}{4\theta} > 0$. When $\gamma_C < \gamma \leq 1$, clearly $Q_T(w;\gamma) > Q_C(w;\gamma)$.

Therefore, $Q_T(w;\gamma) \geq Q_C(w;\gamma)$ on $w < \frac{1+\gamma}{2}$, with the equality holds if and only if $\gamma < \gamma_0$ and $\theta = 1$. □

Proof of Corollary 1. From Proposition C.3, γ_0 is the threshold level of γ that decides whether the buyer's order quantity follows Q_h or Q_l.

From the proof for trade credit quantity $Q_T(w;\gamma)$ in Proposition C.1, when the buyer uses $Q_T = Q_l$, i.e., $\gamma < \gamma_0$, the associated values of w satisfies condition (C.9). Thus, the buyer pays off trade credit in full in both supplier- and buyer-dominant markets. When the buyer uses $Q_T = Q_h$, i.e., $\gamma \geq \gamma_0$, the associated values of w satisfies condition (C.10). Thus, the buyer pays off trade credit in full $wQ_h(w)$ only in buyer-dominant markets. Its trade credit payment in supplier-dominant market is $(\frac{1-\gamma}{2} - Q_h(w))^+ Q_h(w) = p_b Q_T(w;\gamma)$, completing the proof. □

Proof of Corollary 2. The buyer pays different trade credit amounts in supplier- and buyer-dominant markets only if $Q_T = Q_h$ from Corollary 1. From Proposition C.1, $Q_T = Q_h$ on $w \in [\frac{1-(1+\sqrt{2})\gamma}{2}, \frac{1+\gamma}{2})$, which is equivalent to $\gamma \geq \gamma_0$ and $\gamma > 2w - 1$. Thus, a necessary condition is $\gamma \geq \gamma_0$.

Under $Q_T = Q_h$, the buyer's order quantity $Q_h(w) = \frac{1-2w+\gamma}{4}$ increases in γ. Because the buyer pays the full amount $wQ_h(w)$ in buyer-dominant market, its payment increases in γ, completing the proof. □

Proof of Theorem 1. Following the analysis of the COD model in the proof of Proposition C.2, by case (i), when $D > \frac{(1+\gamma)^2}{16} + wq_b$, the supplier defaults with probability one. In the regions where q_d^h is the global maximizer across the four cases, we obtain that:

- By case (iv), when $D < wq_b$ and $\gamma \in [\sqrt{2}-1, 1]$, the default risk is 0 (belonging to scenario (C-0) in Table 3).
- By case (ii) and case (iii), when $wq_b + \frac{1-2\gamma-\gamma^2}{16} < D < wq_b + \frac{(1+\gamma)^2}{16}$ and $\gamma \in [0, \sqrt{2}-1]$, or when $wq_b < D < wq_b + \frac{(1+\gamma)^2}{16}$ and $\gamma \in [\sqrt{2}-1, 1]$, the supplier defaults in the buyer-dominant and low market scenarios, with probability $1 - \frac{\theta}{2}$ (scenario (C-2) in Table 3).

In the regions where q_d^l is the global maximizer across the four cases, we obtain that:

- By case (iv), when $D < wq_b$ and $\gamma \in [0, \sqrt{2}-1]$, the default risk is 0 (belonging to scenario (C-0) in Table 3).
- By case (iii), when $wq_b < D < wq_b + \frac{1-2\gamma-\gamma^2}{16}$ and $\gamma \in [0, \sqrt{2}-1]$, the supplier defaults in the low market scenario, with probability $1-\theta$ (scenario (C-1) in Table 3).

Theorem 1 follows by substituting q_b with the buyer's optimal order quantity $Q_C(w;\gamma)$. □

Proof of Theorem 2. Under $w > \frac{2-\sqrt{2}}{8+2\sqrt{2}}$, the second and third cases in Proposition C.4 apply so we will discuss accordingly.

On $w \in [\frac{2-\sqrt{2}}{8+2\sqrt{2}}, \frac{3\sqrt{2}-4}{2})$, on which $\frac{2w+1}{3} < \sqrt{2}-1$, Table OA1, the top rows of Table OA2, and Table OA3 apply. In particular, the upper bound of γ in the top rows of Table OA2 reduces to $\frac{2w+1}{3}$, under condition (C.15a). The lower bound of γ in Table OA3 also becomes $\frac{2w+1}{3}$, under condition (C.15b).

- Combining the first row in Tables OA1–OA3 yields the first row in Table 4, i.e., case (T-1.1).
- The third row in Table OA3 yields the second row in Table 4, i.e., case (T-1.2).
- The second row in Table OA1 yields the third row in Table 4, i.e., first row in case (T-2).
- The second row in Table OA2, with upper bound $\frac{1+2w}{3}$ and lower bound $\frac{1-2w}{\sqrt{2}+1} = \gamma_0$ from condition (C.15a), is the fourth row in Table 4. Here $\underline{D}(w,\gamma) = wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$.
- The last row in Table OA3, with lower bound $\frac{1+2w}{3}$ from condition (C.15b), is the last row in Table 4.

On $w \in [\frac{3\sqrt{2}-4}{2}, 1]$, on which $\frac{2w+1}{3} \geq \sqrt{2}-1$, Table OA1, Table OA2, and the bottom two rows in Table OA3 apply. In particular, the lower bound of γ in the bottom two rows of Table OA3 becomes $\frac{2w+1}{3}$, under condition (C.15b).

- Combining the first row in Tables OA1–OA3 yields the first row in Table 4, i.e., case (T-1.1). (Note that Table OA1 no longer applies on $w \geq \frac{1}{2}$. But this does not affect the results.)
- Combining the third rows in Tables OA2 and OA3 yields the second row in Table 4, i.e., case (T-1.2).
- The second row in Table OA1 yields the third row in Table 4, i.e., first row in case (T-2). (Note this case is an empty set under $w \geq \frac{1}{2}$.)
- The second and fourth rows in Table OA2, combined with condition (C.15a), yields the third row in Table 4. Here $\underline{D}(w,\gamma) = wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$ on $\gamma \in (\gamma_0, \sqrt{2}-1]$ and is $wQ_h(w)$ on $\gamma \in (\sqrt{2}-1, \frac{1+2w}{3}]$.

- The last row in Table OA3, combined with condition (C.15b), is the last row in Table 4.

Therefore, defining $\underline{D}(w,\gamma)$ as follows yields the theorem:

$$\underline{D}(w,\gamma)=\begin{cases} wQ_h(w)+\frac{1-2\gamma-\gamma^2}{16}, & \text{if } \gamma\in(\gamma_0,\sqrt{2}-1\wedge\frac{1+2w}{3}],\\ wQ_h(w) & \text{if } \gamma\in(\sqrt{2}-1,\frac{1+2w}{3}]. \end{cases} \tag{B.1}$$

□

Proof of Corollary 3. From Theorems 1 and 2, in region (C-0) where the default probability is 0 under COD, the default probability is at least $1-\theta$ under trade credit. Thus, trade credit increases supplier risk under low debt $D\le wQ_C(w)$.

Now consider the case of high debt. Let $\bar{D}^C(w,\gamma)$ denote the debt upper bound for default region with probability $1-\frac{\theta}{2}$ under COD so

$$\bar{D}^C(w,\gamma)=\frac{(1+\gamma)^2}{16}+wQ_C(w).$$

Define $\bar{D}(w,\gamma)$ as the counterpart debt upper bound under trade credit. From Theorem 2,

$$\bar{D}(w,\gamma)=\begin{cases} \frac{(1+\gamma)^2}{16}+wQ_l(w) & \text{if } \gamma\le\gamma_0,\\ \frac{1-2w^2+4w\gamma-\gamma^2}{8} & \text{if } \gamma\in(\gamma_0,\frac{1+2w}{3}],\\ \frac{(1+\gamma)^2}{16} & \text{if } \gamma\in(\frac{1+2w}{3},1]. \end{cases} \tag{B.2}$$

Similarly, define the debt upper bounds for region (C-1) and the combined regions (T-1.1) and (T-1.2):

$$\bar{D}_1^C(w,\gamma)=\begin{cases} \frac{1-2\gamma-\gamma^2}{16}+wQ_l(\frac{w}{\theta}), & \text{if } \gamma\le\sqrt{2}-1,\\ 0 & \text{if } \gamma\in(\sqrt{2}-1,1]. \end{cases} \tag{B.3a}$$

$$\bar{D}_1^T(w,\gamma)=\begin{cases} \frac{1-2\gamma-\gamma^2}{16}+wQ_T(w), & \text{if } \gamma\le\sqrt{2}-1,\\ wQ_h(w), & \text{if } \gamma\in(\sqrt{2}-1,1]. \end{cases} \tag{B.3b}$$

Therefore, under COD, the supplier's default risk is $1-\theta$ on $D\in(wQ_c(w),\bar{D}_1^C(w,\gamma)]$ (region C-1) and is $1-\frac{\theta}{2}$ on $D\in(\bar{D}_1^C(w,\gamma),\bar{D}^C(w,\gamma)]$ (region C-2). Under trade credit, the supplier's default risk is $1-\theta$ on $D\le\bar{D}_1^T(w,\gamma)$ (regions T-1.1 and T-1.2) and is $1-\frac{\theta}{2}$ on $D\in(\bar{D}_1^T(w,\gamma),\bar{D}(w,\gamma)]$ (region T-2).

Because $Q_T(w)\ge Q_l(\frac{w}{\theta})$ (Corollary 1), $\bar{D}_1^T(w,\gamma)\ge\bar{D}_1^C(w,\gamma)$. Thus, on $D\in(wQ_c(w),\bar{D}_1^T(w,\gamma)]$, the supplier's default probability is weakly lower under trade credit than COD. When the debt level $D\in(\bar{D}_1^T(w,\gamma),\bar{D}(w,\gamma)]$, the supplier default probability is always $1-\frac{\theta}{2}$ under trade credit. Under COD, and focusing on the region where the supplier has a lower-than-one default probability with $D<\bar{D}^C(w,\gamma)$, the supplier's default probability is also $1-\frac{\theta}{2}$. Therefore, trade credit weakly decreases supplier risk on $D\in(wQ_C(w),\bar{D}(w,\gamma)]$. Strict risk reduction occurs only if $D\le\bar{D}_1^T(w,\gamma)$. This proves part 2 of the corollary.

On $D > wQ_C(w)$, supplier risk increases under trade credit if and only if the upper bound under trade credit is *below* that under COD. Then under $D \in (\bar{D}(w,\gamma), \bar{D}^C(w,\gamma)]$, the supplier defaults with probability one under trade credit and with probability $1-\frac{\theta}{2} < 1$ under COD. Thus, a necessary condition for risk increase under trade credit is that the supplier's debt level exceeds that the trade credit upper bound: $D > \bar{D}(w,\gamma)$, establishing the high-debt result in part 1 of the corollary. □

Proof of Corollary 4. From the proof of Corollary 3, trade credit strictly decreases bankruptcy risk only in regions (T-1.1) and (T-1.2). When $1-\theta$ approaches zero so θ approaches one, $Q_T(w)$ approaches $Q_C(w)$ on low γ values, $\gamma < \gamma_0$, where the low-order piece is used $Q_T = Q_C = Q_l$. As a result, the upper bound of region (T-1.1) decreases and approaches that of region (C-1) on low values of γ: $\bar{D}_1^T(w,\gamma) = \bar{D}_1^C(w,\gamma)$ (see equ. (B.3)). The supplier has the same bankruptcy risk under trade credit and COD on $\gamma < \gamma_0$ in region (T-1.1). The risk mitigation effect of trade credit approaches zero. On $\gamma > \gamma_0$, however, Q_h is used under trade credit. Because $Q_h(w) > Q_C(w)$, the debt upper bounds $\bar{D}_1^T(w,\gamma) > \bar{D}_1^C(w,\gamma)$. Thus, risk mitigation effect of trade credit remains. □

Online Appendix C. Equilibrium Analysis

Define the following quantities for the buyer's order:

$$Q_h(w) := \frac{1}{4}(1-2w+\gamma), \quad Q_l(w) := \frac{1}{4}(1-2w).$$

Define the following quantities for the supplier's order:

$$q_d^l := \frac{1}{4}, \qquad q_d^h := \frac{1+\gamma}{4}. \tag{C.1}$$

C.1. COD model

The buyer's optimal order quantity in the COD model is characterized below.

Proposition C.1 *Given the wholesale price w and market overlap γ, the buyer's order quantity under COD, $Q_C(w;\gamma)$ is given by:*

$$Q_C(w;\gamma) = \begin{cases} Q_h(\frac{2w}{\theta}), & \text{if } \frac{w}{\theta} < \frac{(1+\gamma-\sqrt{2})^+}{4-2\sqrt{2}}, \\ Q_l(\frac{w}{\theta}), & \text{if } \frac{(1+\gamma-\sqrt{2})^+}{4-2\sqrt{2}} \le \frac{w}{\theta} < \frac{1}{2}, \\ 0, & \text{if } \frac{w}{\theta} \ge \frac{1}{2}. \end{cases} \tag{C.2}$$

Proof of Proposition C.1. First, consider the case when $\theta > 2w$.

(i). When $q_b \in [0, \frac{1-\gamma}{2}]$, the buyer's payoff is:

$$\pi_{b,1}^C = \frac{\theta}{2}\left(\frac{1+\gamma}{2} - q_b\right) q_b + \frac{\theta}{2}\left(\frac{1-\gamma}{2} - q_b\right) q_b - w q_b. \tag{C.3}$$

Since $\pi_{b,1}^C$ is concave, the optimal solution is given by the interior solution $Q_l\left(\frac{w}{\theta}\right) = \frac{1-2w/\theta}{4}$ if $Q_l\left(\frac{w}{\theta}\right) < \frac{1-\gamma}{2}$ (i.e., $\gamma < \frac{1}{2} + \frac{w}{\theta}$). Otherwise, the optimal solution is at the boundary $\frac{1-\gamma}{2}$.

(ii). When $q_b \in [\frac{1-\gamma}{2}, \frac{1+\gamma}{2}]$, the buyer's payoff is:

$$\pi_{b,2}^C = \frac{\theta}{2}\left(\frac{1+\gamma}{2} - q_b\right) q_b - w q_b. \tag{C.4}$$

By the concavity of $\pi_{b,2}^C$, the optimal solution is given by the interior solution $Q_h\left(\frac{2w}{\theta}\right) = \frac{1+\gamma-4w/\theta}{4}$ if $Q_h\left(\frac{2w}{\theta}\right) \in \left(\frac{1-\gamma}{2}, \frac{1+\gamma}{2}\right)$ (i.e., $\gamma > \frac{4w+\theta}{3\theta}$). Otherwise, the optimal is at the boundary $\frac{1-\gamma}{2}$.

When $\theta > 2w$, we have $\frac{4w+\theta}{3\theta} < \frac{1}{2} + \frac{w}{\theta}$. So, when $\gamma \in [0, \frac{4w+\theta}{3\theta}]$, π_b^C is unimodal with q_b^* given by $Q_l\left(\frac{w}{\theta}\right)$. Similarly, when $\gamma \in [\frac{1}{2} + \frac{w}{\theta}, 1]$, π_b^C is unimodal with q_b^* given by $Q_h\left(\frac{2w}{\theta}\right)$. When $\gamma \in \left(\frac{4w+\theta}{3\theta}, \frac{1}{2} + \frac{w}{\theta}\right)$, π_b^C is bimodal. Comparing the two local optima, we have $\pi_{b,1}^C\left[Q_l\left(\frac{w}{\theta}\right)\right] < \pi_{b,2}^C\left[Q_h\left(\frac{2w}{\theta}\right)\right]$ if and only if $\gamma < \gamma_C := \sqrt{2} - 1 + \frac{2w(2-\sqrt{2})}{\theta}$. To summarize, the buyer's optimal order quantity is given by $Q_l\left(\frac{w}{\theta}\right)$ when $\gamma \in [0, \gamma_C]$ (or equivalently $\frac{(1+\gamma-\sqrt{2})^+}{4-2\sqrt{2}} \le \frac{w}{\theta} < \frac{1}{2}$) and is given by $Q_h\left(\frac{2w}{\theta}\right)$ when $\gamma \in [\gamma_C, 1]$ (or equivalently $\frac{w}{\theta} < \frac{(1+\gamma-\sqrt{2})^+}{4-2\sqrt{2}}$).

Next, consider the case when $\theta \le 2w$. Clearly, $\pi_{b,1}^C$ is decreasing when $q_b \in [0, \frac{1-\gamma}{2}]$. Since $\frac{4w+\theta}{3\theta} > 1$, $\pi_{b,2}^C$ is decreasing when $q_b \ge \frac{1-\gamma}{2}$. Therefore, the buyer's optimal order quantity is 0. □

The next result characterizes the supplier's optimal order quantity in the COD model.

Proposition C.2 *Given the wholesale price w, the supplier's direct channel quantity under COD satisfies:*

$$q_d^* = \begin{cases} q_d^l = \frac{1}{4}, & \text{if } \gamma \in [0, \sqrt{2}-1], D \in [0, wQ_C(w) + \frac{1-2\gamma-\gamma^2}{16}], \\ q_d^h = \frac{1+\gamma}{4}, & \text{if } \gamma \in [0, \sqrt{2}-1], D \in (wQ_C(w) + \frac{1-2\gamma-\gamma^2}{16}, wQ_C(w) + \frac{(1+\gamma)^2}{16}] \\ & \quad \text{or } \gamma \in (\sqrt{2}-1, 1], D \in [0, wQ_C(w) + \frac{(1+\gamma)^2}{16}], \\ 0, & \text{otherwise.} \end{cases}$$

Proof of Proposition C.2. Consider the following four cases:

(i). If $wq_b - D \le -\frac{(1+\gamma)^2}{16}$: $\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D$ and $\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D$ in (7) are always negative. So, the supplier defaults in all market scenarios.

(ii). If $-\frac{(1+\gamma)^2}{16} \le wq_b - D \le -\frac{(1-\gamma)^2}{16}$: $\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D$ in (7) is always negative. So, the supplier defaults in the buyer-dominant market and low market scenarios. In the supplier-dominant market scenario, if $q_d \ge \frac{1+\gamma}{2}$, the supplier always defaults. So, q_d satisfies $q_d < \frac{1+\gamma}{2}$ and the supplier's payoff becomes:

$$\pi_{s,1}^C(q_d) = \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right]^+ \tag{C.5}$$

Denote the two roots of the equation $\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D = 0$ as $q_d^{(1)} = \frac{1+\gamma-\sqrt{16(wq_b-D)+(1+\gamma)^2}}{4}$ and $q_d^{(2)} = \frac{1+\gamma+\sqrt{16(wq_b-D)+(1+\gamma)^2}}{4}$. Clearly, $0 < q_d^{(1)} < q_d^{(2)} < \frac{1+\gamma}{2}$. So, $\pi_s^C = 0$ when $q_d \le q_d^{(1)}$ and $q_d \ge q_d^{(2)}$, and is positive and concave when $q_d \in (q_d^{(1)}, q_d^{(2)})$. The supplier's optimal order quantity is given by $q_d^h = \frac{1+\gamma}{4}$. The default probability is $1 - \frac{\theta}{2}$.

(iii). If $-\frac{(1-\gamma)^2}{16} \le wq_b - D \le 0$: The supplier always defaults in the low market scenario. If $\frac{1-\gamma}{2} \le q_d \le \frac{1+\gamma}{2}$, the supplier always defaults in the buyer-dominant market scenario, too. The supplier's payoff is given by (C.5). If $q_d \le \frac{1-\gamma}{2}$, the supplier's payoff becomes:

$$\pi_{s,2}^C(q_d) = \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right]^+ + \frac{\theta}{2}\left[\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D\right]^+ \tag{C.6}$$

Denote the two roots of the equation $\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D = 0$ as $q_d^{(3)} = \frac{1-\gamma-\sqrt{16(wq_b-D)+(1-\gamma)^2}}{4}$ and $q_d^{(4)} = \frac{1-\gamma+\sqrt{16(wq_b-D)+(1-\gamma)^2}}{4}$. If $q_d < q_d^{(3)}$ or $q_d > q_d^{(4)}$, the supplier defaults in the buyer-dominant and low market scenarios, and the payoff is given by (C.5). When $q_d \in [q_d^{(3)}, q_d^{(4)}]$, the supplier's payoff is given by (C.6).

It can be shown that $0 < q_d^{(1)} < q_d^{(3)} < q_d^{(4)} < q_d^{(2)}$, $q_d^{(4)} < \frac{1-\gamma}{2}$, and $q_d^{(2)} < \frac{1+\gamma}{2}$. So,

$$\pi_s^C = \begin{cases} 0, & \text{when } q_d < q_d^{(1)} \\ \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right], & \text{when } q_d^{(1)} \le q_d \le q_d^{(3)} \\ \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right] + \frac{\theta}{2}\left[\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D\right], & \text{when } q_d^{(3)} \le q_d \le q_d^{(4)} \\ \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right], & \text{when } q_d^{(4)} \le q_d \le q_d^{(2)} \\ 0, & \text{when } q_d > q_d^{(2)} \end{cases}$$

The unconstrained maximizers of (C.5) and (C.6) are $q_d^h = \frac{1+\gamma}{4}$ and $q_d^l = \frac{1}{4}$, respectively. We can show that $q_d^h > q_d^{(3)}$ (so π_s^C is nondecreasing in q_d when $q_d < q_d^{(3)}$) and $q_d^l > q_d^{(3)}$.

- If $q_d^{(3)} < q_d^l < q_d^{(4)}$ and $q_d^{(3)} < q_d^h < q_d^{(4)}$ (or equivalently $\gamma < \frac{1}{3}$, $\frac{-1+2\gamma+3\gamma^2}{16} < wq_b - D < 0$), then π_s^C is unimodal and the global maximum is achieved at q_d^l.
- If $q_d^{(3)} < q_d^l < q_d^{(4)}$ and $q_d^{(4)} < q_d^h < q_d^{(2)}$, then π_s^C is bimodal. Comparing $\pi_{s,1}^C(q_d^h)$ and $\pi_{s,2}^C(q_d^l)$, we obtain that the global maximum is achieved at q_d^l when $\gamma < \frac{1}{3}, \frac{-1+2\gamma+\gamma^2}{16} < wq_b - D < \frac{-1+2\gamma+3\gamma^2}{16}$ or when $\frac{1}{3} < \gamma < \sqrt{2}-1, \frac{-1+2\gamma+\gamma^2}{16} < wq_b - D < 0$; the global maximum is achieved at q_d^h when $\gamma < \sqrt{2}-1, \frac{-1+2\gamma}{16} < wq_b - D < \frac{-1+2\gamma+\gamma^2}{16}$ or when $\sqrt{2}-1 < \gamma < \frac{1}{2}, \frac{-1+2\gamma}{16} < wq_b - D < 0$.
- If $q_d^l > q_d^{(4)}$ (or equivalently when $\gamma < \frac{1}{2}, -\frac{(1-\gamma)^2}{16} < wq_b - D < \frac{-1+2\gamma}{16}$ or when $\frac{1}{2} < \gamma < 1, -\frac{(1-\gamma)^2}{16} < wq_b - D < 0$), then we must have $q_d^{(4)} < q_d^h < q_d^{(2)}$. This implies that π_s^C is unimodal with the maximum achieved at q_d^h.

When q_d^h is optimal, the default probability is $1 - \frac{\theta}{2}$. When q_d^l is optimal, the default probability is $1 - \theta$.

(iv). If $wq_b - D > 0$: The supplier does not default in the low market scenario. When $q_d \in [0, \frac{1-\gamma}{2}]$, π_s^C is given by $\pi_{s,2}^C(q_d) + (1-\theta)(wq_b - D)$. When $q_d \in [\frac{1-\gamma}{2}, \frac{1+\gamma}{2}]$, π_s^C is given by $\pi_{s,1}^C(q_d) + \left(1 - \frac{\theta}{2}\right)(wq_b - D)$. Define $q_d^{(1)}$ to $q_d^{(4)}$ and q_d^h, q_d^l as in (ii) and (iii). It can be shown that when $wq_b - D > 0$, the following is true: $q_d^{(1)} < 0$, $q_d^{(3)} < 0$, $\frac{1-\gamma}{2} < q_d^{(4)} < q_d^{(2)}$, and $q_d^{(2)} > \frac{1+\gamma}{2}$. So,

$$\pi_s^C = \begin{cases} \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right] + \frac{\theta}{2}\left[\left(\frac{1-\gamma}{2} - q_d\right) q_d + wq_b - D\right] & \text{if } q_d \le \frac{1-\gamma}{2}, \\ \quad + (1-\theta)(wq_b - D), & \\ \frac{\theta}{2}\left[\left(\frac{1+\gamma}{2} - q_d\right) q_d + wq_b - D\right] + \left(1 - \frac{\theta}{2}\right)(wq_b - D), & \text{if } \frac{1-\gamma}{2} \le q_d \le \frac{1+\gamma}{2}, \\ 0, & \text{if } q_d > \frac{1+\gamma}{2}. \end{cases}$$

- If $\gamma < \frac{1}{3}$: we have $0 < q_d^l < \frac{1-\gamma}{2}$ and $0 < q_d^h < \frac{1-\gamma}{2}$. So, π_s^C is unimodal with the maximum achieved at q_d^l.
- If $\frac{1}{3} \le \gamma \le \frac{1}{2}$: we have $0 < q_d^l < \frac{1-\gamma}{2}$ and $\frac{1-\gamma}{2} < q_d^h < \frac{1+\gamma}{2}$. So, π_s^C is bimodal. Comparing the two local maximum, we obtain that q_d^l is the global maximizer when $\frac{1}{3} < \gamma < \sqrt{2}-1$, and q_d^h is the global maximizer when $\sqrt{2}-1 < \gamma < \frac{1}{2}$.
- If $\frac{1}{2} < \gamma < 1$: we have $q_d^l > \frac{1-\gamma}{2}$ and $\frac{1-\gamma}{2} < q_d^h < \frac{1+\gamma}{2}$. So, π_s^C is unimodal with the maximum achieved at q_d^h.

The default probability is 0 when q_d^h and q_d^l are achieved.

Finally, combining the regions where q_d^l is the global maximizer across the four cases, we obtain the condition $\gamma \in [0, \sqrt{2}-1], D \in [0, wQ_C(w;\gamma) + \frac{1-2\gamma-\gamma^2}{16}]$. Combining the regions where q_d^h is the global maximizer across the four cases, we obtain the condition $\gamma \in [0, \sqrt{2}-1], D \in (wQ_C(w;\gamma) + \frac{1-2\gamma-\gamma^2}{16}, wQ_C(w;\gamma) + \frac{(1+\gamma)^2}{16}]$ or $\gamma \in (\sqrt{2}-1, 1], D \in [0, wQ_C(w;\gamma) + \frac{(1+\gamma)^2}{16}]$. □

C.2. Trade credit model

The buyer's optimal order quantity under trade credit is characterized below.

Proposition C.3 *Given the wholesale price w and market overlap γ, the buyer's order quantity under trade credit, $Q_T(w;\gamma)$ is given by:*

$$Q_T(w;\gamma)=\begin{cases} Q_l(w), & \text{if } w<\frac{1}{2}-\frac{1+\sqrt{2}}{2}\gamma,\\ Q_h(w), & \text{if } \frac{1}{2}-\frac{1+\sqrt{2}}{2}\gamma\le w<\frac{1+\gamma}{2},\\ 0, & \text{if } w\ge\frac{1+\gamma}{2}. \end{cases} \tag{C.7}$$

Proof of Proposition C.3. Given wholesale price w, the buyer's expected payoff π_b^T is a piecewise function of order quantity q_b (equ. (4)). Specifically, if $q_b\le\frac{1-\gamma}{2}-w$, then the buyer can pay off wq_b in full under both supplier- and buyer-dominant markets. Otherwise, the buyer defaults in the supplier-dominant market and repays wq_b only in the buyer-dominant market.

$$\pi_b^T=\begin{cases} \frac{\theta}{2}(\frac{1-\gamma}{2}-q_b)q_b+\frac{\theta}{2}(\frac{1+\gamma}{2}-q_b)q_b-\theta wq_b, & \text{if } q_b\le\frac{1-\gamma}{2}-w,\\ \frac{\theta}{2}(\frac{1+\gamma}{2}-q_b-w)q_b, & \text{if } q_b\in(\frac{1-\gamma}{2}-w,\frac{1+\gamma}{2}-w],\\ 0 & \text{if } q_b>\frac{1+\gamma}{2}-w. \end{cases} \tag{C.8}$$

It is easy to show that the unconstrained optimum of the first piece of π_b^T in equ. (C.8) is $Q_l(w)=\frac{1-2w}{4}$. The unconstrained optimum of the second piece of π_b^T in equ. (C.8) is $Q_h(w)=\frac{1-2w+\gamma}{4}$.

Feasibility condition of $Q_l(w)$ is

$$Q_l(w)\le\frac{1-\gamma}{2}-w \qquad\Longleftrightarrow\qquad 0<w\le\frac{1-\gamma}{2}. \tag{C.9}$$

Thus, if $w>\frac{1-\gamma}{2}$, then $Q_l(w)>\frac{1-\gamma}{2}-w$. Concavity of π_b^T on $q_b\le\frac{1-\gamma}{2}-w$ implies that π_b^T is monotone increasing on this internal. Thus, the optimal order quantity must be greater than $\frac{1-\gamma}{2}-w$, in the next pieces.

Feasibility condition of $Q_h(w)$ is

$$\frac{1-\gamma}{2}-w<Q_h(w)\le\frac{1+\gamma}{2}-w \qquad\Longleftrightarrow\qquad \frac{(1-3\gamma)^+}{2}\le w<\frac{1+\gamma}{2}. \tag{C.10}$$

Note that if $w\ge\frac{1+\gamma}{2}$, then $\pi_b^T\equiv 0$ for all $q_b\ge 0$ from equ. (C.8). The buyer receives zero payoff with probability one due to the high wholesale price. In this case, we assume that it never orders. Thus, we shall focus on the region $w<\frac{1+\gamma}{2}$. Note that $Q_h(w)=\frac{1-2w+\gamma}{4}<\frac{1+\gamma}{2}-w$ for all $w<\frac{1+\gamma}{2}$. Thus, if $Q_h(w)$ is infeasible, then it must be that $Q_h(w)$ violates the lower bound in (C.10): $Q_h(w)\le\frac{1-\gamma}{2}-w$, implying that π_b^T is monotone decreasing on $q_b>\frac{1-\gamma}{2}-w$.

Because $Q_h(w)>Q_l(w)$, $Q_h(w)\le\frac{1-\gamma}{2}-w$ implies that $Q_l(w)<\frac{1-\gamma}{2}-w$ so $Q_l(w)$ is feasible. Thus, $Q_l(w)$ is the global optimum for π_b^T if $Q_h(w)$ is infeasible on $w<\frac{1+\gamma}{2}$. Similarly, infeasibility of $Q_l(w)$, i.e., $Q_l(w)>\frac{1-\gamma}{2}-w$, implies that $Q_h(w)$ must be feasible given $Q_h(w)>Q_l(w)$ on $w<\frac{1+\gamma}{2}$. Thus, $Q_h(w)$ is the global optimum for π_b^T if $Q_h(w)$ is infeasible on $w<\frac{1+\gamma}{2}$.

Because the feasibility conditions of $Q_l(w)$ and $Q_h(w)$ overlaps on $w \in [\frac{1-3\gamma}{2}, \frac{1-2\gamma}{2}]$, the buyer's expected payoff π_b^T is bi-modal under these w values. Comparing the values of π_b^T under $q_b = Q_l(w)$ and $q_b = Q_h(w)$ on $w \in [\frac{1-3\gamma}{2}, \frac{1-2\gamma}{2}]$ yields that $Q_l(w)$ generates a higher payoff on $w \le \frac{1-(1+\sqrt{2})\gamma}{2}$ and $Q_h(w)$ generates a higher payoff otherwise.

Therefore, the optimal order quantity is $Q_l(w)$ on $w < \frac{1-(1+\sqrt{2})\gamma}{2}$ and is $Q_h(w)$ on $w \in [\frac{1-(1+\sqrt{2})\gamma}{2}, \frac{1+\gamma}{2})$, completing the proof. □

Next, we derive the supplier's optimal order quantity under trade credit.

Proposition C.4 *Given the wholesale price w, the supplier's direct channel quantity under trade credit satisfies:*

1. *If $w \le \frac{2-\sqrt{2}}{8+2\sqrt{2}}$, then*

$$q_d^* = \begin{cases} q_d^l & \text{if } \gamma \le \sqrt{2}-1,\ D \le wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \\ q_d^h & \text{if } \gamma \le \sqrt{2}-1,\ D \in [wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{(1+\gamma)^2}{16}] \\ & \quad \text{or } \gamma \in (\sqrt{2}-1, 1],\ D \le \frac{(1+\gamma)^2}{16}. \end{cases} \tag{C.11}$$

2. *If $w \in (\frac{2-\sqrt{2}}{8+2\sqrt{2}}, \frac{3\sqrt{2}-4}{2}]$, then*

$$q_d^* = \begin{cases} q_d^l & \text{if } \gamma \le \sqrt{2}-1,\ D \le wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \\ q_d^h & \text{if } \gamma \le \frac{1+2w}{3},\ D \in [wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{1-2w^2+4w\gamma-\gamma^2}{8}] \\ & \quad \text{or } \gamma \in (\frac{1+2w}{3}, \sqrt{2}-1],\ D \in [wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{(1+\gamma)^2}{16}], \\ & \quad \text{or } \gamma \in (\sqrt{2}-1, 1],\ D \le \frac{(1+\gamma)^2}{16}. \end{cases} \tag{C.12}$$

3. *If $w \in (\frac{3\sqrt{2}-4}{2}, 1]$, then*

$$q_d^* = \begin{cases} q_d^l & \text{if } \gamma \le \sqrt{2}-1,\ D \le wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \\ q_d^h & \text{if } \gamma \le \sqrt{2}-1,\ D \in [wQ_T(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{1-2w^2+4w\gamma-\gamma^2}{8}] \\ & \quad \text{or } \gamma \in (\sqrt{2}-1, \frac{1+2w}{3}],\ D \le \frac{1-2w^2+4w\gamma-\gamma^2}{8}, \\ & \quad \text{or } \gamma \in (\frac{1+2w}{3}, 1],\ D \le \frac{(1+\gamma)^2}{16}. \end{cases} \tag{C.13}$$

Proof of Proposition C.4. We compute the optimal direct channel quantity by discussing three cases of the buyer's trade credit payment: (i) buyer repays trade credit in full whenever market $A = 1$, (ii) buyer partially pays trade credit in the supplier-dominant market ($A = 1, X_s = 1$), and (iii) buyer pays zero in the supplier-dominant market ($A = 1, X_s = 1$).

The following lemma presents the result in case (i). The proof of all lemmas are at the end of this subsection.

Lemma C.1 *Under $w < \frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma$ (or equivalently $\gamma < \frac{1-2w}{\sqrt{2}+1}$) so buyer's order quantity $Q_T(w;\gamma) = Q_l(w)$, the supplier's optimal direct channel quantity q_d^* and the associated default scenarios are given by Table OA1.*

Table OA1 **Supplier direct channel quantity and default scenarios under trade credit with $w < \frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma$ (equivalently $\gamma < \frac{1-2w}{1+\sqrt{2}}$)**

	Competition		Supplier default?		
q_d^*	intensity γ	Supplier debt level D	Low	Buyer	Supplier
q_d^l	$\gamma < \frac{1-2w}{1+\sqrt{2}}$	$D \le wQ_l(w) + \frac{1-2\gamma-\gamma^2}{16}$	Yes	No	No
q_d^h	$\gamma < \frac{1-2w}{1+\sqrt{2}}$	$D \in [wQ_l(w) + \frac{1-2\gamma-\gamma^2}{16}, wQ_l(w) + \frac{(1+\gamma)^2}{16}]$	Yes	Yes	No

Market scenarios "Low", "Supplier", and "Buyer" are the low market, supplier-dominant, and buyer-dominant markets defined in Table 1.

In cases (ii) and (iii), the buyer uses the high-order piece $Q_T = Q_h$ so $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma, \frac{1+\gamma}{2})$ (Proposition C.1). Cases (ii) and (iii) correspond to where the buyer makes nonzero and zero payment in the supplier-dominant market when it defaults. From Corollary 1, the buyer's payment in supplier-dominant market reduces to zero when $p_d = (\frac{1-\gamma}{2} - Q_h(w))^+ = 0$, which is equivalent to

$$\frac{1-\gamma}{2} \le Q_h(w) \qquad \Longleftrightarrow \qquad w \le \frac{3\gamma - 1}{2}. \tag{C.14}$$

Thus, the condition for case (ii) to occur is $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma \vee \frac{3\gamma-1}{2}, \frac{1+\gamma}{2})$, which is a nonempty interval iff $w > \frac{2-\sqrt{2}}{8+2\sqrt{2}} \approx 0.054$. The condition for case (iii) is $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma, \frac{3\gamma-1}{2} \wedge \frac{1+\gamma}{2})$, which is nonempty for all $w \in (0,1)$.

For ease of discussion, we rewrite the conditions for cases (ii) and (iii) in terms of γ:

$$\text{Case (ii):} \qquad w > \frac{2-\sqrt{2}}{8+2\sqrt{2}}, \ \frac{1-2w}{1+\sqrt{2}} \vee (2w-1) \le \gamma < \frac{1+2w}{3}, \tag{C.15a}$$

$$\text{Case (iii):} \qquad \frac{1-2w}{1+\sqrt{2}} \vee \frac{1+2w}{3} \le \gamma \le 1. \tag{C.15b}$$

The following lemmas present the results in cases (ii) and (iii). The proofs are at the end of this subsection.

Lemma C.2 *Under condition (C.15a) so buyer's order quantity $Q_T(w;\gamma) = Q_h(w)$ and makes nonzero trade credit payment in the supplier-dominant market, the supplier's optimal direct channel quantity q_d^* and the associated default scenarios are given by Table OA2.*

Lemma C.3 *Under condition (C.15b) so buyer's order quantity $Q_T(w;\gamma) = Q_h(w)$ and makes zero trade credit payment in the supplier-dominant market, the supplier's optimal direct channel quantity q_d^* and the associated default scenarios are given by Table OA3.*

On $w \le \frac{2-\sqrt{2}}{8+2\sqrt{2}}$, case (ii) does not exist so Lemma C.2 is irrelevant. Furthermore, $\frac{1+2w}{3} \ge \frac{1-2w}{1+\sqrt{2}}$ in condition (C.15b), which reduces to $\frac{1+2w}{3} \le \gamma \le 1$. Combining Lemmas C.1 and C.3 yields equ. (C.11).

Table OA2 Supplier direct channel quantity and default scenarios under trade credit with condition (C.15a)

	Competition		Supplier default?		
q_d^*	intensity γ	Supplier debt level D	Low	Buyer	Supplier
q_d^l	$\gamma \le \sqrt{2}-1$	$D \le wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$	Yes	No	No
q_d^h	$\gamma \le \sqrt{2}-1$	$D \in [wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{1-2w^2+4w\gamma-\gamma^2}{8}]$	Yes	Yes	No
	$\gamma > \sqrt{2}-1$	$D \le wQ_h(w)$	Yes	No	No
	$\gamma > \sqrt{2}-1$	$D \in [wQ_h(w), \frac{1-2w^2+4w\gamma-\gamma^2}{8}]$	Yes	Yes	No

Market scenarios "Low", "Supplier", and "Buyer" are the low market, supplier-dominant, and buyer-dominant markets defined in Table 1.

Table OA3 Supplier direct channel quantity and default scenarios under trade credit with condition (C.15b)

	Competition		Supplier default?		
q_d^*	intensity γ	Supplier debt level D	Low	Buyer	Supplier
q_d^l	$\gamma \le \sqrt{2}-1$	$D \le wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$	Yes	No	No
q_d^h	$\gamma \le \sqrt{2}-1$	$D \in [wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{(1+\gamma)^2}{16}]$	Yes	Yes	No
	$\gamma > \sqrt{2}-1$	$D \le wQ_h(w)$	Yes	No	No
	$\gamma > \sqrt{2}-1$	$D \in [wQ_h(w), \frac{(1+\gamma)^2}{16}]$	Yes	Yes	No

Market scenarios "Low", "Supplier", and "Buyer" are the low market, supplier-dominant, and buyer-dominant markets defined in Table 1.

Note that $\frac{1+2w}{3} > \sqrt{2}-1$ is equivalent to $w > \frac{3\sqrt{2}-4}{2} \approx 0.12$. Thus, on $w \le \frac{3\sqrt{2}-4}{2}$, the upper bound on γ in case (ii) (see (C.15a)) is less than $\sqrt{2}-1$. Only the top two rows in Table OA2 apply. All rows in Table OA3 apply. Combining them with Table OA1 yields equ. (C.12).

On $w > \frac{3\sqrt{2}-4}{2}$, all rows in Table OA2 apply and the bottom two rows in Table OA3 apply. Combining them with Table OA1 yields equ. (C.13), completing the proof. □

Proofs for this subsection:

Proof of Lemma C.1. In this case, the buyer's order quantity $Q_T(w) = Q_l(w)$ from Proposition C.1 and Corollary 1. The associated parameter region is $w < \frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma$ (Proposition C.1), which implies that $\gamma < \frac{1}{1+\sqrt{2}}$. The supplier's expected payoff is a piecewise function depending on whether it can fully repay the debt D. Because the supplier receives the same trade credit payment $wQ_l(w)$ in the supplier- and buyer-dominant markets, the supplier has higher total revenue in the supplier-dominant market. Thus, if the supplier is solvent in the buyer-dominant market, it must be solvent in the supplier-dominant market. Therefore, we only need to discuss the following two cases: (1) Solvency in both market scenarios, and (2) Solvency only in supplier-dominant market.

Case (1): Solvency in both supplier- and buyer-dominant markets. In this case, if $q_d \le \frac{1-\gamma}{2}$ so that the supplier's market price in the buyer-dominant market is $\frac{1-\gamma}{2} - q_d$ (see Table 1), then the supplier's

expected payoff is

$$\pi_{s1}^T = \frac{\theta}{2}[(\frac{1-\gamma}{2} - q_d)q_d + wQ_l(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + wQ_l(w) - D]. \tag{C.16}$$

Its unconstrained optimizer is $q_d^l = \frac{1}{4}$. The feasibility condition requires that $q_d^l \le \frac{1-\gamma}{2}$ and $(\frac{1-\gamma}{2} - q_d^l)q_d^l + wQ_l(w) - D > 0$, which is equivalent to

$$\gamma \le \frac{1}{2}, \quad D < \frac{1}{16}(1 + 4w - 8w^2 - 2\gamma) = wQ_l(w) + \frac{1}{16}(1 - 2\gamma). \tag{C.17}$$

If $\gamma > \frac{1}{2}$, then $q_d^l > \frac{1-\gamma}{2}$ so the supplier's expected payoff is monotone increasing on $q_d \le \frac{1-\gamma}{2}$. Thus, the optimal direct quantity q_d^* satisfies $q_d^* > \frac{1-\gamma}{2}$. If $\gamma \le \frac{1}{2}$ but the debt level D exceeds the upper bound in equ. (C.17), then this belongs to case (i2) the supplier defaults in the buyer-dominant market.

If $q_d \in (\frac{1-\gamma}{2}, \frac{1+\gamma}{2}]$, then the market price $p_s = 0$ in the buyer-dominant market and $p_s = \frac{1+\gamma}{2} - q_d$ in the supplier-dominant market. Thus, the supplier's expected payoff is

$$\pi_{s2}^T = \frac{\theta}{2}[wQ_l(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + wQ_l(w) - D]. \tag{C.18}$$

Its unconstrained optimizer is $q_d^h = \frac{1+\gamma}{4}$, which automatically satisfies $q_d^h < \frac{1+\gamma}{2}$. Thus, the feasibility condition reduces to $q_d^h > \frac{1-\gamma}{2}$ and $wQ_l(w) - D > 0$, which is equivalent to

$$\gamma \in (\frac{1}{3}, 1], \quad D < \frac{1}{4}(w - 2w^2) = wQ_l(w). \tag{C.19}$$

If $\gamma \le \frac{1}{3}$, then $q_d^h \le \frac{1-\gamma}{2}$ so the supplier's expected payoff is monotone decreasing on $q_d > \frac{1-\gamma}{2}$. Thus, the optimal direct quantity q_d^* satisfies $q_d^* < \frac{1-\gamma}{2}$. If $\gamma > \frac{1}{3}$ but the debt level D exceeds the upper bound in equ. (C.19), then this belongs to case (i2) where the supplier defaults in the buyer-dominant market.

Case (2): Solvency in supplier-dominant markets only. In this case, the market price p_s must satisfy $\frac{1+\gamma}{2} - q_d > 0$, as otherwise the supplier's market price $p_s = 0$ in both the supplier- and buyer-dominant markets. Given that it also receives the same trade credit, it collects the same revenue in these market scenarios, which implies that it cannot default in only one. The supplier's expected payoff is

$$\tilde{\pi}_{s2}^T = \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + wQ_l(w) - D]. \tag{C.20}$$

Clearly, $\tilde{\pi}_{s2}^T$ differs from π_{s2}^T by only a constant. Thus, $q_d^h = \frac{1+\gamma}{4}$ is its unconstrained optimizer, which automatically guarantees a nonzero market price as $p_s = \frac{1+\gamma}{2} - q_d^h > 0$. The feasibility condition requires that (a) $q_d^h \le \frac{1-\gamma}{2}$ and $(\frac{1-\gamma}{2} - q_d)q_d + wQ_l(w) - D \le 0$, or (b) $q_d^h > \frac{1-\gamma}{2}$ and $wQ_l(w) - D \le 0$. These conditions reduce to:

$$(\gamma \le \frac{1}{3},\ D \in [wQ_l(w) + \frac{1 - 2\gamma - 3\gamma^2}{16}, wQ_l(w) + \frac{(1+\gamma)^2}{16})), \tag{C.21a}$$

$$\text{or} \quad (\gamma \in (\frac{1}{3}, 1],\ D \in [wQ_l(w), wQ_l(w) + \frac{(1+\gamma)^2}{16})). \tag{C.21b}$$

Global optimum. When the buyer pays trade credit in full, the supplier's expected payoff takes one of the three functional forms of π_{s1}^T, π_{s2}^T, and $\tilde{\pi}_{s2}^T$, when it does not default with probability one. The supplier is solvent whenever $A=1$ if the debt level is low. Otherwise, the supplier is solvent only in supplier-dominant market. Combining results above yields that:

1. If $\gamma \le \frac{1}{3}$ and $D < wQ_l(w) + \frac{1-2\gamma-3\gamma^2}{16}$, then q_d^l is feasible and the global optimum (as the supplier's payoff is monotone decreasing on $q_d > \frac{1-\gamma}{2}$).
2. If $\gamma \le \frac{1}{3}$ and $D \in (wQ_l(w) + \frac{1-2\gamma-3\gamma^2}{16}, wQ_l(w) + \frac{1-2\gamma}{16})$, then both q_d^l and q_d^h are feasible, with associated payoff functions π_{s1}^T and $\tilde{\pi}_{s2}^T$.
3. If $\gamma \in (\frac{1}{3}, \frac{1}{2}]$ and $D < wQ_l(w)$, then both q_d^l and q_d^h are feasible, with associated payoffs π_{s1}^T and π_{s2}^T.
4. If $\gamma \in (\frac{1}{3}, \frac{1}{2}]$ and $D \in (wQ_l(w), wQ_l(w) + \frac{1-2\gamma}{16})$, then both q_d^l and q_d^h are feasible, with expected payoffs π_{s1}^T and $\tilde{\pi}_{s2}^T$.
5. If $\gamma \in (\frac{1}{3}, \frac{1}{2}]$ and $D \in (wQ_l(w) + \frac{1-2\gamma}{16}, wQ_l(w) + \frac{(1+\gamma)^2}{16})$, then q_d^h is the only feasible solution and the global optimum.
6. If $\gamma > \frac{1}{2}$ and $D < wQ_l(w) + \frac{(1+\gamma)^2}{16}$, then q_d^h is feasible and the global optimum.

Recall that $\gamma < \frac{1}{1+\sqrt{2}}$ in case (i). Given γ, if the debt level is greater than the debt upper bound above, then the supplier defaults with probability one.

Comparing the payoffs in case 2 above yields that q_d^l is optimal on $D < wQ_l(w) + \frac{1-2\gamma-\gamma^2}{16}$ and q_d^h is optimal otherwise. Comparing the payoffs in case 3 yields that q_d^l always yields a higher payoff than q_d^h. Comparing the payoffs in case 4 yields that q_d^l yields a higher payoff on $\gamma \in (\frac{1}{3}, \sqrt{2}-1)$ and $D < wQ_l(w) + \frac{1-2\gamma-\gamma^2}{16}$. Otherwise q_d^h yields a higher payoff. Combining these results yields Table OA1. □

Proof of Lemma C.2. Unlike Lemma C.1, where the buyer repays the same amounts under $A=1$, in this case the buyer pays in full $wQ_h(w)$ in the buyer-dominant market and pays $p_bQ_h(w)$ in the supplier-dominant market. Thus, the supplier may not have the highest revenue in the supplier-dominant market due to the pooling effect. It may be solvent only in the buyer-dominant market, which gives rise to a new case we need to discuss.

Because the buyer's trade credit payment is exogenous to the supplier's direct channel revenue, it is never optimal for the supplier to use a direct channel quantity that leads to zero market price in all market scenarios. Thus, in equilibrium, $q_d^* < \frac{1+\gamma}{2}$. This implies that the supplier's expected payoff takes the following forms:

1. Supplier is solvent when $A=1$, with $q_d < \frac{1-\gamma}{2}$:

$$\pi_{s11}^T = \frac{\theta}{2}[(\frac{1-\gamma}{2} - q_d)q_d + wQ_h(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + p_bQ_h(w) - D]. \tag{C.22}$$

2. Supplier is solvent when $A=1$, with $q_d \geq \frac{1-\gamma}{2}$:

$$\pi_{s21}^T = \frac{\theta}{2}[wQ_h(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + p_b Q_h(w) - D]. \tag{C.23}$$

3. Supplier is solvent only in supplier-dominant market:

$$\tilde{\pi}_{s21}^T = \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d + p_b Q_h(w) - D]. \tag{C.24}$$

4. Supplier is solvent only in buyer-dominant market, with $q_d < \frac{1-\gamma}{2}$ (We note that $q_d \geq \frac{1-\gamma}{2}$ is strictly dominated because the feasibility of $q_d \geq \frac{1-\gamma}{2}$ implies $wQ_h(w) > D$ in this scenario, which implies that the supplier can always do better by setting $q_d < \frac{1-\gamma}{2}$):

$$\pi_{s3}^T = \frac{\theta}{2}[(\frac{1-\gamma}{2} - q_d)q_d + wQ_h(w) - D]. \tag{C.25}$$

The remainder of the proof first analyzes the four pieces in isolation, then combines them together to find the optimal direct quantity.

Case 1. Clearly, the supplier's payoff π_{s11}^T differs from π_{s1}^T in equ. (C.16) by a constant. Thus, the unconstrained optimal solution is q_d^l as for π_{s1}^T. The feasibility condition is $q_d^l \leq \frac{1-\gamma}{2}$, $(\frac{1-\gamma}{2} - q_d^l)q_d^l + wQ_h(w) - D > 0$, and $(\frac{1+\gamma}{2} - q_d^l)q_d^l + p_b Q_h(w) - D > 0$, which reduces to

$$\gamma \leq \frac{1}{2}, \qquad D < \frac{2 - 4w^2 + 8w\gamma - 3\gamma^2}{16} \wedge \frac{1 + 4w - 8w^2 - 2\gamma + 4w\gamma}{16}. \tag{C.26}$$

If $\gamma > \frac{1}{2}$, then $q_d^l > \frac{1-\gamma}{2}$ so the supplier's payoff is increasing in q_d on $q_d < \frac{1-\gamma}{2}$.

Case 2. The supplier's payoff π_{s21}^T differs from π_{s2}^T in equ. (C.18) by a constant. Thus, the unconstrained optimizer is q_d^h, which automatically satisfies $q_d^h < \frac{1+\gamma}{2}$. The feasibility condition is $q_d^h > \frac{1-\gamma}{2}$, $(\frac{1+\gamma}{2} - q_d^h)q_d^h + p_b Q_h(w) - D > 0$, and $wQ_h(w) > D$, which reduces to

$$\gamma \geq \frac{1}{3}, \qquad D < wQ_h(w) \wedge \frac{1 - 2w^2 + 4w\gamma - \gamma^2}{8}. \tag{C.27}$$

If $\gamma < \frac{1}{3}$, then $q_d^h < \frac{1-\gamma}{2}$ so the supplier's payoff is decreasing in q_d on $q_d > \frac{1-\gamma}{2}$.

Case 3. The supplier's payoff $\tilde{\pi}_{s21}^T$ differs from $\tilde{\pi}_{s2}^T$ in equ. (C.20) by a constant. Thus, the unconstrained optimizer is q_d^h, which automatically satisfies $q_d^h < \frac{1+\gamma}{2}$. The feasibility condition requires that $(\frac{1+\gamma}{2} - q_d^h)q_d^h + p_b Q_h(w) - D > 0$, and (a) $q_d^h \leq \frac{1-\gamma}{2}$ and $(\frac{1-\gamma}{2} - q_d)q_d + wQ_h(w) - D \leq 0$, or (b) $q_d^h > \frac{1-\gamma}{2}$ and $wQ_h(w) - D \leq 0$. These conditions reduce to:

$$(\gamma \leq \frac{1}{3},\ D \in [\frac{1 + 4w - 8w^2 - 2\gamma + 4w\gamma - 3\gamma^2}{16}, \frac{1 - 2w^2 + 4w\gamma - \gamma^2}{8}), \tag{C.28a}$$

$$\text{or} \quad (\gamma \in (\frac{1}{3}, 1],\ D \in [wQ_h(w), \frac{1 - 2w^2 + 4w\gamma - \gamma^2}{8})). \tag{C.28b}$$

Case 4. The unconstrained optimizer of π_{s3}^T is $q_{d0} = \frac{1-\gamma}{4}$, which automatically satisfies $q_{d0} < \frac{1-\gamma}{2}$. Thus, the feasibility condition is $(\frac{1+\gamma}{2} - q_{d0})q_{d0} + p_b Q_h(w) - D \le 0$ and $(\frac{1-\gamma}{2} - q_{d0})q_{d0} + wQ_h(w) - D > 0$, which reduces to

$$D \in (\frac{1-2w^2+4w\gamma-3\gamma^2}{8}, \frac{1+4w-8w^2-2\gamma+4w\gamma+\gamma^2}{16}). \tag{C.29}$$

Algebra yields that the set in equ. (C.29) is nonempty only if $\gamma > \frac{1}{3}$.

Global optimum. We shall compare the possible optimal values q_d^l, q_d^h, and q_{d0}, accounting for the overall condition that $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma \vee \frac{3\gamma-1}{2}, \frac{1+\gamma}{2})$.

On $\gamma \le \frac{1}{3}$, q_d^l and q_d^h are both feasible on the intersection of debt levels in equ. (C.26) and (C.28), with the respective payoffs π_{s11}^T and $\tilde{\pi}_{s21}^T$. Comparing the payoffs on the intersection region yields that on $\gamma \le \frac{1}{3}$, $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma, \frac{1+\gamma}{2})$, the optimal direct quantity $q_d^* = q_d^l$ if $D < \frac{1+4w-8w^2-2\gamma+4w\gamma-\gamma^2}{16} = wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$. Otherwise, $q_d^* = q_d^h$ on $D \in [wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{1-2w^2+4w\gamma-\gamma^2}{8})$.

On $\gamma \in (\frac{1}{3}, \frac{1}{2}]$, q_d^l is feasible when D is low, and may overlap with q_d^h in either case 2 (equ. (C.27)) or case 3 (equ. (C.28)). Algebra yields that the feasibility region of q_d^l does not overlap with that of q_{d0} in equ. (C.29).

- On the debt level overlap in equ. (C.26) and (C.27), the payoffs associated with q_d^l and q_d^h are π_{s11}^T in (C.22) and π_{s21}^T in (C.23). Comparing the payoffs on the debt overlap yields that the optimal direct quantity $q_d^* = q_d^l$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D < wQ_h(w)$, and $q_d^* = q_d^h$ if $\gamma \in (\sqrt{2}-1, \frac{1}{2}]$ and $D < wQ_h(w)$.
- On the debt level overlap in equ. (C.26) and (C.28), the payoffs associated with q_d^l and q_d^h are π_{s11}^T in (C.22) and $\tilde{\pi}_{s21}^T$ in (C.24). Comparing the payoffs on the debt overlap yields that the optimal direct quantity $q_d^* = q_d^l$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D \in [wQ_h(w), wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16})$. The optimal direct quantity $q_d^* = q_d^h$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D \in (wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, wQ_h(w) + \frac{1-2\gamma}{16})$, or $\gamma \in (\sqrt{2}-1, \frac{1}{2}]$ and $D \in [wQ_h(w), wQ_h(w) + \frac{1-2\gamma}{16})$.

On $\gamma > \frac{1}{2}$, the feasibility region of q_{d0} in equ. (C.29) is a subset of the feasibility region of q_d^h on $\gamma > \frac{1}{3}$ (see (C.27) and (C.28)), as the lower bound on D of the latter is zero and the upper bound on D of the latter exceeds that in (C.28) on $w \ge \frac{3\gamma-1}{2}$. Thus, we only need to compare the payoff of q_{d0}, which is π_{s3}^T in equ. (C.25), with the payoff of q_d^h under condition (C.26), which is π_{s21}^T, and the payoff of q_d^h under condition (C.28), which is $\tilde{\pi}_{s21}^T$. Comparison yields that q_d^h always dominates q_{d0} on the overlapping region. Thus, q_{d0} is never a global optimum. On $\gamma > \frac{1}{2}$, q_d^h is the global optimum, but there are two default scenarios, corresponding to cases 2 and 3, respectively,

To sum up, under condition $w \in (\frac{1}{2} - \frac{1+\sqrt{2}}{2}\gamma \vee \frac{3\gamma-1}{2}, \frac{1+\gamma}{2})$,

- If $\gamma \le \sqrt{2}-1$ and $D < wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$, then q_d^l is the global optimum. Case 1 is true.
- If $\gamma \le \sqrt{2}-1$ and $D \in [wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{1-2w^2+4w\gamma-\gamma^2}{8})$, then q_d^h is the global optimum. Case 3 is true.

- If $\gamma > \sqrt{2}-1$ and $D < wQ_h(w)$, then q_d^h is the global optimum. Case 2 is true.
- If $\gamma > \sqrt{2}-1$ and $D \in [wQ_h(w), \frac{1-2w^2+4w\gamma-\gamma^2}{8})$, then q_d^h is the global optimum. Case 3 is true.

□

Proof of Lemma C.3. This proof is similar to that of Lemma C.2, except that now in the supplier-dominant market, the supplier receives zero payment from the buyer. Because the buyer's trade credit payment is exogenous to the supplier's direct channel revenue, it is never optimal for the supplier to use a direct channel quantity that leads to zero market price in all market scenarios. Thus, in equilibrium, $q_d^* < \frac{1+\gamma}{2}$. This implies that the supplier's expected payoff takes the following forms:

1. Supplier is solvent when $A=1$, with $q_d < \frac{1-\gamma}{2}$:

$$\pi_{s12}^T = \frac{\theta}{2}[(\frac{1-\gamma}{2} - q_d)q_d + wQ_h(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d - D]. \tag{C.30}$$

2. Supplier is solvent when $A=1$, with $q_d \geq \frac{1-\gamma}{2}$:

$$\pi_{s22}^T = \frac{\theta}{2}[wQ_h(w) - D] + \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d - D]. \tag{C.31}$$

3. Supplier is solvent only in supplier-dominant market:

$$\tilde{\pi}_{s22}^T = \frac{\theta}{2}[(\frac{1+\gamma}{2} - q_d)q_d - D]. \tag{C.32}$$

4. Supplier is solvent only in buyer-dominant market, with $q_d < \frac{1-\gamma}{2}$. The supplier's payoff is π_{s3}^T in equ. (C.25).

The remainder of the proof first analyzes the four pieces in isolation, then combines them together to find the optimal direct quantity.

Case 1. Clearly, the supplier's payoff π_{s12}^T differs from π_{s1}^T in equ. (C.16) by a constant. Thus, the unconstrained optimal solution is q_d^l as for π_{s1}^T. The feasibility condition is $q_d^l \leq \frac{1-\gamma}{2}$, $(\frac{1-\gamma}{2} - q_d^l)q_d^l + wQ_h(w) - D > 0$, and $(\frac{1+\gamma}{2} - q_d^l)q_d^l - D > 0$, which reduces to

$$\gamma \leq \frac{1}{2}, \qquad D < \frac{1+2\gamma}{16} \wedge \frac{1+4w-8w^2-2\gamma+4w\gamma}{16}. \tag{C.33}$$

If $\gamma > \frac{1}{2}$, then $q_d^l > \frac{1-\gamma}{2}$ so the supplier's payoff is increasing in q_d on $q_d < \frac{1-\gamma}{2}$.

Case 2. The supplier's payoff π_{s22}^T differs from π_{s2}^T in equ. (C.18) by a constant. Thus, the unconstrained optimizer is q_d^h, which automatically satisfies $q_d^h < \frac{1+\gamma}{2}$. The feasibility condition is $q_d^h > \frac{1-\gamma}{2}$, $(\frac{1+\gamma}{2} - q_d^h)q_d^h - D > 0$, and $wQ_h(w) > D$, which reduces to

$$\gamma \geq \frac{1}{3}, \qquad D < wQ_h(w) \wedge \frac{(1+\gamma)^2}{16}. \tag{C.34}$$

If $\gamma < \frac{1}{3}$, then $q_d^h < \frac{1-\gamma}{2}$ so the supplier's payoff is decreasing in q_d on $q_d > \frac{1-\gamma}{2}$.

Case 3. The supplier's payoff $\tilde{\pi}^T_{s22}$ differs from $\tilde{\pi}^T_{s2}$ in equ. (C.20) by a constant. Thus, the unconstrained optimizer is q^h_d, which automatically satisfies $q^h_d < \frac{1+\gamma}{2}$. The feasibility condition requires that $(\frac{1+\gamma}{2} - q^h_d)q^h_d - D > 0$, and (a) $q^h_d \le \frac{1-\gamma}{2}$ and $(\frac{1-\gamma}{2} - q_d)q_d + wQ_h(w) - D \le 0$, or (b) $q^h_d > \frac{1-\gamma}{2}$ and $wQ_h(w) - D \le 0$. These conditions reduce to:

$$(\gamma \le \frac{1}{3},\ D \in [\frac{1+4w-8w^2-2\gamma+4w\gamma-3\gamma^2}{16}, \frac{(1+\gamma)^2}{16}), \tag{C.35a}$$

$$\text{or} \quad (\gamma \in (\frac{1}{3}, 1],\ D \in [wQ_h(w), \frac{(1+\gamma)^2}{16}). \tag{C.35b}$$

Case 4. The unconstrained optimizer of π^T_{s3} is $q_{d0} = \frac{1-\gamma}{4}$, which automatically satisfies $q_{d0} < \frac{1-\gamma}{2}$. Thus, the feasibility condition is $(\frac{1+\gamma}{2} - q_{d0})q_{d0} - D \le 0$ and $(\frac{1-\gamma}{2} - q_{d0})q_{d0} + wQ_h(w) - D > 0$, which reduces to

$$D \in (\frac{1+2\gamma-3\gamma^2}{16}, \frac{1+4w-8w^2-2\gamma+4w\gamma+\gamma^2}{16}). \tag{C.36}$$

Algebra yields that the set in equ. (C.36) is nonempty only if $\gamma > \frac{1}{3}$.

Global optimum. We shall compare the possible optimal values q^l_d, q^h_d, and q_{d0}, accounting for the overall condition (C.15b). Because condition (C.15b) implies that $\gamma > \frac{1}{3}$, condition (C.35a) is never true. Next we focus on the remaining cases.

On $\gamma \in (\frac{1}{3}, \frac{1}{2}]$, q^l_d is feasible when D is low, and may overlap with q^h_d in either case 2 (equ. (C.34)) or case 3 (equ. (C.35b)). Algebra yields that the feasibility region of q^l_d does not overlap with that of q_{d0} in equ. (C.36).

- On the debt level overlap in equ. (C.33) and (C.34), the payoffs associated with q^l_d and q^h_d are π^T_{s12} in (C.30) and π^T_{s22} in (C.31). Comparing the payoffs on the debt overlap yields that the optimal direct quantity $q^*_d = q^l_d$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D < wQ_h(w)$, and $q^*_d = q^h_d$ if $\gamma \in (\sqrt{2}-1, \frac{1}{2}]$ and $D < wQ_h(w)$.
- On the debt level overlap in equ. (C.33) and (C.35), the payoffs associated with q^l_d and q^h_d are π^T_{s12} in (C.30) and $\tilde{\pi}^T_{s22}$ in (C.32). Comparing the payoffs on the debt overlap yields that the optimal direct quantity $q^*_d = q^l_d$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D \in [wQ_h(w), wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16})$. The optimal direct quantity $q^*_d = q^h_d$ if $\gamma \in (\frac{1}{3}, \sqrt{2}-1]$ and $D \in (wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, wQ_h(w) + \frac{1-2\gamma}{16})$, or $\gamma \in (\sqrt{2}-1, \frac{1}{2}]$ and $D \in [wQ_h(w), wQ_h(w) + \frac{1-2\gamma}{16})$.

On $\gamma > \frac{1}{2}$, the feasibility region of q_{d0} in equ. (C.36) is a subset of the feasibility region of q^h_d on $\gamma > \frac{1}{3}$ (see (C.34) and (C.35)), as the lower bound on D of the latter is zero and the upper bound on D of the latter exceeds that in (C.35) under condition (C.15b). Thus, we only need to compare the payoff of q_{d0}, which is π^T_{s3} in equ. (C.25), with the payoff of q^h_d under condition (C.34), which is π^T_{s22}, and with the payoff of q^h_d under condition (C.35), which is $\tilde{\pi}^T_{s22}$. Comparison yields that q^h_d always dominates q_{d0} on the overlapping region. Thus, q_{d0} is never a global optimum. On $\gamma > \frac{1}{2}$, q^h_d is the global optimum, but there are two default scenarios, corresponding to cases 2 and 3, respectively,

To sum up, under condition (C.15b),

- If $\gamma \leq \sqrt{2}-1$ and $D < wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}$, then q_d^l is the global optimum. Case 1 is true.
- If $\gamma \leq \sqrt{2}-1$ and $D \in [wQ_h(w) + \frac{1-2\gamma-\gamma^2}{16}, \frac{(1+\gamma)^2}{16})$, then q_d^h is the global optimum. Case 3 is true.
- If $\gamma > \sqrt{2}-1$ and $D < wQ_h(w)$, then q_d^h is the global optimum. Case 2 is true.
- If $\gamma > \sqrt{2}-1$ and $D \in [wQ_h(w), \frac{(1+\gamma)^2}{16})$, then q_d^h is the global optimum. Case 3 is true.

□

Online Appendix D. Supplemental Empirical Analysis and Results

D.1. Buyer-supplier size and bankruptcy risk comparison

Table OA4 T-Test of buyer and supplier size.

Panel A			
	Mean	**Std. Dev.**	**t-stat**
Supplier size	5.70	2.28	6.92
Peer buyer size	7.41	2.38	
Panel B			
	Mean	**Std. Dev.**	**t-stat**
Supplier size	5.70	2.28	12.01
Non-peer buyer size	8.63	1.80	
Panel C			
	Mean	**Std. Dev.**	**t-stat**
Peer buyer size	7.41	2.38	4.85
Non-peer buyer size	8.63	1.80	

Notes: Table OA4 represents the comparison among treated suppliers' average size, their peer buyers' average size and their non-competing buyers' average size. Peer buyer is defined as the buyer sharing the same four-digit SIC code with supplier. Firm size is defined as the logarithm of firm's total assets(in million dollars). We calculate treated firms' average size within the time range and the average sizes of all of their (non)peer buyers. We conduct three t-tests between each two of them. The results show that for each treated supplier, the average size of their non-peer buyers is statistically larger than the average size of their peer buyers, and the average size of their peer buyers is larger than their own average size.

Table OA5 T-Test of peer buyer and supplier bankruptcy risk.

Distance-to-Default			
	Mean	**Std. Dev.**	**t-stat**
Supplier DD	4.35	2.58	4.31
Peer buyer DD	5.68	2.78	
OScore			
	Mean	**Std. Dev.**	**t-stat**
Supplier OScore	-2.99	3.76	2.55
Peer buyer OScore	-3.93	2.72	

Notes: Table OA5 represents the comparison between treated suppliers' average bankruptcy risk and their peer buyers' bankruptcy risk. Peer buyer is defined as the buyer sharing the same four-digit SIC code with supplier. Bankruptcy risk is proxied by distance-to-default and OScore. We calculate their average risk within the time range and the average risk of all of their peer buyers. We conduct two t-tests. The results show that for each treated supplier, their average bankruptcy risk is statistically higher than their peer buyers'.

D.2. Significance of control variables

Table OA6 Significance of Control Variables

	(1)
Dependent variables	Treat
Size	0.439***
	(0.123)
Cash	0.277***
	(0.089)
Sales Growth	0.022
	(0.016)
Inventory	-0.385***
	(0.138)
Market Share	-0.025
	(0.109)
Constant	-2.617***
	(0.115)
Observations	82,275

Notes: This table shows the logistic regression results examining whether and which of our control variables significantly influence the likelihood of a firm selling to industry peers. Robust standard errors clustered at firm level in parentheses: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

D.3. Balance test

Table OA7 Balance Test of Matched Sample

	Treated	Control	Difference	p-value
	(1)	(2)	(3)	
$Size_{t-1}$	-0.155	-0.107	0.048	0.771
	(1.030)	(1.107)		
$Cash_{t-1}$	0.560	0.612	0.052	0.774
	(1.282)	(1.237)		
$Inventory_{t-1}$	-0.475	-0.602	-0.127	0.376
	(0.831)	(0.851)		
$OScore_{t-1}$	-4.203	-3.807	0.396	0.784
	(2.382)	(1.980)		
DD_{t-1}	4.205	4.158	-0.047	0.967
	(2.926)	(3.070)		

Notes: This table presents the balance test of our matched sample. $Size_{t-1}$ comprises treated firms' size 1 quarter before they receive the treatment and their matched control firms' size in the same calendar year quarter. We can conclude that there is no statistical pre-treatment difference between treated group and the control group.

D.4. Variable descriptions

Table OA8 Variable Descriptions

Variable	Measurement	Data source
Independent Variables		
$Treat_i$	An indicator that equals 1 if firm i ever sells to its industry peers	FactSet Revere
$SellPeer_{i,t}$	An indicator that equals 1 if firm i sells to its industry peers in year quarter t and equals 0 otherwise	FactSet Revere
$TCProvision_{i,t}$	The ratio of firm i's accounts receivable to firm i's total sales in year quarter t	Compustat
Dependent Variables		
$DD_{i,t}$	Merton's distance-to-default of firm i in year quarter t	Compustat & CRSP
$OScore_{i,t}$	The Ohlson O-Score of firm i in year quarter t	Compustat
Control Variables		
$Size_{i,t}$	The logarithm of firm i's total assets in year quarter t	Compustat
$Cash_{i,t}$	The ratio of firm i's cash and short-term investments to the total assets in year quarter t	Compustat
$SalesGrowth_{i,t}$	The difference between firm i's total sales in year quarter t and that in year quarter $t-1$ divided by firm i's total sales in year quarter $t-1$	Compustat
$Inventory_{i,t}$	The ratio of firm i' inventory to the total assets in year quarter t	Compustat
$MarketShare_{i,t}$	The ratio of firm i' sales to the total sales of its industry in year quarter t	Compustat
Moderating Variables		
$PreEventLeverage_i$	Eight-quarter pre-event average of long-term debt scaled by total assets	Compustat
$PreEventCash_i$	Eight-quarter pre-event average of cash-to-assets	Compustat
$PreEventMarginCorr_i$	Correlation between focal supplier's gross margin and the average gross margin of its industry peer buyers over the eight pre-event quarters	Compustat
$PreEventETNIC_i$	Industry-demeaned average ETNIC product-similarity measure with industry peer buyers, measured two years prior to the event year	Hoberg and Phillips (2025)

D.5. Summary statistics

Table OA9 Summary Statistics of Panel Sample (n = 82,275)

Variable	Mean	p10	p25	p50	p75	p90
TCProvision	0.682	0.230	0.422	0.583	0.742	0.959
DD	4.789	0.574	2.247	4.384	6.890	9.550
OScore	-4.154	-6.651	-5.607	-4.483	-3.211	-1.741
SellPeer	0.031	0.000	0.000	0.000	0.000	0.000
Size	5.917	3.338	4.432	5.884	7.326	8.533
Cash	0.260	0.018	0.052	0.157	0.404	0.704
Inventory	0.143	0.009	0.062	0.123	0.202	0.294
Market Share	0.007	0.000	0.000	0.000	0.003	0.016
Sales Growth	0.084	-0.206	-0.068	0.021	0.117	0.304
PreEventLeverage	0.175	0.000	0.013	0.107	0.278	0.451
PreEventCash	0.408	0.073	0.130	0.339	0.653	0.884
PreEventMarginCorr	0.149	-0.571	-0.271	0.277	0.524	0.829
PreEventETNIC	-0.010	-0.186	-0.121	0.000	0.073	0.149

Notes: This table reports summary statistics for all variables in their original (unstandardized) form. In subsequent regression analyses, all control variables are standardized to have zero mean and unit variance.

D.6. Placebo Test

To address concerns that our results may be driven by the specific matching technique or the particular selection of treated firms, we conduct a series of randomized, simulation-based placebo tests. For each treated supplier, we randomly select two non-treated suppliers from the same four-digit SIC industry. We then assign the starting year-quarter of the actual treated firm to one of these two as the pseudo-treated firm, designating the other as the pseudo-control. This process is repeated 1,000 times to generate 1,000 placebo samples. For each sample, we re-estimate equation 13, yielding 1,000 placebo estimates of the coefficient of interest for each outcome variable.

Figures OA1 and OA2[1] show that the actual coefficients from Table 6 lie in the tails of the placebo distributions: the actual coefficient is consistently lower than most simulated coefficients when the outcome variable is OScore, and higher than most simulated coefficients when the outcome variable is DD. These results suggest that our findings are unlikely to be driven by chance or by the specific selection of treated firms.

[1] Coefficients below the 0.5th percentile and above the 99.5th percentile are excluded.

Figure OA1 Placebo Test for OScore

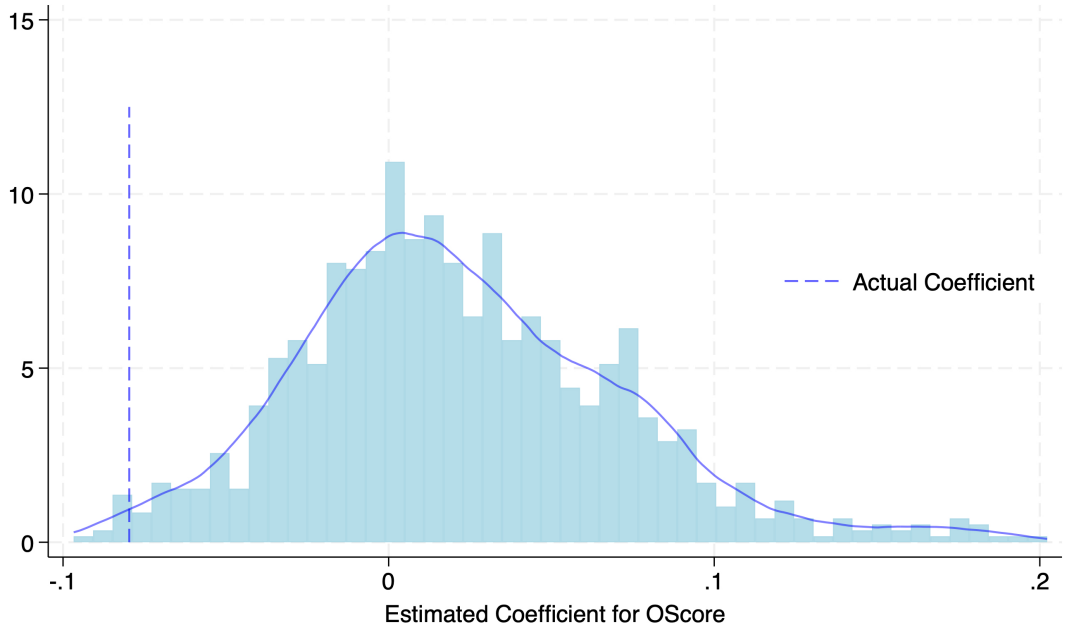
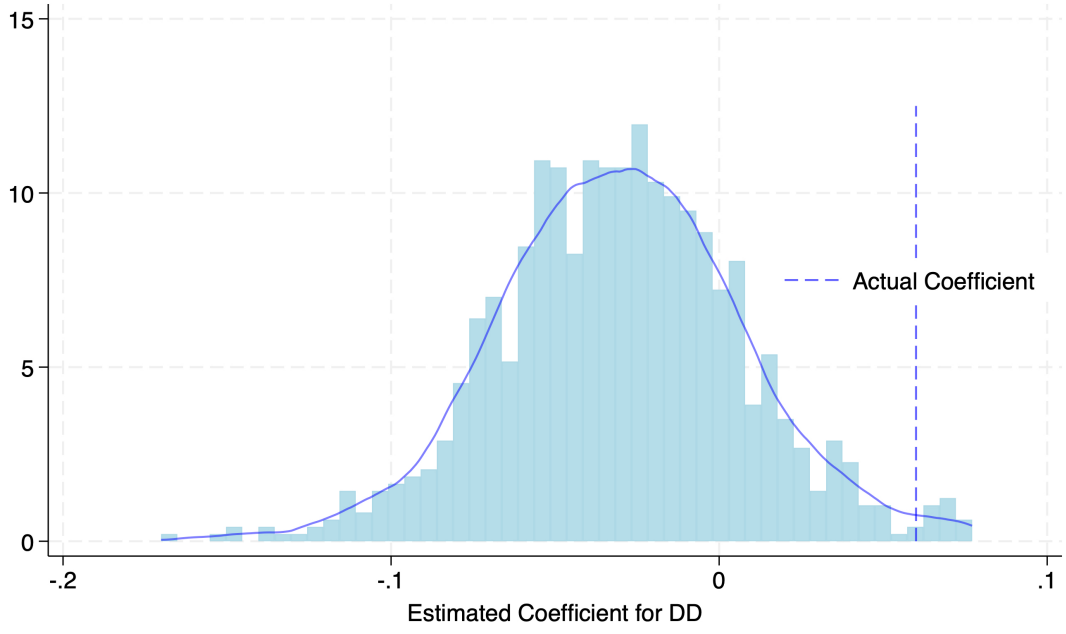


Figure OA2 Placebo Test for DD



References

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