

Online Appendix to “Share Issues versus Share Repurchases”

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1 Proofs of results stated in the Appendix

Proof of Lemma A1: The result is immediate from the fact that, by definition,

$$\frac{\Pi(a, i_1, \theta_1, p_1)}{\Pi(a, i_2, \theta_2, p_2)} = \left(\frac{p_1}{p_2}\right)^\epsilon \left(\frac{a + i_1 + S(i_1, \theta_1)}{a + i_2 + S(i_2, \theta_2)} \cdot \frac{1 + \frac{i_2}{p_2}}{1 + \frac{i_1}{p_1}}\right)^{1-\epsilon}.$$

Proof of Lemma A2: Consider the ODE

$$\frac{\partial a(i)}{\partial i} = -\frac{V(a(i), i, 1) S_i(i, 1)}{\epsilon V(a(i), i, 1) + (1 - \epsilon) i} \quad (\text{O1})$$

for $i \in [I^*, -I_L]$ with boundary condition

$$a(I^*) = a_{\max}. \quad (\text{O2})$$

That $\epsilon < \underline{\epsilon}$ implies for any $i \in [I^*, -I_L]$, $\epsilon < \frac{-i}{a_{\max} + S(i, 1)}$, and hence for any $a(i) \leq a_{\max}$, $\epsilon V(a(i), i, 1) + (1 - \epsilon) i < 0$. By Assumption 1(ii), $\frac{\partial a(i)}{\partial i} = 0$ for $i = I^*$ and $\frac{\partial a(i)}{\partial i} < 0$ for $i > I^*$. Hence, (O1) and (O2) determine a unique function $a(i)$ on $[I^*, i_0]$ for some $i_0 \in (I^*, -I_L]$, with i_0 satisfying either $a(i_0) = a_{\min}$ or $i_0 = -I_L$. By construction, $a(i)$ is strictly decreasing.

Define $a_0 = a(i_0)$. By the Inverse Function Theorem, the inverse function of $a(\cdot)$, denoted by $\hat{i}(\cdot)$, is the unique solution to (9), (10) and (24) on $[a_0, a_{\max}]$. By construction, $\hat{i}(a_0) = i_0$. If $a_0 > a_{\min}$, then $\hat{i}(a_0) = -I_L$. This completes the proof.

Proof of Lemma A3: We define

$$\begin{aligned} \tilde{\pi}(a, i, s, \tilde{a}) &= \ln(\tilde{a} + s)^\epsilon \left(\frac{a + i + s}{1 + \frac{i}{\tilde{a} + s}}\right)^{1-\epsilon} \\ &= \ln(\tilde{a} + s) + (1 - \epsilon) \ln(a + i + s) - (1 - \epsilon) \ln(\tilde{a} + i + s), \end{aligned}$$

so that the log payoff of a type- a firm from mimicking type $\tilde{a} \in A$, $\ln \Pi(a, i(\tilde{a}), \theta(\tilde{a}), P(i(\tilde{a}), \theta(\tilde{a})))$,

can be expressed by $\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$. Then the partial derivatives of $\tilde{\pi}$ satisfy

$$\begin{aligned}\tilde{\pi}_i(a, i, s, \tilde{a}) &= \frac{1-\epsilon}{a+i+s} - \frac{1-\epsilon}{\tilde{a}+i+s} \\ \tilde{\pi}_s(a, i, s, \tilde{a}) &= \frac{1}{\tilde{a}+s} + \frac{1-\epsilon}{a+i+s} - \frac{1-\epsilon}{\tilde{a}+i+s} \\ \tilde{\pi}_{\tilde{a}}(a, i, s, \tilde{a}) &= \frac{1}{\tilde{a}+s} - \frac{1-\epsilon}{\tilde{a}+i+s}.\end{aligned}$$

For $\tilde{a} \in A$ except for countable points,

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) = \tilde{\pi}_i \cdot \frac{\partial i(\tilde{a})}{\partial \tilde{a}} + \tilde{\pi}_s \cdot \frac{dS(i(\tilde{a}), \theta(\tilde{a}))}{d\tilde{a}} + \tilde{\pi}_{\tilde{a}}, \quad (\text{O3})$$

and by Condition (A6),

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})|_{a=\tilde{a}} \quad (\text{O4})$$

Note that in (O3), $\tilde{\pi}_i$ and $\tilde{\pi}_s$ are strictly decreasing in a , and $\tilde{\pi}_{\tilde{a}}$ is invariant to a .

Case: Issue game: By (A6), for $\tilde{a}$ in the interior of A , $\frac{dS(i(\tilde{a}), \theta(\tilde{a}))}{d\tilde{a}} < 0$. For $\tilde{a}$ with $\frac{\partial i(\tilde{a})}{\partial \tilde{a}} \leq 0$,

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$$

strictly increases with a . Hence, by (O4), for each $a \in A$, for $\tilde{a} \geq a$ in A except for countable points,

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \leq 0.$$

This implies for all $a, \tilde{a} \in A$ with $a \neq \tilde{a}$,

$$\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) < \tilde{\pi}(a, i(a), S(i(a), \theta(a)), a).$$

Hence, firm $a \in A$ does not benefit from mimicking any firm $\tilde{a} \in A$.

Case: Repurchase game: Let θ be the constant value of $\theta(a)$. For $\tilde{a} \in A$ with $\frac{\partial i(\tilde{a})}{\partial \tilde{a}} \leq 0$,

$$\frac{dS(i(\tilde{a}), \theta(\tilde{a}))}{d\tilde{a}} = S_i(i(\tilde{a}), \theta) \frac{\partial i(\tilde{a})}{\partial \tilde{a}}.$$

By (O3),

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) = (1-\epsilon) \frac{1 + S_i(i(\tilde{a}), \theta)}{a + i(\tilde{a}) + S(i(\tilde{a}), \theta)} \frac{\partial i(\tilde{a})}{\partial \tilde{a}} + C, \quad (\text{O5})$$

where C is independent of a . By Assumption 1, $1 + S_i(i(\tilde{a}), \theta) > 0$. Hence,

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$$

is non-decreasing in a . By (O4), for each $a \in A$, for $\tilde{a} > (<) a$ in A except for countable points,

$$\frac{d}{d\tilde{a}} \tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \leq (\geq) 0.$$

This implies for all $a, \tilde{a} \in A$,

$$\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \leq \tilde{\pi}(a, i(a), S(i(a), \theta(a)), a).$$

Hence, firm $a \in A$ does not benefit from mimicking any firm $\tilde{a} \in A$. This completes the proof.

Proof of Lemma A4: *We first establish the “only if” part:*

Fix an off-equilibrium choice $(\tilde{i}, \tilde{\theta})$ and a type a whose equilibrium choice $(i(a), \theta(a))$ satisfies (A9). We prove (A10) by showing that any type $\tilde{a} < a$ cannot be associated with $(\tilde{i}, \tilde{\theta})$ under D1. Let p be a price such that

$$\Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p) \geq \Pi^*(\tilde{a}).$$

In equilibrium, type $\tilde{a}$ does not benefit from mimicking type a , implying

$$\Pi^*(\tilde{a}) \geq \Pi(\tilde{a}, i(a), \theta(a), P(i(a), \theta(a))).$$

This implies

$$\Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p) \geq \Pi(\tilde{a}, i(a), \theta(a), P(i(a), \theta(a))).$$

By (A9) and Lemma A1, type a strictly prefers to deviate to $(\tilde{i}, \tilde{\theta})$ at price p :

$$\Pi(a, \tilde{i}, \tilde{\theta}, p) > \Pi(a, i(a), \theta(a), P(i(a), \theta(a))) = \Pi^*(a).$$

Hence $D_{\tilde{a}}(\tilde{i}, \tilde{\theta}) \subsetneq D_a(\tilde{i}, \tilde{\theta})$, and so $(\tilde{i}, \tilde{\theta})$ cannot be associated with type $\tilde{a}$ under D1.

The case (A11) is analogous: a parallel argument to the above yields $D_{\tilde{a}}(i, \theta) \subsetneq D_a(i, \theta)$ for $\tilde{a} > a$, implying $(\tilde{i}, \tilde{\theta})$ cannot be associated with $\tilde{a} > a$, and hence (A12).

Next, we establish the “if” part:

Consider an equilibrium in which the conditions stated in the Lemma hold. Let $(\tilde{i}, \tilde{\theta})$ be an off-equilibrium action. We show that the price $P(\tilde{i}, \tilde{\theta})$ satisfies D1. Define $\tilde{a}$ by

$$P(\tilde{i}, \tilde{\theta}) = \tilde{a} + S(\tilde{i}, \tilde{\theta}). \tag{O6}$$

It suffices to show that $D_a(\tilde{i}, \tilde{\theta}) \subset D_{\tilde{a}}(\tilde{i}, \tilde{\theta})$ for all $a \neq \tilde{a}$, which is equivalent to showing that for any $a \neq \tilde{a}$ and any p ,

$$\Pi(a, \tilde{i}, \tilde{\theta}, p) \geq \Pi^*(a) \tag{O7}$$

implies

$$\Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p) \geq \Pi^*(\tilde{a}). \quad (\text{O8})$$

Case: (O7) holds for some $a < \tilde{a}$ and p

Suppose to the contrary that (O8) is violated, i.e., $\Pi^*(\tilde{a}) > \Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p)$. By the equilibrium condition and continuity, there is $\check{a} \in [a, \tilde{a})$ such that

$$\Pi^*(\check{a}) \geq \Pi(\check{a}, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))) > \Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p). \quad (\text{O9})$$

(O7) and the equilibrium condition together imply

$$\Pi(a, \tilde{i}, \tilde{\theta}, p) \geq \Pi(a, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))). \quad (\text{O10})$$

Observe that

$$\tilde{i} + S(\tilde{i}, \tilde{\theta}) \leq i(\check{a}) + S(i(\check{a}), \theta(\check{a})); \quad (\text{O11})$$

this follows because (O6) implies $P(\tilde{i}, \tilde{\theta}) > \check{a} + S(\tilde{i}, \tilde{\theta})$, and by the Lemma's conditions, not (A12) implies not (A11). Applying Lemma A1, inequalities (O10) and (O11) imply that

$$\Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p) \geq \Pi(\check{a}, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))) = \Pi^*(\check{a}),$$

yielding a contradiction with inequality (O9). This establishes (O8).

Case: (O7) holds for some $a > \tilde{a}$ and p

Suppose to the contrary that (O8) is violated, i.e., $\Pi^*(\tilde{a}) > \Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p)$. By the equilibrium condition and continuity, there is $\check{a} \in (\tilde{a}, a]$ such that

$$\Pi^*(\check{a}) \geq \Pi(\check{a}, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))) > \Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p). \quad (\text{O12})$$

(O7) and the equilibrium condition together imply

$$\Pi(a, \tilde{i}, \tilde{\theta}, p) \geq \Pi(a, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))). \quad (\text{O13})$$

Observe that

$$\tilde{i} + S(\tilde{i}, \tilde{\theta}) \geq i(\check{a}) + S(i(\check{a}), \theta(\check{a})); \quad (\text{O14})$$

this follows because by (O6), $P(\tilde{i}, \tilde{\theta}) < \check{a} + S(\tilde{i}, \tilde{\theta})$, and by the Lemma's conditions, not (A10) implies not (A9). Applying Lemma A1, inequalities (O13) and (O14) imply that

$$\Pi(\tilde{a}, \tilde{i}, \tilde{\theta}, p) \geq \Pi(\check{a}, i(\check{a}), \theta(\check{a}), P(i(\check{a}), \theta(\check{a}))) = \Pi^*(\check{a}),$$

yielding a contradiction with inequality (O12). This establishes (O8) and completes the proof.

Proof of Lemma A5: Suppose on an interval of firm types A , $i(a)$ and $\theta(a)$ are continuous, and prices are fully revealing. Then for $a \in A$,

$$\Pi^*(a) = P(i(a), \theta(a)) = a + S(i(a), \theta(a)).$$

Consider two firm types $a_1, a_2 \in A$ with $a_2 > a_1$. Let their equilibrium choices be denoted by (i_1, θ_1) and (i_2, θ_2) . For $k = 1, 2$, let p_k denote $P(i_k, \theta_k)$, s_k denote $S(i_k, \theta_k)$, and v_k denote $V(a_k, i_k, \theta_k)$. Equilibrium choices imply

$$\Pi(a_1, i_2, \theta_2, p_2) \leq \Pi^*(a_1) \quad (\text{O15})$$

$$\Pi(a_2, i_1, \theta_1, p_1) \leq \Pi^*(a_2). \quad (\text{O16})$$

With (3) and (22), condition (O15) can be written as

$$\begin{aligned} & p_2^\epsilon \left(\frac{v_2 - a_2 + a_1}{1 + \frac{i_2}{p_2}} \right)^{1-\epsilon} \leq p_1 \\ \Leftrightarrow & p_2^{\frac{1}{1-\epsilon}} (v_2 - a_2 + a_1) \leq p_1^{\frac{1}{1-\epsilon}} v_2, \\ \Leftrightarrow & \left(p_2^{\frac{1}{1-\epsilon}} - p_1^{\frac{1}{1-\epsilon}} \right) v_2 \leq p_2^{\frac{1}{1-\epsilon}} (a_2 - a_1), \\ \Leftrightarrow & \frac{p_2^{\frac{1}{1-\epsilon}} - p_1^{\frac{1}{1-\epsilon}}}{a_2 - a_1} \leq \frac{p_2^{\frac{1}{1-\epsilon}}}{v_2}. \end{aligned}$$

Similarly, condition (O16) yields

$$\frac{p_2^{\frac{1}{1-\epsilon}} - p_1^{\frac{1}{1-\epsilon}}}{a_2 - a_1} \geq \frac{p_1^{\frac{1}{1-\epsilon}}}{v_1},$$

and hence

$$\frac{(a_1 + S(i_1, \theta_1))^{\frac{1}{1-\epsilon}}}{V(a_1, i_1, \theta_1)} \leq \frac{(a_2 + S(i_2, \theta_2))^{\frac{1}{1-\epsilon}} - (a_1 + S(i_1, \theta_1))^{\frac{1}{1-\epsilon}}}{a_2 - a_1} \leq \frac{(a_2 + S(i_2, \theta_2))^{\frac{1}{1-\epsilon}}}{V(a_2, i_2, \theta_2)}.$$

Since $i(a)$ and $\theta(a)$ are continuous on A , $V(a, i(a), \theta(a))$ and $a + S(i(a), \theta(a))$ are continuous on A . The squeeze theorem implies

$$\frac{d[a + S(i(a), \theta(a))]^{\frac{1}{1-\epsilon}}}{da} = \frac{[a + S(i(a), \theta(a))]^{\frac{1}{1-\epsilon}}}{V(a, i(a), \theta(a))},$$

which then implies (A5), completing the proof.

Proof of Lemma A6: That

$$\frac{\partial \Pi(a, i, \theta, p)}{\partial p} = \frac{\Pi}{p(p+i)} (\epsilon p + i)$$

has the same sign as $\epsilon p + i$ implies (A19). In the issue game, that $\frac{\partial \Pi(a, i, \theta, p)}{\partial p} > 0$ for $i > 0$ follows $\epsilon p + i > 0$. In the repurchase game, if $\epsilon < \underline{\epsilon}$, then $\epsilon < \frac{-i}{a_{\max} + S(i, 1)}$ for any i . For $p \leq a_{\max} + S(i, \theta)$,

$\epsilon p + i < 0$, and hence $\frac{\partial \Pi(a, i, \theta, p)}{\partial p} < 0$, completing the proof.

Proof of Lemma A7: Consider ODE (O1) for $i \in [I_L, I^*]$ with boundary condition

$$a(I^*) = a_{\min}. \quad (\text{O17})$$

By Assumption 1(ii), $\frac{\partial a(i)}{\partial i} = 0$ for $i = I^*$ and $\frac{\partial a(i)}{\partial i} < 0$ for $i < I^*$. Hence, (O1) and (O17) determine a unique function $a(i)$ on $[i_0, I^*]$ for some $i_0 \in [I_L, I^*)$, with i_0 satisfying either $i_0 > I_L$ and $a(i_0) = a_{\max}$, or $i_0 = I_L$. By construction, $a(i)$ is strictly decreasing.

Let $\hat{a} = a(i_0)$. By the Inverse Function Theorem, the inverse function of $a(\cdot)$, denoted by $\hat{i}(\cdot)$, is the unique solution to (15), (16), and (24) on $[a_{\min}, \hat{a}]$. By construction, $\hat{i}(\hat{a}) = i_0$. If $\hat{a} < a_{\max}$, then $\hat{i}(\hat{a}) = I_L$.

If $I_L = 0$, (24) is zero for $\hat{i}(a) = I_L = 0$, and hence the ODE never reaches I_L . In this case, $\hat{i}(a) > I_L$ for all a , and $\hat{a} = a_{\max}$.

By the above, if $\hat{a} < a_{\max}$, then $I_L > 0$, and (26) is strictly negative. Hence, (18) and (26) determines a unique function $\hat{\theta}(a)$ for $a \geq \hat{a}$.

Finally, we establish that there is enough space on $\theta \in (0, 1]$ for these firms to separate. To do so, suppose to the contrary that there is a firm $a_0 > \hat{a}$ such that $\hat{\theta}(a_0) = 0$. By Assumption (1) in Footnote 7, firm $a_{\max}$ strictly prefers issuing $(I_L, 1)$ at price $a_{\min} + S(I_L, 1)$ to doing nothing and being priced $a_{\max}$:

$$\Pi(a_{\max}, I_L, 1, a_{\min} + S(I_L, 1)) > a_{\max}.$$

By Lemma A1, type a_0 has the same preference:

$$\Pi(a_0, I_L, 1, a_{\min} + S(I_L, 1)) > a_{\max}^\epsilon a_0^{1-\epsilon} > a_0.$$

By Lemma A3, type a_0 weakly prefers issuing I_L with efficiency $\hat{\theta}(a_0) = 0$ at price $a_0 + S(I_L, 0)$ to issuing I_L with efficiency $\hat{\theta}(\hat{a}) = 1$ at price $\hat{a} + S(I_L, 1)$:

$$a_0 + S(I_L, 0) = \Pi(a_0, I_L, 0, a_0 + S(I_L, 0)) \geq \Pi(a_0, I_L, 1, \hat{a} + S(I_L, 1)) > \Pi(a_0, I_L, 1, a_{\min} + S(I_L, 1)),$$

where the final inequality follows from firms preferring to issue at higher prices (Lemma A6). The above pair of inequalities implies $a_0 + S(I_L, 0) > a_0$, contradicting Assumption 1(i) and completing the proof.

Proof of Lemma A8: By Proposition 6(i), all repurchasing firms adopt efficiency $\theta = 1$. Hence $i(a) + S(i(a), 1)$ is weakly decreasing in a by Corollary 1. From Assumption 1(iii) it follows that $i(a)$ is weakly decreasing in a . Hence there exists $\hat{a} \in [a_{\min}, a_{\max}]$ such that firms $a < \hat{a}$ repurchase $-I_L$ and firms $a > \hat{a}$ repurchase $i(a) < -I_L$ with $i(a)$ weakly decreasing.

Claim 1: $i(a)$ is strictly decreasing for $a > \hat{a}$.

Proof of Claim 1: Suppose to the contrary, a non-degenerate interval of firm types A pool on action $(i, 1)$ with $i < -I_L$. Then

$$\inf A + S(i, 1) < P(i, 1) < \sup A + S(i, 1). \quad (\text{O18})$$

Case: $\epsilon P(i, 1) + i \leq 0$. Then by Lemma A6, $\Pi_p(a, i, 1, p) < 0$ for $p < P(i, 1)$. So for any $a \in A$, from (O18),

$$\Pi^*(a) = \Pi(a, i, 1, P(i, 1)) < \Pi(a, i, 1, \inf A + S(i, 1)). \quad (\text{O19})$$

Since $\epsilon(\inf A + S(i, 1)) + i < 0$, by continuity of S , there is $\tilde{i} > i$ such that for $i' \in (i, \tilde{i})$, $\epsilon(\inf A + S(i', 1)) + i' < 0$, and hence

$$\Pi_p(a, i', 1, p) < 0 \quad \text{for } p \leq \inf A + S(i', 1) \quad (\text{O20})$$

by Lemma A6. By Assumption 1(iii), $i' + S(i', 1) > i + S(i, 1)$, implying

$$P(i', 1) \leq \inf A + S(i', 1) \quad (\text{O21})$$

by Lemma A4 if $(i', 1)$ is off-equilibrium and by Corollary 1 if $(i', 1)$ is on-path. Inequalities (O20) and (O21) together imply

$$\Pi(a, i', 1, P(i', 1)) \geq \Pi(a, i', 1, \inf A + S(i', 1)).$$

For $a \in A$, since the RHS of the above inequality is continuous in i' , from (O19), there exists $i' \in (i, \tilde{i})$ such that $\Pi^*(a) < \Pi(a, i', 1, P(i', 1))$, contradicting the equilibrium condition.

Case: $\epsilon P(i, 1) + i > 0$. Then by Lemma A6, $\Pi_p(a, i, 1, p) > 0$ for $p > P(i, 1)$. So for any $a \in A$, from (O18),

$$\Pi^*(a) = \Pi(a, i, 1, P(i, 1)) < \Pi(a, i, 1, \sup A + S(i, 1)). \quad (\text{O22})$$

Since $\epsilon(\sup A + S(i, 1)) + i > 0$, by continuity of S , there is $\tilde{i} < i$ such that for $i' \in (\tilde{i}, i)$,

$$\epsilon(\sup A + S(i', 1)) + i' > 0, \quad (\text{O23})$$

and hence

$$\Pi_p(a, i', 1, p) > 0 \quad \text{for } p \geq \sup A + S(i', 1) \quad (\text{O24})$$

by Lemma A6. Moreover, (O23) implies there exists $\tilde{a} \in A$ such that $V(\tilde{a}, i', 1) > 0$ so $(i', 1)$ is a feasible action for firm $\tilde{a}$. By Assumption 1(iii), $i' + S(i', 1) < i + S(i, 1)$, implying

$$P(i', 1) \geq \sup A + S(i', 1) \quad (\text{O25})$$

by Lemma A4 if $(i', 1)$ is off-equilibrium and by Corollary 1 if $(i', 1)$ is on-path. Inequalities (O24) and (O25) together imply

$$\Pi(\tilde{a}, i', 1, P(i', 1)) \geq \Pi(\tilde{a}, i', 1, \sup A + S(i', 1)).$$

Since the RHS of the above inequality is continuous in i' , from (O22), there exists $i' \in (\tilde{i}, i)$ such that $\Pi^*(\tilde{a}) < \Pi(\tilde{a}, i', 1, P(i', 1))$, contradicting the equilibrium condition and establishing Claim 1.

Claim 2. For $a > \hat{a}$, $i(a)$ is continuous.

The proof of Claim 2 is identical to the proof of Subclaim 2.2 in the proof of Proposition 3. This completes the proof of this Lemma.

Proof of Lemma A9:

Claim 1: There is $a_1 > a_{\min}$ such that (24), (A32) and (A33) determine a unique strategy $\hat{i}(a)$ for each type $a \in [a_{\min}, a_1]$ and (A34) and (A35) are satisfied on this range. $i_1 \equiv \hat{i}(a_1) < I^*$.

Proof of Claim 1: Consider ODE

$$\frac{\partial a(i)}{\partial i} = -\frac{V(a(i), i, 1) S_i(i, 1)}{\epsilon V(a(i), i, 1) + (1 - \epsilon)i} \quad (\text{O26})$$

for $i \leq I^*$ with boundary condition

$$a(I^*) = a_{\min}. \quad (\text{O27})$$

That $\epsilon > \bar{\epsilon}$ implies $\epsilon V(a_{\min}, I^*, 1) + (1 - \epsilon)I^* > 0$. By continuity, there is $\delta_0 > 0$ such that for (a, i) in the closed $2\delta_0$ -neighbourhood of $(a_{\min}, I^*)$,

$$\epsilon V(a, i, 1) + (1 - \epsilon)i > 0,$$

and hence $-\frac{V(a, i, 1) S_i(i, 1)}{\epsilon V(a, i, 1) + (1 - \epsilon)i}$ is continuous and hence Lipschitz continuous in (a, i) in this neighbourhood. By the Picard–Lindelöf theorem, there is $\delta_1 \in (0, \delta_0)$ such that (O26) and (O27) determine a unique $a(i) \in [a_{\min}, a_{\min} + \delta_0]$ for $i \in [I^* - \delta_1, I^*]$. Let $i_1 = I^* - \delta_1$ and $a_1 = a(I^* - \delta_1)$. For $i \in [i_1, I^*)$, $\frac{\partial a(i)}{\partial i} < 0$ for the following reasons: $S_i(i, 1) > 0$ by Assumption 1(ii), and since $(i, a(i))$ is in the $2\delta_0$ -neighbourhood of $(a_{\min}, I^*)$,

$$\epsilon V(a(i), i, 1) + (1 - \epsilon)i > 0$$

and hence $V(a(i), i, 1) > 0$. Hence $a(i)$ is strictly decreasing on $[i_1, I^*]$ and has an inverse function, which is the unique solution $\hat{i}(\cdot)$ to (24), (A32), and (A33) for $a \in [a_{\min}, a_1]$ by the Inverse Function Theorem. For $a \in [a_{\min}, a_1]$, since $(a, \hat{i}(a))$ is in the $2\delta_0$ -neighbourhood of $(a_{\min}, I^*)$, we have (A34), which together with (24) implies (A35). This completes the proof of Claim 1.

Claim 2: (24) and the boundary condition $\hat{i}(a_1) = i_1$ determine a unique $\hat{i}(a)$ for each type $a \in [a_1, a_{\max}]$, and (A34) and (A35) are satisfied on this range.

Proof of Claim 2: Consider

$$A = \{(a, i) : V(a, i, 1) > 0\},$$

and for $(a, i) \in A$ define function

$$f(a, i) = -\frac{\epsilon V(a, i, 1) + (1 - \epsilon) i}{V(a, i, 1) S_i(i, 1)}.$$

Consider the initial value problem

$$\frac{\partial \hat{i}(a)}{\partial a} = f(a, \hat{i}(a)), \quad \hat{i}(a_1) = i_1. \quad (\text{O28})$$

Since f is locally Lipschitz continuous on A , by the Continuation Theorem for ODEs, (O28) has a unique solution on $[a_1, a_{\max}]$.

Suppose to the contrary of (A35) that $B = \{a \in [a_1, a_{\max}] : \frac{\partial \hat{i}(a)}{\partial a} \geq 0\}$ is not empty. By Subclaim 1.1, $\frac{\partial \hat{i}(a)}{\partial a} < 0$ for $a = a_1$, and hence $\inf B > a_1$, and $\frac{\partial \hat{i}(a)}{\partial a} = 0$ for $a = \inf B$ by continuity. This then implies

$$\frac{\partial^2 \hat{i}(a)}{\partial a^2} \Big|_{a=\inf B} = -\frac{\epsilon}{V(a, \hat{i}(a), 1) S_i(\hat{i}(a), 1)} \Big|_{a=\inf B} + C \times \underbrace{\frac{\partial \hat{i}(a)}{\partial a} \Big|_{a=\inf B}}_{=0}$$

with $C < \infty$, and hence $\frac{\partial^2 \hat{i}(a)}{\partial a^2} < 0$. Continuity implies $\frac{\partial^2 \hat{i}(a)}{\partial a^2} < 0$ for all a in a neighbourhood of $\inf B$, and hence $\frac{\partial \hat{i}(a)}{\partial a} > \frac{\partial \hat{i}(a)}{\partial a} \Big|_{a=\inf B} = 0$ for all $a < \inf B$ in this neighbourhood by the Mean Value Theorem. But this contradicts with the definition of B and establishes (A35) for $a \in [a_1, a_{\max}]$. (A35) and (24) then imply (A34). This completes the proof of Claim 2 and hence the proof of the Lemma.

2 Option of doing nothing

We establish in the Lemma below that it is without loss of generality to ignore firms' option of doing nothing ($i = 0$) for the issue game with $\epsilon \in [0, 1)$ and for the repurchase game with $\epsilon \in [0, \underline{\epsilon}]$. (It is also without loss of generality for the repurchase game with $\epsilon \in (\bar{\epsilon}, 1)$ since the proof of Proposition 6(iii) remains identical if firms can choose i from $(-\infty, 0]$.)

Lemma O1. *With the firm's objective function (22), under the assumptions in Footnote 26 (which reduce to the assumptions in Footnote 7 in the case $\epsilon = 0$), if $I_L > 0$, then all firms strictly prefer issuing $(I_L, 1)$ under any price to doing nothing in the issue game with $\epsilon \in [0, 1)$, and all firms strictly prefer repurchasing $(-I_L, 1)$ under any price to doing nothing in the repurchase game with*

$\epsilon \in [0, \underline{\epsilon})$.

This Lemma then implies including doing nothing ($i = 0$) in firms' action space doesn't change the set of equilibria: in any equilibrium all firms must transact $|i| \geq I_L$, and conversely, given firms' strategy, a firm doesn't benefit from deviating to doing nothing ($i = 0$) as long as it doesn't benefit from deviating to $(I_L, 1)$ (the issue game) or $(-I_L, 1)$ (the repurchase game).

Proof of Lemma O1:

In the issue game, by Lemma A6, $\Pi(a, I_L, 1, p)$ is strictly increasing in p . By issuing I_L with the efficient method $\theta = 1$, firm $a_{\max}$ has payoff not lower than $\Pi(a_{\max}, I_L, 1, a_{\min} + S(I_L, 1))$. By doing nothing, type $a_{\max}$ has payoff not higher than

$$\Pi(a_{\max}, 0, 1, a_{\max}) = a_{\max}.$$

Then by Assumption (1), type $a_{\max}$ strictly prefers $(I_L, 1)$ to doing nothing. By Assumption 1(iii),

$$I_L + S(I_L, 1) > S(0, 1).$$

By Lemma A1, all types strictly prefer $(I_L, 1)$ to doing nothing.

In the repurchase game with $\epsilon \in [0, \underline{\epsilon})$, by Lemma A6, $\Pi(a, -I_L, 1, p)$ is strictly decreasing in p . By repurchasing $-I_L$ with the efficient method $\theta = 1$, firm $a_{\min}$ has payoff not lower than $\Pi(a_{\min}, -I_L, 1, a_{\max} + S(-I_L, 1))$. By doing nothing, type $a_{\min}$ has payoff not higher than $\Pi(a_{\min}, 0, 1, a_{\max})$. Then by Assumption (25), type $a_{\min}$ strictly prefers $(-I_L, 1)$ to doing nothing. By Assumption 1(iii),

$$-I_L + S(-I_L, 1) < S(0, 1).$$

By Lemma A1, all types strictly prefer $(-I_L, 1)$ to doing nothing, completing the proof.

3 Option of partial completion

We analyze the perturbed game in which firms have the option to partially complete their announced transaction size as specified in Footnote 14.

Proposition O1. *In the perturbed repurchase (issue) game with the option of partial completion, there is a PBE in which firms play the strategy of Proposition 2 (Proposition 4).*

Proof. In the perturbed game, the firm chooses i, θ, i^A with $|i^A| \leq |i|$ to maximize $\Pi(a, i^A, \theta, P(i, \theta))$.

Step 1. Consider the repurchase game.

For an off-equilibrium announcement (i, θ) , let the belief on firm type follow that specified in the proof of Proposition 2, and the belief on the actual transaction size be loaded on $\arg \max_{i' \in [i, -I_L]} S(i', \theta)$.

That is, the off-equilibrium price is

$$P(i, \theta) = a^E(i, \theta) + \max_{i' \in [i, -I_L]} S(i', \theta) \quad (\text{O29})$$

where $a^E(i, \theta) = a$ if $i + S(i, \theta) = \hat{i}(a) + S(\hat{i}(a), 1)$ for some $a \geq \hat{a}$; $a^E(i, \theta) = \hat{a}$ if $i + S(i, \theta) > \sup_{a \geq \hat{a}} \hat{i}(a) + S(\hat{i}(a), 1)$; $a^E(i, \theta) = a_{\max}$ if $i + S(i, \theta) < \inf_{a \geq \hat{a}} \hat{i}(a) + S(\hat{i}(a), 1)$. Note that $a^E(i, \theta)$ is weakly decreasing in i .

Since $\max_{i' \in [i, -I_L]} S(i', \theta) \geq S(i, \theta)$, the price for each off-equilibrium announcement (i, θ) is not lower than the off-equilibrium price for (i, θ) in the proof of Proposition 2. Since firms prefer lower prices in the repurchase game, it then follows the proof of Proposition 2 that no firm benefits from deviating to announcing any (i, θ) and actually repurchasing $i^A = i$. It remains to show that no firm benefits from deviating to announcing any (i, θ) and completing the repurchase partially ($|i^A| < |i|$, that is, $i^A > i$). This immediately follows the next Claim:

Claim 1: For $i^A > i$, $P(i^A, \theta) \leq P(i, \theta)$.

Since firms prefer lower prices in the repurchase game, for $i^A > i$, by Claim 1,

$$\Pi(a, i^A, \theta, P(i, \theta)) \leq \Pi(a, i^A, \theta, P(i^A, \theta)).$$

Since we have established above that firms do not benefit from deviating to announcing any (i^A, θ) and actually repurchasing i^A , they do not benefit from deviating to announcing any (i, θ) with $i < i^A$ and actually repurchasing i^A either.

Proof of Claim 1: Consider the following cases:

Case: both (i, θ) and (i^A, θ) are on-path. Then $i^A > i \geq I^*$ and $\theta = 1$, and hence $S(i^A, \theta) < S(i, \theta)$ by Assumption 1(ii). Since firms' strategy $i(a)$ is weakly decreasing, $E[a|i^A, \theta] \leq E[a|i, \theta]$. This implies $P(i^A, \theta) \leq P(i, \theta)$.

Case: both (i, θ) and (i^A, θ) are off-path. Since $i(a) + S(i(a), 1)$ strictly decreases on $[\hat{a}, a_{\max}]$, $a^E(i^A, \theta) \leq a^E(i, \theta)$. Since $\max_{i' \in [i^A, -I_L]} S(i', \theta) \leq \max_{i' \in [i, -I_L]} S(i', \theta)$, we have $P(i^A, \theta) \leq P(i, \theta)$ by (O29).

Case: one of (i, θ) and (i^A, θ) is on-path and the other is off-equilibrium. We relabel them as (i_{on}, θ) and (i_{off}, θ) accordingly. Then $i_{\text{on}} \geq I^*$ and $\theta = 1$, and $S(i_{\text{on}}, \theta) = \max_{i' \in [i_{\text{on}}, -I_L]} S(i', \theta)$ by Assumption 1, so $P(i_{\text{on}}, \theta)$ can be expressed as

$$P(i_{\text{on}}, \theta) = E[a|i, \theta] + \max_{i' \in [i_{\text{on}}, -I_L]} S(i', \theta). \quad (\text{O30})$$

Since $i^A > i$,

$$\max_{i' \in [i^A, -I_L]} S(i', \theta) \leq \max_{i' \in [i, -I_L]} S(i', \theta).$$

Moreover, given the definition of a^E , if (i^A, θ) is on-path and (i, θ) is off-path, then $E[a|i^A, \theta] \leq a^E(i, \theta)$; if (i^A, θ) is off-path and (i, θ) is on-path, $a^E(i^A, \theta) \leq E[a|i, \theta]$. In both cases, we have $P(i^A, \theta) \leq P(i, \theta)$ by (O29) and (O30). This completes the proof of Claim 1 and hence step 1.

Step 2. Consider the issue game.

For an off-equilibrium announcement (i, θ) , let the belief on firm type follow that specified in the proof of Proposition 4, and the belief on the actual transaction size be loaded on $\arg \min_{i' \in [I_L, i]} S(i', \theta)$. That is, the off-equilibrium price is

$$P(i, \theta) = a^E(i, \theta) + \min_{i' \in [I_L, i]} S(i', \theta) \quad (\text{O31})$$

where $a^E(i, \theta) = a$ if $i + S(i, \theta) = i(a) + S(i(a), \theta(a))$ for some a ; $a^E(i, \theta) = a_{\min}$ if $i + S(i, \theta) > \sup_a i(a) + S(i(a), \theta(a))$; $a^E(i, \theta) = a_{\max}$ if $i + S(i, \theta) < \inf_a i(a) + S(i(a), \theta(a))$. Note that $a^E(i, \theta)$ is weakly decreasing in i .

Since $\min_{i' \in [I_L, i]} S(i', \theta) \leq S(i, \theta)$, the price for each off-equilibrium announcement (i, θ) is not higher than the off-equilibrium price for (i, θ) in the proof of Proposition 4. Since firms prefer higher prices in the issue game, the next Claim follows the proof of Proposition 4:

Claim 2: No firm benefits from deviating to announcing any (i, θ) and actually issuing $i^A = i$.

It remains to show that no firm benefits from deviating to announcing any (i, θ) and completing the issuance partially ($i^A < i$). Consider such a deviation and the following cases.

Case 1: Both (i, θ) and (i^A, θ) are off-path. Then that $i^A < i$ implies $a^E(i, \theta) \leq a^E(i^A, \theta)$ and $\min_{i' \in [I_L, i]} S(i, \theta) \leq \min_{i' \in [I_L, i]} S(i^A, \theta)$, and hence $P(i, \theta) \leq P(i^A, \theta)$ by (O31). Hence

$$\Pi(a, i^A, \theta, P(i, \theta)) \leq \Pi(a, i^A, \theta, P(i^A, \theta)). \quad (\text{O32})$$

Since firms do not benefit from deviating to announcing (i^A, θ) and actually issuing i^A by Claim 2, they do not benefit from deviating to announcing (i, θ) and actually issuing i^A either.

Case 2: Both (i, θ) and (i^A, θ) are on-path. Since firms' equilibrium strategy entails full separation and full completion, the price of the choice of firm a follows

$$P(i(a), \theta(a)) = a + S(i(a), \theta(a)).$$

Since the equilibrium strategy of firms $a \leq \hat{a}$ satisfies $\theta = 1$ and condition (12) and the strategy of firms $a > \hat{a}$ satisfies $i = I_L$ and condition (19), and both $i(a)$ and $\theta(a)$ are continuous at $\hat{a}$,

$$\frac{d}{da} [a + S(i(a), \theta(a))] = \frac{a + S(i(a), \theta(a))}{V(a, i(a), \theta(a))} > 0. \quad (\text{O33})$$

Since $i(a)$ is weakly decreasing, $P(i, \theta)$ is weakly decreasing in i for on-path (i, θ) . This implies $P(i, \theta) \leq P(i^A, \theta)$ and hence (O32). That firms don't deviate to announcing (i, θ) and actually issuing i^A follows Claim 2.

Case 3: (i, θ) is off-path and (i^A, θ) is on-path. Then either $\theta = 1$ or $i^A = I_L$. In both cases, that $i > i^A$ implies $a^E(i, \theta) \leq E[a|i^A, \theta]$ and $\min_{i' \in [I_L, i]} S(i', \theta) \leq S(i^A, \theta)$, so $P(i, \theta) \leq P(i^A, \theta)$. Hence (O32) holds and that firms don't deviate to announcing (i, θ) and actually issuing i^A follows Claim 2.

Case 4: (i, θ) is on-path and (i^A, θ) is off-path. This implies $\theta = 1$ and $i^A < \hat{i}(a_{\max})$. Let $\tilde{a}$ denote the firm type with $\hat{i}(\tilde{a}) = i$, then

$$P(i, \theta) = \tilde{a} + S(i, 1) \leq a_{\max} + S(\hat{i}(a_{\max}), 1),$$

where the inequality follows (O33). This gives an upper bound to firm $a_{\max}$'s payoff from announcing (i, θ) and actually issuing i^A :

$$\Pi(a_{\max}, i^A, 1, P(i, \theta)) \leq \Pi(a_{\max}, i^A, 1, a_{\max} + S(\hat{i}(a_{\max}), 1)). \quad (\text{O34})$$

Since

$$\frac{\partial}{\partial i^A} \ln \Pi(a_{\max}, i^A, 1, a_{\max} + S(\hat{i}(a_{\max}), 1)) = \frac{1 + S_i(i^A, 1)}{V(a_{\max}, i^A, 1)} - \frac{1}{a_{\max} + i^A + S(\hat{i}(a_{\max}), 1)}$$

is strictly positive for $i^A < \hat{i}(a_{\max})$ because $S_i(i^A, 1) > 0$ and $S(i^A, 1) < S(\hat{i}(a_{\max}), 1)$, we know

$$\Pi(a_{\max}, i^A, 1, a_{\max} + S(\hat{i}(a_{\max}), 1)) < \Pi(a_{\max}, \hat{i}(a_{\max}), 1, a_{\max} + S(\hat{i}(a_{\max}), 1)).$$

This together with (O34) implies firm $a_{\max}$ prefers its equilibrium action (announcing $(\hat{i}(a_{\max}), 1)$ and completing the full amount of issuance) to announcing (i, θ) and actually issuing i^A . Hence all firms prefer firm $a_{\max}$'s equilibrium action to announcing (i, θ) and actually issuing i^A by Lemma A1. Since we have established no firm benefits from mimicking another firm, it follows that no firm benefits from deviating to announcing (i, θ) and actually issuing i^A . This completes the proof. $\square$

4 Details for Section 6.2

Proposition O2. *With firm's objective function (4), the issue game with θ restricted to 1 has a unique D1 equilibrium. There is a firm type $\check{a} \leq \hat{a}$ (with $\hat{a}$ defined as in Section 3.3) such that firms with $a < \check{a}$ issue $\hat{i}(a)$ determined by (12), (15) and (16), and firms with $a > \check{a}$ issue $i = I_L$. $\check{a} < \hat{a}$ if and only if $\hat{a} < a_{\max}$. The payoffs of firms $a \in (\check{a}, \hat{a})$ are strictly higher than in the issue game where firms can choose θ from $[0, 1]$.*

Proof. If $\hat{a} = a_{\max}$, then all firms use $\theta = 1$ in the unique D1 equilibrium of the issue game where firms can choose θ from $[0, 1]$ by Proposition 5. It is apparent that restricting θ to 1 does not change the result. In the rest of this proof, we consider the case $\hat{a} < a_{\max}$. For $\tilde{a} \leq \hat{a}$, define

$$g(\tilde{a}) = \Pi(\tilde{a}, I_L, 1, E[a|a \geq \tilde{a}] + S(I_L, 1)) - [\tilde{a} + S(\hat{i}(\tilde{a}), 1)].$$

Claim 1: $g(\hat{a}) > 0$, and there is a unique $\check{a} \in [a_{\min}, \hat{a})$ that satisfies

$$g(a) > 0 \text{ for } a \in (\check{a}, \hat{a}), \quad g(a) < 0 \text{ for } a \in (a_{\min}, \check{a}).$$

Proof of Claim 1: Since $\hat{a} < a_{\max}$, $\hat{i}(\hat{a}) = I_L$ by definition of $\hat{a}$. Since firms prefer higher price in the issue game,

$$\begin{aligned} \Pi(\hat{a}, I_L, 1, E[a|a \geq \hat{a}] + S(I_L, 1)) &> \Pi(\hat{a}, I_L, 1, \hat{a} + S(I_L, 1)) \\ &= \hat{a} + S(I_L, 1), \end{aligned}$$

implying $g(\hat{a}) > 0$. Hence it suffices to show that

$$g(a_1) \leq 0 \quad \text{implies} \quad g(a_2) < 0 \quad \text{for} \quad a_2 < a_1. \quad (\text{O35})$$

Suppose $g(a_1) \leq 0$, implying firm a_1 weakly prefers $(\hat{i}(a_1), 1)$ at price $a_1 + S(\hat{i}(a_1), 1)$ to $(I_L, 1)$ at price $E[a|a \geq a_1] + S(I_L, 1)$. By Assumption 1,

$$\hat{i}(a_1) + S(\hat{i}(a_1), 1) \geq I_L + S(I_L, 1).$$

Then firm $a_2 < a_1$ also weakly prefers $(\hat{i}(a_1), 1)$ at price $a_1 + S(\hat{i}(a_1), 1)$ to $(I_L, 1)$ at price $E[a|a \geq a_1] + S(I_L, 1)$ by Lemma A1. Since $E[a|a \geq a_1] > E[a|a \geq a_2]$, this further implies firm a_2 strictly prefers $(\hat{i}(a_1), 1)$ at price $a_1 + S(\hat{i}(a_1), 1)$ to $(I_L, 1)$ at price $E[a|a \geq a_2] + S(I_L, 1)$. By Lemma A3, firm a_2 weakly prefers $(\hat{i}(a_2), 1)$ at price $a_2 + S(\hat{i}(a_2), 1)$ to $(\hat{i}(a_1), 1)$ at price $a_1 + S(\hat{i}(a_1), 1)$. Hence firm a_2 strictly prefers $(\hat{i}(a_2), 1)$ at price $a_2 + S(\hat{i}(a_2), 1)$ to $(I_L, 1)$ at price $E[a|a \geq a_2] + S(I_L, 1)$, and hence $g(a_2) < 0$. This establishes Claim 1.

Step 1. We show the existence of the D1 equilibrium with $\check{a}$ determined as in Claim 1 and with the following off-equilibrium belief: for an off-equilibrium i , if $i \geq I^*$, the belief loads on type $a_{\min}$; if $i \leq \hat{i}(\check{a})$, the belief loads on type $\check{a}$.

Under this belief, the price for an off-equilibrium i is

$$P(i, 1) = \begin{cases} a_{\min} + S(i, 1), & i \geq I^* \\ \check{a} + S(i, 1), & i \leq \hat{i}(\check{a}). \end{cases} \quad (\text{O36})$$

The price satisfies D1 by Lemma A4.

Substep 1.1. We argue firms don't mimic each other.

That types $a < \check{a}$ do not mimic each other follows Lemma A3. It suffices to argue that in the case $\check{a} > a_{\min}$, types $a > \check{a}$ and $a < \check{a}$ do not mimic each other. In this case, $g(\check{a}) = 0$ by definition of $\check{a}$, implying firm $\check{a}$ is indifferent between I_L and $\hat{i}(\check{a})$. Hence firm $\check{a}$ prefers I_L to mimicking any type $a < \check{a}$ by Lemma A3. Firms $a > \check{a}$ have the same preference by Lemma A1, and hence do not benefit from mimicking firms $a < \check{a}$.

On the other hand, that type $\check{a}$ is indifferent between I_L and $\hat{i}(\check{a})$ implies firms $a < \check{a}$ prefer $\hat{i}(\check{a})$ to I_L by Lemma A1. This then implies firms $a < \check{a}$ prefer their equilibrium choice $\hat{i}(a)$ to deviating to I_L (mimicking types $a > \check{a}$) by Lemma A3. This completes Substep 1.1.

Substep 1.2. We argue firms do not deviate to any off-equilibrium $i \geq I^*$.

It suffices to show that firm $a_{\min}$ does not deviate to any off-equilibrium $i \geq I^*$, because it will then follow this claim and Lemma A1 that all firms prefer mimicking firm $a_{\min}$ to deviating to $i \geq I^*$, and since we have established that firms do not benefit from mimicking type $a_{\min}$ in Substep 1.1, it follows that they do not benefit from deviating to $i \geq I^*$.

Note that firm $a_{\min}$ receives its fair price if it issues $i \geq I^*$ by (O36). Since $S(I^*, 1) \geq S(i, 1)$, firm $a_{\min}$ strictly prefers I^* to $i > I^*$. Hence it suffices to prove firm $a_{\min}$ weakly prefers its equilibrium choice to I^* . If firm $a_{\min}$'s equilibrium choice is I^* , there is nothing to prove. If firm $a_{\min}$'s equilibrium choice is not I^* , then it's I_L and $\check{a} = a_{\min}$. In this case, $g(a_{\min}) \geq 0$ by the definition of $\check{a}$, implying firm $a_{\min}$ weakly prefers its equilibrium choice I_L to I^* . This completes Substep 1.2.

Substep 1.3. We argue firms do not deviate to any off-equilibrium $i \in [I_L, \hat{i}(\check{a})]$.

Consider an off-equilibrium $i \in [I_L, \hat{i}(\check{a})]$. Firm $\check{a}$ is fairly priced both when issuing $\hat{i}(\check{a})$ and when issuing i by (O36), and since $\hat{i}(\check{a})$ induces higher surplus, firm $\check{a}$ weakly prefers $\hat{i}(\check{a})$ to i . Firms $a < \check{a}$ have the same preference by Lemma A1, and since they don't benefit from deviating to $\hat{i}(\check{a})$ by Lemma A3, they don't benefit from deviating to i .

By Claim 1, and by continuity of g , we know $g(\check{a}) \geq 0$, implying firm $\check{a}$ weakly prefers I_L to $\hat{i}(\check{a})$. By transitivity, firm $\check{a}$ weakly prefers I_L to i . Firms $a > \check{a}$ have the same preference by Lemma A1, and hence do not benefit from deviating to i .

By the above, regardless of whether firm $\check{a}$'s equilibrium choice is $\hat{i}(\check{a})$ or I_L it doesn't benefit from deviating to i , completing Substep 1.3.

Step 2. We show the uniqueness of the D1 equilibrium.

By Corollary 1, $i(a)$ is weakly decreasing, and hence there is type $\tilde{a}$ such that firms $a > \tilde{a}$ choose I_L and $a < \tilde{a}$ choose higher sizes. Similar to the proof of Proposition 5, one can show that types $a < \tilde{a}$ fully separate and their choices are continuous. If type $a_{\min}$ chooses some $i > I_L$ with $i \neq I^*$, it receives $a_{\min} + S(i, 1)$ in equilibrium, and hence benefits from deviating to I^* , which leads to weakly

better market belief and strictly higher surplus. This implies $i(a_{\min}) = I^*$ as long as $\tilde{a} > a_{\min}$. By Lemma A5, the choices of firms $a < \tilde{a}$ satisfy ODE (12). By Lemma A4, off-equilibrium prices are uniquely determined by (O36).

Step 3. We show the payoffs of firms $a \in (\check{a}, \hat{a})$ are strictly higher than in the issue game where firms can choose θ from $[0, 1]$.

Let $\Pi^R(a)$ denote the equilibrium payoff of firm a in the issue game where θ is restricted to 1 and $\Pi^U(a)$ the equilibrium payoff of firm a in the issue game where $\theta \in [0, 1]$ are available. Since $\Pi^R(\check{a}) = \Pi^U(\check{a})$, to prove $\Pi^R(a) > \Pi^U(a)$ for $a \in (\check{a}, \hat{a})$, it suffices to prove

$$\frac{d}{da} \ln \Pi^R(a) > \frac{d}{da} \ln \Pi^U(a) \quad \text{for } a \in (\check{a}, \hat{a}). \quad (\text{O37})$$

For $a \in (\check{a}, \hat{a})$,

$$\begin{aligned} \frac{d}{da} \ln \Pi^R(a) &= \frac{d}{da} \ln \Pi(a, I_L, 1, P(I_L, 1)) \\ &= \frac{d}{da} \ln V(a, I_L, 1) \\ &= \frac{1}{V(a, I_L, 1)}, \end{aligned}$$

and since $\Pi^U(a) = a + S(\hat{i}(a), 1)$, and by (12)

$$\frac{d}{da} \ln \Pi^U(a) = \frac{1}{V(a, \hat{i}(a), 1)}.$$

Since $V(a, \hat{i}(a), 1) > V(a, I_L, 1)$ by Assumption 1(iii), (O37) follows, completing the proof. $\square$

5 More details for the repurchase microfoundation

We verify that the firm value function under the repurchase microfoundation in Section 4 given by (A42) in Appendix B satisfies Assumption 1. To show $S_\theta > 0$, it suffices to show that $\frac{\partial S}{\partial(\frac{1}{n})} < 0$. Differentiating (A43),

$$\frac{\partial S}{\partial(\frac{1}{n})} = - \left[|i| + \gamma \kappa \left(\frac{1}{n} \right)^{\gamma-1} |i|^\gamma - \frac{\left(\frac{|i|}{n} + \kappa \left(\frac{|i|}{n} \right)^\gamma \right) \beta T e^{-\beta \frac{T}{n}}}{1 - e^{-\beta \frac{T}{n}}} \right] \frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}.$$

Since $\gamma > 1$, it suffices to show that

$$|i| + \kappa \left(\frac{1}{n} \right)^{\gamma-1} |i|^\gamma > \frac{\left(\frac{|i|}{n} + \kappa \left(\frac{|i|}{n} \right)^\gamma \right) \beta T e^{-\beta \frac{T}{n}}}{1 - e^{-\beta \frac{T}{n}}},$$

i.e.,

$$1 > \frac{\beta \frac{T}{n} e^{-\beta \frac{T}{n}}}{1 - e^{-\beta \frac{T}{n}}},$$

i.e.,

$$e^{\beta \frac{T}{n}} > 1 + \beta \frac{T}{n},$$

which holds because $\beta \frac{T}{n} > 0$.

It is immediate that V is decreasing and concave in $|i|$, and hence increasing and concave in i . Concavity of V in i implies S is single-peaked in i .

To show $I^* < 0$, it suffices to show $S_{|i|}(0, 1) > 0$, where $S_{|i|}$ denotes the partial derivative of S with respect to $|i|$. Note

$$S_{|i|} = 1 - \left(\frac{1}{n} + \gamma \kappa \left(\frac{1}{n} \right)^\gamma |i|^{\gamma-1} \right) \frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}},$$

and hence

$$S_{|i|}(0, 1) = 1 - \frac{1}{N} \cdot \frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{N}}}.$$

It suffices to show

$$1 - e^{-\beta \frac{T}{N}} > \frac{1}{N} (1 - e^{-\beta T}).$$

This holds with equality for $\frac{1}{N} = 0, 1$. The LHS is strictly concave in $\frac{1}{N}$, and the RHS is linear in $\frac{1}{N}$. Hence, the inequality holds given $\frac{1}{N} \in (0, 1)$.