Appendix of "Estimating the Pricing Kernel: A Simple Regression-Based Dimension-Reduction Approach"

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A Proof of Theorem 1

Notation

Before presenting the proof, we introduce the notation used throughout the analysis. Boldface symbols denote vectors or matrices, whereas nonbold symbols denote scalars. The notation $\overset{\mathcal{D}}{=}$ indicates equality in distribution. For an N-dimensional vector $\boldsymbol{x} \in \mathbb{R}^N$, we define the $\boldsymbol{A}$-norm as $\|\boldsymbol{x}\|_{\boldsymbol{A}}^2 = \boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{A} \in \mathbb{R}^{N\times N}$ is symmetric and positive definite. The ex-ante maximum squared Sharpe ratio is denoted by $\zeta \equiv \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$. The notation $Y_T = o_p(f(T))$ means that $Y_T/f(T) \xrightarrow{P} 0$, and $Y_T = O_p(f(T))$ means that for any $\epsilon > 0$, there exists a finite $M > 0$ such that $P(|Y_T/f(T)| > M) < \epsilon$ for all T. For notational simplicity, the dependence of $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, $\boldsymbol{\omega}^\star$, and ζ on N, and that of $\widetilde{\boldsymbol{\omega}}$ on T and N, is suppressed throughout.

The solution to the expected quadratic utility maximization problem (4) is

$$\boldsymbol{\omega}^\star \equiv \frac{1}{\gamma}\frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{1+\zeta}. \tag{A.1}$$

The expected return and variance of $\boldsymbol{\omega}^\star$ are given by

$$r^\star \equiv (\boldsymbol{\omega}^\star)'\,\boldsymbol{\mu} = \frac{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\gamma(1+\zeta)} = \frac{\zeta}{\gamma(1+\zeta)}, \tag{A.2}$$

$$(\sigma^\star)^2 \equiv (\boldsymbol{\omega}^\star)'\,\boldsymbol{\Sigma}\boldsymbol{\omega}^\star = \frac{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\gamma^2(1+\zeta)^2} = \frac{\zeta}{\gamma^2(1+\zeta)^2}. \tag{A.3}$$

For a $T \times N$ sample of realized excess returns

$$\boldsymbol{R} \equiv \begin{pmatrix} R_{11} & \cdots & R_{1N} \\ \vdots & \ddots & \vdots \\ R_{T1} & \cdots & R_{TN} \end{pmatrix},$$

the return series of the theoretically MVE portfolio $\boldsymbol{\omega}^\star$ and of REP $\widetilde{\boldsymbol{\omega}}$ are denoted by

$$\boldsymbol{R}^\star \equiv \boldsymbol{R}\boldsymbol{\omega}^\star, \quad \text{and} \quad \widetilde{\boldsymbol{R}} \equiv \boldsymbol{R}\widetilde{\boldsymbol{\omega}}, \quad \text{respectively.} \tag{A.4}$$

Let $\widetilde{\boldsymbol{r}}$ denote the one-period-ahead return of the N assets, and $\widetilde{r}_{\text{REP}} \equiv \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}$ denote the OOS return of REP. The expected return and variance of the OOS return of REP, *conditional* on $\widetilde{\boldsymbol{\omega}}$, are

$$\mathbb{E}\left(\widetilde{r}_{\text{REP}}|\widetilde{\boldsymbol{\omega}}\right) = \widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}, \tag{A.5}$$

$$\text{Var}\left(\widetilde{r}_{\text{REP}}|\widetilde{\boldsymbol{\omega}}\right) = \widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}, \tag{A.6}$$

and the *unconditional* mean and variance of the OOS return of REP are

$$\mathbb{E}\left(\widetilde{r}_{\text{REP}}\right) = \mathbb{E}\left(\mathbb{E}\left(\widetilde{r}_{\text{REP}}|\widetilde{\boldsymbol{\omega}}\right)\right) = \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right), \tag{A.7}$$

$$\text{Var}\left(\widetilde{r}_{\text{REP}}\right) = \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}\right) + \text{Var}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right). \tag{A.8}$$

Let $\text{SR}(\cdot) \equiv \frac{\mathbb{E}(\cdot)}{\sqrt{\text{Var}(\cdot)}}$ denote the *unconditional* Sharpe ratio of a portfolio. For convenience, we restate the following time series regression of the N assets with respect to REP,

$$\widetilde{\boldsymbol{r}} = \boldsymbol{\alpha}_{\text{REP}} + \boldsymbol{\beta}_{\text{REP}}\widetilde{r}_{\text{REP}} + \boldsymbol{\varepsilon}_{\text{REP}}, \tag{A.9}$$

where $\boldsymbol{\beta}_{\text{REP}}$ are the betas of the N assets with respect to REP, and $\boldsymbol{\alpha}_{\text{REP}}$ and $\boldsymbol{\varepsilon}_{\text{REP}}$ are $N \times 1$ vectors of α's and residuals, respectively.

Claims

We prove Theorem 1 by sequentially verifying the following three claims.

Claim 1. *The expected OOS return and variance of REP, conditional on the portfolio weights* $\widetilde{\boldsymbol{\omega}}$*, converge* in probability *to those of the theoretically MVE portfolio,* $\boldsymbol{\omega}^\star$*, as* $T, N \to \infty$ *and*

$\sqrt{\log(N)/T} \to 0$. *That is,*

$$\left|\boldsymbol{\mu}'\widetilde{\boldsymbol{\omega}} - \boldsymbol{\mu}'\boldsymbol{\omega}^\star\right| \xrightarrow{P} 0, \tag{A.10}$$

$$\left|\sqrt{\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}} - \sqrt{(\boldsymbol{\omega}^\star)'\,\boldsymbol{\Sigma}\boldsymbol{\omega}^\star}\right| \xrightarrow{P} 0. \tag{A.11}$$

Claim 2. *The* unconditional *mean, variance, and Sharpe ratio of the OOS return of REP converge to those of the theoretically MVE portfolio, $\boldsymbol{\omega}^\star$, as $T, N \to \infty$ and $\sqrt{\log(N)/T} \to 0$. That is,*

$$\mathbb{E}\left(\widetilde{r}_{\text{REP}}\right) - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu} \to 0, \tag{A.12}$$

$$\text{Var}\left(\widetilde{r}_{\text{REP}}\right) - (\boldsymbol{\omega}^\star)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star \to 0, \tag{A.13}$$

$$\text{SR}^2\left(\widetilde{r}_{\text{REP}}\right) - \text{SR}^2\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) \to 0. \tag{A.14}$$

Claim 3. *The pricing errors relative to REP vanish uniformly across assets; that is,*

$$\max_{1\le j\le N} |\alpha_{\text{REP},j}| \to 0,$$

as $T, N \to \infty$ and $\sqrt{\log(N)/T} \to 0$, where $\alpha_{\text{REP},j}$ denotes the jth element of the regression intercept $\boldsymbol{\alpha}_{\text{REP}}$ in Equation (A.9).

A.1 Proof of Claim 1

A.1.1 Lemmas for Claim 1

We list several lemmas that will be used in the proof of Claim 1.

Lemma A.1. (Lemma 3 in Chatterjee (2014)). *Suppose that $\zeta_i \sim N(0, \sigma_i^2)$ for $i = 1, \cdots, m$, which need not be independent. Then*

$$\mathbb{E}\left(\max_{1\le i\le m} |\zeta_i|\right) \le \max_{1\le i\le m} \sigma_i\sqrt{2\log(2m)}. \tag{A.15}$$

Lemma A.2. (Lemma 2 in Ao et al. (2019)). *Suppose that $\xi_i \sim \chi^2(T)$ for $i = 1, \cdots, m$, which*

need not be independent. If $\sqrt{\log(2m)/2T} \leq 1/4$, *then*

$$\mathbb{E}\left(\max_{1\leq i\leq m}|\xi_i - T|\right) \leq 2\sqrt{2T\log(2m)}. \tag{A.16}$$

Lemma A.3. (Lemma 3 in Ao et al. (2019)). *Suppose that* $X_j \sim N(0,1)$ *for* $j = 1, \cdots, p$, *and* $Y_k \sim N(0,1)$ *for* $k = 1, \cdots, q$. *The two sequences* $(X_j)_{j=1}^p$ *and* $(Y_k)_{k=1}^q$ *are independent, but* X_j*'s do not need to be independent, nor do* Y_k*'s. Let* $\left(X_j^t\right)_{t=1}^T$ *and* $\left(Y_k^t\right)_{t=1}^T$ *be i.i.d. copies of* $(X_j)_{j=1}^p$ *and* $(Y_k)_{k=1}^q$, *respectively. Further assume that* $\log(2pq)/T \leq 1/2$. *Then*

$$\mathbb{E}\left(\max_{j,k}\left|\sum_{t=1}^T X_j^t Y_k^t\right|\right) \leq 2\sqrt{T\log(2pq)}. \tag{A.17}$$

Lemma A.4. (Lemma 4 in Ao et al. (2019)). *Suppose that* $\boldsymbol{X}_t = (X_{t,1}, \cdots, X_{t,N})', t = 1, \cdots, T$, *are i.i.d. random vectors from the multivariate normal distribution* $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ *with* $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_N)'$ *and* $\boldsymbol{\Sigma} = (\sigma_{jk})_{1\leq j,k\leq N}$. *For* $j, k = 1, \cdots, N$, *define* $Y_{j,k} \equiv \mathbb{E}(X_{1,j}X_{1,k}) - \frac{1}{T}\sum_{t=1}^T X_{t,j}X_{t,k}$. *If* $\max_{1\leq j\leq N}|\mu_j| \leq M$ *and* $\max_{1\leq j\leq N}|\sigma_{jj}| \leq L$, *then*

$$\mathbb{E}\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right) \leq 2L\sqrt{\frac{2\log(2N)}{T}} + \left(2L + 2M\sqrt{2L}\right)\sqrt{\frac{\log(2N^2)}{T}}. \tag{A.18}$$

Lemma A.5. *Suppose that* $(\widetilde{X}_1, \ldots, \widetilde{X}_N, \widetilde{Y})'$ *is jointly normally distributed. For each* $j = 1, \ldots, N$, *the pair* $(\widetilde{X}_j, \widetilde{Y})'$ *has mean* $\boldsymbol{\mu}_{jY} = (\mu_j, \mu_Y)'$ *and covariance matrix*

$$\boldsymbol{\Sigma}_{jY} = \begin{pmatrix} \sigma_j^2 & \sigma_{jY} \\ \sigma_{jY} & \sigma_Y^2 \end{pmatrix}, \tag{A.19}$$

$j = 1, \cdots, N$. *Further assume that there exists* $L < \infty$ *such that* $\max_{1\leq j\leq N}\sigma_j^2 \leq L$. *Let* $(X_{t1}, \ldots, X_{tN}, Y_t)'$, $t = 1, \ldots, T$, *be i.i.d. copies of* $(\widetilde{X}_1, \ldots, \widetilde{X}_N, \widetilde{Y})'$. *If* $\log(2N)/T \leq 1/2$, *then*

$$\mathbb{E}\left(\max_{1\leq j\leq N}\left|\sum_{t=1}^T\left[\left(X_{tj} - \mu_j\right)\left(Y_t - \mu_Y\right) - \sigma_{jY}\right]\right|\right) \leq \sigma_Y\sqrt{LT}\left(\sqrt{2} + 2\sqrt{\log(2N)}\right). \tag{A.20}$$

Proof. The proof is adopted from the Internet Appendix (pp. 10-11) of Ao et al. (2019). Denote the correlation between $\widetilde{X}_j$ and $\widetilde{Y}$ by $\rho_j \equiv \sigma_{jY}/(\sigma_j \sigma_Y)$. Note that

$$\begin{pmatrix} \frac{\widetilde{X}_j - \mu_j}{\sigma_j} \\ \frac{\widetilde{Y} - \mu_Y}{\sigma_Y} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & \rho_j \\ \rho_j & 1 \end{pmatrix} \right). \tag{A.21}$$

Let $(Z_t)_{t=1}^T$ and $(W_{tj})_{t=1}^T, j = 1, \cdots, N$, be i.i.d. copies from the standard normal distribution and the two sequences are independent from each other. The variables W_{tj} may be correlated across j. Then let

$$\frac{Y_t - \mu_Y}{\sigma_Y} \overset{\mathcal{D}}{=} Z_t, \tag{A.22}$$

$$\frac{X_{tj} - \mu_j}{\sigma_j} \overset{\mathcal{D}}{=} m_j Z_t + n_j W_{tj}, \tag{A.23}$$

where m_j and n_j are constants to be determined such that Z_t and $m_j Z_t + n_j W_{tj}$ follow the bivariate normal distribution as in (A.21). The means of the two random variables on the RHS of (A.22) and (A.23) are both 0, and their variances are

$$\begin{aligned} \mathrm{Var}\left(Z_t\right) &= 1, \\ \mathrm{Var}\left(m_j Z_t + n_j W_{tj}\right) &= m_j^2 + n_j^2, \quad j = 1, ..., N. \end{aligned}$$

The covariance of the two random variables is

$$\mathrm{Cov}\left(Z_t, m_j Z_t + n_j W_{tj}\right) = m_j, \quad j = 1, ..., N.$$

It then follows that m_j and n_j must satisfy the following conditions

$$\begin{cases} m_j^2 + n_j^2 = 1, \\ m_j = \rho_j, \end{cases} \quad \Rightarrow \quad \begin{cases} m_j = \rho_j, \\ n_j = \sqrt{1 - \rho_j^2}. \end{cases} \tag{A.24}$$

Hence,

$$\begin{aligned}\sum_{t=1}^{T}\left[\left(X_{tj}-\mu_j\right)\left(Y_t-\mu_Y\right)-\sigma_{jY}\right] &= \sigma_j\sigma_Y\sum_{t=1}^{T}\left(\frac{X_{tj}-\mu_j}{\sigma_j}\cdot\frac{Y_t-\mu_Y}{\sigma_Y}-\frac{\sigma_{jY}}{\sigma_j\sigma_Y}\right)\\ &\overset{\mathcal{D}}{=} \sigma_j\sigma_Y\sum_{t=1}^{T}\left[\left(\rho_j Z_t+\sqrt{1-\rho_j^2}W_{tj}\right)Z_t-\rho_j\right]\\ &= \sigma_j\sigma_Y\rho_j\sum_{t=1}^{T}\left(Z_t^2-1\right)+\sigma_j\sigma_Y\sqrt{1-\rho_j^2}\sum_{t=1}^{T}Z_tW_{tj}\\ &\equiv V_{j,1}+V_{j,2}. \end{aligned} \tag{A.25}$$

Then

$$\max_{1\le j\le N}\left|\sum_{t=1}^{T}\left[\left(X_{tj}-\mu_j\right)\left(Y_t-\mu_Y\right)-\sigma_{jY}\right]\right| \le \max_{1\le j\le N}\left|V_{j,1}\right|+\max_{1\le j\le N}\left|V_{j,2}\right|. \tag{A.26}$$

For $V_{j,1}$,

$$\begin{aligned}\mathbb{E}\left(\max_{1\le j\le N}\left|V_{j,1}\right|\right) &\le \sigma_Y\mathbb{E}\left(\max_{1\le j\le N}\left(\left|\sigma_j\right|\left|\rho_j\right|\left|\sum_{t=1}^{T}\left(Z_t^2-1\right)\right|\right)\right)\\ &\le \sigma_Y\mathbb{E}\left(\sqrt{L}\times 1\times\left|\sum_{t=1}^{T}\left(Z_t^2-1\right)\right|\right)\\ &= \sigma_Y\sqrt{L}\mathbb{E}\left(\left|\sum_{t=1}^{T}Z_t^2-T\right|\right)\\ &\le \sigma_Y\sqrt{L}\sqrt{\mathbb{E}\left[\left(\sum_{t=1}^{T}Z_t^2-T\right)^2\right]}, \end{aligned} \tag{A.27}$$

where the second inequality follows from the variance bound in Lemma A.5 and the last inequality follows from Jensen's inequality. To calculate the expectation in the last inequality, let $S\equiv\sum_{t=1}^{T}Z_t^2\sim\chi^2(T)$, then $\mathbb{E}(S)=T,\mathrm{Var}(S)=2T$. Then

$$\mathbb{E}\left[\left(\sum_{t=1}^{T}Z_t^2-T\right)^2\right]=\mathbb{E}\left[\left(S-T\right)^2\right]=\mathrm{Var}\left(S\right)=2T, \tag{A.28}$$

and thus

$$\mathbb{E}\left(\max_{1\le j\le N}\left|V_{j,1}\right|\right)\le\sigma_Y\sqrt{L}\times\sqrt{2T}=\sigma_Y\sqrt{2LT}. \tag{A.29}$$

For $V_{j,2}$,

$$\begin{aligned}\mathbb{E}\left(\max_{1\leq j\leq N}|V_{j,2}|\right) &\leq \sigma_Y \mathbb{E}\left(\max_{1\leq j\leq N}\left(|\sigma_j|\left|\sqrt{1-\rho_j^2}\right|\left|\sum_{t=1}^{T} Z_t W_{tj}\right|\right)\right)\\ &\leq \sigma_Y\sqrt{L}\mathbb{E}\left(\max_{1\leq j\leq N}\left|\sum_{t=1}^{T} Z_t W_{tj}\right|\right)\\ &\leq 2\sigma_Y\sqrt{LT\log(2N)}, \end{aligned} \tag{A.30}$$

where the last inequality follows from Lemma A.3.

Finally, by (A.29) and (A.30), we have

$$\begin{aligned}\mathbb{E}\left(\max_{1\leq j\leq N}\left|\sum_{t=1}^{T}\left[\left(X_{tj}-\mu_j\right)\left(Y_t-\mu_Y\right)-\sigma_{jY}\right]\right|\right) &\leq \mathbb{E}\left(\max_{1\leq j\leq N}|V_{j,1}|+\max_{1\leq j\leq N}|V_{j,2}|\right)\\ &\leq \sigma_Y\sqrt{LT}\left(\sqrt{2}+2\sqrt{\log(2N)}\right). \end{aligned} \tag{A.31}$$

□

A.1.2 Proof of Claim 1

Proof. The proof can be divided into three parts.

Part 1. First, we show that

$$\frac{1}{T}\mathbb{E}\left(\left\|\boldsymbol{R}^\star-\widetilde{\boldsymbol{R}}\right\|_2^2\right)\leq O\left(\sqrt{\frac{\log(N)}{T}}\right)+O\left(\frac{1}{\sqrt{T}}\right). \tag{A.32}$$

Given the asset excess return matrix $\boldsymbol{R}$, define

$$\boldsymbol{S}_N=\left\{\boldsymbol{R}\boldsymbol{\omega}:|\omega_1|+\cdots+|\omega_N|\leq\lambda\right\}. \tag{A.33}$$

By definition, $\boldsymbol{S}_N$ is a compact convex subset of $\mathbb{R}^T$. Recall that $\widetilde{\boldsymbol{R}}\equiv\boldsymbol{R}\widetilde{\boldsymbol{\omega}}$ is the projection of the dependent variable $(1/\gamma)\mathbf{1}$ onto $\boldsymbol{S}_N$, where $\mathbf{1}$ is a $T\times 1$ vector of ones. By Assumption A4, $\boldsymbol{R}^\star\equiv\boldsymbol{R}\boldsymbol{\omega}^\star$ is also in $\boldsymbol{S}_N$. Therefore, the angle between $(\boldsymbol{R}^\star-\widetilde{\boldsymbol{R}})$ and $[(1/\gamma)\mathbf{1}-\widetilde{\boldsymbol{R}}]$ is obtuse, and

consequently,

$$\left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\left(\frac{1}{\gamma}\mathbf{1}-\widetilde{\boldsymbol{R}}\right)\leq 0. \tag{A.34}$$

Hence, by (A.34) and the definition of $r^{\star}$, we have

$$\begin{aligned}
\left\|\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right\|_2^2 &= \left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\left(\boldsymbol{R}^{\star}-\frac{1}{\gamma}\mathbf{1}+\frac{1}{\gamma}\mathbf{1}-\widetilde{\boldsymbol{R}}\right)\\
&\leq \left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\left(\boldsymbol{R}^{\star}-\frac{1}{\gamma}\mathbf{1}\right)\\
&= \left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\left(\boldsymbol{R}^{\star}-r^{\star}\mathbf{1}\right)+\left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\left(r^{\star}\mathbf{1}-\frac{1}{\gamma}\mathbf{1}\right)\\
&= \left(\boldsymbol{\omega}^{\star}-\widetilde{\boldsymbol{\omega}}\right)'\boldsymbol{R}'\left(\boldsymbol{R}\boldsymbol{\omega}^{\star}-\mathbf{1}\boldsymbol{\mu}'\boldsymbol{\omega}^{\star}\right)-\frac{1}{\gamma(1+\zeta)}\left(\boldsymbol{R}^{\star}-\widetilde{\boldsymbol{R}}\right)'\mathbf{1}\\
&= \sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\left(R_{tk}-\mu_k\right)R_{tj}-\frac{1}{\gamma(1+\zeta)}\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}R_{tj}\\
&= \sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\left[\left(R_{tk}-\mu_k\right)\left(R_{tj}-\mu_j\right)-\sigma_{jk}\right]\\
&\quad+\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\left(R_{tk}-\mu_k\right)\mu_j\\
&\quad+\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\sigma_{jk}-\frac{1}{\gamma(1+\zeta)}\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}R_{tj}\\
&= \sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)D_{j,1}+\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)D_{j,2}+D_3,
\end{aligned} \tag{A.35}$$

where the third equality follows from the definition of $r^{\star}$, and

$$D_{j,1}\equiv\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\left[\left(R_{tk}-\mu_k\right)\left(R_{tj}-\mu_j\right)-\sigma_{jk}\right], \tag{A.36}$$

$$D_{j,2}\equiv\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\left(R_{tk}-\mu_k\right)\mu_j, \tag{A.37}$$

$$D_3\equiv\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}\sum_{k=1}^{N}\omega_k^{\star}\sigma_{jk}-\frac{1}{\gamma(1+\zeta)}\sum_{j=1}^{N}\left(\omega_j^{\star}-\widetilde{\omega}_j\right)\sum_{t=1}^{T}R_{tj}. \tag{A.38}$$

We examine the three terms one by one.

(*i*) Denote $R_t^{\star}=\sum_{k=1}^{N}\omega_k^{\star}R_{tk}, t=1,\cdots,T$, which follow i.i.d. normal distributions with mean

$r^\star$ and variance $(\sigma^\star)^2$. For $D_{j,1}$ (A.36), we can rewrite it as

$$\begin{aligned} D_{j,1} &\equiv \sum_{t=1}^{T}\left[(R_{tj}-\mu_j)\sum_{k=1}^{N}\omega_k^\star\left(R_{tk}-\mu_k\right)-\sum_{k=1}^{N}\omega_k^\star\sigma_{jk}\right] \\ &= \sum_{t=1}^{T}\left[(R_{tj}-\mu_j)\left(R_t^\star-r^\star\right)-\operatorname{Cov}\left(\sum_{k=1}^{N}\omega_k^\star R_{tk},R_{tj}\right)\right] \\ &= \sum_{t=1}^{T}\left[(R_{tj}-\mu_j)\left(R_t^\star-r^\star\right)-\operatorname{Cov}\left(R_t^\star,R_{tj}\right)\right], \end{aligned} \tag{A.39}$$

where the second equality follows from the definition of $r^\star$.

By Lemma A.5, we have

$$\mathbb{E}\left(\max_{1\le j\le N}|D_{j,1}|\right)\le\sigma^\star\sqrt{LT}\left(\sqrt{2}+2\sqrt{\log(2N)}\right). \tag{A.40}$$

(*ii*) For $D_{j,2}$ (A.37), note that $D_{j,2}$ is the sum of i.i.d. normal random variables, so $D_{j,2}$ is also normally distributed. It is easy to verify that $\mathbb{E}\left(D_{j,2}\right)=0$ and $\operatorname{Var}\left(D_{j,2}\right)=T\mu_j^2\left(\sigma^\star\right)^2$.

Thus, $D_{j,2}\sim N\left(0,T\mu_j^2\left(\sigma^\star\right)^2\right)$ and

$$\begin{aligned} \mathbb{E}\left(\max_{1\le j\le N}|D_{j,2}|\right) &\le \left(\max_{1\le j\le N}\left|\sqrt{T}\mu_j\sigma^\star\right|\right)\cdot\sqrt{2\log\left(2N\right)} \\ &\le \sigma^\star M\sqrt{2T\log\left(2N\right)}, \end{aligned} \tag{A.41}$$

where the first inequality follows from Lemma A.1 and the second from Assumption A2.

(*iii*) For D_3 (A.38), using the fact that $\boldsymbol{\omega}^\star$ is the theoretically optimal portfolio, we separate the terms associated with $\widetilde{\omega}_j$ and $\omega_j^\star$, and rewrite D_3 as

$$\begin{aligned} D_3 &= \sum_{j=1}^{N}\widetilde{\omega}_j\left[\sum_{t=1}^{T}\left(\frac{1}{\gamma\left(1+\zeta\right)}R_{tj}-\sum_{k=1}^{N}\omega_k^\star\sigma_{jk}\right)\right]+\sum_{t=1}^{T}\sum_{j=1}^{N}\sum_{k=1}^{N}\omega_j^\star\omega_k^\star\sigma_{jk}-\frac{1}{\gamma\left(1+\zeta\right)}\sum_{t=1}^{T}\sum_{j=1}^{N}\omega_j^\star R_{tj} \\ &= \underbrace{\sum_{j=1}^{N}\widetilde{\omega}_j\left[\sum_{t=1}^{T}\left(\frac{1}{\gamma\left(1+\zeta\right)}R_{tj}-\sum_{k=1}^{N}\omega_k^\star\sigma_{jk}\right)\right]}_{D_{3a}}+\underbrace{T\left(\sigma^\star\right)^2-\frac{1}{\gamma\left(1+\zeta\right)}\sum_{t=1}^{T}\sum_{j=1}^{N}\omega_j^\star R_{tj}}_{D_{3b}}. \end{aligned} \tag{A.42}$$

For D_{3a} in (A.42),

$$\begin{aligned} D_{3a} &= \frac{1}{\gamma(1+\zeta)} \sum_{j=1}^{N} \widetilde{\omega}_j \sum_{t=1}^{T} R_{tj} - \sum_{t=1}^{T} (\boldsymbol{\omega}^\star)' \boldsymbol{\Sigma} \widetilde{\boldsymbol{\omega}} \\ &= \frac{1}{\gamma(1+\zeta)} \sum_{j=1}^{N} \widetilde{\omega}_j \sum_{t=1}^{T} R_{tj} - \sum_{t=1}^{T} \frac{1}{\gamma(1+\zeta)} \boldsymbol{\mu}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma} \widetilde{\boldsymbol{\omega}} \\ &= \frac{1}{\gamma(1+\zeta)} \sum_{j=1}^{N} \widetilde{\omega}_j \sum_{t=1}^{T} (R_{tj} - \mu_j), \end{aligned} \tag{A.43}$$

where the second equality is obtained by using the definition $\boldsymbol{\omega}^\star = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\gamma(1+\zeta)}$. Note that the random variable $X_j \equiv \sum_{t=1}^{T}(R_{tj}-\mu_j)$ is normally distributed with mean $\mathbb{E}(X_j) = 0$ and variance $\mathrm{Var}(X_j) = T\sigma_{jj}$. Then

$$\begin{aligned} \mathbb{E}(|D_{3a}|) &= \frac{1}{\gamma(1+\zeta)} \mathbb{E}\left(\left| \sum_{j=1}^{N} \widetilde{\omega}_j X_j \right| \right) \\ &\leq \frac{1}{\gamma(1+\zeta)} \mathbb{E}\left(\sum_{j=1}^{N} |\widetilde{\omega}_j| \max_{1\leq j\leq N} |X_j| \right) \\ &\leq \frac{1}{\gamma(1+\zeta)} \cdot \lambda \cdot \left[\max_{1\leq j\leq N} \left| \sqrt{\mathrm{Var}(X_j)} \right| \cdot \sqrt{2\log(2N)} \right] \\ &\leq \frac{\lambda}{\gamma(1+\zeta)} \sqrt{2LT\log(2N)}, \end{aligned} \tag{A.44}$$

where the second inequality follows from the LASSO penalty and Lemma A.1, and the last inequality from Assumption A3.[15]

For D_{3b} in (A.42), recall that $\sum_{j=1}^{N} R_{tj}\omega_j^\star = R_t^\star$ is normally distributed with mean $r^\star$ and variance $(\sigma^\star)^2$. Let $Z_t \sim N(0,1), t = 1, \cdots, T$, then $\sum_{j=1}^{N} \omega_j^\star R_{tj} \overset{\mathcal{D}}{=} r^\star + \sigma^\star Z_t$. Thus,

$$D_{3b} \overset{\mathcal{D}}{=} T(\sigma^\star)^2 - \frac{1}{\gamma(1+\zeta)} \sum_{t=1}^{T} (r^\star + \sigma^\star Z_t) = -\frac{\sqrt{\zeta}}{\gamma^2(1+\zeta)^2} \sum_{t=1}^{T} Z_t, \tag{A.45}$$

where the last equality is obtained by substituting the definition for $r^\star$ and $(\sigma^\star)^2$.

Note that $\sum_{t=1}^{T} Z_t \sim N(0,T)$, and consider $\sqrt{T}Z \overset{\mathcal{D}}{=} \sum_{t=1}^{T} Z_t$ where Z is a standard normal

[15]Note that the theorem still holds when λ in the LASSO-regularized regression (10) is larger than the upper bound of $\|\boldsymbol{\omega}^\star\|_1$ in Assumption A4. For simplicity, we assume that they are equal.

random variable, then

$$\mathbb{E}\left(|D_{3b}|\right) = \frac{\sqrt{\zeta}}{\gamma^2\left(1+\zeta\right)^2}\mathbb{E}\left(\left|\sqrt{T}Z\right|\right) \leq \frac{\sqrt{T\zeta}}{\gamma^2\left(1+\zeta\right)^2}, \tag{A.46}$$

where the last inequality follows because $\mathbb{E}\left(|Z|\right) = \sqrt{\frac{2}{\pi}} < 1$.

Combining (A.44) and (A.46), we have

$$\mathbb{E}\left(|D_3|\right) \leq \mathbb{E}\left(|D_{3a}|\right) + \mathbb{E}\left(|D_{3b}|\right) \leq \frac{\lambda}{\gamma\left(1+\zeta\right)}\sqrt{2LT\log\left(2N\right)} + \frac{\sqrt{T\zeta}}{\gamma^2\left(1+\zeta\right)^2}. \tag{A.47}$$

Combining (A.35), (A.40), (A.41) and (A.47), we have

$$\begin{aligned}
\mathbb{E}\left(\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2\right) &\leq \mathbb{E}\left[\sum_{j=1}^N \left(\omega_j^\star - \widetilde{\omega}_j\right) D_{j,1} + \sum_{j=1}^N \left(\omega_j^\star - \widetilde{\omega}_j\right) D_{j,2} + D_3\right] \\
&\leq \mathbb{E}\left[\sum_{j=1}^N \left(\left|\omega_j^\star\right| + \left|\widetilde{\omega}_j\right|\right) \cdot \max_{1\leq j\leq N}\left|D_{j,1}\right|\right] + \mathbb{E}\left[\sum_{j=1}^N \left(\left|\omega_j^\star\right| + \left|\widetilde{\omega}_j\right|\right) \cdot \max_{1\leq j\leq N}\left|D_{j,2}\right|\right] + \mathbb{E}\left(|D_3|\right) \\
&\leq (\lambda+\lambda)\cdot\mathbb{E}\left(\max_{1\leq j\leq N}\left|D_{j,1}\right|\right) + (\lambda+\lambda)\cdot\mathbb{E}\left(\max_{1\leq j\leq N}\left|D_{j,2}\right|\right) + \mathbb{E}\left(|D_3|\right) \\
&\leq 2\lambda\left[\sigma^\star\sqrt{2LT} + 2\sigma^\star\sqrt{LT\log\left(2N\right)}\right] + 2\lambda\left[\sigma^\star M\sqrt{2T\log\left(2N\right)}\right] \\
&\quad + \frac{\lambda}{\gamma\left(1+\zeta\right)}\sqrt{2LT\log\left(2N\right)} + \frac{\sqrt{T\zeta}}{\gamma^2\left(1+\zeta\right)^2}.
\end{aligned} \tag{A.48}$$

Part 2. We now proceed to the second part of the proof where we examine the discrepancy between the one-period future returns by $\boldsymbol{\omega}^\star$ and $\widetilde{\boldsymbol{\omega}}$.

Let $\mathcal{F}$ be the σ-algebra generated by $(R_{tj})_{1\leq t\leq T, 1\leq j\leq N}$. Let $\widetilde{\boldsymbol{r}} \equiv (\widetilde{r}_1, \cdots, \widetilde{r}_N)'$ be one-period future returns of the N assets. Denote the future returns of the portfolios $\boldsymbol{\omega}^\star$ and $\widetilde{\boldsymbol{\omega}}$ by

$$Q^\star = \sum_{j=1}^N \omega_j^\star \widetilde{r}_j = \left(\boldsymbol{\omega}^\star\right)'\widetilde{\boldsymbol{r}}, \qquad \widetilde{Q} = \sum_{j=1}^N \widetilde{\omega}_j \widetilde{r}_j = \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}, \tag{A.49}$$

respectively. By the i.i.d. sampling assumption,

$$\mathbb{E}\left(\widetilde{\boldsymbol{r}}|\mathcal{F}\right) = \mathbb{E}\left(\widetilde{\boldsymbol{r}}\right) = \mathbb{E}\left(\boldsymbol{r}\right) = \boldsymbol{\mu}, \tag{A.50}$$

$$\mathbb{E}\left(\widetilde{\boldsymbol{r}}\widetilde{\boldsymbol{r}}'|\mathcal{F}\right) = \mathbb{E}\left(\widetilde{\boldsymbol{r}}\widetilde{\boldsymbol{r}}'\right) = \mathbb{E}\left(\boldsymbol{r}\boldsymbol{r}'\right) = \boldsymbol{\Sigma} + \boldsymbol{\mu}\boldsymbol{\mu}'. \tag{A.51}$$

Note that $\widetilde{\boldsymbol{\omega}}$ is independent of $\widetilde{\boldsymbol{r}}$ since it is estimated from the observed data up to T, i.e., $\boldsymbol{R}_t, t = 1, \cdots, T$. Note also that $\boldsymbol{\omega}^\star$ is constant with respect to the σ-algebra. Hence, the expected discrepancy between the future returns is

$$\begin{aligned}
\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right] &= \mathbb{E}\left[\left(\left(\boldsymbol{\omega}^\star\right)'\widetilde{\boldsymbol{r}} - \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right)^2 |\mathcal{F}\right] \\
&= \left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)'\mathbb{E}\left(\boldsymbol{r}\boldsymbol{r}'\right)\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right) \qquad \text{(A.52)} \\
&= \left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)'\left(\boldsymbol{\Sigma} + \boldsymbol{\mu}\boldsymbol{\mu}'\right)\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right) \\
&\geq \left\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right\|_{\boldsymbol{\Sigma}}^2. \qquad \text{(A.53)}
\end{aligned}$$

Note that we can rewrite (A.52) as

$$\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right] = \left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)'\mathbb{E}\left(\boldsymbol{r}\boldsymbol{r}'\right)\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right) = \sum_{j=1}^{N}\sum_{k=1}^{N}\left(\omega_j^\star - \widetilde{\omega}_j\right)\left(\omega_k^\star - \widetilde{\omega}_k\right)\mathbb{E}\left(r_j r_k\right). \tag{A.54}$$

Note also that

$$\begin{aligned}
\frac{1}{T}\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2 &= \frac{1}{T}\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)'\left(\boldsymbol{R}'\boldsymbol{R}\right)\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right) \\
&= \sum_{j=1}^{N}\sum_{k=1}^{N}\left[\left(\omega_j^\star - \widetilde{\omega}_j\right)\left(\omega_k^\star - \widetilde{\omega}_k\right)\left(\frac{1}{T}\sum_{t=1}^{T}R_{tj}R_{tk}\right)\right]. \qquad \text{(A.55)}
\end{aligned}$$

So

$$\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right] - \frac{1}{T}\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2 = \sum_{j=1}^{N}\sum_{k=1}^{N}\left(\omega_j^\star - \widetilde{\omega}_j\right)\left(\omega_k^\star - \widetilde{\omega}_k\right)\left[\mathbb{E}\left(r_j r_k\right) - \frac{1}{T}\sum_{t=1}^{T}R_{tj}R_{tk}\right]. \tag{A.56}$$

Define

$$Y_{j,k} = \mathbb{E}\left(r_j r_k\right) - \frac{1}{T}\sum_{t=1}^{T}R_{tj}R_{tk}. \tag{A.57}$$

By Assumption A4 and the LASSO constraint, we have

$$\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right] - \frac{1}{T}\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2 = \sum_{j=1}^{N}\sum_{k=1}^{N}\left(\omega_j^\star - \widetilde{\omega}_j\right)\left(\omega_k^\star - \widetilde{\omega}_k\right) Y_{j,k}$$
$$\leq \sum_{j=1}^{N}\sum_{k=1}^{N}\left(\left|\omega_j^\star\right| + \left|\widetilde{\omega}_j\right|\right)\left(\left|\omega_k^\star\right| + \left|\widetilde{\omega}_k\right|\right)\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right)$$
$$\leq 4\lambda^2\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right). \tag{A.58}$$

Rearranging terms gives

$$\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right] \leq \frac{1}{T}\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2 + 4\lambda^2\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right). \tag{A.59}$$

Therefore,

$$\mathbb{E}\left(\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\|_{\boldsymbol{\Sigma}}^2\right) \leq \mathbb{E}\left\{\mathbb{E}\left[\left(Q^\star - \widetilde{Q}\right)^2 |\mathcal{F}\right]\right\}$$
$$\leq \frac{1}{T}\mathbb{E}\left(\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2\right) + 4\lambda^2\mathbb{E}\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right). \tag{A.60}$$

By (A.48) and Lemma A.4, we know that

$$\frac{1}{T}\mathbb{E}\left(\left\|\boldsymbol{R}^\star - \widetilde{\boldsymbol{R}}\right\|_2^2\right) \leq O\left(\sqrt{\frac{\log(N)}{T}}\right) + O\left(\frac{1}{\sqrt{T}}\right), \tag{A.61}$$

$$\mathbb{E}\left(\max_{1\leq j,k\leq N}|Y_{j,k}|\right) \leq O\left(\sqrt{\frac{\log(N)}{T}}\right). \tag{A.62}$$

Thus,

$$\mathbb{E}\left(\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\|_{\boldsymbol{\Sigma}}^2\right) \leq O\left(\sqrt{\frac{\log(N)}{T}}\right) + O\left(\frac{1}{\sqrt{T}}\right). \tag{A.63}$$

Hence, by Assumption A5 ($T, N \to \infty$ at rate $\log(N)/T \to 0$) and Markov's inequality,

$$\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\|_{\boldsymbol{\Sigma}}^2 \xrightarrow{P} 0, \tag{A.64}$$

In addition, $\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\|_{\boldsymbol{\Sigma}} \xrightarrow{P} 0$ as well by continuous mapping.

Part 3. The third part completes the proof for Claim 1.

(i) For the discrepancy of the expected returns of $\widetilde{\boldsymbol{\omega}}$ and $\boldsymbol{\omega}^\star$,

$$\begin{aligned}\left|\boldsymbol{\mu}'\widetilde{\boldsymbol{\omega}} - \boldsymbol{\mu}'\boldsymbol{\omega}^\star\right| &= \left|\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-\frac{1}{2}} \cdot \boldsymbol{\Sigma}^{\frac{1}{2}}\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)\right| \\ &\leq \sqrt{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}} \cdot \sqrt{\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)'\boldsymbol{\Sigma}\left(\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right)} \\ &\leq \sqrt{M}\left\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right\|_{\boldsymbol{\Sigma}}, \end{aligned} \tag{A.65}$$

where the first and second inequalities follow from the Cauchy-Schwarz inequality and Assumption A2, respectively. Then by (A.64), the first conclusion of Claim 1 (Equation A.10) is obtained.

(ii) For the discrepancy between the volatility of $\widetilde{\boldsymbol{\omega}}$ and $\boldsymbol{\omega}^\star$, by triangle inequality for the norm $\|\cdot\|_{\boldsymbol{\Sigma}}$, we have

$$\left|\sqrt{\left(\widetilde{\boldsymbol{\omega}}\right)'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}} - \sqrt{\left(\boldsymbol{\omega}^\star\right)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star}\right| \equiv \left|\|\boldsymbol{\omega}^\star\|_{\boldsymbol{\Sigma}} - \|\widetilde{\boldsymbol{\omega}}\|_{\boldsymbol{\Sigma}}\right| \leq \left\|\boldsymbol{\omega}^\star - \widetilde{\boldsymbol{\omega}}\right\|_{\boldsymbol{\Sigma}}. \tag{A.66}$$

Again by (A.64), the second conclusion of Claim 1 (Equation A.11) is obtained. □

A.2 Proof of Claim 2

Notation for Claim 2

Throughout the analysis for Claim 2, the convergence refers to the asymptotics under the double limit $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$. Let $\{a_{T,N}\}$ denote a *double sequence* in T and N. A *linear subsequence* $\left\{a_{T_p,N_p}\right\}$ of a double sequence $\{a_{T,N}\}$ is defined by a subsequence with increasing indices $\{T_p\}$ and $\{N_p\}$ out of the sequence of positive integers. Given a number L, if $\lim_{p\to\infty} a_{T_p,N_p} = L$ for all linear subsequences $\left\{a_{T_p,N_p}\right\}$ such that $\lim_{p\to\infty}\sqrt{\log(N_p)/T_p} = 0$, then we say that $a_{T,N} \to L$ as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$. A double sequence of random variables $\{X_{T,N}\}$ is said to *converge in probability* to X at rate $\sqrt{\log(N)/T} \to 0$, denoted by $X_{T,N} \xrightarrow{P} X$ if $P\left(|X_{T,N} - X| > \tau\right) \to 0$ as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$, for all $\tau > 0$.

A.2.1 Lemmas and Theorem for Claim 2

Some useful lemmas and theorems are given as follows and their proofs are standard and are available upon request.

Lemma A.6. *If $X_{T,N} \xrightarrow{P} X$ as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$, and $|X_{T,N}| \leq Y$ almost surely (i.e., $P(|X_{T,N}| \leq Y) = 1$) for all T, N, then $|X| \leq Y$ almost surely.*

Lemma A.7. *Suppose $X_{T,N}$ is a double sequence of random variables and $Y_{T,N}$ is a non-random double sequence. If $X_{T,N} \xrightarrow{P} X$ and $Y_{T,N} \to Y$ as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$, then $X_{T,N} + Y_{T,N} \xrightarrow{P} X + Y$.*

Theorem A.1. (Dominated Convergence Theorem under Double Limits) *If $X_{T,N} \xrightarrow{P} X$ as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$, and $|X_{T,N}| \leq Y$ almost surely for all T, N, then for any positive integer ν such that $\mathbb{E}(Y^\nu) < \infty$,*

$$\mathbb{E}\left(X_{T,N}^\nu\right) \to \mathbb{E}\left(X^\nu\right),$$

as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$.

A.2.2 Proof of Claim 2

Proof. The proof can be divided into three parts.

Part 1: The unconditional expected return.

By Assumption A2 and the LASSO constraint (10) in Theorem 1,

$$|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}| = \left|\sum_{j=1}^{N} \widetilde{\omega}_j \mu_j\right| \leq \sum_{j=1}^{N} |\widetilde{\omega}_j|\,|\mu_j| \leq \left(\sum_{j=1}^{N} |\widetilde{\omega}_j|\right) \cdot \max_{1\leq j\leq N} |\mu_j| \leq M\lambda < \infty, \tag{A.67}$$

for all T, N. Similarly, Assumptions A2 and A4 imply that $|(\boldsymbol{\omega}^\star)'\boldsymbol{\mu}| \leq M\lambda$. By (A.10) in Claim 1,

$$\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu} \xrightarrow{p} 0.$$

Moreover,

$$\left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu}\right| \leq 2M\lambda$$

for all T, N. Applying Theorem A.1 to this difference with $\nu = 1$ gives

$$\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right) - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu} \to 0. \tag{A.68}$$

Recall that $\widetilde{\boldsymbol{r}}$ is the one-period future return and $\widetilde{\boldsymbol{\omega}}$ is measurable with respect to the σ-algebra $\mathcal{F}$. The first term of the LHS of (A.68) is equal to $\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right)$ since $\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) = \mathbb{E}\left[\widetilde{\boldsymbol{\omega}}'\mathbb{E}\left(\widetilde{\boldsymbol{r}}|\mathcal{F}\right)\right] = \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right)$, and the second term is equal to $\mathbb{E}\left[(\boldsymbol{\omega}^\star)'\,\widetilde{\boldsymbol{r}}\right]$. Hence,

$$\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) - \mathbb{E}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) \to 0. \tag{A.69}$$

as $T, N \to \infty$ at rate $\sqrt{\log\left(N\right)/T} \to 0$.

Part 2: The unconditional variance.

Applying the law of total variance to the OOS return of REP in Equation (A.8), we have

$$\begin{aligned}
\mathrm{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) &= \mathbb{E}\left[\mathrm{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}|\mathcal{F}\right)\right] + \mathrm{Var}\left[\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}|\mathcal{F}\right)\right] \\
&= \mathbb{E}\left[\widetilde{\boldsymbol{\omega}}'\mathrm{Var}\left(\widetilde{\boldsymbol{r}}|\mathcal{F}\right)\widetilde{\boldsymbol{\omega}}\right] + \mathrm{Var}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right) \\
&= \underbrace{\mathbb{E}\left[\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}\right]}_{I} + \underbrace{\mathbb{E}\left\{\left[\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right)\right]^2\right\}}_{II}.
\end{aligned} \tag{A.70}$$

(i) For I, by Assumption A3 and the LASSO constraint in (10),

$$\begin{aligned}
\left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}\right| &= \left|\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{\omega}_i\widetilde{\omega}_j\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}\rho_{ij}\right| \\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\left|\widetilde{\omega}_i\right|\left|\widetilde{\omega}_j\right|\left|\sqrt{\sigma_{ii}}\right|\left|\sqrt{\sigma_{jj}}\right| \\
&\leq \left(\sum_{i=1}^{N}\left|\widetilde{\omega}_i\right|\right)\cdot\left(\sum_{j=1}^{N}\left|\widetilde{\omega}_j\right|\right)\cdot\max_{1\leq i\leq N}\left|\sqrt{\sigma_{ii}}\right|\cdot\max_{1\leq j\leq N}\left|\sqrt{\sigma_{jj}}\right|
\end{aligned}$$

$$\leq \lambda^2 L < \infty. \tag{A.71}$$

Assumptions A3 and A4 also imply that

$$(\boldsymbol{\omega}^\star)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star \leq \lambda^2 L.$$

Hence, by (A.11) in Claim 1,

$$\left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}} - (\boldsymbol{\omega}^\star)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star\right| \leq 2\lambda\sqrt{L}\left|\sqrt{\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}} - \sqrt{(\boldsymbol{\omega}^\star)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star}\right| \xrightarrow{p} 0. \tag{A.72}$$

Note that the absolute difference (LHS of (A.72)) is bounded by $2\lambda^2 L$ for all T, N. Applying Theorem A.1 to this difference with $\nu = 1$, and using $\text{Var}((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}) = (\boldsymbol{\omega}^\star)'\boldsymbol{\Sigma}\boldsymbol{\omega}^\star$, gives

$$\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\omega}}\right) - \text{Var}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) \to 0. \tag{A.73}$$

as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$.

(ii) For II, by (A.10) in Claim 1 and (A.68), i.e., $\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu} \xrightarrow{P} 0$, and $\mathbb{E}(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}) - (\boldsymbol{\omega}^\star)'\boldsymbol{\mu} \to 0$, respectively, in conjunction with Lemma A.7, we obtain

$$\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right) \xrightarrow{P} 0, \tag{A.74}$$

as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$. Note also that

$$\left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right)\right| \leq \left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right| + \mathbb{E}\left(\left|\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right|\right) \leq 2M\lambda < \infty.$$

Then by Theorem A.1 and letting $\nu = 2$,

$$\mathbb{E}\left\{\left[\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu} - \mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\boldsymbol{\mu}\right)\right]^2\right\} \to 0. \tag{A.75}$$

Combining (A.70), (A.73), and (A.75), we obtain

$$\operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) - \operatorname{Var}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) \to 0. \tag{A.76}$$

as $T, N \to \infty$ at rate $\sqrt{\log(N)/T} \to 0$.

Part 3: The unconditional Sharpe ratio.

Before proceeding, we state two useful identities. Denote $c = \gamma(1+\zeta)$ and $\boldsymbol{d} = \widetilde{\boldsymbol{\omega}} - \boldsymbol{\omega}^\star$. Recalling that $\boldsymbol{\omega}^\star = c^{-1}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, we have

$$\operatorname{Var}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) = \frac{\zeta}{c^2}. \tag{A.77}$$

Moreover, because $\widetilde{\boldsymbol{\omega}}$ is independent of $\widetilde{\boldsymbol{r}}$,

$$\operatorname{Cov}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}, \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) = \frac{\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right)}{c}. \tag{A.78}$$

First, note that ζ is the maximum attainable squared Sharpe ratio in the asset universe. Thus,

$$\operatorname{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) \leq \zeta. \tag{A.79}$$

Next, we show that the squared Sharpe ratio loss of REP vanishes asymptotically. The variance of the return difference between REP and the optimal portfolio $\boldsymbol{\omega}^\star$ can be expressed as follows:

$$\begin{aligned} \operatorname{Var}\left(\boldsymbol{d}'\widetilde{\boldsymbol{r}}\right) &= \operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) + \operatorname{Var}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) - 2\operatorname{Cov}\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}, \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) \\ &= \operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) + \frac{\zeta}{c^2} - \frac{2}{c}\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right). \end{aligned} \tag{A.80}$$

Multiplying both sides of Equation (A.80) by c^2, adding and subtracting the squared Sharpe ratio of REP, and completing the square, we obtain

$$\begin{aligned} c^2\operatorname{Var}\left(\boldsymbol{d}'\widetilde{\boldsymbol{r}}\right) &= \zeta - \operatorname{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) + \operatorname{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) + c^2\operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) - 2c\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) \\ &= \zeta - \operatorname{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) + \frac{\left[\mathbb{E}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) - c\operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right)\right]^2}{\operatorname{Var}\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right)}. \end{aligned} \tag{A.81}$$

The squared term in the final line of Equation (A.81) is nonnegative. Combining this with Equation (A.79) yields

$$0 \leq \zeta - \mathrm{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) \leq c^2 \operatorname{Var}\left(\boldsymbol{d}'\widetilde{\boldsymbol{r}}\right). \tag{A.82}$$

By the law of total variance,

$$\begin{aligned} \operatorname{Var}\left(\boldsymbol{d}'\widetilde{\boldsymbol{r}}\right) &= \mathbb{E}\left(\boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}\right) + \operatorname{Var}\left(\boldsymbol{d}'\boldsymbol{\mu}\right) \\ &\leq \mathbb{E}\left(\boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}\right) + \mathbb{E}\left[(\boldsymbol{d}'\boldsymbol{\mu})^2\right] \\ &\leq (1+\zeta)\mathbb{E}\left(\boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}\right), \end{aligned} \tag{A.83}$$

where the last steps follows from the Cauchy–Schwarz inequality. Since $\zeta \leq M$, applying Equation (A.63) yields

$$0 \leq \mathrm{SR}^2\left((\boldsymbol{\omega}^\star)'\widetilde{\boldsymbol{r}}\right) - \mathrm{SR}^2\left(\widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) \leq \gamma^2(1+M)^3\mathbb{E}\left(\boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}\right) \to 0, \tag{A.84}$$

as $T, N \to \infty$ and $\sqrt{\log(N)/T} \to 0$. □

A.3 Proof of Claim 3

Proof. According to Gibbons et al. (1989), the following identity holds

$$\boldsymbol{\alpha}_{\mathrm{REP}}'\boldsymbol{\Sigma}_{\varepsilon_{\mathrm{REP}}}^{-1}\boldsymbol{\alpha}_{\mathrm{REP}} = \phi^2\left(\widetilde{\boldsymbol{r}}, \widetilde{r}_{\mathrm{REP}}\right) - \phi^2\left(\widetilde{r}_{\mathrm{REP}}\right), \tag{A.85}$$

provided that $\boldsymbol{\Sigma}_{\varepsilon_{\mathrm{REP}}}$ is a positive definite covariance matrix, where $\boldsymbol{\alpha}_{\mathrm{REP}}$ is the $N \times 1$ vector of regression intercepts and $\boldsymbol{\Sigma}_{\varepsilon_{\mathrm{REP}}}$ is the covariance matrix of the regression residuals from the time series regression (A.9). $\phi^2(\cdot)$ represents the ex-ante maximum squared Sharpe ratio generated by the assets in the parenthesis.

First, note that, by definition, $\phi^2(\widetilde{r}_{\mathrm{REP}}) = \mathrm{SR}^2(\widetilde{r}_{\mathrm{REP}})$. In addition, since $\widetilde{r}_{\mathrm{REP}}$ is a linear

combination of $\widetilde{\boldsymbol{r}}$ with weight $\widetilde{\boldsymbol{\omega}}$ and $\widetilde{\boldsymbol{\omega}}$ is independent of $\widetilde{\boldsymbol{r}}$, we have the following relation

$$\phi^2(\widetilde{\boldsymbol{r}}, \widetilde{r}_{\mathrm{REP}}) \equiv \phi^2\left(\widetilde{\boldsymbol{r}}, \widetilde{\boldsymbol{\omega}}'\widetilde{\boldsymbol{r}}\right) = \phi^2\left(\widetilde{\boldsymbol{r}}\right) = \mathrm{SR}^2\left((\boldsymbol{\omega}^\star)'\,\widetilde{\boldsymbol{r}}\right), \tag{A.86}$$

where the last equality follows because $\boldsymbol{\omega}^\star$ is the theoretically optimal portfolio and has the maximum Sharpe ratio.

Combining (A.85) and (A.86) with the convergence of the squared Sharpe ratio difference to zero in (A.84) yields

$$\boldsymbol{\alpha}'_{\mathrm{REP}}\boldsymbol{\Sigma}^{-1}_{\varepsilon_{\mathrm{REP}}}\boldsymbol{\alpha}_{\mathrm{REP}} \to 0, \tag{A.87}$$

as $T, N \to \infty$ and $\sqrt{\log(N)/T} \to 0$. Consequently, for each j, the Cauchy–Schwarz inequality gives

$$\begin{aligned} |\alpha_{\mathrm{REP},j}|^2 &= \left|\left(\boldsymbol{\Sigma}^{1/2}_{\varepsilon_{\mathrm{REP}}}\mathbf{e}_j\right)'\left(\boldsymbol{\Sigma}^{-1/2}_{\varepsilon_{\mathrm{REP}}}\boldsymbol{\alpha}_{\mathrm{REP}}\right)\right|^2 \\ &\le \Sigma_{\varepsilon_{\mathrm{REP}},jj}\,\boldsymbol{\alpha}'_{\mathrm{REP}}\boldsymbol{\Sigma}^{-1}_{\varepsilon_{\mathrm{REP}}}\boldsymbol{\alpha}_{\mathrm{REP}}, \end{aligned} \tag{A.88}$$

where $\mathbf{e}_j$ denotes the jth standard basis vector in $\mathbb{R}^N$.

Because the variance of a regression residual cannot exceed the variance of the dependent variable,

$$\Sigma_{\varepsilon_{\mathrm{REP}},jj} = \mathrm{Var}(\varepsilon_{\mathrm{REP},j}) \le \sigma_{jj} \le L. \tag{A.89}$$

It then follows that the maximum absolute entry of $\boldsymbol{\alpha}_{\mathrm{REP}}$ satisfies

$$\max_{1\le j\le N} |\alpha_{\mathrm{REP},j}| \le \sqrt{L\boldsymbol{\alpha}'_{\mathrm{REP}}\boldsymbol{\Sigma}^{-1}_{\varepsilon_{\mathrm{REP}}}\boldsymbol{\alpha}_{\mathrm{REP}}} \to 0, \tag{A.90}$$

as $T, N \to \infty$ and $\sqrt{\log(N)/T} \to 0$. Therefore, the pricing errors relative to REP vanish uniformly across the assets. □

B Cross-validation Procedure of the REP approach

In this section, we describe the 10-fold CV procedure used to select the tuning parameter ξ for REP. Suppose that the estimation sample consists of 120 monthly observations. We partition the sample into 10 equal-sized folds, each containing 12 observations. In CV iteration i, nine folds are used as the training sample to estimate the LASSO solution path, while the remaining fold serves as the validation sample. For each ξ along the solution path, we evaluate the validation-sample mean squared error (MSE) of the corresponding estimated portfolio $\boldsymbol{\omega}_i(\xi)$,

$$\mathrm{MSE}_i(\xi) = \frac{1}{12}\sum_{j=1}^{12}\left(\frac{1}{\gamma} - \boldsymbol{\omega}_i(\xi)'\boldsymbol{R}_{ij}\right)^2, \tag{B.1}$$

where $\boldsymbol{R}_{ij}$ denotes the vector of asset excess returns for the j-th observation in the validation fold of iteration i.

For each iteration i, we record the value of ξ_i, as defined in Equation (15), that minimizes the validation-sample MSE. This procedure is repeated across all 10 training–validation splits, and the selected tuning parameter is obtained by averaging the resulting values $(\xi_i)_{i=1}^{10}$. To further enhance robustness, we repeat the entire 10-fold CV procedure 10 times using different random partitions of the sample and set the final value of ξ equal to the average of the 10 resulting tuning parameters. Finally, we estimate REP using the full estimation sample of 120 observations and the selected value of ξ.

Importantly, the performance of the REP approach is not sensitive to the specific implementation of the CV procedure. For example, one may alternatively use λ directly as the tuning parameter and select the value of λ that minimizes the average validation-sample MSE across the 10 folds. We find that such alternative CV implementations lead to quantitatively similar results.

C Supplementary Results

In this section, we report supplementary results and tables that complement the main results.

C.1 Information on the 174 Characteristics and Corresponding Factors

We begin with the large set of firm characteristics considered in Hou et al. (2020). Among these, we are able to match 221 characteristics in the database of Chen and Zimmermann (2022). We then apply three filters to these characteristics. First, we exclude 34 quarterly variants of characteristics for which annual counterparts are already included. Second, we remove five variables that are dummy variables (e.g., secured-debt and convertible indicators) or take only a small number of distinct values. Finally, we exclude eight characteristics with more than 80% missing observations in our stock–month panel from January 1970 to December 2022. These eight characteristics are ProbInformedTrading, Governance, CustomerMomentum, MS, PredictedFE, RDcap, CitationsRD, and PatentsRD. After applying these filters, we obtain the 174 characteristics used in the paper. The following tables report the definitions of these characteristics and summary statistics of the corresponding factors. For further details, see the database of Chen and Zimmermann (2022).

Table C.1: Descriptions of the 174 Characteristics

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
EarningsSurprise	Earnings Surprise	Foster, Olsen and Shevlin (1984)	1
AnnouncementReturn	Earnings announcement return	Chan, Jegadeesh and Lakonishok (1996)	1
REV6	Earnings forecast revisions	Chan, Jegadeesh and Lakonishok (1996)	1
Mom6m	Momentum (6 month)	Jegadeesh and Titman (1993)	1
Mom12m	Momentum (12 month)	Jegadeesh and Titman (1993)	1
IndMom	Industry Momentum	Grinblatt and Moskowitz (1999)	1
RevenueSurprise	Revenue Surprise	Jegadeesh and Livnat (2006)	1
ChTax	Change in Taxes	Thomas and Zhang (2011)	1
AnalystRevision	EPS forecast revision	Hawkins, Chamberlin, Daniel (1984)	1
NumEarnIncrease	Earnings streak length	Loh and Warachka (2012)	1
High52	52 week high	George and Hwang (2004)	1

Continued on next page

Table C.1 – continued from previous page

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
ResidualMomentum6m	6 month residual momentum	Blitz, Huij and Martens (2011)	1
ResidualMomentum	Momentum based on FF-3 residuals	Blitz, Huij and Martens (2011)	1
retConglomerate	Conglomerate return	Cohen and Lou (2012)	1
BEME	Book-to-market ratio	Rosenberg, Reid, and Lanstein (1985)	2
AM	Total assets to market	Fama and French (1992)	2
LRreversal	Long-run reversal	De Bondt and Thaler (1985)	2
EP	Earnings-to-Price Ratio	Basu (1977)	2
sfe	Earnings Forecast to price	Elgers, Lo and Pfeiffer (2001)	2
CF	Cash flow to market	Lakonishok, Shleifer, Vishny (1994)	2
DivYieldAnn	Last year's dividends over price	Naranjo, Nimalendran, Ryngaert (1998)	2
PayoutYield	Payout Yield	Boudoukh et al. (2007)	2
NetPayoutYield	Net Payout Yield	Boudoukh et al. (2007)	2
MeanRankRevGrowth	Revenue Growth Rank	Lakonishok, Shleifer, Vishny (1994)	2
sgr	Annual sales growth	Lakonishok, Shleifer, Vishny (1994)	2
EntMult	Enterprise Multiple	Loughran and Wellman (2011)	2
SP	Sales-to-price	Barbee, Mukherji and Raines (1996)	2
cfp	Operating Cash flows to price	Desai, Rajgopal, Venkatachalam (2004)	2
IntanBM	Intangible return using BM	Daniel and Titman (2006)	2
IntrinsicValue	Intrinsic or historical value	Frankel and Lee (1998)	2
AnalystValue	Analyst Value	Frankel and Lee (1998)	2
EBM	Enterprise component of BM	Penman, Richardson and Tuna (2007)	2
NetDebtPrice	Net debt to price	Penman, Richardson and Tuna (2007)	2
EquityDuration	Equity Duration	Dechow, Sloan and Soliman (2004)	2
Leverage	Market leverage	Bhandari (1988)	2
Investment	Investment to revenue	Titman, Wei and Xie (2004)	3
AssetGrowth	Asset growth	Cooper, Gulen and Schill (2008)	3
InvestPPEInv	change in ppe and inv/assets	Lyandres, Sun and Zhang (2008)	3
NOA	Net Operating Assets	Hirshleifer et al. (2004)	3
dNoa	change in net operating assets	Hirshleifer, Hou, Teoh, Zhang (2004)	3
GrLTNOA	Growth in long term operating assets	Fairfield, Whisenant and Yohn (2003)	3
grcapx1y	Investment growth (1 year)	Anderson and Garcia-Feijoo (2006)	3
grcapx	Change in capex (two years)	Anderson and Garcia-Feijoo (2006)	3
grcapx3y	Change in capex (three years)	Anderson and Garcia-Feijoo (2006)	3
ShareIss1Y	Share issuance (1 year)	Pontiff and Woodgate (2008)	3

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Table C.1 – continued from previous page

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
ChInvIA	Change in capital inv (ind adj)	Abarbanell and Bushee (1998)	3
CompEquIss	Composite equity issuance	Daniel and Titman (2006)	3
CompositeDebtIssuance	Composite debt issuance	Lyandres, Sun and Zhang (2008)	3
InvGrowth	Inventory Growth	Belo and Lin (2012)	3
ChInv	Inventory Growth	Thomas and Zhang (2002)	3
Accruals	Accruals	Sloan (1996)	3
TotalAccruals	Total accruals	Richardson et al. (2005)	3
DelCOA	Change in current operating assets	Richardson et al. (2005)	3
DelCOL	Change in current operating liabilities	Richardson et al. (2005)	3
ChNNCOA	Change in Net Noncurrent Op Assets	Soliman (2008)	3
ChNCOA	Change in Noncurrent Operating Assets	Soliman (2008)	3
ChNCOL	Change in Noncurrent Operating Liab	Soliman (2008)	3
DelNetFin	Change in net financial assets	Richardson et al. (2005)	3
DelSTI	Change in short-term investment	Richardson et al. (2005)	3
DelLTI	Change in long-term investment	Richardson et al. (2005)	3
DelFINL	Change in financial liabilities	Richardson et al. (2005)	3
DelEqu	Change in equity to assets	Richardson et al. (2005)	3
AbnormalAccruals	Abnormal Accruals	Xie (2001)	3
PctAcc	Percent Operating Accruals	Hafzalla, Lundholm, Van Winkle (2011)	3
AbnormalAccrualsPercent	Percent Abnormal Accruals	Hafzalla, Lundholm, Van Winkle (2011)	3
XFIN	Net external financing	Bradshaw, Richardson, Sloan (2006)	3
NetEquityFinance	Net equity financing	Bradshaw, Richardson, Sloan (2006)	3
NetDebtFinance	Net debt financing	Bradshaw, Richardson, Sloan (2006)	3
RoE	net income / book equity	Haugen and Baker (1996)	4
ChangeRoE	Change in Return on equity	Balakrishnan, Bartov and Faurel (2010)	4
roaq	Return on assets (qtrly)	Balakrishnan, Bartov and Faurel (2010)	4
ChangeRoA	Change in Return on assets	Balakrishnan, Bartov and Faurel (2010)	4
RetNOA	Return on Net Operating Assets	Soliman (2008)	4
PM	Profit Margin	Soliman (2008)	4
AssetTurnover	Asset Turnover	Soliman (2008)	4
CapTurnover	Capital turnover	Haugen and Baker (1996)	4
GP	gross profits / total assets	Novy-Marx (2013)	4
GPlag	gross profits / total assets	Novy-Marx (2013)	4
OperProf	operating profits / book equity	Fama and French (2006)	4

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Table C.1 – continued from previous page

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
OperProfLag	operating profits / book equity	Fama and French (2006)	4
OperProfRD	Operating profitability R&D adjusted	Ball et al. (2016)	4
OperProfRDLagAT	Oper prof R&D adj lagged assets	Ball et al. (2016)	4
CBOperProf	Cash-based operating profitability	Ball et al. (2016)	4
CBOperProfLagAT	Cash-based oper prof lagged assets	Ball et al. (2016)	4
PS	Piotroski F-score	Piotroski (2000)	4
FailureProbability	Failure probability	Campbell, Hilscher and Szilagyi (2008)	4
FailureProbabilityJune	Failure probability	Campbell, Hilscher and Szilagyi (2008)	4
ZScore	Altman Z-Score	Dichev (1998)	4
Tax	Taxable income to income	Lev and Nissim (2004)	4
BookLeverage	Book leverage (annual)	Fama and French (1992)	4
OrgCapNoAdj	Org cap w/o industry adjustment	Eisfeldt and Papanikolaou (2013)	5
OrgCap	Organizational capital	Eisfeldt and Papanikolaou (2013)	5
AdExp	Advertising Expense	Chan, Lakonishok and Sougiannis (2001)	5
GrAdExp	Growth in advertising expenses	Lou (2014)	5
RD	R&D over market cap	Chan, Lakonishok and Sougiannis (2001)	5
rd_sale	R&D to sales	Chan, Lakonishok and Sougiannis (2001)	5
OPLeverage	Operating leverage	Novy-Marx (2011)	5
hire	Employment growth	Bazdresch, Belo and Lin (2014)	5
BrandInvest	Brand capital investment	Belo, Lin and Vitorino (2014)	5
AOP	Analyst Optimism	Frankel and Lee (1998)	5
Herf	Industry concentration (sales)	Hou and Robinson (2006)	5
HerfAsset	Industry concentration (assets)	Hou and Robinson (2006)	5
HerfBE	Industry concentration (equity)	Hou and Robinson (2006)	5
FirmAge	Firm age based on CRSP	Barry and Brown (1984)	5
PriceDelayRsq	Price delay r square	Hou and Moskowitz (2005)	5
PriceDelaySlope	Price delay coeff	Hou and Moskowitz (2005)	5
PriceDelayTstat	Price delay SE adjusted	Hou and Moskowitz (2005)	5
GrSaleToGrInv	Sales growth over inventory growth	Abarbanell and Bushee (1998)	5
GrSaleToGrReceivables	Change in sales vs change in receiv	Abarbanell and Bushee (1998)	5
GrGMToGrSales	Gross margin growth to sales growth	Abarbanell and Bushee (1998)	5
GrSaleToGrOverhead	Sales growth over overhead growth	Abarbanell and Bushee (1998)	5
ETR	Effective Tax Rate	Abarbanell and Bushee (1998)	5
LaborforceEfficiency	Laborforce efficiency	Abarbanell and Bushee (1998)	5

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Table C.1 – continued from previous page

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
nanalyst	Number of analysts	Elgers, Lo and Pfeiffer (2001)	5
tang	Tangibility	Hahn and Lee (2009)	5
realestate	Real estate holdings	Tuzel (2010)	5
KZ	Kaplan Zingales index	Lamont, Polk and Saa-Requejo (2001)	5
WW	Whited-Wu index	Whited and Wu (2006)	5
Cash	Cash to assets	Palazzo (2012)	5
AccrualQualityJune	Accrual Quality in June	Francis, LaFond, Olsson, Schipper (2005)	5
AccrualQuality	Accrual Quality	Francis, LaFond, Olsson, Schipper (2005)	5
OrderBacklog	Order backlog	Rajgopal, Shevlin, Venkatachalam (2003)	5
EarningsPersistence	Earnings persistence	Francis, LaFond, Olsson, Schipper (2004)	5
EarningsPredictability	Earnings Predictability	Francis, LaFond, Olsson, Schipper (2004)	5
EarningsSmoothness	Earnings Smoothness	Francis, LaFond, Olsson, Schipper (2004)	5
EarningsValueRelevance	Value relevance of earnings	Francis, LaFond, Olsson, Schipper (2004)	5
EarningsTimeliness	Earnings timeliness	Francis, LaFond, Olsson, Schipper (2004)	5
EarningsConservatism	Earnings conservatism	Francis, LaFond, Olsson, Schipper (2004)	5
AssetLiquidityBook	Asset liquidity over book assets	Ortiz-Molina and Phillips (2014)	5
AssetLiquidityMarket	Asset liquidity over market	Ortiz-Molina and Phillips (2014)	5
ForecastDispersion	EPS Forecast Dispersion	Diether, Malloy and Scherbina (2002)	5
MomSeasonShort	Return seasonality last year	Heston and Sadka (2008)	5
Mom12mOffSeason	Momentum without the seasonal part	Heston and Sadka (2008)	5
MomSeason	Return seasonality years 2 to 5	Heston and Sadka (2008)	5
MomOffSeason	Off season long-term reversal	Heston and Sadka (2008)	5
MomSeason06YrPlus	Return seasonality years 6 to 10	Heston and Sadka (2008)	5
MomOffSeason06YrPlus	Off season reversal years 6 to 10	Heston and Sadka (2008)	5
MomSeason11YrPlus	Return seasonality years 11 to 15	Heston and Sadka (2008)	5
MomOffSeason11YrPlus	Off season reversal years 11 to 15	Heston and Sadka (2008)	5
MomSeason16YrPlus	Return seasonality years 16 to 20	Heston and Sadka (2008)	5
MomOffSeason16YrPlus	Off season reversal years 16 to 20	Heston and Sadka (2008)	5
BrandCapital	Brand capital to assets	Belo, Lin and Vitorino (2014)	5
size	Market capitalization	Banz (1981)	6
IdioVol3F	Idiosyncratic risk (3 factor)	Ang et al. (2006)	6
IdioVolAHT	Idiosyncratic risk (AHT)	Ali, Hwang, and Trombley (2003)	6
IdioVolCAPM	Idiosyncratic risk (CAPM)	Ang et al. (2006)	6
IdioVolQF	Idiosyncratic risk (q factor)	Ang et al. (2006)	6

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Table C.1 – continued from previous page

(1)	(2)	(3)	(4)
Acronym	Brief Introduction	Literature	Group
Beta	CAPM beta	Fama and MacBeth (1973)	6
BetaFP	Frazzini-Pedersen Beta	Frazzini and Pedersen (2014)	6
BetaDimson	Dimson Beta	Dimson (1979)	6
std_turn	Share turnover volatility	Chordia, Subrahmanyam, Anshuman (2001)	6
DolVol	Past trading volume	Brennan, Chordia, Subrahmanyam (1998)	6
VolSD	Volume Variance	Chordia, Subrahmanyam, Anshuman (2001)	6
Illiquidity	Amihud's illiquidity	Amihud (2002)	6
zerotradeAlt1	Days with zero trades	Liu (2006)	6
zerotrade	Days with zero trades	Liu (2006)	6
zerotradeAlt12	Days with zero trades	Liu (2006)	6
MaxRet	Maximum return over month	Bali, Cakici, and Whitelaw (2011)	6
ReturnSkew	Return skewness	Bali, Engle and Murray (2015)	6
ReturnSkewCAPM	Idiosyncratic skewness (CAPM)	Bali, Engle and Murray (2015)	6
ReturnSkew3F	Idiosyncratic skewness (3F model)	Bali, Engle and Murray (2015)	6
ReturnSkewQF	Idiosyncratic skewness (Q model)	Bali, Engle and Murray (2015)	6
Coskewness	Coskewness	Harvey and Siddique (2000)	6
BetaTailRisk	Tail risk beta	Kelly and Jiang (2014)	6
betaRR	Return-market return illiquidity beta	Acharya and Pedersen (2005)	6
betaCC	Illiquidity-illiquidity beta (beta2i)	Acharya and Pedersen (2005)	6
betaRC	Return-market illiquidity beta	Acharya and Pedersen (2005)	6
betaCR	Illiquidity-market return beta (beta4i)	Acharya and Pedersen (2005)	6
betaNet	Net liquidity beta (betanet,p)	Acharya and Pedersen (2005)	6
SRreversal	Short-run reversal	Jegadeesh (1990)	6
DownsideBeta	Downside beta	Ang, Chen and Xing (2006)	6
BidAskSpread	Bid-ask spread	Amihud and Mendelson (1986)	6
BetaBDLeverage	Broker-Dealer Leverage Beta	Adrian, Etula and Muir (2014)	6
BetaLiquidityPS	Pastor-Stambaugh liquidity beta	Pastor and Stambaugh (2003)	6

Table C.2: Summary Statistics of the 174 Characteristic-based Factors

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
EarningsSurprise	1	636	0.1630	0.1815	0.5041	1.1198	-2.21	22.72
AnnouncementReturn	1	617	0.3120	0.3414	0.6018	1.7962	0.40	8.91
REV6	1	556	0.2128	0.2695	0.8994	0.8198	-1.52	11.27
Mom6m	1	636	0.3227	0.4174	1.7675	0.6324	-1.29	17.98
Mom12m	1	636	0.3953	0.4801	1.7533	0.7811	-0.87	12.53
IndMom	1	636	0.2179	0.2267	1.3780	0.5478	0.31	12.20
RevenueSurprise	1	636	0.1065	0.1146	0.5044	0.7317	-1.23	11.30
ChTax	1	636	0.1808	0.1683	0.6125	1.0226	-0.39	7.77
AnalystRevision	1	562	0.1191	0.1442	0.3780	1.0916	-2.86	34.14
NumEarnIncrease	1	636	0.2017	0.1861	0.6139	1.1379	-0.09	7.69
High52	1	636	0.3179	0.5034	2.2666	0.4858	-1.17	12.31
ResidualMomentum6m	1	636	0.1578	0.1777	0.8878	0.6156	-0.26	8.59
ResidualMomentum	1	636	0.2582	0.2539	0.9394	0.9521	-0.09	7.75
retConglomerate	1	563	0.4358	0.3133	1.5887	0.9503	1.45	15.88
BEME	2	636	0.2151	0.1689	1.2614	0.5908	0.38	5.57
AM	2	636	0.0935	0.0678	1.3674	0.2368	-0.35	7.32
LRreversal	2	636	-0.0923	0.0184	1.0946	-0.2920	-1.07	7.30
EP	2	636	0.1405	0.1056	1.1772	0.4133	-0.08	6.82
sfe	2	561	0.0796	0.0345	1.6792	0.1641	-0.49	10.50
CF	2	636	0.2132	0.1528	1.4790	0.4993	-0.32	9.52
DivYieldAnn	2	636	0.1629	0.1077	1.5202	0.3712	-0.05	5.63
PayoutYield	2	636	0.1077	0.0606	0.8281	0.4503	0.74	6.98
NetPayoutYield	2	636	0.1994	0.1578	1.0104	0.6835	-0.04	5.74
MeanRankRevGrowth	2	636	-0.0026	0.0285	0.6347	-0.0144	-0.39	4.00
sgr	2	636	-0.1805	-0.1843	0.7305	-0.8558	0.08	4.50
EntMult	2	636	-0.1247	-0.1184	0.8821	-0.4896	0.63	8.74
SP	2	636	0.1680	0.0923	1.2887	0.4517	1.30	14.66
cfp	2	636	0.2058	0.1845	1.2682	0.5621	0.46	9.71

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
IntanBM	2	636	-0.0474	0.0233	1.2701	-0.1292	-0.29	10.99
IntrinsicValue	2	558	0.0656	0.0199	1.3900	0.1635	-0.42	9.75
AnalystValue	2	558	0.0739	-0.0245	1.3359	0.1917	0.23	9.92
EBM	2	636	0.0628	0.0542	0.4279	0.5081	-0.19	6.40
NetDebtPrice	2	636	0.0100	-0.0530	1.2269	0.0281	-0.54	10.72
EquityDuration	2	636	-0.2238	-0.2003	1.3266	-0.5843	0.52	7.38
Leverage	2	636	0.0795	0.0495	1.2998	0.2119	-0.34	6.85
Investment	3	636	-0.0546	-0.0505	0.4743	-0.3991	-0.29	6.71
AssetGrowth	3	636	-0.2657	-0.2343	0.7556	-1.2181	-0.28	5.26
InvestPPEInv	3	636	-0.2004	-0.2008	0.7757	-0.8948	0.23	4.50
NOA	3	636	-0.1624	-0.1535	0.8733	-0.6441	-0.06	5.12
dNoa	3	636	-0.2406	-0.2067	0.6842	-1.2183	0.05	4.90
GrLTNOA	3	636	0.0331	0.0279	0.5931	0.1933	0.50	6.24
grcapx1y	3	636	-0.0374	-0.0596	0.4125	-0.3142	0.81	8.32
grcapx	3	636	-0.1194	-0.1167	0.4809	-0.8602	-0.07	5.42
grcapx3y	3	636	-0.1112	-0.0906	0.4573	-0.8422	-0.38	4.89
ShareIss1Y	3	636	-0.2447	-0.2114	0.7614	-1.1133	0.13	11.15
ChInvIA	3	636	-0.0575	-0.0607	0.5523	-0.3605	-0.21	6.63
CompEquIss	3	636	-0.1199	-0.0938	0.7693	-0.5400	-1.18	11.37
CompositeDebtIssuance	3	636	-0.0488	-0.0717	0.4284	-0.3949	0.25	4.92
InvGrowth	3	636	-0.1160	-0.1434	0.6795	-0.5915	0.06	6.03
ChInv	3	636	-0.1333	-0.1355	0.5796	-0.7965	0.10	4.88
Accruals	3	636	-0.0961	-0.1005	0.6155	-0.5411	-0.05	5.36
TotalAccruals	3	636	-0.0625	-0.0218	0.6257	-0.3458	-1.07	8.75
DelCOA	3	636	-0.1380	-0.0965	0.7309	-0.6542	0.07	3.90
DelCOL	3	636	-0.0915	-0.0901	0.7026	-0.4510	0.00	4.37
ChNNCOA	3	636	-0.0952	-0.0922	0.4243	-0.7775	0.92	10.25
ChNCOA	3	636	-0.2135	-0.1810	0.5625	-1.3150	-0.81	6.88

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
ChNCOL	3	636	-0.1338	-0.1209	0.4932	-0.9395	-0.08	5.18
DelNetFin	3	636	0.1039	0.1169	0.4645	0.7751	-0.47	6.10
DelSTI	3	636	-0.0186	0.0218	0.4535	-0.1418	-0.25	7.13
DelLTI	3	636	-0.0311	-0.0349	0.3748	-0.2873	-0.90	8.29
DelFINL	3	636	-0.1577	-0.1879	0.4594	-1.1892	0.68	6.75
DelEqu	3	636	-0.1733	-0.1188	0.8753	-0.6859	-0.38	5.36
AbnormalAccruals	3	612	-0.0682	-0.0737	0.3955	-0.5970	-0.80	7.25
PctAcc	3	636	-0.0668	-0.0792	0.4840	-0.4778	-0.02	5.13
AbnormalAccrualsPercent	3	612	-0.0681	-0.0787	0.2964	-0.7958	0.75	7.88
XFIN	3	612	-0.3279	-0.2560	1.2467	-0.9110	-0.30	6.93
NetEquityFinance	3	612	-0.1873	-0.1501	1.0923	-0.5942	0.45	8.07
NetDebtFinance	3	612	-0.1177	-0.1212	0.5099	-0.7995	-0.84	15.01
RoE	4	636	0.1170	0.1846	0.9932	0.4079	-1.02	8.91
ChangeRoE	4	636	0.1300	0.1332	0.5502	0.8187	-1.55	20.64
roaq	4	636	0.2359	0.2899	1.4443	0.5658	-1.08	10.54
ChangeRoA	4	636	0.1267	0.1556	0.6463	0.6793	-1.05	10.81
RetNOA	4	636	0.0105	0.0290	0.6053	0.0600	-1.07	9.68
PM	4	636	0.1495	0.1954	1.1096	0.4666	-1.07	8.03
AssetTurnover	4	636	0.0794	0.0645	0.6254	0.4396	0.57	6.17
CapTurnover	4	636	0.0380	0.0273	0.8302	0.1586	0.14	3.44
GP	4	636	0.1508	0.1237	0.8775	0.5953	-0.02	3.35
GPlag	4	636	0.0594	0.0731	0.9499	0.2168	0.14	3.54
OperProf	4	636	0.1324	0.1201	0.8892	0.5159	0.39	8.30
OperProfLag	4	636	0.0650	0.0417	0.8094	0.2781	-0.35	8.65
OperProfRD	4	636	0.2294	0.2328	1.0079	0.7885	0.23	6.41
OperProfRDLagAT	4	636	0.1340	0.1240	0.9113	0.5095	0.14	3.94
CBOperProf	4	636	0.2744	0.2813	0.8960	1.0610	0.19	8.63
CBOperProfLagAT	4	636	0.2068	0.2254	0.8080	0.8867	-0.01	5.17

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
PS	4	612	-0.0203	0.0107	0.8218	-0.0857	-0.61	7.55
FailureProbability	4	609	-0.1056	-0.1197	2.0848	-0.1754	1.68	19.96
FailureProbabilityJune	4	606	-0.1801	-0.0994	1.8393	-0.3392	-1.20	27.21
ZScore	4	636	-0.0897	-0.0847	0.9258	-0.3354	1.72	23.01
Tax	4	636	0.1146	0.1212	0.6961	0.5703	-0.83	12.51
BookLeverage	4	636	0.0187	-0.0250	1.0895	0.0596	-0.27	7.09
OrgCapNoAdj	5	636	0.0840	0.0791	1.0610	0.2741	0.91	10.95
OrgCap	5	636	0.0890	0.0568	0.6947	0.4436	1.54	17.94
AdExp	5	636	0.0250	-0.0046	0.8909	0.0974	1.36	15.53
GrAdExp	5	636	-0.0693	-0.0295	0.6021	-0.3986	-0.42	5.49
RD	5	636	0.1851	0.0524	1.3398	0.4787	1.43	9.43
rd_sale	5	636	-0.0081	-0.1131	1.2182	-0.0230	2.04	20.31
OPLeverage	5	636	0.1004	0.0565	0.8084	0.4303	0.34	4.49
hire	5	636	-0.1694	-0.1466	0.7252	-0.8094	-0.29	4.82
BrandInvest	5	636	0.0524	0.0671	1.0146	0.1789	-0.61	9.85
AOP	5	558	-0.0220	-0.0179	0.6527	-0.1168	0.14	5.39
Herf	5	636	-0.0022	-0.0001	0.7428	-0.0103	-0.32	14.76
HerfAsset	5	636	0.0127	0.0129	0.7925	0.0556	-0.56	14.51
HerfBE	5	636	-0.0218	-0.0364	0.6438	-0.1174	-0.66	13.59
FirmAge	5	636	0.1116	0.1275	1.1617	0.3327	-0.02	7.81
PriceDelayRsq	5	636	-0.0455	0.0467	1.1067	-0.1423	-0.21	4.47
PriceDelaySlope	5	636	-0.0119	-0.0040	0.4871	-0.0844	-0.12	7.56
PriceDelayTstat	5	636	0.0203	0.0483	0.6781	0.1038	0.28	7.54
GrSaleToGrInv	5	636	0.0624	0.0617	0.4326	0.4995	0.29	6.94
GrSaleToGrReceivables	5	636	0.0270	0.0495	0.3692	0.2532	-0.03	4.99
GrGMToGrSales	5	636	0.0721	0.0820	0.4421	0.5652	-0.48	4.95
GrSaleToGrOverhead	5	636	-0.0558	-0.0316	0.5397	-0.3580	-0.28	5.78
ETR	5	636	-0.0238	-0.0381	0.3762	-0.2191	0.43	5.65

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
LaborforceEfficiency	5	636	-0.0665	-0.0712	0.4596	-0.5015	0.25	5.32
nanalyst	5	563	0.0269	0.0394	0.9758	0.0953	-0.12	5.87
tang	5	636	0.0712	0.0464	0.9318	0.2648	1.36	15.88
realestate	5	636	0.0238	0.0590	0.5000	0.1651	-0.36	5.06
KZ	5	636	-0.0110	-0.0378	0.6031	-0.0630	0.27	4.35
WW	5	636	-0.1279	-0.1278	1.7855	-0.2481	0.47	8.58
Cash	5	615	0.0593	0.0877	1.6428	0.1251	1.31	17.61
AccrualQualityJune	5	636	-0.0835	-0.0828	1.1861	-0.2437	0.39	6.34
AccrualQuality	5	636	-0.0843	-0.0912	1.1382	-0.2567	0.24	5.48
OrderBacklog	5	624	-0.0033	0.0068	0.6508	-0.0177	-0.09	3.64
EarningsPersistence	5	636	0.0131	0.0133	0.6623	0.0684	-0.05	5.72
EarningsPredictability	5	636	-0.0987	-0.1092	0.4448	-0.7689	0.02	6.05
EarningsSmoothness	5	636	-0.0123	0.0153	1.3704	-0.0311	0.60	9.73
EarningsValueRelevance	5	636	0.0525	0.0514	0.4723	0.3853	-0.23	4.71
EarningsTimeliness	5	636	0.0131	0.0154	0.3668	0.1237	0.11	5.78
EarningsConservatism	5	636	-0.0121	-0.0245	0.2535	-0.1650	0.71	19.19
AssetLiquidityBook	5	636	-0.0089	-0.0236	1.3625	-0.0225	1.14	14.73
AssetLiquidityMarket	5	636	0.2482	0.1416	1.1420	0.7529	0.76	7.98
ForecastDispersion	5	563	-0.1320	-0.1501	0.8288	-0.5517	0.59	6.57
MomSeasonShort	5	636	0.1595	0.1557	1.0570	0.5226	-0.26	12.03
Mom12mOffSeason	5	636	0.3588	0.4407	1.8961	0.6555	-1.23	19.43
MomSeason	5	636	0.1076	0.1317	0.7293	0.5110	0.24	10.19
MomOffSeason	5	636	-0.2300	-0.1489	1.1301	-0.7051	-0.78	8.67
MomSeason06YrPlus	5	636	0.1431	0.1132	0.6131	0.8087	0.13	10.41
MomOffSeason06YrPlus	5	636	-0.1280	-0.1399	0.7034	-0.6301	-0.02	5.19
MomSeason11YrPlus	5	636	0.1135	0.0962	0.5406	0.7271	0.22	6.98
MomOffSeason11YrPlus	5	636	-0.0413	-0.0458	0.6063	-0.2357	-0.84	7.32
MomSeason16YrPlus	5	636	0.1225	0.0968	0.6002	0.7072	0.11	7.60

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
MomOffSeason16YrPlus	5	636	-0.0364	-0.0163	0.6878	-0.1832	-0.05	4.20
BrandCapital	5	636	0.0026	-0.0119	0.6267	0.0145	0.17	4.41
size	6	636	0.0328	0.1547	1.3425	0.0846	-0.78	6.04
IdioVol3F	6	636	-0.4312	-0.5553	2.1414	-0.6975	0.82	9.67
IdioVolAHT	6	636	-0.3460	-0.4094	2.4833	-0.4827	0.58	8.16
IdioVolCAPM	6	636	-0.4425	-0.5461	2.1826	-0.7024	0.82	9.91
IdioVolQF	6	601	-0.4208	-0.5321	2.0643	-0.7061	0.84	10.72
Beta	6	636	-0.0724	-0.0804	2.4182	-0.1037	0.72	8.69
BetaFP	6	636	-0.1169	-0.2132	2.2254	-0.1819	0.52	6.41
BetaDimson	6	636	-0.0791	-0.1078	1.4706	-0.1862	0.36	10.80
std_turn	6	636	-0.1996	-0.2019	1.6874	-0.4097	0.56	9.01
DolVol	6	636	-0.0187	-0.0407	1.2416	-0.0521	0.13	5.07
VolSD	6	636	-0.1062	-0.1484	0.9751	-0.3774	0.72	8.30
Illiquidity	6	636	-0.0107	-0.0288	0.8829	-0.0421	0.36	5.21
zerotradeAlt1	6	636	0.0355	0.0062	0.9540	0.1289	0.14	6.32
zerotrade	6	636	0.0688	0.0534	0.9885	0.2412	0.10	5.73
zerotradeAlt12	6	636	0.0723	0.0838	0.9529	0.2627	-0.13	4.56
MaxRet	6	636	-0.4518	-0.5123	1.8560	-0.8434	0.78	11.42
ReturnSkew	6	636	-0.0747	-0.0471	0.5638	-0.4588	0.75	18.23
ReturnSkewCAPM	6	636	-0.0451	-0.0044	0.5468	-0.2856	0.83	23.21
ReturnSkew3F	6	636	-0.0173	0.0043	0.5045	-0.1185	0.38	24.25
ReturnSkewQF	6	601	-0.0204	0.0009	0.4709	-0.1498	0.29	24.27
Coskewness	6	636	-0.1621	-0.1239	0.8353	-0.6723	-0.20	16.93
BetaTailRisk	6	636	0.0881	0.0114	1.3048	0.2338	0.72	6.46
betaRR	6	636	-0.0455	-0.0608	1.9717	-0.0800	0.51	7.40
betaCC	6	636	-0.0043	-0.0068	0.7623	-0.0196	0.15	5.36
betaRC	6	636	0.0370	0.0652	1.3945	0.0919	-0.99	9.11
betaCR	6	636	-0.0159	-0.0139	0.5456	-0.1007	0.32	6.09

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Table C.2 – continued from previous page

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acronym	Group	Nobs	Mean (%)	Median (%)	Std (%)	SR	Skew	Kurt
betaNet	6	636	0.0058	-0.0065	0.7229	0.0279	-0.05	5.24
SRreversal	6	636	-0.2778	-0.2050	1.6113	-0.5972	-0.56	15.58
DownsideBeta	6	636	-0.0954	-0.0979	2.2367	-0.1477	0.14	7.56
BidAskSpread	6	636	-0.2728	-0.3043	1.8870	-0.5007	0.89	10.43
BetaBDLeverage	6	596	0.0633	0.0390	0.8189	0.2679	0.74	11.46
BetaLiquidityPS	6	636	0.0314	0.0600	0.6774	0.1605	-0.64	8.41

Statistics reported in Table C.2 are computed using available monthly observations from January 1970 to December 2022. Means, medians, and standard deviations are expressed as monthly percentages, while Sharpe ratios are annualized. Nobs denotes the number of available monthly observations.

C.2 Risk-adjusted Performance of REP and Pairwise Alpha Tests

This subsection performs additional analysis on the OOS performance of REP. We begin by examining the risk-adjusted performance of REP relative to eight prominent factor models in the asset pricing literature. These include the CAPM (Sharpe, 1964; Lintner, 1965), the Fama-French three-factor (FF-3), five-factor (FF-5), and six-factor (FF-6) models (Fama and French, 1993, 2015, 2018), the four-factor model of Carhart (1997) (Carhart-4), the q-factor model augmented with an expected investment growth factor from Hou et al. (2021) (HXZ-q5), the mispricing factor model of Stambaugh and Yuan (2017) (SY-4), and the behavioral factor model of Daniel et al. (2020) (DHS-3). Additionally, we combine all unique factors from these eight models into one composite benchmark, labeled "ALL".

Table C.3 reports the risk-adjusted OOS performance of REP. Across all benchmark models, REP delivers positive and statistically significant alphas, with significance at the 1% level or better. These results indicate that the superior performance of REP is not captured by existing factor

models. The ALL model yields the highest explanatory power, with an R^2 of 16.63%, followed by the HXZ-q5 model with an R^2 of 10.62%. In contrast, traditional factor models based on the market, size, value, and momentum explain only a small fraction of the variation in returns of REP.

We next compare the MVE portfolios estimated by the approaches reported in Table 2. Specifically, we run pairwise univariate OLS regressions of monthly excess returns on the MVE portfolio constructed by one method against those of another over the OOS period from January 1980 to December 2022.[16] Table C.4 reports the regression intercepts ("alphas"), which measure abnormal performance of one MVE portfolio relative to another, along with Newey–West t-statistics, which are proportional to the information ratio of the test portfolio relative to the benchmark (Bessembinder et al., 2022). The methods listed along rows correspond to test portfolios, while those along columns serve as benchmark models.

The first row of Table C.4 shows that, when treated as the test portfolio, REP delivers large positive alphas relative to all benchmark models, with t-statistics exceeding 4.9, consistent with its highest OOS Sharpe ratio reported in Table 2. Conversely, when REP is used as the benchmark (the first column), only five alternative strategies yield t-statistics above 3. Moreover, although some strategies earn statistically significant alphas relative to REP, the economic magnitudes are much smaller—typically only a fraction of the alphas earned by REP against them. For example, the alpha of REP relative to MV-PC-40 is 3.6%, whereas the reverse alpha is only 1.7%.

Given that some strategies earn significant alphas relative to REP, a natural question is how much the Sharpe ratio of REP can be improved by optimally combining it with such strategies. Following Treynor and Black (1973), we compute the implied maximum Sharpe ratio based on pairwise univariate regressions. Specifically, the implied maximum Sharpe ratio is given by

$$\max SR_{\text{Implied}} = \sqrt{SR_{\text{REP}}^2 + IR_{\text{Benchmark}}^2}, \tag{C.1}$$

where SR_{REP} denotes the OOS Sharpe ratio of REP, and $IR_{\text{Benchmark}}$ is the information ratio of

[16] Due to space constraints, we omit a few strategies—Market, $1/N$, and MV-PC-3. The results are robust to their inclusion.

the benchmark relative to REP, defined as

$$IR_{\text{Benchmark}} = \frac{\alpha_{\text{Benchmark}}}{\sigma(\epsilon)}, \tag{C.2}$$

with $\alpha_{\text{Benchmark}}$ and $\sigma(\epsilon)$ obtained from the regression

$$r_{\text{Benchmark}} = \alpha_{\text{Benchmark}} + \beta_{\text{Benchmark}} r_{\text{REP}} + \epsilon. \tag{C.3}$$

Because this calculation uses information from the entire OOS period, the maximum Sharpe ratio should be interpreted as an ex-post measure.

Table C.5 reports the results, with each row corresponding to one benchmark method. Combining REP with alternative benchmarks yields only modest gains, with Sharpe ratio improvements below 6%. The largest increase, 5.98%, arises when combining REP with the MVE portfolio based on 40 PCs. In contrast, the implied improvements relative to the benchmark strategies themselves are substantially larger.

We further examine whether an ex-ante combination strategy that aggregates all MVE portfolios reported in Table C.4 can deliver superior performance. In contrast to the preceding analysis, we implement a purely ex-ante approach by forming a simple equal-weighted portfolio of all strategies. The resulting combination achieves an OOS Sharpe ratio of 3.47 (unreported), slightly below that of REP. This evidence suggests that simple model averaging does not yield incremental gains and that REP already captures a large share of attainable mean–variance efficiency. More sophisticated ex-ante combination schemes may yield further improvements, which we leave for future research.

C.3 Properties of Individual Stock Weights

Table C.6 reports the summary statistics of the estimated stock-level weights, $\widehat{\boldsymbol{w}}_t$, implied under each approach, which are computed by substituting the estimated factor weights, $\widehat{\boldsymbol{\theta}}_t \equiv \left(\widehat{\theta}_{m,t}, \widehat{\boldsymbol{\theta}}'_{c,t}\right)'$, into Equation (18). Following Brandt et al. (2009), we report several statistics of $\widehat{\boldsymbol{w}}_t$ to summarize

the properties of the stock-level weights. These statistics are computed for each cross-section and then averaged over time.

Column two reports the average absolute position per stock, $\sum_i |w_i|/N_t$, which measures sparsity at the stock level. REP has an average absolute position of 0.17% per stock, comparable to MAXSER and the MVE portfolio based on either the 40 PCs or the 10 RP-PCA factors (all around 0.15%), and markedly lower than that of the plug-in Markowitz rule and the MVE portfolios using shrinkage-based covariance estimators. The last four columns show that REP holds average aggregate long and short positions of 4.22 and –3.71, respectively, with 63.06% of stocks held long and 36.94% short. The resulting net exposure is 0.51, higher than MAXSER's 0.20. Aside from this difference, the weight statistics of REP broadly resemble those of MAXSER. Overall, REP delivers well-behaved weights and a degree of sparsity in line with benchmarks that exhibit strong OOS performance in Table 2.

C.4 Simulation Analysis

We conduct a simulation analysis to evaluate the ability of REP to achieve mean-variance efficiency and compare its performance with other benchmark approaches considered in our empirical analysis. The simulation design closely follows Ao et al. (2019), who simulate i.i.d. observations of monthly excess returns for $N = 100$ stocks based on the following data-generating process

$$\boldsymbol{r}_t = \boldsymbol{\alpha} + \boldsymbol{B}\boldsymbol{F}_t + \boldsymbol{\epsilon}_t, \tag{C.4}$$

where $\boldsymbol{F}_t$ is a 3×1 vector of time-t excess returns of the factors, $\boldsymbol{B}$ is the 100×3 matrix of loadings of the 100 stocks on the three factors, and $\boldsymbol{\alpha}$ is a 100×1 vector representing the non-zero alphas of the stocks with respect to the factors. The parameters of the data-generating process are calibrated from real data. Specifically, the 100 stocks are randomly sampled from the S&P 500 index constituents, and the three factors are the factors from the FF-3 model. Both $\boldsymbol{F}_t$ and $\boldsymbol{\epsilon}_t$ are assumed to follow multivariate normal distributions and are independent of each other. The

calibrated quantities include $\boldsymbol{\alpha}$ and $\boldsymbol{B}$ for the 100 assets with respect to the FF-3 model, the mean $\boldsymbol{\mu_F}$ and covariance matrix $\boldsymbol{\Sigma_F}$ of the FF-3 factors, and the covariance matrix $\boldsymbol{\Sigma_\epsilon}$ of the error terms. For details on the calibration, refer to Ao et al. (2019, p. 2910). The theoretical maximum Sharpe ratio of the 100 stocks is 1.5217 (annualized).

Next, the returns for the 100 stocks are simulated using the data-generating process in equation (C.4). For each sample size T, we simulate $T + 1$ observations and estimate the portfolio weights based on T observations. The remaining observation is used to compute the OOS return of the strategy. This process is repeated $S = 10,000$ times, yielding a series of 10,000 OOS returns for each strategy. We evaluate the performance of all portfolio strategies based on the distribution of their OOS returns. To assess statistical significance, we test the differences in Sharpe ratios between REP and the other benchmark portfolios using the tests proposed by Jobson and Korkie (1981) and Memmel (2003). Following Ao et al. (2019), we consider sample sizes of $T = 120$ and $T = 240$, and also include $T = 80$ to evaluate the performance of REP in a high-dimensional setting where the number of assets exceeds the sample size.

The simulation results, presented in Table C.7, demonstrate that REP outperforms nearly all benchmark strategies in terms of Sharpe ratios across all sample sizes, with the exception of the MV rule based on 5 RP-PCA factors (MV-RP-PC-5) when $T = 80$. Specifically, when $T = 80$ months, REP generates an annualized Sharpe ratio of 0.6475, slightly underperforming the MV-RP-PC-5 strategy, which has a Sharpe ratio of 0.6482. Nonetheless, except for the two RP-PCA strategies, we can significantly reject the null hypothesis that the Sharpe ratio of REP is less than or equal to that of other competing rules at the 5% level or better. More importantly, REP achieves the best performance when $T = 120$ or $T = 240$ months, with the Sharpe ratio test strongly rejecting the null hypothesis that the OOS Sharpe ratio of REP is less than or equal to that of any benchmark portfolios at least at 1% level.

Regarding benchmark approaches, we observe that MV-RP-PC-5 (MAXSER) achieves the second-best performance when $T = 120$ (240) months. The JKBJ portfolio underperforms all other methods, including the MV-Sample, and even delivers a negative OOS Sharpe ratio when

$T = 120$. This underperformance is primarily attributable to the additional weight normalization step in the JKBJ procedure. In untabulated results, we show that removing this normalization step significantly improves performance, bringing the JKBJ portfolio much closer to MV-Sample. This finding highlights the detrimental impact of weight normalization on OOS performance, consistent with Ao et al. (2019).

Among the portfolio rules in the second category, the plug-in mean-variance portfolio (MV-Sample) performs the worst, while MV-NLS-FF-3 performs the best. Both the nonlinear shrinkage portfolio (MV-NLS) and the Bayesian shrinkage rule of Kozak et al. (2020) outperform the linear shrinkage rule MV-LS. It is also reasonable that MV-NLS-FF-3 performs better than MV-NLS, as it incorporates additional factor structure information from the FF-3 model. For the portfolio rules in the third category, we observe that as the sample size increases, the MV rules with more standard principal components (PCs) tend to outperform those with fewer PCs. However, the reverse is true for RP-PCA strategies. Additionally, we find that RP-PCA is more efficient at extracting latent pricing factors than standard PCA, as its OOS performance consistently dominates that of standard PCA across all sample sizes, consistent with the findings of Lettau and Pelger (2020). Notably, the use of just five latent factors extracted by RP-PCA outperforms the 40 factors extracted by standard PCA.

In summary, Table C.7 shows that the REP rule achieves the best performance, outperforming all other competing rules in terms of OOS Sharpe ratio, with only one exception.

C.5 Nonlinear Characteristic Factors

This subsection presents a simple extension of our baseline analysis. In addition to the 174 characteristic factors used in the baseline results, we incorporate an additional 174 characteristics-based factors constructed from quadratic transformations of the same characteristics, yielding a total of 348 characteristic factors plus the market. As before, the characteristic weights used to construct each factor are cross-sectionally standardized to have zero mean and unit variance each month. We then re-examine the OOS performance of the MVE portfolio estimated by REP and

the competing benchmark approaches based on this expanded factor set. Table C.8 shows that REP continues to deliver the highest OOS Sharpe ratio, which increases from 3.59 to 3.84 when the nonlinear characteristic factors are included. This result suggests that allowing for nonlinear transformations of characteristics can further enhance the mean-variance efficiency of the estimated pricing kernel, consistent with the insights of prior literature (Gu et al., 2020; Kozak et al., 2020; Chen, Pelger, and Zhu, 2024).

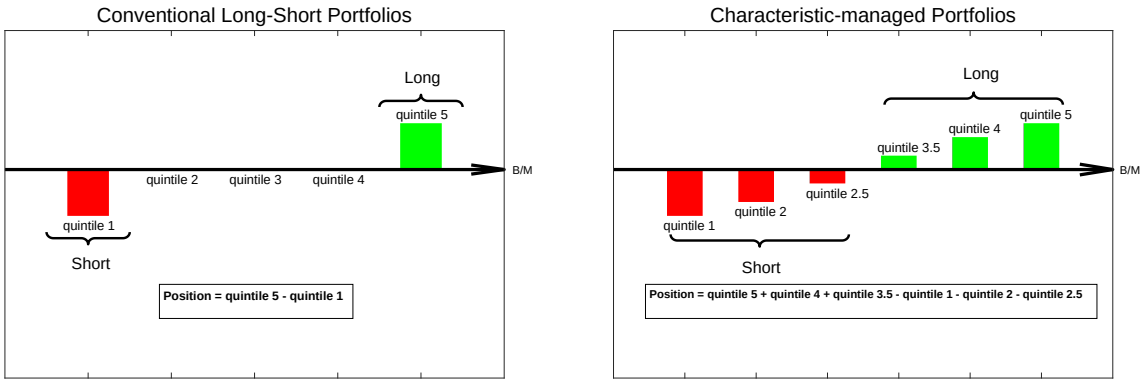
C.6 Real-time REP

This subsection examines how the performance of the estimated MVE factor portfolios changes once *inclusion bias* is taken into account. Using the publication dates of the 174 characteristics from Chen and Zimmermann (2022), we reconstruct the MVE factor portfolio based only on characteristics that had already been published by each year, rather than the full set of factors that become available ex post at the end of the sample. All other settings follow those in Table 2. This design reflects a more realistic investment opportunity set faced by typical investors. Table C.9 reports the results.

We find that OOS Sharpe ratios decline for most methods. Excluding simple benchmarks (Market, $1/N$, JKBJ, and EW-CM), the average reduction in Sharpe ratios relative to the baseline results in Table 2 is 28%, while the decline for REP is slightly larger at 31%. This sizable decline reflects the smaller real-time investment universe—particularly during the early OOS period (1980–2000), when fewer than 45 characteristics were available—and the associated reduction in the maximum attainable Sharpe ratio. Despite this adjustment, REP achieves the highest OOS Sharpe ratio of 2.47 and significantly outperforms most competing benchmarks at the 5% level. Its performance advantage relative to other methods narrows, as expected, given the limited dimensionality of the investment universe in the early part of the sample.

These findings contribute to the anomaly literature. While many anomalies weaken or lose statistical significance when evaluated individually using post-publication data (McLean and Pontiff, 2016), we show that jointly exploiting the information contained in all available published char-

acteristics continues to generate substantial gains in mean–variance efficiency. This highlights the value of academic anomaly research when factors are evaluated collectively rather than in isolation (Jensen, Kelly, and Pedersen, 2023).



(a) B/M Long-short Portfolio

(b) B/M-managed Portfolio

Figure C.1: **Difference between long-short portfolio and characteristic-managed portfolio**

Figure C.1 visually illustrates the difference between the CM portfolios and conventional long-short portfolios. Panel (a) of Figure C.1 shows a long-short portfolio sorted by book-to-market ratios, where stocks in the top quintile are held long and those in the bottom quintile are held short. Panel (b) demonstrates that a B/M-managed portfolio is equivalent to going long on stocks with book-to-market ratios above the mean of the cross-section and shorting those below the mean. The positions in individual stocks are given by their cross-sectional standardized book-to-market ratios.

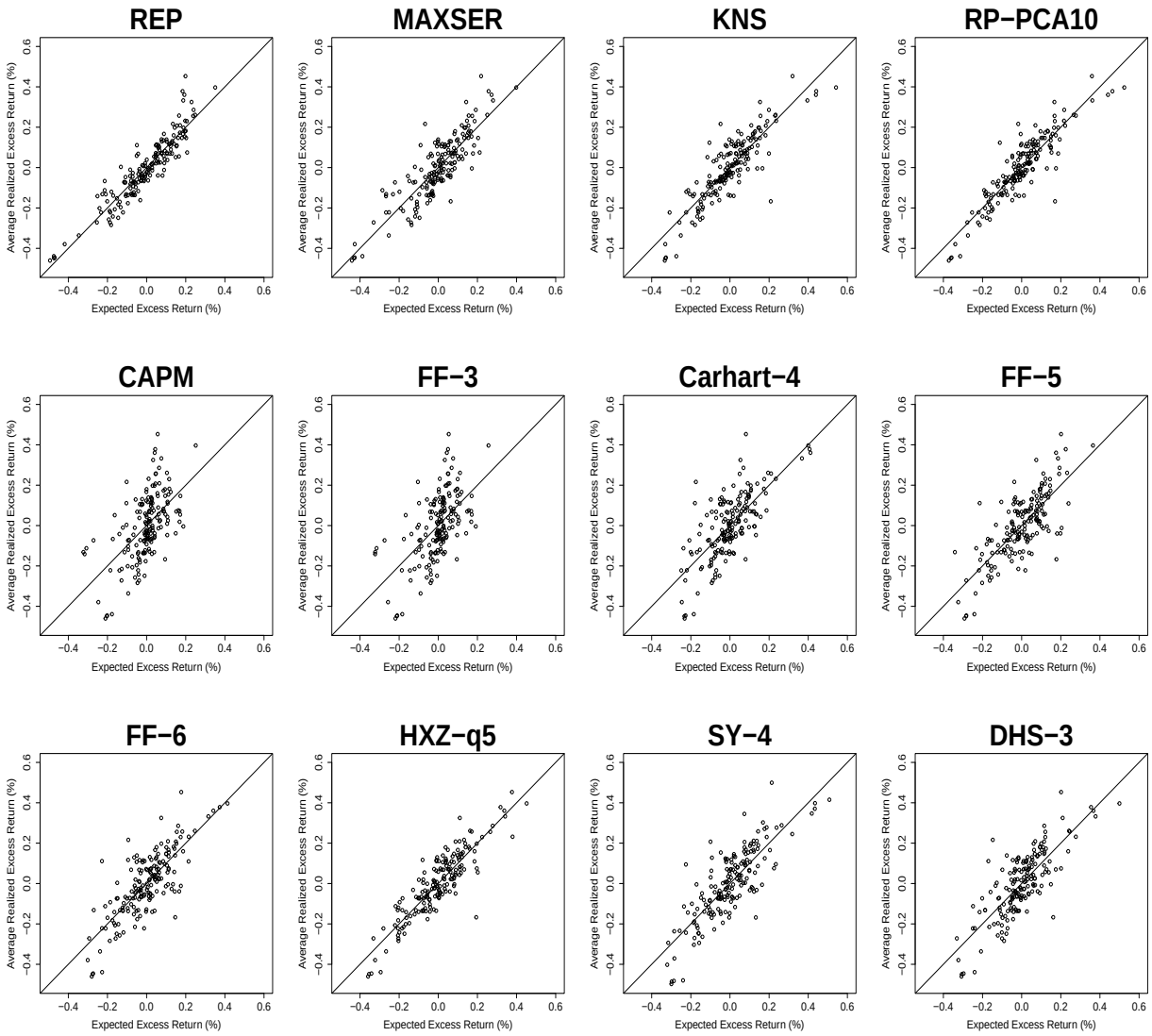


Figure C.2: **Pricing the 174 Characteristic-based Factors**

Figure C.2 illustrates the OOS performance of the MVE portfolios estimated by REP, MAXSER, and KNS, as well as that of the 10-factor RP-PCA model and eight prominent factor models, in pricing the 174 characteristic factors. The eight factor models include the CAPM (Sharpe, 1964; Lintner, 1965), the Fama-French three-factor (FF-3), five-factor (FF-5), and six-factor (FF-6) models (Fama and French, 1993, 2015, 2018), the four-factor model of Carhart (1997) (Carhart-4), the q-factor model of Hou et al. (2021) (HXZ-q5), the mispricing factor model of Stambaugh and Yuan (2017) (SY-4), and the behavioral factor model of Daniel et al. (2020) (DHS-3). In each subfigure, the vertical axis plots the average realized returns of the 174 factors, while the horizontal axis shows their expected returns as implied by each model. The full sample spans January 1980 to December 2022. The balanced sample is constructed by excluding months in which any of the 174 characteristic factors or the factors from the pricing model have missing return data.

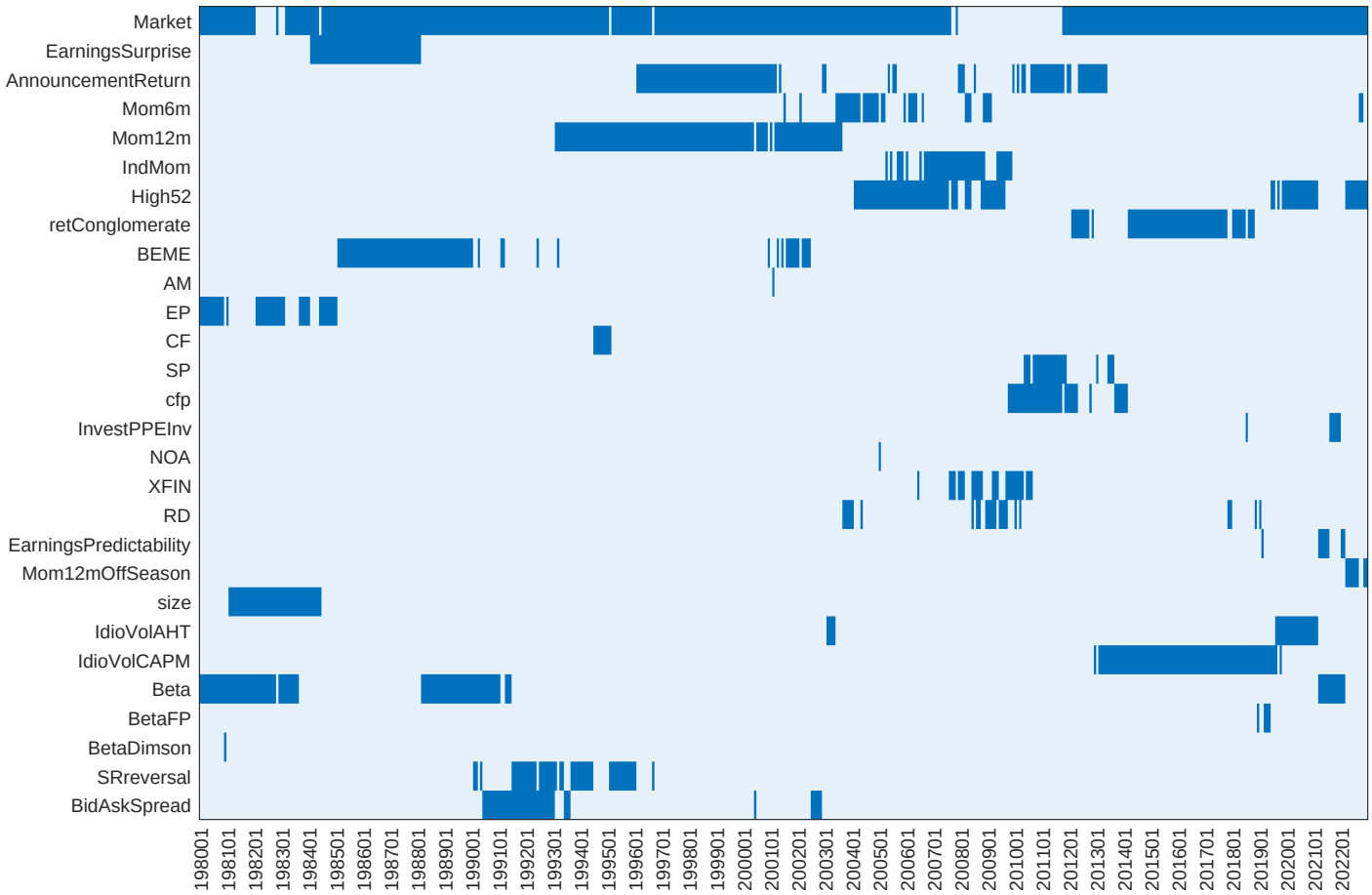


Figure C.3: **A Real-time Three-Factor Model**

Figure C.3 presents the constituent factors selected for the real-time three-factor model. The real-time model is constructed using REP to select the top three factors from a pool consisting of the market factor and characteristic-based factors published by the year of the portfolio formation date. The sample period spans from January 1980 to December 2022.

Table C.3: **Risk-adjusted OOS Performance of REP**

Table C.3 reports the time series regression of the OOS return of the REP rule with respect to nine factor models, including the CAPM of Sharpe (1964) (CAPM), the Fama-French three- (FF-3), five- (FF-5), and six-factor (FF-6) models (Fama and French, 1993, 2015, 2018), the FF-3 model augmented by the momentum factor (Carhart, 1997) (Carhart-4), the augmented q-factor model of Hou et al. (2021) (HXZ-q5), the mispricing factor model of Stambaugh and Yuan (2017) (SY-4), the behavioral factor model of Daniel et al. (2020) (DHS-3), and the union of all unique factors in the eight models (ALL). The "ALL" model includes MKT, SMB, HML, MOM, RMW, CMA, IA, ROE, EG, MGMT, PERF, PEAD, and FIN. MKT, SMB, and HML originate from FF-3, and MOM is from Carhart-4. Column one of the table lists the reported items, including the monthly risk-adjusted return in percent (α's), the factor loadings (betas), and the R^2 statistics. Columns two through ten report the regression estimates with respect to these nine factor models. The quantities in parentheses are the Newey-West t-statistics of the α's or factor betas. The sample periods are January 1980–December 2022 for CAPM, FF-3, Carhart-4, FF-5, FF-6, and HXZ-q5; January 1980–December 2016 for SY-4; July 1982–December 2022 for DHS-3; and July 1982–December 2016 for ALL.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	CAPM	FF-3	Carhart-4	FF-5	FF-6	HXZ-q5	SY-4	DHS-3	ALL
α (%)	6.34%	6.34%	6.19%	5.96%	5.88%	5.41%	6.07%	5.51%	5.42%
	(21.15)	(21.68)	(20.22)	(19.86)	(18.77)	(16.58)	(17.47)	(16.74)	(15.27)
MKT	0.10	0.14	0.18	0.23	0.26	0.31	0.32	0.29	0.35
	(1.52)	(2.20)	(2.76)	(3.54)	(3.66)	(4.30)	(4.10)	(4.01)	(4.50)
SMB		-0.27	-0.27	-0.12	-0.13	0.01	0.03		0.05
		(-2.85)	(-2.75)	(-1.17)	(-1.26)	(0.07)	(0.30)		(0.46)
HML		-0.01	0.07	-0.26	-0.17				-0.04
		(-0.06)	(0.57)	(-2.10)	(-1.43)				(-0.22)
MOM			0.19		0.14				-0.07
			(2.19)		(1.57)				(-0.56)
RMW				0.55	0.52				0.55
				(3.38)	(3.16)				(2.18)
CMA				0.52	0.45				-0.07
				(2.58)	(2.22)				(-0.15)
IA						0.19			0.31
						(1.20)			(0.74)
ROE						0.03			-0.31
						(0.23)			(-1.48)
EG						0.98			0.65
						(5.04)			(3.19)
MGMT							0.57		0.27
							(4.10)		(1.13)
PERF							0.30		0.08
							(3.02)		(0.60)
PEAD								0.81	0.76
								(4.83)	(4.18)
FIN								0.38	-0.06
								(3.92)	(-0.30)
R^2	0.52%	2.11%	3.75%	6.73%	7.59%	10.62%	8.73%	10.16%	16.63%

Table C.4: **Pairwise Alpha Tests for the MVE portfolios**

Table C.4 reports the results of pairwise univariate OLS regressions of monthly excess returns on one model's MVE portfolio against those of another model's MVE portfolio over the OOS period from January 1980 to December 2022. The methods listed along the rows correspond to the dependent variables, while those listed along the columns serve as explanatory variables of the regressions. Panel A reports the regression intercepts (monthly "alphas") in percentage terms, which measure the abnormal performance of one MVE portfolio relative to another. Panel B reports the associated Newey–West t-statistics with one lag. Statistical significance at the 1%, 5%, and 10% levels is denoted by superscripts "a", "b", and "c" in Panel A, respectively. To save space, MV-PC and MV-RP-PC are abbreviated as PC and RP-PC, respectively.

	Panel A: Alpha (%)															
Method	REP	JKBJ	MAXSER	EW-CM	MV-Sample	KNS	MV-LS	MV-LS-CC	MV-NLS	MV-NLS-FF-3	PC-5	PC-10	PC-20	PC-40	RP-PC-5	RP-PC-10
REP		6.4a	2.8a	6.3a	3.8a	4.0a	1.4a	1.9a	1.2a	1.3a	5.6a	5.3a	4.6a	3.6a	4.0a	3.2a
JKBJ	−0.5		−0.4	−0.5	−0.5	−0.1	−0.4	0.0	−0.4	−0.2	−0.4	−0.3	−0.3	−0.1	−0.4	−0.2
MAXSER	0.9a	5.3a		5.3a	2.1a	2.3a	1.0a	1.5a	1.2a	1.1a	4.0a	3.7a	2.8a	2.1a	2.3a	1.5a
EW-CM	0.4	0.8a	0.8a		0.7a	1.0a	0.5c	0.7a	0.6c	0.6b	1.1a	1.1a	1.1a	1.1a	1.0a	1.1a
MV-Sample	1.5b	8.1a	1.3c	8.0a		4.8a	1.6a	2.3a	1.4a	1.6a	7.0a	6.5a	5.4a	4.5a	4.9a	3.5a
KNS	1.8a	6.4a	1.6a	6.5a	3.9a		0.9b	1.5a	1.6a	1.3a	4.0a	3.3a	2.1a	1.1a	1.4a	0.6c
MV-LS	0.5	7.9a	2.5a	7.8a	4.1a	3.7a		0.8a	0.3	0.4c	6.6a	6.1a	5.0a	3.6a	4.3a	2.8a
MV-LS-CC	0.8	9.0a	3.2a	8.9a	4.8a	4.3a	0.3		0.5	0.0	7.7a	7.1a	5.8a	4.1a	5.0a	3.0a
MV-NLS	0.4	9.4a	3.4a	9.3a	4.9a	5.0a	0.5b	1.2a		0.3	8.1a	7.6a	6.4a	4.8a	5.6a	3.9a
MV-NLS-FF-3	0.7b	9.0a	3.4a	9.0a	4.9a	4.8a	0.9a	1.0a	0.5a		7.7a	7.2a	6.0a	4.3a	5.1a	3.4a
PC-5	0.6c	2.1a	0.1	2.2a	1.3a	−0.2	0.5	0.8b	0.7b	0.6c		0.2	0.1	0.1	−0.8a	−0.1
PC-10	0.7b	2.5a	0.3	2.6a	1.5a	−0.1	0.5	0.9b	0.8b	0.6c	0.8a		0.1	0.1	−0.8a	−0.4
PC-20	1.0a	3.4a	0.6b	3.6a	2.0a	0.4b	0.8b	1.1a	1.1a	0.9b	2.0a	1.4a		0.1	0.2	−0.7a
PC-40	1.7a	4.8a	1.8a	4.9a	3.3a	1.8a	1.6a	1.9a	1.9a	1.6a	3.6a	3.2a	2.1a		1.8a	0.7a
RP-PC-5	1.1a	3.7a	0.9a	3.8a	2.3a	0.8a	1.0a	1.3a	1.2a	1.0a	1.9a	1.4a	1.0a	0.6a		0.3
RP-PC-10	1.4a	4.9a	1.4a	5.0a	2.9a	1.6a	1.1a	1.3a	1.4a	1.1a	3.6a	2.9a	1.6a	0.8a	1.5a	
	Panel B: t-statistics															
REP		21.53	6.80	20.94	10.72	10.80	5.39	5.75	5.15	4.91	15.49	15.58	12.22	10.41	10.98	8.48
JKBJ	-0.67		-0.65	-0.72	-0.68	-0.24	-0.61	-0.04	-0.65	-0.49	-0.62	-0.56	-0.47	-0.20	-0.72	-0.34
MAXSER	2.85	17.74		17.65	5.46	7.61	3.35	5.09	3.78	3.71	14.51	14.43	9.07	6.55	7.91	4.97
EW-CM	1.40	4.25	3.11		2.82	4.25	1.77	2.75	1.93	2.18	5.01	5.01	4.97	4.42	4.13	4.53
MV-Sample	2.30	16.18	1.90	15.81		7.67	3.22	4.57	2.78	2.97	11.44	11.76	8.89	7.88	7.97	5.84
KNS	4.23	13.83	3.96	13.90	7.32		2.10	2.90	3.62	2.86	12.87	12.88	7.74	3.78	5.72	1.79
MV-LS	1.46	18.87	6.31	18.74	7.07	9.10		3.40	1.32	1.68	15.24	15.14	10.01	8.17	9.60	6.09
MV-LS-CC	1.58	18.79	6.04	18.91	7.13	8.89	1.11		1.26	-0.17	15.24	15.26	9.98	7.38	9.37	5.43
MV-NLS	1.21	19.36	5.74	18.77	7.25	9.22	2.49	4.66		1.34	14.31	14.82	10.97	8.84	9.96	6.92
MV-NLS-FF-3	2.03	19.98	6.68	19.44	7.93	9.71	3.35	4.82	2.62		15.44	15.96	11.67	8.77	10.35	7.03
PC-5	1.69	8.96	0.41	9.32	4.17	-0.88	1.41	2.22	2.14	1.76		1.03	0.46	0.28	-4.55	-0.47
PC-10	2.00	10.39	0.99	11.14	4.34	-0.65	1.55	2.25	2.28	1.73	4.11		0.33	0.25	-7.00	-1.57
PC-20	3.00	13.24	2.35	13.90	5.50	1.99	2.45	3.13	3.15	2.49	8.50	8.20		0.42	0.92	-4.17
PC-40	4.76	17.45	5.62	17.17	8.24	7.92	4.72	5.50	5.43	4.70	13.81	12.93	8.86		7.83	3.35
RP-PC-5	3.44	15.02	3.67	15.32	7.00	4.06	3.56	4.14	3.99	3.30	9.81	13.67	6.18	3.02		1.22
RP-PC-10	4.01	17.57	5.76	18.03	7.42	7.24	3.81	4.42	4.21	3.43	13.89	15.01	10.39	4.11	7.22	

Table C.5: **Sharpe Ratios and Percentage Gains from Combinations**

Table C.5 reports the Sharpe ratios derived from the pairwise univariate regressions introduced in Table C.4. Column two reports the OOS Sharpe ratios of the MVE factor portfolios estimated by REP and competing benchmark methods based on the market and 174 characteristic factors over the OOS period from January 1980 to December 2022. Column three reports the implied maximum Sharpe ratios obtained by optimally combining REP with the MVE factor portfolios estimated by competing benchmark methods. Specifically, the implied maximum Sharpe ratio is constructed from pairwise univariate regressions of each benchmark MVE portfolio return on the return of REP, and is computed as

$$SR_{Implied} = \sqrt{SR^2_{\text{REP}} + IR^2_{\text{Benchmark}}},$$

where SR_{REP} denotes the OOS Sharpe ratio of REP, and $IR_{\text{Benchmark}}$ denotes the information ratio of the benchmark MVE portfolio relative to REP. The information ratio is defined as

$$IR = \alpha_{\text{Benchmark}}/\sigma(\epsilon)_{\text{Benchmark}},$$

where $\alpha_{\text{benchmark}}$ is the intercept and $\sigma(\epsilon)_{\text{Benchmark}}$ is the standard deviation of the regression residuals from the univariate regression of the benchmark MVE portfolio returns on the REP returns. Columns four and five report the percentage improvements in Sharpe ratio relative to REP and to the corresponding benchmark methods, respectively. We take the absolute value of the Sharpe ratio of JKBJ when computing the percentage improvement.

(1)	(2)	(3)	(4)	(5)
			SR improvement relative to (%)	
Method	OOS SR	Implied SR	REP	Benchmark
REP	3.59	-	-	-
JKBJ	−0.08	3.59	0.05	4309.34
MAXSER	2.98	3.65	1.92	22.80
EW-CM	0.65	3.60	0.38	451.63
MV-Sample	2.53	3.63	1.21	43.42
KNS	2.58	3.68	2.67	42.59
MV-LS	3.27	3.61	0.63	10.50
MV-LS-CC	3.15	3.62	0.81	14.80
MV-NLS	3.29	3.60	0.38	9.27
MV-NLS-FF-3	3.35	3.62	1.08	8.36
MV-PC-5	1.51	3.61	0.67	138.96
MV-PC-10	1.77	3.62	1.05	105.24
MV-PC-20	2.26	3.66	2.11	62.16
MV-PC-40	2.97	3.80	5.98	28.03
MV-RP-PC-5	2.58	3.69	2.83	42.82
MV-RP-PC-10	2.99	3.73	4.09	24.91

Table C.6: **Summary Statistics of the Individual Stock Weights of the Estimated MVE Portfolios**

Table C.6 presents the summary statistics of the individual stock weights for the portfolios estimated by REP and alternative benchmarks, using the market and 174 characteristic factors, over the OOS period from January 1980 to December 2022. These statistics are computed for each cross-section, and their time series averages are then taken. Columns two through four report the average absolute weights of individual stocks, as well as the maximum and minimum weights (expressed as percentages). Columns five and six provide the aggregate long and short positions in the cross-section. Finally, columns seven and eight report the percentages of stocks with non-negative and negative weights, respectively, within the cross-section.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\sum_i \lvert w_i \rvert / N_t$ (%)	$\max_i w_i$ (%)	$\min_i w_i$ (%)	$\sum_i w_i I(w_i \geq 0)$	$\sum_i w_i I(w_i < 0)$	$\sum_i I(w_i \geq 0)/N_t$ (%)	$\sum_i I(w_i < 0)/N_t$ (%)
Regression-based MV portfolios and other simple rules							
REP	0.17	1.97	−1.77	4.22	−3.71	63.06	36.94
JKBJ	0.10	0.90	−0.85	2.55	−2.64	56.30	43.70
MAXSER	0.15	1.44	−1.54	3.66	−3.46	63.15	36.85
Market	0.02	3.15	0.00	1.00	0.00	100.00	0.00
1/N	0.02	0.02	0.02	1.00	0.00	100.00	0.00
EW-CM	0.02	3.15	−0.02	1.04	−0.04	81.32	18.68
MV portfolios based on sample and shrinkage-based covariance matrices							
MV-Sample	0.34	2.98	−3.38	8.45	−8.20	61.44	38.56
KNS	0.12	0.79	−1.22	3.33	−3.26	64.38	35.62
MV-LS	0.18	2.17	−1.62	4.78	−4.19	62.63	37.37
MV-LS-CC	0.27	2.05	−2.66	6.85	−6.52	59.78	40.22
MV-NLS	0.24	2.53	−2.22	6.35	−5.69	60.92	39.08
MV-NLS-FF-3	0.27	2.79	−2.68	7.06	−6.36	60.95	39.05
MV portfolios based on leading PCs							
MV-PC-3	0.05	0.29	−0.44	1.20	−1.20	49.33	50.67
MV-PC-5	0.07	0.44	−0.62	1.68	−1.66	60.33	39.67
MV-PC-10	0.09	0.59	−0.82	2.12	−2.12	49.86	50.14
MV-PC-20	0.12	0.79	−1.13	2.87	−2.86	53.74	46.26
MV-PC-40	0.15	1.08	−1.48	3.77	−3.73	57.47	42.53
MV-RP-PC-5	0.10	0.64	−0.96	2.40	−2.35	63.80	36.20
MV-RP-PC-10	0.14	0.90	−1.34	3.37	−3.29	62.78	37.22

Table C.7: **Simulation Results**

This table reports simulation results for portfolio rules based on 10,000 samples, where each sample contains $T+1$ i.i.d. observations of excess returns on $N = 100$ assets. For each sample, T observations are used to estimate the portfolio rules, and the remaining one is used to calculate the OOS return of the portfolio rules. The training sample size T takes values of 80, 120, and 240. The data generating process of stock returns is calibrated from real data using the FF-3 model, following Ao et al. (2019). The theoretical maximum annualized Sharpe ratio is 1.5217. Column one lists the abbreviations of the portfolio rules. The portfolio rule "Factor" refers to the plug-in mean-variance rule based on the simulated Fama-French three factors. The other columns report the annualized OOS Sharpe ratios and the p-values of the Sharpe ratio test of Jobson and Korkie (1981) with the correction of Memmel (2003), whose null hypotheses are that the Sharpe ratio of REP is less than or equal to that of the benchmark rules. REP is constructed with a risk aversion of $\gamma = 10$. The target monthly volatility σ is set to 0.04 for Factor, MAXSER, and the MVE rules in the second and third categories, except KNS, which does not require a volatility input.

The theoretical maximum Sharpe ratio: 1.5217						
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Portfolio	$T = 80$		$T = 120$		$T = 240$	
	Sharpe ratio	p-value	Sharpe ratio	p-value	Sharpe ratio	p-value
Regression-based MV portfolios and other simple rules						
REP	0.6475	-	0.9258	-	1.1384	-
JKBJ	0.0484	< 0.0001	-0.0318	< 0.0001	0.0264	< 0.0001
MAXSER	0.5658	0.0010	0.7128	< 0.0001	1.0426	< 0.0001
Factor	0.1443	< 0.0001	0.2320	< 0.0001	0.2804	< 0.0001
1/N	0.4601	< 0.0001	0.4593	< 0.0001	0.4628	< 0.0001
MV portfolios based on sample and shrinkage-based covariance matrices						
MV-Sample	0.3664	< 0.0001	0.3113	< 0.0001	0.7114	< 0.0001
KNS	0.4881	< 0.0001	0.5660	< 0.0001	0.7674	< 0.0001
MV-LS	0.4770	< 0.0001	0.5461	< 0.0001	0.7655	< 0.0001
MV-LS-CC	0.5830	0.0071	0.6591	< 0.0001	0.8351	< 0.0001
MV-NLS	0.5617	< 0.0001	0.6912	< 0.0001	0.8563	< 0.0001
MV-NLS-FF-3	0.5995	0.0333	0.7108	< 0.0001	0.8866	< 0.0001
MV portfolios based on leading PCs						
MV-PC-3	0.4995	< 0.0001	0.5910	< 0.0001	0.5928	< 0.0001
MV-PC-5	0.5100	0.0002	0.6061	< 0.0001	0.6178	< 0.0001
MV-PC-10	0.5298	0.0012	0.6329	< 0.0001	0.6420	< 0.0001
MV-PC-20	0.5095	0.0001	0.6546	< 0.0001	0.6645	< 0.0001
MV-PC-40	0.4992	< 0.0001	0.6278	< 0.0001	0.7661	< 0.0001
MV-RP-PC-5	0.6482	0.5094	0.7815	< 0.0001	0.9523	< 0.0001
MV-RP-PC-10	0.6348	0.3190	0.7639	< 0.0001	0.9427	< 0.0001

Table C.8: **Linear and Quadratic Characteristic Factors**

Table C.8 summarizes the OOS investment performance of the MVE factor portfolios estimated by REP and alternative benchmarks, using the market factor and a total of 348 linear and quadratic characteristic-based factors over the OOS period from January 1980 to December 2022. Column one lists the abbreviations of the portfolio rules, while column two reports the annualized Sharpe ratios for each rule during the OOS period. Column three presents the p-values of the Sharpe ratio test (Jobson and Korkie, 1981; Memmel, 2003), where the null hypothesis is that the Sharpe ratio of REP is less than or equal to that of the corresponding portfolio rule listed in column one. Columns four through eight report the annualized average returns, annualized standard deviations, skewness, kurtosis, and maximum drawdowns of each portfolio rule over the OOS period.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	SR (Ann.)	$p(\text{SR}_{\text{REP}} \leq \text{SR})$	Mean (Ann.%)	Std (Ann.%)	Skewness	Kurtosis	MaxDD (%)
Regression-based MV portfolios and other simple rules							
REP	3.84		80.01	20.85	0.13	4.43	21.35
JKBJ	−0.01	0.00	−1.01	75.70	−3.15	219.51	1566.65
MAXSER	2.82	0.00	62.80	22.30	0.07	4.97	26.28
Market	0.53	0.00	8.27	15.65	−0.66	5.00	49.99
1/N	0.44	0.00	8.89	20.12	−0.36	5.76	59.59
EW-CM	0.64	0.00	9.59	15.05	−0.66	5.13	48.71
MV portfolios based on sample and shrinkage-based covariance matrices							
MV-Sample	2.46	0.00	92.49	37.61	−0.07	4.68	63.07
KNS	2.37	0.00	114.02	48.06	1.19	5.28	41.76
MV-LS	3.35	0.00	132.49	39.61	0.19	4.85	74.42
MV-LS-CC	3.30	0.00	159.47	48.31	0.28	4.75	47.47
MV-NLS	3.49	0.00	152.29	43.65	0.26	4.81	65.52
MV-NLS-FF-3	3.52	0.02	142.77	40.52	−0.06	4.72	51.30
MV portfolios based on leading PCs							
MV-PC-3	0.61	0.00	9.94	16.33	−0.11	9.55	56.67
MV-PC-5	1.40	0.00	22.85	16.34	−0.30	7.52	34.24
MV-PC-10	1.75	0.00	29.19	16.66	0.36	5.29	48.28
MV-PC-20	2.08	0.00	36.91	17.75	0.04	6.61	45.52
MV-PC-40	2.86	0.00	51.98	18.15	0.08	4.29	22.23
MV-RP-PC-5	2.62	0.00	46.00	17.56	0.15	3.78	33.36
MV-RP-PC-10	3.11	0.00	63.71	20.46	−0.26	6.75	36.56

Table C.9: **The OOS Performance of Real-Time MVE Portfolios**

Table C.9 reports the OOS investment performance of REP and alternative benchmark approaches for estimating MVE factor portfolios over the period January 1980 to December 2022. A characteristic factor is included in the portfolio only if it has been published by that year. Column one lists the abbreviations of the portfolio rules, while column two reports the annualized Sharpe ratios for each rule during the OOS period. Column three presents the p-values of the Sharpe ratio test (Jobson and Korkie, 1981; Memmel, 2003), where the null hypothesis is that the Sharpe ratio of REP is less than or equal to that of the corresponding portfolio rule listed in column one. Columns four through eight report the annualized average returns, annualized standard deviations, skewness, kurtosis, and maximum drawdowns of each portfolio rule over the OOS period.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	SR (Ann.)	$p(\text{SR}_{\text{REP}} \leq \text{SR})$	Mean (Ann.%)	Std (Ann.%)	Skewness	Kurtosis	MaxDD (%)
Regression-based MV portfolios and other simple rules							
REP	2.47		50.40	20.38	0.18	4.15	36.87
MAXSER	2.05	0.00	38.49	18.81	−0.02	6.37	43.77
JKBJ	0.13	0.00	15.81	117.36	2.10	170.95	1748.23
Market	0.53	0.00	8.27	15.65	−0.66	5.00	49.99
1/N	0.44	0.00	8.89	20.12	−0.36	5.76	59.59
EW-CM	0.60	0.00	9.44	15.78	−0.71	5.15	48.91
MV portfolios based on sample and shrinkage-based covariance matrices							
MV-Sample	1.56	0.00	49.53	31.71	−0.16	16.14	82.64
KNS	1.89	0.00	39.85	21.12	0.46	6.08	32.47
MV-LS	2.00	0.00	38.24	19.16	−0.87	11.77	65.03
MV-LS-CC	2.07	0.00	46.45	22.40	−1.10	16.08	59.32
MV-NLS	2.12	0.00	48.72	22.98	−0.51	7.01	59.22
MV-NLS-FF-3	2.18	0.00	49.69	22.82	−0.70	7.39	56.51
MV portfolios based on leading PCs							
MV-PC-3	0.75	0.00	12.92	17.26	−0.42	12.22	52.86
MV-PC-5	1.35	0.00	22.67	16.81	−0.35	6.65	31.94
MV-PC-10	1.52	0.00	26.14	17.16	−0.17	4.56	41.76
MV-PC-20	1.79	0.00	32.21	17.99	0.11	4.14	54.01
MV-PC-40	2.26	0.07	43.63	19.33	0.34	3.58	28.01
MV-RP-PC-5	1.93	0.00	32.25	16.70	−0.07	4.85	30.21
MV-RP-PC-10	2.15	0.02	36.91	17.19	−0.08	4.88	40.26

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