

Note on “Guarantees in Auctions: The Auction House as Negotiator and Managerial Decision Maker”

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Appendix

Note on "Guarantees in Auctions: The Auction House as Negotiator and Managerial Decision Maker" by Eric A. Greenleaf, Jun Ma, Wanhua Qiu, Ambar G. Rao, and Atanu R. Sinha.

SUMMARY OF NOTATION: We use the same notation as in "Guarantees in Auctions: The Auction House as Negotiator and Managerial Decision Maker" by Eric A. Greenleaf, Ambar G. Rao, and Atanu R. Sinha (*Management Science*, Vol. 39, no. 9, Sept. 1993, pp. 1130-1145).

Define,

$$E(\pi_{SC}) = \int_{r_S}^{a_h} p_a dF_1(p_a) + F_1(r_S) \int_{r_{US}}^{d_h} p_d dF_2(p_d) + F_1(r_S) \cdot F_2(r_{US}) \cdot r_{US} . \quad (A1)$$

$$E(\pi_{HC}) = c[E(\pi_{SC}) - F_1(r_S)F_2(r_{US})r_{US}] . \quad (A2)$$

$$E(\pi_{SG}) = g + \int_g^{a_h} (p_a - g)(1 - c') dF_1(p_a) . \quad (A3)$$

$$E(\pi_{HG}) = -E(\pi_{SG}) + \int_{r_H}^{a_h} p_a (1 + c) dF_1(p_a) + F_1(r_H) \left\{ \int_{r_{UH}/(1+c)}^{d_h} p_d (1 + c) dF_2(p_d) + F_2\left(\frac{r_{UH}}{1+c}\right) r_{UH} \right\} . \quad (A4)$$

LEMMA 1: (a) *The optimal reserve r_S^* for the seller in a conventional auction is*

$$r_S^* = E(p_d) + \int_{d_l}^{r_{US}} (r_{US} - p_d) dF_2(p_d), \text{ which implies that } r_S^* \geq E(p_d) \text{ and } r_S^* \geq r_{US}, \text{ the inequalities}$$

being strict when $r_{US} > d_l$.

(b) *The optimal reserve r_H^* for the auction house in a guaranteed auction is*

$$r_H^* = E(p_d) + \int_{d_l}^{r_{UH}/(1+c)} \left(\frac{r_{UH}}{1+c} - p_d\right) dF_2(p_d), \text{ which implies that}$$

$$r_H^* > \frac{E(p_d)}{1+c} \quad \& \quad r_H^* > \frac{r_{UH}}{1+c} .$$

PROOF OF LEMMA 1(a): Same as in GRS.

PROOF OF LEMMA 1(b): By differentiating $E(\pi_{HG})$ w.r.t. r_H and simplifying,

$$\begin{aligned}
r_H^* &= E(p_d) + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d(1+c)) dF_2(p_d) \\
&= E(p_d) + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d) dF_2(p_d) - \frac{c}{1+c} \left\{ \int_{d_l}^{r_{UH}/(1+c)} p_d dF_2(p_d) \right\} \\
&= E(p_d) + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d) dF_2(p_d) - \frac{c}{1+c} \left\{ \int_{d_l}^{d_h} p_d dF_2(p_d) - \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) \right\} \\
&= E(p_d) + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d) dF_2(p_d) - \frac{c}{1+c} \left\{ E(p_d) - \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) \right\} \\
&= E(p_d) \left\{ 1 - \frac{c}{1+c} \right\} + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d) dF_2(p_d) + \frac{c}{1+c} \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) \\
&= \frac{E(p_d)}{1+c} + \frac{1}{1+c} \int_{d_l}^{r_{UH}/(1+c)} (r_{UH} - p_d) dF_2(p_d) + \frac{c}{1+c} \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d).
\end{aligned}$$

Since the two integrals on r.h.s. are > 0 , we have, $r_H^* > \frac{E(p_d)}{1+c}$.

Simplifying further we have,

$$\begin{aligned}
r_H^* &= \int_{d_l}^{d_h} p_d dF_2(p_d) - \int_{d_l}^{r_{UH}/(1+c)} (p_d - \frac{r_{UH}}{1+c}) dF_2(p_d) \\
&= \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + \int_{d_l}^{r_{UH}/(1+c)} \frac{r_{UH}}{1+c} dF_2(p_d) \\
&= \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + \frac{r_{UH}}{1+c} \left[\int_{d_l}^{r_{UH}/(1+c)} dF_2(p_d) \right] \\
&= \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + \frac{r_{UH}}{1+c} \left[\int_{d_l}^{d_h} dF_2(p_d) - \int_{r_{UH}/(1+c)}^{d_h} dF_2(p_d) \right] \\
&= \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + \frac{r_{UH}}{1+c} \left[1 - \int_{r_{UH}/(1+c)}^{d_h} dF_2(p_d) \right]
\end{aligned}$$

$$= \frac{r_{UH}}{1+c} + \int_{r_{UH}/(1+c)}^{d_h} \left(p_d - \frac{r_{UH}}{1+c} \right) dF_2(p_d) > \frac{r_{UH}}{1+c}. \quad \square$$

LEMMA 2: If $r_{US} = r_{UH} = r_U$, then $r_S^* - r_H^* \geq 0$, the inequality being strict when $r_U > d_l \geq 0$.

PROOF OF LEMMA 2:

$$\begin{aligned} r_S^* - r_H^* &= \int_{d_l}^{r_U} (r_U - p_d) dF_2(p_d) - \int_{d_l}^{r_U/(1+c)} \left(\frac{r_U}{1+c} - p_d \right) dF_2(p_d) \\ &= \int_{d_l}^{r_U} r_U dF_2(p_d) - \int_{d_l}^{r_U} p_d dF_2(p_d) - \int_{d_l}^{r_U/(1+c)} \frac{r_U}{1+c} dF_2(p_d) + \int_{d_l}^{r_U/(1+c)} p_d dF_2(p_d) \\ &= \int_{d_l}^{r_U} \left(\frac{r_U}{1+c} + \frac{cr_U}{1+c} \right) dF_2(p_d) - \int_{d_l}^{r_U/(1+c)} \left(\frac{r_U}{1+c} \right) dF_2(p_d) - \int_{r_U/(1+c)}^{r_U} p_d dF_2(p_d) \\ &= \int_{r_U/(1+c)}^{r_U} \frac{r_U}{1+c} dF_2(p_d) + c \int_{d_l}^{r_U} \frac{r_U}{1+c} dF_2(p_d) - \int_{r_U/(1+c)}^{r_U} p_d dF_2(p_d) \\ &= \int_{r_U/(1+c)}^{r_U} \left(\frac{r_U}{1+c} - p_d \right) dF_2(p_d) + \int_{r_U/(1+c)}^{r_U} \frac{cr_U}{1+c} dF_2(p_d) + \int_{d_l}^{r_U/(1+c)} \frac{cr_U}{1+c} dF_2(p_d) \\ &= \int_{r_U/(1+c)}^{r_U} \left(\frac{r_U}{1+c} + \frac{cr_U}{1+c} - p_d \right) dF_2(p_d) + \int_{d_l}^{r_U/(1+c)} \frac{cr_U}{1+c} dF_2(p_d) \\ &= \int_{r_U/(1+c)}^{r_U} (r_U - p_d) dF_2(p_d) + \int_{d_l}^{r_U/(1+c)} \frac{cr_U}{1+c} dF_2(p_d) > 0, \end{aligned}$$

since both the integrals are ≥ 0 . When $r_U > d_l \geq 0$, the inequalities are strict. \square

LEMMA 3 (Revised): *The sum of the expected payoffs to the auction house and the seller in a guaranteed auction is greater than or equal to the sum of the expected payoffs in a conventional auction. The inequality is strict when $r_U > d_l \geq 0$.*

PROOF OF LEMMA 3:

Using (A1) and (A2) the total expected payoff in conventional auction is

$$E(\pi_{SC}) + E(\pi_{HC}) = (1+c) \left[\int_{r_S}^{a_h} p_a dF_1(p_a) + F_1(r_S) \cdot \int_{r_{US}}^{d_h} p_d dF_2(p_d) + F_1(r_S)F_2(r_{US})r_{US} \right] \\ - cF_1(r_S)F_2(r_{US})r_{US}$$

Using (A3) and (A4) the total expected payoff in guaranteed auction is

$$E(\pi_{SG}) + E(\pi_{HG}) = (1+c) \left[\int_{r_H}^{a_h} p_a dF_1(p_a) + F_1(r_H) \cdot \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) \right] + F_1(r_H)F_2\left(\frac{r_{UH}}{1+c}\right)r_{UH}.$$

Note that both the above expressions are independent of g and c' . From above,

$$E(\pi_{SG}) + E(\pi_{HG}) - [E(\pi_{SC}) + E(\pi_{HC})] = (1+c) \left[\int_{r_S}^{a_h} p_a dF_1(p_a) - \int_{r_S}^{a_h} p_a dF_1(p_a) \right] \\ + (1+c) \left[F_1(r_H) \left\{ \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + F_2\left(\frac{r_{UH}}{1+c}\right) \cdot \frac{r_{UH}}{1+c} \right\} \right] \\ - (1+c) \left[F_1(r_S) \left\{ \int_{r_{US}}^{d_h} p_d dF_2(p_d) + F_2(r_{US}) \cdot r_{US} \right\} \right] + cF_1(r_S)F_2(r_{US})r_{US}.$$

$$\text{Now } r_S^* = \int_{r_{US}}^{d_h} p_a dF_2(p_d) + F_2(r_{US}) \cdot r_{US}.$$

$$\text{And } r_H^* = \int_{r_{UH}/(1+c)}^{d_h} p_d dF_2(p_d) + F_2\left(\frac{r_{UH}}{1+c}\right) \cdot \frac{r_{UH}}{1+c}.$$

Evaluated at r_S^* , r_H^* , and setting $r_{US} = r_{UH} = r_U$, we have, $E(\pi_{SG}) + E(\pi_{HG}) - [E(\pi_{SC}) + E(\pi_{HC})]$

$$= (1+c) \left[\int_{r_H^*}^{r_S^*} p_a dF_1(p_a) \right] + (1+c) [F_1(r_H^*) \cdot r_H^*] - (1+c) [F_1(r_S^*) \cdot r_S^*] + cF_1(r_S^*)F_2(r_U)r_U$$

$$= (1+c) \left[\int_{r_H^*}^{r_S^*} p_a dF_1(p_a) + r_H^* \cdot F_1(r_H^*) - r_S^* \cdot F_1(r_S^*) \right] + cF_1(r_S^*)F_2(r_U)r_U$$

$$= (1+c) \left[\int_{r_H^*}^{r_S^*} p_a dF_1(p_a) + \int_{r_S^*}^{r_H^*} [F_1(p_a) + p_a f_1(p_a)] dp_a \right] + cF_1(r_S^*)F_2(r_U)r_U$$

[Digression: Integrating by parts the second term in $\int_{r_S^*}^{r_H^*} [F_1(p_a) + p_a f_1(p_a)] dp_a$] we write,

$$\int_{r_S^*}^{r_H^*} [F_1(p_a) + p_a f_1(p_a)] dp_a = \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + p_a F_1(p_a) \Big|_{r_S^*}^{r_H^*} - \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a$$

$$= r_H^* F_1(r_H^*) - r_S^* F_1(r_S^*).]$$

$$= (1+c) \left[\int_{r_H^*}^{r_S^*} p_a dF_1(p_a) + \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \int_{r_S^*}^{r_H^*} p_a dF_1(p_a) \right] + cF_1(r_S^*)F_2(r_U)r_U, \text{ or,}$$

$$E(\pi_{SG}) + E(\pi_{HG}) - [E(\pi_{SC}) + E(\pi_{HC})] = (1+c) \left[\int_{r_S^*}^{r_H^*} F_1(p_a) dp_a \right] + cF_1(r_S^*)F_2(r_U)r_U \quad (\text{A5})$$

$$= (1+c) \left[\int_{r_S^*}^{r_H^*} F_1(r_S^*) dp_a \right] - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] + cF_1(r_S^*)F_2(r_U)r_U$$

$$= (1+c)F_1(r_S^*) \{r_H^* - r_S^*\} + cF_1(r_S^*)F_2(r_U)r_U - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \quad (\text{A6})$$

which using Lemma 2 gives,

$$\begin{aligned} &= F_1(r_S^*) \left[- (1+c) \int_{r_U/(1+c)}^{r_U} (r_U - p_d) dF_2(p_d) - (1+c) \frac{cr_U}{1+c} \{F_2(r_U/(1+c))\} \right] \\ &\quad + cF_1(r_S^*)F_2(r_U)r_U - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \\ &= - (1+c) \int_{r_U/(1+c)}^{r_U} F_1(r_S^*)(r_U - p_d) dF_2(p_d) - F_1(r_S^*)cr_U \{F_2(r_U/(1+c))\} \\ &\quad + cF_1(r_S^*)F_2(r_U)r_U - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \\ &= - (1+c) \int_{r_U/(1+c)}^{r_U} F_1(r_S^*)(r_U - p_d) dF_2(p_d) + F_1(r_S^*)cr_U \{F_2(r_U) - F_2(r_U/(1+c))\} \\ &\quad - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \\ &= - (1+c) \int_{r_U/(1+c)}^{r_U} F_1(r_S^*)(r_U - p_d) dF_2(p_d) + \int_{r_U/(1+c)}^{r_U} F_1(r_S^*)cr_U dF_2(p_d) \\ &\quad - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \\ &= - \left\{ \int_{r_U/(1+c)}^{r_U} [(1+c)F_1(r_S^*)(r_U - p_d) - cr_U F_1(r_S^*)] dF_2(p_d) \right\} \\ &\quad - (1+c) \left[\int_{r_S^*}^{r_H^*} \{F_1(r_S^*) - F_1(p_a)\} dp_a \right] \end{aligned}$$

Now the integrand in the first term is

$$(1+c) F_1(r_S^*) (r_U - p_d) - c r_U F_1(r_S^*)$$

$$= F_1(r_S^*) \{ (1+c)(r_U - p_d) - c r_U \} = F_1(r_S^*) \{ -(1+c)p_d + r_U + c r_U - c r_U \}$$

$$= F_1(r_S^*) \{ -(1+c)p_d + r_U \} < 0,$$

since in the range of integration $p_d > r_U/(1+c)$. Hence the first term is ≥ 0 , the inequality being strict when $r_U > d_l \geq 0$. Since $r_S^* - r_H^* \geq 0$ (Lemma 2), the second term is ≥ 0 . Thus,

$$E(\pi_{SG}) + E(\pi_{HG}) - [E(\pi_{SC}) + E(\pi_{HC})] \geq 0. \quad \square \quad (\text{A7})$$

PROOFS OF PROPOSITIONS:

Before presenting the proofs we develop the following relationships.

Let $K_1 = \{E(\pi_{HG}) - q_b E(\pi_{HC})\}$ and $K_2 = \{E(\pi_{SG}) - [q_b + (1 - q_b)\delta]E(\pi_{SC})\}$. K_1 and K_2 are expected incremental payoffs to the auction house and the seller respectively, and are functions of g and c' .

The Nash bargaining solution is obtained by maximizing $L(g, c') = K_1 \cdot K_2$.

From (A3),

$$E(\pi_{SG}) = g + \int_g^{a_h} p_a (1 - c') dF_1(p_a) - g(1 - c') \int_g^{a_h} dF_1(p_a).$$

$$\frac{\partial E(\pi_{SG})}{\partial g} = 1 - g(1 - c')f_1(g) - (1 - c') \int_g^{a_h} dF_1(p_a) - g(1 - c')(-f_1(g)),$$

by canceling the second and fourth terms,

$$\begin{aligned} &= 1 - (1 - c')(1 - F_1(g)) \\ &= 1 + (1 - c')(F_1(g) - 1). \end{aligned} \quad (\text{A8})$$

$$\frac{\partial}{\partial c'} E(\pi_{SG}) = - \int_g^{a_h} (p_a - g) dF_1(p_a). \quad (\text{A9})$$

Based on (A4),

$$\frac{\partial}{\partial g} E(\pi_{HG}) = -1 + (c' - 1)[F_1(g) - 1]. \quad (\text{A10})$$

$$\frac{\partial}{\partial c'} E(\pi_{HG}) = \int_g^{a_h} (p_a - g) dF_1(p_a). \quad (\text{A11})$$

$$\text{Note } \frac{\partial E(\pi_{SG})}{\partial g} = -\frac{\partial E(\pi_{HG})}{\partial g}. \quad (\text{A12})$$

$$\text{And } \frac{\partial E(\pi_{SG})}{\partial c'} = -\frac{\partial E(\pi_{HG})}{\partial c'}. \quad (\text{A13})$$

PROPOSITION 1 (revised) (a) *In the Nash bargaining solution, the incremental expected payoffs above the respective conflict values are equal for both parties.*

(b) *With low switching costs ($\delta^* \leq \delta \leq 1$ for some δ^*), the seller will threaten to change auction houses if a guarantee cannot be negotiated (i.e., $q_b = 0$).*

(c) *With high switching costs ($0 \leq \delta \leq \delta^*$ for some δ^*), no such threat will be made (i.e., $q_b = 1$).*

PROOF OF PROPOSITION 1(a):

$$L(g, c') = K_1 \cdot K_2$$

$$= \{E(\pi_{HG}) - q_b E(\pi_{HC})\} \{E(\pi_{SG}) - [q_b + (1 - q_b)\delta] E(\pi_{SC})\}.$$

$$\frac{\partial L}{\partial g} = \frac{\partial E(\pi_{HG})}{\partial g} \cdot K_2 + K_1 \cdot \frac{\partial E(\pi_{SG})}{\partial g}$$

$$= [K_1 - K_2] \frac{\partial E(\pi_{SG})}{\partial g}, \quad \text{using (A12)}$$

$$= [K_1 - K_2] [1 + (1 - c') [F_1(g) - 1]], \quad \text{using (A8)}. \quad (\text{A14})$$

$$\frac{\partial L}{\partial c'} = \frac{\partial E(\pi_{HG})}{\partial c'} \cdot K_2 + K_1 \cdot \frac{\partial E(\pi_{SG})}{\partial c'}$$

$$= [K_2 - K_1] \frac{\partial E(\pi_{HG})}{\partial c'}, \quad \text{using (A13)}$$

$$= [K_2 - K_1] \int_g^{a_h} (p_a - g) dF_1(p_a), \quad \text{using (A11)}. \quad (\text{A15})$$

From (A14) and (A15) we see that $\partial L / \partial g = \partial L / \partial c' = 0$ if and only if $K_1 = K_2$. (Note that the conditions $c' = 0$, $g = a_h$ and $p_a = g$, which also yield $\partial L / \partial g = \partial L / \partial c' = 0$, occur on a set of measure zero.)

PROOF OF PROPOSITION 1(b):

At the bargaining solution since $K_1^* = K_2^*$

$$E(\pi_{HG}) - q_b \cdot E(\pi_{HC}) = E(\pi_{SG}) - \{q_b + (1 - q_b)\delta\}E(\pi_{SC}).$$

$$\text{Rearranging } E(\pi_{HG}) - E(\pi_{HC}) = E(\pi_{SG}) - q_b \cdot E(\pi_{SC}) - (1 - q_b)\{\delta E(\pi_{SC}) + E(\pi_{HC})\}.$$

$$\text{From (A5), } E(\pi_{HG}) - E(\pi_{HC}) = (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U - E(\pi_{SG}) + E(\pi_{SC}).$$

Equating above two equations,

$$\begin{aligned} 2E(\pi_{SG}) &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 + q_b)E(\pi_{SC}) + (1 - q_b)\{\delta E(\pi_{SC}) + E(\pi_{HC})\} \\ &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 - q_b)E(\pi_{HC}) + \{1 + q_b + (1 - q_b)\delta\}E(\pi_{SC}) \\ &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 - q_b)E(\pi_{HC}) + \{2 - (1 - q_b) + (1 - q_b)\delta\}E(\pi_{SC}) \\ &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 - q_b)E(\pi_{HC}) + 2E(\pi_{SC}) + \{(\delta - 1)(1 - q_b)\}E(\pi_{SC}) \\ &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 - q_b)E(\pi_{HC}) + 2E(\pi_{SC}) - (1 - \delta)(1 - q_b)E(\pi_{SC}) \\ &= (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + 2E(\pi_{SC}) + (1 - q_b)[E(\pi_{HC}) - (1 - \delta)E(\pi_{SC})]. \end{aligned} \quad (\text{A16})$$

$$2[E(\pi_{SG}) - E(\pi_{SC})] = (1 + c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + (1 - q_b)[E(\pi_{HC}) - (1 - \delta)E(\pi_{SC})]. \quad (\text{A17})$$

From the seller's perspective, a meaningful guarantee implies that the l.h.s. of (A17) must be positive, so the seller is better off with a guaranteed than a conventional auction. Now the seller wants to maximize the l.h.s. through proper selection of q_b . We show that when

$$\delta > \delta^* = 1 - c + \frac{[cF_1(r_S)F_2(r_U)r_U]}{E(\pi_{SC})} \text{ the seller's threat is credible, i.e., } q_b = 0.$$

From (A17) note that by selecting $q_b=0$, the seller maximizes the r.h.s., so long as the expression in the square brackets in the r.h.s. is greater than zero. This happens because the sum of the first two terms in the r.h.s. of (A17) is greater than zero (by A5 and A7). Now the expression in the square brackets in the r.h.s. is greater than zero when $\delta > 1 - [E(\pi_{HC}) / E(\pi_{SC})]$, which using (A2) gives,

$$\begin{aligned} &= 1 - \{c[E(\pi_{SC}) - F_1(r_S)F_2(r_{US})r_{US}] / E(\pi_{SC})\} \\ &= 1 - c + [cF_1(r_S)F_2(r_{US})r_{US}] / E(\pi_{SC}) = \delta^*. \end{aligned} \tag{A18}$$

If $\delta=1$, then from (A17) and using (A5), we get $E(\pi_{SG}) - E(\pi_{SC}) = E(\pi_{HG})$. \square

PROOF OF PROPOSITION 1(c):

When $\delta \leq \delta^*$, then $q_b = 1$, and although the seller does not switch auction houses, he can always opt for a conventional auction. Hence the auction house settles for splitting the difference equally. \square

PROPOSITION 2(a). *For $1 \geq \delta > \delta^*$, the Nash bargaining solution is given by the frontier*

$\{g^*, c'^*\}$ characterized by:

$$c'^* = 1 + \frac{g^* - \frac{1}{2} \left\{ (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1 + \delta + c\} E(\pi_{SC}) \right\}}{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)}.$$

2(b). *On the frontier, c^* and g^* are positively related. As c^* increases so does g^* , but at a decreasing rate.*

PROOF OF PROPOSITION 2(a):

Substituting for $E(\pi_{HC})$ in terms of $E(\pi_{SC})$ in (A17) ($\ominus q_b = 0$, and $r_{US} = r_U$)

$$\begin{aligned} 2E(\pi_{SG}) &= (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + cF_1(r_S^*)F_2(r_U)r_U + 2E(\pi_{SC}) + \\ &\quad [cE(\pi_{SC}) - cF_1(r_S^*)F_2(r_U)r_U - (1-\delta)E(\pi_{SC})] \end{aligned}$$

$$\begin{aligned}
&= (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + E(\pi_{SC}) \{2+c-(1-\delta)\} \\
&= (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}). \tag{A19}
\end{aligned}$$

Substituting from (A3) in (A19),

$$2g + 2 \int_g^{a_h} (p_a - g)(1-c') dF_1(p_a) = (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}),$$

$$\text{or, } 2(1-c') \int_g^{a_h} (p_a - g) dF_1(p_a) = -2g + (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}),$$

$$\text{or, } 2(c'-1) \int_g^{a_h} (p_a - g) dF_1(p_a) = 2g - (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a - \{1+\delta+c\} E(\pi_{SC})$$

$$\text{or, } (c'-1) \int_g^{a_h} (p_a - g) dF_1(p_a) = g - \frac{1}{2} \left\{ (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}) \right\}.$$

Therefore, on the bargaining frontier,

$$c'^* = 1 + \frac{g^* - \frac{1}{2} \left\{ (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}) \right\}}{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)}. \quad \square \tag{A20}$$

PROOF OF PROPOSITION 2(b):

Finally, from (A20), differentiating c'^* w.r.t. g^* , we get,

$$\frac{dc'^*}{dg^*} = \frac{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) - \left[g^* - \frac{1}{2} \left\{ (1+c) \int_{r_S^*}^{r_H^*} F_1(p_a) dp_a + \{1+\delta+c\} E(\pi_{SC}) \right\} \right] \{-[1-F_1(g^*)]\}}{\left[\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) \right]^2}. \tag{A20i}$$

Now, the expression in large square brackets in numerator of (A20i), based on solving for the

corresponding expression in (A20), is $(c'^*-1) \int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)$.

$$\therefore \frac{dc'^*}{dg^*} = \frac{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) - \left[(c'^*-1) \int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) \right] \{-[1-F_1(g^*)]\}}{\left[\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) \right]^2}.$$

Canceling the integral in numerator and denominator,

$$\begin{aligned} \frac{dc'^*}{dg^*} &= \frac{1 - (c'^* - 1) \{- [1 - F_1(g^*)]\}}{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)} = \frac{c'^* - c'^* F_1(g^*) + F_1(g^*)}{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)} \\ &= \frac{c'^* (1 - F_1(g^*)) + F_1(g^*)}{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)}. \end{aligned} \quad (\text{A21})$$

□ both numerator and denominator in (A21) are positive, $dg^*/dc'^* > 0$.

$$\begin{aligned} \frac{dg^*}{dc'^*} &= \frac{\int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a)}{c'^* (1 - F_1(g^*)) + F_1(g^*)}. \\ \therefore \frac{d^2 g^*}{dc'^{*2}} &= - \frac{\left\{ \int_{g^*}^{a_h} (p_a - g^*) dF_1(p_a) \right\} \{1 - F_1(g^*)\}}{[c'^* (1 - F_1(g^*)) + F_1(g^*)]^2} < 0, \end{aligned}$$

unless $g^* = a_h$, which occurs on a set of measure zero. □

PROPOSITION 3: *The seller's expected payoff in a guaranteed auction is no less than in a conventional auction. The auction house's expected payoff in a guaranteed auction could be more or less than its expected payoff in a conventional auction.*

PROOF OF PROPOSITION 3:

The first part of the proposition is obvious, since the seller can always choose to use a conventional auction and will only use a guaranteed auction if it makes him better off. As for the auction house, when $0 \leq \delta < \delta^*$, it splits the revenue gain with the seller (see Propositions 1(a) and (c)), and hence is better off.

Next, we examine cases where guarantees can make the auction house worse off, including some where $\delta = 1$ and $r_u > 0$, and where $\delta < 1$ and $r_u > 0$. When $\delta = 1$ the seller has all the bargaining power and can switch auction houses at no cost. Therefore, in the guaranteed auction, the expected payoff

to the auction house equals the incremental expected payoff the seller gains by using a guaranteed auction instead of a conventional one. That is, $E(\pi_{HG}) = E(\pi_{SG}) - E(\pi_{SC})$ (see the proof of Proposition 1(b) immediately following A18). If $r_u=0$ and $\delta=1$ the auction house's expected payoff clearly decreases. Here, the seller loses nothing by switching and the guaranteed auction does not increase total expected auction revenue (by Lemma 3), so on the bargaining frontier the seller obtains half of $E(\pi_{HC})$. Next, consider auctions where $r_u>0$ and $\delta=1$. Rearranging terms in (A5), and substituting $E(\pi_{HG})$ for $E(\pi_{SG}) - E(\pi_{SC})$, we obtain:

$$[E(\pi_{HG}) - E(\pi_{HC})] = (1+c) \left[\int_{r_s^*}^{r_u^*} F_1(p_a) dp_a \right] + cF_1(r_s^*)F_2(r_u)r_u - E(\pi_{HG}).$$

Thus, if $E[\pi_{HG}] > (<) (1+c) \int_{r_s^*}^{r_u^*} F_1(p_a) dp_a + cF_1(r_s^*)F_2(r_u)r_u$, the auction house's expected payoff decreases (increases) in the guaranteed auction relative to the conventional auction, even if $r_u>0$. By continuity of the functions involved it follows that this can also hold for a range of δ strictly less than 1. Thus, there are auctions where guarantees make the auction house worse off, even if $\delta<1$ and $r_u>0$. \square

PROPOSITION 4: For any mutually acceptable combination of g and c' , the variance of the seller's payoff in a guaranteed auction is less than in a conventional auction.

PROOF OF PROPOSITION 4:

Consider the distribution of payoffs for the seller in the conventional auction. In the guaranteed auction, all payoffs in the conventional auction distribution less than g , are moved upward to a mass point g . Also, all payoffs greater than g are moved downward to a value $g+(p_a - g)(1-c')$. Each of these by itself decreases the variance of the seller's revenue in the guaranteed auction. Since both

effects move mass towards g , their respective impacts on variance cannot offset each other, and their collective impact also reduces this variance. \square