David versus Goliath:

An Analysis of Asymmetric Mixed Strategy Games
and Experimental Evidence
Technical Appendices

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December 2001

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Technical Appendix A

We establish the proposition through a series of lemma. In Lemma A1, we establish that there are no pure strategy equilibria for this game. In Lemma A2, we show that the proposed equilibrium is a valid equilibrium. The proposed equilibrium requires that both firms invest zero with positive probability. Lemma A2 establishes that this must be true for at least one firm in any equilibria of the game. Lemma A3 shows that, if a firm invests zero with positive probability, then the equilibrium proposed in the proposition is the unique equilibrium for that firm. Together with the result of Lemma A2 this implies that the proposed equilibrium must be the unique equilibrium for at least one firm. Lemma A4 and A5 establish that the equilibrium must also be unique for the other firm. Lemma A6 establishes the convergence of the discrete case to the continuous case as $k \to \infty$.

Lemma A1 *There is no pure strategy equilibrium of the game if $k > 1$. However, there exists a mixed strategy solution.*

Proof: The second part follows from Nash's Existence Theorem (Nash 1951). To establish the first part assume that it not true and there is a pure strategy equilibrium. Suppose firm i's equilibrium strategy is to invest x_i^* and assume that $x_i^* \geq x_j^*$. We will consider various cases.

Case 1: $x_i^* = x_j^*$

If $x_i^* = 0$ then both firms make zero profits and firm i can do better by investing $\frac{c}{k}$ and make profits $r_i - \frac{c}{k} > 0$. If $x_i^* > 0$ then both firms are making negative profits and can do better by deviating to zero. So this cannot be an equilibrium.

Case 2: $x_i^* > x_j^*$ *and* $x_i^* < c$

In this case firm j makes zero profits. Suppose firm j deviates to c then it wins the patent and its profits are $r_j - c > 0$. Thus, firm j will deviate and this cannot be an equilibrium.

Case 3: $x_i^* > x_j^* > 0$ *and* $x_i^* = c$.

In this case firm j makes negative profits and can benefit by deviating to investing zero.

Case 4: $c = x_i^* > x_j^*$ and $x_j^* = 0$

In this case firm i can deviate to $\frac{c}{k} < c$ (since $k > 1$) and make profits $r_i - \frac{c}{k} > r_i - c$. Thus, firm i will deviate.

Thus, in all conditions a pure strategy solution cannot be an equilibrium. □

Lemma A2 *The equilibrium proposed in Proposition 1 is a valid Nash equilibrium.*

Proof: Since $p_H(0) > 0$ and $p_L(0) > 0$ in the proposed equilibrium it follows that $\Pi_H = 0$ and $\Pi_L = 0$. In a mixed strategy solution we need that each firm is indifferent among all its strategies in the support of its distribution. Therefore, we must have for firm L:

$$\begin{aligned}
(r_H - \frac{c}{k})p_L(0) - (\frac{c}{k})p_L(\frac{c}{k}) - \cdots - (\frac{c}{k})p_L(c) &= 0 && \text{(A1)}\\
(r_H - \frac{2c}{k})p_L(0) + (r_H - \frac{2c}{k})p_L(\frac{c}{k}) \cdots - (\frac{2c}{k})p_L(c) &= 0 && \text{(A2)}\\
&\vdots && \text{(A3)}\\
p_L(0) + p_L(\frac{c}{k}) + \cdots + p_L(c) &= 1 && \text{(A4)}
\end{aligned}$$

This can be more compactly rewritten as:

$$\begin{aligned}
(r_H - \frac{ic}{k})\sum_{j=0}^{i-1} p_L(\frac{jc}{k}) - \frac{ic}{k}\sum_{j=i}^{k} p_L(\frac{jc}{k}) &= 0 \text{ where } i = 1, 2, \ldots, k && \text{(A5)}\\
\sum_{j=0}^{k} p_L(\frac{jc}{k}) &= 1 && \text{(A6)}
\end{aligned}$$

It is easy to see that the (A6) is satisfied for the equilibrium solution proposed in Proposition 1. We substitute the proposed solution in (A5) to check whether (A5) holds. The left hand side of (A5) after substituting the proposed solution reduces to:

$$\begin{aligned}
&(r_H - \frac{ic}{k})\sum_{j=0}^{i-1}\frac{ic}{kr_H} - (\frac{ic}{k})\sum_{j=0}^{k-1}\frac{ic}{kr_H} - \frac{ic}{k}(1 - \frac{c}{kr_H}) && \text{(A7)}\\
&= r_H\frac{ic}{kr_H} - \frac{ic}{k}\left(\frac{kc}{kr_H}\right) - \frac{ic}{k}(1 - \frac{c}{kr_H}) && \text{(A8)}\\
&= \frac{ic}{k}(1 - \frac{c}{kr_H}) - \frac{ic}{k}(1 - \frac{c}{kr_H}) && \text{(A9)}\\
&= 0 && \text{(A10)}
\end{aligned}$$

which completes the proof. □

Lemma A3 *In equilibrium at least one firm invests zero with positive probability.*

Proof: Suppose not. Let i^* be the minimum value of i such that $p_H(ic/k) > 0$ and correspondingly let j^* be the minimum value of j such that $p_L(jc/k) > 0$. Without loss of generality let $i^* \leq j^*$ then:

$$\Pi_H(\frac{i^*c}{k}) = -\frac{i^*c}{k} < 0 \tag{A11}$$

However, firm H can do better by not investing at all. □

Lemma A4 *The equilibrium proposed in Proposition 1 is the unique equilibrium for at least one firm.*

Proof We know from Lemma A3 that at least one firm must invest zero with positive probability and therefore make zero in equilibrium. Without loss in generality assume that L makes zero in equilibrium. To prove uniqueness it is sufficient to show that the solution of the linear equations given by (A5) and (A6) is unique. These equations can be rewritten in matrix form as:

$$\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 & 1 \\ r_H - c/k & -c/k & \cdots & -c/k & -c/k & -c/k \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_H - c & r_H - c & \cdots & r_H - c & r_H - c & -c \end{pmatrix} \begin{pmatrix} p_L(0) \\ p_L(c/k) \\ . \\ p_L(c) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ . \\ 0 \end{pmatrix} \tag{A12}$$

If the determinant of the first matrix is non zero then we know that the solution is unique. Since the value of a determinant remains unchanged if we add a multiple of any column to another column, we can evaluate the determinant of the matrix by subtracting column $(i+1)$ from column (i). In this case the relevant determinant becomes:

$$\begin{vmatrix} 0 & 0 & \cdots & 0 & 1 & 1 \\ r_H & 0 & \cdots & 0 & 0 & -c/k \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & r_H & -c \end{vmatrix} = (-1)^k \begin{vmatrix} r_H & 0 & \cdots & 0 & 0 & 0 \\ 0 & r_H & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & r_H \end{vmatrix} = (-1)^k (r_H)^k \neq 0 \tag{A13}$$

where the first equality follows by expanding the determinant using the first row and the second equality follows from the properties of diagonal matrix. Thus, the equilibrium solution in this case is unique. □

Lemma A5 *Let i^* be such that it is the minimum value of i for which $p_H(ic/k) > 0$ and let j^* be defined similarly. If $i^* > 0$ then $p_L(i^*c/k) = 0$. Similarly, if $j^* > 0$ then $p_H(j^*c/k) = 0$.*

Proof: If H does not place any weight on zero then from Lemma A3 it follows that $p_L(0) > 0$. Now if L invests $i^*\frac{c}{k}$ then it wins with probability zero (since it either ties with H or invests less than H). Thus, by investing i^*c/k firm L makes $-i^*c/k < 0$. Thus, $p_L(i^*c/k) = 0$. A similar argument works for firm H. □

Lemma A6 *The equilibrium proposed in Proposition 1 is the unique equilibrium of the game.*

Proof: We know from Lemma A4 that at least for one firm the proposed equilibrium is the unique solution. We also know that this unique solution must be the one specified in Proposition 1. Without loss in generality assume that for firm L the proposed solution is the unique solution. Then there are two possibilities: firm H invests zero with positive probability or firm H does not invest an amount zero. In the first case, from Lemma A4 the solution proposed in Proposition 1 is the unique solution for firm H. If however $p_H(0) = 0$ then from Lemma A5 it follows that the equilibrium strategy for firm L must place a weight zero on i^*c/k where i^* is the minimum value of i for which $p_H(ic/k) > 0$. However, the unique solution for firm L requires that $p_L(ic/k) > 0$ for all i. Thus, in equilibrium $p_H(0) > 0$. Therefore the equilibrium proposed in Proposition 1 is also the unique equilibrium for firm H. □

Lemma A7 *The c.d.f. for the discrete game converges to:*

$$F_H(x) = \begin{cases} x/r_L & 0 \leq x < c \\ 1 & x \geq c. \end{cases} \tag{A14}$$

$$F_L(x) = \begin{cases} x/r_H & 0 \leq x < c \\ 1 & x \geq c \end{cases} \tag{A15}$$

as $k \to \infty$ which is the equilibrium solution for the continuous case.

Proof: Using the equilibrium solution for the discrete case it follows that:

$$\text{Prob}(x_L \leq x; k) = F_L^k(x) = \left(\frac{c}{kr_H}\right)K(x) \tag{A16}$$

where $K(x)$ is the integer which is closest to xc/k subject to the condition that $K(x) \leq xk/c$. It is easy to see that:

$$\left(\frac{xk}{c} - 1\right) \leq K(x) \leq \frac{xk}{c} \tag{A17}$$

This implies that:

$$\left(\frac{c}{kr_H}\right)\left(\frac{xk}{c} - 1\right) \leq F_L^k(x) \leq \left(\frac{c}{kr_H}\right)\left(\frac{xk}{c}\right) \tag{A18}$$

which becomes:

$$\frac{x}{r_H} - \frac{cr_H}{k} \leq F_L^k(x) \leq \frac{x}{r_H} \tag{A19}$$

Taking the limit as $k \to \infty$ the result follows. An identical argument establishes the c.d.f for firm H.□

Technical Appendix B

We establish the proposition through a series of lemmas. We first establish that there is no pure strategy equilibrium for this game. Then, we establish that the lower end of the support for both the firms must be zero and the upper end of support must be c. Later we establish that the mixed strategy equilibrium must be such that the c.d.f. for both firms are strictly increasing on their support. Finally we establish that the equilibrium profits are zero and both firms have no mass point at zero but a mass point at c. Using these observations we derive the equilibrium mixed strategy distributions.

Lemma B1 *There exists no pure strategy equilibrium in the game.*

Proof: Suppose firm i's equilibrium strategy is to invest x_i^* and assume that $x_i^* \geq x_j^*$. We will consider various cases.

Case 1: $x_i^* = x_j^*$

If $x_i^* = 0$ then both firms make zero profits and firm i can do better by investing ϵ and make profits $r_i - \epsilon > 0$. If $x_i^* > 0$ then both firms are making negative profits and can do better by deviating to zero. So this cannot be an equilibrium.

Case 2: $x_i^* > x_j^*$ *and* $x_i^* < c$

In this case firm j makes zero profits. Suppose firm j deviates to $x_i^* + \epsilon < c$ then it wins the patent and its profits are $r_j - x_i^* - \epsilon > r_j - c > 0$. Thus, firm j will deviate and this cannot be an equilibrium.

Case 3: $x_i^* > x_j^* > 0$ *and* $x_i^* = c$.

In this case firm j makes negative profits and can benefit by deviating to investing zero.

Case 4: $c = x_i^* > x_j^*$ *and* $x_j^* = 0$

In this case firm i can deviate to ϵ and make profits $r_i - \epsilon > r_i - c$. Thus, firm i will deviate. Thus, in all conditions a pure strategy solution cannot be an equilibrium. □

Now we will consider the case when firm i uses a mixed strategy with support S_i and the cumulative distribution function F_i. We will denote the mass points of the distribution at x by $\alpha_i(x)$. We define:

$$\begin{aligned} \overline{s}_i &= \sup(S_i) && \text{(B1)} \\ \underline{s}_i &= \inf(S_i) && \text{(B2)} \end{aligned}$$

Lemma B2 *If $F_i(x)$is strictly increasing over the interval (a,b) then $F_j(x)$, $j \neq i$ is also strictly increasing over the interval.*

Proof: Assume this is not true. Consider the situation in which $F_H(x)$ is strictly increasing over (a,b) and $F_L(x)$ is constant. We first consider the case when $\alpha_L(a) = 0$. Then we have:

$$\begin{aligned} \Pi_H^*(a; F_L(x)) &= F_L(a)(r_H - a) + (1 - F_L(a))(-a) && \text{(B3)} \\ &= F_L(a)r_H - a && \text{(B4)} \\ &= F_L(b)r_H - a && \text{(B5)} \\ &> F_L(b)r_H - b && \text{(B6)} \\ &= \Pi_H^*(b; F_L(x)) && \text{(B7)} \end{aligned}$$

Thus, firm H's profits at a are higher than at b which implies that b cannot be in the support of 1 which contradicts the equilibrium assumption. Now consider the case when $\alpha_L(a) > 0$. The arguments above do not necessarily apply since we may have $\text{Prob}(x_L < a) \neq \text{Prob}(x_L < b)$. However, since F_L is monotonically increasing by assumption, there must exist an ϵ such that $0 < \epsilon$ and $a + \epsilon < b$ such that $\alpha_L(a + \epsilon) = 0$ and the arguments as before imply that $F_H(x)$ must also be strictly increasing over the interval. □

Lemma B3 $\min\left(\underline{s}_H, \underline{s}_L\right) = 0$.

Proof: Assume without loss in generality that $\underline{s}_H \leq \underline{s}_L$. In this case:

$$\Pi_H(\underline{s}_H, F_L) = -\underline{s}_H \leq 0 \tag{B8}$$

Therefore in this case we must have $\underline{s}_H = 0$. □

Lemma B4 $\underline{s}_H = \underline{s}_L = 0$

Proof: Without loss of generality assume that $\underline{s}_H < \underline{s}_L$. From previous lemma $\underline{s}_H = 0$. First consider the case when $\alpha_H(0) = 0$. In this case, F_H is strictly increasing for some interval $(0, a)$ and from Lemma B2 it follows that $\underline{s}_L = 0$. If $\alpha_H(0) > 0$ and F_H is strictly increasing for some interval $[0, a)$ then again from Lemma B2 the result follows. So the only case we need to consider is when $\alpha_H(0) > 0$ and $F_H(x)$ is constant for $(0, \underline{s}_L)$ where $\underline{s}_L > 0$. We have two possibilities: $\alpha_H(\underline{s}_L) > 0$ or $\alpha_H(\underline{s}_L) = 0$. If $\alpha_H(\underline{s}_L) > 0$ then:

$$\Pi_H(\underline{s}_H; F_L) = -\underline{s}_H < 0 = \Pi_H(0; F_L) \tag{B9}$$

which violates the equilibrium condition. Thus, we must have $\alpha_H(\underline{s}_L) = 0$ and therefore there exists an ϵ such that $0 < \epsilon < \underline{s}_L$ such that $F_H(\epsilon) = F_H(\underline{s}_L)$. Thus, we have:

$$\begin{aligned}
\Pi_L(\underline{s}_L; F_H) &\leq F_H(\underline{s}_L) r_L - \underline{s}_L && \text{(B10)}\\
&= F_H(\epsilon) r_L - \underline{s}_L && \text{(B11)}\\
&< F_H(\epsilon) r_L - \epsilon && \text{(B12)}\\
&= \Pi_L(\epsilon; F_H) && \text{(B13)}
\end{aligned}$$

which violates the equilibrium assumption. □

Lemma B5 $\overline{s}_H = \overline{s}_L = \overline{s}$

Proof: Suppose (without loss in generality) that $\overline{s}_H < \overline{s}_L$ then there exists an ϵ such that $\overline{s}_H + \epsilon < \overline{s}_L$ and we have:

$$\begin{aligned}
\Pi_L(\overline{s}_L; F_H) &= r_L - \overline{s}_L && \text{(B14)}\\
&< r_L - \overline{s}_H - \epsilon && \text{(B15)}\\
&= \Pi_L(\overline{s}_H + \epsilon; F_H) && \text{(B16)}
\end{aligned}$$

which contradicts the equilibrium assumption. □

Lemma B6 *$F_i(x)$ is strictly increasing for $x \in [0, \overline{s}]$.*

Proof: If one of the distribution functions is increasing then from Lemma B2 both are. Therefore, we consider the case when both are constant over some interval (a, b) where a and b have positive densities. We first consider the case when $\alpha_i(a) = 0$ for at least one firm. Let $\alpha_H(a) = 0$. In this case $\text{Prob}(x_H < a) = \text{Prob}(x_H < b)$:

$$\begin{aligned}
\Pi_L(a; F_H) &= \text{Prob}(x_H < a)r_L - a && \text{(B17)}\\
&> \text{Prob}(x_H < a)r_L - b && \text{(B18)}\\
&= \text{Prob}(x_H < b)r_L - b && \text{(B19)}\\
&= \Pi_L(b; F_H) && \text{(B20)}
\end{aligned}$$

which violates the equilibrium assumption. To complete the proof we now consider the case when $\alpha_H(a) > 0$ and $\alpha_L(a) > 0$. In this case:

$$\Pi_L(a; F_H) = \text{Prob}(x_H < a)r_L - a \tag{B21}$$

Consider a strategy in which firm L charges $a + \epsilon$ with positive probability then for small ϵ we have:

$$\begin{aligned}
\Pi_L(a + \epsilon; F_H) &= \text{Prob}(x_H < a + \epsilon)r_L - a - \epsilon && \text{(B22)}\\
&> \text{Prob}(x_H < a)r_L - a - \epsilon && \text{(B23)}\\
&\simeq \text{Prob}(x_H < a)r_L - a && \text{(B24)}\\
&= \Pi_L(a; F_H) && \text{(B25)}
\end{aligned}$$

Thus, firm L can benefit by deviating which violates the equilibrium assumption. □

Lemma B7 *The equilibrium profits are zero. Further,* $\alpha_i(0) = 0$ *for* $i = 1, 2$.

Proof: The first part follows from Lemma B4. To see the second part consider the case when $\alpha_H(0) > 0$. Then, we have for small $\epsilon > 0$:

$$\Pi_L(\epsilon; F_H) \geq \alpha_H(0)r_H - \epsilon > 0 \tag{B26}$$

which contradicts the equilibrium assumption. □

Lemma B8 $\overline{s}_H = \overline{s}_L = c$. *Further,* $\alpha_i(c) > 0$ *for all* i.

Proof: To see the first part assume that it is not true and let $\overline{s}_H = \overline{s}_L = c$. We have:

$$\Pi_H(c; F_L) = r_H - c_1 > 0 = \Pi^*_H \tag{B27}$$

which contradicts the equilibrium assumption. To see that $\alpha_i(c) > 0$, we note that if it is not true and $\alpha_L(c) = 0$ then again $\Pi^*_H = r_H - c_1 > 0$ which contradicts Lemma B7. □

Lemma B9 $\alpha_i(s) = 0$ *for all* $s \in [0, c)$.

Proof: First note that mass points at zero are ruled out by Lemma B7. Therefore, consider a situation such that $\alpha_H(s) > 0$ where $s \in (0, c)$. From previous lemma we know that the equilibrium strategy has positive density at $s - \epsilon$ and $s + \epsilon$ for small ϵ. We have:

$$\Pi_L(s - \epsilon; F_H) = \text{Prob}(x < s - \epsilon) r_H - s + \epsilon \tag{B28}$$

and:

$$\Pi_L(s + \epsilon; F_H) = \text{Prob}(x_H < s + \epsilon) r_H - s - \epsilon \tag{B29}$$

This implies that:

$$\Pi_L(s + \epsilon; F_H) - \Pi_L(s - \epsilon; F_H) \simeq \alpha_H(s) r_H - 2\epsilon > 0 \tag{B30}$$

which contradicts the equilibrium assumption. □

We have therefore established that the equilibrium involves a mixed strategy with a continuous strictly increasing distribution over $[0, c)$ and mass points at c. Further, the equilibrium profits for both firms are zero. Therefore, for firm H we must have:

$$(r_L - x) F_H(x) - x\Big(1 - F_H(x)\Big) = 0 \tag{B31}$$

which implies that:

$$F_H(x) = \begin{cases} x/r_L & 0 \leq x < c \\ 1 & x \geq c. \end{cases} \tag{B32}$$

Note that firm H has a mass point $1 - c/r_L$ at $x = c$. Similarly, we see that:

$$F_L(x) = \begin{cases} x/r_H & 0 \leq x < c \\ 1 & x \geq c \end{cases} \tag{B33}$$

Again firm L has a mass point $1 - c/r_H$ at $x = c$.

Technical Appendix C

Equilibrium solution for the case with ties

First, it easy to see that there is no pure strategy solution. Further, for the parameters chosen there is no dominated strategy. Since 0 is not dominated each firm makes 0 profits. The relevant equations are:

$$(r_i - c/2)p_j(0) + (tr_i - c/2)p_j(c/2) - c/2p_j(c) = 0 \quad \text{(C1)}$$

$$(r_i - c)p_j(0) + (r_i - c)p_j(c/2) + (tr_i - c)p_j(c) = 0 \quad \text{(C2)}$$

$$p_j(0) + p_j(c/2) + p_j(c) = 1 \quad \text{(C3)}$$

where p represents the equilibrium probabilities and $i = \{L, H\}$ and $j = \{H, L\}$.

Substituting $c = 2$, $t = 1/2$, $r_L = 2.2$ and $r_H = 2.9$ in the above equations and solving we get:

$$p_L = (p_L(0), p_L(1), p_L(2)) = (0.3103, 0.0689, 0.6207)$$

$$p_H = (p_H(0), p_H(1), p_H(2)) = (0.0909, 0.7272, 0.1818)$$

The mean investment for L is given by:

$$\text{Mean}(L) = p_L(0) + p_L(c/2)c/2 + p_L(c)c = 1.3103 \quad \text{(C4)}$$

The probability that firm L wins is given by:

$$\text{Prob(L wins)} = p_L(c/2)p_H(0) + p_L(c)p_H(0) + p_H(c/2) = 0.5141 \quad \text{(C5)}$$

Similarily, we can calculate the probability that H wins which turns out to be 0.2946 and the mean investment of H which is 1.0909.

Analysis for the case when $c_H > c_L$ and $r_H = r_L = r$

Using arguments similar to that in Lemma B1 we can establish that there are no pure strategy solutions. We will drive the equilibrium solution for the case when the strategy space is continuous. We proceed to show that the equilibrium proposed in the text is a valid Nash equilibrium. The proposed equilibrium is:

$$F_L(x) = \begin{cases} (r - c_L)/r & \text{if } x = 0 \\ (r - c_L + x)/r & x \in (0, c_L] \\ 1 & \text{otherwise.} \end{cases} \tag{C6}$$

and:

$$F_H(x) = \begin{cases} x/r & x \in [0, c_L) \\ 1 & x \geq c_L \end{cases} \tag{C7}$$

Note that since firm L has a mass point at zero it makes zero profits. If firm H invests $x < c$ its expected profits are:

$$\Pi_H(x) = \text{Prob}(x_L < x)r - x \tag{C8}$$

$$= r\left(\frac{r - c_L + x}{r}\right) - x \tag{C9}$$

$$= r - c_L \tag{C10}$$

If firm H invests c_L it wins with probability one and makes profits of $(r - c_L)$. Thus, firm H makes $(r - c_L)$ for all possible investment levels in its support and therefore cannot benefit by deviating. Also note that firm H cannot do better by investing any amount greater than c_L since in this case its profits will be less than $(r - c_L)$.

The expected profits for firm L if it invests x are:

$$\Pi_L(x) = \text{Prob}(x_H < x)r - x \tag{C11}$$

$$= \left(\frac{c_L}{r}\right)r - c_L \tag{C12}$$

$$= 0 \tag{C13}$$

Thus, firm L makes zero profits for all levels of feasible investment. This establishes that the proposed equilibrium is a valid equilibrium.

To see that the H stochastically dominates L we notice that:

$$F_L(x) - F_H(x) = \frac{r - c_L}{r} > 0 \tag{C14}$$

Finally note that the probability that H will win is:

$$\text{Prob(H wins)} = \left(\frac{r - c_L}{r}\right) + \int_0^{c_L} \left(\frac{1}{r}\right)\left(\frac{r - x + c_L}{r}\right) dx \tag{C15}$$

The first term is the probability that firm H will invest c_L and the second term is the probability that firm H invests $x < c_L$ and firm L will invest less than x. This simplifies to:

$$\text{Prob(H wins)} = 1 - \frac{c_L^2}{2r^2} \tag{C16}$$

Similarly:

$$\text{Prob(L wins)} = \frac{c_L^2}{2r^2} \tag{C17}$$

We have:

$$\text{Prob(H wins)} - \text{Prob(L wins)} = \frac{(r - c_L)(r + c_L)}{r^2} > 0 \tag{C18}$$

Thus, in this case (i.e., when $c_H > c_L$) firm H is more likely to win the patent.

Technical Appendix D

Instructions for Subjects

You will participate today in a decision making experiment concerning competition between two firms. A research foundation interested in organizational behavior has contributed the funds for this study.

We are interested in studying how two firms compete with each other in developing and patenting a new product. We are simulating this common situation in the laboratory. You will represent a firm. Another subject will represent the competing firm.

The experiment involves many trials. At the beginning of each trial, you will be provided with some investment capital and then asked how much of it you wish to invest in the new product development project. The competing firm will also be provided the same amount of investment capital and asked to make similar investment decision

The rules of this investment game are simple. The capital invested by each firm in the development of the new product is non-recoverable. Therefore, once invested the money is lost irrespective of the outcome of the competition. The firm that invests more capital in the product development research will succeed in developing the new product, and receive a fixed reward. The fixed reward represents profit that you earn from marketing the patented new product. The losing firm receives nothing.

Experimental Procedure. As discussed above, 2 firms are competing to develop a product. At the beginning of each trial each player will be given some investment capital. All the players will receive the same investment capital and it remains unchanged from trial to trial. The investment capital will be stated in terms of an experimental currency called "francs," and at the end of the experiment your earnings will be converted to US dollars.

Once each player is allotted (endowed) some investment capital, he/she must decide how much to invest in his/her new product development research. You may invest any number of francs (including zero), provided your investment does not exceed your endowment (investment capital allotted for the trial). After all the players have made their investment decisions, privately and anonymously, the computer will compare the total investments made by the two competing players. The player who invests the larger amount will succeed in developing and patenting the new technology product, and will receive a reward of known size (in francs). The losing player will receive nothing. In this game ties will be counted as loses, as no firm can be considered a winner. In other words, if the investments made by both players are equal, then no reward will be given to any of the players. *Note that different players receive different rewards for winning the competition.*

As you can see from the game description, the individual payoffs for a trial are computed as follows:

Payoff of the winning player	=	endowment for the trial - investment made by the firm in the trial + reward
Payoff of the losing player	=	endowment for the trial - investment made by the firm in the trial

At the end of each trial the computer will display the following information:

1. The investments made by the winning and the losing players
2. The player winning the competition
3. Your payoff for the trial.

It is important to note that only you know your investment decisions, and you are taking these decisions under complete anonymity. On each trial you will compete against a different person in this room.

We are providing below two examples to help you understand how your payoff is computed at the end of each trial. In the first example both the players invest different amount of capital. In the second example both players invest the same amount of capital.

Example: Suppose the capital endowed to each subject at the beginning of a trial is 2 francs. Your reward for winning the competition is 4 francs, while your competitor's reward for winning the competition is 7 francs. Also suppose that you invest 2 francs and your competitor invests 1 francs. You invested more for developing the new product, so you win the competition. Your payoff in this trial will be:

Your payoff	= endowment - your investment + reward
	= 2 - 2 + 4 = 4 francs.
Your competitor's payoff	= endowment - your competitor's investment
	+ reward
	= 2 - 1 + 0 = 1 francs.

Now imagine that your competitor invests 2 francs as well. In this case there is a tie and the reward will not be awarded to either of the competing players. Your payoff in case of a tie will be as follows:

Your payoff	= endowment - your investment + reward
	= 2 - 2 + 0 = 0 francs.
Your competitor's payoff	= endowment - your partner's investment + reward
	= 2 - 2 + 0 = 0 francs.

This concludes the description of the decision task ahead of you. Paper and pencil are placed beside the computer terminal so that you may record the investments made by your group and the other group. At the end of the experiment, your accumulated payoff will be converted to US dollars at the conversion rate of 100 francs = 7.5 dollar. You will be asked to sign a receipt for the money, and complete a brief questionnaire before leaving the lab. We are required to retain some biographical information about you, as we are paying you for participating in this experiment. However, during the course of this experiment you will remain anonymous. If you have any questions, please raise your hand and the supervisor will assist you.

After all the participants have understood the instructions, we will start the computerized experiment. In order to help you become familiar with the decision task, you will go through five practice trials.

Thank you for participating in this experiment!

Technical Appendix E

Modeling the Choice of Product Standards

In high-technology markets product standards play an important role in determining the success of a product category (Robertson and Gatignon 1986) We present a parsimonious model of product standardization that involves asymmetric mixed strategies.[1] Consider a situation in which two firms (L,H) need to choose a product standard and are involved in negotiations to agree on a common standard. Choosing the same technology is beneficial to both firms. Yet, each firm has a vested interest in promoting its own technology as the standard. If firms fail to agree on a common standard they receive a payoff of zero. If technology L is agreed upon, then firm L has a payoff $a + r_L$ while firm H makes a where $a > 0$ and $r_L > 0$. Similarly, when technology H is chosen firm H's payoff is $a + r_H$ and firm L makes a. Thus, r_L and r_H represent the additional benefit that accrues to firm L and H, if their respective technology is chosen. To model asymmetric firms we assume that $r_H > r_L$. We model the process of standardization as a game of negotiation. Each firm makes an offer to the other firm that it is willing to adopt either Technology L or Technology H. If both firms agree on a common technology then a standard is set and firms receive payoffs based on the chosen technology. Otherwise, the game ends with no standardization and each firm receives zero payoff.[2] This model has a unique (non-degenerate) mixed strategy equilibrium.[3] Suppose firm L chooses L with probability $p_L(L)$ and chooses H with probability $p_L(H)$ then

[1] An elaborate model of standard setting is beyond the scope of this paper. See Katz and Shapiro (1985, 1986), Farrell and Saloner (1985, 1986, 1988), and Besen and Farrell (1994) for diverse models of product standards.

[2] For a dynamic version of this game, though involving symmetric players, see Farrell and Saloner (1985).

[3] There are also two asymmetric pure strategy equilibrium. However, pure strategy equilibrium in games of coordination are less credible since they assume that firms can a priori choose one of the two equilibria and also that there would be no coordination failure. See also Cooper, Dejong, Forsythe and Ross(1990), Van Huyck, Battalio and Beil(1990) and Cooper, Dejong, Forsythe and Ross(1996) for experimental evidence that subjects find it difficult to coordinate to an equilibrium.

firm L's strategy should be such that firm H is indifferent between choosing H and L. If firm H chooses H its expected payoffs are:

$$\Pi_H(H) = (a + r_H)(1 - p_L(L)) \tag{E1}$$

where $(1 - p_L(L))$ is the probability that firm L will choose H. Similarly, firm H's expected payoffs when it chooses L are:

$$\Pi_H(L) = ap_L(L) \tag{E2}$$

Equating the two profits in (E1) and (E2) we get $p_L(L)$ which is given by:

$$p_L(L) = \frac{a + r_H}{2a + r_H} \tag{E3}$$

Similarly, firm H chooses H with probability $p_H(H)$ which is given by:

$$p_H(H) = \frac{a + r_L}{2a + r_L} \tag{E4}$$

Since $r_H > r_L$, it follows that $p_H(H) < p_L(L)$. Consequently, firm L's technology is more likely to be chosen. Thus, we find that if firms are allowed to non cooperatively choose product standards, then the firm with less vested interest in a particular standard is more likely to prevail.

Finally, we note that the above game is the asymmetric version of the well known "Game of Chicken" and the "War of Attrition" (Fudenberg and Tirole 1991, p. 119). This structure can therefore be used in a variety of situations where firms need to coordinate their actions – for example in models of negotiation, models of entry and exit, and models of licensing.

Technical Appendix F

Modeling Consumer Search

Research in consumer information search has a long history in marketing (see Moorthy, Ratchford and Talukdar 1997 for a recent paper and a review of this literature[1]). Our purpose is to highlight some implications of asymmetric mixed strategy games in consumer search. Consider a market with one dominant firm and a continuum of homogeneous consumers. The dominant firm produces a product which has a valuation v for the consumer. There are also multiple fringe firms in the market. These fringe firms sell a product of lesser value v_L at its marginal cost.[2] For simplicity, we equate the marginal cost for the product to zero. The consumer is aware of the dominant firm, but needs to search in order to locate the fringe firms. The consumer has an option either to invest in searching which costs her c, or to refrain from searching. We assume that the consumer can only search once and cannot search for fringe store after it has visited the dominant firm. This is a strong assumption but our purpose is not to develop a general model of search but to illustrate how mixed strategies can potentially play a role in such situations.[3] For a more elaborate model of search which involve mixed strategies see, for example, Wilde (1992). If the consumer knows the location of the fringe firm and the firm charges a price greater than $v - v_L$ she prefers to buy from the fringe firms; otherwise, she buys from the dominant firm. However, if the consumer has not searched she is willing to pay up to v for the product. Thus, the firm has two options: either it can charge v or $v - v_L$. If the firm charges v and the consumer refuses to buy then the firm incurs inventory carrying costs K. In this case it is easy to see that there is only a mixed strategy solution. In equilibrium, the firm charges a low price (i.e., $v - v_L$) with probability

[1]See also Stigler(1961), Rothschild (1973), Weitzman (1979), Ratchford (1982), and Wilde(1992).

[2]This approach of modeling dominant firms and fringe firms is similar to the approach used by Salop and Scheffman(1983).

[3]This assumption has some empirical support. Empirical studies show that even for major purchases many consumers only visit one store (Wilkie and Dickson 1985).

$p_f(L)$ and the consumer searches with probability $q_c(s)$. In equilibrium, $q_c(s)$ is such that the firm is indifferent between charging the high and the low price. If the consumer searches with probability $q_c(s)$ and the firm charges a high price, then the firm's expected profits are:

$$\Pi_f(H) = -Kq_c(s) + (1 - q_c(s))v \tag{F1}$$

where the first term in (F1) represents the (negative) profits when the firm charges a high price and the consumer is aware of the fringe firm, and the second term is the profit that the firm makes if the consumer is unaware of the fringe firm. If the firm charges a low price its expected profits are:

$$\Pi_f(L) = (v - v_L) \tag{F2}$$

Equating the profits in (F1) and (F2) we have:

$$q_c(s) = \frac{v_L}{K + v} \tag{F3}$$

Similarly, we get:

$$p_f(L) = \frac{v_L - c}{v_L} \tag{F4}$$

Thus, the dominant firm's actions are independent of its own inventory carrying costs. However, as the search cost of the consumer increases the firm charges a higher price more frequently. Similarly, the consumer's search strategy is independent of his/her search costs but the consumer searches less as the inventory cost of the dominant firm increases. The model can also be adapted to consider enforcement of copyright laws, and monitoring issues in the context of sales force and partners in strategic alliances.

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