Deconstructing the Pioneer's Advantage: Examining Vintage Effects and Consumer Valuations of Quality and Variety

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Appendix

PROPOSITION A1. *For the sequential entry, variety-quality model, define a measure of the cost-of-quality difference between the two firms as:*

$$Q = \frac{b^2}{4a}\left(\frac{1}{c_1} - \frac{1}{c_2}\right)$$

Note that Q is positive if the pioneer (Firm 1) has a lower cost-of-quality than the later entrant (Firm 2). The unique, pure-strategy equilibrium is:

For $Q \le -25/16$, *the later entrant captures the entire market (the pioneer will exit the market).*

For $Q \ge 5/2$, *the pioneer captures the entire market (the later entrant will not enter).*

For $-25/16 < Q < 5/2$, *the equilibrium quality levels, prices and demand levels are:*

$$q_1^* = b/2c_1 \qquad q_2^* = b/2c_2$$

$$p_1^* - c_1 q_1^{*2} = a\left(2 + y_1^* + y_2^* + Q\right)/3 \qquad p_2^* - c_2 q_2^{*2} = a\left(4 - y_1^* - y_2^* - Q\right)/3$$

$$Demand_1^* = \left(2 + y_1^* + y_2^* + Q\right)/6 \qquad Demand_2^* = \left(4 - y_1^* - y_2^* - Q\right)/6$$

The equilibrium variety levels are given by:

For $-25/16 < Q \le \sqrt{18} - 7/2$: $\quad y_1^* = 15 - Q - 6\sqrt{6 - Q}$ *and* $y_2^* = 1$

For $\sqrt{18} - 7/2 \le Q \le 2\left(\sqrt{18} - 3\right)/3$: $\quad y_1^* = 0.5$ *and* $y_2^* = \sqrt{18} - Q - 5/2$

For $2\left(\sqrt{18} - 3\right)/3 \le Q < 5/2$: $\quad y_1^* = 0.5$ *and* $y_2^* = 3\sqrt{4Q + 26} - Q - 29/2$

Given the equilibrium quality levels, the parameter Q can be expressed as

$$Q = \frac{b}{2a}\left(q_1^* - q_2^*\right)$$

PROOF OF PROPOSITION A1. Solve by backward induction, considering only pure strategies.[i] For the pricing subgame (the last stage of the game), the equilibrium solution is similar to that in d'Aspremont et al. (1979) for the linear Hotelling model; namely, the equilibrium prices must be such that neither firm has an incentive to price-undercut its rival to capture the entire market, and each firm will have a positive market share in equilibrium. First find the demand faced by each firm, given every consumer buys a single unit. The consumer indifferent between the two products is located at:

$$\theta_0 = \frac{1}{2a}\left(ay_1 + ay_2 + p_2 - p_1 + bq_1 - bq_2\right) \qquad \text{for } y_1 \le \theta_0 \le y_2$$

Firm 1 will capture the entire market if $U_1(\theta) > U_2(\theta)\ \forall\theta$, and likewise firm 2 will capture the entire market if $U_2(\theta) > U_1(\theta)\ \forall\theta$. Firm 1's demand is therefore

$$D_1 = \begin{cases} 1 & \text{if } p_1 < p_2 - a(y_2 - y_1) + b(q_1 - q_2) \\ 0 & \text{if } p_1 > p_2 + a(y_2 - y_1) + b(q_1 - q_2) \\ \theta_0 & \text{otherwise} \end{cases}$$

and demand of firm 2 is obviously $D_2 = 1 - D_1$. The two firms will have no incentive to deviate in prices in the pricing stage if:

$$\pi_1\left(y_1, y_2, q_1, q_2, p_1^*, p_2^*\right) > p_2^* - a(y_2 - y_1) + b(q_1 - q_2) - c_1 q_1^2$$
$$\pi_2\left(y_1, y_2, q_1, q_2, p_1^*, p_2^*\right) > p_1^* - a(y_2 - y_1) - b(q_1 - q_2) - c_2 q_2^2$$

Assuming these conditions hold, each firm sets price to maximize profits, which gives

$$p_1^* = \left[a(2 + y_1 + y_2) + b(q_1 - q_2) + 2c_1 q_1^2 + c_2 q_2^2\right]/3$$
$$p_2^* = \left[a(4 - y_1 - y_2) - b(q_1 - q_2) + 2c_2 q_2^2 + c_1 q_1^2\right]/3$$

The corresponding profit functions then become

$$\pi_1\left(y_1, y_2, q_1, q_2, p_1^*, p_2^*\right) = \frac{1}{18a}\left[a(2 + y_1 + y_2) + b(q_1 - q_2) - c_1 q_1^2 + c_2 q_2^2\right]^2$$

$$\pi_2\left(y_1, y_2, q_1, q_2, p_1^*, p_2^*\right) = \frac{1}{18a}\left[a(4 - y_1 - y_2) - b(q_1 - q_2) - c_2 q_2^2 + c_1 q_1^2\right]^2$$

Consider now the location and quality stage for Firm 2. The optimal quality level is:

$$q_2^* = b/2c_2$$

Firm 2's first-order condition of maximizing profits with respect to variety location y is strictly negative, such that it would like to locate as close to Firm 1 as possible. Similarly, for Firm 1's location and quality stage, the optimal quality is:

$$q_1^* = b/2c_1$$

and Firm 1 wishes to locate as close to Firm 2 as possible. Profits for both firms are strictly concave in quality, and there is no strategic distortion in that optimal quality levels are strictly a function of a firm's own cost-of-quality. Therefore, if the firms' variety locations are such that the pricing non-deviation constraints hold, a pure-strategy equilibrium will exist. We thus find optimal variety locations such that the price non-deviation constraints are satisfied and that neither firm has an incentive to deviate in variety in order to steal the entire market. We add the reasonable technical assumption that if a firm deviates in its variety location such that it then has an incentive to try and capture the entire market, the opposing firm will retaliate in the pricing stage by cutting price to marginal cost (e.g., see Novshek 1980). This condition merely states that a firm will not deviate in variety location and attempt to capture the entire market unless and until the opposing firm's price retaliation threat is no longer credible. A firm's retaliation threat is not credible if the firm would need to price below marginal cost to dissuade the competing firm from deviating.

Define the parameter Q as

$$Q = \left[b(q_1 - q_2) - c_1 q_1^2 + c_2 q_2^2\right]/a$$

The non-deviation conditions for (p_1^*, p_2^*) to be a price equilibrium can be written as:

$$(2 + y_1 + y_2 + Q)^2 \geq 12(2 + y_1 - 2y_2 + Q) \qquad \text{and}$$
$$(4 - y_1 - y_2 - Q)^2 \geq 12(1 + 2y_1 - y_2 - Q)$$

It is easy to show that the second condition, corresponding to Firm 2 having no incentive to price undercut, is critical whenever

$$y_1 + y_2 \geq 1 + Q/2$$

Further note that, although Firm 1 would like to make y_1+y_2 as large as possible, Firm 1 will never choose $y_1 > 1/2$. If it did so, Firm 2 would locate on the opposite side of the variety line ($y_2 < 1/2$) and Firm 1 will be worse off. Consistent with our 2-firm game with accommodating entry, we only consider variety locations where $y_1 \leq 1/2$ and $y_2 \geq 1/2$.

Firm 1's preemptive first-mover advantage lies in gaining an advantageous variety location over its later entrant competitor. Consider a range of Q such that Firm 2 is forced to take the extreme variety position $y_2 = 1$. Firm 1 will choose a variety level such that the second non-deviation constraint binds, or

$$y_1^2 + (2Q - 30)y_1 + (9 + 6Q + Q^2) = 0 \Rightarrow y_1^* = 15 - Q - 6\sqrt{6 - Q}$$

This optimal variety location for Firm 1 is valid until the firm can locate in the middle of the variety line, which occurs at $Q = \sqrt{18} - 7/2$. For smaller values of Q, the threat of retaliation gives Firm 2 no incentive to deviate in variety location as long as $Q > -25/16$. For $Q \le -25/16$ the pioneer has such a quality disadvantage that its threat of price retaliation is no longer credible, and Firm 2 can locate at the variety midpoint and capture the entire market (Firm 1 will exit). Thus, for the range $-25/16 < Q \le \sqrt{18} - 7/2$, $y_2^* = 1$ and $y_1^* = 15 - Q - 6\sqrt{6 - Q}$, and it is easy to verify that the non-deviation conditions hold and that both firms get positive profits.

For $Q \ge \sqrt{18} - 7/2$ Firm 1 stays at the variety middle by assumption, such that the binding non-deviation constraint is used to determine Firm 2's variety location.[ii] For the region of Q where the second non-deviation condition still binds we find that

$$y_1^* = 1/2 \text{ and } y_2^* = \sqrt{18} - Q - 5/2 \text{ for } \sqrt{18} - 7/2 \le Q \le 2(\sqrt{18} - 3)/3$$

For larger values of Q, the critical constraint switches to Firm 1 having no incentive to price undercut. We then find that

$$y_1^* = 1/2 \text{ and } y_2^* = 3\sqrt{4Q + 26} - Q - 29/2 \text{ for } 2(\sqrt{18} - 3)/3 \le Q < 5/2$$

noting that Firm 2 cannot receive positive profits whenever $Q \ge 5/2$. One can again verify that the non-deviation constraints are satisfied, and that neither firm has an incentive to deviate in variety locations since the threat of price retaliation is credible. Since the non-deviation conditions are satisfied and one of the constraints always binds, uniqueness is assured by strictly concave profit functions in price and quality. Comparative statics results of Propositions 1a and 1b follow directly from the optimal solution. Q.E.D.

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[i] The issue of mixed-strategy equilibria for Hotelling and other models with discontinuous payoff functions is considered by Dasgupta and Maskin (1986).

[ii] Our assumption that the pioneer stays at the variety midpoint results in profits being a constant in this region of Q. Note this is clearly due to the manner in which the variety distribution is modeled. Alternatively, we could assume that the pioneer may locate at $y > 0.5$ and introduce (at very low cost) one or more products at the lower variety levels, preventing the later entrant from entering there. This would make the later entrant always locate at $y = 1$, while the pioneer's optimal location (and profits) will be increasing in Q. See Tyagi (2000) for comparison. The intuition and fundamental results of the model remain unchanged, however. This constant-profit occurrence is typical of linear variety on a finite line. Indeed, for the linear Hotelling model discussed in d'Aspremont et al. (1979), assuming the two firms are located symmetrically on the Hotelling line in the outer quartiles (such that price equilibrium exists), the firms will have identical profits regardless of which locations are considered.