

Costly Bidding in Online Markets for IT Services

August 2003

Eli M. Snir

esnir@mail.cox.smu.edu

SMU, Cox School of Business, ITOM Department
374 Crow
Dallas, TX 75275-0333

Lorin M. Hitt

lhitt@wharton.upenn.edu

University of Pennsylvania, Wharton School
571 Jon M. Huntsman Hall
Philadelphia, PA 19104

We would like to thank Eric Clemons, David Croson, Paul Kleindorfer, Moti Levi, Dennis Yao and seminar participants at Case Western Reserve University, INSEAD, MIT, Southern Methodist University, Stanford, the Technion – Israel Institute of Technology, the University of Texas at Dallas, the Twelfth Annual Workshop on Information Systems and Economics (WISE), two anonymous reviewers, and the Special Issue Editors and Associate Editors for helpful comments on previous drafts of this paper. All remaining errors and omissions are solely our responsibility. This material is based on work supported by the National Science Foundation under Grant No. IIS-9733877.

Appendix 2: Proofs

Proof of Proposition 1:

The following bidding strategy of price as a function of quality, $p^*(q)$, is shown to be an equilibrium¹⁷ which satisfies conditions (a), (b) and (c) in Proposition 1:

$$p^*(q) = \left\{ \begin{array}{ll} \text{No-bid} & q < q_m \\ vq_m & q = q_m \\ cq + \frac{(v-c) \int_{q_m}^q [F(z)]^{n-1} dz}{[F(q)]^{n-1}} + \frac{c_T}{[F(q)]^{n-1}} & q > q_m \end{array} \right\} \quad (\text{A.1})$$

where q_m is defined by: $(vq_m - cq_m)[F(q_m)]^{n-1} - c_T = 0$

We begin by showing that the surplus offered by the vendor to the buyer under this strategy is increasing in vendor quality, which guarantees that the highest quality vendor always wins the auction. Let buyer surplus be represented by: $S(q, p^*(q)) = vq - p^*(q)$, then for $q > q_m$

$$S(q, p^*(q)) = vq - cq - \frac{(v-c) \int_{q_m}^q [F(z)]^{n-1} dz + c_T}{[F(q)]^{n-1}} \quad (\text{A.2})$$

$S(q, p^*(q))$ is continuous at $q=q_m$ from the definition of q_m and for other values of q by inspection.

Taking the derivative w.r.t. q yields:

$$\begin{aligned} \frac{\partial S(q, p^*(q))}{\partial q} &= \\ (v-c) - \frac{(v-c)([F(q)]^{n-1})^2 - [(v-c) \int_{q_m}^q [F(z)]^{n-1} dz + c_T](n-1)[F(q)]^{n-2} f(q)}{([F(q)]^{n-1})^2} &= \\ + \frac{[(v-c) \int_{q_m}^q [F(z)]^{n-1} dz + c_T]}{[F(q)]^n} (n-1)f(q) &> 0 \end{aligned}$$

This observation, which commonly arises or is assumed in the auctions literature (Riley and Samuelson, 1981; Samuelson, 1985; McAfee and McMillan, 1987; and in this setting see Carr, 2003), enables a simple characterization of the probability of a vendor winning the auction. A vendor of quality q wins the auction, only if all other vendors are of lower quality. This occurs with probability $P_q(q, p_q, F(q), n) = [F(q)]^{n-1}$. The profit for a vendor of type q is therefore:

$$\mathbf{p}(q) = (p(q) - cq)P_q(q, p_q, F(q), n) - c_T$$

$$\text{or: } \mathbf{p}(q) = (p(q) - cq)[F(q)]^{n-1} - c_T$$

Substituting bid price from A.1 into the profit expression yields:

$$\mathbf{p}(p^*(q)) = \left\{ \begin{array}{ll} 0 & q < q_m \\ 0 & q = q_m \\ (v-c) \int_{q_m}^q [F(z)]^{n-1} dz & q > q_m \end{array} \right.$$

which is continuous in q . Continuity at $q=q_m$ is immediate.

Taking the derivative of profit with respect to quality shows that $\frac{\partial \mathbf{p}}{\partial q} = 0$ for $q \leq q_m$ and

$$\frac{\partial \mathbf{p}(p^*(q))}{\partial q} = (v-c)[F(q)]^{n-1} > 0 \text{ for } q > q_m. \text{ Thus, under this strategy, profits are weakly}$$

monotone in quality. There exists a lowest quality level that earns non-negative profits given by the solution to $(vq_m - cq_m)[F(q_m)]^{n-1} - c_T = 0$. Profits are monotonically increasing above this threshold. Note that a vendor of quality q_m offers a price equal to the buyer's reservation value (vq_m), extracting all surplus from the buyer, if awarded the contract.

Finally, we complete the proof by showing that indeed the price strategy asserted in A.1 is a Nash equilibrium. Consider a vendor of type q who is choosing an optimal bidding strategy when all other $(n-1)$ vendors are bidding according to $p^*(q)$ as described by A.1. Because buyer surplus is monotone and continuous in quality under strategy $p^*(q)$ for all bidding vendors, there exists a mapping between vendor quality and buyer surplus. A vendor of quality q , in choosing his best response, can choose to offer the buyer surplus equivalent to their actual quality (q) or

¹⁷ The equilibrium can be constructed following the methodology in Snir (2000).

offer surplus equivalent to any other quality (denote this alternate quality by w). For $p^*(q)$ to be an equilibrium, the optimal choice for vendor of type q , masquerading as w , is to choose $w=q$ (i.e., $\frac{\partial \mathbf{p}(q,w)}{\partial w} = 0$ at $w=q$). As before, our candidate vendor wins the auction only when all the other vendors bid such that they offer less surplus. Because of the mapping from vendor quality to buyer surplus, this happens with probability $[F(w)]^{n-1}$. Therefore, the vendor faces a probability $[F(w)]^{n-1}$ of winning the contract. A vendor at or below the threshold q_m earns negative expected profit for any acceptable bid, so we can restrict our attention to $q \geq q_m$. In this context, the profit for a vendor of type q , when bidding as w , can be written as:

$$\mathbf{p}(w,q) = vq[F(w)]^{n-1} - S(w)[F(w)]^{n-1} - cq[F(w)]^{n-1} - c_T$$

The first term is the social value from vendor q bidding as w . The second term is the buyer's expected surplus from this bid, and the third term is the vendor's expected cost. The expected profit is derived by subtracting bidding cost. Rearranging:

$$\mathbf{p}(w,q) = (v-c)q[F(w)]^{n-1} - B(w) - c_T \quad (\text{A.3})$$

$$\text{Where } B(w) \equiv S(w)[F(w)]^{n-1}$$

We need to show that:

$$\frac{\partial \mathbf{p}(w,q)}{\partial w} = 0 = (v-c)q \frac{d}{dw} [F(w)]^{n-1} - B'(w) \quad \text{at } w=q \quad (\text{A.4})$$

Note that at $w=q$:

$$B(q) = (vq - p(q))[F(q)]^{n-1}$$

Inserting $p^*(q)$ we have

$$B(q) = (v-c)q[F(q)]^{n-1} + (v-c) \int_{q_m}^q [F(z)]^{n-1} dz - c_T$$

Therefore:

$$B'(q) = (v-c)[F(q)]^{n-1} + (v-c)q \frac{d}{dq} [F(q)]^{n-1} - (v-c)[F(q)]^{n-1}$$

$$B'(q) = (v-c)q \frac{d}{dq} [F(q)]^{n-1} \quad (\text{A.5})$$

Substituting A.5 back into A.4 yields the required result.

To complete the proof we need to show that $\mathbf{p}(w,q)$ attains a global maximum at $w = q$. From A.4 and A.5:

$$\frac{\partial \mathbf{p}(w, q)}{\partial w} = (v - c)(q - w) \frac{d}{dw} [F(w)]^{n-1} = (v - c)(q - w) F'(w) [F(w)]^{n-2}$$

By assumption $v > c$, $n > 1$, $F(w) > 0$, $F'(w) > 0$. Therefore, $\text{sign} \frac{\partial \mathbf{p}(w, q)}{\partial w} = \text{sign}(q - w)$ which implies that $w < q$ the profit function is increasing, $w > q$ the profit function is decreasing, and zero only at $w = q$. Therefore the profit function attains a unique global maximum at $w = q$.

Q.E.D.

Proof of Proposition 2:

In the equilibrium described in Proposition 1, higher quality vendors offer more surplus to the buyer, and the highest quality vendor is awarded the contract. From equation (3) in subsection III.B.:

$$\mathbf{p}(q_m) = (v q_m - c q_m) [F(q_m)]^{n-1} - c_T = 0$$

To show the $q_m > q_L$ we show that for the lowest quality expected profit from bidding according to $p^*(q)$ is negative, while for the highest quality, expected profit is positive. Coupled with monotonicity and continuity of $\mathbf{p}(q_m)$ (from Proposition 1) and the Fixed-point Theorem, this assures that there exists a break-even quality $q_m \in (\underline{q}, \bar{q}]$.

For the lowest quality $F(\underline{q}) = 0$, $\mathbf{p}(\underline{q}) = -c_T$. For the highest quality $F(\bar{q}) = 1$. When only the highest quality participates, $\mathbf{p}(\bar{q}) = (v \bar{q} - c \bar{q}) - c_T > 0$ by assumption.

The other results arise from investigating the expected revenue for the break-even quality bidder, $(v q_m - c q_m) [F(q_m)]^{n-1}$, and the cost, c_T , of submitting a bid.

Define a given parameter vector (n^0, c^0, v^0, c_T^0) such that $\mathbf{p}(q_m^0) = 0$.

If $n^1 > n^0$ then $[F(q_m^0)]^{n^1-1} < [F(q_m^0)]^{n^0-1}$ and $\mathbf{p}(q_m^0) < 0$

If $c^1 > c^0$ then $(v q_m^0 - c^1 q_m^0) < (v q_m^0 - c^0 q_m^0)$ and $\mathbf{p}(q_m^0) < 0$

If $v^1 < v^0$ then $(v^1 q_m^0 - c q_m^0) < (v^0 q_m^0 - c q_m^0)$ and $\mathbf{p}(q_m^0) < 0$

If $c_T^1 > c_T^0$ then $\mathbf{p}(q_m^0) < 0$

For any of these changes (an increase in n , c , c_T or a decrease in ν), the vendor of quality q_m^0 earns negative profit from participating. From the monotonicity of $\mathbf{p}(q)$ and $\mathbf{p}(\bar{q}) > 0$, the break-even quality level increases under each of these changes.

QED