Online Appendix for "Mass Customization versus Mass Production: Variety and Price Competition"

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Proof of Proposition 1. First, the market segment captured by a standard product is contiguous: if we take any two locations served by a product, all locations in between must also be served by the same product. Second, the delivered price at boundary points between any two adjacent market segments and at end points (0 and 1) must be at least $\bar{p}$. Suppose not. In both cases (interior point or end point) the associated product(s) can be replaced by products that are priced higher and that cover exactly the same market area(s). Third, market segments captured by standard products must be of equal length and all products must be priced the same. Suppose not; i.e., in MP's optimal product line design, there exist two neighboring products that cover market segments of unequal length a and $l-a$ ($a \neq l/2$), and that are priced differently at $p_1 = \bar{p} - da/2$ and $p_2 = \bar{p} - d\left(l-a\right)/2$, respectively. (Higher prices would not allow the given segment lengths; lower prices are not optimal.) Without loss of generality, assume $0 < a < l-a < l < 1$. MP's profit from these two products is $\triangle\pi_p(a) = (\bar{p} - c_p)\lambda l - \frac{\lambda d}{2}\left[a^2 + (l-a)^2\right]$. We can rearrange these two segments without affecting any other segment by varying a. Solving $\max\limits_{0 \leq a \leq l/2} \triangle\pi_p(a)$ gives $a^* = l/2$. That is, MP is better off by replacing the two unequal segments with two equal segments. This is accomplished by equalizing the prices at $p_1 = p_2 = \bar{p} - dl/4$ while keeping the total coverage of the market segments the same. This improvement affects no other segments. Therefore the optimal product line design cannot have unequal segments. There may be alternative optima for the product locations. The vector of locations symmetric around the center, $\mathbf{x} = \left(\frac{1}{2n}, \frac{3}{2n}, \cdots, \frac{2n-1}{2n}\right)$, is optimal; but in some regions of parameter values, $\mathbf{x}$ (or subsets of $\mathbf{x}$) can be shifted within the bounds of the product space (at most by $\pm\left|\frac{1}{2n} - \left(\frac{\bar{p}-p_1}{d}\right)\right|$) without reducing profits. ■

Proof of Proposition 2. MP's pricing problem can be restated as: $\max_{p_p} \pi_p(f, n, p_p) = (p_p - c_p)\lambda n y_p - nf - F_p(f)$ subject to $\bar{p} - d/2n \leq p_p \leq \bar{p}$ and $y_p = 2(\bar{p} - p_p)/d$. The expression for y_p in the second constraint is valid only for the range of prices in the first constraint. This restriction is justified by two simple observations. First, MP would never reduce p_p below $\bar{p} - d/2n$ because it already captures the entire market at that price. Second, with $p_p > \bar{p}$, MP attracts no demand. Substituting y_p, as defined by the second constraint above, into π_p, one can verify that π_p is concave in p_p. The result then follows from the Lagrangean method. ■

Proof of Proposition 3. Our solution strategy is to decompose the problem into two sub-problems based on the two regions of the objective function, solve them separately, and combine the solutions. The two sub-problems are:

$(M1)$: $\max \pi_p(f, n, p_p^*) = (\bar{p} - c_p - \frac{d}{2n})\lambda - nf - F_p(f)$ subject to $n \geq \frac{d}{\bar{p}-c_p}$ and $n \geq 1$.

$(M2)$: $\max \pi_p(f, n, p_p^*) = \frac{\lambda n}{2d}(\bar{p} - c_p)^2 - nf - F_p(f)$ subject to $n \leq \frac{d}{\bar{p}-c_p}$ and $n \geq 1$.

Let n_1^* and n_2^* be the optimal solutions to $(M1)$ and $(M2)$. The objective function of $(M1)$ is strictly concave in n, while that of $(M2)$ is linear in n. The Lagrangean method gives:

$$n_1^* = \begin{cases} \sqrt{\frac{\lambda d}{2f}} & \text{, if } \; f < \min\left\{\frac{\lambda d}{2}, \frac{\lambda(\bar{p}-c_p)^2}{2d}\right\} \\ \max\left(1, \frac{d}{\bar{p}-c_p}\right) & \text{, otherwise} \end{cases}$$

$$n_2^* = \begin{cases} \frac{d}{\bar{p}-c_p} & \text{, if } f < \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \leq c_p + d \\ \in \left[1, \frac{d}{\bar{p}-c_p}\right] & \text{, if } f = \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \leq c_p + d \\ 1 & \text{, if } f > \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \leq c_p + d \end{cases}$$

The result follows from combining these, choosing the better solution in case of overlap. ■

Proof of Proposition 4. Suppose that MP knows MC's price, p_c ($c_c \leq p_c \leq \bar{p}$), hence solves the following problem to set its own price:

$$\max_{c_p \leq p_p \leq \bar{p}} \quad \pi_p(p_c, f, n, p_p) = (p_p - c_p)\lambda n y_p - nf - F_p(f)$$

$$st. \quad y_p = \begin{cases} 1/n & \text{, if } p_p \leq p_c - d/2n \\ 2(p_c - p_p)/d & \text{, if } p_c - d/2n \leq p_p \leq p_c \\ 0 & \text{, if } p_p \geq p_c \end{cases}$$

Employing the same arguments as in Proposition 2, this problem can be further restricted to the mid-region where $p_c - d/2n \leq p_p \leq p_c$. Using the Lagrangean method, we obtain MP's best response function:

$$p_p^*(n, p_c) = \begin{cases} p_c - d/2n & \text{, if } p_c \geq c_p + d/n \\ (p_c + c_p)/2 & \text{, if } c_p < p_c < c_p + d/n \\ \in [c_p, \bar{p}] & \text{, if } p_c \leq c_p \end{cases}$$

Suppose now that MC knows MP's price, p_p ($c_p \leq p_p \leq \bar{p}$), hence solves:

$$\max_{c_c \leq p_c \leq \bar{p}} \quad \pi_c(p_c, f, n, p_p) = (p_c - c_c)\lambda y_c - F_c$$

$$st. \quad y_c = \begin{cases} 1 & \text{, if } p_c \leq p_p \\ 1 - 2n(p_c - p_p)/d & \text{, if } p_p \leq p_c \leq p_p + d/2n \text{ and } p_c \leq \bar{p} \\ 0 & \text{, if } p_p + d/2n \leq p_c \leq \bar{p} \end{cases}$$

Using exactly the same approach, we obtain MC's best response function:

$$p_c^*(n,p_p)=\begin{cases} p_p & \text{, if } p_p \geq c_c + d/2n \\ \frac{d}{4n}+\frac{p_p+c_c}{2} & \text{, if } c_c - d/2n < p_p < c_c + d/2n \text{ , } p_p < 2\bar{p} - c_c - d/2n \\ \bar{p} & \text{, if } p_p \geq 2\bar{p} - c_c - d/2n \\ \in [c_c, \bar{p}] & \text{, if } p_p \leq c_c - d/2n \end{cases}$$

Given the two best response functions, we find the equilibrium pairs $(\hat{p}_c, \hat{p}_p)$ by solving $\hat{p}_p = p_p^*(n, \hat{p}_c)$ and $\hat{p}_c = p_c^*(n, \hat{p}_p)$ simultaneously. ■

Proof of Proposition 5. We decompose the problem into four sub-problems based on the regions $(\eta 1)$ - $(\eta 4)$, solve these separately, and combine their solutions. The sub-problem that corresponds to region (ηk), labeled (Ck) for $k = 1, 2, 3, 4$, can be stated as follows.

$(C1)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \lambda(c_c - c_p - \frac{d}{2n}) - nf - F_p(f)$ st. $n \geq \frac{d}{c_c - c_p}$ and $n \geq 1$.

$(C2)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \frac{\lambda n}{2d}(\bar{p} - c_p)^2 - nf - F_p(f)$ st. $1 \leq n \leq \frac{d}{3\bar{p} - 2c_c - c_p}$.

$(C3)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \frac{2\lambda n}{9d}(\frac{d}{2n} + c_c - c_p)^2 - nf - F_p(f)$ st. $1 \leq n \leq \max\left[\frac{d}{c_c - c_p}, \frac{-d}{2(c_c - c_p)}\right]$ and $n \geq \frac{d}{3\bar{p} - 2c_c - c_p}$.

$(C4)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = -nf - F_p(f)$ st. $n \geq \frac{-d}{2(c_c - c_p)}$ and $n \geq 1$.

It is easy to see that the objective functions of $(C1)$, $(C2)$, $(C3)$ and $(C4)$ are strictly concave, linear, strictly convex and linear in n, respectively. The Lagrangean method yields the following, where $\hat{n}_k$ represents the optimal solution for (Ck):

$$\hat{n}_1=\begin{cases} \sqrt{\frac{\lambda d}{2f}} & \text{, if } c_c - c_p > 0 \text{ and } f < \frac{\lambda d}{2} \text{ and } f < \frac{\lambda(c_c - c_p)^2}{2d} \\ \frac{d}{c_c - c_p} & \text{, if } 0 < c_c - c_p \leq d \text{ and } f \geq \frac{\lambda(c_c - c_p)^2}{2d} \\ 1 & \text{, if } c_c - c_p > d \text{ and } f \geq \frac{\lambda d}{2} \end{cases}$$

$$\hat{n}_2 = \begin{cases} \frac{d}{3\bar{p}-2c_c-c_p} & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f < \frac{\lambda(\bar{p}-c_p)^2}{2d} \\ \in \left[1, \frac{d}{3\bar{p}-2c_c-c_p}\right] & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f = \frac{\lambda(\bar{p}-c_p)^2}{2d} \\ 1 & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f > \frac{\lambda(\bar{p}-c_p)^2}{2d} \end{cases}$$

$$\hat{n}_3 = \begin{cases} \frac{d}{c_c-c_p} & \text{, if } \{ 0 < c_c-c_p \le d \text{ , } 3\bar{p}-2c_c-c_p \le d \text{ , } f < \frac{\lambda(c_c-c_p)^2}{2d} \} \\ & \quad \text{or } \{ 0 < c_c-c_p \le d \text{ , } 3\bar{p}-2c_c-c_p > d \text{ ,} \\ & \quad\quad \text{and } f < \frac{2\lambda(c_c-c_p)}{9d}(c_c-c_p-\frac{d}{4}) \} \\ \frac{d}{3\bar{p}-2c_c-c_p} & \text{, if } \{ 0 < c_c-c_p \le d \text{ , } 3\bar{p}-2c_c-c_p \le d \text{ , } f \ge \frac{\lambda(c_c-c_p)^2}{2d} \} \\ & \quad \text{or } \{ -\frac{d}{2} \le c_c-c_p \le 0 \text{ , } 3\bar{p}-2c_c-c_p \le d \} \\ 1 & \text{, if } \{ 0 < c_c-c_p \le d \text{ , } 3\bar{p}-2c_c-c_p > d \text{ ,} \\ & \quad\quad \text{and } f \ge \frac{2\lambda(c_c-c_p)}{9d}(c_c-c_p-\frac{d}{4}) \} \\ & \quad \text{or } \{ -\frac{d}{2} \le c_c-c_p \le 0 \text{ , } 3\bar{p}-2c_c-c_p > d \} \end{cases}$$

$$\hat{n}_4 = \begin{cases} \frac{d}{2(c_p-c_c)} & \text{, if } -\frac{d}{2} < c_c-c_p < 0 \\ 1 & \text{, if } c_c-c_p \le -\frac{d}{2} \end{cases}$$

Combining these four solutions by choosing the best one wherever they overlap, we obtain the optimal number of standard products under duopoly competition. The resulting optimal variety by MP is given in the proposition under four parameter regions; associated profit expressions are as follows:

Case 1: High reservation price: $\bar{p} > \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	$\pi_c^{duop}(f) \equiv \pi_c(\hat{p}_c, f, \hat{n}, \hat{p}_p)$	$\pi_p^{duop}(f) \equiv \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$
$(\delta 1)$	$-F_c$	$\lambda (c_c - c_p) - \sqrt{2\lambda df} - F_p(f)$
$(\delta 2)$	$-F_c$	$\lambda \left(c_c - c_p - \frac{d}{2}\right) - f - F_p(f)$
$(\delta 3)$	$\frac{2\lambda}{9d} \left[d - (c_c - c_p)\right]^2 - F_c$	$\frac{2\lambda}{9d} \left(c_c - c_p + \frac{d}{2}\right)^2 - f - F_p(f)$
$(\delta 4)$	$\lambda (c_p - c_c) - F_c$	$-f - F_p(f)$

Case 2: Low reservation price: $\bar{p} \leq \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	$\pi_c^{duop}(f) \equiv \pi_c(\hat{p}_c, f, \hat{n}, \hat{p}_p)$	$\pi_p^{duop}(f) \equiv \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$
$(\varepsilon 1)$	$-F_c$	$\lambda (c_c - c_p) - \sqrt{2\lambda df} - F_p(f)$
$(\varepsilon 2)$	$\frac{2\lambda(\bar{p} - c_c)^2}{3\bar{p} - 2c_c - c_p} - F_c$	$\frac{\lambda(\bar{p} - c_p)^2}{2(3\bar{p} - 2c_c - c_p)} - \frac{df}{(3\bar{p} - 2c_c - c_p)} - F_p(f)$
$(\varepsilon 3)$	$\lambda(\bar{p} - c_c) \left(1 - \frac{\hat{n}(\bar{p} - c_p)}{d}\right) - F_c$	$-F_p(f)$
$(\varepsilon 4)$	$\lambda(\bar{p} - c_c) \left(1 - \frac{\bar{p} - c_p}{d}\right) - F_c$	$\frac{\lambda(\bar{p} - c_p)^2}{2d} - f - F_p(f)$

■

Proof of Proposition 6. We first characterize MP's best response to no entry by MC. In this case, MP sets its marginal cost of variety f so as to maximize $\pi_p^{monop}(f) = \pi_p(f, n^*, p_p^*)$ given in Proposition 3. We decompose the problem into two sub-problems based on the regions $(\mu 1)$ and $(\mu 2)$, the only regions in Proposition 3 that can lead to a non-negative profit for MP. The two sub-problems - labeled $(D1)$ and $(D2)$, respectively - can be stated as follows.

$(D1)$: $\pi_p(f, n^*, p_p^*) = \lambda (\bar{p} - c_p) - \sqrt{2\lambda df} - F_p(f)$ st. $(\mu 1)$.

$(D2)$: $\pi_p(f, n^*, p_p^*) = \lambda \left(\bar{p} - c_p - \frac{d}{2}\right) - f - F_p(f)$ st. $(\mu 2)$.

The objective functions of $(D1)$ and $(D2)$ are decreasing-increasing-decreasing and strictly concave in f, respectively. Therefore, for $(D1)$ the larger of two roots (if they exist) and for $(D2)$ the unique root (if it exists) of the first order condition gives the respective solutions, $f^*_{(\mu 1)}$ and$f^*_{(\mu 2)}$ below. The result obtains when these solutions are combined by choosing the better one wherever they overlap.

$$f^*_{(\mu 1)} = \begin{cases} 0 & \text{, if } \frac{\theta^2}{\lambda d} \leq 2\gamma \text{ or } \{ \frac{\theta^2}{\lambda d} > 2\gamma \text{ and } \ln(\gamma + f_0) - \ln(\gamma) < \frac{2 f_0}{\gamma + f_0} \} \\ f_1 & \text{, if } \frac{\theta^2}{\lambda d} > 2\gamma \text{ and } \ln(\gamma + f_0) - \ln(\gamma) \geq \frac{2 f_0}{\gamma + f_0} \end{cases}$$

$$f^*_{(\mu 2)} = f_2 \equiv \max \left\{ \frac{\lambda d}{2}, \theta - \gamma \right\}$$

where $f_1 \equiv \min \left\{ \frac{\lambda d}{2}, \frac{\lambda (\bar{p} - c_p)^2}{2d}, \frac{\theta^2}{\lambda d} - \gamma + \sqrt{\left(\frac{\theta^2}{\lambda d}\right)^2 - 2\gamma \left(\frac{\theta^2}{\lambda d}\right)} \right\}$.

As a result, MP's best response to no entry by MC is to enter the market and choose cost of variety f^* if $\pi_p^{monop}(f^*) \geq 0$; MP's optimal decisions and profits are:

Region	**Cost of Variety**, f^*	**Variety**, n^*	**Price**, p_p^*
$(\tau 1)$	0	∞	$\bar{p}$
$(\tau 2)$	f_1	$\sqrt{\frac{\lambda d}{2 f_1}}$	$\bar{p} - \sqrt{\frac{d f_1}{2\lambda}}$
$(\tau 3)$	f_2	1	$\bar{p} - \frac{d}{2}$

Region	**MP's Monopoly Profit**, $\pi_p^{monop}(f^*) = \pi_p(f^*, n^*, p_p^*)$
$(\tau 1)$	$\lambda (\bar{p} - c_p) - \kappa + \theta \ln(\gamma)$
$(\tau 2)$	$\lambda (\bar{p} - c_p) - \kappa - \theta \left[\frac{2 f_1}{\gamma + f_1} - \ln(\gamma + f_1) \right]$
$(\tau 3)$	$\lambda \left(\bar{p} - c_p - \frac{d}{2}\right) - f_2 - [\kappa - \theta \ln(\gamma + f_2)]$

where $f_1 \equiv \min \left\{ \frac{\lambda d}{2}, \frac{\lambda (\bar{p} - c_p)^2}{2d}, \frac{\theta^2}{\lambda d} - \gamma + \sqrt{\left(\frac{\theta^2}{\lambda d}\right)^2 - 2\gamma \left(\frac{\theta^2}{\lambda d}\right)} \right\}$, $f_2 \equiv \max \left\{ \frac{\lambda d}{2}, \theta - \gamma \right\}$, $0 \leq f_1 \leq$

f_2, and the parameter regions are defined as:

$$(\tau 1) \equiv \{ \theta \leq \sqrt{2\lambda d\gamma} \text{ or } [\ \theta > \sqrt{2\lambda d\gamma}\ ,\ \ln(\gamma + f_1) - \ln(\gamma) < \tfrac{2f_1}{\gamma+f_1}\ ]\ \}$$
$$\text{and } \{\ \bar{p} \leq c_p + d \text{ or } [\ \bar{p} > c_p + d\ ,\ \ln(\gamma + f_2) - \ln(\gamma) < \tfrac{f_2+\lambda d/2}{\theta}\ ]\ \}$$

$$(\tau 2) \equiv \{\ \theta > \sqrt{2\lambda d\gamma}\ ,\ \ln(\gamma + f_1) - \ln(\gamma) \geq \tfrac{2f_1}{\gamma+f_1}\ \} \text{ and } \{\ \bar{p} \leq c_p + d \text{ or}$$
$$[\ \bar{p} > c_p + d\ ,\ \ln(\gamma + f_2) - \ln(\gamma) < \tfrac{f_2+\lambda d/2}{\theta} - \tfrac{2f_1}{\gamma+f_1}\ ]\ \}$$

$$(\tau 3) \equiv \bar{p} > c_p + d\ , \text{ and}$$
$$\{\ \{\ \{\ \theta \leq \sqrt{2\lambda d\gamma} \text{ or } [\ \theta > \sqrt{2\lambda d\gamma}\ ,\ \ln(\gamma + f_1) - \ln(\gamma) < \tfrac{2f_1}{\gamma+f_1}\ ]\ \}$$
$$\text{and } \ln(\gamma + f_2) - \ln(\gamma) \geq \tfrac{f_2+\lambda d/2}{\theta}\ \}$$
$$\text{or } \{\ \theta > \sqrt{2\lambda d\gamma}\ ,\ \ln(\gamma + f_1) - \ln(\gamma) \geq \tfrac{2f_1}{\gamma+f_1}\ ,$$
$$\text{and } \ln(\gamma + f_2) - \ln(\gamma) \geq \tfrac{f_2+\lambda d/2}{\theta} - \tfrac{2f_1}{\gamma+f_1}\ \}\ \}$$

Next, we characterize MP's best response to entry by MC. In this case, MP maximizes $\pi_p^{duop}(f) = \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$, which is a result of Proposition 5 and is given in its proof. We decompose MP's problem into two sub-problems based on the regions $(\delta 3)$ and $(\varepsilon 2)$, the only regions in Proposition 5 that can lead to non-negative profits for both firms. The two sub-problems - labeled $(E1)$ and $(E2)$, respectively - can be stated as follows.

$(E1)$: $\pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p) = \frac{2\lambda}{9d}\left(c_c - c_p + \frac{d}{2}\right)^2 - f - F_p(f)$ st. $(\delta 3)$.

$(E2)$: $\pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p) = \frac{\lambda(\bar{p}-c_p)^2}{2(3\bar{p}-2c_c-c_p)} - \frac{df}{(3\bar{p}-2c_c-c_p)} - F_p(f)$ st. $(\varepsilon 2)$.

The objective functions of $(E1)$ and $(E2)$ are both concave in f. Therefore, the first order condition is necessary and sufficient for global optimality of each sub-problem; and gives the respective solutions, $\hat{f}_{(\delta 3)}$ and $\hat{f}_{(\varepsilon 2)}$ below, which yield the result when combined (there are no overlaps in this case because ($\delta 3$) and ($\varepsilon 2$) are mutually exclusive).

$$\hat{f}_{(\delta 3)} = \begin{cases} \max\left\{0, \bar{f}, \theta - \gamma\right\} & \text{, if } \; 0 < c_c - c_p \leq d \\ \max\left\{0, \theta - \gamma\right\} & \text{, if } \; -\frac{d}{2} \leq c_c - c_p \leq 0 \end{cases}$$

$$\hat{f}_{(\varepsilon 2)} = \begin{cases} \max\left\{0, \breve{f}\right\} & \text{, if } \; 0 < c_c - c_p < d \; \text{ and } \; f_2 < \breve{f} \\ \max\left\{0, f_3\right\} & \text{, if } \; \{\, 0 < c_c - c_p < d \; \text{ and } \; \breve{f} \leq f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d} \,\} \\ & \quad \text{or } \{\, -\frac{d}{2} < c_c - c_p \leq 0 \; \text{ and } \; f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d} \,\} \\ \frac{\lambda(\bar{p}-c_p)^2}{2d} & \text{, if } \; \{\, 0 < c_c - c_p < d \; \text{ and } \; f_3 \geq \frac{\lambda(\bar{p}-c_p)^2}{2d} \,\} \\ & \quad \text{or } \{\, -\frac{d}{2} < c_c - c_p \leq 0 \; \text{ and } \; f_3 \geq \frac{\lambda(\bar{p}-c_p)^2}{2d} \,\} \end{cases}$$

where $f_3 \equiv \frac{\theta}{d}\left(3\bar{p} - 2c_c - c_p\right) - \gamma$.

As a result, MP's best response to entry by MC is to enter the market and choose cost of variety $\hat{f}$ if $\pi_p^{duop}(\hat{f}) \geq 0$; the firms' equilibrium decisions and profits are:

Region	**Cost of Variety**, $\hat{f}$	**Variety**, $\hat{n}$	**Price Equilibrium**, $(\hat{p}_c, \hat{p}_p)$
($\upsilon 1$)	0	∞	$(\min(c_c, c_p), \min(c_c, c_p))$
($\upsilon 2$)	$\frac{\lambda(\bar{p}-c_p)^2}{2d}$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$(\bar{p}, \frac{\bar{p}+c_p}{2})$
($\upsilon 3$)	$\max\left\{\breve{f}, f_3\right\}$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$(\bar{p}, \frac{\bar{p}+c_p}{2})$
($\upsilon 4$)	$\max\left\{\theta - \gamma, \bar{f}\right\}$	1	$\left(\frac{d}{3} + \frac{2c_c+c_p}{3}, \frac{d}{6} + \frac{c_c+2c_p}{3}\right)$

Region	**MC's Duopoly Profit**, $\pi_c^{duop}(\hat{f}) = \pi_c(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$
($\upsilon 1$)	$-F_c$, if $c_p \leq c_c$ $\lambda\left(\bar{p} - c_c\right) - F_c$, if $c_c < c_p$
($\upsilon 2$)	$\frac{2\lambda(\bar{p}-c_c)^2}{3\bar{p}-2c_c-c_p} - F_c$
($\upsilon 3$)	$\frac{2\lambda(\bar{p}-c_c)^2}{3\bar{p}-2c_c-c_p} - F_c$
($\upsilon 4$)	$\frac{2\lambda}{9d}\left[d - (c_c - c_p)\right]^2 - F_c$

Region	MP's Duopoly Profit, $\pi_p^{duop}(\hat{f}) = \pi_p(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$
$(\upsilon 1)$	$\lambda(\bar{p} - c_p) - [\kappa - \theta \ln(\gamma)]$, if $c_p \leq c_c$ $-[\kappa - \theta \ln(\gamma)]$, if $c_c < c_p$
$(\upsilon 2)$	$-\left[\kappa - \theta \ln(\gamma + \frac{\lambda(\bar{p}-c_p)^2}{2d})\right]$
$(\upsilon 3)$	$\frac{\lambda(\bar{p}-c_p)^2}{2(3\bar{p}-2c_c-c_p)} - \frac{d \max\left(\breve{f}, f_3\right)}{3\bar{p}-2c_c-c_p} - \kappa + \theta \ln\left[\gamma + \max\left(\breve{f}, f_3\right)\right]$
$(\upsilon 4)$	$\frac{2\lambda}{9d}\left(c_c - c_p + \frac{d}{2}\right)^2 - \max\left(\theta - \gamma, \bar{f}\right) - \kappa + \theta \ln\left[\gamma + \max\left(\theta - \gamma, \bar{f}\right)\right]$

where $f_3 \equiv \frac{\theta}{d}(3\bar{p} - 2c_c - c_p) - \gamma$, and the parameter regions are defined as:

$$(\upsilon 1) \equiv \quad \{ \ \bar{p} > \tfrac{d}{3} + \tfrac{2c_c+c_p}{3} \ , \theta \leq \gamma \ , \{ \ [\ 0 < c_c - c_p \leq d \ , \ \bar{f} \leq 0 \ ] \text{ or } [\ -\tfrac{d}{2} \leq c_c - c_p \leq 0 \ ] \ \} \ \}$$

$$\text{or } \{ \ \bar{p} \leq \tfrac{d}{3} + \tfrac{2c_c+c_p}{3} \ , \{ \ [\ 0 < c_c - c_p \leq d \ , \ f_3 \leq \breve{f} \leq 0 \ ]$$

$$\text{or } [\ -\tfrac{d}{2} \leq c_c - c_p \leq 0 \ , \ \breve{f} \leq \ f_3 \leq 0 \ ] \ \} \ \}$$

$$(\upsilon 2) \equiv \quad \bar{p} \leq \tfrac{d}{3} + \tfrac{2c_c+c_p}{3} \ , -\tfrac{d}{2} \leq c_c - c_p \leq d \ , \ f_3 \geq \tfrac{\lambda(\bar{p}-c_p)^2}{2d}$$

$$(\upsilon 3) \equiv \quad \bar{p} \leq \tfrac{d}{3} + \tfrac{2c_c+c_p}{3} \ , \{ \ [\ 0 < c_c - c_p \leq d \ , \ \breve{f} > 0 \ ]$$

$$\text{or } [\ -\tfrac{d}{2} \leq c_c - c_p \leq d \ , \ 0 < f_3 < \tfrac{\lambda(\bar{p}-c_p)^2}{2d} \ ] \ \}$$

$$(\upsilon 4) \equiv \quad \bar{p} > \tfrac{d}{3} + \tfrac{2c_c+c_p}{3} \ , \{ \ [\ 0 < c_c - c_p \leq d \ , \ \bar{f} > 0 \ ] \text{ or } [\ -\tfrac{d}{2} \leq c_c - c_p \leq d \ , \ \theta > \gamma \ ] \ \}$$

Finally, the entry game equilibrium can be explicitly but tediously characterized by using the following logical statements and the profit expressions given in the proofs of Propositions 5 and 6 (we assume that the firms enter when they are indifferent between entering and not entering):

Condition		Outcome
$\pi_p^{duop}(\hat{f}) \geq 0$, $\pi_c^{duop}(\hat{f}) \geq 0$	$\Longrightarrow$	Duopoly
$\pi_p^{duop}(\hat{f}) \geq 0$, $\pi_c^{duop}(\hat{f}) < 0$	$\Longrightarrow$	MP Monopoly
$\pi_p^{monop}(f^*) \geq 0$, $\pi_c^{monop} < 0$	$\Longrightarrow$	MP Monopoly
$\pi_p^{duop}(\hat{f}) < 0$, $\pi_c^{duop}(\hat{f}) \geq 0$	$\Longrightarrow$	MC Monopoly
$\pi_p^{monop}(f^*) < 0$, $\pi_c^{monop} \geq 0$	$\Longrightarrow$	MC Monopoly
$\pi_j^{duop}(\hat{f}) < 0 \leq \pi_j^{monop}$ for $j = p, c$	$\Longrightarrow$	MP or MC Monopoly
$\pi_p^{monop}(f^*) < 0$, $\pi_c^{monop} < 0$	$\Longrightarrow$	Market Breakdown

In the only remaining case, if both firms earn negative profit under competition but non-negative profit under monopoly, then both MC and MP monopoly are Nash equilibria. ■

Figures 1-3 below provide graphical illustrations of the sets of regions ($\eta 1$)-($\eta 4$), ($\delta 1$)-($\delta 4$) and ($\varepsilon 1$)-($\varepsilon 4$) for a numerical example. The parameter values used are: $\bar{p} = 40$, $c_p = 20$, $\lambda = 1$; $n = 1$ for Figure 1; $d = 10$ for Figures 1-2, and $d = 100$ for Figure 3.

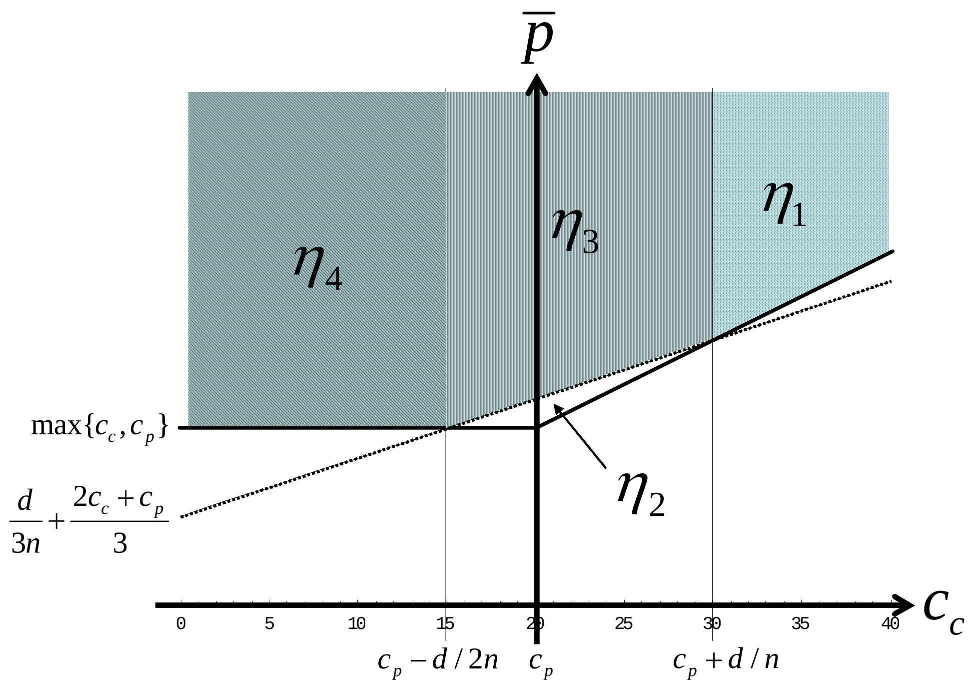

Figure 1: Illustration of Regions for Proposition 4

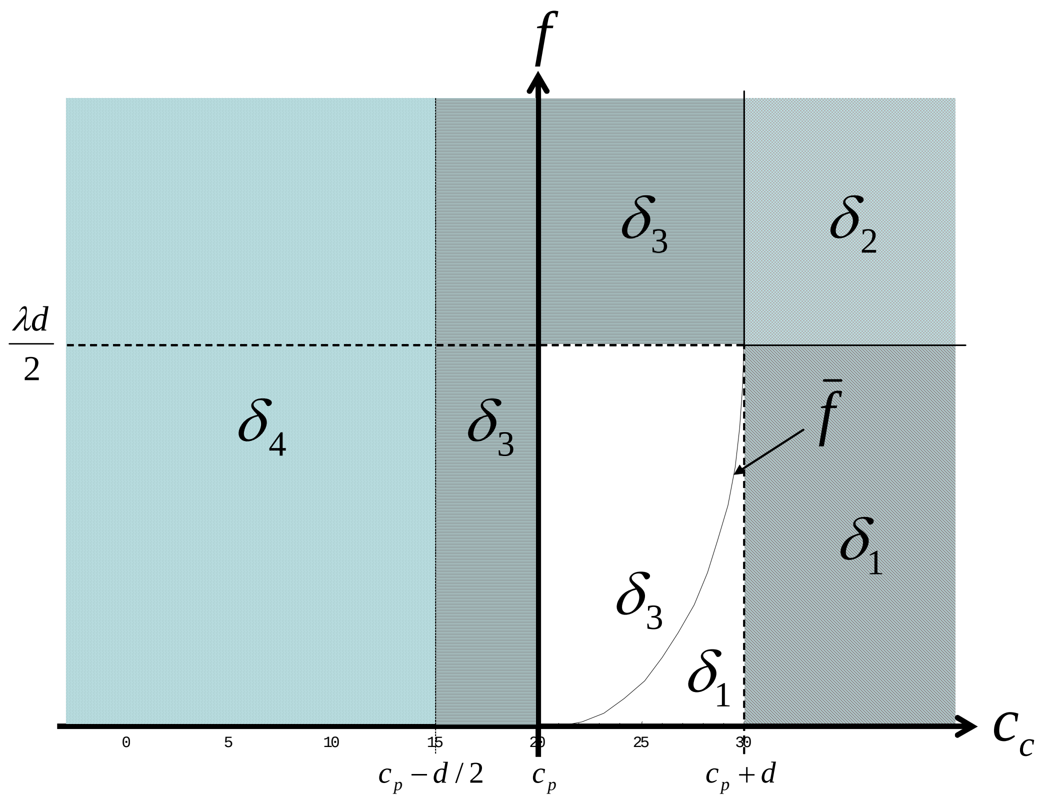

Figure 2: Illustration of Regions for Proposition 5, Case 1

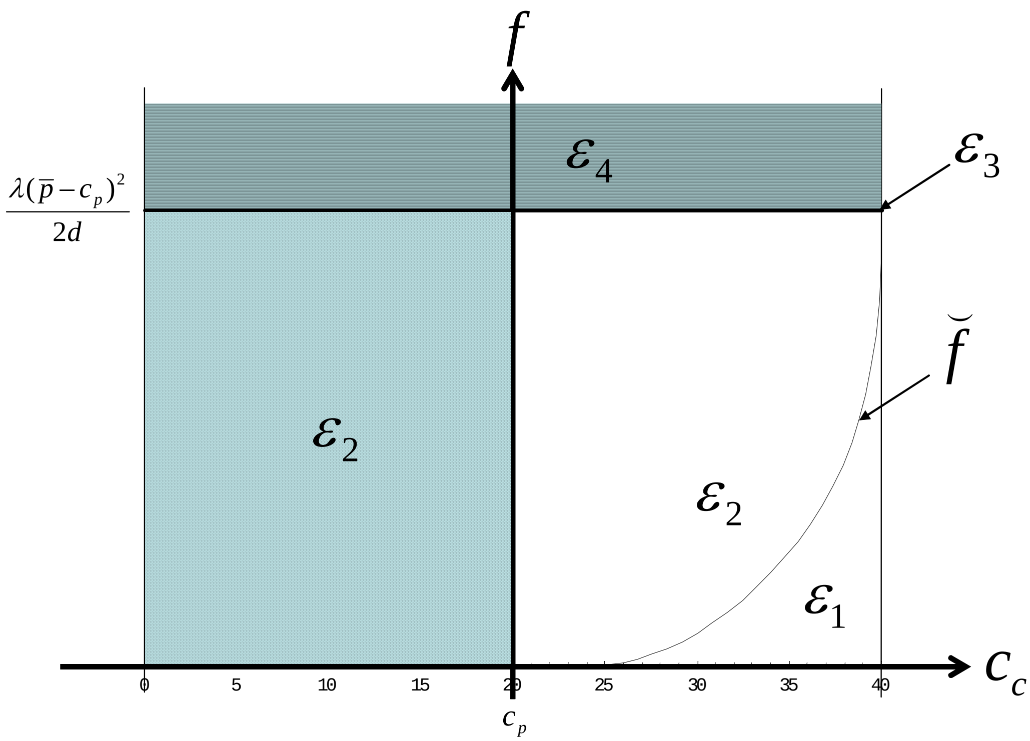

Figure 3: Illustration of Regions for Proposition 5, Case 2