Online Supplement for "Feasting on Leftovers: Strategic Use of Shortages in Price Competition Among Differentiated Products"

Proofs of Propositions

Proof of Proposition 1 under high-to-low arrivals. We follow analogous steps to the proof under independent arrivals, as given in the Appendix of the paper.

Lemma EC.1 *Denote $R^H(\rho)$ as the subset of (Q_1, Q_2) that satisfies all of the following conditions: (i) $Q_1 + (2-\rho)Q_2 \leq 1$, (ii) $Q_1 + 2Q_2 \geq 1$, (iii) $(1-Q_2)Q_2 \geq \alpha^2/4$, and (iv) $Q_1\alpha^2 + 4\beta Q_1 Q_2 > (1-\rho Q_2)^2 Q_2$, where $\alpha = 1 - Q_1$ and $\beta = (1-\rho)Q_2$. If $2/3 < \rho < 1$, then $R^H(\rho)$ contains an open subset and $(S_1^N(\rho), S_2^N(\rho)) \in R^H(\rho)$.*

The proof uses simple algebra and is omitted.

Recall that in regime (a), given by $p_2 \geq \rho p_1$, only the leader has primary demand, in regime (b), given by $p_1 - (1-\rho) \leq p_2 \leq \rho p_1$, both firms have primary demand, and, in regime (c), given by $p_2 \leq p_1 - (1-\rho)$, only the follower has primary demand.

Lemma EC.2 *Under high-to-low arrivals, the profits to each firm are given as follows, as functions of their respective prices.*

$$\pi_1 = \begin{cases} p_1 \min(1-p_1, Q_1), & \text{in regime (a)}, \\ p_1 \min\left[1 - \frac{p_1-p_2}{1-\rho} + \left(\frac{\rho p_1 - p_2}{1-\rho} - Q_2\right)^+, Q_1\right], & \text{in regime (b)}, \\ p_1 \min\left[(1-p_1-Q_2)^+, Q_1\right], & \text{in regime (c)}, \end{cases} \tag{EC.1}$$

and

$$\pi_2 = \begin{cases} p_2 \min\left[\left(1 - \frac{p_2}{\rho} - Q_1\right)^+, Q_2\right], & \text{in regime (a)}, \\ p_2 \min\left[\frac{\rho p_1 - p_2}{\rho(1-\rho)} + \left(1 - \frac{p_1-p_2}{1-\rho} - Q_1\right)^+, Q_2\right], & \text{in regime (b)}, \\ p_2 \min\left(1 - \frac{p_2}{\rho}, Q_2\right), & \text{in regime (c)}. \end{cases} \tag{EC.2}$$

The proof entails using definitions and simple algebra and is therefore omitted.

Similar to the proof for independent arrivals, we partition the regimes so that, within each sub-regime, the profit expressions for each firm are simplified. Sub-regime $(a1)$ adds the requirement that $Q_1 \geq 1 - p_1$, so the leader meets all of her demand. Sub-regime $(a2)$ requires $1 - p_2/\rho \leq Q_1 \leq 1 - p_1$, so the leader has unmet demand but those customers are not willing to buy from the follower. Sub-regime $(a3)$ requires $Q_1 \leq 1 - p_2/\rho \leq Q_1 + Q_2$, so the leader provides leftovers to the follower who satisfies all of his demand. Sub-regime $(a4)$ requires $Q_1+Q_2 \le 1-p_2/\rho$, so the leader provides leftovers to the follower who does not satisfies them all. Hence the leader's profits are

$$\pi_1 = \begin{cases} p_1(1-p_1), & \text{in sub-regime (a1),} \\ p_1 Q_1, & \text{in sub-regimes (a2), (a3) and (a4),} \end{cases} \tag{EC.3}$$

and the follower's profits are

$$\pi_2 = \begin{cases} 0, & \text{in sub-regimes (a1) and (a2),} \\ p_2\left(\alpha - \frac{p_2}{\rho}\right), & \text{in sub-regime (a3),} \\ p_2 Q_2, & \text{in sub-regime (a4).} \end{cases} \tag{EC.4}$$

Sub-regime $(b1)$ adds the requirements that $Q_1 \ge 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \ge (\rho p_1 - p_2)/[\rho(1-\rho)]$, so both firms satisfy their primary demands. Sub-regime $(b2)$ requires $Q_1 \le 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \ge \alpha - p_2/\rho$, so the leader provides leftovers to the follower who satisfies all of his demand. Sub-regime $(b3)$ requires $Q_1 \le 1-(p_1-p_2)/(1-\rho)$ and $(\rho p_1 - p_2)/[\rho(1-\rho)] \le Q_2 \le 1-p_2/\rho - Q_1$, so the leader leaves leftovers to the follower, who satisfies all of his primary demand but not all demand. Sub-regime $(b4)$ adds $Q_2 \le (\rho p_1 - p_2)/\rho(1-\rho)$ and $Q_1 \ge 1-p_1-Q_2$, so the follower leaves leftovers to the leader who satisfies all of her demand. Sub-regime $(b5)$ requires $Q_2 \le (\rho p_1 - p_2)/\rho(1-\rho)$ and $1-(p_1-p_2)/(1-\rho) \le Q_1 \le 1-p_1-Q_2$, so the follower provides leftovers to the leader, who satisfies all primary demand but not all demand. Sub-regime $(b6)$ requires $Q_1 \le 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$, so neither firm satisfies all of its primary demand. Therefore, the leader's profits in each sub-regime can be written as

$$\pi_1 = \begin{cases} p_1\left(1 - \frac{p_1-p_2}{1-\rho}\right), & \text{in sub-regime (b1),} \\ p_1(1-p_1-Q_2), & \text{in sub-regime (b4),} \\ p_1 Q_1, & \text{in sub-regimes (b2), (b3), (b5) and (b6),} \end{cases} \tag{EC.5}$$

and the follower's profits are given at (A.6).

For regime (c), sub-regime $(c1)$ adds the requirement that $Q_2 \ge 1-p_2/\rho$, so the follower meets all of his primary demand. Sub-regime $(c2)$ requires $1-p_1 \le Q_2 \le 1-p_2/\rho$, so the follower has unmet demand but those customers are not willing to buy from the leader. Sub-regime $(c3)$ requires $Q_2 \le 1-p_1 \le Q_1+Q_2$, so the follower provides leftovers to the leader who satisfies all of her demand. Sub-regime $(c4)$ requires $Q_1+Q_2 \le 1-p_1$, so the follower provides leftovers to the leader who does not satisfies them all. Therefore the leader's profits for each sub-regime are

$$\pi_1 = \begin{cases} 0, & \text{in sub-regimes (c1) and (c2),} \\ p_1\left(1-p_1-Q_2\right), & \text{in sub-regime (c3),} \\ p_1 Q_1, & \text{in sub-regime (c4),} \end{cases} \tag{EC.6}$$

while the follower's profits are

$$\pi_2 = \begin{cases} p_2\left(1 - \frac{p_2}{\rho}\right), & \text{in sub-regime (c1),} \\ p_2 Q_2, & \text{in sub-regimes (c2), (c3) and (c4).} \end{cases} \tag{EC.7}$$

Lemma EC.3 provides the conditional optimal response of the follower to the leader's price, conditioned on each regime (or a set of sub-regimes) that the follower can feasibly select under $R^H(\rho)$.

Lemma EC.3 *If $(Q_1, Q_2) \in R^H(\rho)$ under high-to-low arrivals, then the optimal profits of the follower are given as follows, within the regimes in which the follower can restrict himself.*

(a) Within regime (a), if $p_1 \geq \alpha$, then $\pi_2^* = 0$.

(b) Within regime (a), if $p_1 < \alpha$, then[29]

$$\pi_2^* = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq \frac{\alpha}{2} \quad [a3]\,, \\ \rho p_1(\alpha - p_1) & \text{if} \quad \frac{\alpha}{2} \leq p_1 \quad [a3]\,. \end{cases} \tag{EC.8}$$

(c) Within regime (b), if $p_1 \geq \alpha$, then the follower's optimal profits are given at (A.10).

(d) Within regime (b), if $p_1 \leq \alpha$ and $p_2 \geq p_1 - (1-\rho)\alpha$, the follower's optimal profits are given at (A.11).

(e) Within regime (b), if $p_1 \leq \alpha$ and $p_2 \leq p_1 - (1-\rho)\alpha$, the follower's optimal profits are given at (A.12).

(f) Within regime (c), if $Q_2 \leq 1/2$, then

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if } p_1 \geq 1 - \rho Q_2 \quad [c1]\,, \\ (p_1 - (1-\rho))\, Q_2 & \text{if } p_1 \leq 1 - \rho Q_2 \quad [c2],\ [c3] \ \text{or } [c4]\,. \end{cases} \tag{EC.9}$$

(g) Within regime (c), if $Q_2 \geq 1/2$, then

$$\pi_2^* = \begin{cases} \frac{\rho}{4} & \text{if} \quad 1 - \frac{\rho}{2} \leq p_1 \quad [c1]\,, \\ (p_1 - (1-\rho))\, \frac{1-p_1}{\rho} & \text{if} \quad 1 - \rho Q_2 \leq p_1 \leq 1 - \frac{\rho}{2} \quad [c1]\,, \\ (p_1 - (1-\rho))\, Q_2 & \text{if} \quad p_1 \leq 1 - \rho Q_2 \quad [c2],\ [c3] \ \text{or } [c3]\,. \end{cases} \tag{EC.10}$$

We follow the analogous proof for independent arrivals; for parts (a) and (b), we utilize the condition (ii) of $R^H(\rho)$; for parts (c), (d) and (e), we use the conditions (i) and (ii) in $R^H(\rho)$.

Let $p_2^*(p_1)$ denote the optimal price response by the follower as a function of the leader's price.

Lemma EC.4 *If $(Q_1, Q_2) \in R^H(\rho)$ under high-to-low arrivals, then the optimal profits of the follower and the resulting profits of the leader, are given as follows, as functions of the leader's price.*

(a) If $p_1 \geq \alpha$ and $Q_2 \leq 1/2$, the follower's optimal profits are given at (A.15).

[29] The chosen sub-regime is provided in square brackets

(b) If $p_1 \geq \alpha$ and $Q_2 \geq 1/2$, the follower's optimal profits are given at (A.16).

(c) If $p_1 \geq \alpha$, then, the leader's corresponding profits are given at (A.17).

(d) If $p_1 \leq \alpha$ and $Q_2 \leq 1/2$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ \rho(p_1-\beta)Q_2 & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.11}$$

where

$$p_1^H = \frac{\alpha^2}{4Q_2} + \beta. \tag{EC.12}$$

(e) If $p_1 \leq \alpha$ and $Q_2 \geq 1/2$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ \rho(p_1-\beta)Q_2 & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if} \quad 1-\rho Q_2 \leq p_1 \leq 1-\frac{\rho}{2} \quad [c1]\,, \\ \frac{\rho}{4} & \text{if} \quad 1-\frac{\rho}{2} \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.13}$$

(f) If $p_1 \leq \alpha$, then,

$$\pi_1(p_1, p_2^*(p_1)) = \begin{cases} p_1 Q_1 & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ p_1(1-\rho Q_2 - p_1) & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.14}$$

As noted in the paper, the leader's equilibrium price p_1^H is the largest price she can charge and still have the follower prefer to sell only to leftover customers rather than to engage in direct competition. We compare the follower's optimal profits, derived in lemma EC.3, across different regimes. We utilize the conditions (iii) to prove (EC.11), and (ii) to prove (EC.13).

Finally, consider the leader's decision on p_1 to maximize $\pi_1(p_1,\, p_2^*(p_1))$. From the conditions, (i) and (ii), we have $p_1^H \leq \alpha$. Further, from the condition (iv) in $R^H(\rho)$, we have $Q_1(\beta+\alpha^2/(4Q_2)) > (1-\rho Q_2)^2/4$. Hence $p_1^* = p_1^H$ and the corresponding equilibrium $p_2^H(p_1^H) = \rho\alpha/2$. Thus, the Baseline Leftovers Equilibrium arises and is unique. □

Proof of Proposition 1 under low-to-high arrivals. We follow analogous steps to the proof under independent arrivals, as given in the Appendix of the paper.

Lemma EC.5 *Denote $R^L(\rho)$ as the subset of (Q_1, Q_2) that satisfies all of the following conditions: (i) $Q_2 \leq 1/2$, (ii) $Q_2 \leq Q_1$, (iii) $Q_1 \geq \frac{(1-\rho)^2}{1+\rho^2}$, (iv) $Q_1 + 2Q_2 \geq 1$,*
(v) $Q_1 + 2\beta \leq 1$, (vi) $\beta(4Q_1 + (5-4\rho)Q_2 - 2) \leq \alpha Q_1$,
and (vii) $4Q_1(Q_1-\rho Q_2) + Q_2(2-5Q_2+(1+\rho)^2 Q_2) < 1$, where $\alpha = 1-Q_1$ and $\beta = (1-\rho)Q_2$. If $0 < \rho < 1$, $R^L(\rho)$ contains an open subset and $(S_1^N(\rho), S_2^N(\rho)) \in R^L(\rho)$.

The proof uses simple algebra and is omitted.

Recall that in regime (a), only the leader has primary demand, in regime (b), both firms have primary demand, and, in regime (c), only the follower has primary demand.

Lemma EC.6 *Under low-to-high arrivals, the profits to each firm are given as follows, as functions of their respective prices.*

$$\pi_1 = \begin{cases} p_1 \min(1-p_1,\, Q_1), & \textit{in regime (a)}, \\ p_1 \min\left[1 - \frac{p_1-p_2}{1-\rho} + \min\left\{\frac{\rho p_1 - p_2}{1-\rho},\, \left(\frac{\rho p_1 - p_2}{\rho(1-\rho)} - Q_2\right)^+\right\}, Q_1\right], & \textit{in regime (b)}, \\ p_1 \min\left[1-p_1,\, \left(1 - \frac{p_2}{\rho} - Q_2\right)^+,\, Q_1\right], & \textit{in regime (c)}, \end{cases} \tag{EC.15}$$

and

$$\pi_2 = \begin{cases} p_2 \min\left[1 - \frac{p_2}{\rho},\, (1-p_1-Q_1)^+,\, Q_2\right], & \textit{in regime (a)}, \\ p_2 \min\left[\frac{\rho p_1 - p_2}{\rho(1-\rho)} + \left(1 - \frac{p_1-p_2}{1-\rho} - Q_1\right)^+,\, Q_2\right], & \textit{in regime (b)}, \\ p_2 \min\left(1 - \frac{p_2}{\rho},\, Q_2\right), & \textit{in regime (c)}. \end{cases} \tag{EC.16}$$

The proof entails using definitions and simple algebra and is therefore omitted.

Similar to the proof of independent arrivals, we partition the regimes. Sub-regime $(a1)$ adds the requirement that $Q_1 \geq 1 - p_1$, so the leader meets all of her demand. Sub-regime $(a2)$ requires $Q_1 \leq 1 - p_1$, $Q_2 \geq 1 - p_2/\rho$ and $1 - p_2/\rho \leq \alpha - p_1$, so the follower's potential demand, if he were the monopolist, is less than his capacity and also his potential demand is less than the leftovers from the leader. Sub-regime $(a3)$ requires $Q_1 \leq 1 - p_1$, $Q_2 \leq 1 - p_2/\rho$ and $Q_2 \leq \alpha - p_1$, so the leader provides leftovers to the follower who does not satisfy them all. Sub-regime $(a4)$ adds $Q_1 \leq 1 - p_1$, $\alpha - p_1 \leq 1 - p_2/\rho$ and $Q_2 \geq \alpha - p_1$, so the leftovers from the leader are smaller than the follower's capacity as well as his potential demand. Hence the leader's profits are

$$\pi_1 = \begin{cases} p_1(1-p_1), & \text{in sub-regime (a1)}, \\ p_1 Q_1, & \text{in sub-regimes (a2), (a3) and (a4)}, \end{cases} \tag{EC.17}$$

and the follower's profits are

$$\pi_2 = \begin{cases} 0, & \text{in sub-regime (a1)}, \\ p_2\left(1 - \frac{p_2}{\rho}\right), & \text{in sub-regime (a2)}, \\ p_2 Q_2, & \text{in sub-regime (a3)}, \\ p_2(\alpha - p_1), & \text{in sub-regime (a4)}. \end{cases} \tag{EC.18}$$

For regime (b), sub-regime $(b1)$ adds the requirements that $Q_1 \geq 1 - (p_1 - p_2)/(1-\rho)$ and $Q_2 \geq (\rho p_1 - p_2)/[\rho(1-\rho)]$, so both firms satisfy their primary demands. Sub-regime $(b2)$ requires $Q_1 \leq 1 - (p_1 - p_2)/(1-\rho)$ and $Q_2 \geq \alpha - p_2/\rho$, so the leader leaves leftovers for the follower who satisfies all of his demand. Sub-regime $(b3)$ requires $Q_1 \leq 1 - (p_1 - p_2)/(1-\rho)$ and $(\rho p_1 - p_2)/[\rho(1-$

$\rho)] \le Q_2 \le \alpha - p_2/\rho$, so the leader leaves leftovers for the follower, who satisfies all of his primary demand but not all demand. Sub-regime $(b4)$ adds $Q_2 \le (\rho p_1 - p_2)/\rho$ and $Q_1 \ge 1 - p_1$, so the follower leaves enough leftovers for the leader so that she has all of her potential customers and she satisfies all of her demand. Sub-regime $(b5)$ requires $Q_2 \le (\rho p_1 - p_2)/\rho$ and $1-(p_1-p_2)/(1-\rho) \le Q_1 \le 1-p_1$, so the follower leaves enough leftovers for the leader so that the leader has all of her potential customers, and she satisfies all of her primary demand but not all of her demand. Sub-regime $(b6)$ adds $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $Q_1 \ge 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demand and she satisfies all of her demand. Sub-regime $(b7)$ adds $(\rho p_1 - p_2)/\rho \le Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $1-(p_1-p_2)/(1-\rho) \le Q_1 \le 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demand, and she satisfies her primary demand but not all of her demand. Sub-regime $(b8)$ requires $Q_1 \le 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$, so neither firm satisfies all of its primary demand. Therefore, the leader's profits in each sub-regime can be written as

$$\pi_1 = \begin{cases} p_1\left(1 - \frac{p_1-p_2}{1-\rho}\right), & \text{in sub-regime (b1)}, \\ p_1(1-p_1), & \text{in sub-regime (b4)}, \\ p_1\left(1 - \frac{p_2}{\rho} - Q_2\right), & \text{in sub-regime (b6)}, \\ p_1 Q_1, & \text{in sub-regimes (b2), (b3), (b5), (b7) and (b8)}, \end{cases} \tag{EC.19}$$

and the follower's profits are

$$\pi_2 = \begin{cases} p_2\,\frac{\rho p_1 - p_2}{\rho(1-\rho)}, & \text{in sub-regime (b1)}, \\ p_2\left(\alpha - \frac{p_2}{\rho}\right), & \text{in sub-regime (b2)}, \\ p_2\,Q_2, & \text{in sub-regimes (b3), (b4), (b5), (b6), (b7) and (b8)}. \end{cases} \tag{EC.20}$$

For regime (c), sub-regime $(c1)$ adds the requirement that $Q_2 \ge 1 - p_2/\rho$, so the follower meets all of his primary demand. Sub-regime $(c2)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \le 1 - p_2/\rho - Q_2$ and $Q_1 \ge 1 - p_1$, so the follower leaves enough leftovers for the leader, larger than the leader's potential demands, and the leader satisfies them all. Sub-regime $(c3)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \le 1 - p_2/\rho - Q_2$ and $Q_1 \le 1 - p_1$, so the follower leaves enough leftovers for the leader, larger than the leader's potential demands, but the leader does not satisfy them all. Sub-regime $(c4)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \ge 1 - p_2/\rho - Q_2$ and $Q_1 \ge 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demands and the leader satisfies all her demands. Sub-regime $(c5)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \ge 1 - p_2/\rho - Q_2$ and $Q_1 \le 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demands and the leader does not satisfy all her demands. Therefore the leader's profits for each sub-regime are

$$\pi_1 = \begin{cases} 0, & \text{in sub-regime (c1)}, \\ p_1\left(1-p_1\right), & \text{in sub-regime (c2)}, \\ p_1\left(1-\frac{p_2}{\rho}-Q_2\right), & \text{in sub-regime (c4)}, \\ p_1 Q_1, & \text{in sub-regimes (c3) and (c5)}, \end{cases} \tag{EC.21}$$

while the follower's profits are

$$\pi_2 = \begin{cases} p_2\left(1-\frac{p_2}{\rho}\right), & \text{in sub-regime (c1)}, \\ p_2 Q_2, & \text{in sub-regimes (c2), (c3), (c4) and (c5)}. \end{cases} \tag{EC.22}$$

Lemma EC.7 provides the conditional optimal response of the follower to the price of the leader, conditioned on each regime (or a set of sub-regimes) that the follower can feasibly select.

Lemma EC.7 *If $(Q_1, Q_2) \in R^L(\rho)$ under low-to-high arrivals, then the optimal profits of the follower are given as follows, within the regimes in which the follower can restrict himself.*

(a) Within regime (a),

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if } p_1 \le \alpha - Q_2 & [a3], \\ \rho(p_1+Q_1)(\alpha-p_1) & \text{if } \alpha - Q_2 \le p_1 \le \alpha & [a4], \\ 0 & \text{if } \alpha \le p_1 & [a1]. \end{cases} \tag{EC.23}$$

(b) Within regime (b), if $p_1 \ge \alpha$, then

$$\pi_2^* = \begin{cases} \rho(p_1-\beta)Q_2 & \text{if } p_1 \le 1-\rho Q_2 & [b1], \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 & [b1]. \end{cases} \tag{EC.24}$$

(c) Within regime (b), if $p_1 \le \alpha$ and $p_2 \ge p_1 - (1-\rho)\alpha$, then

$$\pi_2^* = \begin{cases} \rho p_1 Q_2 & \text{if } p_1 \le \alpha - Q_2 & [b3], \\ \rho p_1(\alpha - p_1) & \text{if } \alpha - Q_2 \le p_1 \le \frac{\alpha}{2} & [b2], \\ \frac{\rho\alpha^2}{4} & \text{if } \frac{\alpha}{2} \le p_1 \le \alpha\left(1-\frac{\rho}{2}\right) & [b2], \\ (p_1-\alpha(1-\rho))\,\frac{\alpha-p_1}{\rho} & \text{if } \alpha\left(1-\frac{\rho}{2}\right) \le p_1 & [b2]. \end{cases} \tag{EC.25}$$

(d) Within regime (b), if $p_1 \le \alpha$, $p_2 \le p_1 - (1-\rho)\alpha$ and $Q_1 + (2-\rho)Q_2 \ge 1$, then

$$\pi_2^* = \begin{cases} (p_1-\alpha(1-\rho))\,Q_2 & \text{if } p_1 \le \alpha - \rho Q_2 & [b8], \\ (p_1-\alpha(1-\rho))\,\frac{\alpha-p_1}{\rho} & \text{if } \alpha - \rho Q_2 \le p_1 \le \frac{2\alpha(1-\rho)}{2-\rho} & [b1], \\ \frac{\rho p_1^2}{4(1-\rho)} & \text{if } \frac{2\alpha(1-\rho)}{2-\rho} \le p_1 \le 2\beta & [b1], \\ \rho(p_1-\beta)Q_2 & \text{if } 2\beta \le p_1 \le 1-\rho Q_2 & [b1], \\ (p_1-(1-\rho))\,\frac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 & [b1]. \end{cases} \tag{EC.26}$$

(e) Within regime (b), if $p_1 \le \alpha$, $p_2 \le p_1 - (1-\rho)\alpha$ and $Q_1 + (2-\rho)Q_2 \le 1$, then

$$\pi_2^* = \begin{cases} (p_1-\alpha(1-\rho))\,Q_2 & \text{if } p_1 \le \alpha - \rho Q_2 & [b8], \\ \rho(p_1-\beta)Q_2 & \text{if } \alpha - \rho Q_2 \le p_1 \le 1-\rho Q_2 & [b1], \\ (p_1-(1-\rho))\,\frac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 & [b1]. \end{cases} \tag{EC.27}$$

(f) Within regime (c),

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if } p_1 \geq 1-\rho Q_2 \quad [c1]\,, \\ (p_1-(1-\rho))\,Q_2 & \text{if } p_1 \leq 1-\rho Q_2 \quad [c2],\ \ [c3],\ \ [c4] \text{ or } [c5]\,. \end{cases} \tag{EC.28}$$

The proof is analogous to the case of independent arrivals. For part (a), we utilize the condition (i) of $R^L(\rho)$; for parts (b), (c), (d) and (e), we use the conditions (i), (iv) and (v); for part (f), we use the condition (i) of $R^L(\rho)$.

Lemma EC.8 *If $(Q_1, Q_2) \in R^L(\rho)$ under low-to-high arrivals, then the optimal follower's profits and the resulting leader's profits, are given as follows, as functions of the leader's price.*

(a) If $p_1 \geq \alpha$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \rho(p_1-\beta)Q_2 & \text{if} \quad p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.29}$$

$$\pi_1(p_1,\, p_2^*(p_1)) = \begin{cases} p_1(1-\rho Q_2 - p_1) & \text{if} \quad p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.30}$$

(b) If $p_1 \leq \alpha$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \rho(1-Q_2)Q_2 & \text{if} \quad p_1 \leq \alpha - Q_2 \quad [a3]\,, \\ \rho(p_1+Q_1)(\alpha - p_1) & \text{if} \quad \alpha - Q_2 \leq p_1 \leq p_1^L \quad [a4]\,, \\ \rho(p_1-\beta)Q_2 & \text{if} \quad p_1^L \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.31}$$

$$\pi_1(p_1,\, p_2^*(p_1)) = \begin{cases} p_1 Q_1 & \text{if} \quad p_1 \leq p_1^L \quad [a3] \text{ or } [a4]\,, \\ p_1(1-\rho Q_2 - p_1) & \text{if} \quad p_1^L \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.32}$$

where

$$p_1^L = \frac{1}{2}\left(\alpha - (Q_1+Q_2) + \sqrt{(\alpha-(Q_1+Q_2))^2 + 4\delta}\right)\,, \tag{EC.33}$$

and $\delta = \alpha Q_1 + \beta Q_2$.

As noted in the paper, the leader's equilibrium price p_1^L is the largest price she can charge and still have the follower prefer to sell solely to leftover customers rather than to engage in direct competition using all of his capacity. We use conditions (i) and (v) to prove (EC.29) and (EC.30); we use (iii), (iv), (v) and (vi) to prove (EC.31) and (EC.32).

Finally, for the optimal price for the leader, note that under the condition, (vii), $(1-\rho Q_2)/2 \leq p_1^L$. Hence π_1 is decreasing within the sub-regime $[b1]$. Therefore in equilibrium, $p_1^* = p_1^L$ as in (EC.33) and

$$p_2^*(p_1^L) = \rho(p_1^L + Q_1)\,, \tag{EC.34}$$

with the corresponding profits $\pi_1 = p_1^L Q_1$ and $\pi_2 = \rho(p_1^L + Q_1)(\alpha - p_1^L)$. □

Proof of Proposition 2: Under independent arrivals, given the leader's price p_1, the follower's optimal price p_2 within the Pure Leftovers Regime is $\rho/2$, which does not depend on p_1. Consequently, the follower's leftover demand from the leader is $\left(1-Q_1/(1-p_1)\right)/2$, which decreases as p_1 increases. Under low-to-high arrivals, $L_2^L=\min(1-p_2/\rho,\, \alpha-p_1)$, which is decreasing in p_1 when $p_2 \le \rho(p_1+Q_1)$. Otherwise L_2^L does not depend on p_1. If $\alpha-p_1 \le Q_2$, the optimal p_2 is either $\rho/2$ or $\rho(1-\alpha+p_1)$. In both cases, the follower's leftover demand weakly decreases in p_1. If $\alpha-p_1 \ge Q_2$, his leftover demand does not depend on p_1. Under high-to-low arrivals, given p_1, the optimal p_2 equals to $\rho\alpha/2$, which yields $L_2^H=\alpha/2$ in the Pure Leftovers Regime. Therefore the follower's demand is unchanged under high-to-low arrivals. □

Proof of Proposition 3: First, the leader sets p_1 at which the follower's profits under the Pure Leftovers Regime are the same as his profits under a direct competition in the Baseline Leftovers Equilibrium. Further follower's profits under direct competition are the same across all arrival sequences given p_1, $\pi_2=\rho(p_1-\beta)Q_2$. Consider the Pure Leftovers Regime. For independent arrivals, the follower's leftover demand from the leader is $L_2^I=1-p_2/\rho-Q_1(1-p_2/\rho)/(1-p_1)$. For high-to-low arrivals, it is $L_2^H=\alpha-p_2/\rho$, while for low-to-high arrivals, it is equal to $L_2^L=\min(1-p_2/\rho,\, \alpha-p_1)$. First, since $p_1 \le p_2/\rho$ in the Baseline Leftovers Equilibrium, $L_2^H \le L_2^I$ for any given p_2 under the Pure Leftovers Regime. Second, since $1-p_2/\rho \ge L_2^I$ and $\alpha-p_1-L_2^I=(1-Q_1/(1-p_1))(p_2/\rho-p_1)\ge 0$, we obtain $L_2^I \le L_2^L$ for any given p_2 under the Pure Leftovers Regime. Therefore at $p_1=p_1^I$, the follower strictly prefers to be in the Pure Leftovers Regime under low-to-high arrivals, while he strictly prefers to be in direct competition under high-to-low arrivals. As a result, $p_1^H \le p_1^I \le p_1^L$ as in part (a), which in turn implies $\pi_1^H \le \pi_1^I \le \pi_1^L$ in part (b) since the leader's sales quantity in the Baseline Leftovers Equilibrium is Q_1 for all customer arrival sequences. In addition, the follower's profits under the Pure Leftovers Regime are the same as his profits under direct competition in the Baseline Leftovers Equilibrium for all customer arrival sequences. The follower's profits under a direct competition are the same for all arrivals and this follower's profits function are increasing in p_1 for any p_2. Hence $\pi_2^H \le \pi_2^I \le \pi_2^L$. Furthermore, from the follower's equilibrium price under the Baseline Leftovers Equilibrium, we have $p_2^H=\rho\alpha/2 \le p_2^I=\rho/2$. Since $Q_2 \le \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}$ in $R^L(\rho)$, we also obtain $p_2^I \le p_2^L$. □

Proof of Proposition 4: First, given that $Q_1 \in [0,1]$, p_1^H is decreasing in Q_1. Using the fact that $(1+4Q_2(1+\beta))^2-64Q_2^2\gamma=1+8Q_2(2Q_1+(1-\beta)(2Q_2(1-\beta)-1))$ is increasing in Q_1, it follows that p_1^I is decreasing in Q_1. For low-to-high arrivals, from the condition (i) in $R^L(\rho)$, we have

$$\frac{\partial p_1^L}{\partial Q_1} = -1 + \frac{Q_2}{\sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}} \leq 0\,. \tag{EC.35}$$

Similarly, it follows that π_2^H, π_2^I and π_2^L are decreasing in Q_1. Further, note that $\partial \pi_1^H / \partial Q_1 = \alpha(1 - 3Q_1)/(4Q_2) + \beta$, which is positive when Q_1 is small. Moreover, $\partial^2 \pi_1^H / \partial Q_1^2 = -(2 - 3Q_1)/(2Q_2) \leq 0$ in $R^H(\rho)$, which shows that π_1^H is concave in Q_1. For independent arrivals, we obtain

$$\frac{\partial \pi_1^I}{\partial Q_1} = \frac{1 + 4(1+\beta)Q_2}{8Q_2} - \frac{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma + 8Q_1Q_2}{8Q_2\sqrt{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}\,. \tag{EC.36}$$

When Q_1 is small, $\partial \pi_1^I / \partial Q_1$ is positive. Further it follows that in $R^I(\rho)$, the numerator in (EC.36) is decreasing in Q_1. Therefore, π_1^I is quasi-concave in Q_1. For low-to-high arrivals, similarly we obtain

$$\frac{\partial \pi_1^L}{\partial Q_1} = \frac{1}{2}\left(1 - 4Q_1 - Q_2 + \sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta} + \frac{2Q_1Q_2}{\sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}}\right), \tag{EC.37}$$

which is positive when Q_1 is small. Further, taking a derivative of $\partial \pi_1^L / \partial Q_1$ with respect to Q_1, one can show that in $R^L(\rho)$,

$$\frac{\partial^2 \pi_1^L}{\partial Q_1^2} = -2 + \frac{2Q_2}{\sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}}\left(1 - \frac{Q_1Q_2}{(\alpha - (Q_1 + Q_2))^2 + 4\delta}\right) \leq 0\,. \tag{EC.38}$$

Hence π_1^L is concave in Q_1. Further, note that $p_2^H = \rho\alpha/2$ is decreasing in Q_1 and $p_2^I = \rho/2$ does not depend on Q_1. Lastly, from simple algebra, it follows that $p_2^L = \rho(p_1^L + Q_1)$ is increasing in Q_1. □

Proof of Proposition 5: First, note that from the condition (i) in $R^H(\rho)$, we obtain

$$\frac{\partial p_1^H}{\partial Q_2} = 1 - \rho - \frac{\alpha^2}{4Q_2^2} \leq 0\,. \tag{EC.39}$$

Taking derivative of p_1^I with respect to Q_2, we then have

$$\frac{\partial p_1^I}{\partial Q_2} = \frac{1 - 4\beta(1 - 2Q_1 - 4(1-\beta)Q_2^2)}{8Q_2^2\sqrt{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma}} - \frac{1 - 4\beta Q_2}{8Q_2^2}\,. \tag{EC.40}$$

Under the conditions, (i), (ii) and (iii) in $R^I(\rho)$, we find that $\partial p_1^I / \partial Q_2 < 0$. For p_1^L, we obtain

$$\frac{\partial p_1^L}{\partial Q_2} = \frac{1}{2}\left(-1 + \frac{-1 + 2Q_1 + 5Q_2 - 4\rho Q_2}{\sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}}\right) \leq 0\,, \tag{EC.41}$$

since $-1 + 2Q_1 + 5Q_2 - 4\rho Q_2 \leq \sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}$ in $R^L(\rho)$. Hence the leader's price decreases as Q_2 increases for all three arrival sequences. Further, $\pi_1^s = p_1^s Q_1$, for $s \in \{H, I, L\}$, under the Baseline Leftovers Equilibrium, the leader's profits decrease in Q_2 for all $s \in \{H, I, L\}$. It is straightforward from Table 1 that p_2^H, π_2^H and p_2^I do not depend on Q_2. Taking derivative of S_2^I with respect to Q_2, we obtain

$$\frac{\partial S_2^I}{\partial Q_2} = 1 - 2\beta - \frac{2Q_1 - (1 - 4Q_2(1-\beta))(1-2\beta)}{\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}} \geq 0\,, \tag{EC.42}$$

in $R^I(\rho)$. Hence $\pi_2^I = S_2^I\rho/2$ is increasing in Q_2. Next, consider low-to-high arrivals. First, it follows that in $R^L(\rho)$,

$$\frac{\partial p_2^L}{\partial Q_2} = \frac{\rho}{2}\left(-1 + \frac{2Q_1 + 5Q_2 - 4\rho Q_2 - 1}{\sqrt{(\alpha - (Q_1+Q_2))^2 + 4\delta}}\right) \leq 0\,. \tag{EC.43}$$

Lastly, taking derivative of π_2^L with respect to Q_2 and simplifying, we have

$$\frac{\partial \pi_2^L}{\partial Q_2} = \frac{\rho(\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta} - Q_2)}{2}\left(\frac{1 - 2Q_1 - (5-4\rho)Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}} + 1\right)\,. \tag{EC.44}$$

We then obtain that $\partial\pi_2^L/\partial Q_2 \geq 0$ in $R^L(\rho)$. $\square$

Proof of Proposition 6: For part (a), note that $\partial\pi_1^N/\partial\rho = -1/[2(2-\rho)^2] < 0$ and $\partial(\pi_1^N + \pi_2^N)/\partial\rho = -(2+\rho)/[4(2-\rho)^3] < 0$. Hence π_1^N and $\pi_1^N + \pi_2^N$ are decreasing in ρ. In addition, $\partial\pi_2^N/\partial\rho = (2-3\rho)/[4(2-\rho)^3]$, which is increasing in ρ if $\rho \leq 2/3$, and decreasing afterwards. Further, $\partial p_2^N/\partial\rho = 1/2 - 1/(2-\rho)^2$. Thus p_2^N is increasing in ρ if $\rho \leq 2 - \sqrt{2}$, and decreasing afterwards.

For part (b), from the expressions in Table 1, it follows that π_1^H and π_1^L decrease as ρ increases. For π_1^I, first, note that

$$\frac{\partial p_1^I}{\partial\rho} = -\frac{Q_2}{2} - \frac{Q_2(4(1-\beta)Q_2 - 1)}{2\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}} \leq 0\,. \tag{EC.45}$$

Since $\pi_1^I = p_1^I Q_1$ under the Baseline Leftovers Equilibrium, we obtain that π_1^I is decreasing in ρ. Hence, the leader's profits decrease in ρ for all three arrival sequences. Next, note that $p_2^H = \rho\alpha/2$ and $p_2^I = \rho/2$ are increasing in ρ. In addition, it follows that in $R^L(\rho)$,

$$\frac{\partial p_2^L}{\partial\rho} = \frac{1-Q_2}{2} + \frac{(\alpha-(Q_1+Q_2))^2 + 4\delta - 2\rho Q_2^2}{2\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}} \geq 0\,. \tag{EC.46}$$

Thus, the follower's price increases in ρ for all three arrival sequences. Lastly, note that $S_2^H = \alpha/2$ does not depend on ρ and $S_2^L = (1 + Q_2 - \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta})/2$ increases as ρ increases. Hence, $\pi_2^H = p_2^H S_2^H$ and $\pi_2^L = p_2^L S_2^L$ are increasing in ρ. In addition, we obtain

$$\frac{\partial\pi_2^I}{\partial\rho} = \frac{1}{8}\Bigg(4Q_2(1-(1-2\rho)Q_2) + 1 - \sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma} - \frac{\rho Q_2\,(4Q_2(1-\beta)-1)}{2\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\Bigg) \geq 0\,, \tag{EC.47}$$

under the conditions, (i), (ii) and (iii) in $R^I(\rho)$. Therefore, the follower's profits increase in ρ for all three arrival sequences. For the industry profits under low-to-high arrivals, it follows that

$$\frac{\partial(\pi_1^L+\pi_2^L)}{\partial\rho}=\frac{Q_2}{2}\left(1-2Q_1-(3-4\rho)Q_2+\frac{(\alpha-(Q_1+Q_2))^2+4\delta-2\rho Q_2^2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right). \tag{EC.48}$$

We obtain $(\alpha-(Q_1+Q_2))^2+4\delta-2\rho Q_2^2\geq 0$ in $R^L(\rho)$. Further, condition (v) in $R^L(\rho)$ implies that $2Q_1+(3-4\rho)Q_2\leq 1$. Hence we obtain $\partial(\pi_1^L+\pi_2^L)/\partial\rho\geq 0$. Therefore industry profits increase under low-to-high arrivals. For independent and high-to-low arrivals, we provide a region of parameters where industry profits are non-monotonic in ρ. Under independent arrivals, when $(Q_1,\,Q_2)=(0.5,\,0.4)\in R^I(\rho)$, where $\rho\in[0.75,\,0.9]$, $\pi_1^I+\pi_2^I$ is decreasing and then increasing in ρ afterwards. Under high-to-low arrivals, $\pi_1^H+\pi_2^H$ is decreasing in ρ when $(Q_1,\,Q_2)=(0.5,\,0.4)\in R^H(\rho)$, where $\rho\in[0.75,\,0.83]$, while $\pi_1^H+\pi_2^H$ is increasing in ρ when $(Q_1,\,Q_2)=(0.3,\,0.4)\in R^H(\rho)$, where $\rho\in[0.8,\,0.95]$.

Next, the condition (i) in $R^H(\rho)$ implies $(2-\rho)Q_2(1-(2-\rho)Q_2)\leq 1/4$, from which we obtain $2(2-\rho)^2Q_1Q_2\leq 2(2-\rho)^2Q_2(1-(2-\rho)Q_2)\leq(2-\rho)/2\leq 1$. From this inequality, it follows that

$$\frac{\partial(\pi_1^H-\pi_1^N)}{\partial\rho}=-\frac{2(2-\rho)^2Q_1Q_2-1}{2(2-\rho)^2}\geq 0\,. \tag{EC.49}$$

Further, we obtain that in $R^H(\rho)$

$$\frac{\partial(\pi_2^H-\pi_2^N)}{\partial\rho}=\frac{\alpha^2}{4}-\frac{2-3\rho}{4(2-\rho)^3}\geq 0\,. \tag{EC.50}$$

In addition, for the comparison between independent arrivals and high-to-low arrivals, from $(4Q_2(1-\beta)-1)^2-((1+4Q_2(1+\beta))^2-64Q_2^2\gamma)=-16Q_1Q_2\leq 0$, we obtain

$$\frac{\partial(\pi_1^I-\pi_1^H)}{\partial\rho}=\frac{Q_1Q_2}{2}\left(1-\frac{4Q_2(1-\beta)-1}{\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\right)\geq 0\,. \tag{EC.51}$$

Similarly, from the conditions (i), (ii), and (iii) in $R^I(\rho)$, it follows that $\partial(\pi_2^I-\pi_2^H)/\partial\rho\geq 0$. □

Proof of Proposition 7: For part (a), from the expressions in Table 1, first note that

$$r^I = \frac{4Q_2\rho}{4Q_2(1+\beta)+1-\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}, \tag{EC.52}$$

$$r^H = \frac{2\rho\alpha Q_2}{\alpha^2+4\beta Q_2}, \quad \text{and} \tag{EC.53}$$

$$r^L = \rho\left(1+\frac{2Q_1}{1-2Q_1-Q_2+\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right). \tag{EC.54}$$

We then obtain that $\rho/2=r^N\leq\rho\leq r^H\leq r^I$ from the conditions (i) and (ii) in $R^I(\rho)$ and $R^H(\rho)$. In addition, it follows that $r^I\leq r^L$ in $R^I(\rho)\cap R^L(\rho)$.

For part (b), first note that $r^N = \rho/2$ is increasing in ρ, Q_1 and Q_2. Next, for high-to-low arrivals, p_1^H is decreasing in ρ, whereas p_2^H is increasing in ρ. Consequently, r^H is increasing in ρ. Further, from the condition (i) in $R^H(\rho)$, we obtain

$$\frac{\partial r^H}{\partial Q_1} = \frac{2\rho Q_2\,(\alpha^2 - 4\beta Q_2)}{(\alpha^2 + 4\beta Q_2)^2} \geq 0\,. \tag{EC.55}$$

Note that p_2^H in Table 1 does not depend on Q_2 and that p_1^H is decreasing in Q_2 from (EC.39). As a result, r^H is increasing in Q_2. Next, for independent arrivals, p_1^I is decreasing in ρ from (EC.45) and $p_2^I = \rho/2$ is increasing in ρ. Thus $r^I = p_2^I/p_1^I$ is increasing in ρ. Observe that p_1^I is decreasing in Q_1. Further, p_1^I is decreasing in Q_2 from Proposition 5. $p_2^I = \rho/2$ does not depend on Q_1 or Q_2. Therefore $r^I = p_2^I/p_1^I$ is increasing in Q_1 and Q_2. Lastly, for low-to-high arrivals, p_1^L is decreasing in ρ. Further, from (EC.46), we have $\partial p_2^L/\partial\rho \geq 0$. Hence we obtain $\partial r^L/\partial\rho \geq 0$. From (EC.35), p_1^L is decreasing in Q_1. In addition, p_2^L is increasing in Q_1. Hence, r^L is increasing in Q_1. Finally, in $R^L(\rho)$, $(Q_1 + Q_2 - \alpha)^2 \leq (Q_1 + Q_2 + 4\beta Q_2 - \alpha)^2 \leq (Q_1 + Q_2 - \alpha)^2 + 4\delta$, from which we obtain

$$\frac{\partial r^L}{\partial Q_2} = \frac{2\rho Q_1\left(\alpha - (Q_1+Q_2) - 4\beta Q_2 + \sqrt{(\alpha - (Q_1+Q_2))^2 + 4\delta}\right)}{\sqrt{(\alpha - (Q_1+Q_2))^2 + 4\delta}\left(\alpha - (Q_1+Q_2) + \sqrt{(\alpha - (Q_1+Q_2))^2 + 4\delta}\right)} \geq 0\,.\ \square \tag{EC.56}$$

Proofs for the Remaining Comparative Statics in Table 1: The remaining comparative statics results in Table 1 that we have not proved are $\partial\pi_T^s/\partial Q_1$ and $\partial\pi_T^s/\partial Q_2$ for $s \in \{H, I, L\}$.

For high-to-low arrivals, note that π_1^H is quasi-concave in Q_1, and π_2^H is decreasing in Q_1. As a result, $\pi_T^H = \pi_1^H + \pi_2^H$ is quasi-concave in Q_1. Furthermore, from $\partial\pi_1^H/\partial Q_2 < 0$ and $\partial\pi_2^H/\partial Q_2 = 0$, we obtain $\partial\pi_T/\partial Q_2 < 0$.

For independent arrivals, similarly note that π_1^I is quasi-concave in Q_1, and π_2^I is decreasing in Q_1. As a result, $\pi_T^I = \pi_1^I + \pi_2^I$ is quasi-concave in Q_1. In addition, taking a derivative of π_T^I with respect to Q_2, we obtain

$$\begin{aligned}\frac{\partial\pi_T^I}{\partial Q_2} &= \frac{\rho Q_2(1 + 4(1-\beta)Q_2) + Q_1(1 + 4(1+\beta)Q_2) - (Q_1 + \rho Q_2)\sqrt{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}{8Q_2} \\ &\quad \times \frac{A + B\sqrt{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}{8Q_2^2\sqrt{(1 + 4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}\,,\end{aligned} \tag{EC.57}$$

where

$$A = 8Q_1^2 Q_2 + 4\rho(1-2\beta)Q_2^2 + (1 - 4(1 + 2\rho Q_2)Q_2 + 16\beta(1-\beta)Q_2^2)Q_1\,, \tag{EC.58}$$

and

$$B = 4\beta Q_1 Q_2 - Q_1 + 4\rho(1-2\beta)Q_2^2\,. \tag{EC.59}$$

In $R^H(\rho)$, $B\leq 0$. Further one can show that $A^2\leq((1+4Q_2(1+\beta))^2-64Q_2^2\gamma)B^2$ in $R^H(\rho)$. This proves that $\partial\pi_T^I/\partial Q_2\leq 0$.

For low-to-high arrivals, similarly note that π_1^L is quasi-concave in Q_1, and π_2^L is decreasing in Q_1. As a result, $\pi_T^L=\pi_1^L+\pi_2^L$ is quasi-concave in Q_1. In addition, π_T^L is decreasing in Q_2 at $(Q_1, Q_2)=(0.5, 0.49)$ with $\rho=0.6$ whereas π_T^L is increasing in Q_2 at $(Q_1, Q_2)=(0.5, 0.46)$ with $\rho=0.8$. The Baseline Leftovers Equilibrium arises under both cases. Hence $\partial\pi_T^L/\partial Q_2$ is indeterminate. □

Proof for the Unlimited Capacities Equilibrium : We first obtain the follower's best response given the leader's price p_1;

$$p_2^*(p_1)=\begin{cases}\frac{\rho p_1}{2} & \text{if} \quad p_1\leq\frac{2(1-\rho)}{2-\rho}\,,\\ p_1-(1-\rho) & \text{if} \quad \frac{2(1-\rho)}{2-\rho}\leq p_1\leq 1-\frac{\rho}{2}\,,\\ \frac{\rho}{2} & \text{if} \quad 1-\frac{\rho}{2}\leq p_1\,.\end{cases} \tag{EC.60}$$

Plugging (EC.60) into the leader's profit maximization problem and optimizing, we obtain p_1^N. Then it follows that $p_2^N=p_2^*(p_1^N)$. □

Numerical Examples for Robustness: Leftovers equilibria can emerge in other production cost settings, such as when unit production costs are not proportional to quality. Let c_i denote the unit production cost for Firm i, so that the cost to firm i of selling the quantity x is $c_i x$, where, of course, $x\leq Q_i$. These represent costs of exercising the firm's capacity and differ from the firm's unit capacity cost. If $c_1=0.1$, $c_2=0.064$ and $\rho=0.8$, so the unit production costs are not proportional to ρ, then a Baseline Leftovers Equilibrium arises under independent arrivals when $Q_1=0.5$ and $Q_2=0.42$. The equilibrium outcome is $p_1^I=0.305$ and $p_2^I=0.432$, with the leader leaving 0.195 in leftovers to the follower, who sells 0.129 units.

Leftovers equilibria can also emerge when production costs are not linear in quantity. For example, if Firm i's cost to produce x units is $C_i(x)=c_i x^2$ for $(c_1, c_2)\in\{(0.1, 0.08), (0.1, 0.064), (0.1, 0.01)\}$, a Leftovers Equilibrium arises with $Q_1=0.5$, $Q_2=0.42$ and $\rho=0.8$ under independent arrivals. In addition, if $C_i(x)=c_i\sqrt{x}$, a Leftovers Equilibrium also arises under the same parameter set.

Lastly, Leftovers Equilibria can also arise when the lower quality firm prices first. If the leader has quality 0.8, the follower has quality 1 (so the lower quality firm goes first), and the capacities are 0.42 and 0.50 for the leader and the follower, respectively, then, under independent arrivals, the lower quality leader leaves 0.40 in leftovers for the high quality follower to exploit. The (lower quality) leader's equilibrium price is 0.144 and the (high quality) follower's equilibrium price is 0.5. The leader's demand is 0.82 leaving 0.4 in leftovers and the follower satisfies only 0.24 of those leftovers in equilibrium.