Electronic Companion to "Tax-Effective Supply Chain Decisions under China's Export-Oriented Tax Policies"

Optimality Equations of EI Strategy

In this part, we derive the optimality equations for the Export-Import (EI) strategy. Specifically, we discuss the following cases:

Case 1. $\hat{p}_g \geq \hat{p}_{gc}$. In this case, selling a unit in the overseas market is more profitable than reimporting and selling it in the domestic market. We thus have three subcases as follows:

Subcase 1a. $\hat{p}_g > \hat{s}_g \geq \hat{p}_{gc} > \hat{s}_{gc}$. This is an extreme subcase in which, for an exported unit, salvaging it in the overseas market is more attractive than reimporting it back to the domestic market. (This situation may occur when the cost of reimporting is very expensive, for instance, either b_{gc} or t_P is high.) Thus, regardless of the demand realization, all exported x units will be sold or salvaged in the overseas market. Clearly, in this subcase, P_{EI2} becomes two separate newsvendor problems with the following optimal solutions:

$$x^{EI2} = F_g^{-1}\left[\frac{\hat{p}_g}{\hat{p}_g - \hat{s}_g}\right], \quad y^{EI2} = F_c^{-1}\left[\frac{\hat{p}_c}{\hat{p}_c - \hat{s}_c}\right],$$

while the allocation of x^{EI2} to the two markets is $x_1^{EI2} = x^{EI2}$ and $x_2^{EI2} = 0$. Similarly, the optimal solution to P_{EI1} is

$$x^{EI1} = F_g^{-1}\left[\frac{\hat{p}_g}{\hat{p}_g - \hat{s}_g}\right].$$

Subcase 1b. $\hat{p}_g \geq \hat{p}_{gc} > \hat{s}_g \geq \hat{s}_{gc}$. In this subcase, the exported x units are first used to satisfy the demand in the overseas market. Upon demand realization, a quantity of $x_2 = (x - \xi_g)^+ \wedge (\xi_c - y)^+$ is then reimported to satisfy the unmet demands in the domestic market. Finally, any unsold exported products are salvaged in the overseas market. We can show that the optimal solutions, x^{EI2} and y^{EI2}, to problem P_{EI2} satisfy

$$\hat{p}_g - (\hat{p}_g - \hat{s}_g)\,F_g(x^{EI2}) + (\hat{p}_{gc} - \hat{s}_g)\int_0^{x^{EI2}}\int_{x^{EI2}+y^{EI2}-\xi_g}^{\infty} f_c(\xi_c)\,f_g(\xi_g)\,d\xi_c d\xi_g = 0, \tag{10}$$

$$\hat{p}_c - (\hat{p}_c - \hat{s}_c)\,F_c(y^{EI2}) - (\hat{p}_{gc} - \hat{s}_g)\int_0^{x^{EI2}}\int_{y^{EI2}}^{x^{EI2}+y^{EI2}-\xi_g} f_c(\xi_c)\,f_g(\xi_g)\,d\xi_c d\xi_g = 0. \tag{11}$$

The first term of (10), $\hat{p}_g - (\hat{p}_g - \hat{s}_g)\,F_g(x^{EI2})$, can be viewed as the marginal profit for selling in the overseas market with respect to an increase in x^{EI2}. In the second term, $(\hat{p}_{gc} - \hat{s}_g)$ represents the added net profit for reimporting and selling a unit in the domestic market instead of salvaging that unit in the overseas market. Therefore, the second term of (10) represents the marginal profit generated by reimporting. The interpretation for (11) is similar.

The allocation of x^{EI2} to the two markets in problem P_{EI2} is

$$x_1^{EI2} = x^{EI2} - \left(x^{EI2} - \xi_g\right)^+ \wedge \left(\xi_c - y^{EI2}\right)^+, \quad x_2^{EI2} = \left(x^{EI2} - \xi_g\right)^+ \wedge \left(\xi_c - y^{EI2}\right)^+.$$

For problem P_{EI1}, the optimal solution x^{EI1} satisfies

$$\hat{p}_g - (\hat{p}_g - \hat{s}_g) F_g\left(x^{EI1}\right) + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI1}} \int_{x^{EI1}-\xi_g}^{\infty} f_c(\xi_c) f_g(\xi_g) \, d\xi_c d\xi_g = 0. \tag{12}$$

(The result can be viewed as setting $y = 0$ in problem P_{EI2}.) The allocation of x^{EI1} to the two markets is

$$x_1^{EI1} = x^{EI1} - \left(x^{EI1} - \xi_g\right)^+ \wedge \xi_c, \quad x_2^{EI1} = \left(x^{EI1} - \xi_g\right)^+ \wedge \xi_c.$$

Subcase 1c. $\hat{p}_g \geq \hat{p}_{gc} > \hat{s}_{gc} > \hat{s}_g$. This subcase is similar to the previous one, except that any surplus from the exported x units will be reimported and salvaged in the domestic market. Thus, the optimality equations are the same as those of the previous subcase ((10)-(11) and (12)), except that $\hat{s}_g$ is replaced by $\hat{s}_{gc}$. The allocation of x^{EI2} to the two markets in problem P_{EI2} is

$$x_1^{EI2} = x^{EI2} \wedge \xi_g, \; x_2^{EI2} = (x^{EI2} - \xi_g)^+.$$

The allocation of x^{EI1} in problem P_{EI1} is identical to the above.

Case 2. $\hat{p}_{gc} > \hat{p}_g$. In this case, reimporting and selling a unit in the domestic market is more profitable than selling the unit in the overseas market. We have the following three subcases:

Subcase 2a. $\hat{p}_{gc} > \hat{s}_{gc} \geq \hat{p}_g > \hat{s}_g$. This represents an extreme situation in which selling and salvaging the product in the overseas market is not desirable. All of the products (both x and y) will therefore be sold in the domestic market. The optimal x^{EI2} and y^{EI2} for problem P_{EI2} can be determined by solving

$$\begin{aligned}
&\hat{p}_{gc} - (\hat{p}_{gc} - \hat{s}_{gc}) F_c\left(x^{EI2} + y^{EI2}\right) = 0, \\
&\hat{p}_c - (\hat{p}_c - \hat{s}_c - \hat{p}_{gc} + \hat{s}_{gc}) F_c(y^{EI2}) - (\hat{p}_{gc} - \hat{s}_{gc}) F_c\left(x^{EI2} + y^{EI2}\right) = 0.
\end{aligned}$$

The allocation of x^{EI2} to the two markets is $x_1^{EI2} = 0$ and $x_2^{EI2} = x^{EI2}$.

Problem P_{EI1} in this subcase becomes a newsvendor problem for the domestic market, and x^{EI1} is a newsvendor solution of

$$x^{EI1} = F_c^{-1}\left[\frac{\hat{p}_{gc}}{\hat{p}_{gc} - \hat{s}_{gc}}\right].$$

The allocation of x^{EI1} to the two markets is the same as that in problem P_{EI2}.

Subcase 2b. $\hat{p}_{gc} > \hat{p}_g > \hat{s}_{gc} \geq \hat{s}_g$. In this subcase, the exported x units are first used to satisfy any unmet demand in the domestic market and then sold to meet the demand in the overseas market. Any unsold exported quantity will be salvaged in the domestic market. We can show that the optimal x^{EI2} and y^{EI2} to problem P_{EI2} satisfy

$$\hat{p}_{gc} - (\hat{p}_{gc} - \hat{p}_g) F_c\left(x^{EI2} + y^{EI2}\right) - (\hat{p}_g - \hat{s}_{gc}) \int_0^{x^{EI2}} \int_0^{x^{EI2}+y^{EI2}-\xi_g} f_c(\xi_c) f_g(\xi_g)\, d\xi_c d\xi_g = 0, \tag{13}$$

$$\begin{aligned} &\hat{p}_c - (\hat{p}_c - \hat{s}_c)F_c(y^{EI2}) - (\hat{p}_{gc} - \hat{p}_g)\left(F_c(x^{EI2} + y^{EI2}) - F_c\left(y^{EI2}\right)\right) \\ &\quad - (\hat{p}_g - \hat{s}_{gc}) \int_0^{x^{EI2}} \int_{y^{EI2}}^{x^{EI2}+y^{EI2}-\xi_g} f_c(\xi_c) f_g(\xi_g)\, d\xi_c d\xi_g = 0. \end{aligned} \tag{14}$$

The allocation of x^{EI2} to the two markets is

$$x_1^{EI2} = \left[x^{EI2} - \left(\xi_c - y^{EI2}\right)^+\right]^+ \wedge \xi_g, \quad x_2^{EI2} = x^{EI2} - \left[x^{EI2} - \left(\xi_c - y^{EI2}\right)^+\right]^+ \wedge \xi_g.$$

In problem P_{EI1}, the first-order condition indicates that x^{EI1} is the solution to

$$\hat{p}_{gc} - (\hat{p}_{gc} - \hat{p}_g) F_c\left(x^{EI1}\right) - (\hat{p}_g - \hat{s}_{gc}) \int_0^{x^{EI1}} \int_0^{x^{EI1}-\xi_g} f_c(\xi_c) f_g(\xi_g)\, d\xi_c d\xi_g = 0. \tag{15}$$

The allocation of x^{EI1} to the two markets is given by

$$x_1^{EI1} = \left(x^{EI1} - \xi_c\right)^+ \wedge \xi_g, \quad x_2^{EI1} = x^{EI1} - \left(x^{EI1} - \xi_c\right)^+ \wedge \xi_g.$$

Subcase 2c. $\hat{p}_{gc} > \hat{p}_g > \hat{s}_g > \hat{s}_{gc}$. This subcase is similar to the previous one, except that any exported but unsold products will be salvaged in the overseas market instead of being reimported back to the domestic market. The optimality equations of P_{EI2} and P_{EI1} are therefore identical to those in the previous subcase ((13)-(14) and (15), respectively), except that $\hat{s}_{gc}$ is replaced by $\hat{s}_g$. The allocations of x^{EI1} and x^{EI2} to the two markets are

$$x_1^{EI2} = \left(x^{EI2} - \left(\xi_c - y^{EI2}\right)^+\right)^+, \quad x_2^{EI2} = x^{EI2} \wedge \left(\xi_c - y^{EI2}\right)^+,$$

for problem P_{EI2} and

$$x_1^{EI1} = (x^{EI1} - \xi_c)^+, \quad x_2^{EI1} = x^{EI1} \wedge \xi_c,$$

for problem P_{EI1}.

Justification of Assumption of Proposition 7

We remark that the assumption of Proposition 7, i.e.,

$$\hat{p}_c - \hat{s}_c \geq \hat{p}_{gc} - max(\hat{s}_{gc}, \hat{s}_g), \tag{16}$$

though cumbersome, is not overly restrictive. To see this, recall that $\hat{p}_c$ and $\hat{s}_c$ are the net profit and net salvage value, respectively, for selling/salvaging P in the domestic market directly; $\hat{p}_{gc}$ is the net profit for selling P in the domestic market through an export and reimport operation; and $\hat{s}_g$ is the net value for salvaging P in the overseas market. Since an exported unit can either be salvaged in the overseas market or be reimported and savaged in the domestic market, $max(\hat{s}_{gc}, \hat{s}_g)$ represents the net salvage value for a unit of exported P. Thus, the left-hand side of (16) represents the net profit gained by the firm who is able to sell an extra unit of P in the domestic market from the production amount y initially planned for the domestic market. Similarly, the right-hand side of (16) represents the net profit from selling an extra unit of P in the domestic market from the initial amount x to be exported (through export and reimport operation).

Note that if P is reimported and eventually consumed in the domestic market, certain VAT and tariffs will be levied. Thus $\hat{p}_{gc}$ and $\hat{s}_{gc}$ are typically much smaller than $\hat{p}_c$ and $\hat{s}_c$, respectively (see (3)). Thus, it is straightforward that condition (16) would easily hold in this situation. On the other hand, if P is reimported as a bonded component and thus the selling and salvage prices for the reimported units satisfy (5), it is easy to show that (16) holds in this case as well.

Proofs of Propositions

We present below the proofs of the propositions in the paper.

Proof of Proposition 1: (i) Consider a buyer of product P in the China market who will use P to assemble another product. Let u be the unit production cost and q be the unit revenue at the buyer side. Besides, let h_c^P and h_g^P be the unit inventory holding costs of un-bonded P and bonded P, respectively. Assume the buyer sells z units in total. With Option 1, the buyer purchases P from the domestic market at p_c, and thus his profit is

$$(q + v_0 q - u - h_c^P)z - (p_c + v_0 p_c)z - v_0 q z + v_0 p_c z = (q - u - h_c^P)z - p_c z,$$

where the first term on the left-hand side is the gross sales, the second term is the purchasing cost of P, and the last two terms are the output and input VAT. With Option 2, the buyer imports P at price p_{gc} and thus has a profit of

$$\begin{aligned}&(q+v_0q-u-h_c^P)z-(1+t_P+v_0(1+t_P))p_{gc}z-v_0qz+v_0(1+t_P)p_{gc}z\\=&(q-u-h_c^P)z-(1+t_P)p_{gc}z,\end{aligned}$$

where the second term on the left-hand side is the total cost for importing P (including tariffs and taxes) and the last two terms again represent the output and input VAT.

Comparing the profits under two procurement options, we conclude that the buyer will be indifferent at $(1+t_P)p_{gc}=p_c$. Then, (3) follows.

(ii) Now consider the case in which the buyer of P uses it to assemble an exported product. With Option 1, P is domestically purchased at price p_c. Note that the buyer pays no output VAT since the final product is exported. The profit under this option is thus

$$(q-u-h_c^P)z-(p_c+v_0p_c)z-T_1,$$

where $T_1=-v_0p_cz+(qz-0)(v_0-v_1)$ is the tax payable. On the other hand, with Option 2, the buyer pays neither taxes nor tariffs on the imported P nor output VAT. Thus, the buyer's profit is

$$(q-u-h_c^P-\delta_p)z-p_{gc}z-T_2,$$

where the tax payable T_2 of this option is $T_2=(qz-p_{gc}z)(v_0-v_1)$ and as defined, $\delta_p\geq 0$ represents the added logistics and trading cost to the buyer in this option as opposed to Option 1.

Comparing the two profits under Options 1 and 2, we see that the buyer would be indifferent if $p_c=(1-v_0+v_1)p_{gc}+\delta_p$. Then, (4) follows. □

Proof of Proposition 2: By comparing (2) with (1), we know that when

$$\begin{aligned}&(1-v_0+v_1)p_{gc}-p_c+(t_{C2}+v_0-v_1)w_2\leq b_{gc}-b'_{gc}\text{ and}\\&(1-v_0+v_1)s_{gc}-s_c+(t_{C2}+v_0-v_1)w_2\leq b_{gc}-b'_{gc},\end{aligned}\tag{17}$$

the BI strategy is more attractive than the EI strategy.

With the assumptions of $t_P\geq t_{C2}$ and $s_c\geq(1+t_{C2})w_2$ and the fact that $s_c<p_c$, the left-hand sides of both inequalities in (17) are smaller than or equal to

$$\begin{aligned}&\frac{-v_0+v_1-t_P}{1+t_P}s_c+(t_{C2}+v_0-v_1)w_2\leq\frac{-v_0+v_1-t_{C2}}{1+t_{C2}}s_c+(t_{C2}+v_0-v_1)w_2\\=&\frac{t_{C2}+v_0-v_1}{1+t_{C2}}((1+t_{C2})w_2-s_c)\leq 0\leq b_{gc}-b'_{gc}.\end{aligned}$$

Thus, we conclude that the BI strategy is more attractive than the EI strategy. □

Proof of Proposition 3: Suppose $\delta^* \leq 0$. In this case, we have

$$(1 - v_0 + v_1)p_{gc} - p_c + (t_{C2} + v_0 - v_1)w_2 - (b_{gc} - b'_{gc}) = -\delta_p + \delta^* \leq 0.$$

Similarly, we can show

$$(1 - v_0 + v_1)s_{gc} - s_c + (t_{C2} + v_0 - v_1)w_2 - (b_{gc} - b'_{gc}) \leq 0.$$

By discussions in the proof of Proposition 2 (i.e., (17) holds), we conclude that the BI strategy is more attractive than the EI strategy.

Suppose now $\delta^* > 0$. To show (i), note that when $\delta_p = \delta^*$ and $\delta_s = \delta^*$, we have $\hat{p}_{gc} = \hat{p}'_{gc}$ and $\hat{s}_{gc} = \hat{s}'_{gc}$. Thus, problems defined by (2) and (1) are identical. Clearly, $\pi^{EI}(\delta_p, \delta_s) = \pi^{BI}$.

To prove (ii), first note that the expected profit $\pi^{EI}(\delta_p, \delta_s)$ is a non-increasing function of the costs δ_p and δ_s respectively. Thus, suppose

$$\pi^{EI}(\delta_p^1, \delta_s^1) = \pi^{EI}(\delta_p^2, \delta_s^2) = \pi^{BI}$$

and $\delta_p^1 \leq \delta_p^2$. We have

$$\pi^{EI}(\delta_p^2, \delta_s^2) = \pi^{EI}(\delta_p^1, \delta_s^1) \geq \pi^{EI}(\delta_p^2, \delta_s^1),$$

which implies $\delta_s^2 \leq \delta_s^1$. Therefore, (ii) follows.

Now, for any fixed (δ_p, δ_s^1), say, in the LR sub-region. Suppose $\pi^{EI}(\delta_p^1, \delta_s^1) = \pi^{BI}$. We have $\delta_p < \delta_p^1$ and therefore $\pi^{EI}(\delta_p, \delta_s^1) \geq \pi^{EI}(\delta_p^1, \delta_s^1) = \pi^{BI}$. So (iii) follows. □

Proof of Proposition 4: For ease of exposition, we rewrite the optimization problem P_{EI} as

$$\max_{x \geq 0, y \geq 0} \left[\Pi(x, y) - \mathbf{1}(x)J_g - \mathbf{1}(y)J_c - h_{2g}x - h_{2c}y\right],$$

where

$$\begin{aligned}\Pi(x, y) \quad = \quad & E \max_{x_1 + x_2 = x} \Big[(\hat{p}_c + h_{2c})\, y \wedge \xi_c + (\hat{s}_c + h_{2c})\,(y - \xi_c)^+ + (\hat{p}_g + h_{2g})\, x_1 \wedge \xi_g \\ & + (\hat{s}_g + h_{2g})\,(x_1 - \xi_g)^+ + (\hat{p}_{gc} + h_{2g})\left(x_2 \wedge (\xi_c - y)^+\right) + (\hat{s}_{gc} + h_{2g})\left(x_2 - (\xi_c - y)^+\right)^+\Big].\end{aligned}$$

That is, Π represents the total expected profit that excludes fixed and variable costs of storing the imported component C2. As discussed, the optimal solution to P_{EI} can be obtained by comparing the solutions to the following two optimization problems (which correspond to the EI1 and EI2 strategies):

$$\begin{aligned} P_{EI1}: \quad & \max_{x \geq 0} \left[\Pi(x, 0) - h_{2g}x\right] - J_g, \\ P_{EI2}: \quad & \max_{x \geq 0, y \geq 0} \left[\Pi(x, y) - h_{2g}x - h_{2c}y\right] - J_g - J_c. \end{aligned}$$

Note that it is possible that the constraint of $y \geq 0$ becomes binding in the optimal solution to P_{EI2}. In that case, the optimal solution to P_{EI2} degenerates to an EI1 structure (i.e., $x^{EI2} = x^{EI1}$ and $y^{EI2} = 0$). Below we present two claims.

Claim 1: *Let*

$$\bar{h}_{2c} = \left. \frac{\partial \Pi(x,y)}{\partial y} \right|_{x=x^{EI1}, y=0}.$$

Then, if $h_{2c} \geq \bar{h}_{2c}$, the optimal solution to P_{EI2} is $x^{EI2} = x^{EI1}$ and $y^{EI2} = 0$ (i.e., the same as that to P_{EI1}). Otherwise, we have $y^{EI2} > 0$ in the optimal solution to P_{EI2}.

To prove the claim, we note for P_{EI2} the optimality condition with respect to y is given by

$$\frac{\partial \Pi(x,y)}{\partial y} - h_{2c} + \mu_y = 0,$$

where $\mu_y \geq 0$ is the Lagrange multiplier associated with y. If $\mu_y > 0$, $y^{EI2} = 0$ and thus the optimal solution to P_{EI2} is $x^{EI2} = x^{EI1}$ and $y^{EI2} = 0$. Substituting the solution into the optimality condition stated above, we can obtain the result of Claim 1.

Claim 2: *When $h_{2c} < \bar{h}_{2c}$, $\Pi\left(x^{EI2}, y^{EI2}\right) - h_{2g}x^{EI2} - h_{2c}y^{EI2}$ is decreasing in h_{2c}.*

In the case of $h_{2c} < \bar{h}_{2c}$, as shown in the previous claim, we have $y^{EI2} > 0$. Thus, the result of Claim 2 holds because

$$\begin{aligned} \frac{d\left[\Pi\left(x^{EI2}, y^{EI2}\right) - h_{2g}x^{EI2} - h_{2c}y^{EI2}\right]}{dh_{2c}} &= \frac{\partial\left[\Pi\left(x^{EI2}, y^{EI2}\right) - h_{2g}x^{EI2} - h_{2c}y^{EI2}\right]}{\partial h_{2c}} \\ &= -y^{EI2} < 0, \end{aligned}$$

where the first equality comes from the Envelope Theorem.

In short, Claims 1 and 2 indicate that, when excluding the fixed costs, the optimal profit obtained from the EI2 strategy (i.e., $\Pi\left(x^{EI2}, y^{EI2}\right) - h_{2g}x^{EI2} - h_{2c}y^{EI2}$) is decreasing in h_{2c} when $h_{2c} < \bar{h}_{2c}$ but becomes that of EI1 (i.e., $\Pi\left(x^{EI1}, 0\right) - h_{2g}x^{EI1}$) and thus constant when $h_{2c} \geq \bar{h}_{2c}$.

Now we take into account the fixed costs, that is, J_g in the EI1 strategy and $J_g + J_c$ in EI2. It is straightforward that $\pi^{EI2} = \Pi\left(x^{EI2}, y^{EI2}\right) - h_{2g}x^{EI2} - h_{2c}y^{EI2} - J_g - J_c$ and $\pi^{EI1} = \Pi\left(x^{EI1}, 0\right) - h_{2g}x^{EI1} - J_g$ intersect exactly once at $h_{2c} = \tilde{h}_{2c}$ (which is less than $\bar{h}_{2c}$) and that at the intersection, $y^{EI2} > 0$. Therefore, the proposition follows. □

Prior to proving Propositions 5 to 7, we present a useful lemma below.

Lemma 1 $x^{EI2} + y^{EI2} \geq x^{EI1} \geq x^{EI2}$.

Proof: We prove the lemma by discussing all the cases presented previously. For subcases 1a and 2a, the proof is rather straightforward. Now, consider subcase 1b. We first show $x^{EI1} \geq x^{EI2}$. Note that equation (10) implies that

$$\hat{p}_g - (\hat{p}_g - \hat{s}_g)\, F_g(x^{EI2}) + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI2}} \int_{x^{EI2}-\xi_g}^{\infty} f_g(\xi_g)\, f_c(\xi_c)\, d\xi_c d\xi_g \geq 0.$$

On the other hand, (12) can be rewritten as

$$\hat{p}_g - (\hat{p}_g - \hat{s}_g)\, F_g(x^{EI1}) + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI1}} \int_{x^{EI1}-\xi_g}^{\infty} f_g(\xi_g)\, f_c(\xi_c)\, d\xi_c d\xi_g = 0.$$

Due to the concavity of the expected profit of EI1, we obtain $x^{EI1} \geq x^{EI2}$.

We then show $x^{EI2} + y^{EI2} \geq x^{EI1}$. We have

$$\begin{aligned}
&\hat{p}_g - (\hat{p}_g - \hat{s}_g)\, F_g(x^{EI2} + y^{EI2}) + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI2}+y^{EI2}} \int_{x^{EI2}+y^{EI2}-\xi_g}^{\infty} f_g(\xi_g)\, f_c(\xi_c)\, d\xi_c d\xi_g \\
&= (\hat{p}_g - \hat{s}_g)\left[F_g(x^{EI2}) - F_g(x^{EI2} + y^{EI2})\right] + (\hat{p}_{gc} - \hat{s}_g) \int_{x^{EI2}}^{x^{EI2}+y^{EI2}} \int_{x^{EI2}+y^{EI2}-\xi_g}^{\infty} f_g(\xi_g)\, f_c(\xi_c)\, d\xi_c d\xi_g \\
&\leq (\hat{p}_g - \hat{s}_g)\left[F_g(x^{EI2}) - F_g(x^{EI2} + y^{EI2})\right] + (\hat{p}_{gc} - \hat{s}_g)\left[F_g(x^{EI2} + y^{EI2}) - F_g(x^{EI2})\right] \\
&= (\hat{p}_g - \hat{p}_{gc})\left[F_g(x^{EI2}) - F_g(x^{EI2} + y^{EI2})\right] \leq 0,
\end{aligned}$$

where the first equality comes from applying (10). Comparing the above with (12) and noting the concavity of the expected profit, we conclude that $x^{EI2} + y^{EI2} \geq x^{EI1}$.

We can prove the remaining subcases by following the same logic as used above. For conciseness, we omit the details. □

Proof of Proposition 5: We note that to analyze the sign of $\frac{d\tilde{h}_{2c}}{dh_{2g}}$, it is sufficient to analyze that of $\frac{\partial \pi^{EI2}}{\partial h_{2g}} - \frac{\partial \pi^{EI1}}{\partial h_{2g}}$ due to (7). From the Envelope Theorem, we obtain

$$\begin{aligned}
\frac{\partial \pi^{EI2}}{\partial h_{2g}} &= E\Big[(-1)\, x_1^{EI2} \wedge \xi_g + (-1)\left(x_1^{EI2} - \xi_g\right)^+ + (-1)\left(x_2^{EI2} \wedge (\xi_c - y^{EI2})^+\right) \\
&\quad + (-1)\left(x_2^{EI2} - (\xi_c - y^{EI2})^+\right)^+\Big] \\
&= -E\left[x_1^{EI2} + x_2^{EI2}\right] = -x^{EI2}.
\end{aligned}$$

Similarly, we can show

$$\frac{\partial \pi^{EI1}}{\partial h_{2g}} = -x^{EI1}.$$

Therefore, we conclude

$$\frac{\partial \pi^{EI2}}{\partial h_{2g}} - \frac{\partial \pi^{EI1}}{\partial h_{2g}} = x^{EI1} - x^{EI2} \geq 0,$$

where the inequality comes from Lemma 1. Therefore, we conclude $\frac{d\tilde{h}_{2c}}{dh_{2g}} \geq 0$. $\square$

Proof of Proposition 6: We can show

$$\begin{aligned}\frac{\partial \pi^{EI1}}{\partial v_1} - \frac{\partial \pi^{EI2}}{\partial v_1} = E\big[&(p_g - w_2)\left(x_1^{EI1} \wedge \xi_g - x_1^{EI2} \wedge \xi_g\right) \\ &+ (s_g - w_2)\left(\left(x_1^{EI1} - \xi_g\right)^+ - \left(x_1^{EI2} - \xi_g\right)^+\right) \\ &+ (p_{gc} - w_2)\left(x_2^{EI1} \wedge \xi_c - x_2^{EI2} \wedge (\xi_c - y^{EI2})^+\right) \\ &+ (s_{gc} - w_2)\left(\left(x_2^{EI1} - \xi_c\right)^+ - \left(x_2^{EI2} - (\xi_c - y^{EI2})^+\right)^+\right)\big].\end{aligned}$$

Therefore, in determining the sign of $\frac{\partial \pi^{EI1}}{\partial v_1} - \frac{\partial \pi^{EI2}}{\partial v_1}$, we can analyze the four terms inside the expectation in the above equation. The analysis can be based on the different subcases presented previously.

For subcase 1a, we see that it is a trivial case in which $\frac{\partial \pi^{EI1}}{\partial v_1} - \frac{\partial \pi^{EI2}}{\partial v_1} = 0$ due to $x^{EI1} = x^{EI2} = x_1^{EI1} = x_1^{EI2}$ and $x_2^{EI1} = x_2^{EI2} = 0$. Next, we focus on subcase 1b. In this subcase, the optimal allocation of x^{EI1} in the EI1 structure yields

$$\begin{aligned} x_1^{EI1} \wedge \xi_g &= x^{EI1} \wedge \xi_g, \\ \left(x_1^{EI1} - \xi_g\right)^+ &= \left(x^{EI1} - \xi_g\right)^+ - \left(x^{EI1} - \xi_g\right)^+ \wedge \xi_c, \\ x_2^{EI1} \wedge \xi_c &= \left(x^{EI1} - \xi_g\right)^+ \wedge \xi_c, \\ \left(x_2^{EI1} - \xi_c\right)^+ &= 0, \end{aligned}$$

while the optimal allocation in EI2 gives

$$\begin{aligned} x_1^{EI2} \wedge \xi_g &= x^{EI2} \wedge \xi_g, \\ \left(x_1^{EI2} - \xi_g\right)^+ &= \left(x^{EI2} - \xi_g\right)^+ - \left(x^{EI2} - \xi_g\right)^+ \wedge \left(\xi_c - y^{EI2}\right)^+, \\ x_2^{EI2} \wedge (\xi_c - y^{EI2})^+ &= \left(x^{EI2} - \xi_g\right)^+ \wedge \left(\xi_c - y^{EI2}\right)^+, \\ \left(x_2^{EI2} - (\xi_c - y^{EI2})^+\right)^+ &= 0. \end{aligned}$$

Thus, we have

$$\begin{aligned} (p_g - w_2)\left(x_1^{EI1} \wedge \xi_g - x_1^{EI2} \wedge \xi_g\right) &\geq 0, \\ (s_{gc} - w_2)\left(\left(x_2^{EI1} - \xi_c\right)^+ - \left(x_2^{EI2} - (\xi_c - y^{EI2})^+\right)^+\right) &= 0, \end{aligned}$$

due to $x^{EI1} \geq x^{EI2}$, as shown in the above lemma. On the other hand, note that $\hat{p}_{gc} > \hat{s}_g$, as stated in the condition for this subcase, implies that $p_{gc} - w_2 > s_g - w_2$. Thus, we get

$$\begin{aligned}
&(s_g - w_2)\Big(\big(x_1^{EI1} - \xi_g\big)^+ - \big(x_1^{EI2} - \xi_g\big)^+\Big) + (p_{gc} - w_2)\left(x_2^{EI1} \wedge \xi_c - x_2^{EI2} \wedge (\xi_c - y^{EI2})^+\right) \\
&= (s_g - w_2)\Big(\big(x^{EI1} - \xi_g\big)^+ - \big(x^{EI1} - \xi_g\big)^+ \wedge \xi_c - \big(x^{EI2} - \xi_g\big)^+ + \big(x^{EI2} - \xi_g\big)^+ \wedge \big(\xi_c - y^{EI2}\big)^+\Big) \\
&+ (p_{gc} - w_2)\Big(\big(x^{EI1} - \xi_g\big)^+ \wedge \xi_c - \big(x^{EI2} - \xi_g\big)^+ \wedge \big(\xi_c - y^{EI2}\big)^+\Big) \\
&\geq (s_g - w_2)\Big(\big(x^{EI1} - \xi_g\big)^+ - \big(x^{EI1} - \xi_g\big)^+ \wedge \xi_c - \big(x^{EI2} - \xi_g\big)^+ + \big(x^{EI2} - \xi_g\big)^+ \wedge \big(\xi_c - y^{EI2}\big)^+\Big) \\
&+ (s_g - w_2)\Big(\big(x^{EI1} - \xi_g\big)^+ \wedge \xi_c - \big(x^{EI2} - \xi_g\big)^+ \wedge \big(\xi_c - y^{EI2}\big)^+\Big) \\
&= (s_g - w_2)\Big(\big(x^{EI1} - \xi_g\big)^+ - \big(x^{EI2} - \xi_g\big)^+\Big) \\
&\geq 0,
\end{aligned}$$

where the inequalities come from $p_{gc} - w_2 > s_g - w_2$, $s_g - w_2 \geq 0$ and $x^{EI1} \geq x^{EI2}$. Therefore, we conclude that $\frac{\partial \pi^{EI1}}{\partial v_1} - \frac{\partial \pi^{EI2}}{\partial v_1} \geq 0$ holds for this subcase.

The proof of the remaining subcases follows the same approach as shown above. In summary, we conclude $\frac{d\tilde{h}_{2c}}{dv_1} \leq 0$. □

Proof of Proposition 7: We can show that the desired result holds for all the subcases presented previously. For conciseness, we focus on subcase 1b, in which the optimal objective of EI2 is

$$\begin{aligned}
\pi^{EI2} = {} & -J_g - J_c + \hat{p}_c\mu_c + \hat{s}_c \int_0^{y^{EI2}} (y^{EI2} - \xi_c) f_c(\xi_c)\, d\xi_c - \hat{p}_c \int_{y^{EI2}}^{\infty} \left(\xi_c - y^{EI2}\right) f_c(\xi_c)\, d\xi_c \\
& + \hat{p}_g\mu_g + \hat{s}_g \int_0^{x^{EI2}} (x^{EI2} - \xi_g) f_g(\xi_g)\, d\xi_g - \hat{p}_g \int_{x^{EI2}}^{\infty} \left(\xi_g - x^{EI2}\right) f_g(\xi_g)\, d\xi_g \\
& + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI2}} \int_{y^{EI2}}^{x^{EI2}+y^{EI2}-\xi_g} \left(\xi_c - y^{EI2}\right) f_c(\xi_c)\, f_g(\xi_g)\, d\xi_c d\xi_g \\
& + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI2}} \int_{x^{EI2}+y^{EI2}-\xi_g}^{\infty} \left(x^{EI2} - \xi_g\right) f_c(\xi_c)\, f_g(\xi_g)\, d\xi_c d\xi_g.
\end{aligned}$$

Then, we can prove

$$\frac{\partial \pi^{EI2}}{\partial \mu_c} = (\hat{p}_c - \hat{s}_c)\, F_c(y^{EI2}) + (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI2}} \int_{y^{EI2}}^{x^{EI2}+y^{EI2}-\xi_g} f_c(\xi_c)\, f_g(\xi_g)\, d\xi_c d\xi_g.$$

Similarly, we can show

$$\frac{\partial \pi^{EI1}}{\partial \mu_c} = (\hat{p}_{gc} - \hat{s}_g) \int_0^{x^{EI1}} \int_0^{x^{EI1}-\xi_g} f_c(\xi_c)\, f_g(\xi_g)\, d\xi_c d\xi_g.$$

Thus, we obtain $\frac{\partial \pi^{EI2}}{\partial \mu_c} - \frac{\partial \pi^{EI1}}{\partial \mu_c} \geq 0$ with $x^{EI2} + y^{EI2} \geq x^{EI1}$, as shown in the above lemma, and the assumption of $\hat{p}_c - \hat{s}_c \geq \hat{p}_{gc} - \max(\hat{s}_g, \hat{s}_{gc}) = \hat{p}_{gc} - \hat{s}_g$. Such approach for subcase 1b can be carried over to the proof of other subcases. We omit the details for conciseness. □