On-line Appendix for "Using a Dual-Sourcing Option in the Presence of Asymmetric Information about Supplier Reliability: Competition vs. Diversification"

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A Proofs

Proof of Lemma 1. The supplier's objective function (3) is piecewise linear. We focus on the corner-point solutions: $z = 0$ or q. The result follows. ∎

Proof of Lemma 2. Omitted for brevity. The proof is similar to that of Proposition 3. ∎

Proof of Proposition 1. The proof of this proposition is parallel to that of Proposition 3, except that the feasible region is pruned by the additional constraints $q_1(t_1, t_2) = 0$ or $q_2(t_1, t_2) = 0$. Specifically, when determining the optimal $(q_1, q_2)(t_1, t_2)$, instead of four corner-point solutions, there are only three feasible corner-point solutions: $(q_1, q_2)(t_1, t_2) = (0, 0), (0, D)$ or $(D, 0)$. ∎

Proof of Proposition 2. The procedure for deriving the optimal order quantities of the buyer is similar to the proof of Proposition 3, except that for each of the four maximization problems in (A.10), there is only one feasible corner-point solution: $(q_1, q_2)(t_1, t_2) = (D, D)$. This is the optimal quantity pair for this model. ∎

Proof of Proposition 3. At optimality, from the supplier's optimal profit (3), we have

$$\begin{aligned} &X_n(t_1, t_2) + p_n(t_1, t_2) E \min\Big\{q_n(t_1, t_2),\ \rho_n^{t_n} z_n^{t_n}[(q_n, p_n)(t_1, t_2)]\Big\} \\ &= \pi_n^{t_n}[(X_n, q_n, p_n)(t_1, t_2)] + c^{t_n} z_n^{t_n}[(q_n, p_n)(t_1, t_2)]. \end{aligned} \tag{A.1}$$

Rolling equation (A.1) into (2a), the buyer's maximization problem becomes:

$$\begin{aligned} \max\Bigg\{ &\sum_{t_1, t_2 \in \{H,L\}} \alpha^{t_1}\alpha^{t_2}\Big[\ rE \min\Big\{D,\ \rho_1^{t_1} z_1^{t_1}[(q_1, p_1)(t_1, t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2, p_2)(t_1, t_2)]\Big\} \\ &- \Big\{c^{t_1} z_1^{t_1}[(q_1, p_1)(t_1, t_2)] + K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}\Big\} - \Big\{c^{t_2} z_2^{t_2}[(q_2, p_2)(t_1, t_2)] + K\,\mathbf{1}_{\{q_2(t_1,t_2)>0\}}\Big\}\Big] \\ &- \sum_{t_1=H,L}\Big\{\alpha^{t_1}\Pi_1^{t_1}(t_1)\Big\} - \sum_{t_2=H,L}\Big\{\alpha^{t_2}\Pi_2^{t_2}(t_2)\Big\}\Bigg\} \end{aligned} \tag{A.2a}$$

$$\text{Subject to}\quad \Pi_n^H(H) \geq \Pi_n^H(L), \tag{A.2b}$$

$$\Pi_n^L(L) \geq \Pi_n^L(H), \tag{A.2c}$$

$$\Pi_n^H(H) \geq 0, \tag{A.2d}$$

$$\Pi_n^L(L) \geq 0, \tag{A.2e}$$

$$q_n(t_1,t_2) \geq 0,\ p_n(t_1,t_2) \geq 0,\ \text{for } t_1,t_2 \in \{H,L\}, \quad \text{for } n=1,2, \tag{A.2f}$$

where $\Pi_n^{t_n}(s)$ is supplier-n's expected profit when it reports itself as of type-s, $s \in \{H, L\}$.

To solve problem (A.2), first reduce the *incentive compatibility* and *individual rationality* constraints (A.2b–A.2e) to an equivalent set of constraints, among which there are *monotonicity* constraints for the two suppliers. Then, we temporarily relax the *monotonicity* constraints and solve the relaxed problem. We then show that the optimal solution to the relaxation satisfies the monotonicity constraints and, thus, is optimal for the original problem.

To reduce problem (A.2), we first rearrange the *incentive compatibility* constraints (A.2b) and (A.2c) and the *individual rationality* constraints (A.2d) and (A.2e) for supplier $n = 1, 2$. Define $\overline{n}$ to be the index of the supplier other than supplier n. Applying the definition of a high-type supplier's reliability advantage (Definition 1) under the contract for low-type supplier-n, $(X_n, q_n, p_n)(L, t_{\overline{n}})$, we represent high-type supplier-n's expected profit, when reporting itself as of low-type, as $\Pi_n^H(L) = \Pi_n^L(L) + E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]\Big\}$. Similarly, applying Definition 1 with the contract for high-type supplier-n, $(X_n, q_n, p_n)(H, t_{\overline{n}})$, we represent low-type supplier-n's expected profit, when reporting itself as of high-type, as $\Pi_n^L(H) = \Pi_n^H(H) - E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(H,t_{\overline{n}})]\Big\}$. We substitute these two equalities into the right hand side of the *incentive compatibility* constraints, (A.2b) and (A.2c), obtaining

$$\Pi_n^H(H) \geq \Pi_n^L(L) + E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]\Big\}, \tag{A.3a}$$

$$\Pi_n^L(L) \geq \Pi_n^H(H) - E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(H,t_{\overline{n}})]\Big\}. \tag{A.3b}$$

Furthermore, inequalities (A.3a), $\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})] \geq 0$ and $\Pi_n^L(L) \geq 0$ (constraint (A.2e), *individual rationality* for the low-type) together imply $\Pi_n^H(H) \geq 0$. That is, the *individual rationality* constraint for high-type supplier-n (A.2d) is redundant.

Using the new *incentive compatibility* constraint (A.3) and *individual rationality* constraint (A.2e), we then choose $X_n(t_1,t_2)$ optimally for any given $(q_n,p_n)(t_1,t_2)$. The objective function (A.2a) suggests that the objective is maximized when $X_n(t_n,H)$ and $X_n(t_n,L)$ are chosen such that the expected profit of supplier n of type t_n, $\Pi_n^{t_n}(t_n)$, is minimized. Thus, at the optimal solution, the *individual rationality* constraint (A.2e) reduces to

$$\Pi_n^L(L) = 0. \tag{A.4}$$

Similarly, at the optimal solution the *incentive compatibility* constraint (A.3a) reduces to

$$\Pi_n^H(H) = E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]\Big\}. \tag{A.5}$$

We substitute (A.4) and (A.5) in the *incentive compatibility* constraints (A.3), obtaining

$$(Monotonicity) \quad E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(H,t_{\overline{n}})]\Big\} \geq E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]\Big\},\ n=1,2 \tag{A.6}$$

which is commonly called the *monotonicity* constraint in the information economics literature.

So far, we have reduced the original *incentive compatibility* and *individual rationality* constraints (A.2b–A.2e) to constraints (A.4–A.6). We roll (A.4) and (A.5) into the objective function (A.2a):

$$\begin{aligned}
\max\ &\Bigg\{\sum_{t_1,t_2\in\{H,L\}} \alpha^{t_1}\alpha^{t_2}\Bigg[\ rE\min\Big\{D,\ \rho_1^{t_1}z_1^{t_1}[(q_1,p_1)(t_1,t_2)]+\rho_2^{t_2}z_2^{t_2}[(q_2,p_2)(t_1,t_2)]\Big\}\\
&-\Big\{c^{t_1}z_1^{t_1}[(q_1,p_1)(t_1,t_2)]+K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}\Big\}-\Big\{c^{t_2}z_2^{t_2}[(q_2,p_2)(t_1,t_2)]+K\,\mathbf{1}_{\{q_2(t_1,t_2)>0\}}\Big\}\Bigg]\\
&-\alpha^H E_{t_2}\Big\{\Gamma_1[(q_1,p_1)(L,t_2)]\Big\}-\alpha^H E_{t_1}\Big\{\Gamma_2[(q_2,p_2)(t_1,L)]\Big\}\Bigg\}.
\end{aligned} \tag{A.7}$$

We expand the summation over t_1 and t_2 and the expectations over t_1 and t_2 in the objective function (A.7). The buyer's contract design problem (2) becomes

$$\begin{aligned}
\max_{\substack{(q_n,p_n)(t_1,t_2)\\ n=1,2;\,t_1,t_2\in\{H,L\}}} &\Bigg\{\left(\alpha^H\right)^2\Bigg[rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1,p_1)(H,H)]+\rho_2^H z_2^H[(q_2,p_2)(H,H)]\Big\}\\
&\quad-\Big\{c^H z_1^H[(q_1,p_1)(H,H)]+K\,\mathbf{1}_{\{q_1(H,H)>0\}}\Big\}-\Big\{c^H z_2^H[(q_2,p_2)(H,H)]+K\,\mathbf{1}_{\{q_2(H,H)>0\}}\Big\}\Bigg]\\
&+\left(\alpha^H\alpha^L\right)\Bigg[rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1,p_1)(H,L)]+\rho_2^L z_2^L[(q_2,p_2)(H,L)]\Big\}\\
&\quad-\Big\{c^H z_1^H[(q_1,p_1)(H,L)]+K\,\mathbf{1}_{\{q_1(H,L)>0\}}\Big\}-\Big\{c^L z_2^L[(q_2,p_2)(H,L)]+K\,\mathbf{1}_{\{q_2(H,L)>0\}}\Big\}\Bigg]\\
&+\left(\alpha^L\alpha^H\right)\Bigg[rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,H)]+\rho_2^H z_2^H[(q_2,p_2)(L,H)]\Big\}\\
&\quad-\Big\{c^L z_1^L[(q_1,p_1)(L,H)]+K\,\mathbf{1}_{\{q_1(L,H)>0\}}\Big\}-\Big\{c^H z_2^H[(q_2,p_2)(L,H)]+K\,\mathbf{1}_{\{q_2(L,H)>0\}}\Big\}\Bigg]\\
&+\left(\alpha^L\right)^2\Bigg[rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,L)]+\rho_2^L z_2^L[(q_2,p_2)(L,L)]\Big\}\\
&\quad-\Big\{c^L z_1^L[(q_1,p_1)(L,L)]+K\,\mathbf{1}_{\{q_1(L,L)>0\}}\Big\}-\Big\{c^L z_2^L[(q_2,p_2)(L,L)]+K\,\mathbf{1}_{\{q_2(L,L)>0\}}\Big\}\Bigg]\\
&-\left(\alpha^H\right)^2\Gamma_1[(q_1,p_1)(L,H)]-\alpha^H\alpha^L\Gamma_1[(q_1,p_1)(L,L)]\\
&-\left(\alpha^H\right)^2\Gamma_2[(q_2,p_2)(L,H)]-\alpha^H\alpha^L\Gamma_2[(q_2,p_2)(L,L)]\Bigg\}
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
\text{subject to}\quad & E\Big\{\Gamma_n[(q_n,p_n)(H,t_{\overline{n}})]\Big\}\geq E\Big\{\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]\Big\},\ (Monotonicity)\\
& q_n(t_1,t_2)\geq 0,\ p_n(t_1,t_2)\geq 0,\quad \text{for } n=1,2 \text{ and } t_1,t_2\in\{H,L\}.
\end{aligned} \tag{A.9}$$

This concludes our first major step of reducing problem (A.2).

Now, we carry on with the second major step: to solve the equivalent problem (A.8) to find the optimal $(q_n, p_n)(t_1, t_2)$. We first temporarily drop the *monotonicity* constraint (A.9) and solve problem (A.8) with only nonnegativity constraints. We move $\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]$ in (A.8) to be with other terms that depend on $(q_n, p_n)(L, t_{\overline{n}})$. This allows us to rearrange problem (A.8) as a weighted sum of four maximization problems:

$$\left(\alpha^H\right)^2 \max_{\substack{(q_1,p_1)(H,H)\geq 0 \\ (q_2,p_2)(H,H)\geq 0}} \Big\{ rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1,p_1)(H,H)] + \rho_2^H z_2^H[(q_2,p_2)(H,H)]\Big\} \tag{A.10a}$$

$$- \Big\{c^H z_1^H[(q_1,p_1)(H,H)] + K\,\mathbf{1}_{\{q_1(H,H)>0\}}\Big\} - \Big\{c^H z_2^H[(q_2,p_2)(H,H)] + K\,\mathbf{1}_{\{q_2(H,H)>0\}}\Big\}\Big\}$$

$$+ \left(\alpha^H\alpha^L\right) \max_{\substack{(q_1,p_1)(H,L)\geq 0 \\ (q_2,p_2)(H,L)\geq 0}} \Big\{ rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1,p_1)(H,L)] + \rho_2^L z_2^L[(q_2,p_2)(H,L)]\Big\} \tag{A.10b}$$

$$- \Big\{c^H z_1^H[(q_1,p_1)(H,L)] + K\,\mathbf{1}_{\{q_1(H,L)>0\}}\Big\}$$

$$- \Big\{c^L z_2^L[(q_2,p_2)(H,L)] + K\,\mathbf{1}_{\{q_2(H,L)>0\}} + \frac{\alpha^H}{\alpha^L}\Gamma_2[(q_2,p_2)(H,L)]\Big\}\Big\}$$

$$+ \left(\alpha^L\alpha^H\right) \max_{\substack{(q_1,p_1)(L,H)\geq 0 \\ (q_2,p_2)(L,H)\geq 0}} \Big\{ rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,H)] + \rho_2^H z_2^H[(q_2,p_2)(L,H)]\Big\} \tag{A.10c}$$

$$- \Big\{c^L z_1^L[(q_1,p_1)(L,H)] + K\,\mathbf{1}_{\{q_1(L,H)>0\}} + \frac{\alpha^H}{\alpha^L}\Gamma_1[(q_1,p_1)(L,H)]\Big\}$$

$$- \Big\{c^H z_2^H[(q_2,p_2)(L,H)] + K\,\mathbf{1}_{\{q_2(L,H)>0\}}\Big\}\Big\}$$

$$+ \left(\alpha^L\right)^2 \max_{\substack{(q_1,p_1)(L,L)\geq 0 \\ (q_2,p_2)(L,L)\geq 0}} \Big\{ rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,L)] + \rho_2^L z_2^L[(q_2,p_2)(L,L)]\Big\} \tag{A.10d}$$

$$- \Big\{c^L z_1^L[(q_1,p_1)(L,L)] + K\,\mathbf{1}_{\{q_1(L,L)>0\}} + \frac{\alpha^H}{\alpha^L}\Gamma_1[(q_1,p_1)(L,L)]\Big\}$$

$$- \Big\{c^L z_2^L[(q_2,p_2)(L,L)] + K\,\mathbf{1}_{\{q_2(L,L)>0\}} + \frac{\alpha^H}{\alpha^L}\Gamma_2[(q_2,p_2)(L,L)]\Big\}\Big\}.$$

We solve each of the four maximization problems in (A.10). One can show that setting $z_n^{t_n}[(q_n, p_n)(t_1, t_2)] = q_n(t_1, t_2)$ and $p_n(t_1, t_2) \geq c^{t_n}/\theta^{t_n}$ is without loss of optimality. Next, notice from equation (5) that $\Gamma_n[(X_n, q_n, p_n)(L, t_{\overline{n}})]$ is increasing in $p_n(L, t_{\overline{n}})$. Hence, it is optimal to set $p_n(L, t_{\overline{n}})$ to be its minimum c^L/l, which gives $\Gamma_n[(X_n, q_n, p_n)(L, t_{\overline{n}})] = h\,(\frac{c^L}{l} - \frac{c^H}{h})\, q_n(L, t_{\overline{n}})$.

For each of the four maximization problems in (A.10) we find the optimal order quantities $q_1(t_1, t_2)$ and $q_2(t_1, t_2)$. Because for each of these four maximization problems the objective function is piecewise linear in the order quantities, without loss of optimality, we focus on the corner-point solutions only: $(q_1(t_1, t_2), q_2(t_1, t_2)) \in \big\{(0,0), (D,0), (0,D), (D,D)\big\}$. Comparing the objective function value at these four corner points reveals the optimal order quantities:

Problem (A.10a), $t_1 = t_2 = H$				
When $\psi^{HH} \leq 0$:	$q_1(H,H) = D$	$q_2(H,H) = 0$	$p_1(H,H) \geq \frac{c^H}{h}$	$p_2(H,H) \geq 0$
When $\psi^{HH} > 0$:	$q_1(H,H) = D$	$q_2(H,H) = D$	$p_1(H,H) \geq \frac{c^H}{h}$	$p_2(H,H) \geq \frac{c^H}{h}$
Problem (A.10b), $t_1 = H$ and $t_2 = L$				
When $\psi^{HL} \leq 0$:	$q_1(H,L) = D$	$q_2(H,L) = 0$	$p_1(H,L) \geq \frac{c^H}{h}$	$p_2(H,L) \geq 0$
When $\psi^{HL} > 0$:	$q_1(H,L) = D$	$q_2(H,L) = D$	$p_1(H,L) \geq \frac{c^H}{h}$	$p_2(H,L) = \frac{c^L}{l}$
Problem (A.10c), $t_1 = L$ and $t_2 = H$				
The solution is identical to the solution for problem (A.10b), except that the indices of the two suppliers are swapped.				
Problem (A.10d), $t_1 = L$ and $t_2 = L$				
When $\psi^{L} \leq 0$:	$q_1(L,L) = 0$	$q_2(L,L) = 0$	$p_1(L,L) \geq 0$	$p_2(L,L) \geq 0$
When $\psi^{LL} \leq 0 < \psi^{L}$:	$q_1(L,L) = D$	$q_2(L,L) = 0$	$p_1(L,L) = \frac{c^L}{l}$	$p_2(L,L) \geq 0$
When $\psi^{LL} > 0$:	$q_1(L,L) = D$	$q_2(L,L) = D$	$p_1(L,L) = \frac{c^L}{l}$	$p_2(L,L) = \frac{c^L}{l}$

In Lemma 4, we show that the optimal solution to the relaxation problem (A.10) in the above table satisfies the *monotonicity* constraint (A.6) for supplier 1 and supplier 2, as long as we restrict $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$ whenever $q_n(L, t_{\overline{n}}) = D$ at the optimal solution. Thus, with the additional restriction on $p_n(H, t_{\overline{n}})$, the optimal solution to problem (A.10) is also optimal for the original problem (A.2). Furthermore, for all $p_n(t_1, t_2)$ in the above solution that are within an interval, we fix it to its lower bound.

We now compute the optimal fixed payments using (A.1), (A.4) and (A.5). Without loss of optimality, for $t_{\overline{n}} \in \{H, L\}$ we choose the optimal payments X_n^* to be such that

$$\pi_n^H[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})] = \Gamma_n[(q_n^*, p_n^*)(L, t_{\overline{n}})] \text{ and } \pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] = 0. \tag{A.11}$$

That is,

$$\begin{aligned} X_n^*(H, t_{\overline{n}}) =& \Gamma_n[(q_n^*, p_n^*)(L, t_{\overline{n}})] + c^{t_n} z_n^{t_n}[(q_n^*, p_n^*)(H, t_{\overline{n}})] \\ & - p_n^*(H, t_{\overline{n}}) E \min\left[q_n^*(H, t_{\overline{n}}),\ \rho_n^{t_n} z_n^{t_n}[(q_n^*, p_n^*)(H, t_{\overline{n}})]\right], \end{aligned} \tag{A.12}$$

$$\begin{aligned} X_n(L, t_{\overline{n}}) =& c^{t_n} z_n^{t_n}[(q_n^*, p_n^*)(L, t_{\overline{n}})] \\ & - p_n^*(L, t_{\overline{n}}) E \min\left[q_n^*(L, t_{\overline{n}}),\ \rho_n^{t_n} z_n^{t_n}[(q_n^*, p_n^*)(L, t_{\overline{n}})]\right]. \end{aligned} \tag{A.13}$$

Substituting p_n^* in the above equations, we have $X_n^* = 0$.

Finally, we show that the optimal contract menu satisfies the dominant-strategy *incentive compatibility* and *individual rationality* constraints, which are for $n = 1, 2$ and $t_{\overline{n}} \in \{H, L\}$

(IC) $\pi_n^H[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})] \geq \pi_n^H[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})], \quad \pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] \geq \pi_n^L[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})]$

(IR) $\pi_n^H[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})] \geq 0, \quad \pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] \geq 0.$

By the standard procedure, we can show that these constraints are equivalent to

$$\Gamma_n[(q_n^*, p_n^*)(H, t_{\overline{n}})] \geq \pi_n^H[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})] - \pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] \geq \Gamma_n[(q_n^*, p_n^*)(L, t_{\overline{n}})]$$

$\pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] \geq 0$

Using (A.11) and (A.15), we can verify that the above constraints hold for the optimal contract. ■

Lemma 4. *The optimal solutions to the relaxation* (A.10) *(in the table above) satisfy the monotonicity constraints* (A.6) *for supplier* 1 *and supplier* 2*, if we restrict* $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$ *whenever* $q_n(L, t_{\overline{n}}) = D$*, for* $t_{\overline{n}} \in \{H, L\}$*.*

Proof. It is sufficient to show that, for $n = 1, 2$ and $t_{\overline{n}} = H, L$, with the additional restriction $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$, the optimal solution to (A.10) satisfies

$$\Gamma_n[(q_n, p_n)(H, t_{\overline{n}})] \geq \Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]. \tag{A.15}$$

When $\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})] = 0$ (i.e., $q_n(L, t_{\overline{n}}) = 0$), the inequality (A.15) holds trivially. We now focus on the case when $q_n(L, t_{\overline{n}}) = D$.

From equation (5), $\Gamma_n(q, p)$ is increasing in both q and p. Therefore, it suffices to show that $q_n(H, t_{\overline{n}}) \geq q_n(L, t_{\overline{n}})$ and $p_n(H, t_{\overline{n}}) \geq p_n(L, t_{\overline{n}})$ for all r. First, it can be verified that the optimal solution to (A.10) satisfies $q_n(H, t_{\overline{n}}) \geq q_n(L, t_{\overline{n}})$ for all r. Next, recall that when $q_n(L, t_{\overline{n}}) = D$, $p_n(L, t_{\overline{n}}) = \frac{c^L}{l}$ is optimal. Since the assumption in the lemma gives $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$ whenever $q_n(L, t_{\overline{n}}) = D$, we have $p_n(H, t_{\overline{n}}) \geq p_n(L, t_{\overline{n}})$. Inequality (A.15) follows. ■

Proof of Proposition 4. The proof is straightforward and is omitted for brevity. ■

Proof of Proposition 5. Comparing the buyer's optimal expected profits in the dual-sourcing model (17) and in the single-sourcing model (9), we obtain the value of the dual-sourcing option under asymmetric information:

$$(\alpha^H)^2(\psi^{HH})^+ + (\alpha^H\alpha^L)(\psi^{HL} - \Phi)^+ + (\alpha^L\alpha^H)[\psi^H - (\psi^L - \Phi)^+ + (\psi^{HL} - \Phi)^+] + (\alpha^L)^2(\psi^{LL} - \Phi)^+. \tag{A.16}$$

Following the definition of $\tilde{O}^{t_1,t_2}$, we obtain $\tilde{O}^{HH} = (\psi^{HH})^+$, $\tilde{O}^{HL} = (\psi^{HL} - \Phi)^+$,

$$\tilde{O}^{LH} = \begin{cases} \max\{\psi^{LH}, \psi^H - (\psi^L - \Phi)\} \\ \max\{\psi^{LH} + \psi^L - \Phi, \psi^H\} \end{cases} \quad \text{and } \tilde{O}^{LL} = \begin{cases} \max\{\psi^{LL} - \Phi, 0\} & \text{if } \psi^L - \Phi > 0 \\ \max\{\psi^L - \Phi + \psi^{LL} - \Phi, 0\} & \text{if } \psi^L - \Phi \leq 0 \end{cases}.$$

First, we show $\tilde{O}^{LH} = \psi^H - (\psi^L - \Phi)^+ + (\psi^{HL} - \Phi)^+$. The upper case of $\tilde{O}^{LH}$ equals $\psi^H - (\psi^L - \Phi) + \max\{\psi^{LH} - \psi^H + (\psi^L - \Phi), 0\}$. The lower case of $\tilde{O}^{LH}$ can be written as $\psi^H + \max\{\psi^{LH} - \psi^H + \psi^L - \Phi, 0\}$. One can verify $\psi^{HL} - \Phi = \psi^{LH} - \psi^H + \psi^L - \Phi$. Therefore, $\tilde{O}^{LH} = \psi^H - (\psi^L - \Phi)^+ + (\psi^{HL} - \Phi)^+$. Then, we show $\tilde{O}^{LL} = (\psi^{LL} - \Phi)^+$. In the lower case, $\psi^L - \Phi \leq 0$ implies $\psi^{LL} - \Phi \leq 0$, and hence $\tilde{O}^{LL} = 0 = (\psi^{LL} - \Phi)^+$. ■

Proof of Proposition 6. From equation (21), the value of information when $\psi^{HL} - \Phi \le 0 < \psi^{HL}$ and $\psi^{LL} - \Phi > 0$ (i.e., $\max\{r^{HL}, \tilde{r}^{LL}\} < r \le \tilde{r}^{HL}$) is

$$2(\alpha^L)^2\Phi + 2(\alpha^H\alpha^L)\psi^{HL} = 2(\alpha^H\alpha^L)\left\{h(c^L/l - c^H/h)D + [l(1-h)r - c^L]D - K\right\}.$$

Taking derivative over h, one can verify the above formula is decreasing in h when $r > \frac{c^L}{l^2}$. ■

Proof of Proposition 7. Omitted for brevity, the proof is similar to that of Proposition 3. ■

Proposition 8. *1. Under symmetric information, the value of the dual-sourcing option equals*

$$O = \alpha^H\alpha^H \cdot O^{HH} + \alpha^H\alpha^L \cdot O^{HL} + \alpha^L\alpha^H \cdot O^{LH} + \alpha^L\alpha^L \cdot O^{LL} \tag{A.17}$$

where for all combination of types $O^{t_1t_2}$ is the maximum between the corresponding competition benefits and diversification benefits (given in the left panel of Figure 4 when $\Phi = 0$).

2. For type combinations (H,H), (H,L), and (L,L), $O^{HH} = \tilde{O}^{HH}$, $O^{HL} \ge \tilde{O}^{HL}$, $O^{LL} \ge \tilde{O}^{LL}$. For type combination (L,H), there exists a threshold $\hat{r}$ such that for $r > \hat{r}$, $O^{LH} > \tilde{O}^{LH}$, and for $r \le \hat{r}$, $O^{LH} \le \tilde{O}^{LH}$.

3. There exists a threshold $r_0 \in (r^{LL}, \tilde{r}^{HL})$, such that, for $r > r_0$ the total value of the dual-sourcing option is larger under symmetric information than under asymmetric information ($O \ge \tilde{O}$), and for $r^L < r < r_0$ the converse is true.

Proof of Proposition 8. One can verify the first and second statements of the proposition using the expressions of $\tilde{O}^{t_1,t_2}$ and O^{t_1,t_2}. We omit the details.

We focus on the third statement. The value of the dual-sourcing option under asymmetric information is in (A.16). Rolling in the expressions of ψ^t and ψ^{t_1,t_2} into O^{t_1,t_2} in equation (A.17), we obtain the following expression:

$$(\alpha^H)^2(\psi^{HH})^+ + (\alpha^H\alpha^L)(\psi^{HL})^+ + (\alpha^L\alpha^H)[\psi^H - (\psi^L)^+ + (\psi^{HL})^+] + (\alpha^L)^2(\psi^{LL})^+. \tag{A.18}$$

The difference between the buyer's value of the dual-sourcing option under asymmetric and symmetric information, (A.16) minus (A.18), is

$$\begin{aligned}
&\alpha^L\alpha^H[(\psi^L)^+ - (\psi^L - \Phi)^+] + 2\alpha^H\alpha^L[(\psi^{HL} - \Phi)^+ - (\psi^{HL})^+] + (\alpha^L)^2[(\psi^{LL} - \Phi)^+ - (\psi^{LL})^+] = \\
&(\alpha^L\alpha^H)\left([(l\,r - c^L)D - K]\mathbf{1}_{\{r>r^L\}} - \left[(l\,r - c^L)D - K - \tfrac{\alpha^H}{\alpha^L}h\left(\tfrac{c^L}{l} - \tfrac{c^H}{h}\right)D\right]\mathbf{1}_{\{r>\tilde{r}^L\}}\right) \\
&+ 2\left(\alpha^H\alpha^L\right)\Big(\left\{\left[l(1-h)r - c^L\right]D - K - \tfrac{\alpha^H}{\alpha^L}h\left(\tfrac{c^L}{l} - \tfrac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>\tilde{r}^{HL}\}} \\
&\qquad\qquad - \left\{\left[l(1-h)r - c^L\right]D - K\right\}\mathbf{1}_{\{r>r^{HL}\}}\Big) \\
&+ \left(\alpha^L\right)^2\Big(\left\{\left[l(1-l)r - c^L\right]D - K - \tfrac{\alpha^H}{\alpha^L}h\left(\tfrac{c^L}{l} - \tfrac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>\tilde{r}^{LL}\}} \\
&\qquad\qquad - \left\{\left[l(1-l)r - c^L\right]D - K\right\}\mathbf{1}_{\{r>r^{LL}\}}\Big).
\end{aligned} \tag{A.19}$$

We treat (A.19) as a function of r and show there exists a unique $r_0 \in (r^{LL}, \tilde{r}^{HL})$, at which the curve changes from non-negative to strictly negative. Because the lower bound of the interval, r^{LL}, could be either greater or smaller than $\tilde{r}^L$, we consider two cases: $r^{LL} \geq \tilde{r}^L$ and $r^{LL} < \tilde{r}^L$.

Case $\boldsymbol{r^{LL} \geq \tilde{r}^L}$. We prove the result by tracing the value of (A.19) for $r > r^L$. From the tables describing the thresholds $r^L, \ldots, r^{HL}$ and $\tilde{r}^L, \ldots, \tilde{r}^{HL}$ in Table 1, and the assumption of the case, we get $r^L < \tilde{r}^L \leq r^{LL} < \{\tilde{r}^{LL}, r^{HL}\} < \tilde{r}^{HL}$. One can check that (A.19) is equal to zero at $r = r^L$, strictly positive and strictly increasing in r for $r \in (r^L, \tilde{r}^L]$, constant with respect to r for $r \in (\tilde{r}^L, r^{LL}]$, decreasing in r for $r \in (r^{LL}, \tilde{r}^{HL}]$ (regardless of the ordering of $\tilde{r}^{LL}$ and r^{HL}) and constant in r thereafter. Also, (A.19) is negative at $r = \tilde{r}^{HL}$. Thus, there must exist $r_0 \in (r^{LL}, \tilde{r}^{HL})$ such that (A.19) changes from non-negative to strictly negative at r_0.

Case $\boldsymbol{r^{LL} < \tilde{r}^L}$. Again using the definitions of the thresholds and the assumption of the case, we know $r^L < r^{LL} < \tilde{r}^L < \tilde{r}^{LL} < \tilde{r}^{HL}$. We utilize two sub-cases, depending on r^{HL}.

Sub-case $r^{LL} < r^{HL} \leq \tilde{r}^L$. One can check that (A.19) is zero at $r = r^L$, and strictly positive and strictly increasing in r for $r \in (r^L, r^{LL}]$. As for $r \in (r^{LL}, \tilde{r}^L]$, there are three possibilities for (A.19): increasing throughout, increasing until r^{HL} and then decreasing thereafter, or decreasing throughout. Additionally, (A.19) is decreasing in r for $r \in (\tilde{r}^L, \tilde{r}^{HL}]$ and constant in r thereafter. Therefore, there exists $\tilde{r}^* \in [r^{LL}, \tilde{r}^L]$ such that (A.19) is strictly positive at $r = \tilde{r}^*$ and decreasing for $r > \tilde{r}^*$. Furthermore, (A.19) is negative at $r = \tilde{r}^{HL}$. Thus, there must exist $r_0 > \tilde{r}^*$ such that (A.19) changes from non-negative to strictly negative at r_0. Sub-case $\tilde{r}^L < \{\tilde{r}^{LL}, r^{HL}\} < \tilde{r}^{HL}$ is similar to the previous subcase. ■