Online Supplement for "Optimizing Organic Waste to Energy Operations"

A Auxiliary Derivations and Results

A.1 Auxiliary Notation and Derivations

Rearranging (5), we can write

$$n = m(1+\xi)\beta\Lambda/\tau. \tag{A.1}$$

Using this, the increase in the long-run average social welfare given by (4) can be expressed as

$$W(\beta, m, \xi) = \beta\Lambda\left[\left[\nu_e\, g(m)\, + \nu_L\right][1-u] - c_H(\beta) - (c_K/\tau)(1+\xi)m\right].$$

Define an auxiliary function $Z(\cdot)$ as

$$Z(\beta, m, \xi; p, c) = \beta\Lambda\left[\left[p\, g(m)\, + c\right][1-u] - c_H(\beta) - (c_K/\tau)(1+\xi)m\right]. \tag{A.2}$$

Then

$$W(\beta, m, \xi) = Z(\beta, m, \xi; \nu_e, \nu_L). \tag{A.3}$$

Therefore, setting $p = \nu_e$ and $c = \nu_L$, we can work with the function $Z(\cdot)$ to find socially optimal β, m, and ξ. Similarly, combining (2), (3) and (A.1), long-run average profit rate for the WTE firm can be expressed as

$$\Pi(\beta, m, \xi) = (1-\gamma)\beta\Lambda\left[\left[p_e\, g(m)\, + c_L\right][1-u] - c_H(\beta) - (c_K/\tau)(1+\xi)m\right],$$

and from (A.2), this can be written as

$$\Pi(\beta, m, \xi) = (1-\gamma)\, Z(\beta, m, \xi; p_e, c_L). \tag{A.4}$$

Therefore, $Z(\cdot)$ can also be used to find the profit-maximizing operating strategy of the WTE firm.

In the rest of this section, we derive auxiliary results related to the function $Z(\cdot)$, which can be used to represent both the social welfare function and the profit function of the WTE firm. By the first-order condition of $Z(\beta, m, \xi)$, given by (A.2), with respect to ξ we can characterize optimal percentage excess capacity as:

$$\xi(\beta, m) = \frac{\sigma^2}{2q\mu\beta\Lambda} \cdot \ln\left\{\frac{2q\mu\,\beta\Lambda\tau[p\, g(m)\, + c]}{c_K m\sigma^2}\right\}. \tag{A.5}$$

In what follows, we will drop ξ from $Z(\beta, m, \xi)$ whenever $\xi = \xi(\beta, m)$. Note that, substituting (A.5) in (7) yields

$$u(\beta, m) = \frac{c_K m\sigma^2}{2q\mu\,\beta\Lambda\tau[p\, g(m)\, + c]}. \tag{A.6}$$

Combining (A.2), (A.5) and (A.6), the function $Z(\cdot)$ can be written as a function β and m as

$$Z(\beta, m) = \beta\Lambda\left[p\, g(m)\, + c - c_H(\beta) - \tfrac{c_K m}{\tau}\right] - \tfrac{c_K m\sigma^2}{2q\mu\tau}\left(1 + \ln\left\{\tfrac{2q\mu\,\beta\Lambda\tau[p\, g(m)\, + c]}{c_K m\sigma^2}\right\}\right). \tag{A.7}$$

Furthermore, write

$$Z(\beta) = \max_m Z(\beta, m).$$

A.2 Auxiliary Results for the Urban Setting

The following result (Lemma A.1) is used in the proof of Lemma A.2, which provides a unified proof for Propositions 2 and 4. Likewise, Lemma A.3 provides a unified proof for Propositions 3 and 5.

Lemma A.1 *For $\beta_1 \in (0,1]$, if $Z(\beta_1) > 0$ and $c_H'(\beta_1) < 0$, then* $\left.\frac{\partial Z(\beta,m)}{\partial \beta}\right|_{\beta=\beta_1, m=m(\beta_1)} > 0.$

Proof. It follows from (A.7) that

$$\frac{\partial Z(\beta,m)}{\partial \beta} = \Lambda\left[p\,g(m) + c - c_H(\beta) - \frac{c_K m}{\tau}\right] - \beta\Lambda c_H'(\beta) - \frac{c_K m \sigma^2}{2q\mu\tau\beta}.$$

Then it suffices to show that

$$\Lambda\left[p\,g(m(\beta_1)) + c - c_H(\beta_1) - \frac{c_K m(\beta_1)}{\tau}\right] - \beta_1\Lambda c_H'(\beta_1) - \frac{c_K m(\beta_1)\sigma^2}{2q\mu\tau\beta_1} > 0. \tag{A.8}$$

Using (A.7), $Z(\beta_1) > 0$ implies

$$\beta_1\Lambda\left[p\,g(m(\beta_1)) + c - c_H(\beta_1) - \tfrac{c_K m(\beta_1)}{\tau}\right] - \tfrac{c_K m(\beta_1)\sigma^2}{2q\mu\tau}\left(1 + \ln\left\{\tfrac{2q\mu\beta_1\Lambda\tau[p\,g(m(\beta_1)) + c]}{c_K m(\beta_1)\sigma^2}\right\}\right) > 0.$$

Dividing this statement by $\beta_1 > 0$ and moving second part to the right-hand-side yields

$$\Lambda\left[p\,g(m(\beta_1)) + c - c_H(\beta_1) - \tfrac{c_K m(\beta_1)}{\tau}\right] > \tfrac{c_K m(\beta_1)\sigma^2}{2q\mu\tau\beta_1}\left(1 + \ln\left\{\tfrac{2q\mu\beta_1\Lambda\tau[p\,g(m(\beta_1)) + c]}{c_K m(\beta_1)\sigma^2}\right\}\right).$$

From this, we can derive the following lower bound on the left-hand-side of (A.8):

$$\frac{c_K m(\beta_1)\sigma^2}{2q\mu\tau\beta_1}\ln\left\{\frac{2q\mu\beta_1\Lambda\tau[p\,g(m(\beta_1)) + c]}{c_K m(\beta_1)\sigma^2}\right\} - \beta_1\Lambda c_H'(\beta_1).$$

Thus, using (A.5) it suffices to show that $(c_K/\tau)\,m(\beta_1)\,\xi(\beta_1, m(\beta_1)) \geq \beta_1 c_H'(\beta_1)$, which is true for $c_H'(\beta_1) < 0$. ■

Lemma A.2 *If $Z(1) > 0$ and $c_H'(\beta) < 0$ for all $\beta \in (0,1]$, then $\beta = 1$ maximizes $Z(\beta)$.*

Proof. Suppose that there exists $\beta_1 < 1$ such that $Z(\beta_1) > 0$. Otherwise, $\beta = 1$ is optimal since $Z(1) > 0$ by assumption. Then, it suffices to show that $Z(\beta)$ increases in β for all $\beta \in [\beta_1, 1]$.

If there exists $\beta_\epsilon > \beta_1$ such that $Z(\beta_2) > Z(\beta_1)$ for all $\beta_2 \in (\beta_1, \beta_\epsilon)$, then $Z(\beta)$ increases in β at β_1. Note by optimality that $Z(\beta_2) \geq Z(\beta_2, m(\beta_1))$ for all $\beta_2 > \beta_1$. From this, it suffices to show that there exists $\beta_\epsilon > \beta_1$ such that

$$Z(\beta_2, m(\beta_1)) > Z(\beta_1) \quad \text{for all } \beta_2 \in (\beta_1, \beta_\epsilon). \tag{A.9}$$

By Lemma A.1 we know that

$$\left.\frac{\partial Z(\beta,m)}{\partial \beta}\right|_{\beta=\beta_1, m=m(\beta_1)} > 0,$$

and by continuity of $\partial Z(\beta,m)/\partial\beta$ in β_1, there exists $\beta_\epsilon > \beta_1$ such that

$$\left.\frac{\partial Z(\beta,m)}{\partial\beta}\right|_{\beta=\beta_2, m=m(\beta_1)} > 0 \quad \text{for all } \beta_2 \in (\beta_1, \beta_\epsilon).$$

Then by the fundamental theorem of calculus, for $\beta_2 \in (\beta_1,\beta_\epsilon)$ we have that

$$Z(\beta_2, m(\beta_1)) - Z(\beta_1, m(\beta_1)) = \int_{\beta_1}^{\beta_2} \left.\frac{\partial Z(\beta,m)}{\partial\beta}\right|_{m=m(\beta_1)} d\beta > 0,$$

which yields (A.9), proving the claim that $Z(\beta)$ increases in β at β_1. This argument can be repeated for all $\beta \in (\beta_1, 1]$ to show that $Z(\beta)$ is increasing on $[\beta_1, 1]$, completing the proof. ■

Lemma A.3 *Suppose optimal coverage level β^* is known and let $\tilde{u}(\xi)$ denote $u(\beta^*,\xi)$. For a given percentage excess capacity $\xi > 0$, the optimal retention time $m^*(\xi)$ is given by*

$$m^*(\xi) = \max\left\{ h\left(\frac{c_K\,(1+\xi)}{p\,\tau\,(1-\tilde{u}(\xi))}\right), \underline{m} \right\}. \tag{A.10}$$

Proof. Substituting β^* in (A.2) yields

$$\tilde{Z}(\beta^*, m, \xi) = \left\{ [p\,g(m) + c]\,[1-\tilde{u}(\xi)] - c_H(\beta^*) - \frac{c_K\,m\,(1+\xi)}{\tau} \right\} \beta^*\Lambda.$$

Fix $\xi > 0$. Note that $\tilde{Z}(\beta^*, m, \xi)$ is a concave function of m. Therefore, the following first-order condition is sufficient to characterize the optimal retention time:

$$\frac{\partial \tilde{Z}(\beta^*, m, \xi)}{\partial m} = p\,g'(m)\beta^*\Lambda[1-\tilde{u}(\xi)] - \frac{c_K\,(1+\xi)\beta^*\Lambda}{\tau} = 0. \tag{A.11}$$

First we rule out $m^*(\xi) = \overline{m}$. Otherwise, since $g'(\overline{m}) = 0$, the first-order condition gives $\xi \le -1$, which is a contradiction. Then ignoring the boundary condition $m^*(\xi) \ge \underline{m}$, inverting (A.11) gives

$$m^*(\xi) = h\left(\frac{c_K\,(1+\xi)}{p\,\tau\,(1-\tilde{u}(\xi))}\right),$$

which is optimal if it is greater than or equal to $\underline{m}$. Otherwise we get a boundary solution, i.e. $m = \underline{m}$. Combining these give (A.10), concluding the proof. ■

A.3 Auxiliary Results for the Rural Setting

The following result (Lemma A.4) is used in the proof of Lemma A.5, which provides a unified proof for the first results in Propositions 7 and 8. Likewise, Lemma A.6 provides a unified proof for the second results in Propositions 7 and 8.

Lemma A.4 *For $\beta_1 \in (0,1]$, if $Z(\beta_1) > 0$ and*

$$p\,g(\underline{m}) + c - \frac{c_K\overline{m}}{\tau}\left(1 + \frac{\sigma^2}{2q\mu\underline{\beta}\Lambda}\right) > 3L, \tag{A.12}$$

where $\underline{\beta}$ is the argmin of $c_H(\beta)$, then $\left.\frac{\partial Z(\beta,m)}{\partial\beta}\right|_{\beta=\beta_1, m=m(\beta_1)} > 0.$

Proof. For $\beta_1 < \underline{\beta}$, $c_H'(\beta_1) < 0$ and the proof is similar to the proof of Lemma A.2, in which $Z(1)$ is replaced by $Z(\underline{\beta})$. In what follows, we show the result for $\beta_1 \in (\underline{\beta}, 1]$. From (A.7) the first order condition of $Z(\beta, m)$ with respect to β is

$$\frac{\partial Z(\beta, m)}{\partial \beta} = \Lambda \left[p\, g(m) \; + c - c_H(\beta) - \frac{c_K m}{\tau} \right] - \beta \Lambda c_H'(\beta) - \frac{c_K m \sigma^2}{2 q \mu \tau \beta}.$$

Then it suffices to show that for $\beta_1 \in (\underline{\beta}, 1]$

$$\Lambda \left[p\, g(m(\beta_1)) \; + c - c_H(\beta_1) - \frac{c_K m(\beta_1)}{\tau} \right] - \frac{c_K m(\beta_1) \sigma^2}{2 q \mu \tau \beta_1} > \beta_1 \Lambda c_H'(\beta_1).$$

Using (15), this can be written as

$$\Lambda \left[p\, g(m(\beta_1)) \; + c - \frac{H}{\beta_1 \Lambda} - L\beta_1^2 - \frac{c_K m(\beta_1)}{\tau} \right] - \frac{c_K m(\beta_1) \sigma^2}{2 q \mu \tau \beta_1} > \Lambda \left[-\frac{H}{\beta_1 \Lambda} + 2L\beta_1^2 \right]. \tag{A.13}$$

Simplifying and rearranging (A.13), it suffices to show that

$$p\, g(m(\beta_1)) \; + c - \frac{c_K m(\beta_1)}{\tau} \left(1 + \frac{\sigma^2}{2 q \mu \beta_1 \Lambda} \right) > 3L\beta_1^2.$$

Bounding the lhs from below and the rhs from above gives the condition in (A.12). ■

Lemma A.5 *If $Z(1) > 0$ and $p\, g(\underline{m}) + c - (c_K/\tau)\, \overline{m}\, (1 + \sigma^2/2q\mu\underline{\beta}\Lambda) > 3L$, then $\beta = 1$ is optimal.*

Proof. Suppose that there exists $\beta_1 < 1$ such that $Z(\beta_1) > 0$. Otherwise, $\beta = 1$ is optimal since $Z(1) > 0$ by assumption. Then, it suffices to show that $Z(\beta)$ increases in β for all $\beta \in [\beta_1, 1]$.

If there exists $\beta_\epsilon > \beta_1$ such that

$$Z(\beta_2) > Z(\beta_1) \qquad \text{for all } \beta_2 \in (\beta_1, \beta_\epsilon),$$

then $Z(\beta)$ increases in β at β_1. Note by optimality that

$$Z(\beta_2) \geq Z(\beta_2, m(\beta_1)) \qquad \text{for all } \beta_2 > \beta_1.$$

From this, it suffices to show that there exists $\beta_\epsilon > \beta_1$ such that

$$Z(\beta_2, m(\beta_1)) > Z(\beta_1) \qquad \text{for all } \beta_2 \in (\beta_1, \beta_\epsilon). \tag{A.14}$$

By Lemma A.4 we know that

$$\left. \frac{\partial Z(\beta, m)}{\partial \beta} \right|_{\beta = \beta_1, m = m(\beta_1)} > 0,$$

and by continuity of $\partial Z(\beta, m)/\partial \beta$ in β_1, there exists $\beta_\epsilon > \beta_1$ such that

$$\left. \frac{\partial Z(\beta, m)}{\partial \beta} \right|_{\beta = \beta_2, m = m(\beta_1)} > 0 \qquad \text{for all } \beta_2 \in (\beta_1, \beta_\epsilon).$$

Then by the fundamental theorem of calculus, for $\beta_2 \in (\beta_1, \beta_\epsilon)$ we have that

$$Z(\beta_2, m(\beta_1)) - Z(\beta_1, m(\beta_1)) = \int_{\beta_1}^{\beta_2} \left. \frac{\partial Z(\beta, m)}{\partial \beta} \right|_{m=m(\beta_1)} d\beta > 0,$$

which yields (A.14), proving the claim that $Z(\beta)$ increases in β at β_1. This argument can be repeated for all $\beta \in (\beta_1, 1]$ to show that $Z(\beta)$ is increasing on $[\beta_1, 1]$, completing the proof. ■

Lemma A.6 *If $p\, g(\overline{m}) + c - (c_K/\tau)\, \underline{m}\, (1 + \sigma^2/2q\mu\Lambda) < 3L$, then $\beta < 1$ is optimal.*

Proof. For $\beta < 1$ to be optimal it suffices to show that

$$\left. \frac{\partial Z(\beta, m)}{\partial \beta} \right|_{\beta=1, m=m(1)} < 0. \tag{A.15}$$

It follows from (A.7) that

$$\frac{\partial Z(\beta, m)}{\partial \beta} = \Lambda \left[p\, g(m) + c - c_H(\beta) - \frac{c_K m}{\tau} \right] - \beta \Lambda c_H'(\beta) - \frac{c_K m \sigma^2}{2q\mu\tau\beta}.$$

From this, (A.15) can be written as

$$\Lambda \left[p\, g(m(1)) + c - c_H(1) - \frac{c_K m(1)}{\tau} \right] - \frac{c_K m(1) \sigma^2}{2q\mu\tau} < \Lambda c_H'(1).$$

Using (15), this can be written as

$$\Lambda \left[p\, g(m(1)) + c - \frac{H}{\Lambda} - L - \frac{c_K m(1)}{\tau} \right] - \frac{c_K m(1) \sigma^2}{2q\mu\tau} < \Lambda \left[-\frac{H}{\Lambda} + 2L \right]. \tag{A.16}$$

Simplifying and rearranging (A.16), it suffices to show that

$$p\, g(m(1)) + c - \frac{c_K m(1)}{\tau} \left(1 + \frac{\sigma^2}{2q\mu\Lambda} \right) < 3L.$$

Bounding the left-hand-side from above gives the condition in the statement of the lemma. ■

B Proofs.

Proof of Lemma 1. WG's gain from trade can be written as

$$G(\beta) \quad = \quad c_L \beta \Lambda - [f(\beta, m, n) + c_H(\beta)]\, \beta \Lambda, \tag{B.1}$$

where the first term represents the cost of disposal at landfill and the second term is the cost of disposal at the WTE firm. If the WTE firm and WG split the gains from their transaction according to their relative bargaining power, $(1-\gamma)$ and γ, respectively, the tip fee is the solution to

$$G(\beta) = \gamma\, [\, G(\beta) + \Pi(\beta, m, n)]\,. \tag{B.2}$$

Rearranging, (B.2) can be written as $(1-\gamma)\, G(\beta) = \gamma\, \Pi(\beta, m, n)$, and using (2) and (B.1) yields

$$(1-\gamma)\, \beta\Lambda\, [c_L - f(\beta, m, n) - c_H(\beta)] = \gamma\, \beta\Lambda\, [f(\beta, m, n) + p_e\, g(m)\, [1-u] \, - c_L\, u - c_K n/\beta\Lambda]\,.$$

Solving this for $f(\beta, m, n)$ gives the expression in (3). ■

Proof of Proposition 1. Courcoubetis and Weber (1996) considers a large deviations analysis of the buffer overflow probability of a many-server finite-buffer queue, which is fed by many input streams. Their notation and results can be mapped to our setting as follows. Theorem 1 of Courcoubetis and Weber (1996) gives

$$\lim_{N\to\infty} \frac{1}{N} \log u = -\inf_{m} \sup_{\theta} \left\{\theta\, (b + mc) - m\phi_m\,(\theta)\right\}, \tag{B.3}$$

where $\phi_m\,(\theta)$ is the asymptotic logarithmic moment generating function, N is the number of sources, b and c are the buffer space per source and the service rate per source, respectively. To avoid confusion with the retention time m, we replace the auxiliary variable m with α and rewrite (B.3) in our notation as

$$\lim_{\beta K\to\infty} \frac{1}{\beta K} \log u = -\inf_{\alpha} \sup_{\theta} \left\{\theta\, (q\mu + \alpha(1+\xi)\mu) - \alpha\phi_\alpha\,(\theta)\right\}. \tag{B.4}$$

Recall that $q\mu$ is the buffer space per source and $(1+\xi)\mu$ is the service rate per source in our setting. Similarly, βK is the number of sources. For iid sources, asymptotic logarithmic moment generating function simply reduces to logarithmic moment generating function. More specifically, for Gaussian sources with mean μ and variance σ^2, we have

$$\phi_\alpha\,(\theta) = \log \mathbb{E}[\exp\left\{\theta N(\mu, \sigma^2)\right\}] = \theta\mu + \sigma^2\theta^2/2. \tag{B.5}$$

Substituting (B.5) and multiplying both sides by β/μ, (B.4) yields

$$\lim_{\Lambda\to\infty} \frac{1}{\Lambda} \log u = -\inf_{\alpha} \sup_{\theta} \left\{\beta\,\theta\,(q + \alpha\,\xi) + \frac{\beta\,\alpha\,\sigma^2\theta^2}{2\mu}\right\}. \tag{B.6}$$

The optimization steps in (B.6) are rather straightforward and yield

$$\lim_{\Lambda\to\infty} \frac{1}{\Lambda} \log u = -\frac{2q\mu\,\beta\,\xi}{\sigma^2},$$

completing the proof. ■

Proof of Proposition 2. By (A.3), the condition of the proposition is equivalent to $Z(1) > 0$ with $(p, c) = (\nu_e, \nu_L)$. Note that $c'_H(\beta) < 0$ for all $\beta \in (0, 1]$ in the urban setting. Then, by Lemma A.2 full coverage is the socially optimal strategy. ■

Proof of Proposition 3. The proof follows from the proof of Lemma A.3 with $(p, c) = (\nu_e, \nu_L)$ and $\beta^* = 1$. ■

Proof of Proposition 4. By (A.4), the condition of the proposition is equivalent to $Z(1) > 0$ with $(p, c) = (p_e, c_L)$. Note that $c'_H(\beta) < 0$ for all $\beta \in (0, 1]$ in the urban setting. Then, by Lemma A.2 it is profit maximizing for the firm to offer full coverage. ■

Proof of Proposition 5. The proof follows from the proof of Lemma A.3 with $(p, c) = (p_e, c_L)$ and $\beta^* = 1$. ■

Proof of Proposition 6. The Renewable Energy Credit (REC) can be given to the firm as a premium on the price of electricity, which increases the price, p_e, by δ_e. Accounting for δ_e in (11), the WTE firm's optimal retention time m_p is given by

$$m_p = \max\left\{h\left(\frac{c_K\,(1+\xi_p)}{(p_e+\delta_e)\,\tau\,(1-\tilde{u}(\xi_p))}\right), \underline{m}\right\}. \tag{B.7}$$

We want to find δ_e^* that induces the WTE firm to choose $m_p = m_s$. Substituting $m_s > \underline{m}$ for m_p in (B.7) and solving for δ_e gives

$$\delta_e^* = \frac{c_K(1+\xi_p)}{g'(m_s)\tau(1-\tilde{u}(\xi_p))} - p_e. \tag{B.8}$$

This is the amount of subsidy given by the social planner per kWh of electricity generated by the WTE firm. Thus, per period cost of price premium mechanism to the social planner is given by

$$P(m_s, \xi_p) = \Lambda\,(1-\tilde{u}(\xi_p))\,g(m_s)\,\delta_e^*. \tag{B.9}$$

Combining (B.8) and (B.9) yields

$$P(m_s, \xi_p) = \Lambda\,(1-\tilde{u}(\xi_p))\,g(m_s)\,\left(\frac{c_K(1+\xi_p)}{g'(m_s)\tau(1-\tilde{u}(\xi_p))} - p_e\right). \tag{B.10}$$

Next, consider the lump sum mechanism. The lump sum subsidy decreases firm's per period (per digester) capital cost, c_K, by δ_K. Accounting for δ_K in (11), the WTE firm's optimal retention time m_l is given by

$$m_l = \max\left\{h\left(\frac{(c_K-\delta_K)\,(1+\xi_l)}{p_e\,\tau\,(1-\tilde{u}(\xi_l))}\right), \underline{m}\right\}. \tag{B.11}$$

Setting $m_l = m_s$ in (B.11) and solving it for δ_K gives the amount of subsidy δ_K^* that induces the WTE firm to choose m_s as

$$\delta_K^* = c_K - \frac{\tau\,p_e\,g'(m_s)(1-\tilde{u}(\xi_l))}{(1+\xi_l)}. \tag{B.12}$$

Per period cost of the lump sum mechanism to the social planner is $n\,\delta_K^*$, which by (5) for $\beta = 1$ is given by

$$L(m_s, \xi_l) = \Lambda\,(1+\xi_l)\, m_s\, \delta_K^*/\tau. \tag{B.13}$$

Combining (B.12) and (B.13) yields

$$L(m_s, \xi_l) = \frac{\Lambda\,(1+\xi_l)\, m_s}{\tau} \left(c_K - \frac{\tau\, p_e\, g'(m_s)(1-\tilde{u}(\xi_l))}{(1+\xi_l)} \right). \tag{B.14}$$

Giving lump sum subsidies is more cost effective for the social planner than offering a price premium if $P(m_s, \xi_p)/L(m_s, \xi_l) > 1$, which using (B.10) and (B.14) can be expressed as

$$\frac{\Lambda\,(1-\tilde{u}(\xi_p))\, g(m_s) \left(\frac{c_K(1+\xi_p)}{g'(m_s)\tau(1-\tilde{u}(\xi_p))} - p_e \right)}{\frac{\Lambda\,(1+\xi_l)\, m_s}{\tau} \left(c_K - \frac{\tau\, p_e\, g'(m_s)(1-\tilde{u}(\xi_l))}{(1+\xi_l)} \right)} > 1.$$

Rearranging the terms yields the condition stated in (14). ■

Proof of Proposition 7. The conditions of the first result are equivalent to those of Lemma A.5 with $(p, c) = (\nu_e, \nu_L)$, and the first result follows from Lemma A.5. On the other hand, the condition of the second result is equivalent to that of Lemma A.6 with $(p, c) = (\nu_e, \nu_L)$. Thus, the second result follows from Lemma A.6. ■

Proof of Proposition 8. The conditions of the first result are equivalent to those of Lemma A.5 with $(p, c) = (p_e, c_L)$, and the first result follows from Lemma A.5. On the other hand, the condition of the second result is equivalent to that of Lemma A.6 with $(p, c) = (p_e, c_L)$. Thus, the second result follows from Lemma A.6. ■

Proof of Proposition 9. Suppose $L > 0$. Otherwise $\beta_f = 1$ from Lemma A.2 since for $L \leq 0$ $c_H(\beta)$ is decreasing. If arrivals are deterministic, the optimal solution must satisfy $n = \beta\Lambda m/\tau$, i.e., the system is perfectly balanced. Setting $\xi = 0$ and $u = 0$, (A.4) yields

$$\Pi(\beta, m) = (1-\gamma)\beta\Lambda\left[p_e g(m) + c_L - c_H(\beta) - (c_K/\tau)m\right]. \tag{B.15}$$

Choose $m \geq \underline{m}$ so as to maximize $\Pi(\beta, m)$ for fixed $\beta > 0$, which is clearly a concave problem. By the first order condition we write $p_e g'(m) - c_K/\tau = 0$. Solving this for m and restricting it to be in $[\underline{m}, \overline{m}]$ yield the first result of the proposition.

Next, we characterize the optimal coverage level. Substituting m_f, (B.15) can be expressed as a function of β only: $\Pi(\beta) = (1-\gamma)\beta\Lambda\left[p_e\, g(m_f) + c_L + c_L - c_H(\beta) - (c_K/\tau)\, m_f\right]$. For concavity of $\Pi(\beta)$, it suffices to show that $\Pi''(\beta) < 0$, i.e., $-(1-\gamma)\Lambda\left[\,2\,c_H'(\beta) + \beta\, c_H''(\beta)\,\right] < 0$. From (15), this simplifies to $-(1-\gamma)\Lambda\, 6\, L\, \beta < 0$, which is true for $L > 0$. By the first order condition of $\Pi(\beta)$ we write $(1-\gamma)\Lambda\left[\,p_e^2\tau/c_K + c_L - c_H(\beta) - c_H'(\beta)\beta\,\right] = 0$. Solving for β, substituting (15) and restricting β_f to be less than 1 conclude the proof. ■

Proof of Proposition 10. It suffices to show that inducing the WTE firm to choose $\hat{m} \in (m_f, m_s]$ using the price premium mechanism is more costly than using the lump sum mechanism.

Consider first the price premium mechanism, which increases the price, p_e, by δ_e. Accounting for δ_e in (21), profit maximizing retention time m_p and coverage level β_p can be expressed as

$$m_p = \frac{(p_e + \delta_e)^2\,\tau^2}{c_K^2}, \quad \beta_p = \sqrt{\frac{\frac{(p_e+\delta_e)^2\,\tau}{c_K} + c_L}{3L}}. \tag{B.16}$$

We want to find $\hat{\delta}_e$ that induces the WTE firm to choose $\hat{m} > \underline{m}$. Setting $m_p = \hat{m}$ in (B.16) and solving for δ_e gives

$$\hat{\delta}_e = (c_K/\tau)\sqrt{\hat{m}} - p_e. \tag{B.17}$$

This is the amount of subsidy given by the social planner per kWh of electricity generated by the WTE firm. Thus, per period cost of price premium mechanism to the social planner is given by

$$P(\hat{m}) = \beta_p\,\Lambda\,\hat{\delta}_e\,g(\hat{m}). \tag{B.18}$$

Combining (B.16)-(B.18) yields

$$P(\hat{m}) = \sqrt{\frac{(c_K/\tau)\hat{m} + c_L}{3L}}\,\Lambda\,\left(\frac{c_K}{\tau}\sqrt{\hat{m}} - p_e\right)\,2\sqrt{\hat{m}}. \tag{B.19}$$

Next, consider the lump sum mechanism. The lump sum subsidy decreases firm's per period (per digester) capital cost, c_K, by δ_K. Accounting for δ_K, profit maximizing retention time m_l and coverage level β_l can be expressed as

$$m_l = \frac{p_e^2\,\tau^2}{(c_K - \delta_K)^2}, \quad \beta_l = \sqrt{\frac{p_e^2\,\tau/(c_K - \delta_K) + c_L}{3L}}. \tag{B.20}$$

Setting $m_l = \hat{m}$ in (B.20) and solving it for δ_K gives the amount of subsidy $\hat{\delta}_K$ that induces the WTE firm to choose $\hat{m}$ as

$$\hat{\delta}_K = c_K - p_e\tau/\sqrt{\hat{m}}. \tag{B.21}$$

Per period cost of the lump sum mechanism to the social planner is $n\hat{\delta}_K$, which is given by $L(\hat{m}) = \beta_l\,\Lambda\,\hat{m}\,\hat{\delta}_K/\tau$. Using (B.20) and (B.21), this gives

$$L(\hat{m}) = \sqrt{\frac{p_e\sqrt{\hat{m}} + c_L}{3L}}\,\frac{\Lambda}{\tau}\,\hat{m}\,\left(c_K - \frac{p_e\tau}{\sqrt{\hat{m}}}\right). \tag{B.22}$$

Then, the ratio of $L(\beta_s)$ and $P(\beta_s)$ can be written from (B.19) and (B.22) as

$$\frac{L(\hat{m})}{P(\hat{m})} = \frac{\sqrt{\frac{p_e\sqrt{\hat{m}}+c_L}{3L}}\,\frac{\Lambda}{\tau}\,\hat{m}\,\left(c_K - \frac{p_e\tau}{\sqrt{\hat{m}}}\right)}{\sqrt{\frac{\frac{c_K\hat{m}}{\tau}+c_L}{3L}}\,\Lambda\,\left(\frac{c_K}{\tau}\sqrt{\hat{m}} - p_e\right)\,2\sqrt{\hat{m}}} = \frac{\sqrt{p_e\sqrt{\hat{m}} + c_L}}{2\sqrt{\frac{c_K\hat{m}}{\tau} + c_L}} < \frac{1}{2}, \tag{B.23}$$

since $\sqrt{\hat{m}} > \sqrt{m_f} = p_e\,\tau/c_K$. From (B.23) it is clear that $P(\hat{m}) > 2\;L(\hat{m})$. ∎

Proof of Proposition 11. It suffices to show that inducing the WTE firm to choose $\hat{\beta} \in (\beta_f, \beta_s]$ using the lump sum mechanism is more costly than using the price premium mechanism.

Consider first the price premium mechanism, which increases the price, p_e, by δ_e. We want to find $\hat{\delta}_e$ that induces the WTE firm to choose $\beta_p = \hat{\beta}$. Then, from (21) we can write

$$\hat{\beta} = \sqrt{\frac{(p_e + \hat{\delta}_e)^2 \, \tau / c_K + c_L}{3L}},$$

which yields

$$\hat{\delta}_e = \sqrt{(3\,\hat{\beta}^2 L - c_L)\, c_K/\tau} - p_e. \tag{B.24}$$

Then the WTE firm's optimal retention time m_p is given by

$$m_p = (p_e + \hat{\delta}_e)^2\, \tau^2 / c_K^2 = (3\,\hat{\beta}^2 L - c_L)\, \tau / c_K. \tag{B.25}$$

Per period cost of price premium mechanism to the social planner is given by

$$P(\hat{\beta}) = \hat{\beta}\, \Lambda\, \hat{\delta}_e\, g(m_p). \tag{B.26}$$

Combining (B.24)–(B.26) yields

$$P(\hat{\beta}) = \hat{\beta}\, \Lambda \left(\sqrt{(3\,\hat{\beta}^2 L - c_L)\, c_K/\tau} - p_e \right) 2 \sqrt{(3\,\hat{\beta}^2 L - c_L)\, \tau c_K}. \tag{B.27}$$

Next, consider the lump sum mechanism. Accounting for δ_K, the WTE firm's optimal coverage level β_l can be written from (20) as

$$\beta_l = \sqrt{\frac{p_e^2\, \tau / (c_K - \delta_K) + c_L}{3L}}. \tag{B.28}$$

Setting $\beta_l = \hat{\beta}$ in (B.28) and solving it for δ_K gives the amount of subsidy $\hat{\delta}_K$ that induces the WTE firm to choose $\hat{\beta}$ as

$$\hat{\delta}_K = c_K - \frac{p_e^2 \tau}{3\,\hat{\beta}^2 L - c_L}. \tag{B.29}$$

Then the WTE firm's optimal retention time m_l is given by

$$m_l = p_e^2 \tau^2 / (c_K - \hat{\delta}_K)^2 = [3\,\hat{\beta}^2 L - c_L]^2 / p_e^2. \tag{B.30}$$

Per period cost of the lump sum mechanism to the social planner is $n\,\hat{\delta}_K$, which is given by

$$L(\hat{\beta}) = \hat{\beta}\, \Lambda\, m_l\, \hat{\delta}_K / \tau. \tag{B.31}$$

Combining (B.29)–(B.31) yields

$$L(\hat{\beta}) = \hat{\beta}\, \frac{\Lambda}{\tau}\, \frac{[3\,\hat{\beta}^2 L - c_L]^2}{p_e^2} \left(c_K - \frac{p_e^2 \tau}{3\,\hat{\beta}^2 L - c_L} \right). \tag{B.32}$$

By (B.27) and (B.32), $L(\hat{\beta})/P(\hat{\beta})$ can be expressed (via straightforward calculation) as

$$\frac{L(\hat{\beta})}{P(\hat{\beta})} = \frac{\sqrt{(3\,\hat{\beta}^2 L - c_L)\, c_K/\tau} \left(\sqrt{(3\,\hat{\beta}^2 L - c_L)\, c_K/\tau} + p_e \right)}{p_e\,(p_e + p_e)}.$$

This ratio is greater than 1 and thus $L(\hat{\beta}) > P(\hat{\beta})$, because $\hat{\beta} > \beta_f = \sqrt{(p_e^2\, \tau / c_K + c_L)/3L}$. ■

References

Courcoubetis, C. and R. Weber (1996). Buffer overflow asymptotics for a buffer handling many traffic sources. *Journal of Applied Probability 33*, 886–903.