

## Online Appendix

### A.1. Additional Proofs

PROPOSITION O1. *Under delegation, the optimal price that S1 offers to S2 is a function of his cost  $c_1$  and the price  $w_1$  that M offers to him:*

$$w_2^d(c_1, w_1) = \begin{cases} \underline{c}_2 & \text{if } w_1 - c_1 < \underline{c}_2 \\ k_2^{-1}(w_1 - c_1) & \text{if } \underline{c}_2 \leq w_1 - c_1 \leq k_2(\bar{c}_2) \\ \bar{c}_2 & \text{otherwise} \end{cases} .$$

The function  $w_2^d(c_1, w_1)$  is decreasing in  $c_1$  and increasing in  $w_1$ .

**Proof of Proposition O1:** Consider a S1 with cost  $c_1$  who has been offered a per-unit price of  $w_1$  by M. Let  $\Pi_{S1}(c_1, w_1, w_2)$  be S1's objective function as defined in  $\mathbf{P}_{\text{dM}}$ . Then

$$\frac{d\Pi_{S1}(c_1, w_1, w_2)}{dw_2} = (w_1 - w_2 - c_1 - h_2(w_2))f_2(w_2)Q^*(w_1).$$

The first term in the derivative,  $w_1 - c_1 - w_2 - h_2(w_2)$ , is a decreasing function of  $w_2$ . Since  $f_2(w_2) > 0$  by assumption and  $Q^*(w_1) > 0$ , we immediately conclude that  $\Pi_{S1}(c_1, w_1, w_2)$  is unimodal in  $w_2$ . We now consider three cases depending on the value of  $w_1 - c_1$  to characterize the optimal price  $w_2^d(c_1, w_1)$ : (i) When  $w_1 - c_1 < \underline{c}_2$ , for feasible values of  $w_2$  ( $w_2 \geq \underline{c}_2$ ) we have  $w_1 - c_1 - w_2 - h_2(w_2) < 0$  implying  $w_2^d(c_1, w_1) = \underline{c}_2$ . (ii) When  $w_1 - c_1 > k_2(\bar{c}_2)$ , for feasible values of  $w_2$  ( $w_2 \leq \bar{c}_2$ ) we get  $w_1 - c_1 - w_2 - h_2(w_2) > 0$ , implying  $w_2^d(c_1, w_1) = \bar{c}_2$ . (iii) When  $\underline{c}_2 \leq w_1 - c_1 \leq k_2(\bar{c}_2)$ ,  $w_2^d$  satisfies the first-order condition  $w_2^d + h_2(w_2^d) = w_1 - c_1$ , which is equivalent to  $w_2^d = k_2^{-1}(w_1 - c_1)$ . This completes the proof of Proposition O1.  $\blacksquare$

**Proof of Lemma A1:** For  $w \leq a + \underline{c}_2$ , we have  $k_2^{-1}(w - a) \leq \underline{c}_2$  as  $k_2^{-1}(\cdot)$  inherits the increasing property of  $k_2(\cdot)$ . This implies  $k_2^{-1}(w - c_1) \leq \underline{c}_2$  for every  $a \leq c_1 \leq b$ , resulting in the integral in question being equal to 0. Similarly,  $w \geq b + k_2(\bar{c}_2)$  implies  $k_2^{-1}(w - b) \geq \bar{c}_2$ , and hence the integrand is equal to 1 over the limits of the integral, hence the result. Otherwise (i.e., when  $a + \underline{c}_2 < w < b + k_2(\bar{c}_2)$ ), we have:

$$\begin{aligned} \int_a^b F_2(k_2^{-1}(w - c_1)) dc_1 &= \int_a^b \int_{\underline{c}_2}^{k_2^{-1}(w - c_1)} f_2(c_2) dc_2 dc_1 = \int_{\underline{c}_2}^{\min(k_2^{-1}(w - a), \bar{c}_2)} \int_a^{\min(w - k_2(c_2), b)} f_2(c_2) dc_1 dc_2 \\ &= \int_{\underline{c}_2}^{\min(k_2^{-1}(w - a), \bar{c}_2)} f_2(c_2) (\min(w - k_2(c_2), b) - a) dc_2 \\ &= \int_{\underline{c}_2}^{\max(k_2^{-1}(w - b), \underline{c}_2)} (b - a) f_2(c_2) dc_2 + \int_{\max(k_2^{-1}(w - b), \underline{c}_2)}^{\min(k_2^{-1}(w - a), \bar{c}_2)} (w - k_2(c_2) - a) f_2(c_2) dc_2 \\ &= (b - a) F_2(\max(k_2^{-1}(w - b), \underline{c}_2)) + (w - a) (F_2(\min(k_2^{-1}(w - a), \bar{c}_2)) - F_2(\max(k_2^{-1}(w - b), \underline{c}_2))) \\ &\quad - \min(k_2^{-1}(w - a), \bar{c}_2) F_2(\min(k_2^{-1}(w - a), \bar{c}_2)) + \max(k_2^{-1}(w - b), \underline{c}_2) F_2(\max(k_2^{-1}(w - b), \underline{c}_2)) . \end{aligned}$$

The last two terms of the last equality are by observing that for  $\underline{c}_2 \leq a \leq b \leq \bar{c}_2$ , we have

$$\begin{aligned} \int_a^b F_2(c_2) dc_2 &= \int_a^b \int_{\underline{c}_2}^{c_2} f_2(u) du dc_2 = \int_{\underline{c}_2}^a \int_a^b f_2(u) dc_2 du + \int_a^b \int_u^b f_2(u) dc_2 du \\ &= \int_{\underline{c}_2}^a f_2(u) (b - a) du + \int_a^b f_2(u) (b - u) du = (b - a) F_2(a) + b(F_2(b) - F_2(a)) - \int_a^b u f_2(u) du, \end{aligned}$$

which implies  $\int_a^b k_2(c_2) f_2(c_2) dc_2 = \int_a^b (c_2 f_2(c_2) + F_2(c_2)) dc_2 = bF_2(b) - aF_2(a)$ . Rearranging the terms completes the proof.  $\blacksquare$

**Proof of Theorem 1:** (i) To prove the claim in the second part of Theorem 1(i), first we assume that  $c_i \sim U[\underline{c}, \bar{c}]$ . Since we have identical distributions for  $c_i$ , we drop the subscript  $i$  from  $F_i$  and  $k_i$ .

Under control, as the supplier's cost distributions are identical, M sets  $w_1 = w_2$ . To see this, assume that there exists an equilibrium in which  $w_2 > w_1$ . Observe that since  $F(c_i)$  is assumed to be log-concave (see §2), we have  $\frac{1}{2} \ln F(w_1) + \frac{1}{2} \ln F(w_2) \leq \ln F\left(\frac{w_1+w_2}{2}\right)$ , which, after rearranging terms, implies  $F(w_1)F(w_2) \leq F^2\left(\frac{w_1+w_2}{2}\right)$ . Using this inequality and the problem formulation  $\mathbf{P}_c$ , we observe that M could alternatively set  $w'_1 = w'_2 = \frac{w_1+w_2}{2}$  and thus increase her profit. Therefore, we can rewrite  $\mathbf{P}_c$  as (let  $w = w_1 + w_2$ ):

$$\max_{2\underline{c} \leq w \leq \min(2\bar{c}, \bar{w})} \Pi_M^c(w) := F^2\left(\frac{w}{2}\right) [R(Q^*(w)) - wQ^*(w) - \underline{\pi}] + \underline{\pi}. \quad (\text{O1})$$

Under delegation, we begin by solving for the optimal  $w_2^d$ . For simplicity, we drop the subscript 1 from  $w_1$ . Observe that for  $c \sim U[\underline{c}, \bar{c}]$ , we have  $k(c) = 2c - \underline{c}$  and  $k^{-1}(c) = \frac{c+\underline{c}}{2}$ . Using Equation (1), we find that

$$w_2^d(c_1, w) = \begin{cases} \underline{c} & \text{if } w - c_1 \leq \underline{c} \\ \frac{w - c_1 + \underline{c}}{2} & \text{if } \underline{c} \leq w - c_1 < k(\bar{c}) \\ \bar{c} & \text{otherwise.} \end{cases}$$

Hence we can rewrite the objective function of  $\mathbf{P}_d$ :

$$\begin{aligned} \Pi_M^d(w) &:= E_{c_1} [F(w_2^d(c_1, w))] [R(Q^*(w)) - wQ^*(w) - \underline{\pi}] + \underline{\pi}, \text{ where} \\ E_{c_1} [F(w_2^d(c_1, w))] &= \int_{\underline{c}}^{\max\{\underline{c}, w-2\bar{c}+\underline{c}\}} dF(c_1) + \int_{\max\{\underline{c}, w-2\bar{c}+\underline{c}\}}^{\min\{w-\underline{c}, \bar{c}\}} F\left(\frac{w-c_1+\underline{c}}{2}\right) dF(c_1). \end{aligned} \quad (\text{O2})$$

Using the functional forms of the probability density and the cumulative distribution functions of the uniform distribution, we find that

$$F^2\left(\frac{w}{2}\right) = \frac{(w-2\underline{c})^2}{4(\bar{c}-\underline{c})^2} \quad \text{and} \quad E_c [F(w_2^d(c_1, w))] = \begin{cases} (w-2\underline{c})^2/(4(\bar{c}-\underline{c})^2) & \text{if } w \leq \underline{c} + \bar{c} \\ (2w - \bar{c} - 3\underline{c})/(4(\bar{c}-\underline{c})) & \text{if } \underline{c} + \bar{c} < w \leq 2\bar{c} \\ (5\bar{c} - 3\underline{c} - w)(w - \bar{c} - \underline{c})/(4(\bar{c}-\underline{c})^2) & \text{if } 2\bar{c} < w \leq 3\bar{c} - \underline{c}. \end{cases}$$

It is trivial to show that  $F^2\left(\frac{w}{2}\right) \geq E_c [F(w_2^d(c_1, w))]$ , and is strictly so if and only if  $w > \underline{c} + \bar{c}$ . Hence we need to show that optimal total price under delegation is greater than  $\underline{c} + \bar{c}$ . Observe that:

$$\frac{d\Pi_M^c(w)}{dw} = F\left(\frac{w}{2}\right) f\left(\frac{w}{2}\right) [R(Q^*(w)) - (w + h(w/2))Q^*(w) - \underline{\pi}].$$

Since the first two terms are positive, and the third term is decreasing in  $w$ , we can conclude that the optimal price is strictly greater than  $\underline{c} + \bar{c}$  if the following inequality holds:

$$R(Q^*(\underline{c} + \bar{c})) - \frac{3\bar{c} + \underline{c}}{2} Q^*(\underline{c} + \bar{c}) > \underline{\pi}. \quad (\text{O3})$$

Using the envelope theorem, one can show that  $\underline{\pi}$  is strictly decreasing in  $\bar{w}$ . Hence if  $\bar{w}$  is high enough, inequality (O3) holds. This shows that the optimal total price under delegation is greater than  $\underline{c} + \bar{c}$ , hence M's optimal profit is strictly higher under control.

(ii) To prove the claim in the second part of Theorem 1(ii), first we assume that  $c_i \sim Tr[\underline{c}, m, \bar{c}]$ , where  $m$  is the mode of the triangular distribution. Recall that M's profits under control and delegation are as given in (O1) and (O2). Note also that,  $w = w_1 + w_2$  under control and  $w = w_1$  under delegation. For the triangular distribution, we have:

$$F(c) = \begin{cases} \frac{(c-\underline{c})^2}{(\bar{c}-\underline{c})(m-\underline{c})} & \text{if } \underline{c} \leq c \leq m \\ 1 - \frac{(\bar{c}-c)^2}{(\bar{c}-\underline{c})(\bar{c}-m)} & \text{if } m < c \leq \bar{c} \end{cases} \quad \text{and} \quad k(c) = c + h(c) = \begin{cases} c + \frac{(c-\underline{c})}{2} & \text{if } \underline{c} \leq c \leq m \\ c + \frac{(\bar{c}-c)(\bar{c}-m) - (\bar{c}-c)^2}{2(\bar{c}-c)} & \text{if } m < c \leq \bar{c} \end{cases}.$$

Notice that  $k(\bar{c}) \rightarrow \infty$ . Therefore, using Equation (1), we find  $w_2^d(c_1, w)$  can either be equal to  $\underline{c}$  or to  $k_2^{-1}(w - c_1)$  for triangular distribution. Depending on the relative values of  $c$  and  $m$ ,  $k_2^{-1}(w - c_1)$  is equal to either  $\frac{\underline{c}+2(w-c_1)}{3}$  or to  $\frac{2\bar{c}+(w-c_1)-\sqrt{3\bar{c}(\bar{c}-\underline{c}-m)+3\bar{c}m+(w-c_1-\bar{c})^2}}{3}$ . Therefore, the optimal wholesale price offered by S1 is as follows:

$$w_2^d(c_1, w) = \begin{cases} \frac{2\bar{c}+(w-c_1)-\sqrt{3\bar{c}(\bar{c}-\underline{c}-m)+3\bar{c}m+(w-c_1-\bar{c})^2}}{3} & \text{if } c_1 < w - \frac{3m-\underline{c}}{2} \\ \frac{\underline{c}+2(w-c_1)}{3} & \text{if } w - \frac{3m-\underline{c}}{2} \leq c_1 < w - \underline{c} \\ \underline{c} & \text{if } c_1 \geq w - \underline{c}. \end{cases}$$

Assume that  $\bar{w} \leq m + \underline{c}$ . Due to feasibility, we have  $w \leq \bar{w} \leq m + \underline{c} \leq \frac{3m+\underline{c}}{2}$ . Hence

$$E_c [F(w_2^d(c_1, w))] = \int_{\underline{c}}^{w-\underline{c}} F\left(\frac{\underline{c}+2(w-c_1)}{3}\right) dF(c_1) + \int_{\underline{c}}^{w-\underline{c}} F(\underline{c}) dF(c_1).$$

In our case,  $\frac{w}{2} \leq \frac{m+\underline{c}}{2} \leq m$  and  $\frac{\underline{c}+2(w-c_1)}{3} \leq m$  for  $w - \frac{3m-\underline{c}}{2} \leq c_1$ . By algebraic manipulation and using the functional forms for the triangular distribution,  $\frac{F^2(\frac{w}{2})}{E_c[F(w_2^d(c_1, w))]} = \frac{27}{32}$ . Since  $R(Q^*(w)) - wQ^*(w) - \underline{\pi} \geq 0$  for  $w \leq \bar{w}$ , we conclude that  $\Pi_M^d(w) > \Pi_M^c(w)$  for every  $w \leq m + \underline{c}$ . This completes the proof of the second part of Theorem 1(ii). ■

## A.2. Extensions to Quantity Discount Contracts

In this section, we show how our results with price-only contracts hold exactly as stated in the paper for incremental quantity discount contracts with two price blocks. We start with the detailed formulation under this family of contracts. We then present the proofs.

**A.2.1. Model Formulation** Under an incremental quantity discount contract with two price blocks, the buyer offers to pay  $T_i$  to supplier  $i$  for the right to procure up to  $Q_i^H$  units at unit price  $w_i^H$ , and additional units up to  $(Q_i^L - Q_i^H)$  at unit price  $w_i^L$ , where  $w_i^L \leq w_i^H$ . We focus on the analysis of the case with  $Q_1^L > Q_1^H$ ; otherwise, quantity discount contracts will effectively reduce to price-only contracts, which have already been addressed.

To accommodate this family of contracts, we modify the sequences of events for control and delegation provided in §2 as follows:

**Control Scenario:** M offers a quantity discount contract  $\{T_1, w_1^H, w_1^L, Q_1^H, Q_1^L\}$  to S1 and  $\{T_2, w_2^H, w_2^L, Q_2^H, Q_2^L\}$  to S2. If  $w_i^H \leq c_i$ , supplier  $i$  rejects M's offer. Otherwise, supplier  $i$  commits to a maximum quantity he would deliver under the contract (i.e., he chooses  $Q_i^L$  or  $Q_i^H$ , and in turn the per-unit price). If both suppliers accept her offer, then M requests quantity  $Q$  from both suppliers and pays them accordingly, and she purchases additional units from her alternative source if necessary. If either one of the suppliers rejects the contract offered by M, M purchases only from her alternative source.

**Delegation Scenario:** M offers a quantity discount contract  $\{T_1, w_1^H, w_1^L, Q_1^H, Q_1^L\}$  to S1. If  $w_1^H \leq c_1$ , S1 rejects M's offer right away without extending a contract to S2, so M buys only from the alternative source at price  $\bar{w}$  per unit. Otherwise, the events unfold as follows: S1 offers a quantity discount contract  $\{T_2, w_2^H, w_2^L, Q_2^H, Q_2^L\}$  to S2. M knows that S1 will use a quantity discount contract with two price blocks, but cannot observe the parameters of the contract offered to S2. S2 accepts S1's offer if and only if  $w_2^H \geq c_2$  and commits to a maximum quantity he would deliver under the contract (i.e., he chooses  $Q_2^L$  or  $Q_2^H$ , and in turn the per-unit price). If S2 accepts, then S1 accepts the contract offered by M and commits to a maximum quantity he would deliver under the contract (i.e., he chooses  $Q_1^L$  or  $Q_1^H$ , and in turn the per-unit price). M purchases  $Q$  units from S1, and S1 purchases  $Q$  units from S2. Payments occur accordingly. M procures additional capacity from the alternative source if necessary. If S2 rejects S1's offer, then S1 is forced to reject M's offer, and M buys only from the alternative source.

Given these sequences, we will now characterize M's contract-optimization problem in the control and then delegation scenarios by assuming  $T_i = 0$ . We will then show that this assumption can be made without loss of generality.

**Control Scenario:** A supplier who received a contract  $\{T_i = 0, w_i^H, w_i^L, Q_i^H, Q_i^L\}$  would accept the contract and choose to supply up to  $Q_i^H$  units if and only if

$$\begin{aligned} (w_i^H - c_i)Q_i^H &\geq (w_i^H - c_i)Q_i^H + (w_i^L - c_i)(Q_i^L - Q_i^H) = w_i^H Q_i^H + w_i^L(Q_i^L - Q_i^H) - c_i Q_i^L \\ (w_i^H - c_i)Q_i^H &\geq 0 \end{aligned}$$

or, equivalently,  $w_i^L \leq c_i \leq w_i^H$ . Similarly, the supplier would accept a contract and be willing to supply up to  $Q_i^L$  units if and only if  $c_i \leq w_i^L$ . Notice that the supplier's decision is independent of quantities  $Q_i^L$  and  $Q_i^H$ . In addition, since the components are perfectly complementary, M should offer the same quantities to both suppliers. Hence we drop the subscripts and denote  $Q^k := Q_i^k$  for  $i \in \{1, 2\}$  and  $k \in \{L, H\}$ .

Given the suppliers' decisions, M must choose the optimal quantity to purchase from the suppliers. First notice that, as we show in §2, M would buy an additional  $Q_A = (Q^*(\bar{w}) - Q)^+$  units from the alternative source after procuring  $Q$  units from S1 and S2. Also observe that M sets  $w_1^H + w_2^H \leq \bar{w}$ . Now, there are two scenarios to consider. In the first scenario, both suppliers agree to provide up to  $Q^L$  units and M's problem can be formulated as:

$$\max_{Q \leq Q^L} [R(\max(Q, Q^*(\bar{w}))) - (w_1^H + w_2^H) \min(Q, Q^H) - (w_1^L + w_2^L)(Q - Q^H)^+ - \bar{w}(Q^*(\bar{w}) - Q)^+].$$

In the second scenario, at least one supplier agrees to provide only up to  $Q^H$  units and M solves

$$\max_{Q \leq Q^H} [R(\max(Q, Q^*(\bar{w}))) - (w_1^H + w_2^H)Q - \bar{w}(Q^*(\bar{w}) - Q)^+].$$

Given these formulations, and the fact that the suppliers' decisions are independent of the quantities, it is trivial to show that during contract design M should set  $Q^L = Q^*(w_1^L + w_2^L)$ .

In the first scenario, there are three cases. If  $Q^H \leq Q^*(\bar{w})$ , then the objective function has a single local maximum at  $Q^L$ . If  $Q^*(\bar{w}) \leq Q^H \leq Q^*(w_1^H + w_2^H)$ , then the objective function again has a single local maximum at  $Q^L$ . If  $Q^*(w_1^H + w_2^H) \leq Q^H \leq Q^L$ , then the objective function has two local maxima, at  $Q^*(w_1^H + w_2^H)$  and  $Q^L$ . In order to show that the latter is the global maximum, we need to show that:

$$R(Q^*(w_1^H + w_2^H)) - (w_1^H + w_2^H)Q^*(w_1^H + w_2^H) \leq R(Q^*(w_1^L + w_2^L)) - (w_1^L + w_2^L)Q^*(w_1^L + w_2^L) - Q^H(w_1^H + w_2^H - w_1^L - w_2^L), \quad (\text{O4})$$

In order to prove that this inequality holds, we define  $P(w) := \max_{q \geq 0} [R(q) - w(q - Q^H)]$ . Using the envelope theorem and the fact that  $Q^H \leq Q^*(w_1^H + w_2^H) < Q^*(w_1^L + w_2^L)$ , we find that  $P(w)$  is a decreasing function. Thus, we have

$$R(Q^*(w_1^H + w_2^H)) - (w_1^H + w_2^H)(Q^*(w_1^H + w_2^H) - Q^H) \leq R(Q^*(w_1^L + w_2^L)) - (w_1^L + w_2^L)(Q^*(w_1^L + w_2^L) - Q^H).$$

Rewriting this inequality shows that the inequality (O4) holds. Combining all three cases, we conclude that M's optimal order quantity is  $Q^L$  in the first scenario.

In the second scenario, there are two cases to consider. If  $Q^H \leq Q^*(w_1^H + w_2^H)$ , then the objective function has a single maximum at  $Q^H$ . Otherwise, the local maximum is at  $Q^*(w_1^H + w_2^H)$ . Hence we conclude that M's optimal quantity is  $\min(Q^H, Q^*(w_1^H + w_2^H))$  in this scenario.

Now, observe that M's profit is decreasing in  $Q^H$  in the first scenario, and it does not change in the second scenario as long as  $Q^H > Q^*(w_1^H + w_2^H)$ . Hence we conclude that during contract design M would set  $Q^H \leq Q^*(w_1^H + w_2^H)$ .

Given the equilibrium behaviors of the suppliers and the optimal order quantities by M from the suppliers and the alternative source, one can show that M's contract-optimization problem can be formulated as:

$$\max_{\{w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}} F_1(w_1^L) F_2(w_2^L) \left( \begin{aligned} & R(Q^L) - (w_1^H + w_2^H)Q^H - (w_1^L + w_2^L)(Q^L - Q^H) \\ & - (R(\max(Q^H, Q^*(\bar{w}))) - (w_1^H + w_2^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+) \end{aligned} \right) \\ + F_1(w_1^H) F_2(w_2^H) (R(\max(Q^H, Q^*(\bar{w}))) - (w_1^H + w_2^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi}) + \underline{\pi}.$$

To make the profit comparisons under delegation and control easier, one can rewrite the above problem as:

$$\mathbf{P}_c^{\text{QDC}} : \max_{\{w_S^H, w_S^L, Q_S^H, Q_S^L\}} F_{1.2}(w_S^L) \left( \begin{aligned} & R(Q^L) - w_S^H Q^H - w_S^L (Q^L - Q^H) \\ & - (R(\max(Q^H, Q^*(\bar{w}))) - w_S^H Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+) \end{aligned} \right) \\ + F_{1.2}(w_S^H) (R(\max(Q^H, Q^*(\bar{w}))) - w_S^H Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi}) + \underline{\pi},$$

where  $F_{1.2}(w) := \max_x F_1(x)F_2(w-x)$ .

**Delegation Scenario:** To derive M's optimal contract under delegation, we must first consider S1's optimal contract design problem. S1, contingent on his cost  $c_1$  and the contract  $(T_1 = 0, w_1^H, w_1^L, Q_1^H, Q_1^L)$  offered by M, selects the contract  $(T_2 = 0, w_2^H, w_2^L, Q_2^H, Q_2^L)$  to maximize his expected profit. Trivially, S1 always sets  $Q_2^L = Q_1^L$  due to perfect complementarity. S2's optimal decision can be derived similar to the control scenario. Hence one can write S1's problem as follows:

$$\mathbf{P}_{\text{dm}}^{\text{QDC}} : \max_{\{w_2^H, w_2^L, Q_2^H\}} F_2(w_2^L) (w_1^H Q_1^H + w_1^L (Q_1^L - Q_1^H) - c_1 Q_1^L - w_2^H Q_2^H - w_2^L (Q_1^L - Q_2^H)) \\ + (F_2(w_2^H) - F_2(w_2^L)) (w_1^H \min(Q_1^H, Q_2^H) + w_1^L \max(Q_2^H - Q_1^H, 0) - c_1 Q_2^H - w_2^H Q_2^H).$$

Given this formulation, the following proposition provides a solution for S1's optimal contract.

**PROPOSITION O2.** *Under delegation, an optimal quantity discount contract with two price blocks (and  $T_2 = 0$ ) that S1 offers to S2 is a function of his cost  $c_1$  and the contract that M offers to him:*

$$w_2^{k,d}(c_1, w_1^H, w_1^L, Q_1^H, Q_1^L) = \begin{cases} c_2 & \text{if } w_1^k - c_1 < c_2 \\ k_2^{-1}(w_1^k - c_1) & \text{if } c_2 \leq w_1^k - c_1 \leq k_2(\bar{c}_2) \\ \bar{c}_2 & \text{otherwise} \end{cases}$$

and  $Q_2^{k,d}(c_1, w_1^H, w_1^L, Q_1^H, Q_1^L) = Q_1^k$ , for  $k \in \{L, H\}$ .

*Proof:* The proof is completed through the following steps:

**STEP 1:** We claim that for any parameter setting, there are three candidates for optimal  $Q_2^H$ :  $0, Q_1^H$ , or  $Q_1^L$ . In order to show this, first observe that the derivative of S1's expected profit with respect to  $Q_2^H$  could be written as:

$$F_2(w_2^L)(w_2^L - w_2^H) + \begin{cases} (F_2(w_2^H) - F_2(w_2^L))(w_1^H - c_1 - w_2^H) & \text{if } Q_2^H < Q_1^H, \\ (F_2(w_2^H) - F_2(w_2^L))(w_1^L - c_1 - w_2^H) & \text{otherwise.} \end{cases}$$

Since both equations are independent of  $Q_2^H$ , one can conclude that the optimal decision is either at the boundaries (i.e.,  $0$  or  $Q_2^L = Q_1^L$ ) or at the possible inflection point ( $Q_1^H > 0$ ).

**STEP 2:** After a set of trivial algebraic manipulations, one can show that the following formulas hold for S1's profit:

$$\begin{aligned} & F_2(w_2^L) (w_1^H Q_1^H + w_1^L (Q_1^L - Q_1^H) - c_1 Q_1^L - w_2^L Q_1^L) \text{ when } Q_2^H = 0, \\ & F_2(w_2^H) (w_1^H Q_1^H + w_1^L (Q_1^L - Q_1^H) - c_1 Q_1^L - w_2^H Q_1^L) \text{ when } Q_2^H = Q_2^L = Q_1^L, \\ & F_2(w_2^L) (w_1^H Q_1^H + w_1^L (Q_1^L - Q_1^H) - c_1 Q_1^L - w_2^L Q_1^L) \text{ when } Q_2^H = Q_1^H \text{ and } w_2^H = w_2^L. \end{aligned}$$

First, observe that S1 could achieve the same profit under any equilibrium with  $Q_2^H = 0$  by setting  $Q_2^H = Q_1^H$  and  $w_2^H = w_2^L$ . Similarly, S1 will receive the same profit under any equilibrium with  $Q_2^H = Q_2^L$  by setting  $Q_2^H = Q_1^H$  and  $w_2^L = w_2^H$ .

STEP 3: Given STEP 2, without loss of generality, one can assume that  $Q_2^H = Q_1^H$ . If  $Q_1^H = 0$ , then the quantity discount contract reduces to a price-only contract and the result follows trivially from Proposition O1. Therefore, we will assume  $Q_1^H > 0$  in the rest of the proof. To find the optimal prices under such an equilibrium, we take the derivative of S1's expected profit function with respect to  $w_2^L$ :

$$\begin{aligned} & f_2(w_2^L) (w_1^H Q_1^H + w_1^L (Q_1^L - Q_1^H)) - c_1 Q_1^L - w_2^H Q_1^H - w_2^L (Q_1^L - Q_1^H) + F_2(w_2^L) (Q_1^H - Q_1^L) \\ & - f_2(w_2^L) (w_1^H - c_1 - w_2^H) Q_1^H \\ & = (w_1^L - c_1 - k_2(w_2^L)) f_2(w_2^L) (Q_1^L - Q_1^H), \end{aligned}$$

and with respect to  $w_2^H$ :

$$\begin{aligned} & F_2(w_2^L) (-Q_1^H) + f_2(w_2^H) (w_1^H - c_1 - w_2^H) Q_1^H + (F(w_2^H) - F(w_2^L)) (-Q_1^H) \\ & = (w_1^H - c_1 - k_2(w_2^H)) f_2(w_2^H) Q_1^H. \end{aligned}$$

The first term in the first derivative  $(w_1^L - c_1 - k_2(w_2^L))$  is a decreasing function of  $w_2^L$ . Since  $f_2(w_2^L) > 0$  and  $Q_1^L - Q_1^H > 0$  by assumption, we immediately conclude that S1's profit function is unimodal in  $w_2^L$ . Similarly, S1's profit function is unimodal in  $w_2^H$ . Hence the optimal prices satisfy the first order conditions, implying the optimality of the stated expressions in the proposition. ■

Using Proposition O2 that provides the solution to S1's contract design problem, and by following similar arguments in the control scenario with respect to M's optimal order quantities, one can write M's contract design problem (with  $T_1 = 0$ ) as

$$\begin{aligned} \mathbf{P}_d^{\text{QDC}} : \quad & \max_{\{w_1^H, w_1^L, Q_1^H, Q_1^L\}} E_{c_1} [F_2(k_2^{-1}(w_1^L - c_1))] \left( - (R(\max(Q_1^H, Q^*(\bar{w}))) - w_1^H Q_1^H - \bar{w} (Q^*(\bar{w}) - Q_1^H)^+) \right) \\ & + E_{c_1} [F_2(k_2^{-1}(w_1^H - c_1))] (R(\max(Q_1^H, Q^*(\bar{w}))) - w_1^H Q_1^H - \bar{w} (Q^*(\bar{w}) - Q_1^H)^+ - \underline{\pi}) \\ & + \underline{\pi}. \end{aligned}$$

Given these formulations, we now revisit the assumption of  $T_i = 0$  and show that for the study of quantity discount contracts we can make this assumption without loss of generality.

**PROPOSITION O3.** *Under both control and delegation,  $T_i = 0$  is optimal for the family of quantity discount contracts  $\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}$ .*

*Proof:* To prove this claim, we show that M would receive the same expected profit by replacing a contract with  $T_i > 0$  with another contract wherein  $T_i = 0$  under both delegation and control.

Under control, M must choose a pair of contracts, one for each supplier. We denote M's contract by  $\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}$ . Given the structure of payments, a supplier would accept to supply up to  $Q_i^H$  units if and only if

$$\begin{aligned} T_i + (w_i^H - c_i) Q_i^H & \geq T_i + w_i^H Q_i^H + w_i^L (Q_i^L - Q_i^H) - c_i Q_i^L, \\ T_i + (w_i^H - c_i) Q_i^H & \geq 0, \end{aligned}$$

or, equivalently,  $w_i^L \leq c_i \leq T_i/Q_i^H + w_i^H$ . Similarly, the supplier would be willing to supply up to  $Q_i^L$  units if and only if  $c_i \leq w_i^L = \min(w_i^L, T_i/Q_i^L + w_i^L)$ . Given the suppliers' optimal decisions and M's optimal order quantities, M's problem can be formulated as:

$$\begin{aligned} & \max_{\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}} F_1(w_1^L) F_2(w_2^L) [\lambda^L(\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}) - \lambda^H(\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2})] \\ & + F_1\left(\frac{T_1}{Q_1^H} + w_1^H\right) F_2\left(\frac{T_2}{Q_2^H} + w_2^H\right) [\lambda^H(\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}) - \underline{\pi}] + \underline{\pi}, \end{aligned}$$

where

$$\lambda^L(\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}) := R(\max(\min(Q_1^L, Q_2^L), Q^*(\bar{w}))) - \sum_{i=1}^2 (T_i + w_i^H Q_i^H + w_i^L (Q_i^L - Q_i^H))$$

$$\begin{aligned} & -\bar{w}(Q^*(\bar{w}) - \min(Q_1^L, Q_2^L))^+, \\ \lambda^H(\{T_i, w_i^H, w_i^L, Q_i^H, Q_i^L\}_{i=1,2}) & := R(\max(\min(Q_1^H, Q_2^H), Q^*(\bar{w}))) - \sum_{i=1}^2 (T_i + w_i^H Q_i^H) \\ & -\bar{w}(Q^*(\bar{w}) - \min(Q_1^H, Q_2^H))^+. \end{aligned}$$

Let  $\tilde{w}_i^H := T_i/Q_i^H + w_i^H$ . Since the components are perfectly complementary and it can easily be shown that the suppliers' decisions in this reformulation are independent of the quantities, M should offer the same quantities to both suppliers. Hence we can drop the subscripts from the quantities in the above formulation. One can easily show that the equation above after the aforementioned transformations reduces to  $\mathbf{P}_c^{\text{QDC}}$ . This shows that M would receive the same profit via the contract  $\{0, \tilde{w}_i^H, w_i^L, Q^H, Q^L\}_{i=1,2}$ .

Under delegation, we need to consider S1's contract design problem, given a contract  $\{T_1, w_1^H, w_1^L, Q_1^H, Q_1^L\}$  offered by M. Due to perfect complementarity and a slight modification of STEP 2 in Proposition O2, one can show that  $Q_2^k = Q_1^k$ , for  $k \in \{L, H\}$ . Hence S1 solves the following optimization problem:

$$\begin{aligned} \max_{\{T_2, w_2^H, w_2^L\}} & F_2(w_2^L) (T_1 - T_2 + (w_1^H - w_2^H)Q_1^H + (w_1^L - w_2^L)(Q_1^L - Q_1^H) - c_1 Q_1^L) \\ & + F_2\left(\frac{T_2}{Q_1^H} + w_2^H\right) (T_1 - T_2 + (w_1^H - w_2^H - c_1)Q_1^H). \end{aligned}$$

Using the transformation  $\tilde{w}_i^H := T_i/Q_1^H + w_i^H$  and rearranging terms, one can show that this problem reduces to  $\mathbf{P}_{\text{dM}}^{\text{QDC}}$  with  $Q_2^H = Q_1^H$ . Hence we can use Proposition O2 to find S1's optimal contract. Using similar definitions and transformations as in the control scenario, one can similarly show that M's problem similarly reduces to  $\mathbf{P}_d^{\text{QDC}}$ . ■

## A.2.2. Results

**THEOREM O1.** *Control is strictly optimal if M knows  $c_1$  but is uncertain about  $c_2$ . Delegation is optimal if M knows  $c_2$ .*

*In the case that M is uncertain about both  $c_1$  and  $c_2$ ,*

(i) *If  $f_1(c_1)$  is decreasing in  $c_1$ , then control is optimal and may be strictly so.*

(ii) *If  $f_1(c_1)$  is a unimodal density function with mode  $m_1 \geq \bar{w} - \underline{c}_2$ , then delegation is optimal and may be strictly so.*

*Proof:* We begin by assuming that M knows  $c_1$  exactly. Under control, M offers a price-only contract to S1 with  $w_1 = c_1$ . Hence M's problem under control reduces to:

$$\max_{\{w^H, w^L, Q^H, Q^L\}} F_2(w^L) \left( \begin{aligned} & R(Q^L) - (c_1 + w^H)Q^H - (c_1 + w^L)(Q^L - Q^H) \\ & - (R(\max(Q^H, Q^*(\bar{w}))) - (c_1 + w^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+) \end{aligned} \right) \\ + F_2(w^H) (R(\max(Q^H, Q^*(\bar{w}))) - (c_1 + w^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi}) + \underline{\pi}.$$

As a comparison, M's problem under delegation after defining  $w^k := w_1^k - c_1$  ( $k \in \{L, H\}$ ) is

$$\max_{\{w^H, w^L, Q^H, Q^L\}} F_2(k_2^{-1}(w^L)) \left( \begin{aligned} & R(Q^L) - (c_1 + w^H)Q^H - (c_1 + w^L)(Q^L - Q^H) \\ & - (R(\max(Q^H, Q^*(\bar{w}))) - (c_1 + w^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+) \end{aligned} \right) \\ + F_2(k_2^{-1}(w^H)) (R(\max(Q^H, Q^*(\bar{w}))) - (c_1 + w^H)Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi}) + \underline{\pi}.$$

A direct comparison of these problems shows that M's profit under control is greater, as  $k_2^{-1}(w) \leq w$ . Moreover, since  $\bar{w} > c_1 + \underline{c}_2$ , we must have that under delegation the optimal  $w^H > \underline{c}_2$ ; hence we can claim that the inequality is strict.

When both  $c_1$  and  $c_2$  are uncertain, for the first part of (i), we compare problems  $\mathbf{P}_d^{\text{QDC}}$  and  $\mathbf{P}_c^{\text{QDC}}$  and observe that if the probabilities that M could order up to  $Q^L$  and  $Q^H$  units under control

are greater, M would receive higher expected profit under control. Using the trivial modifications, one can show that for any  $F_1 \in \mathfrak{S}_1$ ,

$$F_{1,2}(w^k) \geq E_{c_1} [F_2(k_2^{-1}(w^k - c_1))],$$

hence completing the proof of the first part. For strict optimality, assume that  $c_1$  and  $c_2$  have a symmetric uniform distribution. From the proof of Theorem 1(ii), control is strictly optimal if the optimal  $w_S^H > \underline{c} + \bar{c}$ . The derivative of M's profit under control with respect to  $w_S^H$  is

$$F\left(\frac{w_S^H}{2}\right) f\left(\frac{w_S^H}{2}\right) \left[ R(\max(Q^H, Q^*(\bar{w}))) - \left(w_S^H + h\left(\frac{w_S^H}{2}\right)\right) Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi} \right].$$

Notice that the first two terms are positive and the third term is decreasing in  $w_S^H$ . Hence the proof is complete if we can show that the third term is positive for  $w_S^H = \underline{c} + \bar{c}$ . If  $Q^H \leq Q^*(\bar{w})$ , this term reduces to  $Q^H(\bar{w} - (3\bar{c} + \underline{c})/2)$  which is strictly positive when  $\bar{w} > (3\bar{c} + \underline{c})/2$ . Otherwise, this term will be equal to  $R(Q^H) - (3\bar{c} + \underline{c})Q^H/2 - \underline{\pi}$ . Since  $\underline{\pi}$  is strictly decreasing in  $\bar{w}$ , we conclude that this term is positive for sufficiently high  $\bar{w}$ . This completes the proof of (i).

For the first part of (ii), we use the result in Theorem 1 to claim that for every  $F_1 \in \Psi_1$  and  $k \in \{L, H\}$ , we have:

$$F_{1,2}(w^k) \leq E_{c_1} [F_2(k_2^{-1}(w^k - c_1))].$$

Using these inequalities and multiplying the one for  $k = L$  by  $R(Q^L) - w_S^H Q^H - w_S^L(Q^L - Q^H) - (R(\max(Q^H, Q^*(\bar{w}))) - w_S^H Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+)$  and the one for  $k = H$  by  $R(\max(Q^H, Q^*(\bar{w}))) - w_S^H Q^H - \bar{w}(Q^*(\bar{w}) - Q^H)^+ - \underline{\pi}$ , we conclude that for any  $\{w_S^L, w_S^H, Q^L, Q^H\}$  quadruple, M's profit under delegation is higher (see problems  $\mathbf{P}_d^{\text{QDC}}$  and  $\mathbf{P}_c^{\text{QDC}}$ ), and hence delegation is optimal. For the second part of (ii), from the proof of Theorem 1(ii), note that we have  $F_{1,2}(w^k) = F^2(w^k/2)$  and

$$F^2(w^k/2) < E_{c_1} [F_2(k_2^{-1}(w^k - c_1))],$$

for  $k \in \{L, H\}$ . This shows that when  $c_1$  and  $c_2$  have a symmetric triangular distribution with  $m_1 \geq \bar{w} - \underline{c}_2$ , then delegation is strictly optimal.  $\blacksquare$

### A.3. Correlation in Suppliers' Costs

In this subsection only, we assume that the joint distribution of  $c_1$  and  $c_2$  is bivariate normal with correlation coefficient  $\rho$ . Our main result is that delegation is optimal when the suppliers' costs have a strong - either positive or negative - correlation.

**PROPOSITION O4.** *M has greater expected profit with delegation than with control if  $\rho \in [-1, \underline{\rho}] \cup [\bar{\rho}, 1]$ , where  $-1 < \underline{\rho} \leq \bar{\rho} < 1$ .*

**Proof:** Let the joint distribution of  $c_1$  and  $c_2$  follow a bivariate normal distribution with  $E[c_i] = \mu_{c_i}$ ,  $\text{Var}[c_i] = \sigma_{c_i}^2$ , and the probability density function

$$f(c_1, c_2) = \frac{1}{2\pi\sigma_{c_1}\sigma_{c_2}\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left\{ \left( \frac{c_1 - \mu_{c_1}}{\sigma_{c_1}} \right)^2 + \left( \frac{c_2 - \mu_{c_2}}{\sigma_{c_2}} \right)^2 - 2\rho \left( \frac{c_1 - \mu_{c_1}}{\sigma_{c_1}} \right) \left( \frac{c_2 - \mu_{c_2}}{\sigma_{c_2}} \right) \right\} \right].$$

Knowing his own cost  $c_1$ , S1 believes that  $c_2$  is normally distributed with mean  $\mu_{c_2}(c_1) := \mu_{c_2} + \rho\sigma_{c_2}\frac{c_1 - \mu_{c_1}}{\sigma_{c_1}}$  and variance  $\sigma_{c_2}^2(c_1) := (1 - \rho^2)\sigma_{c_2}^2$ . Let  $F(\cdot|c_1)$  ( $f(\cdot|c_1)$ ) denote the cumulative distribution function (probability density function) of this distribution, and define  $k(c_2|c_1) := c_2 + \frac{F(c_2|c_1)}{f(c_2|c_1)}$ .

Consider the case with perfect correlation ( $|\rho| = 1$ ). In this case, we have  $\sigma_{c_2}(c_1) = 0$  implying that S1 exactly knows  $c_2$ . Thus, S1 sets  $w_2^d(c_1, w_1)$  equal to  $c_2$  if  $w_1 - c_1 \geq c_2$  and equal to  $\underline{c}_2$  otherwise. Let  $w^c$  be the solution to M's problem under control (see  $\mathbf{P}'_c$ ). Since we have that

$$\max_{\substack{w_1 + w_2 \leq w^c \\ c_i \leq w_i \leq \bar{c}_i \quad i=1,2}} F_1(w_1)F_2(w_2) \leq F_S(w^c) = E_{c_1} [F_2(w_2^d(c_1, w^c))], \quad (\text{O5})$$

a direct comparison of problems  $\mathbf{P}'_c$  and  $\mathbf{P}_d$  concludes that M's optimal profit under delegation is greater. Finally, if  $w^c < \bar{c}_1 + \bar{c}_2$  (i.e., if M procures only from the alternative source with strictly positive probability under control), the inequality in (O5) is strict (as  $f_i(c_i) > 0$  for all  $c_i \in [\underline{c}_i, \bar{c}_i]$ ) implying M's profit is strictly greater under delegation. As  $\bar{c}_i$  is unbounded from above for the normal distribution,  $w_1^c < \bar{c}_1 + \bar{c}_2$  trivially. Hence the inequality is strict. We next show that M's expected profit functions under delegation and control are continuous in  $\rho$ . Using these properties, we can conclude that M's expected profit is greater with delegation in a neighborhood of  $|\rho| = 1$ , i.e., when  $\rho \in [-1, \underline{\rho}] \cup [\bar{\rho}, 1]$  where  $\underline{\rho}$  and  $\bar{\rho}$  are defined appropriately. Note that since M's profit under delegation (and control) is continuous and strictly greater when  $|\rho| = 1$ , we must have  $-1 < \underline{\rho}$  and  $\bar{\rho} < 1$ .

In order to show that M's expected profit under delegation is continuous in  $\rho$ , we first prove that the expected probability that S2 agrees to participate, i.e.,  $E_{c_1, c_2}[Pr(c_2 \leq w_2^d)]$ , is continuous in  $\rho$ . Using the proof of Proposition O1, we have  $w_2^d = k^{-1}(w_1 - c_1|c_1)$ . Thus the expected probability that S2 agrees to participate, given the optimal price of S1  $w_2^d$  and for any price  $w_1$  offered by M, can be written as:

$$\begin{aligned} E_{c_1, c_2}[Pr(c_2 \leq w_2^d)] &= E_{c_1, c_2}[Pr(c_1 + k(c_2|c_1) \leq w_1)] \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{1}_{\left\{c_1 + c_2 + \left(\int_{-\infty}^{c_2} f(x_2|c_1) dx_2\right) / f(c_2|c_1) \leq w_1\right\}} f(c_1, c_2) dc_1 dc_2, \end{aligned} \quad (\text{O6})$$

where  $f(c_2|c_1) = \frac{1}{\sigma_{c_2(c_1)}\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{c_2 - \mu_{c_2(c_1)}}{\sigma_{c_2(c_1)}}\right)^2\right\}$ . Define  $z_1 := \frac{c_1 - \mu_{c_1}}{\sigma_{c_1}}$  and  $z_2 := \frac{c_2 - \mu_{c_2(c_1)}}{\sigma_{c_2(c_1)}}$ . Since this implies  $c_1 = \mu_{c_1} + z_1\sigma_{c_1}$  and  $c_2 = \mu_{c_2} + z_1\rho\sigma_{c_2} + z_2\sigma_{c_2}\sqrt{1 - \rho^2}$ , the Jacobian is  $\left|\begin{array}{cc} \sigma_{c_1} & 0 \\ \rho\sigma_{c_2} & \sigma_{c_2}\sqrt{1 - \rho^2} \end{array}\right| = \sigma_{c_1}\sigma_{c_2}\sqrt{1 - \rho^2}$ . Using the standardized variables  $z_1$  and  $z_2$ , we can rewrite the quantity in equation (O6) as

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{1}_{\left\{\sigma_{c_1}z_1 - \mu_{c_1} + \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2} - \mu_{c_2} - \rho\sigma_{c_2}z_1 \leq w_1\right\}} \phi(z_1)\phi(z_2) dz_1 dz_2 = E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2}}{|\sigma_{c_1} - \rho\sigma_{c_2}|} \right) \right]$$

where  $\check{k}(z_2) = z_2 + \frac{\Phi(z_2)}{\phi(z_2)}$  and  $\Phi(z_2)(\phi(z_2))$  is the standard normal cumulative distribution function (probability density function). (We use the identity  $\Phi(-z) = 1 - \Phi(z)$  in the transformation.) As  $\Phi(\cdot) \leq 1$  and  $E_{z_2}[1] = 1 < \infty$ , we can use the dominated convergence theorem to show that for any  $\rho \in [-1, 1]$ ,

$$\begin{aligned} \lim_{\rho \rightarrow \rho_0} E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2}}{|\sigma_{c_1} - \rho\sigma_{c_2}|} \right) \right] &= E_{z_2} \left[ \lim_{\rho \rightarrow \rho_0} \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2}}{|\sigma_{c_1} - \rho\sigma_{c_2}|} \right) \right] \\ &= E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho_0^2}}{|\sigma_{c_1} - \rho_0\sigma_{c_2}|} \right) \right]. \end{aligned}$$

Hence the expected probability that S2 agrees to participate is continuous in  $\rho$  by definition.

To complete the proof that M's expected profit under delegation is continuous in  $\rho$ , observe that

$$\begin{aligned} &\lim_{\rho \rightarrow \rho_0} \left\{ \max_{w_1 \leq \bar{w}} E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2}}{|\sigma_{c_1} - \rho\sigma_{c_2}|} \right) \right] (\lambda(w_1) - \underline{\pi}) \right\} \\ &= \max_{w_1 \leq \bar{w}} \lim_{\rho \rightarrow \rho_0} E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c_1} + \mu_{c_2} - \check{k}(z_2)\sigma_{c_2}\sqrt{1 - \rho^2}}{|\sigma_{c_1} - \rho\sigma_{c_2}|} \right) \right] (\lambda(w_1) - \underline{\pi}) \end{aligned}$$

$$= \max_{w_1 \leq \bar{w}} E_{z_2} \left[ \Phi \left( \frac{w_1 + \mu_{c1} + \mu_{c2} - \check{k}(z_2)\sigma_{c2}\sqrt{1-\rho_0^2}}{|\sigma_{c1} - \rho_0\sigma_{c2}|} \right) \right] (\lambda(w_1) - \pi).$$

We can show the continuity of M's expected profit under control in  $\rho$  in a similar manner. ■

In all numerical experiments with price-only contracts, we have observed that M has strictly greater expected profit with control than with delegation when  $\rho \in (\underline{\rho}, \bar{\rho})$ . This suggests that the condition in Proposition O4 is necessary as well as sufficient for delegation to be optimal under price-only contracts. In some cases,  $\underline{\rho} = \bar{\rho}$  and delegation is always strictly more profitable for M than control.

Next, we show that Proposition O4 holds for quantity discount contracts.

**PROPOSITION O5.** *M has greater expected profit with delegation than with control if  $\rho \in [-1, \underline{\rho}] \cup [\bar{\rho}, 1]$ , where  $-1 < \underline{\rho} \leq \bar{\rho} < 1$ .*

*Proof:* Consider the case with perfect correlation ( $|\rho| = 1$ ). As we have shown in the proof of Proposition O4, this implies that S1 knows  $c_2$  exactly. In this case, he uses a price-only contract and sets  $w_2^d = c_2$  if  $w_1 - c_1 \geq c_2$  and  $w_2^d = \underline{c}_2$  otherwise. Let  $w_S^{k,c}$  ( $k \in \{L, H\}$ ) be the optimal prices under control. By definition and optimality, we have:

$$F_{1,2}(w_S^{k,c}) = \max_x F_1(x)F_2(w_S^{k,c}) \leq F_S(w_S^{k,c}) = E_{c_1}[F_2(w_2^d)]$$

for  $k \in \{L, H\}$ . A trivial comparison of problems  $\mathbf{P}_c^{\text{QDC}}$  and  $\mathbf{P}_d^{\text{QDC}}$  reveals that M achieves a greater profit under delegation. In order to conclude that it is strictly so, note that  $w_S^{H,c} < \bar{c}_1 + \bar{c}_2$  as  $\bar{c}_i$  is unbounded from above for the normal distribution and  $f_i(c_i) > 0$  for all  $c_i \in [\underline{c}_i, \bar{c}_i]$ . In order to complete the proof, it is sufficient to show that M's expected profit functions under delegation and control are continuous in  $\rho$ , as noted in the proof of Proposition O4.

We first observe that the expected probabilities that S2 agrees to supply up to  $Q_2^H$  or  $Q_2^L$  units are continuous in  $\rho$ . The proofs of these claims are similar to the proof in Proposition O4 that the expected probability that S2 agrees to participate in price-only contracts is continuous in  $\rho$ . If these probabilities are continuous in  $\rho$ , a weighted sum of them is continuous as well, where weights are M's profits given the suppliers' decisions. (Since these weights are independent of  $\rho$ , and lim and max and lim and sum operators are interchangeable.) Observe that with this definition of weights, the sum is M's actual expected profit under delegation and control. This completes the proof. ■