Technical Appendices

A-1. Proof for Theorem 1

Use backward induction. Since the distributor's terminal cost function is $V^d_{N+1}(x_d) = -(c^d+\delta-u)x_d$ with a slope of $-(c^d+\delta-u)$, based on Theorem 4.3 of Porteus (2002), a base-stock policy with base-stock level $y^* = \Phi^{-1}(\frac{p^d}{p^d+h^d})$ is optimal for period k for $n \le k \le N$, since $c^d_k = c^d+\delta\ \forall k \ge n$. Considering period $n-1$, we have $G'^d_{n-1}(y^*) = L'^d_{n-1}(y^*) + (c^d-u) - (c^d+\delta-u) \le 0$ since $L'^d_{n-1}(y^*) = 0$, which implies $y^{d*}_{n-1} \ge y^*$ is the optimal stock level for period $n-1$. Next consider period $n-2$, we have $G'^d_{n-2}(y^{d*}_{n-1}) = L'^d_{n-2}(y^{d*}_{n-1}) + c^d - c^d \ge 0$ since $L'^d_{n-2}(y^{d*}_{n-1}) \ge L'^d_{n-2}(y^*) = 0$ which implies $y^{d*}_{n-2} \le y^*_{n-1}$. By solving $G'^d_{n-2}(y^{d*}_{n-2}) = 0$ we get $y^{d*}_{n-2} = y^* = \Phi^{-1}(\frac{p^d}{p^d+h^d})$. Then by backward induction, we can get $y^{d*}_k = \phi^{-1}(\frac{p^d}{p^d+h^d}) = y^*$ for $1 \le k \le n-2$. ∎

A-2. Proof for Theorem 2

Since the total production quantity through the finite horizon does not depend on the stock level, to show the property of the optimal stock level we can omit the production cost $-c(y-x)$ in each period. Thus, we rewrite $V^i_k(x) = \min_{y\ge x}\{G^i_k(y)\}$, where

$$G^i_k(y) = \begin{cases} L(y) + \int_0^\infty V^i_{k+1}(y-D)^+\phi(D)dD & \text{if } i=0^- \text{ or } 0^+ \text{ and } k \ne n-1 \\ L(y-\bar{y}+y^*) + \int_0^\infty V^0_n(y-(D+\bar{y}-y^*))^+\phi(D)dD & \text{if } k=n-1 \\ p_i\left[L^i(y) + \int_0^\infty V^{0^+}_{k+1}(y-\eta_i)^+\theta_i(\eta_i)d\eta_i\right] & \\ \quad +(1-p_i)\left[hy^+ + \int_0^\infty V^{i+D}_{k+1}(y)\phi(D)dD\right] & \text{otherwise.} \end{cases}$$

To prove the optimality of the produce-up-to policy, we will use backward induction again. It can be easily shown that $L(y)$ and $L^i(y)$ are convex by definition. Since we know $V^i_{N+1}(x)$ is convex in x for any state i, we know $G^i_N(y)$ is convex in y by convexity of $V^i_{N+1}(x)$, whether i equals 0^+ or not. Based on the fact that if $f(y)$ is convex then $g(x) = min_{y\ge x}f(y)$ is also convex, we know $V^i_N(x) = \min_{y\ge x}\{G^i_N(y)\}$ is also convex in x for any i. Now assume in period k, $G^i_k(y)$ and $V^i_k(x)$ are convex $\forall i$. We will prove that $G^i_{k-1}(y)$ and $V^i_{k-1}(x)$ are convex $\forall i$.

Consider first the case when $i=0^-$ or 0^+ and $k \ne n$, we have $G^i_{k-1}(y) = L(y) + \int_0^\infty V^i_k(y-D)^+\phi(D)dD$, then

$$\begin{aligned} G'^i_{k-1}(y) &= L'(y) + (\int_0^\infty V^i_k(y-D)^+\phi(D)dD)' \\ &= L'(y) + (\int_0^y V^i_k(y-D)\phi(D)dD)' + (\int_y^\infty V^i_k(0)\phi(D)dD)' \\ &= L'(y) + \int_0^y V'^i_k(y-D)\phi(D)dD \end{aligned}$$

$$
\begin{aligned}
G''^{i}_{k-1}(y) &= L''_(y) + (\int_0^y V'^{i}_k(y-D)\phi(D)dD)' \\
&= L''_(y) + \int_0^y V''^{i}_k(y-D)\phi(D)dD + V'^{i}_k(0)\phi(y) \\
&\geq V'^{i}_k(0)\phi(y)
\end{aligned}
$$

Since $V^i_k(x) = \min_{y\geq x}\{G^i_k(y)\}$, by definition, we can see $V'^{i}_k(x) \geq 0$. So $G''^{i}_{k-1}(y) \geq 0$ and we have shown $G^i_{k-1}(y)$ is convex. Next we will use the similar method to prove that $G^i_{k-1}(y)$ is convex for other state, i and $k \neq n$

$$
\begin{aligned}
G'^{i}_{k-1}(y) &= p_i L'^{i}(y) + p_i \left(\int_0^\infty V^{0^+}_k(y-\eta_i)^+\theta_i(\eta_i)d\eta_i\right)' + (1-p_i)\left(h(y^+)' + \int_0^\infty V'^{i+D}_k(y)\phi(D)dD\right) \\
&= p_i L'^{i}(y) + p_i \int_0^y V'^{0^+}_k(y-\eta_i)\theta_i(\eta_i)d\eta_i + (1-p_i)\left(h(y^+)' + \int_0^\infty V'^{i+D}_k(y)\phi(D)dD\right) \\
G''^{i}_{k-1}(y) &= p_i L''^{i}(y) + p_i \int_0^y V''^{0^+}_k(y-\eta_i)\theta_i(\eta_i)d\eta_i + p_i V'^{0^+}_k(0)\theta_i(y) \\
&\quad +(1-p_i)\left(h(y^+)'' + \int_0^\infty V''^{i+D}_k(y)\phi(D)dD\right) \\
&\geq p_i L''^{i}(y) + p_i V'^{0^+}_k(0)\theta_i(y) \geq 0
\end{aligned}
$$

This proves $G^i_{k-1}(y)$ is convex and hence $V^i_{k-1}(x) = \min_{y\geq x}\{G^i_{k-1}(y)\}$ is also convex $\forall i \neq 0^-, 0^+$ and $k \neq n$.

Finally, consider $k = n$ and $\forall i$, i.e., given that $G^i_n(y)$ and $V^i_n(x)$ are convex $\forall i$, we want to prove that $G^i_{n-1}(y)$ and $V^i_{n-1}(x)$ are convex $\forall i$.

$$
G^i_{n-1}(y) = L(y - \bar{y} + y^*) + \int_0^\infty V^0_n(y - (D + \bar{y} - y^*))^+\phi(D)dD \tag{A-1}
$$

$$
\begin{aligned}
G'^{i}_{n-1}(y) &= L'(y - \bar{y} + y^*) + \left(\int_0^\infty V^0_n(y - (D + \bar{y} - y^*))^+\phi(D)dD\right)' \\
&= L'(y - \bar{y} + y^*) + \int_0^{y-\bar{y}+y^*} V'^{0}_n(y - (D + \bar{y} - y^*))\phi(D)dD \\
&\quad +V^0_n(0)\phi(y - \bar{y} + y^*) - V^0_n(0)\phi(y - \bar{y} + y^*) \\
G''^{i}_{n-1}(y) &= L''(y - \bar{y} + y^*) + \int_0^{y-\bar{y}+y^*} V''^{0}_n(y - (D + \bar{y} - y^*))\phi(D)dD + V'^{0}_n(0)\phi(y - \bar{y} + y^*) \geq 0
\end{aligned}
$$

So, by backward induction, we have shown that $G^i_k(\cdot)$ and $V^i_k(\cdot)$ are convex $\forall i, k$. Hence, the produce-up-to policy is the optimal policy.

Next we prove $y^0 \leq y^1 \leq y^2 \cdots \leq y^{\bar{y}-y^*-1} \leq y^{0^+}$. By applying Theorem 4.3 of Porteus (2002), it is easy to show that after the distributor resumes ordering (i.e., $i = 0^+$), a stationary base-stock level of $y^{0^+} = \Phi^{-1}(\frac{p}{p+h})$ will be the optimal produce-up-to level. Since $\Theta_0(\cdot) \leq_{st} \Theta_1(\cdot) \leq_{st} \cdots \leq_{st} \Theta_{\bar{y}-y^*-1}(\cdot) \leq_{st} \Phi(\cdot)$, the demand from period n to the ending period N is stochastically increasing. Hence, the myopic policy is optimal and $\forall k \geq n$ with state $i = 0, 1, \cdots, \bar{y} - y^* - 1$, the optimal y^i must satisfy $L'^{i}(y^i) + (hy^+)' = 0$. With the property that $\Theta_i \leq_{st} \Theta_{i+1}$, we can solve $L'^{i}(y^i) + (hy^+)' = 0$ to get that $y^0 \leq y^1 \leq y^2 \cdots \leq y^{\bar{y}-y^*-1} \leq y^{0^+}$. ∎

A-3. Proof for Corollary 1

From Theorem 2, we know for $y^{0^+}(\bar{y}_1) = y^{0^+}(\bar{y}_2) = \Phi^{-1}(\frac{p}{p+h})$. For the transitional periods, let $p_i^1 = Prob\{i + D > \bar{y_1} - y^* | i < \bar{y_1} - y^*\}$ and $p_i^2 = Prob\{i + D > \bar{y_2} - y^* | i < \bar{y_2} - y^*\}$. Then we know $p_i^1 \geq p_i^2$ for any i. and $\Theta_i(\cdot|\bar{y}_1) \geq_{st} \Theta_i(\cdot|\bar{y}_2)$ given $\bar{y}_1 \leq \bar{y}_2$ (details omitted). From Theorem 2 we know a myopic policy is optimal during the transitional periods. By using the property that $\Theta_i(\cdot|\bar{y}_1) \geq_{st} \Theta_i(\cdot|\bar{y}_2)$, we get $y^i(\bar{y}_1) \geq y^i(\bar{y}_2)$ for $i \geq 0$. ∎

A-4. Proof for Theorem 3

If $\delta_1 \leq \delta_2$, we have $y^{d*}_{n-1}(\delta_1) \leq y^{d*}_{n-1}(\delta_2)$. We know $y^{0^+}(\delta_1) = y^{0^+}(\delta_2) = \Phi^{-1}(\frac{p}{p+h})$. For the transitional periods, we know $p_i(\delta_1) \geq p_i(\delta_2)$ and $\Psi_i(\cdot|\delta_1) \geq_{st} \Psi_i(\cdot|\delta_2)$ by knowing $y^{d*}_{n-1}(\delta_1) \leq y^{d*}_{n-1}(\delta_2)$. In the Investment Buying model, we have a similar result as Theorem 2 that during the transitional periods, the myopic policy is optimal. By using the property that $\Psi_i(\cdot|\delta_1) \geq_{st} \Psi_i(\cdot|\delta_2)$, we get $y^i(\delta_1) \geq y^i(\delta_2)$ for any $i \geq 0$. ∎

A-5. Extensions

In addition to the above numerical analysis presented in the paper, we investigate four additional issues and their impact on the supply chain profits, pareto-improving fee ranges, and value of information. This study provide us more insights into the industry practice and also help us establish the robustness of our results.

A-5.1 Impact of the Mean Demand

Different products may face different volumes of demand. To separate from the effect of demand variance (which was already shown in section 6), we expanded the computational study by using the demand distributions Erlang (15, 2), Erlang (15, 2)+5, Erlang (15, 2)+10, Erlang (15, 2)+15..., hence keeping the demand variance the same, but capturing mean demand per period to be 35, 40, 45,...through 75 in addition to the 30 that was already in our study. We do this for the case with $\delta = 10\%$ as cases with other δ values have similar insights.

Figures A-1(a) and A-1(b) show how the average percentage of supply chain profit increase of FFS over IB and the pareto-improving fee ranges change with mean demand. Observe that: (1) As the average demand increases, the percentage of improvement of FFS over IB decreases, indicating that, the benefits of FFS model is bigger for products with smaller average demand. This is because the supply chain transparency enabled by FFS is more valuable at lower mean demands as the demand variance then plays a larger role in defining supply chain efficiency. (2) As the mean demand increases, the upper fee decreases, but the lower fee does not change much. This is because the lower fee is related to the break-even compensation to the distributor for not using investment buying, which is directly connected to the mean demand. Thus, as the mean demand increases, the break-even compensation also increases, leaving the *per-unit* lower fee roughly unchanged. On the other hand, the upper fee captures the change in the manufacturer profit which consists of (i) reduction in investment buying; and (ii) value of information. Reduction in investment buying is directly related to the mean demand (as mentioned above) while the value of information is a function of demand variance. Thus, as the mean demand increases, the first quantity increases while the second quantity remains the same, leading to a decreasing trend of the *per-unit* upper fee.

To summarize, given similar demand variance, for products with lower mean demand (potentially more expensive products like bio-pharmaceuticals), the benefit of FFS over IB is higher, distributor could potentially charge a higher fee, and the value of information is roughly the same (figure omitted).

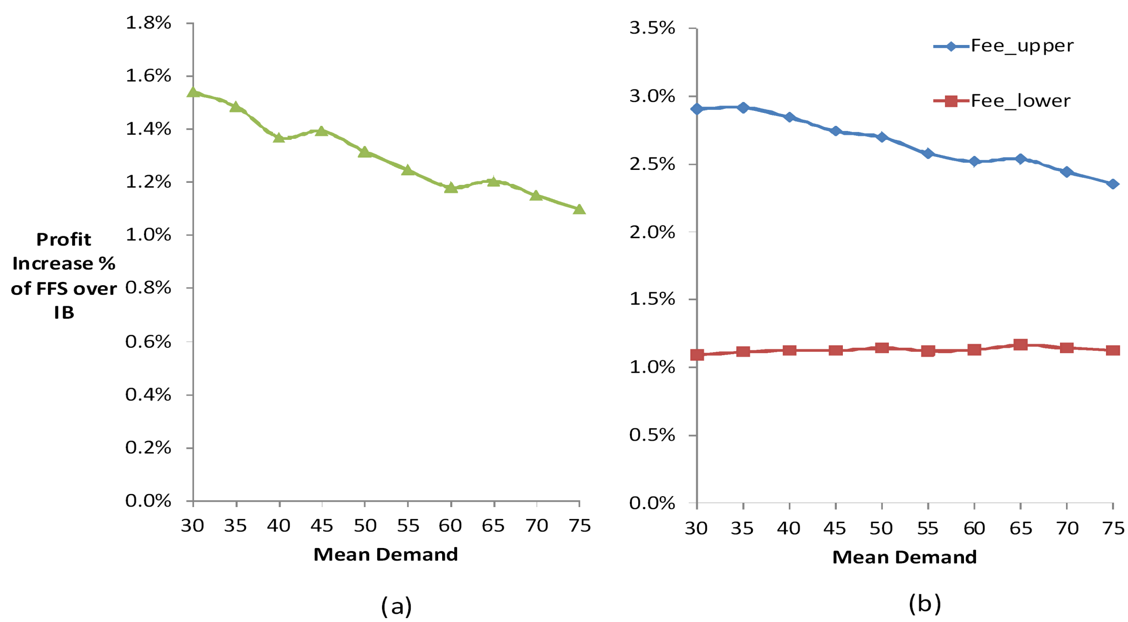

Figure A-1: The impact of mean demand on the profit increase of FFS over IB (a) and the pareto-improving fee ranges (b)

A-5.2 Impact of fixed ordering Cost

Generally speaking, in the pharmaceutical industry, the cost of a distributor placing an order is not significant. Nearly all transactions between a manufacturer and a distributor are conducted using Electronic Data Interchange (EDI). In addition, all pharmaceutical manufacturers use bar codes at the pallet, case, shelf-pack, and individual item levels. This enables very high levels of efficiency in order receipt and handling by distributors. According to HDMA (Healthcare Distribution and Management Association), 100% of distributors use warehouse management systems (WMS) and the distributor operating expenses average only 1.12% of revenues, i.e., very small. Therefore, we do not specifically model ordering cost. But the periodic review set-up still helps us to capture the impact of fixed ordering cost. Specifically, in our periodic review model, an ordering cycle is defined as one period. In practice, the ordering cycle is usually one or two weeks, which is adopted with the balance of holding and penalty costs and the ordering/transaction cost. Thus, given a specific finite horizon, say one year (as used in our numerical study), a decrease in the number of periods in the horizon indicates an increase in the length of an ordering cycle, corresponding to an increase of the ordering cost. Thus, by investigating how the number of periods in the one-year finite horizon affects the supply chain profits, fee ranges, and the value of information, we can see the impact of ordering cost.

Specifically, we re-ran the computational study with N=36, 48, and 60, compared to N=24 in the original study. These correspond to ordering every 1.5, 1.0, and 0.8 weeks (6 days) as compared to every 2 weeks (N=24) that was in the study. We choose these numbers because in practice, the ordering cycle almost never exceeds two weeks.

Fig A-2(a) shows that as N decreases (ordering cost increases), supply chain profit increase of FFS over IB increases, i.e., the benefit of switching from IB to FFS increases. This is because the supply chain is able to better recover from the shock created by the price increase when ordering more frequently, hence the transition to FFS is not that beneficial. Moreover, Fig A-2(b) shows that as the ordering cost increases (N decreases), the upper fee increases and the lower fee decreases, leading to a larger pareto-improving fee range. This is because when ordering cost increases, the distributor's ordering pattern under FFS is closer to that under the IB system. As a result, he needs a smaller fee to compensate him for switching from IB to FFS (i.e. lower fee decreases). Similarly, this effect would lead the manufacturer to offering a smaller fee to the retailer when switching from IB to FFS. On the other hand, as N decreases, the value of information to the manufacturer increases (see Fig A-2c), enabling the manufacturer to offer a larger fee to the distributor. This latter effect outweighs the former effect, hence leading to an increase in the upper fee when ordering cost increases (N decreases).

In summary, for companies with a higher fixed ordering cost, there is a bigger benefit switching from IB to FFS. In addition, the upper fee will be higher while the lower fee will be lower, leading to a larger pareto-improving fee range. Finally, the company would forego a bigger benefit when not using the inventory information shared by the distributor.

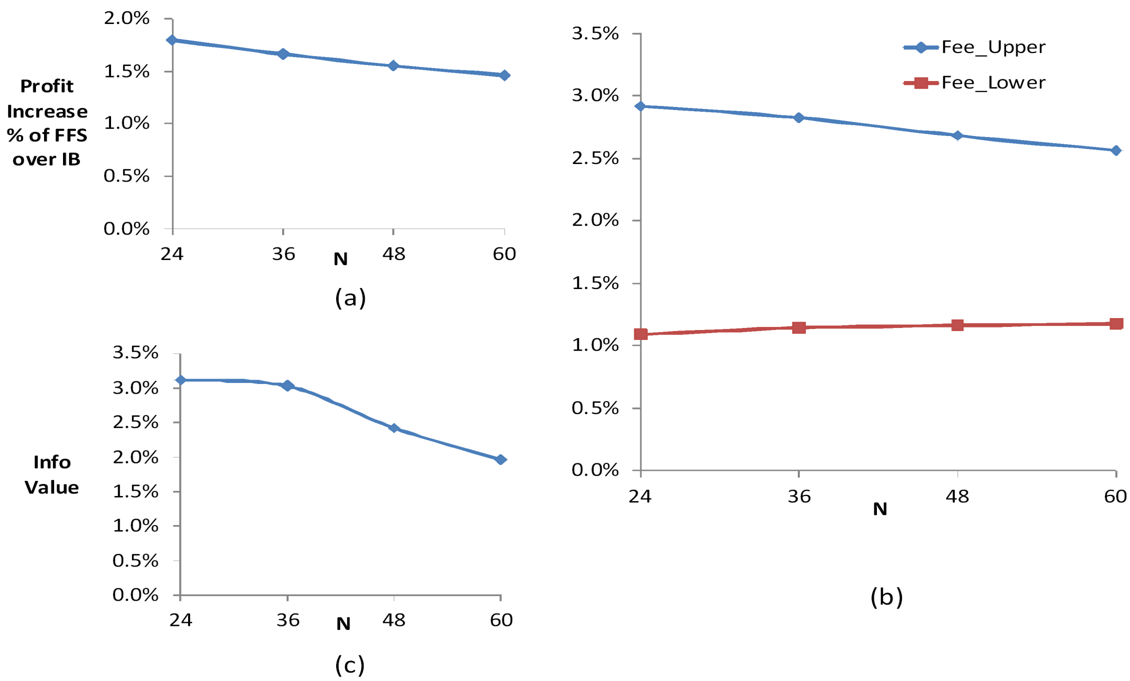

Figure A-2: The impact of fixed cost on the profit increase of FFS over IB (a), the pareto-improving fee ranges (b), and the value of information (c)

A-5.3 Impact of Holding and Penalty Cost Increase after price increase

In the previous sections, we have assumed that holding and penalty costs are not changed after price increase. In reality, these costs will be affected by price increase. In this section, we ran further numerical experiments on the impact of holding and penalty cost change after price increase. The change of the holding and penalty costs for both the manufacturer and the distributor are based on the price change δ and their relationship to WAC. For example, when $\delta = 15\%$, h, p, h_d, p_d all increase 15% after price increase. Theoretically, holding costs should only increase for products purchased at higher price, yet tracking products based on purchase prices is very difficult. Hence, in the computational study, holding and penalty costs are increased for all products after price increase in period n. By doing this, we overestimate the impact of the holding cost hence our results provide an upper-bound of such impact. We chose to run experiments for the cases with $\delta = 15\%$ since these are the cases with the maximum change to the holding and penalty costs in the study, hence would again provide an upper bound of this impact.

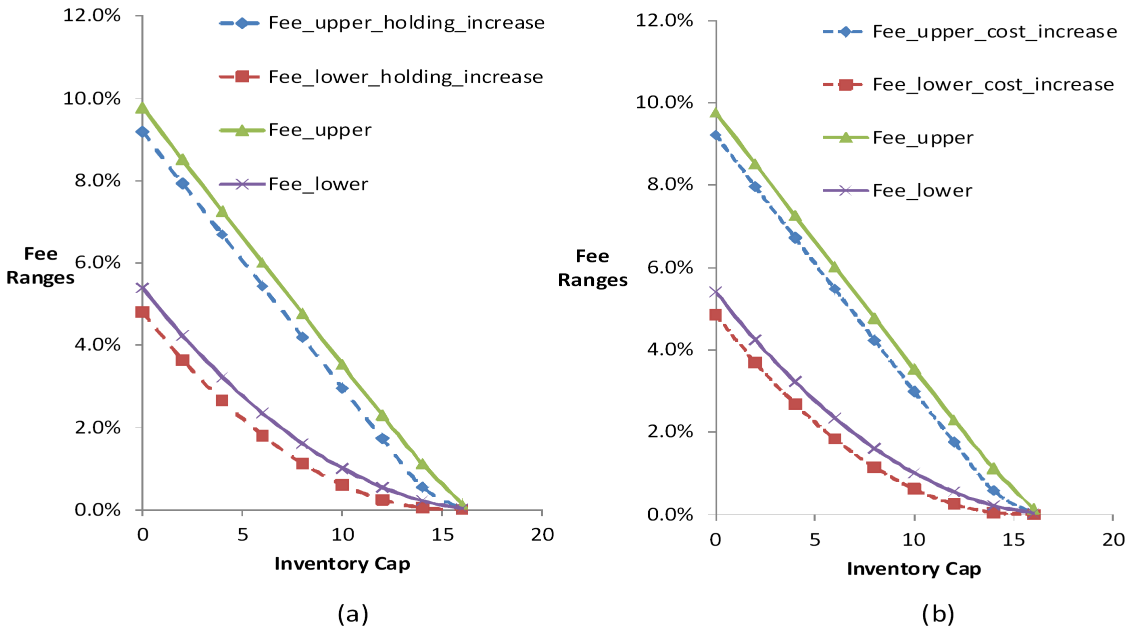

Figure A-3: Pareto-improving fee ranges with only holding cost increase (a) and with holding and penalty cost increases (b)

To see the impact of holding cost and penalty cost, we re-ran the experiments first changing only the holding cost after price increase (penalty cost remained the same). Fig A-3(a) shows that when considering only holding cost increase, both the upper fee and lower fee decrease slightly (by at most 0.6%). This is because distributors will order less in investment buying anticipating holding cost increases after price increase. Hence, the manufacturer needs to compensate the distributor less for switching from IB to FFS (upper and lower fees decrease). On the contrary, when penalty cost increases after the price increase, distributors will order more in investment buying (due to the greater penalty during the transitional periods and the following periods if they stock out), leading the upper and lower fees to increase, opposite to the effect of holding cost. Since there is only a small chance of incurring penalty cost (due to the high service level), the effect of holding cost outweighs that of the penalty cost, hence the overall effect will have similar trend as the holding cost. Fig A-3(b) confirms this, showing that considering both the holding and penalty cost increase, the upper and lower fees decrease slightly (by most 0.5%).

In summary, given these results provide an upper bound, the impact of the increase of holding and penalty costs after price increase seems to be not very significant (fees decrease by at most 0.5% and the value of information remains almost the same (figure not shown)).

A-5.4 Different Demand Distributions

In this section, we investigate whether our results hold true for distributions other than the Erlang and Exponential distributions. Specifically, we expand our study to include uniform and normal demands. For uniform distribution, we ran experiments on demand following $U[10, 50]$ and $U[20, 40]$; for normal distribution, we ran experiments on demand following $N(30, 5)$ and $N(30, 10)$. All distributions have a mean demand of 30 and we ran all experiments with $WAC = 200$, $\delta = 10\%$, and $\bar{y}$ levels starting from y^* until it reaches y^{d*}_{n-1}. Our results show that supply chain profit increases when switching from IB to FFS for all the distributions and the average profit increase of FFS over IB is comparable for all three distributions that we tested, i.e., 1.48% for normal distribution, 1.46% for the uniform distribution, and 1.53% for the Erlang distributions in the original study. This result is not surprising since imposing an inventory cap increases the efficiency of the supply chain by reducing excessive inventory without sacrificing the service level. Hence, we expect our results also hold for other demand distributions.