Online Supplement

Result A1. (Wu et al. 2011) The optimal capacities for the IDM and foundry to maximize the system's expected profit, $K_c = (K_s, K_f)$, are summarized as follows: If $\frac{p-c_f}{v_f} > \frac{p-c_s}{v_s}$, $K_c = \left(0, F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)\right)$. If $\frac{p-c_f}{v_f} \le \frac{p-c_s}{v_s}$, $K_c = \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right), F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)\right)$.

Proposition A1. *For an $\alpha-$contract (w,α), the capacity equilibrium is as follows.*

(a) Assume $w > c_s$ and $0 \le \alpha < 1$ hold.

(a1) Assume $\frac{p-c_s}{v_s} v_f + c_f \ge c_s + v_s$ holds. Then:

(a1.1) $\Omega^* = \arg\max\left[Z_s(\Omega_1), Z_s(\Omega_2)\right]$, *if* $w \in (\max[c_s, c_f + v_f],\ c_s + v_s]$,

(a1.2) $\Omega^* = \arg\max\left[Z_s(\Omega_2), Z_s(\Omega_3)\right]$, *if* $w \in \left(c_s + v_s, \min\left[\frac{p-c_s}{v_s} v_f + c_f,\ p\right]\right]$ *and* $\frac{w-c_f-v_f}{w-c_f} \ge \frac{w-c_s-v_s}{w-c_s}$,

(a1.3) $\Omega^* = \Omega_2$, *if* $w \in \left(c_s + v_s, \min\left[\frac{p-c_s}{v_s} v_f + c_f,\ p\right]\right]$ *and* $\frac{w-c_f-v_f}{w-c_f} < \frac{w-c_s-v_s}{w-c_s}$,

(a1.4) $\Omega^* = \Omega_3$, *if* $w \in \left(\min\left[\frac{p-c_s}{v_s} v_f + c_f,\ p\right], p\right]$.

(a2). Assume $\frac{p-c_s}{v_s} v_f + c_f < c_s + v_s$ holds. Then:

(a2.1) $\Omega^* = \arg\max\left[Z_s(\Omega_1), Z_s(\Omega_2)\right]$, *if* $w \in \left(\max[c_s, c_f + v_f],\ \max\left[c_s, \frac{p-c_s}{v_s} v_f + c_f\right]\right]$,

(a2.2) $\Omega^* = \Omega_1$, *if* $w \in \left(\max\left[c_s, \frac{p-c_s}{v_s} v_f + c_f\right],\ c_s + v_s\right]$,

(a2.3) $\Omega^* = \Omega_3$, *if* $w \in (c_s + v_s,\ p]$.

(b) Assume $\alpha = 1$ or $w \le c_s$ holds (the foundry first case).

(b1) $\Omega^* = \Omega_2|_{\alpha=1}$, *if* $w \in \left(c_f + v_f,\ \min\left[\frac{p-c_s}{v_s} v_f + c_f,\ p\right]\right]$,

(b2) $\Omega^* = \Omega_1$, *if* $w \in \left(\min\left[\frac{p-c_s}{v_s} v_f + c_f,\ p\right],\ p\right]$.

Proposition A2. *Assume $w > c_s + r$ holds.*

(a) If $\frac{p-w}{p-w+r} \ge \frac{w-c_f-v_f}{w-c_f}$, there are five possible capacity and capacity reservation equilibria: $\Theta_i, i = 1, 2, ..., 5$.

(a.1) If $w \le c_s + v_s$, $\frac{p-w}{p-w+r} \ge \frac{p-c_s-v_s}{p-c_s}$ is implied, then $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_2)\right]$.

(a.2) If $w > c_s + v_s$ and $\frac{w-c_f-v_f}{w-c_f} > \frac{w-c_s-v_s}{w-c_s-r}$, there are two cases.

(a.2.1) $\frac{p-w}{p-w+r} \ge \frac{p-c_s-v_s}{p-c_s}$: $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_4)\right]$.

(a.2.2) $\frac{p-w}{p-w+r} < \frac{p-c_s-v_s}{p-c_s}$: $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_4), Z_s(\Theta_5)\right]$.

(a.3) If $w > c_s + v_s$ and $\frac{w-c_f-v_f}{w-c_f} \le \frac{w-c_s-v_s}{w-c_s-r}$, there are four cases.

(a.3.1) $\frac{w-c_s-v_s}{w-c_s} < \frac{w-c_f-v_f}{w-c_f} \le \frac{p-w}{p-w+r} < \frac{w-c_s-v_s}{w-c_s-r}$: $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_5)\right]$.

(a.3.2) $\frac{w-c_s-v_s}{w-c_s} < \frac{w-c_f-v_f}{w-c_f} \le \frac{w-c_s-v_s}{w-c_s-r} \le \frac{p-w}{p-w+r}$: $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_4)\right]$.

(a.3.3) $\frac{w-c_f-v_f}{w-c_f} \le \frac{w-c_s-v_s}{w-c_s} < \frac{w-c_s-v_s}{w-c_s-r} \le \frac{p-w}{p-w+r}$: $\Theta^* = \Theta_4$.

(a.3.4) $\frac{w-c_f-v_f}{w-c_f} \le \left(\frac{w-c_s-v_s}{w-c_s}, \frac{p-w}{p-w+r}\right) < \frac{w-c_s-v_s}{w-c_s-r}$: $\Theta^* = \Theta_5$.

(b) If $\frac{p-w}{p-w+r} < \frac{w-c_f-v_f}{w-c_f}$, there are three possible capacity and capacity reservation equilibria: $\Theta_i, i = 1, 3, 5$.

(b.1) If $w \le c_s + v_s$, then $\Theta^* = \Theta_1$.

(b.2) If $w > c_s + v_s$, then there are three cases.

(b.2.1) $\frac{w-c_f-v_f}{w-c_f} \in \left[\frac{p-c_s-v_s}{p-c_s}, \infty\right)$: $\Theta^* = \Theta_3$.

(b.2.2) $\frac{w-c_f-v_f}{w-c_f} \in \left[\frac{w-c_s-v_s}{w-c_s}, \frac{p-c_s-v_s}{p-c_s}\right)$: $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_5)\right]$.

(b.2.3) $\frac{w-c_f-v_f}{w-c_f} \in \left[0, \frac{w-c_s-v_s}{w-c_s}\right)$: $\Theta^* = \Theta_5$.

PROPOSITION A3. *Assume $c_s < w \le c_s + r$ holds. The capacity and capacity reservation equilibrium is as follows.*

(a) If $\frac{p-c_s-v_s}{p-c_s} \ge \frac{w-c_f-v_f}{w-c_f} \ge \frac{w-c_s-v_s}{w-c_s}$, *then there are three cases.*

(a.1) If $w > c_s + v_s$, *then* $\Theta^* = \arg\max\left[Z_s(\Theta_3), Z_s(\Theta_5)\right]$.

(a.2) If $w \le c_s + v_s$ *and* $w < c_s + r$, *then there are three cases.*

(a.2.1) $\frac{c_s+v_s-w}{c_s+r-w} \le \frac{w-c_f-v_f}{w-c_f}$: $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_6)\right]$.

(a.2.2) $\frac{w-c_f-v_f}{w-c_f} < \frac{c_s+v_s-w}{c_s+r-w} \le \frac{p-c_s-v_s}{p-c_s}$: $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_6)\right]$.

(a.2.3) $\frac{p-c_s-v_s}{p-c_s} < \frac{c_s+v_s-w}{c_s+r-w}$: $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_2)\right]$.

(a.3) If $w \le c_s + v_s$ *and* $w = c_s + r$, *then* $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_2)\right]$.

(b) If $\frac{p-c_s-v_s}{p-c_s} \ge \frac{w-c_s-v_s}{w-c_s} \ge \frac{w-c_f-v_f}{w-c_f}$, *then* $\Theta^* = \Theta_5$.

(c) If $\frac{w-c_f-v_f}{w-c_f} \ge \frac{p-c_s-v_s}{p-c_s} \ge \frac{w-c_s-v_s}{w-c_s}$, *then there are two cases.*

(c.1) If $w > c_s + v_s$, *then* $\Theta^* = \Theta_3$.

(c.2) If $w \le c_s + v_s$, *then there are two cases.*

(c.2.1) $\frac{p-w}{p-w+r} < \frac{w-c_f-v_f}{w-c_f}$, *then* $\Theta^* = \Theta_1$.

(c.2.2) $\frac{p-w}{p-w+r} \ge \frac{w-c_f-v_f}{w-c_f}$, *then* $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_2)\right]$.

PROPOSITION A4. *Assume $w \le c_s$ holds. The capacity and capacity reservation equilibrium is as follows.*

(a) If $\frac{w-c_f-v_f}{w-c_f} \le \frac{p-c_s-v_s}{p-c_s}$, *then*

(a.1) $\Theta^* = \arg\max\left[Z_s(\Theta_2), Z_s(\Theta_7)\right]$, *for* $\frac{p-c_s-v_s}{p-c_s} < \frac{c_s+v_s-w}{c_s+r-w}$,

(a.2) $\Theta^* = \arg\max\left[Z_s(\Theta_6), Z_s(\Theta_7)\right]$, *for* $\frac{w-c_f-v_f}{w-c_f} < \frac{c_s+v_s-w}{c_s+r-w} \le \frac{p-c_s-v_s}{p-c_s}$,

(a.3) $\Theta^* = \Theta_7$, *for* $\frac{c_s+v_s-w}{c_s+r-w} \le \frac{w-c_f-v_f}{w-c_f}$.

(b) If $\frac{w-c_f-v_f}{w-c_f} > \frac{p-c_s-v_s}{p-c_s}$, *then*

(b.1) $\Theta^* = \Theta_1$, *for* $\frac{p-w}{p+r-w} \le \frac{w-c_f-v_f}{w-c_f}$,

(b.2) $\Theta^* = \arg\max\left[Z_s(\Theta_1), Z_s(\Theta_2)\right]$, *for* $\frac{p-w}{p+r-w} > \frac{w-c_f-v_f}{w-c_f}$.

Proof of Proposition 1. (a) Assume $w > c_s$. Consider $0 < \alpha < 1$ first. If $K_f \le \frac{\alpha}{1-\alpha} K_s$, then the foundry's and the IDM's profits under different demand realizations are represented as follows.

Realized Demand	IDM's Profit	Foundry's Profit
$d \in \left(0, \frac{K_f}{\alpha}\right]$	$pd - c_s(1-\alpha)d - w\alpha d - K_s v_s$,	$(w - c_f)\alpha d - v_f K_f$,
$d \in \left(\frac{K_f}{\alpha}, K_s + K_f\right]$	$pd - c_s(d - K_f) - wK_f - K_s v_s$,	$(w - c_f)K_f - v_f K_f$,
$d \in (K_s + K_f,\ \infty]$	$p(K_s + K_f) - c_s K_s - wK_f - K_s v_s$,	$(w - c_f)K_f - v_f K_f$.

Integrating across three intervals leads to Equations (1) and (2).

If $K_f \ge \frac{\alpha}{1-\alpha} K_s$, then the expected profits are derived similarly.

If $\alpha = 0$, then it is straightforward to verify that the expected profits are expressed as (3) and (4) with $\alpha = 0$. Likewise, if $\alpha = 1$, then they are expressed as (1) and (2) with $\alpha = 1$. (b) $w \le c_s$ is the foundry first case. The corresponding expected profits are the same as that in $\alpha(1)-$contract. ■

Proof of Lemma 1. (a) Assume that $w > c_s$ and $0 < \alpha < 1$ hold.

Consider firstly $K_s \in [(1-\alpha) K_w,\ \infty)$. For $K_f \in \left[0, \frac{\alpha}{1-\alpha} K_s\right]$, using (1), we have $\frac{dZ_f}{dK_f} = (w - c_f - v_f) - (w - c_f) F\left(\frac{K_f}{\alpha}\right)$, and $\frac{d^2 Z_f}{dK_f^2} < 0$. Therefore, the local optimum in this interval is verified as $K_f = \alpha K_w$ by concavity. For $K_f \in \left[\frac{\alpha}{1-\alpha} K_s, \infty\right)$, i.e., $K_f \geq \frac{\alpha}{1-\alpha} K_s \geq \frac{\alpha}{1-\alpha} \cdot (1-\alpha) K_w = \alpha K_w$, $K_s + K_f \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. For $K_f \geq \frac{\alpha}{1-\alpha} K_s$, using (3) we have $\frac{dZ_f}{dK_f} = -(w - c_f) F(K_s + K_f) + (w - c_f - v_f) \leq 0$ over interval $\left[\frac{\alpha}{1-\alpha} K_s, \infty\right)$. Therefore, for $K_s \in [(1-\alpha) K_w,\ \infty)$, $K_f^* = \alpha K_w$ by combining the analysis over the two intervals.

Consider secondly $K_s \in [0,\ (1-\alpha) K_w]$. For $K_f \in \left[0, \frac{\alpha}{1-\alpha} K_s\right]$, $K_f \leq \frac{\alpha}{1-\alpha} K_s \leq \frac{\alpha}{1-\alpha} (1-\alpha) K_w = \alpha K_w$, and thus $\frac{dZ_f}{dK_f} = (w - c_f - v_f) - (w - c_f) F\left(\frac{K_f}{\alpha}\right) \geq 0$ over interval $\left[0, \frac{\alpha}{1-\alpha} K_s\right]$. If $K_f \in \left[\frac{\alpha}{1-\alpha} K_s, \infty\right]$, then $\frac{dZ_f}{dK_f} = (w - c_f - v_f) - (w - c_f) F(K_s + K_f)$, and thus the local optimum $K_f = K_w - K_s$ by concavity. Therefore, for $K_s \in [0,\ (1-\alpha) K_w]$, by combining the analysis over the two intervals, we obtain $K_f^* = K_w - K_s$.

Assume $w > c_s$ and $\alpha = 0$ hold. Then using (3) with $\alpha = 0$, $\frac{dZ_f}{dK_f} = -(w - c_f) F(K_s + K_f) + (w - c_f - v_f)$. Therefore, $K_f^* = K_w - K_s$ for $K_s \in [0,\ K_w]$. $K_f^* = 0$ otherwise. Assume that $w > c_s$ and $\alpha = 1$ hold. Then using (1) with $\alpha = 1$, we have $\frac{dZ_f}{dK_f} = (w - c_f - v_f) - (w - c_f) F(K_f)$. Hence, $K_f^* = K_w$.

(b) Assume $w \leq c_s$ holds. This is the foundry first case. Same as the case of $\alpha = 1$, $K_f^* = K_w$. ■

Proof of Proposition 2. Based on the foundry's capacity decision in Lemma 1, Z_s in Proposition 1 for the case of $w > c_s$ can be rewritten as

$$Z_s = \begin{cases} (p - c_s - \alpha(w - c_s)) \int_0^{\frac{K_s}{1-\alpha}} \overline{F(t)} dt + (p - w) \int_{\frac{K_s}{1-\alpha}}^{K_w} \overline{F(t)} dt - v_s K_s, & \text{for } \alpha \leq \frac{K_w - K_s}{K_w}, \\ (p - c_s) \int_0^{K_s + \alpha K_w} \overline{F(t)} dt - \alpha(w - c_s) \int_0^{K_w} \overline{F(t)} dt - v_s K_s, & \text{for } \alpha \geq \frac{K_w - K_s}{K_w}. \end{cases}.$$

Over each segment, the IDM chooses α and K_s to maximize Z_s. Mathematically, it does not matter whether α or K_s is optimized first.

(1) Over the segment of $\alpha \leq \frac{K_w - K_s}{K_w}$ (referred to as the first segment), for any given K_s, $\frac{dZ_s}{d\alpha} = -(w - c_s) \int_0^{\frac{K_s}{1-\alpha}} \overline{F(t)} dt + (w - c_s) \overline{F}\left(\frac{K_s}{1-\alpha}\right) \frac{K_s}{1-\alpha} \leq -(w - c_s) \frac{K_s}{1-\alpha} \overline{F}\left(\frac{K_s}{1-\alpha}\right) + (w - c_s) \overline{F}\left(\frac{K_s}{1-\alpha}\right) \frac{K_s}{1-\alpha} = 0$. Thus, $\alpha = 0$ is weakly optimal over the segment of $\alpha \leq \frac{K_w - K_s}{K_w}$. Specifically, $\frac{dZ_s}{d\alpha} = 0$ for $K_s = 0$ and $\frac{dZ_s}{d\alpha} < 0$ for $K_s > 0$.

Given $\alpha = 0$, $\frac{dZ_s}{dK_s} = (w - c_s - v_s) - (w - c_s) F(K_s)$.

If $w \leq c_s + v_s$, $\frac{dZ_s}{dK_s} < 0$ and $K_s = 0$. Since Z_s with $K_s = 0$ in the first segment is equal to Z_s with $\alpha = 1$ and $K_s = 0$ in the second segment (from the expression of Z_s), then the first segment is weakly dominated by the second segment when $w \leq c_s + v_s$.

If $w > c_s + v_s$, $K_s = \min[F^{-1}\left(\frac{w - c_s - v_s}{w - c_s}\right), (1-\alpha) K_w]$, and $\alpha = 0$ is strictly optimal over the segment of $\alpha \leq \frac{K_w - K_s}{K_w}$.

(2) Over the segment of $\alpha \geq \frac{K_w - K_s}{K_w}$ (referred to as the second segment), suppose α is determined first, and $K_s(\alpha)$ is chosen secondly to maximize Z_s. Then $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} = \frac{\partial Z_s}{\partial \alpha}|_{K_s(\alpha)} + \frac{dK_s(\alpha)}{d\alpha} \frac{\partial Z_s}{\partial K_s}|_{K_s(\alpha)}$, where $\frac{\partial Z_s}{\partial K_s} = (p - c_s - v_s) - (p - c_s) F(K_s + \alpha K_w)$. Over the segment of $\alpha \geq \frac{K_w - K_s}{K_w}$, $F(K_s + \alpha K_w) \geq \frac{w - c_f - v_f}{w - c_f}$.

(2.1) If $\frac{w - c_f - v_f}{w - c_f} > \frac{p - c_s - v_s}{p - c_s}$ (i.e., $w > c_f + \frac{v_f(p - c_s)}{v_s}$), $\frac{\partial Z_s}{\partial K_s} < 0$ and $K_s = 0$. Once $K_s = 0$, the second segment $\alpha \geq \frac{K_w - K_s}{K_w}$ becomes $\alpha \geq 1$, the first segment becomes $\alpha \leq 1$. So, the second segment is weakly dominated by the first segment.

(2.2) If $\frac{w - c_f - v_f}{w - c_f} \leq \frac{p - c_s - v_s}{p - c_s}$ (i.e., $w \leq c_f + \frac{v_f(p - c_s)}{v_s}$), K_s is chosen so that $\frac{\partial Z_s}{\partial K_s} = 0$, i.e., $K_s = K_p - \alpha K_w$. Thus, $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} = \frac{\partial Z_s}{\partial \alpha}|_{K_s(\alpha)} = -(w - c_s - v_s) K_w + (w - c_s) \int_o^{K_w} F(t)\, dt = (w - c_s) K_w \left(\frac{-(w - c_s - v_s)}{w - c_s} + I_f\right)$, which is a constant. If $c_s < w \le c_s + v_s$, then $\frac{dZ_s}{d\alpha} > 0$ and $\alpha = 1$ over this segment. If $w > c_s + v_s$, then $\frac{dZ_s}{d\alpha} > 0$ and $\alpha = 1$ for $I_f > \frac{w-c_s-v_s}{w-c_s}$ and $\frac{dZ_s}{d\alpha} \le 0$ for $I_f \le \frac{w-c_s-v_s}{w-c_s}$. The monotonicity of Z_s in α over the second segment, coupled with that Z_s decreases in α over the first segment, implies $\alpha^* = 0$ or 1.

Given $\alpha = 1$, $K_f = K_w$ and $K_s = \max[K_p - K_w, 0]$, so $\Omega^* = (\max[K_p - K_w, 0], K_w)$.

Given $\alpha = 0$, we examine the IDM's optimal capacity conditional on $K_s \le K_w$ and conditional on $K_s \ge K_w$, respectively. If conditional on $K_s \le K_w$, $K_s = \min[F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_w]$ as derived in (1) and $K_f = \max[K_w - F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), 0]$. If conditional on $K_s \ge K_w$, $\alpha \ge \frac{K_w - K_s}{K_w}$ always holds since $\alpha \ge 0$. As derived in (2), if $\frac{w-c_f-v_f}{w-c_f} > \frac{p-c_s-v_s}{p-c_s}$, $\frac{dZ_s}{dK_s} < 0$; if $\frac{w-c_f-v_f}{w-c_f} \le \frac{p-c_s-v_s}{p-c_s}$, $K_s = K_p$ and $K_f = 0$. Thus, given $\alpha = 0$, $\Omega^* = \left(F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_w - F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)\right)$ if $\frac{w-c_f-v_f}{w-c_f} > \frac{p-c_s-v_s}{p-c_s}$; $\Omega^* = \left(F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_w - F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)\right)$ or $(K_p, 0)$ if $\frac{w-c_f-v_f}{w-c_f} \le \frac{p-c_s-v_s}{p-c_s}$.

(3) Next, we combine the two segments of α. We consider two cases of $c_s + v_s > c_f + \frac{v_f(p-c_s)}{v_s}$ and $c_s + v_s \le c_f + \frac{v_f(p-c_s)}{v_s}$, respectively.

(3.1) $c_s + v_s > c_f + \frac{v_f(p-c_s)}{v_s}$:

If $w > c_s + v_s$, then $\alpha = 0$ is a local optimum over the first segment and the second segment is weakly dominated by the first segment, so $\alpha^* = 0$. If $w \le c_f + \frac{v_f(p-c_s)}{v_s}$, the first segment is weakly dominated by the second segment and $\alpha = 1$ is a local optimum over the second segment, so $\alpha^* = 1$. If $c_s + v_s \ge w > c_f + \frac{v_f(p-c_s)}{v_s}$, (1) and (2) imply $K_s = 0$ for any α, so the value of α is irrelevant. Without loss of generality, we set α to either 0 or 1.

(3.2) $c_s + v_s \le c_f + \frac{v_f(p-c_s)}{v_s}$:

If $w > c_f + \frac{v_f(p-c_s)}{v_s}$, then $\alpha = 0$ is a local optimum over the first segment and the second segment is weakly dominated by the first segment, so $\alpha^* = 0$. If $w \le c_s + v_s$, the first segment is weakly dominated by the second segment and $\alpha = 1$ is a local optimum over the second segment, so $\alpha^* = 1$.

If $c_f + \frac{v_f(p-c_s)}{v_s} \ge w > c_s + v_s$, we analyze as follows.
$Z_s|_{\alpha=1} = Z_s|_{\alpha=1, K_s=K_p-K_w, K_f=K_w} = (p-w)\int_0^{K_w} \overline{F(t)}dt + (p-c_s)\int_{K_w}^{K_p} \overline{F(t)}dt - v_s(K_p - K_w)$,
$Z_s|_{\alpha=0} = \max\left[Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}, Z_s|_{\alpha=0} = Z_s|_{\alpha=0, K_s=K_p, K_f=0}\right]$, where
$Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s} = (p-c_s)\int_0^{F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)} \overline{F(t)}dt + (p-w)\int_{F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)}^{K_w} \overline{F(t)}dt$
$- v_s F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)$, and $Z_s|_{\alpha=0, K_s=K_p, K_f=0} = (p-c_s)\displaystyle\int_0^{K_p} \overline{F(t)}dt - v_s K_p$.
$d\left(Z_s|_{\alpha=1} - Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}\right)/dw = \frac{dK_w}{dw}\left(-(p-c_s)\frac{v_f}{w-c_f} + v_s\right)$. $\frac{dK_w}{dw} > 0$ and $w < c_f + \frac{v_f(p-c_s)}{v_s}$ imply $-(p-c_s)\frac{v_f}{w-c_f} + v_s < 0$, so $d\left(Z_s|_{\alpha=1} - Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}\right)/dw < 0$. Further, it can be shown $Z_s|_{\alpha=1} < Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}$ at $w = c_f + \frac{v_f(p-c_s)}{v_s}$ and $Z_s|_{\alpha=1} > Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}$ at $w = c_s + v_s$. By the intermediate value theorem, there exists a threshold $\hat{w} \in \left(c_s + v_s, c_f + v_f + \frac{v_f(p-c_s-v_s)}{v_s}\right)$ so that $Z_s|_{\alpha=1} - Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s} > 0$ for $c_s + v_s < w < \hat{w}$ and $Z_s|_{\alpha=1} - Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s} < 0$ for $\hat{w} < w < c_f + v_f + \frac{v_f(p-c_s-v_s)}{v_s}$.

Since $Z_s|_{\alpha=0} \ge Z_s|_{\alpha=0, K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), K_f=K_w-K_s}$, $Z_s|_{\alpha=1} - Z_s|_{\alpha=0} < 0$ for $\hat{w} < w < c_f + v_f + \frac{v_f(p-c_s-v_s)}{v_s}$. For $c_s + v_s < w < \hat{w}$, we still need to compare $Z_s|_{\alpha=1}$ and $Z_s|_{\alpha=0, K_s=K_p, K_f=0}$. $Z_s|_{\alpha=1} - Z_s|_{\alpha=0, K_s=K_p, K_f=0} = K_w(w-c_s)\left(I_f - \frac{w-c_s-v_s}{w-c_s}\right)$. Thus, $Z_s|_{\alpha=1} > Z_s|_{\alpha=0, K_s=K_p, K_f=0}$ if $I_f > \frac{w-c_s-v_s}{w-c_s}$

and $Z_s|_{\alpha=1} < Z_s|_{\alpha=0,K_s=K_p,K_f=0}$ if $I_f < \frac{w-c_s-v_s}{w-c_s}$. This, together with the result that $Z_s|_{\alpha=1} - Z_s|_{\alpha=0,K_s=F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right),K_f=K_w-K_s} > 0$ for $c_s+v_s < w < \hat{w}$, implies $Z_s|_{\alpha=1} > Z_s|_{\alpha=0}$ if $I_f > \frac{w-c_s-v_s}{w-c_s}$ and $Z_s|_{\alpha=1} < Z_s|_{\alpha=0}$ if $I_f < \frac{w-c_s-v_s}{w-c_s}$ for $c_s+v_s < w < \hat{w}$.

Combining (3.1) and (3.2) completes the proof. ■

Proof of Proposition 3. According to Proposition 2, $\alpha^* = 0$ or 1. If $\alpha^* = 0$, then $\alpha-$contract degenerates to wholesale price contract (with a single parameter w) in the horizontal setting, which cannot coordinate the centralized capacity profile, as shown in Wu et al. (2011). If $\alpha^* = 1$, then $\alpha-$contract can coordinate the production decision only when $c_s > c_f$. If $MOC_f > MOC_s$, the centralized system has only the foundry build capacity $F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$. However, for $w < p$, the foundry's optimal capacity K_w is strictly less than $F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$. Thus, for $MOC_f > MOC_s$, $\alpha(1)-$contract with $w < p$ cannot coordinate the centralized capacity profile. For $MOC_s \geq MOC_f$, $\alpha(1)-$contract with $w = \frac{v_f c_s - c_f v_s}{v_f - v_s}$ can coordinate the centralized capacity investment and production decisions, because it leads to $K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) = F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)$ and $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)$, which are centralized capacity decisions. However, the IDM's expected profit under this coordinating contract is less than that when he relies on his own capacity, as shown below.

Denote the IDM's expected profit when he relies on his own capacity as Z_s^i. $Z_s^i = (p-c_s)\int_0^{K_s^i}(1-F(x))dx - v_s K_s$, where $K_s^i = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)$. Denote the IDM's expected profit under the $\alpha(1)-$contract with $w = \frac{v_f c_s - c_f v_s}{v_f - v_s}$ as $Z_s^{\alpha 1}$. $Z_s^{\alpha 1} = (p-w)\int_0^{K_f^{\alpha 1}}(1-F(x))dx + (p-c_s)\int_{K_f^{\alpha 1}}^{K_s^{\alpha 1}+K_f^{\alpha 1}}(1-F(x))dx - v_s K_s^{\alpha 1}$, where $K_f^{\alpha 1} = F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)$, $K_s^{\alpha 1} = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)$. $Z_s^{\alpha 1} - Z_s^i = -(w-c_s)\int_0^{K_f^{\alpha 1}}(1-F(x))dx - v_s K_s^{\alpha 1} + v_s K_s^i = -(w-c_s)\int_0^{K_f^{\alpha 1}}(1-F(x))dx + v_s K_f^{\alpha 1} = -\left(\frac{v_f c_s - c_f v_s}{v_f - v_s} - c_s\right)\int_0^{K_f^{\alpha 1}}(1-F(x))dx + v_s K_f^{\alpha 1} = \frac{v_s}{v_f - v_s}(v_f + c_f - v_s - c_s) K_f^{\alpha 1} + \frac{v_s}{v_f - v_s}\left((c_s - c_f)\int_0^{K_f^{\alpha 1}} F(x)\, dx\right)$. The conditions of $c_s > c_f$ and $MOC_s \geq MOC_f$ imply $v_f > v_s$. $\int_0^{K_f^{\alpha 1}} F(x)\, dx < F\left(K_f^{\alpha 1}\right) K_f^{\alpha 1} = \frac{c_s+v_s-c_f-v_f}{c_s-c_f} K_f^{\alpha 1}$. Thus, $Z_s^{\alpha 1} - Z_s^i < 0$. That is, the only coordinating $\alpha-$contract does not maximize the IDM's expected profit. Alternatively speaking, the IDM's profit-maximizing $\alpha-$contract cannot coordinate the centralized capacity profile. ■

Proof of Proposition 4. We prove the case of $w > c_s + r$. The rest cases can be shown similarly. For $w > c_s + r$, the capacity of IDM is used first. The following table lists the IDM's and the foundry's profits under different demand realizations.

Realized Demand	IDM's Profit	Foundry's Profit
$d \in (0, K_s]$	$(p-c_s)d - rR - v_s K_s$	$rR - v_f K_f$
$d \in (K_s, K_s+R]$	$pd - c_s K_s - w(d-K_s)$ $-r(R+K_s-d) - v_s K_s$	$(w-c_f)(d-K_s) +$ $r(R+K_s-d) - v_f K_f$
$d \in (K_s+R, K_s+K_f]$	$pd - c_s K_s - w(d-K_s) - v_s K_s$	$(w-c_f)(d-K_s) - v_f K_f$
$d \in (K_s+K_f,\ \infty)$	$p(K_s+K_f) - c_s K_s - wK_f - v_s K_s$	$(w-c_f)K_f - v_f K_f$

After integration across the four demand intervals, we obtain Z_f and Z_s as shown in the proposition. $\frac{dZ_f}{dK_f} = -(w-c_f)F(K_s+K_f) + (w-c_f-v_f)$, and $\frac{d^2 Z_f}{dK_f^2} = -(w-c_f)\ f(K_s+K_f) < 0$. From the concavity of Z_f, it is straightforward to verify the expression of K_f. ■

Proof of Proposition 5. Based on Lemma 1, $K_f = K_w - K_s$ for $\alpha \leq \frac{K_w - K_s}{K_w}$ (referred to as Segment 1) and $K_f = \alpha K_w$ for $\alpha \geq \frac{K_w - K_s}{K_w}$ (referred to as Segment 2). To make sure $K_f \geq b$, we have over Segment 1

$$K_s \leq K_w - b, \tag{16}$$

and over Segment 2

$$\alpha \geq \frac{b}{K_w}. \tag{17}$$

(1) First, we examine over Segment 1, i.e., $\alpha \leq \frac{K_w - K_s}{K_w}$. If $a+b > K_w$, then $K_w - b < a \leq K_s$, which contradicts (16), so there is no feasible solution over segment 1. In order for a feasible solution over Segment 1 to exist, we require $a+b \leq K_w$. Let the superscript "$l*$" denote "local optimal".

As is shown in the proof of Proposition 2, over Segment 1, $\frac{dZ_s}{d\alpha} \leq 0$ for any given K_s, so $\alpha^{l*} = 0$ over Segment 1. Given $\alpha^{l*} = 0$, $\frac{dZ_s}{dK_s} = (w - c_s - v_s) - (w - c_s) F(K_s)$. Thus, $K_s^{l*} = \min\left[(1-\alpha)K_w, \max[a, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)]\right]$.

(2) Second, we examine over Segment 2. As is shown in the proof of Proposition 2, over Segment 2, $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} = \frac{\partial Z_s}{\partial \alpha}|_{K_s(\alpha)} + \frac{dK_s(\alpha)}{d\alpha}\frac{\partial Z_s}{\partial K_s}|_{K_s(\alpha)}$, where $\frac{\partial Z_s}{\partial K_s} = (p - c_s - v_s) - (p - c_s) F(K_s + \alpha K_w)$. Over this segment, $\alpha \geq \frac{K_w - K_s}{K_w}$, i.e., $F(K_s + \alpha K_w) \geq \frac{w - c_f - v_f}{w - c_f}$.

(2.1) If $\frac{w-c_f-v_f}{w-c_f} > \frac{p-c_s-v_s}{p-c_s}$ (or $K_w > K_p$), $\frac{\partial Z_s}{\partial K_s} < 0$ and $K_s^{l*} = a$. Given $K_s = a$, $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} = \frac{\partial Z_s}{\partial \alpha}|_{K_s = a} = (p - c_s)\overline{F(a + \alpha K_w)} K_w - (w - c_s)\int_o^{K_w} \overline{F(t)} dt$. By the definition of α^o, $\frac{dZ_s}{d\alpha}|_{\alpha = \alpha^o} = 0$. Therefore, $\alpha^{l*} = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

(2.2) If $\frac{w-c_f-v_f}{w-c_f} \leq \frac{p-c_s-v_s}{p-c_s}$ (or $K_w \leq K_p$), K_s is chosen so that $K_s + \alpha K_w = \max[K_p, a + \alpha K_w]$.

(2.2.1) If focusing on α so that $a + \alpha K_w \leq K_p$ (i.e., $\alpha \leq \frac{K_p - a}{K_w}$), then there is no feasible solution if $a + b > K_p$ considering the constraint $\alpha \geq \frac{b}{K_w}$ over Segment 2. If $a + b \leq K_p$, then there is feasible solution and $K_s + \alpha K_w = K_p$, which implies $\frac{\partial Z_s}{\partial K_s} = 0$ and $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} = \frac{\partial Z_s}{\partial \alpha}|_{K_s(\alpha)} = -(w - c_s - v_s) K_w + (w - c_s)\int_o^{K_w} F(t)\, dt = (w - c_s) K_w \left(\frac{-(w-c_s-v_s)}{w-c_s} + I_f\right)$. If $I_f > \frac{w-c_s-v_s}{w-c_s}$, then focus on α so that $a + \alpha K_w = K_p$. In this case, $K_s = a$. Given $K_s = a$, it can be shown similarly in (2.1) that $\alpha^{l*} = \min[1, \frac{K_p - a}{K_w}, \max[\frac{b}{K_w}, \alpha^o]]$. If $I_f \leq \frac{w-c_s-v_s}{w-c_s}$, $\frac{dZ_s(\alpha, K_s(\alpha))}{d\alpha} \leq 0$ over Segment 2.

(2.2.2) If focusing on α so that $a + \alpha K_w \geq K_p$ (i.e., $\alpha \geq \frac{K_p - a}{K_w}$), $\frac{\partial Z_s}{\partial K_s}|_{K_s = a} \leq 0$. Thus, $K_s^{l*} = a$. Given $K_s = a$, $\alpha^{l*} = \min[1, \max[\frac{K_p - a}{K_w}, \frac{b}{K_w}, \alpha^o]]$.

Now we combine the analysis over the two segments and find the global optimal α^*.

(a) $a + b > K_w$: There is no feasible solution over Segment 1.

(a1) $K_w > K_p$: By (2.1) $\alpha^{l*} = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$ over segment 2. Thus, the global optimal α^* equals $\min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

(a2) $K_w \leq K_p$:

(a2.1) $a + b \geq K_p$: By (2.2.1) and (2.2.2), $\alpha^* = \min[1, \max[\frac{K_p - a}{K_w}, \frac{b}{K_w}, \alpha^o]] = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

(a2.2) $K_w < a + b < K_p$: Combining (2.2.1) and (2.2.2), if $I_f > \frac{w-c_s-v_s}{w-c_s}$, $\alpha^* = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$; if $I_f \leq \frac{w-c_s-v_s}{w-c_s}$, $\alpha^* = \min[\frac{b}{K_w}, \frac{K_p - a}{K_w}]$ or $\min[1, \max[\frac{K_p - a}{K_w}, \frac{b}{K_w}, \alpha^o]]$, which is equivalent to $\alpha^* = \frac{b}{K_w}$ or $\min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

(b) $a + b \leq K_w$: $\alpha^{l*} = 0$ over Segment 1.

(b1) $K_w > K_p$: $\alpha^* = 0$ or $\min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

(b2) $K_w \leq K_p$: If $I_f > \frac{w-c_s-v_s}{w-c_s}$, $\alpha^* = 0$ or $\min[1, \max[\frac{b}{K_w}, \alpha^o]]$; if $I_f \leq \frac{w-c_s-v_s}{w-c_s}$, $\alpha^* = 0$ or $\min[1, \max[\frac{b}{K_w}, \alpha^o]]$.

Next, we derive the capacity equilibrium for a given α. If $\alpha^* = 0$, we have shown $K_s^* = \max[a, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)]$ and $K_f^* = K_w -$ $\max[a, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)]$ (over Segment 1). If $\alpha^* = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$ or $\frac{b}{K_w}$, $K_s^* = \max[a, K_p - \alpha^* K_w]$ and $K_f^* = \alpha^* K_w$ (over Segment 2). ■

Proof of Proposition 6. As has been shown in the proof of optimal centralized capacities without dual sourcing constraint, the centralized capacity investments are $\left(K_{cs}^u, K_{cf}^u\right) = \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right), F^{-1}\left(\frac{c_s+v_s-c_f-v_f}{c_s-c_f}\right)\right)$ for $MOC_s \geq MOC_f$ and $c_s > c_f$, and $\left(K_{cs}^u, K_{cf}^u\right) = \left(0, F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)\right)$ for $MOC_f > MOC_s$.

For $MOC_s \geq MOC_f$ and $c_s > c_f$: It has been shown that the total profit Z_{cs} is jointly concave in K_s and K_f. Thus, it is clear that $(K_{cs}, K_{cf}) = (a, b)$ if $a \geq K_{cs}^u$ and $b \geq K_{cf}^u$. If $a \geq K_{cs}^u$ and $b < K_{cf}^u$, then $K_{cs} = a$. Given $K_{cs} = a$, $\partial Z_{cs}/\partial K_f = p - c_f - v_f - (c_s - c_f) F(K_f) - (p - c_s) F(a + K_f)$. $\partial Z_{cs}/\partial K_f|_{K_f = K_f^o} = 0$. Given the concavity of Z_{cs} in K_f for any given K_s, $K_{cf} = \max[b, K_f^o]$.

The rest cases can be shown similarly. ■

Lemma A1 facilitates the proof of Proposition 7.

Lemma A1. *In the presence of dual sourcing constraint, if* $c_s > c_f$, $K_{cs} = a$, *and* $K_{cs} + K_{cf} \geq K_p$ *hold, then* $\alpha-$*contract with* $\alpha = 1$ *and* $w = c_f + \frac{v_f}{1-F\left(K_{cf}\right)}$ *is coordinating.*

Proof of Lemma A1. $c_s > c_f$ implies it is optimal for the system to allocate the demand to the foundry first. $\alpha = 1$ guarantees this allocation rule. For $\alpha = 1$, foundry builds $K_w = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ units of capacity. Setting $K_w = K_{cf}$ to replicate the system-optimal capacity profile leads to $w = c_f + \frac{v_f}{1-F\left(K_{cf}\right)}$. For $\alpha = 1$ and foundry's capacity decision K_{cf}, the IDM builds $\max[a, K_p - K_{cf}]$ units of capacity according to Proposition 5. $a + K_{cf} \geq K_p$ implies $\max[a, K_p - K_{cf}] = a$ which is the IDM's capacity decision under the centralized system based on the condition of $K_{cs} = a$. ■

Proof of Proposition 7. $K_{cs}^u + K_{cf}^u = \max[K_p, F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)]$. $a + b \geq K_{cs}^u + K_{cf}^u$ implies $a + b \geq K_p$ and $a + b \geq F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$. In the presence of dual sourcing constraint, $K_{cs} + K_{cf} \geq a + b$. This together with $a + b \geq K_p$ leads to $K_{cs} + K_{cf} \geq K_p$. By the definition of K_s^o in Equation (15), we have $K_s^o + b = K_p$. Thus, $a + b \geq K_p$ imply $a + b \geq K_s^o + b$, i.e., $a \geq K_s^o$. Assume $b \geq K_f^o$ holds. If $c_s > c_f$, then $a \geq K_s^o$ and $b \geq K_f^o$ imply $K_{cs} = a$ and $K_{cf} = b$ by Proposition 6. So far, we have shown conditions in Lemma A1 hold, and thus $\alpha-$contract with $\alpha = 1$ and $w = c_f + \frac{v_f}{1-F(b)}$ is coordinating. Next, we show this contract is also profit-maximizing for the IDM subject to the dual sourcing constraint. According to Proposition 5, $\alpha^* = \min[1, \max[\frac{b}{K_w}, \alpha^o]]$ and $\Omega^* = (a, \alpha^* K_w)$ for $a + b \geq K_p$, where α^o satisfies $F(\alpha^o K_w + a) = \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$. We search the IDM's profit-maximizing $\alpha-$contract conditional on two cases of w: [1] $F(a+b) \geq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$, and [2] $F(a+b) \leq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$.

[1] For w so that $F(a+b) \geq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$ holds: Then, $F(a+b) \geq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s} = F(\alpha^o K_w + a)$, leading to $b \geq \alpha^o K_w$ and $\alpha^* = \min[1, \max[\frac{b}{K_w}, \alpha^o]] = \min[1, \frac{b}{K_w}]$. $K_w \geq b$ must hold due to the foundry's capacity constraint, so $\alpha^* = \min[1, \frac{b}{K_w}] = \frac{b}{K_w}$ and $\Omega^* = (a, b)$. Now, we search among $\alpha-$contracts $\left(\alpha^* = \frac{b}{K_w}, w\right)$ the IDM's profit-maximizing one.

Conditional on $\alpha \geq \frac{K_f}{K_s+K_f} = \frac{b}{a+b}$, $Z_s|_{K_s=a,K_f=b} = (p-c_s)\int_0^{a+b}\overline{F(t)}dt + \alpha(c_s-w)\int_0^{\frac{b}{\alpha}}\overline{F(t)}dt - v_s a$. $\frac{dZ_s|_{K_s=a,K_f=b}}{d\alpha} = \frac{v_f}{\overline{F\left(\frac{b}{\alpha}\right)}}\left(\left(g\left(\frac{b}{\alpha}\right)-1\right)\int_0^{\frac{b}{\alpha}}\overline{F(t)}dt + \overline{F\left(\frac{b}{\alpha}\right)}\frac{b}{\alpha}\right) + (c_s-c_f)\left(\int_0^{\frac{b}{\alpha}}\overline{F(t)}dt - \overline{F\left(\frac{b}{\alpha}\right)}\frac{b}{\alpha}\right) >$ $\frac{v_f}{\overline{F\left(\frac{b}{\alpha}\right)}}\left(\left(g\left(\frac{b}{\alpha}\right)-1\right)\int_0^{\frac{b}{\alpha}}\overline{F(t)}dt + \overline{F\left(\frac{b}{\alpha}\right)}\frac{b}{\alpha}\right)$. Then, there exists a threshold $\hat{b}_1$ so that $\frac{dZ_s|_{K_s=a,K_f=b}}{d\alpha} > 0$ for $b \geq \hat{b}_1$.

Conditional on $\alpha \leq \frac{b}{a+b}$, $Z_s|_{K_s=a,K_f=b} = (p-c_s+\alpha(c_s-w))\int_0^{\frac{a}{1-\alpha}}\overline{F(t)}dt + (p-w)\int_{\frac{a}{1-\alpha}}^{a+b}\overline{F(t)}dt - v_s a$. $\frac{dZ_s|_{K_s=a,K_f=b}}{d\alpha} = \frac{v_f}{\overline{F\left(\frac{b}{\alpha}\right)}}\left(\left(g\left(\frac{b}{\alpha}\right)-1\right)\int_0^{\frac{a}{1-\alpha}}\overline{F(t)}dt + g\left(\frac{b}{\alpha}\right)\int_{\frac{a}{1-\alpha}}^{a+b}\frac{\overline{F(t)}}{\alpha}dt + \overline{F\left(\frac{a}{1-\alpha}\right)}\frac{a}{1-\alpha}\right) + (c_s-c_f)\left(\int_0^{\frac{a}{1-\alpha}}\overline{F(t)}dt - \overline{F\left(\frac{a}{1-\alpha}\right)}\frac{a}{1-\alpha}\right)$. The second term is positive. There exists a threshold $\hat{b}_2$ so that $\frac{dZ_s|_{K_s=a,K_f=b}}{d\alpha} \geq 0$ for $b \geq \hat{b}_2$.

Combining the two intervals of α leads to $\alpha = 1$ if $b \geq \max[\hat{b}_1, \hat{b}_2]$ holds. Since $\frac{b}{K_w} = \alpha^* = 1$, $w = c_f + \frac{v_f}{1-F(b)}$ conditional on $F(a+b) \geq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$. That is, the coordinating $\alpha-$contract ($\alpha = 1$, $w = c_f + \frac{v_f}{1-F(b)}$) indeed maximizes the IDM's expected profit conditional on $F(a+b) \geq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$.

[2] For w so that $F(a+b) \leq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s}$ holds: Then, $F(a+b) \leq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s} = F(a+\alpha^o K_w)$, which leads to $b \leq \alpha^o K_w$. $\alpha^* = \min[1, \max[\frac{b}{K_w}, \alpha^o]] = \min[1, \alpha^o]$ and $\Omega^* = (a, \alpha^* K_w)$. If $\alpha^o \geq 1$, then $\alpha^* = 1$. Now, we search among $\alpha-$contracts, with α and w satisfying $\frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s} = F(a+\alpha K_w)$, the IDM's profit-maximizing one.

Conditional on $\alpha \geq \frac{K_f}{K_s+K_f}$, $Z_s|_{K_s=a,K_f=\alpha K_w} = (p-c_s+\alpha(c_s-w))\int_0^{K_w}\overline{F(t)}dt + (p-c_s)\int_{K_w}^{a+\alpha K_w}\overline{F(t)}dt - v_s a$. $\frac{dZ_s|_{K_s=a,K_f=\alpha K_w}}{d\alpha} = -\alpha\frac{dK_w}{dw}\frac{dw}{d\alpha}\left((1-I_f)g(K_w)(w-c_f) - (w-c_s)(F(K_w)-I_f)\right)$. $\frac{dK_w}{dw} > 0$. Under the assumption of $\frac{dw}{d\alpha^o} < 0$, that is, $\frac{dw}{d\alpha} < 0$ for w and α satisfying $\frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s} = F(a+\alpha K_w)$, then $\frac{dZ_s|_{K_s=a,K_f=\alpha K_w}}{d\alpha} > -\alpha\frac{dK_w}{dw}\frac{dw}{d\alpha}(w-c_s)\left((1-I_f)g(K_w) - (F(K_w)-I_f)\right)$. Since $K_w > b$, there exists a threshold $\hat{b}_3$ so that $\frac{dZ_s|_{K_s=a,K_f=\alpha K_w}}{d\alpha} > 0$ for $b \geq \hat{b}_3$.

Conditional on $\alpha \leq \frac{K_f}{K_s+K_f}$, i.e., $\alpha \leq \frac{\alpha K_w}{a+\alpha K_w}$, we have $F(a+\alpha^o K_w) \leq F(K_w)$. In [2], we have $F(a+b) \leq \frac{p-w\left(1-I_f\right)-c_s I_f}{p-c_s} = F(a+\alpha^o K_w)$, so $F(a+b) \leq F(a+\alpha^o K_w) \leq F(K_w)$. Because $K_w = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) < F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$ for $w < p$, the condition $a+b \geq F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$ contradicts $F(a+b) \leq F(K_w)$. Thus, $\alpha \leq \frac{K_f}{K_s+K_f}$ is infeasible.

To sum up, $\alpha^* = 1$ for $b \geq \max[K_f^o, \hat{b}_1, \hat{b}_2, \hat{b}_3]$. Next, we show given $\alpha = 1$, the w that satisfies $K_w = b$ is optimal. $\frac{dZ_s|_{\alpha=1}}{dw} = \frac{\overline{F(K_w)}}{f(K_w)\left(w-c_f\right)}\left((p-c_s)\overline{F(a+K_w)} - (w-c_s)\overline{F(K_w)}\right) - \int_0^{K_w}\overline{F(t)}dt <$ $\frac{\overline{F(K_w)}}{f(K_w)\left(w-c_f\right)}\left(\frac{v_s}{\overline{F(a+b)}}\overline{F(a+K_w)} - (w-c_s)\overline{F(K_w)}\right) - K_w\overline{F(K_w)} = \frac{\left(\overline{F(K_w)}\right)^2}{f(K_w)\left(w-c_f\right)}\left(\frac{v_s}{\overline{F(a+b)}}\frac{\overline{F(a+K_w)}}{\overline{F(K_w)}} - (w-c_s) - g(K_w)(w-c_f)\right)$. $g(K_w) \geq \frac{v_s}{\overline{F(a+b)}}\frac{\overline{F(a+K_w)}}{\overline{F(K_w)}\left(w-c_f\right)} - \frac{w-c_s}{w-c_f}$ leads to $\frac{dZ_s|_{\alpha=1}}{dw} < 0$. Because $K_w \geq b$, if $g(b)$ is greater than a threshold $g\left(\hat{b}_4\right)$, which is the upper bound of $\frac{v_s}{\overline{F(a+b)}}\frac{\overline{F(a+K_w)}}{\overline{F(K_w)}\left(w-c_f\right)} - \frac{w-c_s}{w-c_f}$, then $\frac{dZ_s|_{\alpha=1}}{dw} < 0$ must hold. Due to the capacity constraint $K_w \geq b$, the IDM's optimal w value satisfies $K_w = b$.

The above analysis shows that given the conditions in the Proposition, where $\hat{b} = \max[K_f^o, \hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4]$, the coordinating $\alpha-$contract $\left(\alpha = 1, w = c_f + \frac{v_f}{1-F(b)}\right)$ is the IDM's profit-maximizing $\alpha-$contract. ■

Lemma A2 facilitates the proof of Proposition 8. It can be shown by using similar optimization approach to that of Proposition A1, and thus its proof is omitted.

Lemma A2. *Assume $c_s < w \leq c_s + r$ holds. The IDM's optimal capacity decision K_s given R in the reservation contract is as follows.*

(a) If $\frac{p-c_s-v_s}{p-c_s} \geq \frac{w-c_f-v_f}{w-c_f} \geq \frac{w-c_s-v_s}{w-c_s}$, then
$K_s = 0$, *for* $R \in \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), \infty\right]$,
$K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - R$, *for* $R \in \left(F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)\right]$,
$K_s = 0$ *or* $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - R$, *for* $R \in \left(F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right]$ *and* $w > c_s + v_s$,
$K_s = F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) - R$ *or* $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - R$, *for* $R \in \left[0, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)\right]$ *and* $w > c_s + v_s$,
$K_s = 0$ *or* $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - R$, *for* $R \in \left[0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right]$ *and* $w \leq c_s + v_s$.

(b) If $\frac{p-c_s-v_s}{p-c_s} \geq \frac{w-c_s-v_s}{w-c_s} \geq \frac{w-c_f-v_f}{w-c_f}$, then
$K_s = 0$, *for* $R \in \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), \infty\right]$,
$K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - R$, *for* $R \in \left[0, F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)\right]$.

(c) If $\frac{w-c_f-v_f}{w-c_f} \geq \frac{p-c_s-v_s}{p-c_s} \geq \frac{w-c_s-v_s}{w-c_s}$, then
$K_s = 0$, *for* $R \in \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), \infty\right]$,
$K_s = 0$, *for* $R \in \left(F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)\right]$ *and* $w > c_s + v_s$,
$K_s = F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) - R$, *for* $R \in \left[0, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)\right]$ *and* $w > c_s + v_s$,
$K_s = 0$, *for* $R \in \left[0, F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)\right]$ *and* $w \leq c_s + v_s$.

Proof of Proposition 8. Since $c_s > c_f$, the demand should be allocated to the foundry first in the centralized system. Thus, the reservation contract (w, r) must satisfy $w \leq c_s$ or $w \leq c_s + r$. In the practical setting, it is rarely the case $w \leq c_s$, so we exclude this case from consideration. Thus, $w \leq c_s + r$ must be satisfied. First, we study the IDM's capacity decision. By Lemma A2 which gives the IDM's capacity decision given any R for the case of no dual sourcing constraint, there are three possible values of K_s depending on the magnitude of R. $R \geq b$ must be satisfied considering the constraint $K_f \geq b$. If taking into account the constraint $K_s \geq a$, then the three possible values of K_s are: a, $\max[a, K_p - R]$, and $\max[a, F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) - R]$. $a + b \geq K_{cs}^u + K_{cf}^u \geq K_p$ leads to $a \geq K_p - b \geq K_p - R$. Thus, all the three possible values of K_s are equal to a. That is, for any R, $K_s^* = a$, which is indeed the centralized decision for the IDM's capacity. Now given $K_s = a$, the IDM decides his capacity reservation decision. We substitute $K_s = a$ and $K_f = R$ into the IDM's profit function Z_s for $c_s \leq w < c_s + r$, and take the derivative with respect to R to derive $\frac{dZ_s}{dR} = (p-w) - (p-c_s)\,F(a+R) + (w-c_s-r)\,F(R)$. Let R^o satisfy $\frac{dZ_s}{dR}|_{R=R^o} = 0$. $\frac{dZ_s}{dR}$ decreases in R, and thus $R^* = \max[b, R^o]$ in order to satisfy the constraint $K_f \geq b$. The centralized capacity decision for the foundry is $K_{cf} = \max[b, K_f^o] = K_f^o$ by $b < K_f^o$. In order for $R^* = K_f^o$, $R^o = K_f^o$ should be satisfied; that is, $(p-w) - (p-c_s)\,F\left(a + K_f^o\right) + (w-c_s-r)\,F\left(K_f^o\right) = 0$. By the definition of K_f^o, we have $r = w - c_f - \frac{w-c_f-v_f}{F\left(K_f^o\right)}$. Substituting r into the condition $w \leq c_s + r$, we derive $w \leq c_f + v_f + (c_s - c_f)\,F\left(K_f^o\right)$. ■

Proof of Proposition 9. By Proposition 4 for the reservation contract with $w > c_s + r$, the system profit is $Z = (p-c_f)\int_{K_s}^{K_s+K_f} \overline{F(t)}\, dt + (p-c_s)\int_0^{K_s} \overline{F(t)} dt - v_s K_s - v_f K_f$. Now we derive the system optimal capacity profile. Since $w > c_s + r$, the IDM uses his own capacity first to satisfy the demand. For given K_s, $\frac{\partial Z}{\partial K_f} = (p-c_f)\,\overline{F(K_s+K_f)} - v_f$ and $\frac{\partial^2 Z}{\partial K_f^2} < 0$. Thus, $K_f = \max[F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right) - K_s, b]$ considering the dual sourcing constraint. Note that $K_s \geq a$ and $a + b > F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$, so $F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right) -$

$K_s - b \le F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right) - a - b < 0$. Thus, $K_f = b$. For $K_f = b$, $\frac{\partial Z}{\partial K_s} = p - c_s - v_s - (p-c_f)F(K_s+b) + (c_s-c_f)F(K_s) < p - c_s - v_s - (p-c_f)F(K_s+b) + (c_s-c_f)F(K_s+b) = p - c_s - v_s - (p-c_s)F(K_s+b)$. $K_s \ge a$ implies $(p-c_s)F(K_s+b) \ge (p-c_s)\frac{p-c_f-v_f}{p-c_f}$. Thus, $\frac{\partial Z}{\partial K_s} < p - c_s - v_s - (p-c_s)\frac{p-c_f-v_f}{p-c_f} < 0$ by $MOC_f > MOC_s$ and $K_s = a$. That is, the system profit is maximized at $K_s = a$ and $K_f = b$ under any reservation contract with $w > c_s + r$. If the capacity decisions are decentralized, the system profit will be lower. Further, under $w > c_s + r$, the production decision is suboptimal for the case of $c_s > c_f$, which implies the system profit under reservation contract with $w > c_s + r$ cannot exceed another contract that leads to $K_s = a$ and $K_f = b$. ■

Proof of Proposition 10. As shown in the proof of Proposition 8, $w \le c_s + r$ must hold to coordinate the production decision, $K_s^* = a$ for any R, $\frac{dZ_s}{dR} = (p-w) - (p-c_s)F(a+R) + (w-c_s-r)F(R)$, $\frac{dZ_s}{dR}$ decreases in R, and $R^* = \max[b, R^o]$, where R^o satisfy $\frac{dZ_s}{dR}|_{R=R^o} = 0$. $R^o > b$ is equivalent to $\frac{dZ_s}{dR}|_{R=b} > 0$, which is further equivalent to $w\overline{F(b)} + (c_s+r)F(b) < p - (p-c_s)F(a+b)$. Since $b < K_f^o$, pushing the IDM's reservation quantity R^o above b and closer to K_f^o improves the system profit. Therefore, reservation contracts that satisfy (i) and (ii) are candidates to improve the system profit. ■

Proof of Proposition A1. (a) Assume that $w > c_s$ and $0 \le \alpha < 1$ hold. We consider separately two intervals of K_s given K_f specified in Lemma 1.

Case 1. $K_s \in \left[0, (1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right]$: Note that $K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s$ by Lemma 1. Then $(1-\alpha)K_f = (1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - (1-\alpha)K_s \ge K_s - (1-\alpha)K_s = \alpha K_s$. Hence using (4) in Lemma 1, we obtain $\frac{dZ_s}{dK_s} = -(w-c_s)F\left(\frac{K_s}{1-\alpha}\right) + w - c_s - v_s$, and $\frac{d^2Z_s}{dK_s^2} < 0$.

Case 1.1. $w \le c_s + v_s$: In this case, $\frac{dZ_s}{dK_s} \le 0$. Therefore Z_s takes the maximal value at $K_s = 0$, and is non-increasing in the interval. Using Lemma 1, the corresponding local optimal capacity pair is $\Omega_1 = \left(0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$.

Case 1.2. $w > c_s + v_s$ and $\frac{w-c_f-v_f}{w-c_f} \ge \frac{w-c_s-v_s}{w-c_s}$: In this case, it is easy to verify that $K_s = (1-\alpha)F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)$ maximizes Z_s by the concavity of Z_s. Using Lemma 1, the corresponding local optimal capacity pair is $\Omega_3 = \left((1-\alpha)F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s\right)$.

Case 1.3. $w > c_s + v_s$ and $\frac{w-c_f-v_f}{w-c_f} < \frac{w-c_s-v_s}{w-c_s}$: In this case, $\frac{dZ_s}{dK_s} > 0$ in this interval. That is, Z_s is increasing in this interval and takes the maximal value at $K_s = (1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$.

Case 2. $K_s \in \left[(1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \infty\right)$: In this case, $K_f = \alpha F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ by Lemma 1, and thus $K_s + K_f \ge F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. Then, $\alpha K_s \ge \alpha(1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) = (1-\alpha)K_f$. If $0 < \alpha < 1$, then using Equation (2), we obtain $\frac{dZ_s}{dK_s} = -(p-c_s)F(K_s+K_f) + p - c_s - v_s$. If $\alpha = 0$, then $K_f = \alpha F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) = 0$. Using Equation (4), we obtain $\frac{dZ_s}{dK_s} = -(p-c_s)F(K_s) + p - c_s - v_s = -(p-c_s)F(K_s+K_f) + p - c_s - v_s$. Therefore, $\frac{dZ_s}{dK_s}$ has the same expression under both $0 < \alpha < 1$ and $\alpha = 0$. $\frac{d^2Z_s}{dK_s^2} < 0$.

Case 2.1. $w \le \frac{p-c_s}{v_s}v_f + c_f$: Note that this condition is equivalent to $\frac{p-c_s-v_s}{p-c_s} \ge \frac{w-c_f-v_f}{w-c_f}$. Then it is easy to verify that Z_s is maximized at $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - K_f$ in this interval by the concavity of Z_s. Using Lemma 1, the corresponding local optimal capacity pair is $\Omega_2 = \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - K_f, \alpha F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$.

Case 2.2. $w > \frac{p-c_s}{v_s}v_f + c_f$: In this case, $\frac{dZ_s}{dK_s} < 0$, and thus Z_s takes the maximal value at $K_s = (1-\alpha)F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, and is decreasing in this interval.

Now we prove (a1) assuming that $\frac{p-c_s}{v_s}v_f + c_f \geq c_s + v_s$ holds.

(a1.1) $c_s < w \leq c_s + v_s \leq \frac{p-c_s}{v_s}v_f + c_f :$ This case corresponds to Cases 1.1 and 2.1. That is, Z_s is decreasing for $K_s \in \left[0, (1-\alpha) F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right]$, and takes the local maximum at $K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - K_f$ for $K_s \in \left[(1-\alpha) F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \infty\right)$. Hence, $\Omega^* = \arg\max\left[Z_s\left(\Omega_1\right), Z_s\left(\Omega_2\right)\right].$

(a1.2) $c_s + v_s < w \leq \min\left\{\frac{p-c_s}{v_s}v_f + c_f, p\right\}$ and $\frac{w-c_f-v_f}{w-c_f} \geq \frac{w-c_s-v_s}{w-c_s}$: This case corresponds to Cases 1.2 and 2.1. Therefore, $\Omega^* = \arg\max\left[Z_s\left(\Omega_2\right), Z_s\left(\Omega_3\right)\right].$

(a1.3) $c_s + v_s < w \leq \min\left\{\frac{p-c_s}{v_s}v_f + c_f, p\right\}$ and $\frac{w-c_f-v_f}{w-c_f} < \frac{w-c_s-v_s}{w-c_s}$: This case corresponds to Cases 1.3 and 2.1. Therefore $\Omega^* = \Omega_2$.

(a1.4) $\min\left\{\frac{p-c_s}{v_s}v_f + c_f, p\right\} < w \leq p :$ $w \leq p$ implies $\frac{p-c_s-v_s}{p-c_s} \geq \frac{w-c_s-v_s}{w-c_s}$. $w \geq \frac{p-c_s}{v_s}v_f + c_f$ is equivalent to $\frac{w-c_f-v_f}{w-c_f} \geq \frac{p-c_s-v_s}{p-c_s}$. Hence, $\frac{w-c_f-v_f}{w-c_f} \geq \frac{w-c_s-v_s}{w-c_s}$ holds. Therefore, this case corresponds to Cases 1.2 and 2.2, and thus $\Omega^* = \Omega_3$.

Similarly, we prove (a2) assuming that $\frac{p-c_s}{v_s}v_f + c_f < c_s + v_s$ holds.

(a2.1) corresponds to Cases 1.1 and 2.1. Therefore $\Omega^* = \arg\max\left[Z_s\left(\Omega_1\right), Z_s\left(\Omega_2\right)\right].$

(a2.2) corresponds to Cases 1.1 and 2.2. Therefore $\Omega^* = \Omega_1$.

(a2.3) corresponds to Cases 1.2 and 2.2. Therefore $\Omega^* = \Omega_3$.

(b) Assume $w \leq c_s$ or $\alpha = 1$, which is the foundry first case. Then $K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ by Lemma 1. After substituting $K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ into Equation (2), we obtain $\frac{dZ_s}{dK_s} = (p - c_s - v_s) - (p - c_s) F(K_s + K_f)$. If $\frac{p-c_s-v_s}{p-c_s} \geq \frac{w-c_f-v_f}{w-c_f}$, then $K_s^* = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - K_f^* = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. If $\frac{p-c_s-v_s}{p-c_s} < \frac{w-c_f-v_f}{w-c_f}$, then $\frac{dZ_s}{dK_s} < 0$ and thus $K_s^* = 0$. ■

Propositions A2, A3, and A4 can be shown by using similar optimization approach to that of Proposition A1, and thus their proofs are omitted.