## Online Supplement: Proofs

This document proves the propositions and states Lemmas 5-10. The proofs of Lemmas 3-10 are available upon request.

**Proof of Proposition 1**. By (18)-(19) the first-best revenue function $\Pi^f$ satisfies

$$\Pi^f(\boldsymbol{\lambda},\mu) = \lambda_1 p_1^{c\mu}(\lambda_1,\mu) + \lambda_2 p_2^{c\mu}(\boldsymbol{\lambda},\mu) = \sum_{i=1}^{2} R_i(\lambda_i) - \frac{\lambda_1(c_1-c_2)}{\mu-\lambda_1} - \frac{(\lambda_1+\lambda_2)c_2}{\mu-\lambda_1-\lambda_2},$$

$$\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu) = R_1'(\lambda_1) - \frac{(c_1-c_2)\mu}{(\mu-\lambda_1)^2} - \frac{c_2\mu}{(\mu-\lambda_1-\lambda_2)^2}, \quad (52)$$

$$\Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu) = R_2'(\lambda_2) - \frac{c_2\mu}{(\mu-\lambda_1-\lambda_2)^2}, \quad (53)$$

$$\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu) - \Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu) = R_1'(\lambda_1) - \frac{(c_1-c_2)\mu}{(\mu-\lambda_1)^2} - R_2'(\lambda_2). \quad (54)$$

$\Pi^f$ is strictly concave and submodular in $\boldsymbol{\lambda}$: $\Pi^f_{\lambda_1\lambda_1}, \Pi^f_{\lambda_2\lambda_2} < \Pi^f_{\lambda_1\lambda_2} < 0$ by (52)-(54) and since $R_i'' < 0$ by *A2.* Therefore $\boldsymbol{\lambda}^f(\mu) = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^f(\boldsymbol{\lambda},\mu)$ is unique: $\Pi^f$ is strictly concave in $\boldsymbol{\lambda}$ and $M(\mu)$ is convex, bounded, and closed, except along $\{\mathbf{0} \le \boldsymbol{\lambda} \le \boldsymbol{\Lambda} : \lambda_1+\lambda_2 = \mu\}$ where $\Pi^f(\boldsymbol{\lambda},\mu) = -\infty$.

Noting that $\mu > \mu_0$ implies $\Pi^f_{\lambda_i}(\mathbf{0},\mu) = \overline{v}_i - c_i/\mu > 0$ for $i = 1,2$, condition $(i)$ is immediate from (20), which we prove next. Since $R_i'(0) = v_i(0) = \overline{v}_i > R_i'(\lambda_i)$ for $\lambda_i > 0$, (52)-(53) yield:

$$\frac{\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu)}{c_1} - \frac{\Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu)}{c_2} = \frac{R_1'(\lambda_1)}{c_1} - \frac{\overline{v}_2}{c_2} < \frac{\overline{v}_1}{c_1} - \frac{\overline{v}_2}{c_2} \text{ if } \lambda_1 > 0 = \lambda_2,$$

$$\frac{\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu)}{c_1} - \frac{\Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu)}{c_2} = \frac{\overline{v}_1}{c_1} - \frac{R_2'(\lambda_2)}{c_2} - \left(1-\frac{c_2}{c_1}\right)\left[\frac{1}{\mu} - \frac{\mu}{(\mu-\lambda_2)^2}\right] > \frac{\overline{v}_1}{c_1} - \frac{\overline{v}_2}{c_2} \text{ if } \lambda_1 = 0 < \lambda_2.$$

Condition $(ii)$ follows from (21), which holds by (54), and since $\Pi^f_{\lambda_1\lambda_2} > \Pi^f_{\lambda_i\lambda_i}$ for $i = 1,2$, implies that: if $\Pi^f_{\lambda_i}(\mathbf{0},\mu) > \Pi^f_{\lambda_j}(\mathbf{0},\mu)$ then $\Pi^f_{\lambda_i}(\boldsymbol{\lambda},\mu) > \Pi^f_{\lambda_j}(\boldsymbol{\lambda},\mu)$ for $\lambda_i = 0 < \lambda_j$ and $i \ne j$.

*Part 1.* If $\frac{\overline{v}_1}{c_1} = \frac{\overline{v}_2}{c_2}$ then $\frac{1}{\mu_0} = \frac{\overline{v}_1}{c_1} = \frac{\overline{v}_2}{c_2}$. Condition $(i)$ implies $\boldsymbol{\lambda}^f(\mu) > \mathbf{0}$ for $\mu > \mu_0$, so $\mu^f = \mu_0$.

*Part 2.* If $\frac{\overline{v}_1}{c_1} > \frac{\overline{v}_2}{c_2}$, then $\frac{1}{\mu_0} = \frac{\overline{v}_2}{c_2} < \frac{\overline{v}_1}{c_1} < \frac{\overline{v}_1-\overline{v}_2}{c_1-c_2}$. Condition $(i)$ implies $\lambda_1^f(\mu) > 0$ for $\mu > \mu_0$. We show $\boldsymbol{\lambda}^f(\mu) > \mathbf{0} \Leftrightarrow \mu > \mu^f$ for $\mu^f > \mu_0$. Let $\overline{\boldsymbol{\lambda}}(\mu) = (\overline{\lambda}_1(\mu), 0) \triangleq \arg\max_{\boldsymbol{\lambda}\in M(\mu),\lambda_2=0} \Pi^f(\boldsymbol{\lambda},\mu)$ where $\overline{\lambda}_1(\mu) > 0$ and $\Pi^f_{\lambda_1}(\overline{\boldsymbol{\lambda}}(\mu),\mu) = 0$. Since $\Pi^f$ is strictly concave in $\boldsymbol{\lambda}$ we have $\lambda_2^f(\mu) > 0$ if and only if

$$\Pi^f_{\lambda_2}(\overline{\boldsymbol{\lambda}}(\mu),\mu) = \overline{v}_2 - \frac{c_2\mu}{(\mu-\overline{\lambda}_1(\mu))^2} > 0 = \Pi^f_{\lambda_1}(\overline{\boldsymbol{\lambda}}(\mu),\mu) = R_1'(\overline{\lambda}_1(\mu)) - \frac{c_1\mu}{(\mu-\overline{\lambda}_1(\mu))^2}. \quad (55)$$

Note that $\Pi^f_{\lambda_2}(\overline{\boldsymbol{\lambda}}(\mu_0),\mu_0) < 0$ since $\frac{1}{\mu_0} = \frac{\overline{v}_2}{c_2}$. Since $\Pi^f_{\lambda_1\lambda_1} < 0$ and $\Pi^f_{\lambda_1\mu} > 0$, (55) implies $d\Pi^f_{\lambda_2}(\overline{\boldsymbol{\lambda}}(\mu),\mu)/d\mu > 0$ and $\lim_{\mu\to\infty} \Pi^f_{\lambda_2}(\overline{\boldsymbol{\lambda}}(\mu),\mu) > 0$. Hence there exists $\mu^f \in (\mu_0,\infty)$ as claimed.

*Part 3.* A similar argument as in Part 2 shows $\boldsymbol{\lambda}^f(\mu) > \mathbf{0} \Leftrightarrow \mu > \mu^f$ where $\frac{1}{\mu^f} \in \left(\frac{\overline{v}_1-\overline{v}_2}{c_1-c_2}, \frac{\overline{v}_1}{c_1}\right)$.■

**Proof of Proposition 3**. *Part 1.* If strategic delay is optimal and $\boldsymbol{\lambda}^s$ is the unique second-best demand vector, then $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^s(\boldsymbol{\lambda},\mu) \in M_2(\mu)$ by Definition 4. Since $\Pi^s(\boldsymbol{\lambda},\mu) = \Pi^{sd}(\boldsymbol{\lambda},\mu)$ for $\boldsymbol{\lambda} \in M_2(\mu)$ by (27), it follows that $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M_2(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu)$. Since $M_2(\mu) = \{\boldsymbol{\lambda} \in M(\mu) \colon \lambda_2 > 0, w_2^{c\mu}(\boldsymbol{\lambda},\mu) < \frac{v_1(\lambda_1)-v_2(\lambda_2)}{c_1-c_2}\}$ by (24), every direction that is feasible at $\boldsymbol{\lambda} \in$

$M_2(\mu)$ for $M(\mu)$ is also feasible for $M_2(\mu)$, hence $\boldsymbol{\lambda}^s$ is a local maximum of $\Pi^{sd}(\boldsymbol{\lambda},\mu)$ on $M(\mu)$. Since $\Pi^{sd}(\boldsymbol{\lambda},\mu)$ is strictly concave in $\boldsymbol{\lambda}$ and $M(\mu)$ is convex, $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu)$.

If $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu) \in M_2(\mu)$, then $\Pi^s(\boldsymbol{\lambda}^s,\mu) = \Pi^{sd}(\boldsymbol{\lambda}^s,\mu) > \Pi^{sd}(\boldsymbol{\lambda},\mu)$ for $\boldsymbol{\lambda}^s \neq \boldsymbol{\lambda} \in M(\mu)$, and $\Pi^s(\boldsymbol{\lambda}^s,\mu) > \Pi^s(\boldsymbol{\lambda},\mu)$ for $\boldsymbol{\lambda}^s \neq \boldsymbol{\lambda} \in M_2(\mu)$. If $\boldsymbol{\lambda} \in M_1(\mu)$ then $\Pi^s(\boldsymbol{\lambda},\mu) = 0 = \Pi^{sd}(\mathbf{0},\mu) < \Pi^s(\boldsymbol{\lambda}^s,\mu)$ since $\boldsymbol{\lambda}^s \neq \mathbf{0}$. If $\boldsymbol{\lambda} \in M_0(\mu)$ then $\lambda_2\left(w_2^{c\mu}(\boldsymbol{\lambda},\mu) - w_2^{sd}(\boldsymbol{\lambda},\mu)\right) \geq 0$, so (27) implies $\Pi^{sd}(\boldsymbol{\lambda}^s,\mu) > \Pi^{sd}(\boldsymbol{\lambda},\mu) \geq \Pi^f(\boldsymbol{\lambda},\mu) = \Pi^s(\boldsymbol{\lambda},\mu)$. Hence, $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^s(\boldsymbol{\lambda},\mu)$.

We next show that $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu) \in M_2(\mu)$ if and only if (29)-(32) hold for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^s$. Suppose $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu) \in M_2(\mu)$. The definition of $M_2(\mu)$ implies (32), $0 < \lambda_2^s \leq \Lambda_2$, and $0 \leq \lambda_1^s \leq \Lambda_1$. We prove $\lambda_2^s < \Lambda_2$ and $0 < \lambda_1^s < \Lambda_1$, which implies (29)-(30), i.e., $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^s,\mu) = \Pi_{\lambda_2}^{sd}(\boldsymbol{\lambda}^s,\mu) = 0$. That $\lambda_2^s < \Lambda_2$ holds since $v_2(\Lambda_2) = 0$ (by *A1*) and (30) imply that $\Pi_{\lambda_2}^{sd}(\boldsymbol{\lambda},\mu) < 0$ if $\lambda_2 = \Lambda_2$. That $\lambda_1^s > 0$ holds since $\partial p_2^{sd}/\partial\lambda_1 > 0$ and $\mu > \mu_0 \geq c_1/\overline{v}_1$ imply by (29) that $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda},\mu) > 0$ if $\lambda_1 = 0$. Since $v_1' < 0$ and (32) implies $v_1(\lambda_1^s) > 0$, we have $\lambda_1^s < \Lambda_1$ by *A1*.

Now suppose that (29)-(32) hold for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^s$. Then (30) implies $\lambda_2^s < \Lambda_2$ and (32) implies $\lambda_1^s < \Lambda_1$ as shown above, so that $\boldsymbol{\lambda}^s \in M_2(\mu) \subset M(\mu)$. Since $\Pi^{sd}(\boldsymbol{\lambda},\mu)$ is strictly concave in $\boldsymbol{\lambda}$ and $M(\mu)$ is convex, (29)-(30) imply $\boldsymbol{\lambda}^s = \arg\max_{\boldsymbol{\lambda}\in M(\mu)} \Pi^{sd}(\boldsymbol{\lambda},\mu) \in M_2(\mu)$.

*Part 2.* If strategic delay is optimal then by Part 1, $\boldsymbol{\lambda}^s \in M_2(\mu)$ and $\Pi^s(\boldsymbol{\lambda}^s,\mu) = \Pi^{sd}(\boldsymbol{\lambda}^s,\mu) > \Pi^{sd}(\boldsymbol{\lambda},\mu)$ for $\boldsymbol{\lambda} \notin M_2(\mu)$. That $\boldsymbol{\lambda}^f(\mu) \in M_2(\mu)$ follows from the definitions of $M_i(\mu)$ in (22)-(24) and (27), which imply $\Pi^f(\boldsymbol{\lambda}^s,\mu) > \Pi^{sd}(\boldsymbol{\lambda}^s,\mu)$ and $\Pi^{sd}(\boldsymbol{\lambda},\mu) \geq \Pi^f(\boldsymbol{\lambda},\mu)$ for $\boldsymbol{\lambda} \notin M_2(\mu)$.■

**Proof of Proposition 4.** By Proposition 3, strategic delay is optimal and $\boldsymbol{\lambda}^s$ is the unique second-best vector if and only if $\boldsymbol{\lambda} = \boldsymbol{\lambda}^s$ satisfies (29)-(32). Conditions (29)-(32) are equivalent to:

$$\frac{\overline{v}_2}{c_2} > \frac{v_1(\lambda_1)}{c_1} \text{ and } \boldsymbol{\lambda} = \boldsymbol{\lambda}^{sd}(\lambda_1)\,, \tag{56}$$

$$\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu) = R_1'(\lambda_1) - \frac{c_1\mu}{(\mu-\lambda_1)^2} + \Lambda_2 \cdot \overline{\lambda}_2^{sd}(\lambda_1)\, c_2 \frac{-v_1'(\lambda_1)}{c_1-c_2} = 0, \tag{57}$$

$$w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu) = \frac{\mu}{(\mu-\lambda_1)(\mu-\lambda_1-\Lambda_2\cdot\overline{\lambda}_2^{sd}(\lambda_1))} < w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) = \frac{v_1(\lambda_1) - v_2(\lambda_2^{sd}(\lambda_1))}{c_1-c_2}, \tag{58}$$

and $\lambda_1 + \lambda_2^{sd}(\lambda_1) < \mu$, where $\overline{\lambda}_2^{sd}(\lambda_1) \triangleq \frac{\lambda_2^{sd}(\lambda_1)}{\Lambda_2}$ is constant in $\Lambda_2 > 0$ (Lemma 3.4). The equivalence holds because: By Lemma 3.1, $\boldsymbol{\lambda}$ satisfies (30), and (31) for $i=2$, if and only if (56) holds; condition (29) holds for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{sd}(\lambda_1)$ if and only if (57) holds, which implies $\lambda_1 > 0$ (since $\mu > \mu_0$), so (31) holds for $i=1$; and for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{sd}(\lambda_1)$, (32) is identical to (58) and $\lambda_1 + \lambda_2^{sd}(\lambda_1) < \mu$.

We show that (56)-(58) are equivalent to the conditions in Parts 1-3 of the Proposition.

*Part 1.* If (37) does not hold, then no $\lambda_1$ satisfies (56) and (57): By Lemma 3.1, $\frac{\overline{v}_2}{c_2} \leq \frac{v_1(\lambda_1^\circ)}{c_1}$ implies $\overline{\lambda}_2^{sd}(\lambda_1^\circ) = 0$, so that $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ)\,,\mu) = 0$ by (36) and (57). Since $\Pi^{sd}(\boldsymbol{\lambda},\mu)$ is strictly concave in $\boldsymbol{\lambda}$ (by *A3*), $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu)$ strictly decreases in $\lambda_1$, so $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu) \neq 0$ if $\lambda_1 \neq \lambda_1^\circ$.

*Part 2.* If (38) does not hold, i.e., $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ)) \leq \mu/(\mu-\lambda_1^\circ)^2$, then no $\lambda_1$ satisfies (57)-(58): For $\lambda_1 < \lambda_1^\circ$ we have $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu) > 0$ since $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ)\,,\mu) \geq 0$ by (57), and $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu)$ strictly decreases in $\lambda_1$. For $\lambda_1 \geq \lambda_1^\circ$ we have $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) \leq w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu)$, because $\mu/(\mu-\lambda_1^\circ)^2 \leq w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ)\,,\mu)$ and $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) - w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu)$ strictly decreases in $\lambda_1$ by Lemma 3.3.

The equivalence (38) $\Leftrightarrow \varepsilon_2(\lambda_2^{sd}(\lambda_1^\circ)) > \varepsilon_1(\lambda_1^\circ)\frac{c_2}{c_1-c_2} + 1$ follows from (33) and (36).

*Part 3.* Suppose (37)-(38) hold. Write $\Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu;\Lambda_2)$ to express the dependence on $\Lambda_2$. Let $\lambda_1(\Lambda_2)$ be the rate that satisfies (57) for a given $\Lambda_2$: $\lambda_1(\Lambda_2) \triangleq \arg\{\lambda_1 \geq 0 : \Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)\,,\mu;\Lambda_2) = 0\}$. We show that $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2))$ satisfies (56) and (58) if and only if $\Lambda_2 < \overline{\Lambda}_2$ for some $\overline{\Lambda}_2 \in (0,\infty)$.

First, note that $\lambda_1(\Lambda_2)$ is unique since $\Pi^{sd}_{\lambda_1}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu; \Lambda_2)$ strictly decreases in $\lambda_1$, and $\lambda_1(0) = \lambda_1^\circ$ since $\Pi^{sd}_{\lambda_1}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ), \mu; 0) = 0$ by (36) and (57). Moreover, $\lambda_1(\Lambda_2)$ is continuous and strictly increasing in $\Lambda_2 \geq 0$: $\Pi^{sd}_{\lambda_1}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu; \Lambda_2)$ is continuous in $(\lambda_1, \Lambda_2)$, strictly decreasing in $\lambda_1$, and strictly increasing in $\Lambda_2$ for $\lambda_1 \geq \lambda_1^\circ$ since $v_1' < 0$ and $\overline{\lambda}_2^{sd}(\lambda_1) > 0$ for $\lambda_1 \geq \lambda_1^\circ$ (where $\frac{\overline{v}_2}{c_2} > \frac{v_1(\lambda_1^\circ)}{c_1}$ by (37) implies $\overline{\lambda}_2^{sd}(\lambda_1^\circ) > 0$ by Lemma 3.1, so that $\overline{\lambda}_2^{sd}(\lambda_1) > 0$ for $\lambda_1 \geq \lambda_1^\circ$ by Lemma 3.2).

Condition (56) holds for all $\Lambda_2$ since $\frac{\overline{v}_2}{c_2} > \frac{v_1(\lambda_1^\circ)}{c_1}$ by (37), $\lambda_1(\Lambda_2) \geq \lambda_1^\circ$, and $v_1' < 0$.

By (38) it follows that condition (58) holds for all $\Lambda_2$ such that

$$w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2)), \mu) \approx \frac{\mu}{(\mu - \lambda_1^\circ)^2} \text{ and } w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2))) \approx w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ)). \tag{59}$$

The relationships in (59) hold for small enough $\Lambda_2 > 0$: $w_2^{c\mu}(\boldsymbol{\lambda}, \mu)$ and $w_2^{sd}(\boldsymbol{\lambda})$ are continuous in $\boldsymbol{\lambda}$, and by continuity of $\lambda_1(\Lambda_2)$ and $\lambda_2^{sd}(\lambda_1)$ we have $\lambda_1(\Lambda_2) \approx \lambda_1(0) = \lambda_1^\circ$ and $\overline{\lambda}_2^{sd}(\lambda_1(\Lambda_2)) \approx \overline{\lambda}_2^{sd}(\lambda_1^\circ)$ for small $\Lambda_2$ where $\overline{\lambda}_2^{sd}(\lambda_1^\circ)$ is constant in $\Lambda_2 > 0$ (Lemma 3.4). Therefore, we have for small $\Lambda_2$: $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2))) \approx w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ))$ where $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1^\circ))$ is independent of $\Lambda_2 > 0$ (Lemma 3.4) and $\Lambda_2 \cdot \overline{\lambda}_2^{sd}(\lambda_1(\Lambda_2)) \approx 0$ so $w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2)), \mu) \approx \frac{\mu}{(\mu-\lambda_1^\circ)^2}$ by (58).

The existence of $\overline{\Lambda}_2 \in (0, \infty)$ holds by continuity and since $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2))) - w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1(\Lambda_2)), \mu)$ strictly decreases in $\Lambda_2$, which follows since $\lambda_1(\Lambda_2) \geq \lambda_1^\circ$ is strictly increasing in $\Lambda_2$, and $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) - w_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu)$ is strictly decreasing in $\lambda_1$ (Lemma 3.3) and nonincreasing in $\Lambda_2$ (Lemma 3.4).■

**Lemma 5** *By Lemma 3,* $\lambda_2^{sd}(\lambda_1) = \arg\max_{\lambda_2 \in [0, \Lambda_2]} \lambda_2 p_2^{sd}(\boldsymbol{\lambda})$ *and* $\boldsymbol{\lambda}^{sd}(\lambda_1) = \left(\lambda_1, \lambda_2^{sd}(\lambda_1)\right)$. *Define*

$$\begin{aligned}
\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1)) &\triangleq \frac{1}{c_1}\left[R_1'(\lambda_1) + \lambda_2^{sd}(\lambda_1)\, c_2 \frac{-v_1'(\lambda_1)}{c_1 - c_2}\right], && (60)\\
\lambda_1^{sd\infty} &\triangleq \min\{\lambda_1 \geq 0 : \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1)) \leq 0\},\ \textit{where } \lambda_1^{sd\infty} > 0, && (61)\\
\mu^*(\lambda_1) &\triangleq \arg\{\mu > \lambda_1 : \Pi_{\lambda_1}^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu) = 0\}\ \textit{for } \lambda_1 \in [0, \lambda_1^{sd\infty}], && (62)\\
\overline{\lambda}_1^{\infty} &\triangleq \min\{\lambda_1 \geq 0 : w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) \leq 0\}, && (63)\\
\overline{w}_2^{c\mu}(\boldsymbol{\lambda}, \mu) &\triangleq \begin{cases} \frac{\mu}{(\mu-\lambda_1)(\mu-\lambda_1-\lambda_2)} \in [0,\infty), & \textit{if } \mu > \lambda_1 + \lambda_2, \\ \infty, & \textit{if } \mu \leq \lambda_1 + \lambda_2. \end{cases} && (64)
\end{aligned}$$

1. *Strategic delay is optimal at capacity* $\mu = \mu^*(\lambda_1)$ *if and only if:*
   *(i)* $\lambda_1 \in (\underline{\lambda}_1, \lambda_1^{sd\infty}]$*, where* $\underline{\lambda}_1 = \min\{\lambda_1 \geq 0 : \frac{v_1(\lambda_1)}{c_1} \leq \frac{\overline{v}_2}{c_2}\}$*; and*
   *(ii)* $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) > \overline{w}_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu^*(\lambda_1))$*, where*

$$\mu^*(\lambda_1) = \lambda_1 + \frac{1 + \sqrt{1 + 4\lambda_1 \cdot \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))}}{2\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))} \textit{ and } \mu^{*\prime}(\lambda_1) > 0 \textit{ for } \lambda_1 \in [0, \lambda_1^{sd\infty}]. \tag{65}$$

2. *If strategic delay optimality condition 1.(ii) holds then* $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) > \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))$.

3. *If* $0 < \lambda_1 < \min(\lambda_1^{sd\infty}, \overline{\lambda}_1^{\infty})$ *then strategic delay optimality condition 1.(ii) satisfies:*

$$\begin{aligned}
w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) &> \overline{w}_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu^*(\lambda_1)) \Leftrightarrow g^{c\mu}(\lambda_1) - g^{sd}(\lambda_1) > 0, && (66)\\
\textit{where } g^{c\mu}(\lambda_1) &\triangleq \frac{\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))}{\overline{w}_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu^*(\lambda_1))} \textit{ and } g^{sd}(\lambda_1) \triangleq \frac{\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))}{w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1))}. && (67)
\end{aligned}$$

**Lemma 6** *Let $\mu_e^{-1} \triangleq \frac{\overline{v}_1-\overline{v}_2}{c_1-c_2}$ and define the functions*

$$
\begin{aligned}
k_1(\boldsymbol{\lambda},\mu) &\triangleq v_1(\lambda_1) - v_2(\lambda_2) - (c_1-c_2)\, w_1^{c\mu}(\lambda_1,\mu), & (68)\\
k_2(\boldsymbol{\lambda},\mu) &\triangleq v_1(\lambda_1) - v_2(\lambda_2) - (c_1-c_2)\, w_2^{c\mu}(\boldsymbol{\lambda},\mu). & (69)
\end{aligned}
$$

1. *$k_1(\mathbf{0},\mu) = k_2(\mathbf{0},\mu)$ and $k_1(\boldsymbol{\lambda},\mu) > k_2(\boldsymbol{\lambda},\mu)$ if $\boldsymbol{\lambda} \neq \mathbf{0}$.*
2. *$\partial k_1/\partial\lambda_1 < 0 < \partial k_1/\partial\lambda_2$, $\partial k_2/\partial\lambda_1 < 0$, and $\partial k_2/\partial\lambda_1 < \partial k_2/\partial\lambda_2$.*
3. *$k_i(\boldsymbol{\lambda}^f(\mu),\mu)$ is continuous in $\mu$ for $i=1,2$.*
4. *If $\lambda_1 = 0 = \lambda_2$ then $\Pi^f_{\lambda_1}(\mathbf{0},\mu) - \Pi^f_{\lambda_2}(\mathbf{0},\mu) = k_i(\mathbf{0},\mu) = (c_1-c_2)\left(\mu_e^{-1} - \mu^{-1}\right)$ for $i=1,2$.*
5. *If $\lambda_1 = 0 < \lambda_2$ then $\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu) - \Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu) > k_i(\boldsymbol{\lambda},\mu)$ for $i=1,2$.*
6. *If $\lambda_1 > 0 = \lambda_2$ then $\Pi^f_{\lambda_1}(\boldsymbol{\lambda},\mu) - \Pi^f_{\lambda_2}(\boldsymbol{\lambda},\mu) < k_i(\boldsymbol{\lambda},\mu)$ for $i=1,2$.*

**Lemma 7** *Fix $\mu > \mu_0$.*

1. *If $\frac{\overline{v}_1}{c_1} = \frac{\overline{v}_2}{c_2}$ then $\frac{\overline{v}_1-\overline{v}_2}{c_1-c_2} = \frac{\overline{v}_1}{c_1} = \frac{\overline{v}_2}{c_2} = \frac{1}{\mu_0} = \frac{1}{\mu^f} = \frac{1}{\mu^s} \geq \frac{1}{\mu^{sd}}$, and $\boldsymbol{\lambda}^s(\mu) > \mathbf{0}$ for $\mu > \mu^s$.*
2. *If $\frac{\overline{v}_1}{c_1} < \frac{\overline{v}_2}{c_2}$ then $\frac{\overline{v}_2}{c_2} > \frac{\overline{v}_1}{c_1} = \frac{1}{\mu_0} > \frac{1}{\mu^f} > \frac{1}{\mu^s} \geq \frac{\overline{v}_1-\overline{v}_2}{c_1-c_2}$ and $\boldsymbol{\lambda}^s(\mu) > \mathbf{0}$ if $\frac{1}{\mu} < \frac{\overline{v}_1-\overline{v}_2}{c_1-c_2}$.*
   - (a) *If $\mu \leq \mu^f$, the first-best is second-best and serves only type-2: $\boldsymbol{\lambda}^s(\mu) = \boldsymbol{\lambda}^f(\mu) \in M_0(\mu)$ with $\lambda_1^s(\mu) = 0 < \lambda_2^s(\mu)$. If $\mu \in (\mu^f, \mu^s)$, then $\boldsymbol{\lambda}^f(\mu) \in M_1(\mu)$ and $\lambda_1^s(\mu) = 0 < \lambda_2^s(\mu)$.*
   - (b) *If $\mu^{sd} < \infty$, then $\mu^s < \mu^{sd}$, and there is $\mu' \in \left(\mu^s, \mu^{sd}\right)$ such that $\mathbf{0} < \boldsymbol{\lambda}^f(\mu') \in M_0(\mu')$.*
3. *If $\frac{\overline{v}_1}{c_1} > \frac{\overline{v}_2}{c_2}$ then $\frac{\overline{v}_1-\overline{v}_2}{c_1-c_2} > \frac{\overline{v}_1}{c_1} > \frac{\overline{v}_2}{c_2} = \frac{1}{\mu_0} > \frac{1}{\mu^f} > \frac{1}{\mu^s}$.*
   - (a) *If $\mu \leq \mu^f$, the first-best is second-best and serves only type-1: $\boldsymbol{\lambda}^s(\mu) = \boldsymbol{\lambda}^f(\mu) \in M_0(\mu)$ with $\lambda_1^s(\mu) > \lambda_2^s(\mu) = 0$. If $\mu \in (\mu^f, \mu^s)$, then $\boldsymbol{\lambda}^f(\mu) \in M_2(\mu)$. If $\mu > \mu^f$ then $\lambda_1^s(\mu) > 0$.*
   - (b) *If $R_1'(\underline{\lambda}_1) > 0$, where $\underline{\lambda}_1 = \arg\{\lambda_1 \in [0,\Lambda_1] : \frac{v_1(\lambda_1)}{c_1} = \frac{\overline{v}_2}{c_2}\} \in (0,\Lambda_1)$, then $\boldsymbol{\lambda}^s(\mu) > \mathbf{0} \Leftrightarrow \mu > \mu^*(\underline{\lambda}_1)$ and*
   $$\mu^f < \mu^s = \mu^{sd} = \mu^*(\underline{\lambda}_1) = \underline{\lambda}_1 + \frac{1 + \sqrt{1 + 4\underline{\lambda}_1 \cdot R_1'(\underline{\lambda}_1)/c_1}}{2R_1'(\underline{\lambda}_1)/c_1}. \quad (70)$$
   - (c) *If $R_1'(\underline{\lambda}_1) \leq 0$, then $\lambda_2^s(\mu) = 0$ for all $\mu$, so that $\mu^s = \mu^{sd} = \infty$.*

*Remark:* Let $\overline{c} \triangleq c_1/c_2 > 1$, $\overline{v} \triangleq \overline{v}_1/\overline{v}_2$, and $\overline{\Lambda} \triangleq \Lambda_1/\Lambda_2$.

**Lemma 8** *Let $v_i(\lambda_i) = \overline{v}_i(1 - \frac{\lambda_i}{\Lambda_i})$, $i=1,2$, and $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c}-1))$ (Assumption A4).*

$$\underline{\lambda}_1 = \max(\Lambda_1(1 - \frac{\overline{c}}{\overline{v}}), 0) \text{ and } \lambda_2^{sd}(\lambda_1) = \frac{\Lambda_2}{2}(1 - \frac{\overline{v}}{\overline{c}}(1 - \frac{\lambda_1}{\Lambda_1})) \cdot I\{\lambda_1 \geqslant \underline{\lambda}_1\}. \quad (71)$$

1. *The function $\Pi^{sd\infty}_{\lambda_1}(\boldsymbol{\lambda}^{sd}(\lambda_1))$ is continuous, strictly decreasing in $\lambda_1 \geq 0$:*

$$\Pi^{sd\infty}_{\lambda_1}(\boldsymbol{\lambda}^{sd}(\lambda_1)) = \frac{\overline{v}_1}{c_1}\left[\left(1 - 2\frac{\lambda_1}{\Lambda_1}\right) + \lambda_2^{sd}(\lambda_1)\frac{\Lambda_1^{-1}}{\overline{c}-1}\right] = \begin{cases} \frac{\overline{v}_1}{c_1}(1 - 2\frac{\lambda_1}{\Lambda_1}), & \lambda_1 < \underline{\lambda}_1, \\ \frac{\overline{v}_1}{c_1}(\alpha - \beta\lambda_1), & \lambda_1 \geq \underline{\lambda}_1, \end{cases} \quad (72)$$

*where $\alpha \triangleq 1 + \left(1 - \frac{\overline{v}}{\overline{c}}\right)\frac{\Lambda_2/\Lambda_1}{2(\overline{c}-1)}$, and $\beta \triangleq \frac{2}{\Lambda_1}(1 - \frac{\overline{v}\Lambda_2/\Lambda_1}{4\overline{c}(\overline{c}-1)}) > 0$,*

$$\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\underline{\lambda}_1))=\begin{cases}\frac{\overline{v}_1}{c_1}\alpha & > \frac{\overline{v}_1}{c_1} = \frac{R_1'(\underline{\lambda}_1)}{c_1} = \frac{v_1(\underline{\lambda}_1)}{c_1}, & \frac{\overline{v}}{\overline{c}} < 1 \text{ (so } \underline{\lambda}_1 = 0), \\ \frac{\overline{v}_1}{c_1}\alpha & = \frac{\overline{v}_1}{c_1} = \frac{R_1'(\underline{\lambda}_1)}{c_1} = \frac{v_1(\underline{\lambda}_1)}{c_1}, & \frac{\overline{v}}{\overline{c}} = 1 \text{ (so } \underline{\lambda}_1 = 0), \\ \frac{\overline{v}_1}{c_1}(\alpha-\beta\underline{\lambda}_1) & = \frac{\overline{v}_1}{c_1}(1-2\frac{\underline{\lambda}_1}{\Lambda_1}) = \frac{R_1'(\underline{\lambda}_1)}{c_1} < \frac{v_1(\underline{\lambda}_1)}{c_1}, & \frac{\overline{v}}{\overline{c}} > 1 \text{ (so } \underline{\lambda}_1 > 0),\end{cases} \tag{73}$$

*and*

$$\overline{v} < 2\overline{c} \Leftrightarrow \underline{\lambda}_1 < \lambda_1^{sd\infty} = \frac{\alpha}{\beta} = \Lambda_1\left(\frac{1}{2} + \frac{\Lambda_2(\overline{c}-\overline{v}/2)}{4\Lambda_1\overline{c}(\overline{c}-1)-\Lambda_2\overline{v}}\right) > \frac{\Lambda_1}{2}. \tag{74}$$

2. *The function $w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1))$ is continuous, strictly decreasing in $\lambda_1 \geq 0$:*

$$w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) = \left\{\begin{matrix}\frac{\overline{v}_1(1-\lambda_1/\Lambda_1)-\overline{v}_2}{c_1-c_2}, & \lambda_1 < \underline{\lambda}_1 \\ \frac{\overline{v}_1}{c_1}(\gamma-\delta\lambda_1), & \lambda_1 \geq \underline{\lambda}_1\end{matrix}\right\}, \quad \gamma \triangleq \frac{2\overline{c}-1-\overline{c}/\overline{v}}{2(\overline{c}-1)}, \quad \delta \triangleq \frac{1}{\Lambda_1}\frac{2\overline{c}-1}{2(\overline{c}-1)} > 0, \tag{75}$$

$$w_2^{sd}(\boldsymbol{\lambda}^{sd}(\underline{\lambda}_1)) = \begin{cases}\frac{\overline{v}_1}{c_1}\gamma & < \frac{v_1(\underline{\lambda}_1)}{c_1} = \frac{\overline{v}_1}{c_1} < \frac{v_2(\lambda_2^{sd}(\underline{\lambda}_1))}{c_2} < \frac{\overline{v}_2}{c_2}, & \frac{\overline{v}}{\overline{c}} < 1 \text{ (so } \underline{\lambda}_1 = 0), \\ \frac{\overline{v}_1}{c_1}\gamma & = \frac{v_1(\underline{\lambda}_1)}{c_1} = \frac{\overline{v}_1}{c_1} = \frac{v_2(\lambda_2^{sd}(\underline{\lambda}_1))}{c_2} = \frac{\overline{v}_2}{c_2}, & \frac{\overline{v}}{\overline{c}} = 1 \text{ (so } \underline{\lambda}_1 = 0), \\ \frac{\overline{v}_1}{c_1}(\gamma-\delta\underline{\lambda}_1) & = \frac{v_1(\underline{\lambda}_1)}{c_1} = \frac{v_2(\lambda_2^{sd}(\underline{\lambda}_1))}{c_2} = \frac{\overline{v}_2}{c_2} < \frac{\overline{v}_1}{c_1}, & \frac{\overline{v}}{\overline{c}} > 1 \text{ (so } \underline{\lambda}_1 > 0),\end{cases} \tag{76}$$

*and*

$$\overline{v} > \overline{c}/(2\overline{c}-1) \Leftrightarrow \underline{\lambda}_1 < \overline{\lambda}_1^{\infty} = \frac{\gamma}{\delta} = \Lambda_1\left(1-\frac{\overline{c}/\overline{v}}{2\overline{c}-1}\right), \tag{77}$$

$$\overline{v} \leq \overline{c}/(2\overline{c}-1) \Rightarrow \overline{\lambda}_1^{\infty} = 0. \tag{78}$$

**Lemma 9** *Let $v_i(\lambda_i) = \overline{v}_i(1-\frac{\lambda_i}{\Lambda_i})$, $i=1,2$, with $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c}-1))$ (Assumption A4) and $\overline{v} < 2\overline{c}$.*

1. *Let $\overline{\Lambda}_2 \triangleq 2\Lambda_1(\overline{c}-\frac{1}{2}-\frac{\overline{c}}{\overline{v}})$. Then $\Lambda_2 < \overline{\Lambda}_2 \Rightarrow \lambda_1^{sd\infty} < \overline{\lambda}_1^{\infty}$, $\Lambda_2 = \overline{\Lambda}_2 \Rightarrow \lambda_1^{sd\infty} = \overline{\lambda}_1^{\infty}$, and $\Lambda_2 > \overline{\Lambda}_2 \Rightarrow \lambda_1^{sd\infty} > \overline{\lambda}_1^{\infty}$.*

2. *If $\overline{c}/(2\overline{c}-1) < \overline{v} < 2\overline{c}$, then $g^{sd}(\underline{\lambda}_1) > 0$. Furthermore:*

$$g^{sd}(\lambda_1) = \frac{\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1))}{w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1))} = \frac{\alpha-\beta\lambda_1}{\gamma-\delta\lambda_1} = \frac{\beta}{\delta}\frac{\lambda_1^{sd\infty}-\lambda_1}{\overline{\lambda}_1^{\infty}-\lambda_1} \text{ for } \lambda_1 \geq \underline{\lambda}_1. \tag{79}$$

   (a) *If $\lambda_1^{sd\infty} < \overline{\lambda}_1^{\infty}$ then $g^{sd}(\lambda_1^{sd\infty}) = 0$, $g^{sd\prime} < 0$ and $g^{sd\prime\prime} < 0$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$.*
   (b) *If $\lambda_1^{sd\infty} = \overline{\lambda}_1^{\infty}$ then $g^{sd} = \frac{\alpha}{\gamma} = \frac{\beta}{\delta} > 0$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$.*
   (c) *If $\lambda_1^{sd\infty} > \overline{\lambda}_1^{\infty}$ then $g^{sd} > 0$, $g^{sd\prime} > 0$, and $g^{sd\prime\prime} > 0$ on $[\underline{\lambda}_1, \overline{\lambda}_1^{\infty}]$.*

3. *If $\overline{v} > \overline{c}$ then $g^{sd}(\underline{\lambda}_1)$ is constant in $\Lambda_2$, and $g^{sd}(\lambda_1)$ increases in $\Lambda_2$ for fixed $\lambda_1 \in (\underline{\lambda}_1, \overline{\lambda}_1^{\infty})$.*

**Lemma 10** *Let $v_i(\lambda_i) = \overline{v}_i(1-\frac{\lambda_i}{\Lambda_i})$, $i=1,2$, with $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c}-1))$ (Assumption A4) and $\overline{v} < 2\overline{c}$.*

$$g_1^{c\mu}(\lambda_1) \triangleq 1 - \frac{\lambda_2^{sd}(\lambda_1)-\lambda_2^{sd}(\underline{\lambda}_1)}{\mu^*(\lambda_1)-\lambda_1} \text{ and } g_2^{c\mu}(\lambda_1) \triangleq -\frac{\lambda_2^{sd}(\underline{\lambda}_1)}{\mu^*(\lambda_1)-\lambda_1}, \quad \lambda_1 \in [0, \lambda_1^{sd\infty}]. \tag{80}$$

1. *$g^{c\mu} \leq 1$ on $[0, \lambda_1^{sd\infty}]$; $g^{c\mu} < 1$ on $(\underline{\lambda}_1, \lambda_1^{sd\infty})$; and $g^{c\mu}(\lambda_1^{sd\infty}) = 1$, where*

$$g^{c\mu}(\lambda_1) = \max\left(0, 1-\frac{\lambda_2^{sd}(\lambda_1)}{\mu^*(\lambda_1)-\lambda_1}\right) = \max(0, g_1^{c\mu}(\lambda_1) + g_2^{c\mu}(\lambda_1)), \quad \lambda_1 \in [0, \lambda_1^{sd\infty}]. \tag{81}$$

2. (a) *$g_1^{c\mu} \leq 1$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$, and $g_1^{c\mu}(\underline{\lambda}_1) = g_1^{c\mu}(\lambda_1^{sd\infty}) = 1$, where*

$$g_1^{c\mu}(\lambda_1) = 1 - \frac{\overline{v}}{2\overline{c}\overline{\Lambda}}\frac{(\lambda_1-\underline{\lambda}_1)}{\mu^*(\lambda_1)-\lambda_1} = 1 - \frac{\overline{v}}{\overline{c}\overline{\Lambda}}\frac{(\lambda_1-\underline{\lambda}_1)\frac{\overline{v}_1}{c_1}(\alpha-\beta\lambda_1)}{1+\sqrt{1+4\lambda_1\cdot\frac{\overline{v}_1}{c_1}(\alpha-\beta\lambda_1)}}, \quad \lambda_1 \in [\underline{\lambda}_1, \lambda_1^{sd\infty}]. \tag{82}$$

   (b) *As $\Lambda_2 \to 0$, $g_1^{c\mu} \to 1$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$. Moreover, $g_1^{c\mu\prime\prime} > 0$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$.*
   (c) *For fixed $\lambda_1 \in (\underline{\lambda}_1, \lambda_1^{sd\infty}]$ the function $g_1^{c\mu}(\lambda_1)$ strictly decreases in $\Lambda_2$.*

3. (a) *If* $\overline{v} < \overline{c}$ *then* $g_2^{c\mu} < 0$ *on* $[0, \lambda_1^{sd\infty})$, $g_2^{c\mu}\left(\lambda_1^{sd\infty}\right) = 0$, *and* $g_2^{c\mu\prime} > 0$ *on* $\left[0, \lambda_1^{sd\infty}\right]$.
   (b) *If* $\overline{v} \geq \overline{c}$ *then* $g_2^{c\mu} = 0$ *on* $[0, \lambda_1^{sd\infty}]$.

**Proof of Proposition 5**. *Part 1.* These claims are proved in Lemma 7.1.

*Part 2.* Let $v_i(\lambda_i) = \overline{v}_i(1 - \frac{\lambda_i}{\Lambda i})$, $i=1,2$, with $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c}-1))$ (Assumption A4) and $\overline{v} = \overline{c}$. By (71) we have $\underline{\lambda}_1 = 0 = \lambda_2^{sd}(\underline{\lambda}_1)$ for $\overline{v} = \overline{c}$. From (73) and (76)

$$\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(0)) = \frac{\overline{v}_1}{c_1} = w_2^{sd}(\boldsymbol{\lambda}^{sd}(0)) \text{ and } g^{sd}(0) = 1. \tag{83}$$

If $\Lambda_2 \geq \Lambda_1 2(\overline{c} - 1.5)$, then $\Lambda_2 \geq \overline{\Lambda}_2$ and $\lambda_1^{sd\infty} \geq \overline{\lambda}_1^{\infty}$ by Lemma 9.1. Therefore $\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\overline{\lambda}_1^{\infty})) \geq 0 = w_2^{sd}(\boldsymbol{\lambda}^{sd}(\overline{\lambda}_1^{\infty}))$, since $\lambda_1^{sd\infty} = \min\{\lambda_1 \geq 0 : \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1)) \leq 0\}$ by (61) and $\overline{\lambda}_1^{\infty} = \min\{\lambda_1 \geq 0 : w_2^{sd}(\boldsymbol{\lambda}^{sd}(\lambda_1)) \leq 0\}$ by (63). Combined with (83), and since $\Pi_{\lambda_1}^{sd\infty}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right)$ and $w_2^{sd}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right)$ are linearly decreasing, it follows that $\Pi_{\lambda_1}^{sd\infty}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right) \geq w_2^{sd}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right)$ on $[0, \lambda_1^{sd\infty}]$. By Lemma 5.2 strategic delay cannot be optimal at any capacity level.

If $\Lambda_2 < \Lambda_1 2(\overline{c} - 1.5)$ then $\Lambda_2 < \overline{\Lambda}_2$ and $\lambda_1^{sd\infty} < \overline{\lambda}_1^{\infty}$ by Lemma 9.1. Hence, $w_2^{sd}\left(\boldsymbol{\lambda}^{sd}\left(\lambda_1^{sd\infty}\right)\right) > \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1^{sd\infty})) = 0 = \overline{w}_2^{c\mu}(\boldsymbol{\lambda}^{sd}(\lambda_1^{sd\infty}), \mu^*(\lambda_1^{sd\infty}))$, where the last equality follows since $\mu^*(\lambda_1^{sd\infty}) = \infty$ by (65). This implies by Lemma 5.1 that strategic delay is optimal for $\mu = \infty$. Next, we prove there is an unique $\underline{x} \in [0, \lambda_1^{sd\infty})$ with $g^{c\mu}(x) - g^{sd}(x) \leq 0$ for $x \in [0, \underline{x}]$ and $g^{c\mu}(x) - g^{sd}(x) > 0$ for $x \in (\underline{x}, \lambda_1^{sd\infty}]$, which implies by Lemma 5.1 and 5.3, and since $\mu^{*\prime} > 0$ by (65), that strategic delay is optimal if and only if $\mu > \mu^{sd} = \mu^*(\underline{x})$.

We have $g^{c\mu} = \max(0, g_1^{c\mu})$ by Lemma 10.1, since $g_2^{c\mu} = 0$ by Lemma 10.3.(b) for $\overline{v} = \overline{c}$. Note

$$g^{c\mu}(0) - g^{sd}(0) = 0, \text{ and } g^{c\mu}(\lambda_1^{sd\infty}) - g^{sd}(\lambda_1^{sd\infty}) = 1. \tag{84}$$

The first equality holds since $g^{sd}(0) = 1$ by (83), and $g^{c\mu}(0) = 1$ because $g_1^{c\mu}(\underline{\lambda}_1) = 1$ by Lemma 10.2.(a) and $\underline{\lambda}_1 = 0$. The second equality holds since $g^{sd}\left(\lambda_1^{sd\infty}\right) = 0$ by Lemma 9.2.(a) and $g_1^{c\mu}\left(\lambda_1^{sd\infty}\right) = 1$ by Lemma 10.2.(a). Since $g^{sd} \geq 0$ on $[0, \lambda_1^{sd\infty}]$ by Lemma 9.2.(a), we have $g^{c\mu}(x) - g^{sd}(x) > 0 \Leftrightarrow g_1^{c\mu}(x) - g^{sd}(x) > 0$. Given (84), and since $g_1^{c\mu\prime\prime} - g^{sd\prime\prime} > 0$ (Lemma 9.2.(a) and Lemma 10.2.(b)), we have two cases. $(i)$ if $g_1^{c\mu\prime}(0) \geq g^{sd\prime}(0)$ then $\underline{x} = 0$, and $\mu^*(0) = \frac{c_1}{\overline{v}_1} = \mu_0$ by (65) and (83). That $g_1^{c\mu\prime}(0) - g^{sd\prime}(0) \geq 0 \Leftrightarrow \frac{\Lambda_2}{\Lambda_1} \leq \frac{2(\overline{c}-1.5)}{1+\overline{v}_1/c_1\Lambda_1(\overline{c}-1)}$ follows since $g^{sd\prime}(0) = \frac{1}{2\overline{\Lambda}}\frac{1-2(\overline{c}-1.5)\overline{\Lambda}}{\Lambda_1(\overline{c}-1)}$ by (79) and $g_1^{c\mu\prime}(0) = -\frac{1}{2\overline{\Lambda}}\frac{\overline{v}_1}{c_1}$ by (82) (for $g^{sd\prime}(0)$ note that $\overline{v} = \overline{c}$ implies $\alpha = 1$ by (72), $\lambda_1^{sd\infty} = 1/\beta$ by (74), $\gamma = 1$ by (75), and $\overline{\lambda}_1^{\infty} = 1/\delta$ by (77); for $g_1^{c\mu\prime}(0)$ use $\alpha = 1$ and $\underline{\lambda}_1 = 0$). $(ii)$ if $g_1^{c\mu\prime}(0) < g^{sd\prime}(0)$ then $\underline{x} \in (0, \lambda_1^{sd\infty})$ and $\mu^*(\underline{x}) > \mu^*(0) = \mu_0$ since $\mu^{*\prime} > 0$.■

**Proof of Proposition 6**. *Part 1.* These claims are proved in Lemma 7.2.

*Part 2.* Let $v_i(\lambda_i) = \overline{v}_i(1 - \frac{\lambda_i}{\Lambda i})$, $i=1,2$, with $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c}-1))$ (Assumption A4) and $\overline{v} < \overline{c}$. By (71) we have $\underline{\lambda}_1 = 0$ and $\lambda_2^{sd}(\underline{\lambda}_1) = \frac{\Lambda_2}{2}(1 - \frac{\overline{v}}{\overline{c}}) > 0$ for $\overline{v} < \overline{c}$. From (73) and (76)

$$\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(0)) > \frac{\overline{v}_1}{c_1} > w_2^{sd}(\boldsymbol{\lambda}^{sd}(0)). \tag{85}$$

If $\overline{c}/(\overline{c} - 1/2) \geq \overline{v}$ or $\Lambda_2 \geq 2\Lambda_1\left(\overline{c} - \frac{1}{2} - \frac{\overline{c}}{\overline{v}}\right)$, then $\Lambda_2 \geq \overline{\Lambda}_2$ and $\lambda_1^{sd\infty} \geq \overline{\lambda}_1^{\infty}$ by Lemma 9.1. By same line of argument as in the proof of Prop. 5, strategic delay cannot be optimal at any capacity.

If $\overline{c} > \overline{v} > \overline{c}/(\overline{c} - 1/2)$ and $\Lambda_2 < 2\Lambda_1\left(\overline{c} - \frac{1}{2} - \frac{\overline{c}}{\overline{v}}\right)$ then $\lambda_1^{sd\infty} < \overline{\lambda}_1^{\infty}$ by Lemma 9.1, so that strategic delay is optimal for $\mu = \infty$ by same line of argument as in the proof of Prop. 5. Next, we prove that there exists an unique $\underline{z} \in \left(0, \lambda_1^{sd\infty}\right)$ with $g^{c\mu}(\underline{z}) = g^{sd}(\underline{z})$ and $g^{c\mu}(z) - g^{sd}(z) > 0$ for $z \in (\underline{z}, \lambda_1^{sd\infty}]$, which implies by Lemma 5.1 and 5.3, and since $\mu^{*\prime} > 0$ by (65), that strategic delay is optimal if and only if $\mu > \mu^{sd} = \mu^*(\underline{z})$.

That there exists $\underline{z} \in \left(0, \lambda_1^{sd\infty}\right)$ with $g^{c\mu}(\underline{z}) = g^{sd}(\underline{z})$ follows since $g^{c\mu} - g^{sd}$ is continuous,

$$g^{c\mu}(0) - g^{sd}(0) < 0, \text{ and } g^{c\mu}(\lambda_1^{sd\infty}) - g^{sd}(\lambda_1^{sd\infty}) = 1. \tag{86}$$

The inequality in (86) holds since $g^{c\mu} = \max(0, g_1^{c\mu} + g_2^{c\mu})$, $g_1^{c\mu}(0) = 1$, and $g_2^{c\mu}(0) < 0$ (Parts 1, 2.(a), and 3.(a) of Lemma 10, respectively, since $\underline{\lambda}_1 = 0 < \lambda_1^{sd\infty}$), and because $g^{sd}(0) > 1$ (by (85), and since $w_2^{sd}(\boldsymbol{\lambda}^{sd}(0)) > 0$ for $\overline{v} > \overline{c}/(\overline{c} - 1/2)$ by (75)). The equality in (86) holds since $g^{c\mu}\left(\lambda_1^{sd\infty}\right) = 1$ by Lemma 10.1, and $g^{sd}\left(\lambda_1^{sd\infty}\right) = 0$ for $\lambda_1^{sd\infty} < \overline{\lambda}_1^{\infty}$ by Lemma 9.2.(a).

Next, we prove the claim: $g^{c\mu}(z) - g^{sd}(z) > 0$ for $z \in (\underline{z}, \lambda_1^{sd\infty}]$. Since $g^{sd} \geq 0$ on $[0, \lambda_1^{sd\infty}]$ by Lemma 9.2.(a), and $g^{c\mu} = \max(0, g_1^{c\mu} + g_2^{c\mu})$, it follows that $g^{c\mu}(z) - g^{sd}(z) > 0 \Leftrightarrow g_1^{c\mu}(z) + g_2^{c\mu}(z) - g^{sd}(z) > 0$, and $g_1^{c\mu}(\underline{z}) + g_2^{c\mu}(\underline{z}) - g^{sd}(\underline{z}) = 0$. Since $g_2^{c\mu\prime} > 0$ by Lemma 10.3.(a), the claim holds if $g_1^{c\mu}(z) - g^{sd}(z) > g_1^{c\mu}(\underline{z}) - g^{sd}(\underline{z})$ for $z \in (\underline{z}, \lambda_1^{sd\infty}]$. This holds since $g_1^{c\mu} - g^{sd}$ is strictly convex ($g^{sd\prime\prime} < 0$ by Lemma 9.2.(a), $g_1^{c\mu\prime\prime} > 0$ by Lemma 10.2.(b)), and moreover

$$g_1^{c\mu}(0) - g^{sd}(0) < 0 < g_1^{c\mu}(\underline{z}) - g^{sd}(\underline{z}) = -g_2^{c\mu}(\underline{z}),$$

because $g_1^{c\mu}(0) = 1 < g^{sd}(0)$ (shown above) and $g_2^{c\mu}(\underline{z}) < 0$ (by Lemma 10.3.(a) since $\underline{z}_1 < \lambda_1^{sd\infty}$).

It remains to prove the lower bound $\mu^*(\underline{x})$ in (50). By Proposition 4, for any given $\mu$, strategic delay is optimal only if it is so for $\Lambda_2 \approx 0$. As $\Lambda_2 \to 0$, we have that $g^{c\mu} \to 1$ on $[0, \Lambda_1/2]$, because $\lambda_2^{sd}(\lambda_1) \to 0$ by (71), and $\lambda_1^{sd\infty} \to \Lambda_1/2$ by (74), so that (72) and (64)-(65) imply:

$$\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\lambda_1)) = \frac{\overline{v}_1}{c_1}\left(1 - 2\frac{\lambda_1}{\Lambda_1}\right) = \frac{\mu^*(\lambda_1)}{(\mu^*(\lambda_1) - \lambda_1)^2} = \overline{w}_2^{c\mu}\left(\boldsymbol{\lambda}^{sd}(\lambda_1), \mu^*(\lambda_1)\right) \text{ for } \lambda_1 \in [0, \Lambda_1/2].$$

It follows from Lemma 5.1 and 5.3 that strategic delay is optimal for $\Lambda_2 \approx 0$ and $\mu = \mu^*(x)$ if and only if $g^{sd}(x) < 1$. Since $g^{sd}(0) > 1$, $g^{sd}\left(\lambda_1^{sd\infty}\right) = 0$, and $g^{sd\prime} < 0$ by Lemma 9.2.(a), the equation $g^{sd}(\underline{x}) = 1$ has an unique solution. Setting $\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\underline{x})) = w_2^{sd}(\boldsymbol{\lambda}^{sd}(\underline{x})) = \frac{\overline{v}_1}{c_1}(\gamma - \delta\underline{x})$ yields

$$\underline{x} = \frac{\Lambda_1}{2}\frac{\overline{c}/\overline{v} - 1}{\overline{c} - 1.5} > 0 \text{ and } \mu^{sd} = \mu^*(\underline{x}) = \underline{x} + \frac{1 + \sqrt{1 + 4\underline{x}\frac{\overline{v}_1}{c_1}(1 - 2\underline{x}/\Lambda_1)}}{2\frac{\overline{v}_1}{c_1}(1 - 2\underline{x}/\Lambda_1)}.$$

That $\mu^*(\underline{x}) > \frac{c_1 - c_2}{\overline{v}_1 - \overline{v}_2}$ follows by noting that $\frac{\overline{v}_1 - \overline{v}_2}{c_1 - c_2} > \frac{\overline{v}_1}{c_1}\left(1 - 2\frac{\underline{x}}{\Lambda_1}\right) = \frac{\mu^*(\underline{x})}{(\mu^*(\underline{x}) - \underline{x})^2} > \frac{1}{\mu^*(\underline{x})}$.■

**Proof of Proposition 7**. *Part 1.* These claims are proved in Lemma 7.3.

*Part 2.* Let $v_i(\lambda_i) = \overline{v}_i(1 - \frac{\lambda_i}{\Lambda_i})$, $i=1,2$, with $\overline{\Lambda} > \overline{v}/(4\overline{c}(\overline{c} - 1))$ (Assumption A4) and $\overline{v} > \overline{c}$. By (71) we have $\underline{\lambda}_1 = \Lambda_1(1 - \frac{\overline{c}}{\overline{v}})$, so that $R_1'(\underline{\lambda}_1) = \overline{v}_1\left(2\frac{\overline{c}}{\overline{v}} - 1\right) > 0 \Leftrightarrow \overline{v} < 2\overline{c}$. By Part 1.(b) strategic delay is not optimal if $\overline{v} \geq 2\overline{c}$. In the rest of the proof $\overline{c} < \overline{v} < 2\overline{c}$.

By (71) we have $\lambda_2^{sd}(\underline{\lambda}_1) = 0$ for $\overline{v} > \overline{c}$. Furthermore

$$\overline{w}_2^{c\mu}\left(\boldsymbol{\lambda}^{sd}(\underline{\lambda}_1), \mu^*(\underline{\lambda}_1)\right) = \frac{\mu^*(\underline{\lambda}_1)}{(\mu^*(\underline{\lambda}_1) - \underline{\lambda}_1)^2} = \Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\underline{\lambda}_1)) < \frac{v_1(\underline{\lambda}_1)}{c_1} = w_2^{sd}(\boldsymbol{\lambda}^{sd}(\underline{\lambda}_1)), \tag{87}$$

where the first equation holds by (64) since $\lambda_2^{sd}(\underline{\lambda}_1) = 0$, the second holds by (65), the inequality holds by (73), and the last equality holds by (76). By (87) and the definitions in (67) we have $g^{c\mu}(\underline{\lambda}_1) = 1 > g^{sd}(\underline{\lambda}_1)$. Let $\overline{\Lambda}_2 \triangleq 2\Lambda_1(\overline{c} - \frac{1}{2} - \frac{\overline{c}}{\overline{v}})$.

*Part 2.(a)* In this case $\Lambda_2 > \overline{\Lambda}_2$, since $\overline{v} \leq \overline{c}/(\overline{c} - \frac{1}{2}) \Rightarrow \overline{\Lambda}_2 \leq 0$. By Lemma 9.1, $\Lambda_2 > \overline{\Lambda}_2 \Rightarrow \lambda_1^{sd\infty} > \overline{\lambda}_1^{\infty}$, so that $\Pi_{\lambda_1}^{sd\infty}(\boldsymbol{\lambda}^{sd}(\overline{\lambda}_1^{\infty})) > w_2^{sd}(\boldsymbol{\lambda}^{sd}(\overline{\lambda}_1^{\infty})) = 0$. By (87), and because $\Pi_{\lambda_1}^{sd\infty}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right)$ and $w_2^{sd}\left(\boldsymbol{\lambda}^{sd}(\lambda_1)\right)$ are linear, they cross once, at some $\underline{x} \in (\underline{\lambda}_1, \overline{\lambda}_1^{\infty})$, and $g^{sd}(\underline{x}) = 1$. Since $\Pi_{\lambda_1}^{sd\infty}\left(\boldsymbol{\lambda}^{sd}(x)\right) \geq w_2^{sd}\left(\boldsymbol{\lambda}^{sd}(x)\right)$ for $x \geq \underline{x}$, by Lemma 5.2 strategic delay is not optimal for $\mu \geq \mu^*(\underline{x})$.

By (87) we have $g^{c\mu}(\underline{\lambda}_1) = 1 > g^{sd}(\underline{\lambda}_1)$. Since $\underline{x} \in (\underline{\lambda}_1, \overline{\lambda}_1^{\infty})$ we have $g^{c\mu}(\underline{x}) < 1$ by Lemma 10.1. It follows that $g^{c\mu}(\underline{\lambda}_1) - g^{sd}(\underline{\lambda}_1) > 0 > g^{c\mu}(\underline{x}) - g^{sd}(\underline{x})$. By continuity $g^{c\mu} - g^{sd}$ has an odd number of roots in $(\underline{\lambda}_1, \underline{x})$. Let $x_s$ be the smallest and $x_l$ the largest root, i.e., $\underline{\lambda}_1 < x_s \le x_l < \underline{x}$, where $g^{c\mu}(x) > g^{sd}(x)$ for $x \in [\underline{\lambda}_1, x_s)$ and $g^{c\mu}(x) \le g^{sd}(x)$ for $x \in [x_l, \underline{x}]$. Since $\mu^{*\prime} > 0$ on $\left[0, \lambda_1^{sd\infty}\right]$ by (65), we have by Lemma 5.3 that strategic delay is optimal for $\mu \in (\mu^*(\underline{\lambda}_1), \mu^*(x_s))$ and not optimal for $\mu \ge \mu^*(x_l)$, so that $\mu^{sd} = \mu^*(\underline{\lambda}_1) < \mu^*(x_s) = \overline{\mu} \le \overline{\overline{\mu}} = \mu^*(x_l) < \mu^*(\underline{x})$.

*Part 2.(b)* If $\overline{c}/(\overline{c} - 0.5) < \overline{v}$ then $\overline{\Lambda}_2 > 0$. For $\Lambda_2 \le \overline{\Lambda}_2$ we characterize the subset(s) of $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$ where $g^{c\mu} - g^{sd} > 0$. (By Lemma 5 this yields the capacity sets for which strategic delay is optimal.) We focus on $g_1^{c\mu} - g^{sd}$, because $g^{c\mu} > g^{sd} \Leftrightarrow g_1^{c\mu} > g^{sd}$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$, which holds since $g^{c\mu} = \max(0, g_1^{c\mu})$ and $g^{sd} \ge 0$. (That $g^{c\mu} = \max(0, g_1^{c\mu})$ holds by Lemma 10: $g^{c\mu} = \max(0, g_1^{c\mu} + g_2^{c\mu})$ by (81), and $g_2^{c\mu} = 0$ on $[0, \lambda_1^{sd\infty}]$ by Part 3.(b). That $g^{sd} \ge 0$ holds by Lemma 9: $\Lambda_2 \le \overline{\Lambda}_2 \Rightarrow \lambda_1^{sd\infty} \le \overline{\lambda}_1^{\infty}$ by Part 1, and $\lambda_1^{sd\infty} \le \overline{\lambda}_1^{\infty} \Rightarrow g^{sd} \ge 0$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$ by Parts 2.(a)-(b).)

For fixed $\Lambda_2 \le \overline{\Lambda}_2$, $g_1^{c\mu} - g^{sd}$ is strictly convex on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$: we have $g^{sd\prime\prime} \le 0$ by Parts 1 and 2.(a)-(b) of Lemma 9, and $g_1^{c\mu\prime\prime} > 0$ by Lemma 10.2.(b). Furthermore we have

$$0 < g_1^{c\mu}(\underline{\lambda}_1) - g^{sd}(\underline{\lambda}_1) \le g_1^{c\mu}(\lambda_1^{sd\infty}) - g^{sd}(\lambda_1^{sd\infty}) \le 1, \tag{88}$$

since $g_1^{c\mu}(\underline{\lambda}_1) = g_1^{c\mu}\left(\lambda_1^{sd\infty}\right) = 1$ by Lemma 10.2.(a), $1 > g^{sd}(\underline{\lambda}_1)$ by (87), and $g^{sd}(\underline{\lambda}_1) \ge g^{sd}\left(\lambda_1^{sd\infty}\right) \ge 0$ since $g^{sd\prime} \le 0$ by Lemma 9.2.(a)-(b).

Hence, for fixed $\Lambda_2 \le \overline{\Lambda}_2$, $g_1^{c\mu} - g^{sd}$ has an unique minimum on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$. Making the dependence on $\Lambda_2$ explicit, let $x_{\min}(\Lambda_2) \triangleq \arg\min_{x \in [\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2)]} \{g_1^{c\mu}(x, \Lambda_2) - g^{sd}(x, \Lambda_2)\}$, where $x_{\min}(\Lambda_2) < \lambda_1^{sd\infty}(\Lambda_2)$ by (88) and since $g_1^{c\mu} - g^{sd}$ is strictly convex on $[\underline{\lambda}_1, \lambda_1^{sd\infty}]$ for fixed $\Lambda_2$. By (79) and (82), $g_1^{c\mu} - g^{sd}$ is continuous on $\left\{(\lambda_1, \Lambda_2) : \Lambda_2 \in (0, \overline{\Lambda}_2], \lambda_1 \in [\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2))\right\}$, so $x_{\min}(\Lambda_2)$ and the value $d(\Lambda_2) \triangleq g_1^{c\mu}(x_{\min}(\Lambda_2), \Lambda_2) - g^{sd}(x_{\min}(\Lambda_2), \Lambda_2)$ are continuous on $(0, \overline{\Lambda}_2]$.

Let $\underline{\Lambda}_2 \triangleq \sup\left\{\Lambda_2 \in (0, \overline{\Lambda}_2] : d(\Lambda_2) > 0\right\}$. Since $d(\Lambda_2)$ is continuous, we have $\underline{\Lambda}_2 > 0$ because $d(\Lambda_2) > 0$ as $\Lambda_2 \to 0$; this holds since $g_1^{c\mu}(\underline{\lambda}_1) - g^{sd}(\underline{\lambda}_1) > 0$ by (88), and $g_1^{c\mu\prime} - g^{sd\prime} > 0$ on $[\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2)]$ as $\Lambda_2 \to 0$ ($g^{sd\prime} < 0$ by Lemma 9.2.(a), and $g_1^{c\mu} \to 1$ as $\Lambda_2 \to 0$ by Lemma 10.2.(b)).

Next, we have $d(\Lambda_2) > 0$ if $\Lambda_2 \in (0, \underline{\Lambda}_2)$, and $d(\Lambda_2) < 0$ if $\Lambda_2 \in \left(\underline{\Lambda}_2, \overline{\Lambda}_2\right)$: this follows because $d(\Lambda_2) \le 0 \Rightarrow d(\Lambda_2') < 0$ for $\Lambda_2' > \Lambda_2$, due to four properties. *(i)* the interval $[\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2)]$ increases in $\Lambda_2$ by (74); *(ii)* $g_1^{c\mu}(\underline{\lambda}_1) - g^{sd}(\underline{\lambda}_1) > 0$ by (88); *(iii)* $g_1^{c\mu}(\underline{\lambda}_1) - g^{sd}(\underline{\lambda}_1)$ is constant in $\Lambda_2$ by Lemma 9.3 and Lemma 10.2.(a); and *(iv)* $g_1^{c\mu}(\lambda_1) - g^{sd}(\lambda_1)$ strictly decreases in $\Lambda_2$ for fixed $\lambda_1 > \underline{\lambda}_1$, by Lemma 9.3 and Lemma 10.2.(c).

For $\Lambda_2 \in (0, \overline{\Lambda}_2)$ we have $\lambda_1^{sd\infty}(\Lambda_2) < \overline{\lambda}_1^{\infty}$ by Lemma 9.1, so strategic delay is optimal for $\mu = \infty$ by Lemma 5.1. We consider in turn the cases, $\Lambda_2 \in (0, \underline{\Lambda}_2)$ and $\Lambda_2 \in (\underline{\Lambda}_2, \overline{\Lambda}_2)$.

Fix $\Lambda_2 \in (0, \underline{\Lambda}_2)$. Then $d(\Lambda_2) > 0$, so $g_1^{c\mu}(\lambda_1, \Lambda_2) - g^{sd}(\lambda_1, \Lambda_2) > 0$ for $\lambda_1 \in \left[\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2)\right]$; by Lemma 5.1 and 5.3 strategic delay is optimal for $\mu \in (\mu^*(\underline{\lambda}_1), \infty)$.

Fix $\Lambda_2 \in (\underline{\Lambda}_2, \overline{\Lambda}_2)$. Then $d(\Lambda_2) < 0$. Since $\left(g_1^{c\mu} - g^{sd}\right)(\lambda_1, \Lambda_2)$ is strictly convex, continuous in $\lambda_1$, and positive at $\underline{\lambda}_1$ and $\lambda_1^{sd\infty}(\Lambda_2)$ by (88), it has two roots $x_s(\Lambda_2) < x_l(\Lambda_2)$ where $\underline{\lambda}_1 < x_s(\Lambda_2) < x_l(\Lambda_2) < \lambda_1^{sd\infty}(\Lambda_2))$. For $\lambda_1 \in \left[\underline{\lambda}_1, \lambda_1^{sd\infty}(\Lambda_2)\right]$, $g_1^{c\mu}(\lambda_1, \Lambda_2) - g^{sd}(\lambda_1, \Lambda_2) > 0$ if and only if $\lambda_1 \in [\underline{\lambda}_1, x_s(\Lambda_2)) \cup (x_l(\Lambda_2), \lambda_1^{sd\infty}(\Lambda_2)]$. Since $\mu^{*\prime} > 0$ by (65), by Lemma 5.1 and 5.3 strategic delay is optimal iff $\mu \in (\mu^*(\underline{\lambda}_1), \mu^*(x_s)) \cup (\mu^*(x_l), \infty]$, and $\mu^*(x_s) = \overline{\mu} < \underline{\underline{\mu}} = \mu^*(x_l)$.■