Online appendix to: Strategic safety stock placement in supply networks with static dual supply

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Appendix A: Optimization algorithm for assembly networks

If the underlying network has a pure assembly structure, $\mathbf{P}$ can be solved by a forward dynamic programming (DP) algorithm. We assign a level code (LC) to each node. In a pure assembly network there is only a single demand node n, i.e. $E = \{n\}$, which we assign level code 1. All other nodes i have $LC(i) := 1 + LC(j)$ with $(i,j) \in \mathcal{A}$. Let N denote the highest level. Further, we define $\mathcal{N}_i$ as the subset of nodes that are connected to i on the subgraph with nodes that have a higher level than i. $\mathcal{N}_i$ is determined as follows:

$$\mathcal{N}_i = \{i\} + \bigcup_{(j,i)\in\mathcal{A}} \mathcal{N}_j \quad . \tag{A.1}$$

The DP proceeds from level N down to 1. It finds the solution to $\mathbf{P}$ for all nodes at the same level by evaluating a functional equation, which is defined below. Thus, when proceeding to the next lower level, the safety stock holding costs for all possible service times of all supplying nodes have already been determined.

A.1. Functional equations

We define two cost functions, C_i^{SS} and C_i^{DS}, one for a single-sourced node and one for a dual-sourced node. Each respective function calculates the minimum safety stock holding cost for the subnetwork with node set $\mathcal{N}_i$. In each cost function, the first term represents the safety stock holding cost of node i resulting from the incoming and outgoing service times. The remaining terms are the cost-to-go functions f_i, which address the nodes in $\mathcal{N}_i$ that are upstream of i and connected to i through their outgoing service times. The function f_i is formally defined after both cost functions have been introduced.

1. **Single-sourced node:** If each item is delivered by a different supplier, i.e. $\delta_{j,i} = 1$, $\forall j : (j,i) \in \mathcal{A}$, then $\delta_i^s = 1$. The minimum cost is a function of a single incoming service time, SI_i, and the outgoing service, S_i (cf. Graves and Willems (2003)). As noted in Proposition 2, $SI_i = SI_i^s = \max\{S_i - T_i^s\ ,\ S_i^s\} = \max\{S_i - T_i^s\ ,\ \max\{S_j \mid j : (j,i) \in \mathcal{A}\}\}$ with $T_i^s = T_{j,i}$ for any $j : (j,i) \in \mathcal{A}$. SI_i^f is irrelevant.

$$C_i^{SS}(SI_i, S_i) = h_i\left(B_i(SI_i^f, SI_i^s, S_i, \delta_i^s, \alpha_i) - \mathbb{E}\left[\tilde{D}_i(SI_i^f, SI_i^s, S_i, \delta_i^s)\right]\right) + \sum_{(j,i)\in\mathcal{A}} f_j(\min\{SI_i, M_j\}) \tag{A.2}$$

M_i is the maximum replenishment lead time for node i, which is defined as $M_i = \max_{(j,i)\in\mathcal{A}}\{M_j + T_{j,i}\}$. The maximum replenishment lead times of the external suppliers are 0 by assumption.

2. **Dual-sourced node:** At a node where the same item is sourced from two different suppliers j and k, i.e. $\delta_{j,i} \in (0,1)$ and $\delta_{k,i} = 1 - \delta_{j,i}$, the minimum cost is a function of both suppliers' outgoing service times, S_j and S_k, and the outgoing service time of node i itself, S_i.

$$\begin{aligned} C_i^{DS}(S_j, S_k, S_i) = h_i &\left(B_i(SI_i^f, SI_i^s, S_i, \delta_i^s, \alpha_i) - \mathbb{E}\left[\tilde{D}_i(SI_i^f, SI_i^s, S_i, \delta_i^s)\right]\right) \\ &+ f_j(\min\{S_j, M_j\}) + f_k(\min\{S_k, M_k\}) \end{aligned} \tag{A.3}$$

Note: $\delta_{j,i}$, S_j, and S_k are related to δ_i^s, S_i^s, and S_i^f and to SI_i^s and SI_i^f as explained in Section 4.1.1.

The cost-to-go function term differs in the single- or dual-supply case. If node i has a single supplier for each required item, we include for each supplier j the minimum safety stock holding cost of the subnetwork with node set $\mathcal{N}_j$ as a function of node j's outgoing service time SI_i. If this outgoing service time is larger than the maximum replenishment lead time of the node j, we adjust it accordingly. This means that all nodes in $\mathcal{N}_j$ do not need to hold any stock and the most upstream nodes in this set should delay their orders with the external suppliers by $M_j - SI_i$ in order to avoid unnecessary inventory.

If node i has two suppliers, j and k, we include the minimum safety stock holding cost for the subnetworks with node sets $\mathcal{N}_j$ and $\mathcal{N}_k$ as a function of the nodes' outgoing service times S_j and S_k, respectively. Similarly, we adjust the service times where necessary.

We solve the following optimization by enumeration to find the functional value $f_i(S_i)$ for a ...

- ... single-sourced node, $\delta_{j,i}=1$, according to:

$$f_i(S_i)=\min_{SI_i}\left\{C_i^{SS}(SI_i,S_i)\right\} \tag{A.4}$$

$$\text{s.t.} \quad \max\{0,S_i-T_{j,i}\}\le SI_i\le M_i-T_{j,i} \quad \text{and } SI_i \text{ integer, for } (j,i)\in\mathcal{A} \tag{A.5}$$

- ... dual-sourced node, $\delta_{j,i}\in(0,1)$, according to:

$$f_i(S_i)=\min_{(S_j,S_k)}\left\{C_i^{DS}(S_j,S_k,S_i)\right\} \quad \text{for } (j,i),(k,i)\in\mathcal{A} \tag{A.6}$$

$$\text{s.t.} \quad \max\{0,S_i-T_{j,i}\}\le S_j\le M_i-T_{j,i} \quad \text{and } S_j \text{ integer, for } (j,i)\in\mathcal{A} \tag{A.7}$$

$$\max\{0,S_i-T_{k,i}\}\le S_k\le M_i-T_{k,i} \quad \text{and } S_k \text{ integer, for } (k,i)\in\mathcal{A} \tag{A.8}$$

The lower bound on S_j, S_k, and SI_i comes from $\mathbf{P}$. The definition of M_i provides the upper bound. If node i is a demand node, then we also constrain S_i by its maximum service time s_i, i.e. $S_i\le s_i$.

A.2. Dynamic program

The dynamic programming algorithm is as follows:

1. For all nodes i with $LC(i):=N$ down to 2, evaluate $f_i(S_i)$ for $S_i=0,1,...,M_i$.
2. For $i:=n$ evaluate $f_i(S_i)$ for $S_i=0,1,...,s_n$.
3. Minimize $f_n(S_n)$ for $S_n=0,1,...,s_n$ to obtain the optimal objective function value.

We find the optimal service times by the standard backtracking procedure for a dynamic program.

Remark: In general acyclic networks with dual supply multiple nodes may need to be evaluated simultaneously, because they are influenced by the same service times. This increases the computational complexity. One possible solution approach to such network structures is to modify the branch and bound algorithm developed in Humair and Willems (2011) for general acyclic networks with single supply such that it can also cope with dual-supply nodes.

Appendix B: Proofs

Proof of Proposition 1. Given the incoming service times definitions in (6), we find that $SI_i^s+T_i^s=\max\{S_i\ ,\ S_i^s+T_i^s\}\ge\max\{S_i\ ,\ S_i^f+T_i^f\}=SI_i^f+T_i^f$ and $SI_i^f+T_i^f=\max\{S_i\ ,\ S_i^f+T_i^f\}\ge S_i$. Thus, (7) holds. Relation (7) comprises all three cases, since Case 1 can be represented by $SI_i^s+T_i^s\ge SI_i^f+T_i^f\ge S_i$, Case 2 reduces to $SI_i^s+T_i^s\ge SI_i^f+T_i^f=S_i$, and Case 3 reduces to $SI_i^s+T_i^s=SI_i^f+T_i^f=S_i$. $\square$

Proof of Proposition 2. In the standard single-supply model, the coverage random variable is given by $D_i(SI_i+T_i-S_i)$ with $SI_i=\max\{S_i-T_i\ ,\ \max\{S_l\mid l:(l,i)\in\mathcal{A}\}\}$. By setting $\delta_i^s=1$, (9) becomes

$$\tilde{D}_i(SI_i^f,SI_i^s,S_i,\delta_i^s=1)=D_i(SI_i^f+T_i^f-S_i)+D_i(SI_i^s+T_i^s-SI_i^f-T_i^f)=D_i(SI_i^s+T_i^s-S_i). \tag{B.1}$$

$T_i^s = T_i$, because due to $\delta_i^s = 1$, only a single process precedes node i, whose duration is independent of the supplier. The node index of the slow supplier of i is found as $\underset{(l,i)\in\mathcal{A}}{\operatorname{argmax}}\{S_l + T_{l,i}\}$. Due to $T_{l,i} = T_i, \forall l : (l,i) \in \mathcal{A}$:

$$S_i^s = S_{\underset{(l,i)\in\mathcal{A}}{\operatorname{argmax}}\{S_l+T_{l,i}\}} = S_{\underset{(l,i)\in\mathcal{A}}{\operatorname{argmax}}\{S_l+T_i\}} = S_{\underset{(l,i)\in\mathcal{A}}{\operatorname{argmax}}\{S_l\}} = \max\{S_l \mid l : (l,i) \in \mathcal{A}\}. \tag{B.2}$$

Consequently, $SI_i^s = \max\{S_i - T_i^s\ ,\ S_i^s\} = \max\{S_i - T_i\ ,\ \max\{S_l \mid l : (l,i) \in \mathcal{A}\}\} = SI_i$. $\square$

Proof of Lemma 1. Define the slow and fast replenishment lead times as $RLT_i^s = SI_i^s + T_i^s$ and $RLT_i^f = SI_i^f + T_i^f$. The objective function consists of the sum of terms of the form

$$C_i(RLT_i^s, RLT_i^f) = h_i \cdot k_i(\alpha_i) \cdot \sigma_i \cdot \sqrt{(RLT_i^f - S_i) + [\delta_i^s]^2 \,(RLT_i^s - RLT_i^f)}. \tag{B.3}$$

The first derivative of C_i with respect to each variable is clearly non-negative:

$$\frac{\partial C_i}{\partial RLT_i^s} = \frac{1}{2} \cdot h_i \cdot k_i \cdot \sigma_i \left(RLT_i^f - S_i + [\delta_i^s]^2 \,(RLT_i^s - RLT_i^f)\right)^{-\frac{1}{2}} \cdot [\delta_i^s]^2 \tag{B.4}$$

$$\frac{\partial C_i}{\partial RLT_i^f} = \frac{1}{2} \cdot h_i \cdot k_i \cdot \sigma_i \left(RLT_i^f - S_i + [\delta_i^s]^2 \,(RLT_i^s - RLT_i^f)\right)^{-\frac{1}{2}} \cdot \left(1 - [\delta_i^s]^2\right), \tag{B.5}$$

since $0 \le \delta_i^s \le 1$ by definition, $RLT_i^s \ge RLT_i^f$ and $RLT_i^f \ge S_i$ due to constraints (17) and (18), and $k_i \ge 0$, since we assume that the planned safety stock level is non-negative (cf. Section 4.1.2). $\square$

Proof of Lemma 2. The proof is by contradiction. Assume $S_j + T_{j,i}$ is the largest replenishment lead time of all supplying nodes of i and $S_k + T_{k,i}$ is the second largest. Suppose that $y_{j,i} = 0$ and $y_{k,i} = 1$. Consequently, constraint (20) together with constraints (17), (19), and Lemma 1 result in $SI_i^f + T_i^f = S_j + T_{j,i} = SI_i^s + T_i^s$. The objective function then reduces to the sum of terms of the form

$$C_i(SI_i^s, SI_i^f) = h_i \cdot k_i(\alpha_i) \cdot \sigma_i \cdot \sqrt{SI_i^f + T_i^f - S_i} = h_i \cdot k_i(\alpha_i) \cdot \sigma_i \cdot \sqrt{SI_i^s + T_i^s - S_i} \tag{B.6}$$

which are independent of δ_i^s. These terms can never be smaller than the ones of the solution with $y_{j,i} = 1$ and $y_{k,i} = 0$, since

$$\sqrt{SI_i^s + T_i^s - S_i} < \sqrt{(SI_i^f + T_i^f - S_i) + [\delta_i^s]^2 \,(SI_i^s + T_i^s - SI_i^f - T_i^f)} \tag{B.7}$$

$$(1 - \delta_i^s)\,(SI_i^s + T_i^s) < (1 - \delta_i^s)\left(SI_i^f + T_i^f\right) \tag{B.8}$$

is not feasible as $SI_i^s + T_i^s \ge SI_i^f + T_i^f$ due to (17). $\square$

Proof of Proposition 3. We prove the first part of the proposition by showing that optimization problem **P** consists of various subproblems where a concave objective function is minimized over a closed, bounded, convex set. For such problems an extreme point property holds (see, e.g., Horst and Tuy (1996)). To facilitate the concavity proof of the objective function, we rewrite it in terms of net replenishment times. Let $\tau_i^s = SI_i^s + T_i^s - S_i$ and $\tau_i^f = SI_i^f + T_i^f - S_i$ denote the net replenishment times from the slow and fast supplier, respectively. Then, the objective function becomes

$$\begin{aligned} C &= \sum_{i=1}^{n} h_i \cdot k_i \cdot \sigma_i \sqrt{(SI_i^f + T_i^f - S_i) + [\delta_i^s]^2 \left(SI_i^s + T_i^s - SI_i^f - T_i^f\right)} \\ &= \sum_{i=1}^{n} h_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\left(1 - [\delta_i^s]^2\right) \cdot \tau_i^f + [\delta_i^s]^2 \cdot \tau_i^s}. \end{aligned} \tag{B.9}$$

The entire optimization problem can be reformulated as

$$\bar{\mathbf{P}} \qquad \min \quad C = \sum_{i=1}^{n} h_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\left(1 - [\delta_i^s]^2\right) \cdot \tau_i^f + [\delta_i^s]^2 \cdot \tau_i^s} \tag{B.10}$$

$$\text{s.t.} \qquad \tau_i^s = SI_i^s + T_i^s - S_i \tag{B.11}$$

$$\tau_i^f = SI_i^f + T_i^f - S_i \tag{B.12}$$

(plus the remaining constraints of **P**).

Due to the binary variable $y_{j,i}$, the feasible region of $\bar{\mathbf{P}}$ is not convex. However, we can divide the optimization problem at each node i into subproblems, whose feasible regions are convex again. At each node we replace constraint (22) by directly fixing the binary variable of one specific supplying node to 1. The feasible region of each subproblem, which is created in this way, is convex.

Note that, if we set $y_{j,i} = 1$ for a node $j, (j,i) \in \mathcal{A}$ that actually does *not* cause the largest replenishment lead time for node i, δ_i^s is specified wrongly. This does not affect the value of the objective function, however, since $\tau_i^f = \tau_i^s$ due to the other constraints and therefore δ_i^s becomes irrelevant, because (B.10) reduces to $C = \sum_{i=1}^n h_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\tau_i^f} = \sum_{i=1}^n h_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\tau_i^s}$.

Lemma 1. *The objective function of each node of each subproblem and thus of the overall optimization problem is jointly concave in τ_i^s and τ_i^f.*

Proof of Lemma 1. The objective function of each node of each subproblem is

$$C_i(\tau_i^s, \tau_i^f) = h_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\left(1 - [\delta_i^s]^2\right) \cdot \tau_i^f + [\delta_i^s]^2 \cdot \tau_i^s}. \tag{B.13}$$

Showing the concavity of (B.13) directly implies that the entire objective function is concave, since the sum of concave functions is concave, too. (B.13) is jointly concave in τ_i^s and τ_i^f, if the Hessian is negative semidefinite.

$$H_{C_i}(\tau_i^s, \tau_i^f) = \begin{bmatrix} \frac{\partial^2 C}{\partial [\tau_i^s]^2} & \frac{\partial^2 C}{\partial \tau_i^s \, \partial \tau_i^f} \\ \frac{\partial^2 C}{\partial \tau_i^s \, \partial \tau_i^f} & \frac{\partial^2 C}{\partial [\tau_i^f]^2} \end{bmatrix} \tag{B.14}$$

This holds true, if the first subdeterminant of the Hessian is smaller or equal to zero and the main determinant is greater or equal to zero. For the first subdeterminant we obtain

$$\frac{\partial^2 C_i}{\partial\left[\tau_i^s\right]^2} = -\frac{1}{4}\cdot h_i\cdot k_i\cdot\sigma_i\left(\left(1-\left[\delta_i^s\right]^2\right)\cdot\tau_i^f+\left[\delta_i^s\right]^2\cdot\tau_i^s\right)^{-\frac{3}{2}}\cdot\left[\delta_i^s\right]^4 \tag{B.15}$$

which is smaller or equal to zero, since the term in parenthesis is non-negative due to $\tau_i^s, \tau_i^f \geq 0$ according to (B.11) and (B.12) together with (17), (18), and (27), and $0 \leq \delta_i^s \leq 1$. The main determinant is given as

$$\begin{aligned}\det(H_{C_i}(\tau_i^s,\tau_i^f)) = & -\frac{1}{4}\cdot h_i\cdot k_i\cdot\sigma_i\left(\left(1-\left[\delta_i^s\right]^2\right)\cdot\tau_i^f+\left[\delta_i^s\right]^2\cdot\tau_i^s\right)^{-\frac{3}{2}}\cdot\left[\delta_i^s\right]^4 \\ & \cdot\left(-\frac{1}{4}\cdot h_i\cdot k_i\cdot\sigma_i\left(\left(1-\left[\delta_i^s\right]^2\right)\cdot\tau_i^f+\left[\delta_i^s\right]^2\cdot\tau_i^s\right)^{-\frac{3}{2}}\cdot\left(1-\left[\delta_i^s\right]^2\right)\cdot\left(1-\left[\delta_i^s\right]^2\right)\right) \\ & -\left(-\frac{1}{4}\cdot h_i\cdot k_i\cdot\sigma_i\left(\left(1-\left[\delta_i^s\right]^2\right)\cdot\tau_i^f+\left[\delta_i^s\right]^2\cdot\tau_i^s\right)^{-\frac{3}{2}}\cdot\left[\delta_i^s\right]^2\cdot\left(1-\left[\delta_i^s\right]^2\right)\right)^2 \\ = & \, 0.\end{aligned} \tag{B.16}$$

□

Due to Lemma 1, for each subproblem an extreme point property holds. Consequently, also the overall optimal values for optimization problem $\bar{\mathbf{P}}$ and in turn $\mathbf{P}$ are found at one of the extreme points of the different feasible regions of the subproblems.

The second part of the proposition follows from showing that the general network single-supply problem, which Lesnaia (2004) has proven to be NP-hard, is actually a special case of our problem. By replacing (17) with $SI_i^s + T_i^s = SI_i^f + T_i^f$, we get the single-supply problem. The term under the square root in the objective function of $\mathbf{P}$ reduces to $SI_i^f + T_i^f - S_i$ and is independent of δ_i^s. Consequently, (25) can be neglected. Since (19) can be rewritten as $SI_i^f + T_i^f \geq S_j + T_{j,i},\ \forall (j,i) \in \mathcal{A}$, (20) is satisfied no matter how $y_{j,i}$ is chosen. Hence, constraints (20)-(22) become irrelevant and thus can be neglected. As (23) and (24) must now hold for any choice of $y_{j,i}$, these constraints can be rewritten as $T_i^f \geq T_{j,i}$ and $SI_i^f \geq S_j,\ \forall (j,i) \in \mathcal{A}$. For a single-supply node it holds that $T_{j,i} = T_i,\ \forall (j,i) \in \mathcal{A}$. Consequently, $T_i^f = T_i$ due to Lemma 1. Moreover, (26) becomes irrelevant, because T_i^s neither influences the objective function nor any of the constraints any more. This leaves exactly the single-supply problem formulation. □

Proof of Lemma 3. Suppose the setting in Figures 6 and 7 with $S_1 + T_{1,3} > S_2 + T_{2,3}$ w.l.o.g.. Stock is held at dual-sourced stage 3, if $S_{3c} < S_1 + T_{1,3}$. The approximate cost are given in (30). We consider the two cases according to the concrete value of S_{3c} separately: (1) $0 \leq S_{3c} < S_2 + T_{2,3}$, (2) $S_2 + T_{2,3} \leq S_{3c} < S_1 + T_{1,3}$.

- **Case 1:** In order to fully benefit from an outgoing service time at stage $3c$ with $0 \leq S_{3c} < S_2 + T_{2,3}$ in terms of a minimal stock requirement at the entire dual-sourced node 3, we need to have an incoming service time $SI_{3c} = \max\{S_{3a}, S_{3b}\}$ that is at least as large as S_{3c}, because the processing time $T_{3c} = 0$. Otherwise, we would forgo some stock reduction potential, because the stocks at stages $3a$ and $3b$ could be reduced by increasing their outgoing service times. Moreover, it is easy to see that in a cost-minimal solution $S_{3a} = S_{3b}$, because in this way the least stock is kept at stages $3a$ and $3b$. (Note that, if the equality of the service times S_{3a} and S_{3b} cannot be achieved, the maximal feasible value for the respective service time is chosen.) Due to Proposition 3 three potentially optimal service-time combinations for a given S_{3c} exist: (i) $S_{3a} = S_{3b} = S_{3c}$, (ii) $S_{3a} = S_1 + T_{1,3}$, $S_{3b} = S_2 + T_{2,3}$, or (iii) $S_{3a} = S_2 + T_{2,3}$, $S_{3b} = S_2 + T_{2,3}$. We find that (i vs. ii)

$$\tilde{C}(S_1, S_2, S_{3a} = S_{3c}, S_{3b} = S_{3c}, S_{3c}) \leq \tilde{C}(S_1, S_2, S_{3a} = S_1 + T_{1,3}, S_{3b} = S_2 + T_{2,3}, S_{3c}) \tag{B.17}$$

$$\delta_{1,3}\sqrt{S_1 + T_{1,3} - S_{3c}} + (1 - \delta_{1,3})\sqrt{S_2 + T_{2,3} - S_{3c}} \leq \sqrt{S_1 + T_{1,3} - S_{3c}} \tag{B.18}$$

$$(1 - \delta_{1,3})\sqrt{S_2 + T_{2,3} - S_{3c}} \leq (1 - \delta_{1,3})\sqrt{S_1 + T_{1,3} - S_{3c}} \tag{B.19}$$

which is true, since $S_1 + T_{1,3} > S_2 + T_{2,3}$ by assumption, and (i vs. iii)

$$\tilde{C}(S_1, S_2, S_{3a} = S_{3c}, S_{3b} = S_{3c}, S_{3c}) \leq \tilde{C}(S_1, S_2, S_{3a} = S_2 + T_{2,3}, S_{3b} = S_2 + T_{2,3}, S_{3c}) \tag{B.20}$$

$$\delta_{1,3}\sqrt{S_1 + T_{1,3} - S_{3c}} + (1 - \delta_{1,3})\sqrt{S_2 + T_{2,3} - S_{3c}} \leq \delta_{1,3}\sqrt{S_1 + T_{1,3} - (S_2 + T_{2,3})} + \sqrt{S_2 + T_{2,3} - S_{3c}} \tag{B.21}$$

$$S_2 + T_{2,3} - S_{3c} \leq \sqrt{S_1 + T_{1,3} - S_{3c}}\sqrt{S_2 + T_{2,3} - S_{3c}} \tag{B.22}$$

which is true, since $S_1 + T_{1,3} > S_2 + T_{2,3}$ by assumption.

- **Case 2:** Along the same line of reasoning as in case 1, we face the same three potentially optimal service time combinations. We find that (i vs. ii)

$$\tilde{C}(S_1, S_2, S_{3a} = S_{3c}, S_{3b} = S_{3c}, S_{3c}) \leq \tilde{C}(S_1, S_2, S_{3a} = S_1 + T_{1,3}, S_{3b} = S_2 + T_{2,3}, S_{3c}) \tag{B.23}$$

$$\delta_{1,3}\sqrt{S_1 + T_{1,3} - S_{3c}} \leq \sqrt{S_1 + T_{1,3} - S_{3c}} \tag{B.24}$$

$$\delta_{1,3} \leq 1 \tag{B.25}$$

which is true for a dual-sourced node, and (i vs. iii)

$$\tilde{C}(S_1, S_2, S_{3a} = S_{3c}, S_{3b} = S_{3c}, S_{3c}) \leq \tilde{C}(S_1, S_2, S_{3a} = S_2 + T_{2,3}, S_{3b} = S_2 + T_{2,3}, S_{3c}) \tag{B.26}$$

$$\delta_{1,3}\sqrt{S_1 + T_{1,3} - S_{3c}} \leq \delta_{1,3}\sqrt{S_1 + T_{1,3} - (S_2 + T_{2,3})} \tag{B.27}$$

$$S_{3c} \leq S_2 + T_{2,3} \tag{B.28}$$

which is true by assumption for this case.

In both cases it is optimal to choose the service times of stages $3a$ and $3b$ equal to the service time of stage $3c$. Since the processing time of stage $3c$ is 0 due to the remodeling, this finding means that stage $3c$ has no time span to cover with safety stock and thus does not carry any stock.

□

Proof of Proposition 4. Suppose the setting in Figures 6 and 7 with $S_1+T_{1,3} > S_2+T_{2,3}$ w.l.o.g.. The cost at node 3 in the exact and the approximate approach are given in (31) and (30), respectively. We consider the two cases in the cost functions separately: (1) $0 \le S_3(=S_{3c}) < S_2+T_{2,3}$, (2) $S_2+T_{2,3} \le S_3(=S_{3c}) \le S_1+T_{1,3}$. Moreover, from Lemma 3 we know that it is optimal in the approximate approach to have $S_{3a}=S_{3b}=S_{3c}$. In the second case it is easy to see from (31) and (30) that both approaches result in identical costs. In the first case the cost of the approximate approach is always worse than the cost of the exact approach for identical surrounding service times, because $C(S_1,S_2,S_3) < \tilde{C}(S_1,S_2,S_{3a}=S_3,S_{3b}=S_3,S_{3c}=S_3)$, i.e.

$$\begin{aligned} & h_3k_3\sigma_3\sqrt{S_2+T_{2,3}-S_3+\delta_{1,3}^2\left(S_1+T_{1,3}-(S_2+T_{2,3})\right)} \\ & \quad < h_3k_3\sigma_3\left(\delta_{1,3}\sqrt{S_1+T_{1,3}-S_3}+\delta_{2,3}\sqrt{S_2+T_{2,3}-S_3}\right) \\ & 0 < 2\delta_{1,3}(1-\delta_{1,3})\sqrt{S_1+T_{1,3}-S_3}\sqrt{S_2+T_{2,3}-S_3}. \end{aligned} \tag{B.29}$$

□

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