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Bid-taker power and supply base diversification

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Online Supplement

Derivations of bidding equilibrium under mechanisms FS and FSR

Under pooling, all suppliers are ex ante symmetric at the time of bidding, and their closed-form equilibrium bidding function under FS/FSR can be readily derived (e.g., Chapter 2.3 of Krishna 2002). Under diversifying, the suppliers have different regional costs, and hence are ex ante asymmetric at the time of bidding. For such a case, it is well-known that there is generally no closed-form solution to the equilibrium functions under FS/FSR; see Chapter 4.3 of Krishna (2002). We provide a general formulation of the bidding equilibrium for the diversifying scenario, in which suppliers 1 to $n_1 \geq 1$ are in region 1 and suppliers n_1+1 to N are in region 2, and describe the approach we use to numerically solve the bidding strategies and evaluate the buyer's expected total cost.

Without loss of generality, we assume $a_1 \leq a_N$ and so $s = a_N - a_1$; namely, region 1 has lower regional cost. We assume there exists a symmetric pure-strategy equilibrium, in which suppliers in the same region use the same bidding strategy and each supplier i's bid increases in his cost x_i. Under mechanisms FS and FSR, the buyer compares $b_i + a_i$ with $b_j + a_j$, which implies that the effect of $\boldsymbol{a}$ on the suppliers' bidding strategies is subsumed by the effect of the regional cost difference. For notational simplicity, let $\beta_{(r)}(x)$ denote the bidding strategy of suppliers in region $r = 1, 2$. Let $\phi_{(1)}$ be the inverse function of $\beta_{(1)}$, and let $\phi_{(2)}$ be the inverse function of $\beta_{(2)} + s$. By definition, $\beta_{(r)}$ maximizes the expected profit of each suppler in region r, so

$$\beta_{(1)}(x) = \arg\max_z \left\{ (z-x)\overline{F}^{n_1-1}(\phi_{(1)}(z))\overline{F}^{N-n_1}(\phi_{(2)}(z)) \right\}, \text{ and} \tag{OS-1a}$$

$$\beta_{(2)}(x) + s = \arg\max_z \left\{ (z-s-x)\overline{F}^{N-n_1-1}(\phi_{(2)}(z))\overline{F}^{n_1}(\phi_{(1)}(z)) \right\}, \tag{OS-1b}$$

where $\bar{F}=1-F$. Let $\underline{z}_{(1)}=\beta_{(1)}(0)$, $\underline{z}_{(2)}=\beta_{(2)}(0)+s$, and $\overline{z}_{(1)}=\beta_{(1)}(1)$. Thus, we have boundary conditions:

$$0=\phi_{(1)}(\underline{z}_{(1)})=\phi_{(2)}(\underline{z}_{(2)});\quad \phi_{(1)}(\overline{z}_{(1)})=1;\quad \phi_{(2)}(\overline{z}_{(1)})=\overline{z}_{(1)}-s. \tag{OS-2}$$

The last equation follows because no supplier from region 2 with $x>\overline{z}_{(1)}-s$ can win the auction, as all suppliers in region 1 bid no more than $\overline{z}_{(1)}$. When $n_1=1$, we must have $\underline{z}_{(1)}=\underline{z}_{(2)}$ in equilibrium. When $n_1\geq 2$, it is reasonable to assume that $\underline{z}_{(1)}\leq\underline{z}_{(2)}$, since region 2 has the higher regional cost. Applying FOCs to equations (OS-1a)-(OS-1b) and using the definitions of $\phi_{(1)}$ and $\phi_{(2)}$, we have:

$$\frac{dF(\phi_{(1)}(z))}{dz}=\frac{\overline{F}(\phi_{(1)}(z))}{(n_1-1)[z-\phi_{(1)}(z)]},\ \text{when } n_1\geq 2 \text{ and } z\in[\underline{z}_{(1)},\underline{z}_{(2)}); \tag{OS-3}$$

and for $z\in[\underline{z}_{(2)},\overline{z}_{(1)}]$ we have

$$\frac{dF(\phi_{(1)}(z))}{dz}=\overline{F}(\phi_{(1)}(z))\left[\frac{\frac{N-n_1}{n_1(N-n_1)-(n_1-1)(N-n_1-1)}}{z-\phi_{(2)}(z)-s}-\frac{\frac{N-n_1-1}{n_1(N-n_1)-(n_1-1)(N-n_1-1)}}{z-\phi_{(1)}(z)}\right]; \tag{OS-4}$$

$$\frac{dF(\phi_{(2)}(z))}{dz}=\overline{F}(\phi_{(2)}(z))\left[\frac{\frac{n_1}{n_1(N-n_1)-(n_1-1)(N-n_1-1)}}{z-\phi_{(1)}(z)}-\frac{\frac{n_1-1}{n_1(N-n_1)-(n_1-1)(N-n_1-1)}}{z-\phi_{(2)}(z)-s}\right]. \tag{OS-5}$$

The ODE set (equations (OS-2)-(OS-5)) applies to both FS and FSR, and the two mechanisms differ solely in $\overline{z}_{(1)}$. In particular, we must have $\overline{z}_{(1)}=1$ under FSR due to the reserve price, while under FS we have $\overline{z}_{(1)}=1$ only when $n_1\geq 2$. This implies that when low-cost region 1 has more than one supplier, the supplier competition makes FS and FSR outcome-equivalent. In contrast, under FS when $n_1=1$, the highest cost-type supplier 1 can choose an optimal bid above 1 such that $\overline{z}_{(1)}=\arg\max_z\{(z-1)\overline{F}^{N-1}(\phi_{(2)}(z))\}$.

Given $\phi_{(1)}$ and $\phi_{(2)}$ we can derive the buyer's expected total procurement cost. Note that the buyer's total cost exceeds a_1+z if and only if $b_i>z$ for each supplier i in region 1 and $b_i+s>z$ for each supplier i in region 2. This implies that the tail distribution of the buyer's total cost is $\overline{F}^{n_1}(\phi_{(1)}(z))\overline{F}^{N-n_1}(\phi_{(2)}(z))$, so the buyer's expected total cost equals

$$a_1+\underline{z}_{(1)}+\int_{\underline{z}_{(1)}}^{\overline{z}_{(1)}}\overline{F}^{n_1}(\phi_{(1)}(z))\overline{F}^{N-n_1}(\phi_{(2)}(z))dz. \tag{OS-6}$$

ODE set (OS-2)-(OS-5) generally has no closed-form solutions. To construct Figures 3 and 4, we numerically solve $\phi_{(1)}$ and $\phi_{(2)}$ and evaluate the buyer's expected profit under FSR, for cases [$N=2$ and $n_1=1$] and [$N=4$ and $n_2=2$], respectively. In short, our methodology is as follows. When $N=2$ and $n_1=1$, we have $\underline{z}_{(1)}=\underline{z}_{(2)}$ and $\overline{z}_{(1)}=1$. The challenge is that $\underline{z}_{(1)}$ is unknown. We search for the value of $\underline{z}_{(1)}$ such that the ODE equations (OS-4)-(OS-5) satisfy the constraint $\overline{z}_{(1)}=1$ (numerically, with a small tolerance level). Note: In this case, (OS-3) is irrelevant. When $N=4$ and $n_1=2$, we only have $\underline{z}_{(1)}<\underline{z}_{(2)}$ and $\overline{z}_{(1)}=1$. This scenario is more challenging because both

$\underline{z}_{(1)}$ and $\underline{z}_{(2)}$ are unknown. Thus, our algorithm needs to search for both parameters simultaneously such that the ODE equations (OS-3)-(OS-5) satsify the constraint $\overline{z}_{(1)} = 1$. In the figures, when the buyer's total expected cost of pooling and diversifying are tied (within a very small tolerance level) we break the tie in favor of pooling (this tends to happen more near $s = 0$).

Derivation of diversification upside (4) and downside (5)

Equation (4) follows because, for all $Mech \in \{RE, RER, FS, FSR, OPT\}$, when $\tilde{a}_1 > \tilde{a}_2$ we have $\pi^{Mech}(\boldsymbol{x}, (\tilde{a}_1, \tilde{a}_1)) = \pi^{Mech}(\boldsymbol{x}, (s, s)) + \tilde{a}_2$ and $\pi^{Mech}(\boldsymbol{x}, (\tilde{a}_1, \tilde{a}_2)) = \pi^{Mech}(\boldsymbol{x}, (s, 0)) + \tilde{a}_2$, per equations (1a), (1b), (2a), (2b) and (3). Similarly, equation (5) follows because when $\tilde{a}_1 \leq \tilde{a}_2$ we have $\pi^{Mech}(\boldsymbol{x}, (\tilde{a}_1, \tilde{a}_1)) = \pi^{Mech}(\boldsymbol{x}, (0, 0)) + \tilde{a}_1$ and $\pi^{Mech}(\boldsymbol{x}, (\tilde{a}_1, \tilde{a}_2)) = \pi^{Mech}(\boldsymbol{x}, (0, s)) + \tilde{a}_1$.

Proof of Lemma 1

Per equations (1a), (1b), (2a), (2b) and (3), we have $\pi^{Mech}(\boldsymbol{x}, (s, 0)) = \pi^{Mech}(\boldsymbol{x}, (0, s))$ for all $\boldsymbol{x}$, s, and $Mech \in \{RE, RER, FS, FSR, OPT\}$. This implies that the expected diversification upside minus downside is positive if and only if inequality (6) holds.

Proof of Proposition 1

Let $\vee$ and $\wedge$ denote the componentwise maximum and minimum operators, respectively. Per (1a) we have $\pi^{RE}(\boldsymbol{x}, (s, s)) = \pi^{RE}(\boldsymbol{x}, (s, 0)) \vee \pi^{RE}(\boldsymbol{x}, (0, s))$ and $\pi^{RE}(\boldsymbol{x}, (0, 0)) \leq \pi^{RE}(\boldsymbol{x}, (s, 0)) \wedge \pi^{RE}(\boldsymbol{x}, (0, s))$ for any $\boldsymbol{x}$ and s. This implies that $\underset{\boldsymbol{x}}{E}\,[\pi^{RE}(\boldsymbol{x}, (s, 0))] + \underset{\boldsymbol{x}}{E}\,[\pi^{RE}(\boldsymbol{x}, (0, s))] \geq \underset{\boldsymbol{x}}{E}\,[\pi^{RE}(\boldsymbol{x}, (s, s)) + \pi^{RE}(\boldsymbol{x}, (0, 0))]$ holds for any F and $s > 0$. This in turn implies, per Lemma 1, that $\underset{\boldsymbol{x}}{E}\,[\pi^{RE}(\boldsymbol{x}, (s, 0))] \geq \frac{1}{2} \underset{\boldsymbol{x}}{E}\,[\pi^{RE}(\boldsymbol{x}, (s, s)) + \pi^{RE}(\boldsymbol{x}, (0, 0))]$ holds for any F and $s > 0$. Therefore, regardless of F and G, inequality (6) can never hold, and pooling is always optimal by Lemma 1.

Proof of Proposition 2

Regardless of $Mech \in \{RER, FSR\}$ and s, per equation (1b) and (2b) we have $\pi^{Mech}(\boldsymbol{x}, (s, s)) = X_{2:2} + s$, $\pi^{Mech}(\boldsymbol{x}, (0, 0)) = X_{2:2}$, and $\pi^{Mech}(\boldsymbol{x}, (s, 0)) \leq 1$. Therefore, inequality (6) holds if $Prob(s > 2(1 - X_{2:2}))$ is sufficiently large.

Proof of Proposition 3

The standard mechanism design analysis (Myerson 1981) can show that $\underset{(\boldsymbol{x},\boldsymbol{a})}{E}[\pi^{OPT}(\boldsymbol{x}, \boldsymbol{a})] = \underset{(\boldsymbol{x},\boldsymbol{a})}{E}\,[\Upsilon(\boldsymbol{x}, \boldsymbol{a})]$, where $\Upsilon(\boldsymbol{x}, \boldsymbol{a}) \overset{def}{=} \min_{i=1,2}\{\psi(x_i) + a_i\}$. Note that for any $\boldsymbol{x}$ and $s > 0$, we have $\Upsilon(\boldsymbol{x}, (0, 0)) = \Upsilon(\boldsymbol{x}, (s, 0)) \wedge \Upsilon(\boldsymbol{x}, (0, s))$ and $\Upsilon(\boldsymbol{x}, (s, s)) > \Upsilon(\boldsymbol{x}, (s, 0)) \vee \Upsilon(\boldsymbol{x}, (0, s))$. This implies that

$$\underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,[\Upsilon(\boldsymbol{x}, (s, 0))] + \underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,[\Upsilon(\boldsymbol{x}, (0, s))] < \underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,[\Upsilon(\boldsymbol{x}, (s, s)) + \Upsilon(\boldsymbol{x}, (0, 0))]\,, \text{ and hence}$$

$$\underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,\left[\pi^{OPT}(\boldsymbol{x}, (s, 0))\right] + \underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,\left[\pi^{OPT}(\boldsymbol{x}, (0, s))\right] < \underset{(\boldsymbol{x},\boldsymbol{s})}{E}\,\left[\pi^{OPT}(\boldsymbol{x}, (s, s)) + \pi^{OPT}(\boldsymbol{x}, (0, 0))\right]$$

hold for any F and G. Therefore, the proposition follows per Lemma 1.

Extension to N-supplier case and proof of Proposition 4

To extend our analysis to cases with $N \geq 3$ suppliers, we extend our notation by replacing $i = 1, 2$ with $i = 1, \ldots, N$ and redefining $\boldsymbol{x} \overset{def}{=} (x_1, \ldots, x_N)$ and $\boldsymbol{a} \overset{def}{=} (a_1, \ldots, a_N)$. Define $(s;0)$ to mean that $a_i = s$ for $i = 1, \ldots, N_1$ and $a_i = 0$ for $i = N_1 + 1, \ldots, N$; similarly define $(0;s)$, $(s;s)$, and $(0;0)$. Our paper's descriptions of the five mechanisms were written to naturally extend to N-supplier cases.

Part (i) follows, as Proposition 3's proof remains valid for the N-supplier case: Simply replace $i = 1, 2$ by $i = 1, \ldots, N$, and replace (s,s), $(s,0)$, $(0,s)$, and $(0,0)$ by $(s;s)$, $(s;0)$, $(0;s)$, and $(0;0)$, respectively. For part (ii), recall that the buyer selects the best mechanism among {RE, FS, RER, FSR}, and effectively among {RER, FSR} since they dominate {RE, FS}. Denote her expected total cost under the best mechanism by π^*. We have $\pi^*(\boldsymbol{x}, (s;s)) = \pi^{Mech}(\boldsymbol{x}, (s;s)) = s + X_{2:N}$ and $\pi^*(\boldsymbol{x}, (0;0)) = \pi^{Mech}(\boldsymbol{x}, (0;0)) = X_{2:N}$ for $Mech \in \{RER, FSR\}$, $\pi^*(\boldsymbol{x}, (0,s)) \leq \pi^{RER}(\boldsymbol{x}, (0,s)) \leq X_{2:N_1}$ and $\pi^*(\boldsymbol{x}, (s,0)) \leq \pi^{RER}(\boldsymbol{x}, (s,0)) \leq X_{2:N-N_1}$. Thus, per Lemma 1 it is optimal to diversify if $Prob[s + 2X_{2:N} - X_{2:N_1} - X_{2:N-N_1} > 0]$ is sufficiently large; part (ii) thus follows.

For part (iii), when $N \geq 4$ we have $N_1 \geq 2$ and $N - N_1 \geq 2$ (both regions have at least two suppliers). In such cases, $\pi^{RE}(\boldsymbol{x}, \boldsymbol{a}) = second\ min\{x_i + a_i;\ \ i = 1, \ldots, N\} = second\ min\{x_i + a_i, 1 + a_i,\ \ i = 1 \ldots N\} = \pi^{RER}(\boldsymbol{x}, \boldsymbol{a})$, hence mechanisms RE and RER are equivalent. Therefore, the buyer's preference is same as in part (ii), as proved in the previous paragraph.

Last, we discuss the case with mechanism RE and $N_1 = 1$. We have $\pi^{RE}(\boldsymbol{x}, (s;s)) = s + X_{2:N}$, $\pi^{RE}(\boldsymbol{x}, (0;0)) = X_{2:N}$. When $s \geq 1$, we have $\pi^{RE}(\boldsymbol{x}, (0;s)) = X_{2:N-1}$ and $\pi^{RE}(\boldsymbol{x}, (s;0)) = s + X_{1:N-1}$. Therefore, when $N_1 = 1$ we find that the buyer's optimal supply base design strategy depends on both distributions F and G under mechanism RE; in particular, when a large regional cost difference is very likely (i.e., $s \geq 1$), the buyer prefers diversifying if and only if the distribution F is such that $2\overline{X}_{2:N} - \overline{X}_{1:N-1} - \overline{X}_{2:N-1} > 0$.

Proof of Proposition 5

Suppose $s \geq 1$. When $Mech \in \{RER, FSR, OPT\}$, we have $\pi^{Mech}(\boldsymbol{x}, (0;s)) = X_{2:N_1}$ and $\pi^{Mech}(\boldsymbol{x}, (s,0)) = X_{2:N-N_1}$. Thus, due to regional symmetry we have $\underset{(\boldsymbol{x},s)}{E}[\pi^{Mech}(\boldsymbol{x}, \boldsymbol{a}) | s \geq 1] = \frac{1}{2}[\overline{X}_{2:N_1} + \overline{X}_{2:N-N_1}]$, which is minimized by $N_1 = \lfloor \frac{N}{2} \rfloor$ if $\overline{X}_{2:n}$ is convex in $n \in \{1, \ldots, N\}$. When $x + \frac{F(x)}{f(x)}$ is increasing in x we indeed have that $\overline{X}_{2:n}$ is convex in $n \in \{1, \ldots, N\}$. To see this, note that the tail distribution $Prob(X_{2:n} > z) = \overline{F}^n(z) + nF(z)\overline{F}^{n-1}(z)$, and this implies that the expectation $\overline{X}_{2:n} = \int_0^1 [\overline{F}^n(z) + nF(z)\overline{F}^{n-1}(z)]dz = \int_0^1 \overline{F}^n(z) d[z + \frac{F(z)}{f(z)}]$. Since $\overline{F}^n(z)$ is convex in n for all z and $z + \frac{F(z)}{f(z)}$ is increasing in z, we know $\overline{X}_{2:n}$ is convex in n. The same proof applies for $Mech \in \{RE, FS\}$, with a note that $\pi^{Mech}(\boldsymbol{x}, (0;s)) > 1 = \overline{X}_{2:N_1}$ when $N_1 = 1$. Namely, the absence of the reserve price yields a higher cost to the buyer when she chooses $N_1 = 1$; obviously this can only make $N_1 = 1$ a worse strategy than $N_1 = \lfloor \frac{N}{2} \rfloor$. Thus, if $Prob(s \geq 1)$ is sufficiently large, $N_1 = \lfloor \frac{N}{2} \rfloor$ is the optimal diversifying strategy.

Proof of Proposition 6

Note that the buyer's expected total procurement cost by pooling two suppliers in region r equals $\int_{-\infty}^{\infty} z dL_{(r)}(z)$, so the buyer prefers region r^* as doing so clearly minimizes the expected total cost. Denote the total cost distribution for region r as $H_{(r)} \overset{def}{=} F_{(r)} \oplus G_{(r)}$, where $\oplus$ is the convolution operator. Note that the buyer's total procurement cost equals $\max\{x_1 + a_1, x_2 + a_2\}$ under mechanism RE. Thus, if the buyer has one supplier in region 1 and one supplier in region 2, the expected total cost equals $\int zd[H_{(1)}(z)H_{(2)}(z)]$. The buyer's expected total cost would be equal to $\int zd[H_{(r)}(z)H_{(r)}(z)]$, if the buyer were able to use a hypothetical diversifying strategy that uses two copies of region r, $r = 1, 2$. The diversification strategy that has one supplier in region 1 and one supplier in region 2 is dominated by either or both of the hypothetical two-copy diversifying strategies because $2\int_{-\infty}^{\infty} zd[H_{(1)}(z)H_{(2)}(z)] - \int_{-\infty}^{\infty} zd[H_{(1)}(z)H_{(1)}(z)] - \int_{-\infty}^{\infty} zd[H_{(2)}(z)H_{(2)}(z)] = -\int_{-\infty}^{\infty} zd[H_{(1)}(z) - H_{(2)}(z)]^2 = \int_{-\infty}^{\infty} [H_{(1)}(z) - H_{(2)}(z)]^2 dz > 0$, where the last equality uses integration by parts and the fact that $H_{(1)}(-\infty) = H_{(2)}(-\infty) = 0$ and $H_{(1)}(\infty) = H_{(2)}(\infty) = 1$. The proposition part (i) follows because either of the hypothetical two-copy-region diversifying strategy is dominated by the corresponding pooling strategy (per Proposition 1).

Under OPT, the buyer's expected total cost by pooling in region 1, pooling in region 2, and diversifying equals $\bar{X}_{2:2} + E[a_{(1)}]$, $\bar{X}_{2:2} + E[a_{(1)}] + \delta$, and $\underset{(\boldsymbol{x},\boldsymbol{a})}{E}\left[\min\{\psi_{(1)}(x_1) + a_{(1)}, \psi_{(2)}(x_2) + \alpha + \delta + \lambda\epsilon\}\right]$, respectively. It is clear that pooling in region 1 dominates pooling in region 2 and hence the buyer chooses between pooling in region 1 and diversifying. The threshold $\hat{\delta}(\lambda)$ defined in the proposition's part (iii) exists because $\underset{(\boldsymbol{x},\boldsymbol{a})}{E}\left[\min\{\psi_{(1)}(x_1) + a_{(1)}, \psi_{(2)}(x_2) + \alpha + \delta + \lambda\epsilon\}\right]$ increases in δ, and as δ goes to infinity, it approaches $\underset{(\boldsymbol{x},\boldsymbol{a})}{E}\left[\psi_{(1)}(x_1) + a_{(1)}\right] = 1 + E[a_{(1)}]$, which exceeds $\bar{X}_{2:2} + E[a_{(1)}]$ (the buyer's expected total cost by pooling in region 1). Furthermore, the threshold $\hat{\delta}(\lambda)$ increases in λ because $\underset{(\boldsymbol{x},\boldsymbol{a})}{E}\left[\min\{\psi_{(1)}(x_1) + a_{(1)}, \psi_{(2)}(x_2) + \alpha + \delta + \lambda\epsilon\}\right]$ decreases in λ given that increasing λ yields a mean-preserving spread of $a_{(2)}$ and $\min\{\psi_{(1)}(x_1) + a_{(1)}, \psi_{(2)}(x_2) + a_{(2)}\}$ is concave in $a_{(2)}$.

Proof of Proposition 7

Under pooling, all N suppliers' regional costs are always equal (i.e., $a_1 = \ldots = a_N$). In such a case, all suppliers are symmetric; all the five mechanisms yield the same total procurement cost to the buyer per the Revenue Equivalence Theorem (Myerson 1981). However, under diversifying, with positive probability, the suppliers from different regions have different regional costs. In such a case, by the five mechanisms' respective definitions, the buyer's expected total procurement cost can be strictly different under different mechanisms; in particular, it is the lowest under OPT and highest when the buyer can only choose RE. The result follows because the access to a larger set of mechanisms can only decrease the buyer's expected total cost if she diversifies but does not change her expected total cost if she chooses pooling.

Extension to multiplicative regional cost case and proof of Proposition 8

For the multiplicative model, adapt §4's description of the five mechanisms as follows: For RER, FSR, and OPT, the optimal reserve price is $\min\{a_1 u, a_2 u\}$; for RE/RER, replace all $x_i + a_i$ by $a_i x_i$, and remove all $-a_j$; for FS/FSR, remove all $+a_i$; for OPT, redefine the virtual total cost as $a_i\psi(x_i)$, redefine the auction starting price as $\psi(u)\max_i\{a_i\}$, and redefine $Pay(OPT) = \min\{a_j\psi^{-1}(p/a_j), a_j u\}$, where $j = \underset{i=1,\ldots,N}{\arg\min}\{a_i\psi(x_i)\}$ is the winning supplier, and $p = \underset{i=1,\ldots,N, i\neq j}{\min}\{a_i\psi(x_i)\}$ is the auction ending price. Thus, we obtain $\pi^{RE}(\boldsymbol{x},\boldsymbol{a}) = \underset{i=1,\ldots,N}{\text{second min}}\{a_i x_i\}$; $\pi^{RER}(\boldsymbol{x},\boldsymbol{a}) = \underset{i=1,\ldots,N}{\text{second min}}\{a_i x_i, a_i u\}$; $\pi^{FS}(\boldsymbol{x},\boldsymbol{a}) = \underset{i=1,\ldots,N}{\min}\{\beta_i^{FS}(x_i,\boldsymbol{a})\}$; $\pi^{FSR}(\boldsymbol{x},\boldsymbol{a}) = \underset{i=1,\ldots,N}{\min}\{\beta_i^{FSR}(x_i,\boldsymbol{a}), a_i u\}$; $\pi^{OPT}(\boldsymbol{x},\boldsymbol{a}) = \min\{a_j\psi^{-1}(p/a_j), a_j u\}$.

To extend Propositions 1, 3, 4(i), we first adapt Lemma 1. Given a pair of realized regional factors $(\tilde{a}_1, \tilde{a}_2)$, we define $\tilde{\eta} = \frac{\tilde{a}_1}{\tilde{a}_2} \vee \frac{\tilde{a}_2}{\tilde{a}_1}$. Note that the diversification upside (4) is re-written as

$$\left[\pi^{Mech}(\boldsymbol{x},(\tilde{a}_1,\tilde{a}_1)) - \pi^{Mech}(\boldsymbol{x},(\tilde{a}_1,\tilde{a}_2))\right]\mathbb{I}(\tilde{a}_1 \geq \tilde{a}_2) = (\tilde{a}_1 \wedge \tilde{a}_2)\left[\pi^{Mech}(\boldsymbol{x},(\tilde{\eta},\tilde{\eta})) - \pi^{Mech}(\boldsymbol{x},(\tilde{\eta},1))\right]\mathbb{I}(\tilde{a}_1 \geq \tilde{a}_2),$$

and the diversification downside (5) is rewritten as

$$\left[\pi^{Mech}(\boldsymbol{x},(\tilde{a}_1,\tilde{a}_2)) - \pi^{Mech}(\boldsymbol{x},(\tilde{a}_1,\tilde{a}_1))\right]\mathbb{I}(\tilde{a}_1 < \tilde{a}_2) = (\tilde{a}_1 \wedge \tilde{a}_2)\left[\pi^{Mech}(\boldsymbol{x},(1,\tilde{\eta})) - \pi^{Mech}(\boldsymbol{x},(1,1))\right]\mathbb{I}(\tilde{a}_1 < \tilde{a}_2).$$

Therefore, diversifying is optimal if and only if

$$\frac{1}{2}\underset{(\boldsymbol{x},\boldsymbol{a})}{E}\left\{(\tilde{a}_1 \wedge \tilde{a}_2)[\pi^{Mech}(\boldsymbol{x},(\tilde{\eta},\tilde{\eta})) - \pi^{Mech}(\boldsymbol{x},(\tilde{\eta},1)) - \pi^{Mech}(\boldsymbol{x},(1,\tilde{\eta})) + \pi^{Mech}(\boldsymbol{x},(1,1))]\right\} > 0, \tag{OS-7}$$

replacing inequality (6) in Lemma 1.

Propositions 1, 3 and 4(i) extend to the multiplicative regional cost model because inequality (OS-7) always holds under OPT for all N, and can never hold under RE when $N = 2$. To prove this, it is straightforward to adapt the proofs of Propositions 1, 3 and 4(i).

To extend Proposition 2, note that $\pi^{RER}(\boldsymbol{x},(\eta,\eta)) = \eta X_{2:2}$, $\pi^{RER}(\boldsymbol{x},(1,1)) = X_{2:2}$, and $\pi^{RER}(\boldsymbol{x},(\eta,1)), \pi^{RER}(\boldsymbol{x},(1,\eta)) \leq u$. This implies that inequality (OS-7) holds if $Prob[(1+\eta)X_{2:2} - 2u > 0]$ is sufficiently large, giving the condition specified in Proposition 8.

To extend Proposition 4(ii), note that $\pi^*(\boldsymbol{x},(\eta,\eta)) = \pi^{Mech}(\boldsymbol{x},(\eta;\eta)) = \eta X_{2:N}$ and $\pi^*(\boldsymbol{x},(1,1)) = \pi^{Mech}(\boldsymbol{x},(1;1)) = X_{2:N}$ for $Mech \in \{RER, FSR\}$, $\pi^*(\boldsymbol{x},(1,\eta)) \leq \pi^{RER}(\boldsymbol{x},(1,\eta)) \leq X_{2:N_1}$, and $\pi^*(\boldsymbol{x},(\eta,1)) \leq \pi^{RER}(\boldsymbol{x},(\eta,1)) \leq X_{2:N-N_1}$. Thus, per Lemma 1 it is optimal to diversify if $Prob[(1+\eta)X_{2:N} - X_{2:N_1} - X_{2:N-N_1} > 0]$ is sufficiently large; part (ii) thus follows. Part (iii) extends because it is easy to verify that $\pi^{RER}(\boldsymbol{x},\boldsymbol{a}) = \pi^{RE}(\boldsymbol{x},\boldsymbol{a})$ for all $(\boldsymbol{x},\boldsymbol{a})$ whenever $N_1 \geq 2$ and $N - N_1 \geq 2$.

To extend Proposition 5, we simply need to duplicate the proof of that proposition, noting that $\pi^{Mech}(\boldsymbol{x};\boldsymbol{a}) = a_1 X_{2:N_1}$ when $a_2 l \geq a_1 u$, and $\pi^{Mech}(\boldsymbol{x};\boldsymbol{a}) = a_2 X_{2:N-N_1}$ when $a_1 l \geq a_2 u$.

To extend Proposition 6(i) we can duplicate the original proof, using $L_{(r)}(z) = \int_{-\infty}^{\infty} F_{(r)}^2(z/y)dC_{(r)}(y)$. To extend Proposition 6(ii) we can duplicate the original proof after redefining $H_{(r)}$ as the distribution of $a_i x_i$ when $a_i \sim C_{(r)}$ and $x_i \sim F_{(r)}$. We can duplicate Proposition 6(iii)'s proof if we replace $\bar{X}_{2:2} + E[a_{(1)}]$ with $\bar{X}_{2:2}E[a_{(1)}]$, replace $\bar{X}_{2:2} + E[a_{(1)}] + \delta$ with $\bar{X}_{2:2}(E[a_{(1)}] + \delta)$, replace $\psi_{(1)}(x_1) + a_{(1)}$ with $\psi_{(1)}(x_1)a_{(1)}$, replace $\psi_{(2)}(x_2) + a_{(2)}$ with $\psi_{(2)}(x_2)a_{(2)}$, replace $\psi_{(2)}(x_2) + \alpha + \delta + \lambda\epsilon$ with $\psi_{(2)}(x_2)(\alpha + \delta + \lambda\epsilon)$, and replace $1 + E[a_{(1)}]$ with $uE[a_{(1)}]$. Proposition 7 extends since its proof is based on symmetry of suppliers under pooling and asymmetry of suppliers under diversifying, which hold in both additive and multiplicative settings.

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