Service Systems with Finite and Heterogeneous Customer Arrivals

Online Supplement

The Joint Impact of Heterogeneity in Inter-Arrival and Service Times

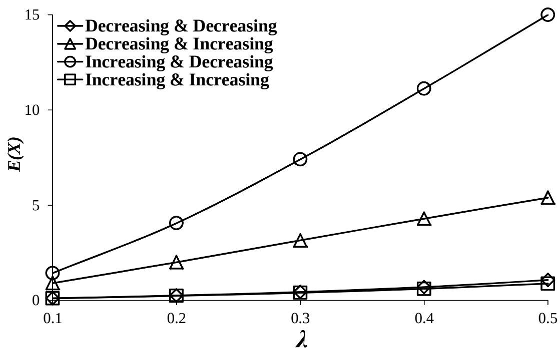
In this section, we consider settings where both the inter-arrival and service times are heterogeneous. In particular, we consider the four scenarios shown in Table S1. Numerical results for the expected waiting time are shown in Figure S1 (results for the expected makespan are omitted for the sake of brevity). We find that combinations of different inter-arrival and service time features lead to significantly different waiting times and makespans. Thus, it is important to explicitly account for both. Again, we find that there are two distinct regimes of operation. When utilization is high ($\rho >> 1$), and a peak in congestion is unavoidable, a combination of inter-arrival and service time features that delays the peak until later in the arrival process reduces expected waiting time the most, which explains why the combination of "Decreasing" inter-arrival times and "Increasing" service times is most preferable, and the combination of "Increasing" inter-arrival times and "Decreasing" service times is least preferable. This ordering is reversed for makespan. On the other hand, when utilization is low ($\rho << 1$), those combinations such as "Decreasing" inter-arrival times and "Decreasing" service times, and "Increasing" inter-arrival times and "Increasing" service times, which avoid peak congestion, tend to reduce expected waiting time the most.

Detailed Derivations for the Performance Measures in Case 3 of Section 6

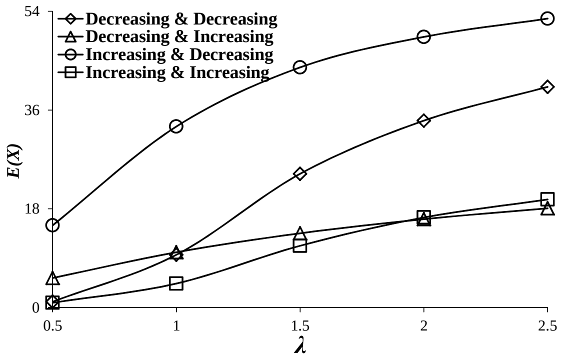
Constant: For this process, we distinguish two cases, $\lambda \geq 1$ and $\lambda < 1$. In the first case, we have $\frac{1}{\lambda_m} \leq 1$ for $m = 2, ..., M$, and therefore $D_m^F = m$. In the second case, we have $\frac{1}{\lambda_m} > 1$ for

Inter-arrival & Service Time Features	Expected Inter-arrival & Service Times
Inter-arrival Time: Decreasing Service Time: Decreasing	$E(T_m) = 2\frac{M-m+1}{M}\frac{1}{\lambda}$ for $m = 2, ..., M$ $E(\varepsilon_m) = 2\frac{M-m+1}{M+1}\frac{1}{\mu}$ for $m = 1, ..., M$
Inter-arrival Time: Decreasing Service Time: Increasing	$E(T_m) = 2\frac{M-m+1}{M}\frac{1}{\lambda}$ for $m = 2, ..., M$ $E(\varepsilon_m) = 2\frac{m}{M+1}\frac{1}{\mu}$ for $m = 1, ..., M$
Inter-arrival Time: Increasing Service Time: Decreasing	$E(T_m) = 2\frac{m-1}{M}\frac{1}{\lambda}$ for $m = 2, ..., M$ $E(\varepsilon_m) = 2\frac{M-m+1}{M+1}\frac{1}{\mu}$ for $m = 1, ..., M$
Inter-arrival Time: Increasing Service Time: Increasing	$E(T_m) = 2\frac{m-1}{M}\frac{1}{\lambda}$ for $m = 2, ..., M$ $E(\varepsilon_m) = 2\frac{m}{M+1}\frac{1}{\mu}$ for $m = 1, ..., M$

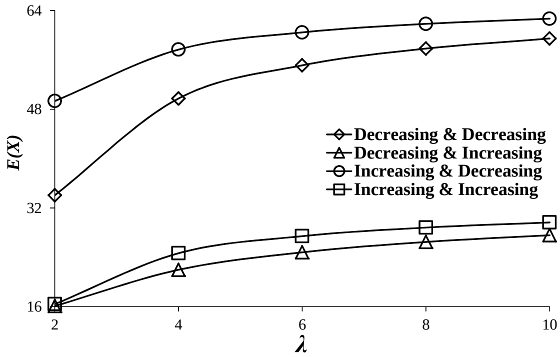
Table S1: Inter-Arrival and Service Time Features



(a) $M = 100, \mu = 1$, Small λ



(b) $M = 100, \mu = 1$, Medium λ



(c) $M = 100, \mu = 1$, Large λ

Figure S1: Impact of Inter-Arrival and Service Time Features on Expected Waiting Time

$m = 2, ..., M$, and therefore $D_m^F = \frac{m+(\lambda-1)}{\lambda}$. Thus, $D_{M(C)}^F = \begin{cases} \frac{M+(\lambda-1)}{\lambda} & \text{for } \lambda \in (2\frac{1}{M}, 1), \\ M & \text{for } \lambda \in [1, 2\frac{M-1}{M}), \end{cases}$

and $E^F(Y)_{(C)} = \begin{cases} \frac{2\lambda M-\lambda}{2\lambda M} & \text{for } \lambda \in (2\frac{1}{M}, 1), \\ \frac{(\lambda-1)M^2+2M-1}{2\lambda M} & \text{for } \lambda \in [1, 2\frac{M-1}{M}). \end{cases}$

Decreasing: For this process, the inter-arrival times are such that $\frac{1}{\lambda_m} \geq 1$ for $m \in [2, \frac{2-\lambda}{2}M + 1]$ (assuming $\frac{\lambda M}{2}$ takes integer values), and $\frac{1}{\lambda_m} < 1$ for $m \in [\frac{2-\lambda}{2}M + 2, M]$. Therefore, $D_m^F = \sum_{j=2}^{m} \frac{1}{\lambda_j} + 1$ for $m \in [2, \frac{2-\lambda}{2}M + 1]$, and $D_m^F = \sum_{j=2}^{\frac{2-\lambda}{2}M+1} \frac{1}{\lambda_j} + (m - \frac{2-\lambda}{2}M)$ for $m \in [\frac{2-\lambda}{2}M + 2, M]$. That is, $D_m^F = \begin{cases} \frac{(2m+\lambda-2)M-m(m-1)}{\lambda M} & \text{for } m \in [2, \frac{2-\lambda}{2}M + 1], \\ \frac{(\lambda-2)^2M+2(\lambda-2)}{4\lambda} + m & \text{for } m \in [\frac{2-\lambda}{2}M + 2, M]. \end{cases}$ Thus,

$D^F_{M(D)} = \frac{(\lambda^2+4)M+(2\lambda-4)}{4\lambda}$, which finally gives $E^F(Y)_{(D)} = \frac{\lambda^3 M^2-(3\lambda^2-24\lambda)M-10\lambda}{24\lambda M}$.

Increasing: For this process, the inter-arrival times are such that $\frac{1}{\lambda_m} < 1$ for $m \in [2, \frac{\lambda}{2}M]$, and $\frac{1}{\lambda_m} \geq 1$ for $m \in [\frac{\lambda}{2}M+1, M]$. We can then see that $\max_{1\leq i\leq m}\{\sum_{j=2}^{i}\frac{1}{\lambda_j}+\sum_{j=i}^{m}\frac{1}{\mu_j}\}$ is equal to either $\sum_{j=2}^{m}\frac{1}{\lambda_j}+\frac{1}{\mu_m}$ or $\sum_{j=1}^{m}\frac{1}{\mu_j}$. Therefore, $D^F_m = \max_{1\leq i\leq m}\{\sum_{j=2}^{m}\frac{1}{\lambda_j}, \sum_{j=1}^{m}\frac{1}{\mu_j}\} = (m-1)\max\{\frac{m}{\lambda M}, 1\}+1$. Let us now distinguish two cases, $\lambda \geq 1$ and $\lambda < 1$. In the first case, $\frac{m}{\lambda M} \leq 1$ for $m = 2, ..., M$, and therefore $D^F_m = m$. In the second case,

$D^F_m = \begin{cases} m & \text{for } m \in [2, \lambda M], \\ \frac{\lambda M+m(m-1)}{\lambda M} & \text{for } m \in [\lambda M+1, M]. \end{cases}$ Therefore, we can obtain

$D^F_{M(I)} = \begin{cases} \frac{M+\lambda-1}{\lambda} & \text{for } \lambda \in (2\frac{1}{M}, 1), \\ M & \text{for } \lambda \in [1, 2\frac{M-1}{M}), \end{cases}$ and $E^F(Y)_{(I)} = \begin{cases} \frac{\lambda^3M^2-(3\lambda^2-6\lambda)M-\lambda}{6\lambda M} & \text{for } \lambda \in (2\frac{1}{M}, 1), \\ \frac{(3\lambda-2)M^2+3M-1}{6\lambda M} & \text{for } \lambda \in [1, 2\frac{M-1}{M}). \end{cases}$