Online Supplement for "Making Better Fulfillment Decisions on the Fly in an Online Retail Environment"

Appendix - Perfect hindsight formulation

The following optimization problem minimizes the cost to fulfill a set of customers' orders over a fixed time horizon. This optimization focuses on a single SKU at a time. A given customer order *k* may be for the SKU by itself, or for the SKU along with other items. We assume that we know all future demand over the time horizon as well as the timing and quantity of each inventory replenishment over this time horizon. For multi-item orders the optimization needs to decide whether to fill with a single shipment or to split into multiple shipments. In the latter case, we impose a significant penalty that is set high enough that this will occur only when it is not possible to fill the multi-item order with a single shipment.

For each customer order there are two sets of binary decision variables: x_{ik}^{Single}, which is set to one if order *k* is fulfilled from FC *i* and FC *i* also has the other items in order *k* (representing a single shipment), and x_{ik}^{Multi}, which is set to one if FC *i* does *not* have the other items in order *k* (representing multiple shipments). For each customer, either her order will be fulfilled in a single shipment or in multiple shipments, but not both. The decision variables X_{it} represent the inventory on-hand in FC *i* on the start of day *t,* with the input parameters X_{i0} denoting the initial inventory. The input parameters X_{it}^{INB} represent the amount of inventory that arrived in the system to FC *i* on day *t*. Recall the input parameters Z_{ik} denote whether or not FC *i* had the other items in order *k* on-hand on day θ_k, where θ_k denotes the day on which customer *k* placed her order. $\mathcal{M}$ denotes a very large number. Here, c_{ik} has a slightly different definition than in section 4.3: it does not know yet whether order *k* was split. Thus, we set $c_{ik} \equiv c_{ijm} / r_k$, where *j* is the region of customer *k*, *m* is the requested delivery time, and r_k is the number of items in customer *k*'s order. (Recall in section 4.3 the value of c_{ik} depended on whether or not order *k* was split into multiple shipments.) The formulation of the optimization problem is:

$$\min_{X,x,z} \quad \sum_{i,k} c_{ik} x_{ik}^{Single} \; + \; 2\sum_{i,k} c_{ik} x_{ik}^{Multi} \; + \; \mathcal{M}\sum_{i,k} x_{ik}^{Multi} \tag{9}$$

$$s.t. \qquad X_{i,t} \; = \; X_{i,t-1} + X_{i,t}^{INB} - \sum_{k:\theta_k = t-1} \left(x_{ik}^{Single} + x_{ik}^{Multi} \right) \qquad \forall\, i > 0 \tag{9-1}$$

$$\sum_{i} \left(x_{ik}^{Single} + x_{ik}^{Multi} \right) \; = \; 1 \qquad \forall\, k \tag{9-2}$$

$$x_{ik}^{Single} \; \le \; Z_{ik} \qquad \forall\, i,k \tag{9-3}$$

$$X_{it} \; \ge \; 0 \qquad \forall\, i,t \tag{9-4}$$

$$x_{ik}^{Single}, x_{ik}^{Multi} \; \in \; \{0,1\} \qquad \forall\, i,k \tag{9-5}$$

Constraints (9-1) ensure that the inventory levels follow mass balance restrictions (what exits cannot exceed the sum of what was present at the start and what enters). These constraints also implicitly require that an order be fulfilled on the day it was placed, something we also require for the myopic and heuristic policies as discussed above in section 4.4. Constraints (9-2) ensure that every order is satisfied either in one shipment or in multiple shipments (but not both). Constraints (9-3) require that if an order for multiple items (that is, an order that requests the specific SKU and some other items) is fulfilled in a single shipment, it be done from an FC that also had the other items in the order on hand. Constraints (9-4) prevent inventory from becoming negative in any facility, while constraints (9-5) require the decision variables x_{ik} to be binary.

We allow the perfect hindsight optimization to split orders as a last resort in order to sidestep the fact that it is possible that *no* feasible fulfillment strategy exists which keeps all of the multi-item orders in a single shipment (for instance, if an order *k* contained a large assortment of eclectic items such that $Z_{ik} = 0$ for all *i*.) When simulating the myopic and heuristic policies (which we do *after* solving the perfect hindsight optimization), we require that these two policies attempt to keep multi-item orders together only if the perfect hindsight optimization could keep them together. If, on the other hand, for a specific customer order *k,* the perfect hindsight optimization split the order, then we automatically treat this order as a split order in the evaluation of both the myopic and heuristic policies. Thus, in our analysis, we highlight the *differences* in each policy's ability to keep multi-item orders together in a single shipment. Once the optimization problem is solved, the perfect hindsight cost for SKU *n* is defined as:

$$\boldsymbol{C}_n^{PH} \; = \; \sum_{i,k} c_{ik} x_{ik}^{Single} + 2\sum_{i,k} c_{ik} x_{ik}^{Multi} \tag{10}$$

where x_{ik} is the solution to (9).