Online Supplement to "PBM Competition in Pharmaceutical Supply Chain: Formulary Design and Drug Pricing"

Appendix A: Proofs

Proof of Theorem 1 (a). By (5) and (6), we have

$$\max_{\vec{y_j}\in\{0,1\}^{\mathbf{N}},\vec{p_j}\in\mathbf{R}^{\mathbf{N}+1}} \quad \pi_j=\frac{\exp\left(\left(A_j+CS_j-B_j\right)/\nu\right)}{\exp\left(u_e^0/\nu\right)+\sum_{k=1}^M\exp\left(\left(A_k+CS_k-B_k\right)/\nu\right)}(B_j-W_j),$$

where W_j is given in (9). Using (7) and (8), we can rewrite PBM j's problem as

$$\max_{\vec{y_j}\in\{0,1\}^{\mathbf{N}},\vec{p_j}\in\mathbf{R}^{\mathbf{N}+1}} \quad \pi_j=\frac{\exp\left(\left(A_j-V_j\right)/\nu\right)}{\exp\left(u_e^0/\nu\right)+\sum_{k=1}^M\exp\left(\left(A_k-V_k\right)/\nu\right)}\left(V_j-U_j\right).$$

Note that U_j only depends on the formulary decision $\vec{y_j}\in\{0,1\}^{\mathbf{N}}$. Hence, the feasible set of U_j is given by Ω_j, and (10) follows directly.

(b). It directly follows from (10) that π_j linearly decreases in U_j. Note that

$$\frac{\partial\ln\pi_j}{\partial V_j}=-\frac{1}{\nu}+\frac{1}{V_j-U_j}+\frac{\exp\left(\left(A_j-V_j\right)/\nu\right)}{\nu\left(\exp\left(u_e^0/\nu\right)+\sum_{k=1}^M\exp\left(\left(A_k-V_k\right)/\nu\right)\right)},\tag{25}$$

and
$$\frac{\partial^2\ln\pi_j}{\partial V_j^2}=-\frac{1}{(V_j-U_j)^2}-\frac{\exp\left(\left(A_j-V_j\right)/\nu\right)\left(\exp\left(u_e^0/\nu\right)+\sum_{k\neq j}\exp\left(\left(A_k-V_k\right)/\nu\right)\right)}{\nu^2\left(\exp\left(u_e^0/\nu\right)+\sum_{k=1}^M\exp\left(\left(A_k-V_k\right)/\nu\right)\right)^2}<0,\tag{26}$$

showing that π_j is strictly log-concave in V_j. Further note that

$$\begin{aligned}V_j=&\left(d_{0|j}(\vec{y_j})(p_{0j}-c_j^g)+\sum_{i=1}^N d_{i|j}(\vec{y_j})(p_{ij}-c_{ij}(y_{ij})-\rho_j r_{ij}y_{ij})\right)S\\&-\frac{\mu}{\alpha}\left(\ln\left(\exp\left(\frac{\gamma_0-\alpha c_j^g+\beta q_0}{\mu}\right)+\sum_{i=1}^N\exp\left(\frac{\gamma_i-\alpha c_{ij}(y_{ij})+\beta q_i}{\mu}\right)\right)\right)S.\end{aligned}\tag{27}$$

For any given original formulary decision vector $\vec{y_j}\in\{0,1\}^{\mathbf{N}}$ (and hence U_j), V_j is a linear combination of the $N+1$ original price decision variables $p_{ij}\in\mathbf{R}$ $(i=0,\dots,N)$, as shown by (27). Therefore, V_j can target any value by varying the original price decision vector $\vec{p_j}$. Conversely, for any given $V_j\in\mathbf{R}$, U_j can target any value in its feasible set Ω_j by choosing the corresponding original formulary decision vector $\vec{y_j}$. At the chosen $\vec{y_j}$, we can keep V_j fixed at the given value by varying the original price decision vector $\vec{p_j}$.

(c). Based on the proof of parts (a) and (b), PBM j can determine its best response sequentially by solving the optimal U_j first and then the optimal V_j. For PBM j, the minimization of U_j is a combinatorial optimization on a finite set: $\min_{\vec{y_j}\in\{0,1\}^{\mathbf{N}}}U_j(\vec{y_j})$, and thus the optimal U_j^* is unique. Since each PBM's optimal aggregate formulary decision U_j^* is independent of each other, PBMs' competition on (U_j,V_j) can be reduced to a competition on V_j only. With U_j^* determined, PBM j only needs to consider the value of V_j in a compact and convex set, $[U_j^*,\overline{V}_j]$, where $\overline{V}_j$ is the unique root of $-\frac{1}{\nu}+\frac{1}{V_j-U_j^*}+\frac{\exp\left(\left(A_j-V_j\right)/\nu\right)}{\nu\left(\exp(u_e^0/\nu)+\exp\left(\left(A_j-V_j\right)/\nu\right)\right)}=0$, because PBM j would make negative profit for any $V_j<U_j^*$ and $\partial\ln\pi_j/\partial V_j<0$ for any $V_j>\overline{V}_j$. The existence of equilibrium directly follows from the concavity of $\ln\pi_j$ in V_j. Furthermore,

$$\frac{\partial^2\ln\pi_j}{\partial V_j^2}+\sum_{k\neq j}\left|\frac{\partial^2\ln\pi_j}{\partial V_j\partial V_k}\right|=-\frac{1}{(V_j-U_j)^2}-\frac{\exp\left(\left(A_j-V_j\right)/\nu\right)\exp\left(u_e^0/\nu\right)}{\nu^2\left(\exp\left(u_e^0/\nu\right)+\sum_{k=1}^M\exp\left(\left(A_k-V_k\right)/\nu\right)\right)^2}<0.\tag{28}$$

By Milgrom and Roberts (1990), a unique equilibrium is guaranteed, and satisfies the first order condition given by (25).

Proof of Proposition 1 Substituting (9) and (2) into (8), we can write PBM j's formulary problem as

$$\begin{aligned}\min_{\vec{y_j}\in\{0,1\}^{\mathbf{N}}} U_j(\vec{y_j}) = &\left(d_{0|j}(\vec{y_j})(w_{0j}-c_j^g)+\sum_{i=1}^{N} d_{i|j}(\vec{y_j})\left(w_{ij}-c_j^n+\left(c_j^n-c_j^p-r_{ij}\right)y_{ij}\right)\right)S\\ &-\frac{\mu}{\alpha}\left(\ln\left(\exp\left(\frac{\gamma_0-\alpha c_j^g+\beta q_0}{\mu}\right)+\sum_{i=1}^{N}\exp\left(\frac{\gamma_i-\alpha c_j^n+\alpha(c_j^n-c_j^p)y_{ij}+\beta q_i}{\mu}\right)\right)\right)S. \end{aligned}\tag{29}$$

By (11), we rewrite (29) into the following form:

$$\min_{\vec{y_j}\in\{0,1\}^{\mathbf{N}}} U_j(\vec{y_j}) = \left(\frac{x_{0j}m_{0j}+\sum_{i=1}^{N}x_{ij}(y_{ij})m_{ij}(y_{ij})}{x_{0j}+\sum_{i=1}^{N}x_{ij}(y_{ij})}-\frac{\mu}{\alpha}\ln\left(x_{0j}+\sum_{i=1}^{N}x_{ij}(y_{ij})\right)\right)S. \tag{30}$$

First, we introduce the following lemma to characterize properties of PBMs' optimal formulary decision.

Lemma 2. *For $j=1,\dots,M$, PBM j puts branded drug i on tier 2 (i.e., $y_{ij}^*=1$) if for any $\vec{y_j}\in\{0,1\}^{\mathbf{N}}$,*

$$\delta_{ij}\le\frac{x_{0j}m_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj})m_{kj}(y_{kj})}{x_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj})}. \tag{31}$$

Proof of Lemma 2 We prove by contradiction. Let $\vec{y_j^*}$ be PBM j's optimal formulary decision vector. Assume to the contrary that (31) holds for any $\vec{y_j}\in\{0,1\}^{\mathbf{N}}$ and $y_{ij}^*=0$. We can construct another formulary decision $\vec{y_j}'$ by setting $y'_{ij}=1$ and $y'_{kj}=y^*_{kj}$ for $k\neq i$. Then we have:

$$\begin{aligned}U_j(\vec{y_j}')=&\left(\frac{x_{0j}m_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*)+x_{ij}(1)m_{ij}(1)-x_{ij}(0)m_{ij}(0)}{x_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj}^*)+x_{ij}(1)-x_{ij}(0)}\right.\\ &\left.-\frac{\mu}{\alpha}\ln\left(x_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj}^*)+x_{ij}(1)-x_{ij}(0)\right)\right)S.\end{aligned}\tag{32}$$

Since $\delta_{ij}=\frac{x_{ij}(1)m_{ij}(1)-x_{ij}(0)m_{ij}(0)}{x_{ij}(1)-x_{ij}(0)}\le\frac{x_{0j}m_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*)}{x_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj}^*)}$ and $x_{ij}(1)>x_{ij}(0)$, we have $U_j(\vec{y_j}')<U_j(\vec{y_j}^*)$, contradicting the optimality of $\vec{y_j}^*$. This completes the proof.

Lemma 2 shows that when assigning branded drugs to tier 2, it is optimal for PBM j to start by choosing the branded drug with the smallest cost change rate, then add the drug with the second smallest cost change rate to tier 2, and so on, until (31) no longer holds. Since PBM j sorts branded drugs by their cost change rates in an ascending order, we can find the threshold index as follows:

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Initialize   $I_j=0$;
FOR   $i=1,\cdots,N$
  IF   $\delta_{ij}\le\frac{x_{0j}m_{0j}+\sum_{k=1}^{i-1}x_{kj}(1)m_{kj}(1)+\sum_{k=i}^{N}x_{kj}(0)m_{kj}(0)}{x_{0j}+\sum_{k=1}^{i-1}x_{kj}(1)+\sum_{k=i}^{N}x_{kj}(0)}$,
    $I_j=i$;
  ELSE   STOP;
  ENDIF
ENDFOR
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Note that

$$\min_{\vec{y_j}\in\{0,1\}^{\mathbf{N}}}\frac{x_{0j}m_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj})m_{kj}(y_{kj})}{x_{0j}+\sum_{k=1}^{N}x_{kj}(y_{kj})}=\frac{x_{0j}m_{0j}+\sum_{k=1}^{I_j}x_{kj}(1)m_{kj}(1)+\sum_{k=I_j+1}^{N}x_{kj}(0)m_{kj}(0)}{x_{0j}+\sum_{k=1}^{I_j}x_{kj}(1)+\sum_{k=I_j+1}^{N}x_{kj}(0)}.$$

By Lemma 2, if $i\le I_j$, it is optimal for PBM j to assign drug i on tier 2 (i.e., $y_{ij}^*=1$).

Proof of Proposition 2 When all branded drugs on PBM j's plan have the same brand-specific attribute γ_i and quality index q_i, we first show that if PBM j puts k branded drugs on tier 2, it is optimal for PBM j to assign k branded drugs with the smallest cost change rate indices on tier 2.

We prove the above statement by contradiction. Let $\vec{y_j^*}$ be PBM j's optimal formulary decision vector. Assume to the contrary of the statement, there exists an $y_{ij}^* = 0$ $(i \leq k)$ and an $y_{lj}^* = 1$ $(l > k)$. We can construct another formulary decision $\vec{y_j}'$ by setting $y_{ij}' = 1$ and $y_{lj}' = 0$ while keeping other branded drugs' formulary decision unchanged. Then we have:

$$U_j(\vec{y_j}') = \left(\frac{x_{0j}m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*) + x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) - x_{lj}(1)m_{lj}(1) + x_{lj}(0)m_{lj}(0)}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}^*) + x_{ij}(1) - x_{ij}(0) - x_{lj}(1) + x_{lj}(0)} \right.$$
$$\left. - \frac{\mu}{\alpha} \ln \left(x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}^*) + x_{ij}(1) - x_{ij}(0) - x_{lj}(1) + x_{lj}(0) \right) \right) S.$$

Note that all branded drugs have the same brand specific attribute and quality index, we have $x_{ij}(1) - x_{ij}(0) = x_{lj}(1) - x_{lj}(0) > 0$. Since PBM j ranks branded drugs in ascending order of their cost change rate, i.e., $\delta_{1j} \leq \delta_{2j} \leq \ldots \leq \delta_{Nj}$, we have $\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)} \leq \delta_{lj} = \frac{x_{lj}(1)m_{lj}(1) - x_{lj}(0)m_{lj}(0)}{x_{lj}(1) - x_{lj}(0)}$, and hence $x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) \leq x_{lj}(1)m_{lj}(1) - x_{lj}(0)m_{lj}(0)$. Therefore, $U_j(\vec{y_j}') \leq U_j(\vec{y_j}^*)$, contradicting the optimality of $\vec{y_j}^*$. This completes the proof of the statement.

By the above statement and Proposition 1, PBM j only needs to compare $N - I_j + 1$ formulary decision vectors, each corresponding to putting k $(k = I_j, \ldots, N)$ branded drugs with smallest cost change rate indices on tier 2. This completes the proof.

Proof of Theorem 2 By the definition of V_j in (7) and the expression of n_j in (5), we can rewrite V_j as a function of PBMs' market share vector, $\vec{n} = (n_1, \ldots, n_M)$, as follows:

$$V_j = A_j - u_e^0 - \nu \left(\ln n_j - \ln(1 - \sum_{k=1}^M n_k) \right), \quad j = 1, \ldots, M. \tag{33}$$

Let n_0 denote the probability of the client not contracting with any PBM, we have $n_0 = 1 - \sum_{k=1}^M n_k$.

Given the optimal formulary decision, the equilibrium V_j^* satisfies the set of first order conditions given by (13). Substituting (33) into (13) and rearranging the terms, we have

$$n_j^* \cdot \exp(\frac{n_j^*}{1 - n_j^*}) = n_0^* \cdot \exp \left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right), \quad j = 1, \ldots, M. \tag{34}$$

By the definition of $H(\cdot)$ function in (14), (16) directly follows. Combining with the fact that $\sum_{j=0}^M n_j^* = 1$, the equilibrium probability of not contracting with any PBM, n_0^*, satisfies the single-variable equation:

$$n_0 + \sum_{j=1}^M H \left(n_0 \cdot \exp \left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) = 1.$$

Since $H(\cdot)$ strictly increases from 0 to 1, the left-hand-side of the above equation strictly increases from 0 to a value greater than 1 as n_0 increases from 0 to 1. Therefore, n_0^* is the unique solution to (15).

Substituting (5), (7) and (13) into (10), PBM j's equilibrium expected profit is given by $\pi_j^* = \frac{\nu n_j^*}{1 - n_j^*}$.

Substituting $n_0^* = 1 - \sum_{k=1}^M n_k^*$ into (33), (18) follows directly.

The client organization's expected utility in the equilibrium is given by

$$\begin{aligned}\overline{v}^* &= E\left(\max\left(\max_j v_j^*, v_0\right)\right) = \nu \ln\left(\exp\left(\frac{u_e^0}{\nu}\right) + \sum_{j=1}^{M} \exp\left(\frac{A_j - V_j^*}{\nu}\right)\right) \\ &= \nu \ln\left(\exp\left(\frac{u_e^0}{\nu}\right)\left(1 + \sum_{j=1}^{M} \frac{n_j^*}{n_0^*}\right)\right) = u_e^0 - \nu \ln n_0^*.\end{aligned}$$

Proof of Lemma 1 (a). Applying the implicit function theorem on the function $H(x)$, we have

$$H'(x) = \frac{(1-H)^2}{(1-H+H^2)\exp(\frac{H}{1-H})} > 0. \tag{35}$$

By (15), (34) and (35), and applying the implicit function theorem on (15), we have

$$\frac{\partial n_0^*}{\partial U_j^*} = \frac{\frac{(1-n_j^*)^2 n_j^*}{\nu\left(1-n_j^*+n_j^{*2}\right)}}{1+\sum_{k=1}^{M} \frac{(1-n_k^*)^2 n_k^*}{(1-n_k^*+n_k^{*2})n_0^*}} > 0. \tag{36}$$

(b). By (16) and (36), we have

$$\begin{aligned}\frac{\partial n_j^*}{\partial U_j^*} &= H'\left(n_0^* \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\left(\frac{\partial n_0^*}{\partial U_j^*} - \frac{n_0^*}{\nu}\right) \\ &= -H'\left(n_0^* \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) \frac{n_0^* + \sum_{k\neq j} \frac{(1-n_k^*)^2 n_k^*}{1-n_k^*+n_k^{*2}}}{\nu\left(1+\sum_{k=1}^{M} \frac{(1-n_k^*)^2 n_k^*}{(1-n_k^*+n_k^{*2})n_0^*}\right)} < 0, \qquad (37) \\ \frac{\partial n_j^*}{\partial U_k^*} &= H'\left(n_0^* \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) \cdot \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) \frac{\partial n_0^*}{\partial U_k^*} > 0, \quad k \neq j. \qquad (38)\end{aligned}$$

By (17), (37) and (38), $\frac{\partial \pi_j^*}{\partial U_j^*} < 0$ and $\frac{\partial \pi_j^*}{\partial U_k^*} > 0$ $(k \neq j)$ directly follow.

(c). By (18), (36) and (37), we have

$$\frac{\partial V_j^*}{\partial U_j^*} = -\nu\left(\frac{1}{n_j^*}\frac{\partial n_j^*}{\partial U_j^*} - \frac{1}{n_0^*}\frac{\partial n_0^*}{\partial U_j^*}\right) > 0. \tag{39}$$

By (18), (34), (36) and (38), we have

$$\frac{\partial V_j^*}{\partial U_k^*} = -\nu\left(\frac{1}{n_j^*}\frac{\partial n_j^*}{\partial U_k^*} - \frac{1}{n_0^*}\frac{\partial n_0^*}{\partial U_k^*}\right) = \frac{\nu n_j^*}{(1-n_j^*+n_j^{*2})n_0^*}\frac{\partial n_0^*}{\partial U_k^*} > 0, \quad k \neq j. \tag{40}$$

(d). The result directly follows from (19) and part (c).

Proof of Proposition 3 (a). Since formulary decision y_{ij} is a binary variable, showing that it is weakly decreasing is equivalent to showing that an increase from 0 to 1 cannot happen when the wholesale price of branded drug i increases. We prove by contradiction. Suppose that the wholesale price of branded drug i increases from w_{ij} to w'_{ij} $(w'_{ij} > w_{ij})$, $y_{ij}^*|_{w_{ij}} = 0$ and $y_{ij}^*|_{w'_{ij}} = 1$. Note that for any given $\vec{y}_j$, we have

$$\frac{\partial U_j(\vec{y}_j)}{\partial w_{ij}} = \frac{x_{ij}(y_{ij})S}{x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj})} > 0, \quad \text{and} \quad \frac{\partial^2 U_j(\vec{y}_j)}{\partial w_{ij}^2} = 0. \tag{41}$$

Therefore,

$$U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w_{ij}} \leq U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w_{ij}} < U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w'_{ij}} \leq U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w'_{ij}},$$

which shows that

$$U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w'_{ij}} - U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w_{ij}} \geq U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w'_{ij}} - U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w_{ij}}. \tag{42}$$

By (41), we have

$$U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w'_{ij}} - U_j\left(\vec{y}_j^*|_{w_{ij}}\right)|_{w_{ij}} = \frac{x_{ij}(y_{ij}^*|_{w_{ij}})(w'_{ij}-w_{ij})S}{x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w_{ij}})}, \tag{43}$$

$$\text{and } U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w'_{ij}} - U_j\left(\vec{y}_j^*|_{w'_{ij}}\right)|_{w_{ij}} = \frac{x_{ij}(y_{ij}^*|_{w'_{ij}})(w'_{ij}-w_{ij})S}{x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w'_{ij}})}. \tag{44}$$

Substitute $y_{ij}^*|_{w_{ij}}=0$, $y_{ij}^*|_{w'_{ij}}=1$, (43) and (44) into (42), we have

$$x_{ij}(0)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w'_{ij}})\right) \geq x_{ij}(1)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w_{ij}})\right) \tag{45}$$

The left hand side of (45) satisfies

$$x_{ij}(0)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w'_{ij}})\right) \leq x_{ij}(0)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(1)\right) = x_{ij}(0)x_{0j}+\sum_{k=1}^{N}\exp\left(\frac{\gamma_i+\gamma_k+\beta\left(q_i+q_k\right)-\alpha\left(c_j^n+c_j^p\right)}{\mu}\right),$$

while the right hand side of (45) satisfies

$$x_{ij}(1)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(y_{kj}^*|_{w_{ij}})\right) \geq x_{ij}(1)\left(x_{0j}+\sum_{k=1}^{N} x_{kj}(0)\right) = x_{ij}(1)x_{0j}+\sum_{k=1}^{N}\exp\left(\frac{\gamma_i+\gamma_k+\beta\left(q_i+q_k\right)-\alpha\left(c_j^n+c_j^p\right)}{\mu}\right).$$

By (45) and $x_{0j}>0$, we have $x_{ij}(0)\geq x_{ij}(1)$. This contradicts with the fact that $x_{ij}(0)<x_{ij}(1)$.

In general, the impact of any other drug's wholesale price on the focal drug's optimal formulary decision is non-monotone, and we have observed this non-monotone effect in numerical examples.

Recall that the optimal formulary decision vector is a dominant choice, and is independent of all other PBMs' decision and cost parameters. Therefore, PBM j's optimal formulary decision is independent of any drug's wholesale price on any competing PBM's plan.

(b). Note that for any given $\vec{y}_j$, (41) holds for any $i=0,\ldots,N$. Consider the case that the wholesale price of any (branded or generic) drug i increases from w_{ij} to w'_{ij} ($w'_{ij}>w_{ij}$). By (41), $U_j^*|_{w'_{ij}}=U_j(\vec{y}_j^*|_{w'_{ij}})|_{w'_{ij}} > U_j(\vec{y}_j^*|_{w'_{ij}})|_{w_{ij}} \geq U_j(\vec{y}_j^*|_{w_{ij}})|_{w_{ij}} = U_j^*|_{w_{ij}}$. Therefore, U_j^* strictly increases in the wholesale price of any (branded or generic) drug charged to PBM j.

Since each PBM's formulary decision is independent of each other, the optimal welfare-adjusted cost of PBM j's plan U_j^* is independent of any (branded or generic) drug's wholesale price charged to any competing PBM.

(c). Since each PBM's formulary decision is independent of each other, the optimal welfare-adjusted cost of PBM j's plan U_j^* is independent of any (branded or generic) drug l's wholesale price charged to PBM k ($k\neq j$), w_{lk}. Therefore, the impact of w_{lk} on the equilibrium welfare-adjusted price of PBM j's plan V_j^* is only through its impact on the optimal welfare-adjusted cost of PBM k's plan U_k^*. By part (c) of Lemma 1 and part (b) of Proposition 3, V_j^* strictly increases in w_{lk} and w_{lj}.

(d). The impact of w_{lk} on the client's expected utility $\overline{v}^*$ directly follows from (19) and part (c) of Proposition 3.

(e). We will show this part in an analogous approach as the above proof in part (c). Since the optimal welfare-adjusted cost of PBM j's plan U_j^* is independent of any (branded or generic) drug l's ($l=0,\ldots,N$)

wholesale price charged to any other PBM k ($k \neq j$), w_{lk}. Therefore, the impact of w_{lk} on PBM j's equilibrium expected market share n_j^* and expected profit π_j^* is only through its impact on the optimal welfare-adjusted cost of PBM k's plan U_k^*. By part (b) of Lemma 1 and part (b) of Proposition 3, the statement follows directly.

Proof of Proposition 4 (a). For any given $\vec{y}_j$ and branded drug i,

$$\frac{\partial U_j(\vec{y}_j)}{\partial q_i} = -\frac{\beta S x_{ij}(y_{ij})}{\mu\left(x_{0j} + \sum_{=1}^{N} x_{ljl}(y_{lj})\right)^2}\left(\frac{\mu}{\alpha}x_{ij}\left(y_{ij}\right) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}\left(y_{ij}\right) + m_{0j}\right) + \sum_{l\,/\,i} x_{lj}\left(y_{lj}\right)_{\;\;=}\left(\frac{\mu}{\alpha} - m_{ij}\left(y_{ij}\right) + m_{lj}\left(y_{lj}\right)\right)\right). \tag{46}$$

Therefore, $sgn\left(\frac{\partial U_j(\vec{y}_j)}{\partial q_i}\right) = -sgn\left(\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right)\right)$. Note that $\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right)$ is convex increasing in q_i, and approaches to $+\infty$ when $q_i \to +\infty$. So $\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right)$ crosses 0 at most once. Denote $q_{ij}(\vec{y}_j)$ as the unique root of $\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right) = 0$ when exists, and $q_{ij}(\vec{y}_j) = -\infty$ otherwise. Set $\overline{q}_{ij} = \left(\max_{\vec{y}_j \in \{0,1\}^N} q_{ij}(\vec{y}_j)\right)^+$ and $\underline{q}_{ij} = \left(\min_{\vec{y}_j \in \{0,1\}^N} q_{ij}(\vec{y}_j)\right)^+$. It directly follows that $\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right) > 0$ for any $\vec{y}_j \in \{0,1\}^N$ when $q_i > \overline{q}_{ij}$, and $\frac{\mu}{\alpha}x_{ij}(y_{ij}) + x_{0j}\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j}\right) + \sum_{l\neq i} x_{lj}(y_{lj})\left(\frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{lj}(y_{lj})\right) < 0$ for any $\vec{y}_j \in \{0,1\}^N$ when $0 \leq q_i < \underline{q}_{ij}$. Therefore, PBM j's optimal welfare-adjusted cost, U_j^*, strictly increases in branded drug i's quality index, q_i, if $0 \leq q_i < \underline{q}_{ij}$, and strictly decreases in q_i if $q_i > \overline{q}_{ij}$.

When $q_{ij}(\vec{y}_j) \neq -\infty$, applying the implicit function theorem, we have $q_{ij}(\vec{y}_j)$ increases in w_{ij} and weakly decreases r_{ij}. Therefore, $\overline{q}_{ij} = \left(\max_{\vec{y}_j \in \{0,1\}^N} q_{ij}(\vec{y}_j)\right)^+$ and $\underline{q}_{ij} = \left(\min_{\vec{y}_j \in \{0,1\}^N} q_{ij}(\vec{y}_j)\right)^+$ both weakly increase in w_{ij} and weakly decrease in r_{ij}.

The impact of generic drug's quality q_0 can be shown in the same way.

(b). By Lemma 1, the impact of drug quality affects any PBM's equilibrium expected welfare-adjusted price, market share, profit, and the client's utility through its impact on all PBMs' optimal welfare-adjusted costs. When all PBMs are symmetric with respect to the cost parameters, the impact of any drug's quality on the optimal welfare-adjusted cost, U_j^*, is the same across all PBMs. Therefore, the impact of any drug's quality on all the quantities of interest mentioned above depends on the aggregate impact of the drug's quality on all PBMs' optimal welfare-adjusted costs. For PBM j, by (37) and (38), we have

$$\sum_{k=1}^{M} \frac{\partial n_j^*}{\partial U_k^*} = -H'\left(n_0^* \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) \exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) \frac{n_0^*}{\nu\left(1 + \sum_{k=1}^{M} \frac{(1-n_k^*)^2 n_k^*}{(1-n_k^* + n_k^{*2}) n_0^*}\right)} < 0. \tag{47}$$

Therefore, the impact of q_i on n_j^* has the opposite directional change as the impact of q_i on U_j^*.

The impact of q_i on V_j^*, π_j^*, and $\overline{v}^*$ can be shown in the same way.

Proof of Proposition 5 Substituting (16) into (15) and by the symmetry of PBMs in the pre-merger model, the equilibrium market share of PBM j, n_j^*, is the unique solution on $(0,1)$ of the following equation:

$$n_j^*\left(M + \exp\left(\frac{n_j^*}{1-n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) = 1, \quad j = 1, \ldots, M. \tag{48}$$

After the merger, it directly follows from (16) and the fact $U_1^{m*} \le U_1^* = U_j^{m*}$ $(j=3,\dots,M)$ that $n_1^{m*} \ge n_j^{m*}$ $(j=3,\dots,M)$. All non-merging PBMs remain symmetric after the merger. By (17), $\pi_1^{m*} \ge \pi_j^{m*}$ $(j=3,\dots,M)$ always holds. Apply (15) and (16) to the post-merger model, we have the following relationships:

$$\begin{aligned} & n_0^{m*} + n_1^{m*} + (M-2)n_j^{m*} = 1, && (49)\\ & n_1^{m*} \exp(\frac{n_1^{m*}}{1-n_1^{m*}}) = n_0^{m*} \exp\left(\frac{A_1 - U_1^{m*} - u_e^0 - \nu}{\nu}\right), && (50)\\ & n_j^{m*} \exp(\frac{n_j^{m*}}{1-n_j^{m*}}) = n_0^{m*} \exp\left(\frac{A_j - U_j^{m*} - u_e^0 - \nu}{\nu}\right), \quad j=3,\dots,M. && (51) \end{aligned}$$

Using $A_1 = A_j$ and $U_j^{m*} = U_j^* = U_1^*$ $(j=3,\dots,M)$, we have n_j^{m*} satisfies the following equation:

$$n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + H\left(n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu}\right)\right) + (M-2)n_j^{m*} = 1, \quad j=3,\dots,M. \tag{52}$$

Note that the left hand side of (52) strictly increases from 0 to a number greater than 1, as n_j^{m*} increases on $[0,1)$, so n_j^{m*} is the unique solution of the above equation. Therefore, $n_j^{m*} \ge n_j^*$ $(j=3,\dots,M)$ iff

$$n_j^* \exp\left(\frac{n_j^*}{1-n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + H\left(n_j^* \exp\left(\frac{n_j^*}{1-n_j^*} + \frac{\Delta U}{\nu}\right)\right) + (M-2)n_j^* \le 1, \tag{53}$$

where n_j^* is given by (48). (53) can be further reduced to $\Delta U \le \Delta\overline{U}$, where

$$\Delta\overline{U} = \nu\left(\frac{2n_j^*}{1-2n_j^*} - \frac{n_j^*}{1-n_j^*} + \ln 2\right). \tag{54}$$

By (17), it directly follows that $\pi_j^{m*} \ge \pi_j^*$ $(j=3,\dots,M)$ iff $\Delta U \le \Delta\overline{U}$.

Similarly, by (49), (50) and (51), we have n_1^{m*} satisfies the following equation:

$$n_1^{m*} \exp\left(\frac{n_1^{m*}}{1-n_1^{m*}} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + n_1^{m*} + (M-2)H\left(n_1^{m*} \exp\left(\frac{n_1^{m*}}{1-n_1^{m*}} - \frac{\Delta U}{\nu}\right)\right) = 1, \tag{55}$$

and n_1^{m*} is the unique solution of the above equation. When $n_1^{m*} = n_1^*$, the left hand side of (55) is:

$$\begin{aligned} & n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + n_1^* + (M-2)H\left(n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{\Delta U}{\nu}\right)\right)\\ & \le n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + n_1^* + (M-2)H\left(n_1^* \exp\left(\frac{n_1^*}{1-n_1^*}\right)\right)\\ & = n_1^*\left(M - 1 + \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) < n_1^*\left(M + \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{A_1 - U_1^* - u_e^0 - \nu}{\nu}\right)\right) = 1. \end{aligned}$$

Therefore, $n_1^{m*} > n_1^*$ always holds. By (17), $\pi_1^{m*} > \pi_1^*$ always holds as well.

Since the left hand side of (55) strictly increases in n_1^{m*}, $n_1^{m*} \ge n_1^* + n_2^* = 2n_j^*$ iff

$$2n_j^* \exp\left(\frac{2n_j^*}{1-2n_j^*} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + 2n_j^* + (M-2)H\left(2n_j^* \exp\left(\frac{2n_j^*}{1-2n_j^*} - \frac{\Delta U}{\nu}\right)\right) \le 1. \tag{56}$$

where n_j^* is given by (48). Note that the left hand side of (56) is a decreasing function of ΔU. Therefore, (56) is equivalent to $\Delta U \ge \Delta\widetilde{U}$, where $\Delta\widetilde{U}$ is the unique solution of

$$2n_j^* \exp\left(\frac{2n_j^*}{1-2n_j^*} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + 2n_j^* + (M-2)H\left(2n_j^* \exp\left(\frac{2n_j^*}{1-2n_j^*} - \frac{\Delta U}{\nu}\right)\right) = 1. \tag{57}$$

Note that when $\Delta U = \Delta\overline{U}$, the left hand side of (57) is given by

$$2n_j^* \exp\left(\frac{n_j^*}{1-n_j^*} - \ln 2 - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + 2n_j^* + (M-2)H\left(2n_j^* \exp\left(\frac{n_j^*}{1-n_j^*} - \ln 2\right)\right)$$

$$= n_j^* \left(M + \exp\left(\frac{n_j^*}{1-n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) = 1.$$

Since the left hand side of (57) is a decreasing function of ΔU, the above equality proves that $\Delta \widetilde{U} = \Delta \overline{U}$.

By (17), $\pi_1^{m*} \geq \pi_1^* + \pi_2^* = 2\pi_j^*$ is equivalent to $n_1^{m*} \geq \frac{2n_j^*}{1+n_j^*}$. Therefore, $\pi_1^{m*} \geq \pi_1^* + \pi_2^*$ iff

$$\frac{2n_j^*}{1+n_j^*} \exp\left(\frac{2n_j^*}{1-n_j^*} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu} \right) + \frac{2n_j^*}{1+n_j^*} + (M-2) H \left(\frac{2n_j^*}{1+n_j^*} \exp\left(\frac{2n_j^*}{1-n_j^*} - \frac{\Delta U}{\nu} \right) \right) \leq 1. \tag{58}$$

where n_j^* is given by (48). Note that the left hand side of (58) is a decreasing function of ΔU. Therefore, (58) is equivalent to $\Delta U \geq \Delta \underline{U}$, where $\Delta \underline{U}$ is the unique solution of

$$\frac{2n_j^*}{1+n_j^*} \exp\left(\frac{2n_j^*}{1-n_j^*} - \frac{\Delta U + A_j - U_j^* - u_e^0 - \nu}{\nu} \right) + \frac{2n_j^*}{1+n_j^*} + (M-2) H \left(\frac{2n_j^*}{1+n_j^*} \exp\left(\frac{2n_j^*}{1-n_j^*} - \frac{\Delta U}{\nu} \right) \right) = 1. \tag{59}$$

It follows from (55) that n_1^{m*} increases in ΔU. Since $\frac{2n_j^*}{1+n_j^*} < 2n_j^*$, we have $\Delta \underline{U} < \Delta \widetilde{U} = \Delta \overline{U}$.

By (19), $\overline{v}^{m*} \geq \overline{v}^*$ iff $n_0^{m*} \leq n_0^*$. Using (51), $n_0^{m*} \leq n_0^*$ is equivalent to

$$n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{A_j - U_j^{m*} - u_e^0 - \nu}{\nu} \right) \leq n_j^* \exp\left(\frac{n_j^*}{1-n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right), \quad j = 3, \ldots, M. \tag{60}$$

It directly follows that the above inequality is equivalent of $n_j^{m*} \leq n_j^*$ $(j = 3, \ldots, M)$. Therefore, $\overline{v}^{m*} \geq \overline{v}^*$ iff $\Delta U \geq \Delta \overline{U}$. By (21) and (22), we have $n_I^{m*} \geq n_I^*$ iff $\Delta U \geq \Delta \overline{U}$.

For the industry profit after the merger, we will show that π_I^{m*} increases in ΔU. By (16), we have $n_1^{m*} = H\left(n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \right)$ for $j = 3, \ldots, M$. Take derivative with respect to ΔU, and use (35),we have

$$\frac{\partial n_1^{m*}}{\partial \Delta U} = \frac{(1-n_1^{m*})^2}{(1 - n_1^{m*} + n_1^{m*2}) \exp\left(\frac{n_1^{m*}}{1-n_1^{m*}} \right)} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \left(\frac{n_j^{m*}}{\nu} + \frac{1 - n_j^{m*} + n_j^{m*2}}{(1-n_j^{m*})^2} \frac{\partial n_j^{m*}}{\partial \Delta U} \right). \tag{61}$$

Apply Implicit Function Theorem to (52), we have for $j = 3, \ldots, M$

$$\frac{\partial n_j^{m*}}{\partial \Delta U} = \frac{ - \frac{(1-n_1^{m*})^2 n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{n_1^{m*}}{1-n_1^{m*}} + \frac{\Delta U}{\nu} \right)}{\nu\left(1-n_1^{m*}+n_1^{m*2}\right)} }{ M - 2 + \frac{1-n_j^{m*}+n_j^{m*2}}{(1-n_j^{m*})^2} \left(\exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) + \frac{(1-n_1^{m*})^2}{1-n_1^{m*}+n_1^{m*2}} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{n_1^{m*}}{1-n_1^{m*}} + \frac{\Delta U}{\nu} \right) \right) } \tag{62}$$

Substitute (17), (61) and (62) into (22), we have: $\partial \pi_I^{m*} / \partial \Delta U =$

$$\frac{ n_j^{m*} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{n_1^{m*}}{1-n_1^{m*}} + \frac{\Delta U}{\nu} \right) \left((M-2) \left(1 - \frac{(1-n_1^{m*})^2}{(1-n_j^{m*})^2} \right) + \frac{1-n_j^{m*}+n_j^{m*2}}{(1-n_j^{m*})^2} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) }{ (1 - n_1^{m*} + n_1^{m*2}) \left(M - 2 + \frac{1-n_j^{m*}+n_j^{m*2}}{(1-n_j^{m*})^2} \left(\frac{(1-n_1^{m*})^2}{1-n_1^{m*}+n_1^{m*2}} \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{n_1^{m*}}{1-n_1^{m*}} + \frac{\Delta U}{\nu} \right) + \exp\left(\frac{n_j^{m*}}{1-n_j^{m*}} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) \right) } > 0.$$

At $\Delta U = 0$, π_I^* is the unique solution of $\frac{\pi_I^*}{M\nu + \pi_I^*} \left(M + \exp\left(\frac{\pi_I^*}{M\nu} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) = 1$, and π_I^{m*} is the unique solution of $\frac{\pi_I^{m*}}{(M-1)\nu + \pi_I^{m*}} \left(M - 1 + \exp\left(\frac{\pi_I^{m*}}{(M-1)\nu} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) = 1$. If $\pi_I^{m*} \geq \pi_I^*$ at $\Delta U = 0$, then $\pi_I^{m*} \geq \pi_I^*$ for any $\Delta U \geq 0$, and we set $\Delta \underline{\underline{U}} = 0$. Otherwise, if $\pi_I^{m*} \leq \pi_I^*$ at $\Delta U = 0$, there exists a threshold $\Delta \underline{\underline{U}} \geq 0$, such that $\pi_I^{m*} \leq \pi_I^*$ if $\Delta U \leq \Delta \underline{\underline{U}}$, and $\pi_I^{m*} \geq \pi_I^*$ if $\Delta U \geq \Delta \underline{\underline{U}}$. Since $\pi_I^{m*} \geq \pi_I^*$ if $\Delta U \in [\Delta \underline{U}, \Delta \overline{U}]$, we have $\Delta \underline{\underline{U}} \leq \Delta \underline{U}$.

Proof of Proposition 6 As shown in the proof of Proposition 5, $\partial \pi_I^{m*}/\partial \Delta U > 0$. By (19), we have $\partial \overline{v}^{m*}/\partial \Delta U = -\left(\nu/n_0^{m*}\right)\left(\partial n_0^{m*}/\Delta U\right)$. By (15), n_0^{m*} satisfies the equation $n_0^{m*} + H\left(n_0^{m*}\exp\left(\frac{A_1 - U_1^* + \Delta U - u_e^0 - \nu}{\nu}\right)\right) + (M-2)H\left(n_0^{m*}\exp\left(\frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)\right) = 1$, and thus $\partial n_0^{m*}/\Delta U < 0$. Therefore, Π^{m*} increases in ΔU. Combine with part (I) and (IV) of Proposition 5, there exists a threshold values of the power index, $\Delta \acute{U}$ which lies between $\Delta \underline{\underline{U}}$ and $\Delta \overline{U}$, such that $\Pi^{m*} \le \Pi^*$ if $0 \le \Delta U \le \Delta \acute{U}$, and $\Pi^{m*} \ge \Pi^*$ if $\Delta U \ge \Delta \acute{U}$.

Proof of Proposition 7 By (18), we have $V_j^* = A_j - u_e^0 - \nu \ln\left(\frac{n_j^*}{n_0^*}\right)$ for $j = 1, \ldots, M$, $V_1^{m*} = A_1 - u_e^0 - \nu \ln\left(\frac{n_1^{m*}}{n_0^{m*}}\right)$ and $V_j^{m*} = A_j - u_e^0 - \nu \ln\left(\frac{n_j^{m*}}{n_0^{m*}}\right)$ for $j = 3, \ldots, M$. Use (16), we have $V_j^{m*} \ge V_j^*$ $(j = 3, \ldots, M)$ iff $n_j^* \le n_j^{m*}$ $(j = 3, \ldots, M)$. By the result of Proposition 5, it follows that $V_j^{m*} \ge V_j^*$ iff $\Delta U \le \Delta \overline{U}$ for $j = 3, \ldots, M$.

Similarly, $V_1^{m*} \ge V_1^* \Leftrightarrow n_1^{m*} \ge \frac{\nu n_j^* + (1 - n_j^*)\Delta U}{\nu + (1 - n_j^*)\Delta U}$, which is equivalent to the following condition using (55):

$$\frac{\nu n_j^* + (1 - n_j^*)\Delta U}{\nu + (1 - n_j^*)\Delta U}\left(\exp\left(\frac{n_j^*}{1 - n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + 1\right) + (M-2)H\left(\frac{\nu n_j^* + (1 - n_j^*)\Delta U}{\nu + (1 - n_j^*)\Delta U}\exp\left(\frac{n_j^*}{1 - n_j^*}\right)\right) \le 1. \tag{63}$$

Note that the left-hand-side of (63) increases in ΔU. Therefore, $V_1^{m*} \ge V_1^* \Leftrightarrow \Delta U \le \Delta \widehat{U}$, where $\Delta \widehat{U}$ is the unique solution of

$$\frac{\nu n_j^* + (1 - n_j^*)\Delta U}{\nu + (1 - n_j^*)\Delta U}\left(\exp\left(\frac{n_j^*}{1 - n_j^*} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right) + 1\right) + (M-2)H\left(\frac{\nu n_j^* + (1 - n_j^*)\Delta U}{\nu + (1 - n_j^*)\Delta U}\exp\left(\frac{n_j^*}{1 - n_j^*}\right)\right) = 1. \tag{64}$$

Note that at $\Delta U = \Delta \overline{U}$ given by (54), and use (48), the left-hand-side of (63) is

$$\begin{aligned}
&\frac{2n_j^* + (1 - 2n_j^*)\ln 2}{1 + (1 - 2n_j^*)\ln 2}\left(\frac{1}{n_j^*} - M + 1\right) + (M-2)H\left(\frac{2n_j^* + (1 - 2n_j^*)\ln 2}{1 + (1 - 2n_j^*)\ln 2}\exp\left(\frac{n_j^*}{1 - n_j^*}\right)\right) \\
&> 2n_j^*\left(\frac{1}{n_j^*} - M + 1\right) + (M-2)H\left(n_j^*\exp\left(\frac{n_j^*}{1 - n_j^*}\right)\right) = 2 - Mn_j^* > 1.
\end{aligned}$$

Therefore, we have $\Delta \widehat{U} < \Delta \overline{U}$.

For the share-weighted industry average aggregate price decision, it directly follows that $V_I^{m*} \ge V_I^*$ if $\Delta U \le \Delta \widehat{U}$, and $V_I^{m*} \le V_I^*$ if $\Delta U \ge \Delta \overline{U}$. When $\Delta \widehat{U} \le \Delta U \le \Delta \overline{U}$, $V_1^{m*} \le V_1^*$ while $V_j^{m*} \ge V_j^*$ $(j = 3, \ldots, M)$. By (18) and (24), we have $V_I^{m*} \ge V_I^*$ iff

$$n_1^{m*}\ln\left(\frac{n_1^{m*}n_0^*}{n_0^{m*}n_j^*}\right) - (M-2)n_j^{m*}\ln\left(\frac{n_0^{m*}n_j^*}{n_j^{m*}n_0^*}\right) \le 0, \quad j = 3, \ldots, M. \tag{65}$$

It follows from (55), (52) and (15) that n_1^{m*} increases in ΔU, n_j^{m*} decreases in ΔU $(j = 3, \ldots, M)$ and n_0^{m*} decreases in ΔU. By (16), we have $\frac{n_0^{m*}}{n_j^{m*}} = \exp\left(\frac{n_j^{m*}}{1 - n_j^{m*}} - \frac{A_j - U_j^* - u_e^0 - \nu}{\nu}\right)$, which decreases in ΔU. Therefore, the left-hand-side of (65) increases in ΔU. Since the left-hand-side of (65) is less than 0 at $\Delta U = \Delta \widehat{U}$, and greater than 0 at $\Delta U = \Delta \overline{U}$. There exists a threshold value $\Delta \check{U} \in [\Delta \widehat{U}, \Delta \overline{U}]$, such that $V_I^{m*} \ge V_I^*$ if $\Delta U \le \Delta \check{U}$, and $V_I^{m*} \le V_I^*$ if $\Delta U \ge \Delta \check{U}$.

Appendix B: Extension

We now extend our base model to the more general setting of multiple client organizations, each with drugs from multiple therapeutical classes. First, we consider the setting of one client organization with drugs from multiple therapeutical classes. When there are L therapeutical classes, PBM j needs to charge the same copayment for all drugs on the same formulary tier and pass the same percentage of rebate to the client organization for all branded drugs that are on the preferred tier. Therefore, the copayment values (c_j^g, c_j^p, and c_j^n) and rebate pass-through rates (ρ_j) do not vary across different therapeutical classes. All other model primitives and decision variables depend on each drug's therapeutical class, so we add subscript k ($k=1,\ldots,L$) to each of them to denote the drug's therapeutical class. It can be verified that Theorem 1 continues to hold with

$$U_j(\vec{y}_{j1},\ldots,\vec{y}_{jL}) = \sum_{k=1}^{L} U_{jk}(\vec{y}_{jk}) = \sum_{k=1}^{L}\left(W_{jk}(\vec{y}_{jk}) - CS_{jk}(\vec{y}_{jk})\right) \tag{66}$$

$$\text{and} \quad V_j(\vec{y}_{j1},\ldots,\vec{y}_{jL},\vec{p}_{j1},\ldots,\vec{p}_{jL}) = \sum_{k=1}^{L} V_{jk}(\vec{y}_{jk},\vec{p}_{jk}) = \sum_{k=1}^{L}\left(B_{jk}(\vec{y}_{jk},\vec{p}_{jk}) - CS_{jk}(\vec{y}_{jk})\right). \tag{67}$$

It follows from Theorem 1 that each PBM obtains its optimal formulary decision vector by minimizing the welfare-adjusted cost of its plan in (66). Note from (66) that the formulary decisions for different therapeutical classes are separable, because drugs in different therapeutical classes are not substitutable. Therefore, each PBM can solve for its optimal formulary decision vector of each therapeutical class separately. With the optimal formulary decision of each PBM determined, the PBMs compete in the reduced price competition by choosing the welfare-adjusted price of its plan, as defined in (67). The equilibrium welfare-adjusted price, the equilibrium expected market share and the expected profit of each PBM, as well as the client organization's expected utility, can be obtained by Theorem 2. It can be verified that all comparative statics results, as well as the result on the impact of PBM mergers, continue to hold in the case with multiple therapeutical classes.

Next, we consider the case with multiple client organizations. It is common in the PBM industry for a PBM to offer a customized plan for each of its client organizations. Since a PBM does not have to commit to the same formulary and pricing schemes for different clients, its formulary and price decisions are completely separable across different clients. Therefore, a PBM can obtain its formulary and price decisions for a client organization, without considering other client organizations, by the procedure outlined above. This result is consistent with the current practice in the PBM industry that different client organizations typically reach out to PBMs requesting a complete design of a coverage plan at different times of a calendar year, and PBMs respond to such requests by customizing a plan for each client organization.

Appendix C: Discussion on the Utility Function of the Client Organization

In the paper, we use the additive random utility function to model the client organization's utility as given in equation (4). As a result, PBM j's market share function (i.e., the probability that the client organization selects PBM j), n_j, is given by the MNL model as in equation (5). Using the definition of the welfare-adjusted price V_j in (7), the market share function for PBM j, n_j, is a function of the welfare-adjusted prices of all PBMs, as follows,

$$n_j(V_j, V_{-j}) = \frac{\exp\left((A_j - V_j)/\nu\right)}{\exp(u_e^0/\nu) + \sum_{k=1}^{M} \exp\left((A_k - V_k)/\nu\right)}, \quad j = 1, \ldots, M.$$

The above MNL model results from the client's additive random utility function (4) and the assumption on the distribution of the random terms in (4).

Alternatively, beyond the basic factors of consumer surplus and plan cost, if other factors (which has been labeled as the term A_j in our model) affect the client's utility function (and hence the client's choice of PBM) in different ways from what we have modeled in this paper, PBM j's market share function $n_j(V_j, V_{-j})$ will be different from the MNL model. For a general function form of $n_j(V_j, V_{-j})$, we can show that, if the PBM j's market share function, $n_j(V_j, V_{-j})$, is decreasing and (weakly) log-concave in the welfare-adjusted price of PBM j's plan, V_j, then the equilibrium of the PBMs' competition on the aggregate formulary and price decisions still exists. The assumption that $n_j(V_j, V_{-j})$ is a decreasing function of V_j reflects the client's preference on the cost-effective quality care provided by a PBM. The assumption that $n_j(V_j, V_{-j})$ is (weakly) log-concave in V_j is common in the literature, which is satisfied by many different forms of demand functions, including linear demand function, logit demand function, and exponential demand function. In addition, every nonnegative concave function is also log-concave. Moreover, if $n_j(V_j, V_{-j})$ satisfies the condition that $\frac{\partial^2 \ln n_j}{\partial V_j^2} + \sum_{k=1, k\neq j}^{M} \left|\frac{\partial^2 \ln n_j}{\partial V_j \partial V_k}\right| \leq 0$, the equilibrium of the PBMs' competition on the aggregate formulary and price decisions is unique. For the general function form of $n_j(V_j, V_{-j})$, the equilibrium results depend on the specific form of $n_j(V_j, V_{-j})$, and may not be explicitly characterized in closed forms.