Online Supplement
Appendix A

This appendix provides the proofs of Propositions 1 to 11. Lemma 1 will be used in the proofs of Lemma 3 and Propositions 2 and 4.

Lemma 1 *If* (6) *holds, then*

$$(1-\tilde{e}_r)K_r''(\tilde{e}_r) - K_r'(\tilde{e}_r) + d_S > 0 \tag{20}$$

and supplier's hiding effort $\tilde{e}_h$ *increases in the buyers's auditing effort* e_a.

Proof of Lemma 1 and Proposition 1: Differentiating (3) and (4) with respect to e_a, we obtain

$$[1 - e_h - e_a(\partial e_h/\partial e_a)](p - c - d_S) - (\partial e_r/\partial e_a)K_r''(e_r) = 0 \tag{21}$$

$$[1 - e_r - e_a(\partial e_r/\partial e_a)](p - c - d_S) - (\partial e_h/\partial e_a)K_h''(e_h) = 0. \tag{22}$$

Using (21)-(22) and the fact that $(\tilde{e}_r, \tilde{e}_h)$ satisfy (3)-(4), we obtain

$$\begin{aligned}
\partial\tilde{e}_r/\partial e_a &= [(1-\tilde{e}_h)K_h''(\tilde{e}_h) - K_h'(\tilde{e}_h)](p-c-d_S)/[K_r''(\tilde{e}_r)K_h''(\tilde{e}_h) - e_a^2(p-c-d_S)^2] && (23)\\
\partial\tilde{e}_h/\partial e_a &= [(1-\tilde{e}_r)K_r''(\tilde{e}_r) - K_r'(\tilde{e}_r) + d_S](p-c-d_S)/[K_r''(\tilde{e}_r)K_h''(\tilde{e}_h) - e_a^2(p-c-d_S)^2]. && (24)
\end{aligned}$$

Because $(\tilde{e}_r, \tilde{e}_h)$ is a best response for the supplier, the Hessian of π_s evaluated at $(e_r, e_h) = (\tilde{e}_r, \tilde{e}_h)$ is negative definite, which implies that the denominator in (23) is strictly positive. This establishes Proposition 1a: $\partial\tilde{e}_r/\partial e_a < 0$ if and only if (6). Next we establish Lemma 1. If (6) holds, then $\partial\tilde{e}_r/\partial e_a < 0$. From (21), this implies $\partial\tilde{e}_h/\partial e_a = [(1-e_h)(p-c-d_S) - (\partial\tilde{e}_r/\partial e_a)K_r''(\tilde{e}_r)]/[e_a(p-c-d_S)] > 0$. Therefore, (24) implies (20). It remains to establish Proposition 1b. Because the supplier's best response responsibility and hiding effort is interior and satisfies (4), $K_h'(\tilde{e}_h) > 0$; further, as noted above, $K_h''(\tilde{e}_h) > 0$. It follows that (7) implies (6).■

Lemma 2 will be used in the proofs of Lemma 3 and Propositions 2, 6 and 11. Let

$$\begin{aligned}
\Gamma &= K_a''(e_a^*)\left[K_r''(e_r^*)K_h''(e_h^*) - (e_a^*)^2(p-c-d_S)^2\right] + (d_B - v + p)(p-c-d_S) \times \\
&\quad \{(1-e_r^*)[(1-e_r^*)K_r''(e_r^*) - K_r'(e_r^*) + d_S] + (1-e_h^*)[(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)]\}. && (25)
\end{aligned}$$

Lemma 2 *If the supplier's responsibility effort* $\tilde{e}_r$ *and hiding effort* $\tilde{e}_h$ *and the marginal cost of auditing effort* $K_a'(e_a)$ *are continuous in the buyer's auditing effort* e_a, *then* $\Gamma > 0$.

Proof of Lemma 2: The buyer's best response auditing effort $\tilde{e}_a(e_r, e_h)$ is the unique solution to $\Upsilon(e_a, e_r, e_h) = 0$, where $\Upsilon(e_a, e_r, e_h) = (1-e_h)(1-e_r)(d_B - v + p) - K_a'(e_a)$. We will show that

$$(\partial/\partial e_a)\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{(e_a,\tilde{e}_r(e_a),\tilde{e}_h(e_a))=(e_a^*,e_r^*,e_h^*)} < 0. \tag{26}$$

To do so, we first establish that

$$\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{e_a=0} > 0 > \Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{e_a=1}. \tag{27}$$

Because $K_a(\cdot)$ is convex, $\pi_b(\cdot)$ is concave. Therefore, if $\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{e_a=0} \leq 0$, then $(e_a, e_r, e_h) = (0, \tilde{e}_r(0), \tilde{e}_h(0))$ is an equilibrium, which contradicts that the unique equilibrium in interior. Similarly, if $\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{e_a=1} \geq 0$, then $(e_a, e_r, e_h) = (1, \tilde{e}_r(1), \tilde{e}_h(1))$ is an equilibrium, which contradicts that the unique equilibrium in interior. This establishes (27). The proof that (26) holds is by contradiction. Suppose that (26) is violated in that the inequality is reversed. Because $\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))$ is continuous in e_a, the first inequality in (27) implies there exists $\ddot{e}_a \in (0, e_a^*)$ such that $\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{e_a=\ddot{e}_a} = 0$. Therefore, $(e_a, e_r, e_h) = (\ddot{e}_a, \tilde{e}_r(\ddot{e}_a), \tilde{e}_h(\ddot{e}_a))$ is an equilibrium, which contradicts that (e_a^*, e_r^*, e_h^*) is the unique equilibrium. Suppose that (26) is violated in that the inequality is replaced with an equality. Because $\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))$ is continuous in e_a, the first inequality in (27) implies that the second inequality in (27) is violated. We conclude that (26) holds. Observe that

$$(\partial/\partial e_a)\Upsilon(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))|_{(e_a, \tilde{e}_r(e_a), \tilde{e}_h(e_a))=(e_a^*, e_r^*, e_h^*)} = -\Gamma/\left[K_r''(e_r^*)K_h''(e_h^*) - (e_a^*)^2(p-c-d_S)^2\right]. \tag{28}$$

Because $(\tilde{e}_r, \tilde{e}_h)$ is a best response for the supplier, the Hessian of π_s evaluated at $(e_r, e_h) = (\tilde{e}_r, \tilde{e}_h)$ is negative definite, which implies that the denominator in (28) is strictly positive. Therefore, (26) implies that $\Gamma > 0$.■

Lemma 3 will be used in the proof of Proposition 5.

Lemma 3 *The supplier's equilibrium responsibility effort e_r^* decreases in the price p and increases in the supplier's production cost c if and only if (6′). If (6′) holds, then the supplier's equilibrium hiding effort e_h^* increases in the p and decreases in c.*

Proof of Lemma 3 and Proposition 2: Differentiating (3), (4) and (5) with respect to d_B, we obtain

$$[(1-e_h)(\partial e_a/\partial d_B) - e_a(\partial e_h/\partial d_B)](p-c-d_S) - (\partial e_r/\partial d_B)K_r''(e_r) = 0 \tag{29}$$

$$[(1-e_r)(\partial e_a/\partial d_B) - e_a(\partial e_r/\partial d_B)](p-c-d_S) - (\partial e_h/\partial d_B)K_h''(e_h) = 0 \tag{30}$$

$$(1-e_r)(1-e_h) - [(1-e_h)(\partial e_r/\partial d_B) + (1-e_r)(\partial e_h/\partial d_B)](d_B - v + p) - (\partial e_a/\partial d_B)K_a''(e_a) = 0. \tag{31}$$

Using (29)-(31) and the fact that (e_a^*, e_r^*, e_h^*) satisfy (3)-(5), we obtain

$$\partial e_a^*/\partial d_B = (1-e_h^*)(1-e_r^*)\left[K_r''(e_r^*)K_h''(e_h^*) - (e_a^*)^2(p-c-d_S)^2\right]/\Gamma \quad (32)$$

$$\partial e_r^*/\partial d_B = [(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)](1-e_h^*)(1-e_r^*)(p-c-d_S)/\Gamma \quad (33)$$

$$\partial e_h^*/\partial d_B = [(1-e_r^*)K_r''(e_r^*) - K_r'(e_r^*) + d_S](1-e_h^*)(1-e_r^*)(p-c-d_S)/\Gamma \quad (34)$$

Similar parallel arguments establish that

$$\partial e_r^*/\partial p = [(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)][e_a^*K_a''(e_a^*) + (1-e_h^*)(1-e_r^*)(p-c-d_S)]/\Gamma$$

$$\partial e_h^*/\partial p = [(1-e_r^*)K_r''(e_r^*) - K_r'(e_r^*) + d_S][e_a^*K_a''(e_a^*) + (1-e_h^*)(1-e_r^*)(p-c-d_S)]/\Gamma$$

$$\partial e_r^*/\partial c = -[(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)]e_a^*K_a''(e_a^*)/\Gamma$$

$$\partial e_h^*/\partial c = -[(1-e_r^*)K_r''(e_r^*) - K_r'(e_r^*) + d_S]e_a^*K_a''(e_a^*)/\Gamma.$$

From Lemma 2, $\Gamma > 0$. Therefore, $\partial e_r^*/\partial d_B < 0$, $\partial e_r^*/\partial p < 0$ and $\partial e_r^*/\partial c > 0$ if and only if (6′). Let (20′) denote inequality (20), where e_r^* replaces $\tilde{e}_r$. Suppose (6′) holds. This implies that (20′) holds (by Lemma 1). Therefore, $\partial e_h^*/\partial p > 0$ and $\partial e_h^*/\partial c < 0$.■

Proof of Proposition 3: When the supplier's budget constraint is binding, the supplier's problem is to choose e_r and e_h to maximize (2) subject to (10), where the inequality binds. When $d_S = 0$, the supplier's problem can be rewritten with a change of variables as

$$\max_{\varepsilon_r \in [0,B]} \{[1 - e_a\rho_r(\varepsilon_r)\rho_h(B-\varepsilon_r)](p-c)\}, \quad (35)$$

where $\varepsilon_i = K_i(e_i)$ and $\rho_i(\varepsilon_i) = 1 - e_i$ for $i \in \{r, h\}$; ε_r denotes the supplier's expenditure on responsibility effort and $\rho_r(\varepsilon_r)$ denotes the probability that the supplier's facility is unsafe; ε_h and $\rho_h(\varepsilon_h)$ are interpreted similarly. In (35), observe that the supplier's problem is to allocate a fixed budget across the two types of effort to maximize the probability of passing the audit, and her optimal allocation is invariant to the buyer's auditing effort e_a and the supplier's margin $p - c$. Therefore, problem (35) can be rewritten as $\min_{\varepsilon_r \in [0,B]} \psi(\varepsilon_r)$, where $\psi(\varepsilon_r) = \rho_r(\varepsilon_r)\rho_h(B-\varepsilon_r)$. Because the supplier's equilibrium responsibility and hiding efforts are strictly positive and because $K_i(0) = 0$ for $i \in \{r, h\}$, the supplier's equilibrium expenditure on responsibility effort is interior $\varepsilon_r \in (0, B)$. The supplier's equilibrium responsibility expenditure ε_r is the solution to the first order condition

$$(\partial/\partial\varepsilon_r)\psi(\varepsilon_r) = \rho_r'(\varepsilon_r)\rho_h(B-\varepsilon_r) - \rho_r(\varepsilon_r)\rho_h'(B-\varepsilon_r) = 0. \quad (36)$$

Because the supplier's equilibrium responsibility expenditure ε_r^* is unique, and because

$(\partial^2/\partial\varepsilon_r^2)\psi(\varepsilon_r^*) > 0$, by the implicit function theorem,

$$\begin{aligned}\partial\varepsilon_r^*/\partial B < 0 \quad &\Longleftrightarrow \quad \rho_r'(\varepsilon_r^*)\rho_h'(B-\varepsilon_r^*) - \rho_r(\varepsilon_r^*)\rho_h''(B-\varepsilon_r^*) > 0 \\ &\Longleftrightarrow \quad \rho_h'(\varepsilon_h^*)^2 - \rho_h(\varepsilon_h^*)\rho_h''(\varepsilon_h^*) > 0 \qquad (37) \\ &\Longleftrightarrow \quad K_h'(e_h^*)/K_h''(e_h^*) > 1 - e_h^*; \qquad (38)\end{aligned}$$

where (37) follows from ε_r^* satisfying (36) and $\varepsilon_h^* = B - \varepsilon_r^*$; and (38) follows from $\rho_h(\varepsilon_h) = 1 - K_h^{-1}(\varepsilon_h)$ and $e_h = K_h^{-1}(\varepsilon_h)$.■

Proof of Proposition 4: Let $\hat{\pi}_b$ denote the right hand side of (18), where $\tilde{e}_i$ replaces e_i for $i \in \{r,h\}$. Then $\hat{\pi}_b$ denotes buyer's expected profit when the buyer commits to the auditing level prior to the supplier's choosing her responsibility and hiding effort. Therefore, when the buyer commits to her auditing effort in advance, the buyer's equilibrium auditing effort $\hat{e}_a$ satisfies the first order condition

$$(\partial/\partial e_a)\hat{\pi}_b = [1-\tilde{e}_h(e_a)][1-\tilde{e}_r(e_a)](d_B - v + p) - K_a'(e_a) + \{v - p + (1 - e_a[1-\tilde{e}_h(e_a)])$$

$$\times (d_B - v + p)\}(\partial\tilde{e}_r/\partial e_a) - e_a[1-\tilde{e}_r(e_a)](d_B - v + p)(\partial\tilde{e}_h/\partial e_a) = 0.$$

Therefore,

$$\begin{aligned}(\partial/\partial e_a)\pi_b|_{e_a=\hat{e}_a} &= [1-\tilde{e}_h(\hat{e}_a)][1-\tilde{e}_r(\hat{e}_a)](d_B - v + p) - K_a'(\hat{e}_a) \\ &= -\{v - p + (1 - \hat{e}_a[1-\tilde{e}_h(\hat{e}_a)])(d_B - v + p)\}(\partial\tilde{e}_r/\partial e_a) \\ &\quad + \hat{e}_a[1-\tilde{e}_r(\hat{e}_a)](d_B - v + p)(\partial\tilde{e}_h/\partial e_a). \qquad (39)\end{aligned}$$

Because $K_a(e_a)$ is convex in e_a, $\pi_b(e_a)$ is concave in e_a. Therefore, (11) holds if and only if $(\partial/\partial e_a)\pi_b|_{e_a=\hat{e}_a} > 0$. From Lemma 1 and Proposition 1, (6$''$) implies $\partial\tilde{e}_r/\partial e_a < 0$ and $\partial\tilde{e}_h/\partial e_a > 0$; from (39), this implies $(\partial/\partial e_a)\pi_b|_{e_a=\hat{e}_a} > 0$.■

Proof of Proposition 5: If (6$'$) holds, then $\partial e_r^*/\partial c > 0$ and $\partial e_h^*/\partial c < 0$ (by Lemma 3). Further,

$$\begin{aligned}(\partial/\partial c)\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = \; &(\partial e_r^*/\partial c)\{e_a^*(1-e_h^*)(v-p) + [1 - e_a^*(1-e_h^*)]d_B\} \\ &- (\partial e_h^*/\partial c)(1-e_r^*)(d_B - v + p) > 0,\end{aligned}$$

where the equality follows from the envelope theorem and the inequality follows from $\partial e_r^*/\partial c > 0$ and $\partial e_h^*/\partial c < 0$. If (6$'$) holds, then $\partial e_r^*/\partial p < 0$ and $\partial e_h^*/\partial p > 0$ (by Lemma 3). Further,

$$(\partial/\partial p)\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = (\partial e_r^*/\partial p)\{v - p + [1 - e_a^*(1-e_h^*)](d_B - v + p)\} - (\partial e_h^*/\partial p)$$

$$\times e_a^*(1-e_r^*)(d_B - v + p) - [1 - e_a^*(1-e_h^*)(1-e_r^*)] < 0,$$

where the equality follows from the envelope theorem and the inequality follows from $\partial e_r^*/\partial p < 0$

and $\partial e_h^*/\partial p > 0.$■

Proof of Proposition 6: Differentiating (3), (4) and (5) with respect to d_S, we obtain

$$1 - e_a(1-e_h) + [(1-e_h)(\partial e_a/\partial d_S) - e_a(\partial e_h/\partial d_S)]\,(p-c-d_S) - (\partial e_r/\partial d_B)K_r''(e_r) = 0 \quad (40)$$

$$-e_a(1-e_r) + [(1-e_r)(\partial e_a/\partial d_S) - e_a(\partial e_r/\partial d_S)]\,(p-c-d_S) - (\partial e_h/\partial d_B)K_h''(e_h) = 0 \quad (41)$$

$$-\,[(1-e_h)(\partial e_r/\partial d_S) + (1-e_r)(\partial e_h/\partial d_S)]\,(d_B - v + p) - (\partial e_a/\partial d_S)K_a''(e_a) = 0. \quad (42)$$

Using (40)-(42) and the fact that (e_a^*, e_r^*, e_h^*) satisfy (3)-(5), we obtain

$$\begin{aligned}\partial e_r^*/\partial d_S = \;& [(1-e_r^*)^2(p-c-d_S)(d_B-v+p) \\ & + K_a''(e_a^*)\{K_h''(e_h^*)[1-e_a^*(1-e_h^*)] + (e_a^*)^2(1-e_r^*)(p-c-d_S)\}]/\Gamma > 0 \\ \partial e_h^*/\partial d_S = \;& -[(1-e_h^*)(1-e_r^*)(p-c-d_S)(d_B-v+p) \\ & + e_a^*K_a''(e_a^*)\{(1-e_r^*)K_r''(e_r^*) + [1-e_a^*(1-e_h^*)](p-c-d_S)\}]/\Gamma < 0,\end{aligned}$$

where the inequality follows because $\Gamma > 0$ (by Lemma 2). Further,

$$\begin{aligned}(\partial/\partial d_S)\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = \;& (\partial e_r^*/\partial d_S)[d_B - e_a^*(1-e_h^*)(d_B-v+p)] - (\partial e_h^*/d_S)e_a^*(1-e_r^*) \\ & \times (d_B-v+p) - [1-e_a^*(1-e_h^*)(1-e_r^*)] > 0,\end{aligned}$$

where the equality follows from the envelope theorem and the inequality follows from $\partial e_r^*/\partial d_S > 0$ and $\partial e_h^*/\partial d_S < 0.$■

Proof of Proposition 7: Under expected penalty y, the equilibrium conditions are (3)-(5), where $(d_S + y)$ replaces d_S and $(d_B - y)$ replaces d_B. Differentiating (3)-(5) with respect to y, we obtain

$$1 - e_a(1-e_h) + [(\partial e_a/\partial y)(1-e_h) - (\partial e_h/\partial y)e_a](p-c-d_S-y) - (\partial e_r/\partial y)K_r''(e_r) = 0 \quad (43)$$

$$-e_a(1-e_r) + [(\partial e_a/\partial y)(1-e_r) - (\partial e_r/\partial y)e_a](p-c-d_S-y) - (\partial e_h/\partial y)K_h''(e_h) = 0 \quad (44)$$

$$-(1-e_r)(1-e_h) - [(\partial e_r/\partial y)(1-e_h) + (\partial e_h/\partial y)(1-e_r)](d_B-y-v+p) - (\partial e_a/\partial y)K_a''(e_a) = 0. \quad (45)$$

Using (43)-(45) and the fact that (e_a^*, e_r^*, e_h^*) satisfies (4) and (5), we obtain

$$\begin{aligned}\partial e_r^*/\partial y \;=\;& \{K_a''(e_a^*)K_h''(e_h^*) - [(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)][e_a^*K_a''(e_a^*) + K_a'(e_a^*) \quad (46)\\ & \times(p-c-d_S-y)/(d_B-v+p)] + (1-e_r^*)^2(p-c-d_S-y)(d_B-v+p)\}/\Gamma\end{aligned}$$

where Γ is as defined in (25), wherein (d_S+y) replaces d_S and (d_B-y) replaces d_B. By straightforward extension of Lemma 2, $\Gamma > 0$. Therefore, $\partial e_r^*/\partial y < 0$ if and only if inequality (12) holds. Next we provide parameters under which instituting a penalty $y > 0$ decreases the buyer's profit and increases the supplier's profit. Suppose $K_a(e_a) = 0.5(e_a)^2$, $K_r(e_r) = 0.25(e_r)^2$, $K_h(e_h) = 20(e_h)^2$, $d_B = 3$, $d_S = 0$, $v = 19$, $p = 17$, and $c = 14$. Without a penalty (i.e., $y = 0$), the equilibrium auditing effort $e_a^* = 0.62$, responsibility effort $e_r^* = 0.36$, hiding effort $e_h^* = 0.03$, buyer's expected profit $\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 0.28$ and supplier's expected profit $\pi_s|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 1.50$. Under penalty $y = 0.46$, the equilibrium efforts and profits are $e_a^* = 0.38$, $e_r^* = 0.28$, $e_h^* = 0.02$, $\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 0.25$ and $\pi_s|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 1.78$.■

Proof of Proposition 8: The likelihood that the facility passes the audit is $1 - e_a(1 - e_h)(1 - e_r) - \theta e_h$, and the likelihood that an unsafe facility passes the audit is $(1 - e_r)[1 - e_a(1 - e_h) - \theta e_h]$. The buyer's expected profit π_b is given by the right hand side of (1) and supplier's expected profit π_s is given by the right hand side of (2), with the addition of $-\theta e_h$ within each of the square brackets in both expressions. Denote (3) with the addition of $-\theta e_h d_S$ in the center expression as (3″), and denote (4) with the addition of $-\theta[p - c - (1 - e_r)d_S]$ in the center expression as (4″). The first order conditions for the supplier's responsibility and hiding effort decisions are (3″) and (4″). Differentiating (3″), (4″) and (5) with respect to θ, we obtain

$$-e_h d_S + (\partial e_a/\partial\theta)(1 - e_h)(p - c - d_S) - (\partial e_h/\partial\theta)[e_a(p - c - d_S) + \theta d_S] - (\partial e_r/\partial\theta)K_r''(e_r) = 0 \quad (47)$$

$$(\partial e_a/\partial\theta)(1 - e_r)(p - c - d_S) - (\partial e_r/\partial\theta)[e_a(p - c - d_S) + \theta d_S] - [p - c - (1 - e_r)d_S] \\ -(\partial e_h/\partial\theta)K_h''(e_h) = 0 \quad (48)$$

$$-[(\partial e_h/\partial\theta)(1 - e_r) + (\partial e_r/\partial\theta)(1 - e_h)](d_B - v + p) - (\partial e_a/\partial\theta)K_a''(e_a) = 0. \quad (49)$$

Using (47)-(49) and the fact that (e_a^*, e_r^*, e_h^*) satisfy (3″), (4″) and (5), we obtain

$$\begin{aligned}\partial e_r^*/\partial\theta &= ([e_a^* K_a''(e_a^*) + K_a'(e_a^*)](p - c)(p - c - d_S) \\ &\quad -d_S\{K_a''(e_a^*)[e_h^* K_h''(e_h^*) + K_h'(e_h^*)] + (1 - e_r^*)^2(d_B - v + p)(p - c - d_S)\})/\Gamma,\end{aligned}$$

where Γ is as defined in (25), wherein $-(e_a^*)^2(p - c - d_S)^2$ is replaced by $-[e_a^*(p - c - d_S) + \theta d_S]^2$, $+d_S$ is replaced by $+(1 - \theta)d_S$ and $-K_h'(e_h^*)$ is replaced by $-K_h'(e_h^*) - \theta(p - c)$. By straightforward extension of Lemma 2, $\Gamma > 0$. Therefore, $\partial e_r^*/\partial y < 0$ if and only if inequality (14) holds. Next we provide parameters under which instituting the penalty of termination for detected hiding $\theta > 0$ decreases the buyer's equilibrium expected profit. Suppose $K_a(e_a) = 0.4(e_a)^2$, $K_r(e_r) = 10(e_r)^2$, $K_h(e_h) = 0.02(e_h)^2$, $d_B = 10.0$, $d_S = 2.0$, $v = 13.50$, $p = 4.0$, and $c = 1.0$. If the buyer does not institute a penalty for detected hiding (i.e., $\theta = 0$), then the equilibrium auditing effort $e_a^* = 0.041$, responsibility effort $e_r^* = 0.100$, hiding effort $e_h^* = 0.927$, buyer's expected profit $\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 0.502$ and supplier's expected profit $\pi_s|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 1.080$. If the buyer institutes the penalty of termination for detected hiding, then under ease of detecting hiding $\theta = 0.1$, the equilibrium efforts and profits are $e_a^* = 0.163, e_r^* = 0.095, e_h^* = 0.712$, $\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 0.431$ and $\pi_s|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = 0.962$. ■

Proof of Proposition 9: The buyer's expected profit π_b is given by the right hand side of (1), with the addition of $+\theta e_h z$; the supplier's expected profit π_s is given by the right hand side of (2), with the addition of $-\theta e_h z$. First, we will show that $\partial e_r^*/\partial z > 0$ and $\partial e_h^*/\partial z < 0$. Denote (4) with the addition of $-\theta z$ in the center expression as (4‴). The first order conditions for the supplier's responsibility and hiding effort decisions are (3) and (4‴). Differentiating (3), (4‴) and (5) with respect to z, we obtain

$$[(\partial e_a/\partial z)(1-e_h) - (\partial e_h/\partial z)e_a](p-c-d_S) - (\partial e_r/\partial z)K_r''(e_r) = 0 \tag{50}$$

$$[(\partial e_a/\partial z)(1-e_r) - (\partial e_r/\partial z)e_a](p-c-d_S) - \theta - (\partial e_h/\partial z)K_h''(e_h) = 0 \tag{51}$$

$$-[(\partial e_h/\partial z)(1-e_r) + (\partial e_r/\partial z)(1-e_h)](d_B - v + p) - (\partial e_a/\partial z)K_a''(e_a) = 0. \tag{52}$$

Using (50)-(52) and the fact that (e_a^*, e_r^*, e_h^*) satisfy (3), (4‴) and (5), we obtain

$$\begin{aligned}\partial e_r^*/\partial z &= [e_a^* K_a''(e_a^*) + (1-e_h^*)(1-e_r^*)(d_B - v + p)](p-c-d_S)\theta/\Gamma \\ \partial e_h^*/\partial z &= -[K_a''(e_a^*)K_r''(e_r^*) + (1-e_h^*)^2(d_B - v + p)(p-c-d_S)]\theta/\Gamma,\end{aligned}$$

where Γ is as defined in (25), wherein $-K_h'(e_h^*)$ is replaced by $-K_h'(e_h^*) - \theta z$. By straightforward extension of Lemma 2, $\Gamma > 0$. Therefore, $\partial e_r^*/\partial z > 0$ and $\partial e_h^*/\partial z < 0$. Consider $\overline{z} > \underline{z}$. Then $\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*),\ z=\overline{z}} = \max_{e_a\in[0,1]} \pi_b|_{(e_r,e_h)=(e_r^*,e_h^*),\ z=\overline{z}} > \max_{e_a\in[0,1]} \pi_b|_{(e_r,e_h)=(e_r^*,e_h^*),\ z=\underline{z}}$ $= \pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*),\ z=\underline{z}}$, where the inequality follows because $e_r^*|_{z=\overline{z}} > e_r^*|_{z=\underline{z}}$, $e_h^*|_{z=\overline{z}} < e_h^*|_{z=\underline{z}}$, and π_b is decreasing in e_h and increasing in e_r and z. The comparative statics for θ follow by a parallel argument.■

Proof of Proposition 10: The first order conditions that characterize the equilibrium in auditing, hiding and responsibility (e_a^*, e_r^*, e_h^*) are given by (3), (4) and (5), where c is replaced by $\underline{p}$. The result follows by argument parallel to that in the proof of Lemma 3 and Proposition 2.■

Proof of Proposition 11: By the envelope theorem,

$$\begin{aligned}(\partial/\partial\gamma)\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} = \quad & (\partial e_r^*/\partial\gamma)\{e_a^*(1-e_h^*)(v-p) + [1-e_a^*(1-e_h^*)]d_B\} \\ & - (\partial e_h^*/\partial\gamma)e_a^*(1-e_r^*)(d_B - v + p).\end{aligned} \tag{53}$$

We will show that $\partial e_r^*/\partial\gamma < 0$ if and only if (16), and $\partial e_h^*/\partial\gamma > 0$ if and only if (16). Differentiating

(3), (4) and (5) with respect to γ, we obtain

$$[(1-e_h)(\partial e_a/\partial\gamma) - e_a(\partial e_h/\partial\gamma)]\,(p-c-d_S) - (\partial e_r/\partial\gamma)K_r''(e_r) = 0 \tag{54}$$

$$[(1-e_r)(\partial e_a/\partial\gamma) - e_a(\partial e_r/\partial\gamma)]\,(p-c-d_S) - \alpha_h[\gamma\ln(e_h)-1](e_h)^{-\gamma-1}$$
$$-\alpha_h\gamma(1+\gamma)(e_h)^{-\gamma-2}(\partial e_h/\partial\gamma) = 0 \tag{55}$$

$$-\,[(1-e_h)(\partial e_r/\partial\gamma) + (1-e_r)(\partial e_h/\partial\gamma)]\,(d_B-v+p) - (\partial e_a/\partial\gamma)K_a''(e_a) = 0. \tag{56}$$

Using (54)-(56) and the fact that (e_a^*, e_r^*, e_h^*) satisfy (3)-(5), we obtain

$$\begin{aligned}\partial e_r^*/\partial\gamma &= (e_h^*)^{-\gamma-1}[e_a^*K_a''(e_a^*) + (1-e_r^*)(1-e_h^*)(d_B-v+p)](p-c-d_S)\alpha_h[\gamma\ln(e_h)-1]/\Gamma \\ \partial e_h^*/\partial\gamma &= -(e_h^*)^{-\gamma-1}[K_a''(e_a^*)K_r''(e_r^*) + (1-e_h^*)^2(d_B-v+p)(p-c-d_S)]\alpha_h[\gamma\ln(e_h)-1]/\Gamma.\end{aligned}$$

Recall from Lemma 2 that $\Gamma > 0$. Therefore, $\partial e_r^*/\partial\gamma < 0$ if and only if (16), and $\partial e_h^*/\partial\gamma > 0$ if and only if (16). The result follows from this observation and (53).■

Appendix B

This appendix provides the proof of **Proposition 12**. In all proofs, the supplier's production cost c is replaced by the price $\underline{p}$, and $d_S = 0$ because the supplier's facility is safe. The first order condition for the buyer's auditing effort (5) becomes $(\partial/\partial e_a)\pi_b = (1-e_h)(1-e_r)[(1-e_r)d_B - v + p] - K_a'(e_a) = 0$. Because the buyer's equilibrium auditing effort $e_a^* > 0$, $(1-e_r^*)d_B - v + p > 0$. Further, if $(\partial/\partial e_r)\pi_b|_{(e_a,e_r,e_h)=(e_a^*,e_r^*,e_h^*)} > 0$, then $e_a^*(1-e_h^*)(v-p) + [1-2e_a^*(1-e_h^*)(1-e_r^*)]d_B > 0$.

Lemma 1 and Proposition 1. The proof is unchanged.

We adapt the definition of Γ, which is used in Lemma 2 and the proofs of subsequent results.

$$\begin{aligned}\Gamma &= K_a''(e_a^*)\left[K_r''(e_r^*)K_h''(e_h^*) - (e_a^*)^2(p-\underline{p})^2\right] + (p-\underline{p}) \times \{[(1-e_r^*)d_B - v + p](1-e_r^*) \times \\ &\quad [(1-e_r^*)K_r''(e_r^*) - K_r'(e_r^*)] + [2(1-e_r^*)d_B - v + p](1-e_h^*)[(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)]\}.\end{aligned}$$

Lemma 2. The proof holds when d_B is replaced by $(1-e_r)d_B$ in the definition of $\Upsilon(e_a, e_r, e_h)$.

Lemma 3 and Proposition 2. The proof holds with the following changes: The left hand side of (31) is replaced with $\{(1-e_r)^2(1-e_h) - [2(1-e_r)d_B - v + p](1-e_h)(\partial e_r/\partial d_B) - [(1-e_r)d_B - v + p](1-e_r)(\partial e_h/\partial d_B)\} - (\partial e_a/\partial d_B)K_a''(e_a)$. In (32)-(34), $(1-e_h^*)(1-e_r^*)$ is replaced by $(1-e_h^*)(1-e_r^*)^2$.

Proposition 3. The proof is unchanged.

Proof of Proposition 5. The proof holds when $[1-e_a^*(1-e_h^*)]d_B$ is replaced by $[1-2e_a^*(1-e_h^*)(1-e_r^*)]d_B$, $(d_B - v + p)$ is replaced by $[(1-e_r^*)d_B - v + p]$, and $\{v - p + [1-e_a^*(1-e_h^*)](d_B - v + p)\}$ is replaced by $e_a^*(1-e_h^*)(v-p) + [1-2e_a^*(1-e_h^*)(1-e_r^*)]d_B$.

Proof of Proposition 9. The proof holds when $(d_B - v + p)$ is replaced by $[(1-e_r^*)d_B - v + p] - (\partial e_r^*/\partial z)(1-e_h)(1-e_r)d_B$ in (52), is replaced by $[(1-e_r^*)d_B - v + p]$ in the expression for $\partial e_r^*/\partial z$, and is replaced by $[2(1-e_r^*)d_B - v + p]$ in the expression for $\partial e_h^*/\partial z$.

Proof of Proposition 10. The proof is unchanged.

Proof of Proposition 11. The proof holds when $(d_B - v + p)$ is replaced by $[(1-e_r^*)d_B - v + p] - (\partial e_r^*/\partial \gamma)(1-e_h)(1-e_r)d_B$ in (56), is replaced by $[(1-e_r^*)d_B - v + p]$ in the expression for $\partial e_r^*/\partial \gamma$, and is replaced by $[2(1-e_r^*)d_B - v + p]$ in the expression for $\partial e_h^*/\partial \gamma$.

Appendix C

This appendix establishes that the results extend to the setting with multiple unsafe supplier states. Consider a generalization of our model formulation in which with probability $\Sigma_{j=1}^{N} P^j(e_r)$ the facility is unsafe; $P^j(e_r) \in [0,1]$ is the probability that the facility is in unsafe state $j \in \{1,..,N\}$; if the buyer sources from a supplier in unsafe state j, the buyer incurs additional expected cost d_B^j. We assume that the buyer does not want to source from an unsafe facility

$$d_B^j > v - p,$$

which implies that if the supplier fails the audit, then the buyer does not source from the supplier. The buyer's expected profit is given by (1) where d_B is replaced by $\Sigma_{j=1}^{N} P^j(e_r) d_B^j$, the buyer's expected cost of sourcing from an unsafe facility, which we assume is weakly decreasing in the supplier's responsibility effort e_r. For ease of exposition, let $d_B(e_r) = \Sigma_{j=1}^{N} P^j(e_r) d_B^j$. The supplier incurs additional expected cost d_S from operating an unsafe facility The implicit assumption that d_S does not vary with the state $j \in \{1,..,N\}$ may reflect the status quo for the many suppliers that face negligible costs $d_S = 0$ for operating an unsafe facility. Alternatively, d_S could represent a maximal penalty for the supplier, such as supplier being forced to go out of business permanently after causing a major harm to workers or the environment.

In all proofs, $d_B(e_r)$ replaces d_B, except as noted otherwise below. The first order conditions are unchanged except that in (5), $d_B(e_r)$ replaces d_B.

Proof of Lemma 1 and Proposition 1. The proof is unchanged.

We adapt the definition of Γ, which is used in Lemma 2 and the proofs of subsequent results, by replacing d_B with $d_B(e_r^*)$ and adding the term $-(1-e_r^*)(1-e_h^*)(p-c-d_S)[(1-e_h^*)K_h''(e_h^*) - K_h'(e_h^*)]d_B'(e_r^*)$.

Proof of Lemma 2. The proof is unchanged.

For Proposition 2, let $d_B^j = \underline{d}_B + \Delta^j$ for $j \in \{1, .., N\}$. The Proposition holds when d_B is replaced by $\underline{d}_B$.

Proof of Lemma 3 and Proposition 2. The proof holds when $(\partial e_i^*/\partial \underline{d}_B)$ replaces $(\partial e_i^*/\partial d_B)$ for $i \in \{a, r, h\}$ throughout, and $(1-e_r)(1-e_h)[1+(\partial e_r/\partial \underline{d}_B)d'_B(e_r)]$ replaces $(1-e_r)(1-e_h)$ in (31).

Proof of Proposition 3. The proof is unchanged.

Proof of Proposition 4. The proof holds with the following changes: On the right hand side of the first displayed equation, $d_B(\tilde{e}_r(e_a))$ replaces d_B and $-[1-\tilde{e}_r(e_a)](1-e_a[1-\tilde{e}_h(e_a)])d'_B(\tilde{e}_r(e_a))(\partial \tilde{e}_r/\partial e_a)$ is added. In the remaining displayed equations, $d_B(\tilde{e}_r(\hat{e}_a))$ replaces d_B. On the right hand side of (39), $+[1-\tilde{e}_r(\hat{e}_a)](1-\hat{e}_a[1-\tilde{e}_h(\hat{e}_a)])d'_B(\tilde{e}_r(\hat{e}_a))(\partial \tilde{e}_r/\partial e_a)$ is added.

Proof of Proposition 5. The proof holds with the following changes: In the center expression of the first displayed equation, $d_B(e_r^*)$ replaces d_B and $-(1-e_r^*)[1-e_a^*(1-e_h^*)]d'_B(e_r^*)(\partial e_r^*/\partial c)$ is added. In the center expression of the second displayed equation, $d_B(e_r^*)$ replaces d_B and $-(1-e_r^*)[1-e_a^*(1-e_h^*)]d'_B(e_r^*)(\partial e_r^*/\partial p)$ is added.

Proof of Proposition 6. The proof holds with the following changes: In all expressions following equation (42), $d_B(e_r^*)$ replaces d_B. In the center expression for the displayed equation for $\partial e_h^*/\partial c$, within the square brackets $-(1-e_r^*)^2(1-e_h^*)(p-c-d_S)d'_B(e_r^*)$ is added. In the center expression of the last displayed equation, $-(1-e_r^*)[1-e_a^*(1-e_h^*)]d'_B(e_r^*)(\partial e_r^*/\partial d_S)$ is added.

Proof of Proposition 7. The proof holds with the following changes: On the left hand side of equation (45), $-(1-e_r)(1-e_h)[1+(\partial e_r/\partial \underline{d}_B)d'_B(e_r)]$ replaces $-(1-e_r)(1-e_h)$. In equation (46), $d_B(e_r^*)$ replaces d_B. In the numerical example, $N=1$ and $d_B^1 = 3$ replaces $d_B = 3$.

Proof of Proposition 8. The proof holds with the following changes: In the center expression for the displayed equation for $\partial e_r^*/\partial \theta$, $d_B(e_r^*)$ replaces d_B. In the numerical example, $N=1$ and $d_B^1 = 10.0$ replaces $d_B = 10.0$.

Proof of Proposition 9. The proof holds with the following changes: On the left hand side of (52), $+(1-e_r)(1-e_h)d'_B(e_r)(\partial e_r/\partial z)$ is added. In the last two displayed equations, $d_B(e_r^*)$ replaces d_B. On the right hand side of the last displayed equation, within the square brackets $-(1-e_r^*)(1-e_h^*)^2(p-c-d_S)d'_B(e_r^*)$ is added.

Proof of Proposition 10. The proof is unchanged.

Proof of Proposition 11. The proof holds with the following changes: In equation (53) and in the last two displayed equations, $d_B(e_r^*)$ replaces d_B. On the right hand side of equation (53), $-(1-e_r^*)[1-e_a^*(1-e_h^*)]d'_B(e_r^*)(\partial e_r^*/\partial\gamma)$ is added. On the left hand side of equation (56), $+(1-e_r)(1-e_h)d'_B(e_r)(\partial e_r/\partial\gamma)$ is added. On the right hand side of the last displayed equation, within the first set of square brackets $-(1-e_r^*)(1-e_h^*)^2(p-c-d_S)d'_B(e_r^*)$ is added.

Appendix D

This appendix provides the proof of the claim in §6 that: If an increase in auditing effort e_a is accompanied by a decrease in the probability of passing the audit and an increase in the probability of a major harm to workers of the environment, then the increase in e_a must be accompanied by a decrease in responsibility effort.

Proof: An increase in e_a is accompanied by a decrease in the probability of passing the audit

$$1-\overline{e}_a[1-\tilde{e}_r(\overline{e}_a)][1-\tilde{e}_h(\overline{e}_a)] < 1-\underline{e}_a[1-\tilde{e}_r(\underline{e}_a)][1-\tilde{e}_h(\underline{e}_a)], \tag{57}$$

where $\overline{e}_a > \underline{e}_a$ and $(\tilde{e}_h(e_a),\tilde{e}_r(e_a))$ denotes the supplier's best response hiding and responsibility efforts under auditing effort e_a. An increase in e_a is accompanied by an increase in the probability of a major harm to workers of the environment

$$[1-\tilde{e}_r(\overline{e}_a)](1-\overline{e}_a[1-\tilde{e}_h(\overline{e}_a)]) > [1-\tilde{e}_r(\underline{e}_a)](1-\underline{e}_a[1-\tilde{e}_h(\underline{e}_a)]). \tag{58}$$

Therefore,

$$0 > \underline{e}_a[1-\tilde{e}_r(\underline{e}_a)][1-\tilde{e}_h(\underline{e}_a)] - \overline{e}_a[1-\tilde{e}_r(\overline{e}_a)][1-\tilde{e}_h(\overline{e}_a)] > \tilde{e}_r(\overline{e}_a) - \tilde{e}_r(\underline{e}_a),$$

where the first inequality follows from (58) and the second from (57).■