Incentive-driven Information Dissemination in Two-tier Supply Chains
Online Appendix: Mathematical Proofs

Derivation of conditional expectations under Assumptions A1 and A2

It follows that (μ, X_1, X_2) is multivariate normal with covariance matrix as follows:

$$\Sigma_{(\mu,X_1,X_2)} = \left[\begin{array}{c|cc} \sigma_\mu & \sigma_\mu + \sigma_\varepsilon & \sigma_\mu + \sigma_\varepsilon \\ \hline \sigma_\mu + \sigma_\varepsilon & \sigma_\mu + \sigma_\varepsilon & \sigma_\mu + \rho \\ \sigma_\mu + \sigma_\varepsilon & \sigma_\mu + \rho & \sigma_\mu + \sigma_\varepsilon \end{array}\right] = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}.$$

By Degroot (1970, p. 55), it follows that $f(\mu|X_1, X_2)$ is normally distributed with the following mean:

$$E(\mu|\, X_1 = x_1, X_2 = x_2) = \Sigma_{12}\Sigma_{22}^{-1}(X_1, X_2)^T = \frac{\sigma_\mu}{2\sigma_\mu+\sigma_\varepsilon+\rho}(x_1 + x_2).$$

Similarly, (μ, X), (μ, X_i) and (X_j, X_i) are bivariate normal with covariance matrices as follows:

$$\Sigma_{(\mu,X)} = \Sigma_{(\mu,X_i)} = \begin{bmatrix} \sigma_\mu & \sigma_\mu \\ \sigma_\mu & \sigma_\mu + \sigma_\varepsilon \end{bmatrix}, \Sigma_{(X_j,X_i)} = \begin{bmatrix} \sigma_\mu + \sigma_\varepsilon & \sigma_\mu + \rho \\ \sigma_\mu + \rho & \sigma_\mu + \sigma_\varepsilon \end{bmatrix}.$$

The posterior distributions of μ and X_j, given X_i, have the following means:

$E(\mu|X = x) = \frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}x$, $E(\mu|X_i = x_i) = \frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}x_i$, $E\left(X_j\middle|X_i = x_i\right) = \frac{\sigma_\mu+\rho}{\sigma_\mu+\sigma_\varepsilon}x_i$. □

Derivation of the operations subgame equilibrium in system B, for given n

By the first-order conditions of (3), $q_i(w|x) = \frac{a_i-\beta a_{3-i}-w_i+\beta w_{3-i}}{2(1-\beta^2)} + \frac{E(\mu|X=x)}{2(1+\beta)}, i = 1,2$. Without vertical information acquisition, with $q_i(w|x)$ and the first-order conditions of $\pi_S^{(0)}$, we have $w_i^{(0)} = \frac{a_i+c_i}{2}$. Then $q_i^{(0)}(x) = \frac{a_i-c_i-\beta(a_{3-i}-c_{3-i})}{4(1-\beta^2)} + \frac{\sigma_\mu}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}x$.

The ex-ante system profit is:

$E\left[\sum_{i=1}^2\left(a_i + \mu - q_i^{(0)} - \beta q_{3-i}^{(0)} - w_i^{(0)}\right)q_i^{(0)}\right] + E\left[\sum_{i=1}^2\left(w_i^{(0)} - c_i\right)q_i^{(0)}(x)\right] =$

$\frac{3[(a_1-c_1)^2+(a_2-c_2)^2-2\beta(a_1-c_1)(a_2-c_2)]}{16(1-\beta^2)} + \frac{\sigma_\mu}{(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}E(\mu X) - \frac{\sigma_u^2}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)^2}E(X^2) =$

$\frac{3[(a_1-c_1)^2+(a_2-c_2)^2-2\beta(a_1-c_1)(a_2-c_2)]}{16(1-\beta^2)} + \frac{\sigma_u^2}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}$, since $E(\mu X) = \sigma_\mu$ and $E(X^2) = \sigma_\mu + \sigma_\varepsilon$.

With $q_i(w|x)$ and the first-order conditions of $\pi_S^{(1)}$, we can follow a similar procedure to establish the equilibrium outcome for the case with vertical information acquisition. □

Derivation of the operations subgame equilibrium in system SC, for given $n = (n_1, n_2)$

By the first-order conditions of (3), we have $q_i(w_1, w_2|x) = \frac{a_i-\beta a_{3-i}-w_i+\beta w_{3-i}}{2(1-\beta^2)} + \frac{E(\mu|X=x)}{2(1+\beta)}, i = 1,2$.

If $n = (0,0)$, with $q_i(w_1, w_2|x)$ and the first-order conditions of (4), we have:

$w_i^{(0,0)} = \frac{(2-\beta^2)a_i-\beta a_{3-i}+2c_i+\beta c_{3-i}}{4-\beta^2} = w_i^0, i = 1,2.$

$q_i^{(0,0)} = \frac{(2-\beta^2)(a_i-c_i)-\beta(a_{3-i}-c_{3-i})}{2(1-\beta^2)(4-\beta^2)} + \frac{\sigma_\mu}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}x = q_i^0 + \frac{1}{2(1+\beta)}Ax.$

The ex-ante profit of the retailer is:

$E\left[\sum_{i=1}^{2}\left(a_i+\mu-q_i^{(0,0)}-\beta q_{3-i}^{(0,0)}-w_i^{(0,0)}\right)q_i^{(0,0)}\right]=$
$\sum_{i=1}^{2}\left(a_i-q_i^0-\beta q_{3-i}^0-w_i^0\right)q_i^0+\frac{\sigma_\mu}{(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}E(\mu X)-2(1+\beta)\left(\frac{\sigma_\mu}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}\right)^2E(X^2)=\pi_R^0+$
$\frac{{\sigma_\mu}^2}{2(1+\beta)(\sigma_\varepsilon+\sigma_\mu)}$.

Similarly, the ex-ante expected profit of supplier i is $E\left[(w_i^{(0,0)}-c_i)q_i^{(0,0)}\right]=(w_i^0-c_i)q_i^0$.

We can follow similar procedures to establish the equilibrium outcomes for the other three cases, $n=\{(1,1),(1,0),(0,1)\}$. Signal price m_i should be deducted (added) from (to) the expected profit of the supplier i (retailer) if $n_i=1$ to obtain the total ex-ante profits. □

Proof of Proposition 1.

By Figure 3, we can derive the suppliers' signal prices m_1 and m_2. The expected ex-ante profits of the suppliers are as shown in Table 3. The equilibria are illustrated in Table A-1.

	$m_2<\frac{3(1-\beta)}{8}\Lambda$	$\frac{3(1-\beta)}{8}\Lambda\le m_2<(\frac{5+3\beta}{8}-\frac{1-4\beta^2}{(2-\beta)^2})\Lambda$	$m_2\ge(\frac{5+3\beta}{8}-\frac{1-4\beta^2}{(2-\beta)^2})\Lambda$
$m_1<\frac{3(1-\beta)}{8}\Lambda$	$\underline{\pi_{S1}^{(0,0)}}, \underline{\pi_{S2}^{(0,0)}}$	$\underline{\pi_{S1}^{(0,1)}}, \pi_{S2}^{(0,1)}$	$\underline{\pi_{S1}^{(0,1)}}, \pi_{S2}^{(0,1)}$
$\frac{3(1-\beta)}{8}\Lambda\le m_1<(\frac{5+3\beta}{8}-\frac{1-4\beta^2}{(2-\beta)^2})\Lambda$	$\pi_{S1}^{(1,0)}, \underline{\pi_{S2}^{(1,0)}}$	$\pi_{S1}^{(1,0)}, \pi_{S2}^{(1,0)}$ $(m_1>m_2)$ $\pi_{S1}^{(0,1)}, \pi_{S2}^{(0,1)}$ $(m_1<m_2)$	$\pi_{S1}^{(0,1)}, \pi_{S2}^{(0,1)}$
$m_1\ge(\frac{5+3\beta}{8}-\frac{1-4\beta^2}{(2-\beta)^2})\Lambda$	$\pi_{S1}^{(1,0)}, \underline{\pi_{S2}^{(1,0)}}$	$\pi_{S1}^{(1,0)}, \pi_{S2}^{(1,0)}$	$\pi_{S1}^{(1,1)}, \pi_{S2}^{(1,1)}$

Note. $\Lambda=\frac{{\sigma_\mu}^2}{2(1+\beta)(\sigma_\varepsilon+\sigma_\mu)}$.

Table A-1. Derivation of NE for System SC

By Table A-1, only one pure strategy NE can be sustained in system SC, where $m_i<\frac{3(1-\beta)}{8}\Lambda$, $n_i=0$. □

Derivation of the operations subgame equilibrium in system RC without horizontal information sharing

We will focus on $q_i(x_i)$ in the form of $q_i(x_i)=q_{i0}+q_{i1}x_i, i=1,2$. The objective functions of the retailers are given in (6). First-order conditions give $q_i(x_i)=\frac{a_i-\beta E[q_{3-i}(x_{3-i})|X_i=x_i]+E[\mu|X_i=x_i]-w_i}{2}$, $i=1,2$.

Under ordering policy $q_i(x_i)=q_{i0}+q_{i1}x_i$ and the first-order conditions, we have:

$$q_{i0}+q_{i1}x_1=\frac{a_i-\beta(q_{3-i,0}+q_{3-i,1}E(X_{3-i}|X_i=x_i)+E(\mu|X_i=x_i)-w_i}{2},\ i=1,2.$$

Applying these two equations for every possible x_1 and x_2, we will have four equations in four unknowns: $2q_{10}=a_1-\beta q_{20}-w_1$, $2q_{20}=a_2-\beta q_{10}-w_2$, $2q_{11}=-\beta q_{21}\frac{\sigma_\mu+\rho}{\sigma_\mu+\sigma_\varepsilon}+\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}$, $2q_{21}=-\beta q_{11}\frac{\sigma_\mu+\rho}{\sigma_\mu+\sigma_\varepsilon}+\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}$. The unique coefficients are obtained by solving these equations.

The equilibrium order quantity $q_i(w_1,w_2|x_i)=\frac{2(a_i-w_i)-\beta(a_{3-i}-w_{3-i})}{4-\beta^2}+Bx_i$, where $B=\frac{\sigma_\mu}{(2+\beta)\sigma_\mu+2\sigma_\varepsilon+\beta\rho}$.

If $n=(0,0)$, with $q_i(w_1,w_2|x_i)$ and the first-order conditions of (8), $w_i^{(0,0)}=\frac{a_i+c_i}{2}$, $i=1,2$. Thus, $q_i^{(0,0)}=q_i^0+Bx_i=\frac{2(a_i-c_i)-\beta(a_{3-i}-c_{3-i})}{2(4-\beta^2)}+\frac{\sigma_\mu}{(2+\beta)\sigma_\mu+2\sigma_\varepsilon+\beta\rho}x_i$. We can follow similar procedures to establish the outcomes for the other three cases.

When the retailers share information, we can follow a similar procedure to obtain the outcomes, except that we now focus on $q_i(x_1, x_2)$ in the form of $q_i(x_i) = q_{i0} + q_{i1}x_1 + q_{i2}x_2, i = 1{,}2$. With all the outcomes, we can then calculate the ex-ante profits by substitutions. □

Proof of Lemma 1.

The results can be proved by comparing the relevant ex-ante profits as shown in Table 4. □

Proof of Lemma 2.

By Figure 4 and Table 4, the supplier's maximum ex-ante profits in areas I – IV are:

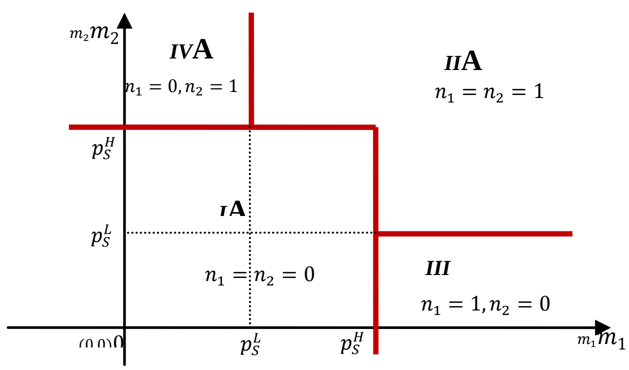
$$\pi_S^I = \pi_S^{(0,0)}, \pi_S^{II} = \pi_S^{(0,0)} + \frac{\sigma_u^2}{2[\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu)]} - p_N^L - p_N^H,$$

$$\pi_S^{III} = \pi_S^{IV} = \pi_S^{(0,0)} + \frac{{\sigma_\mu}^2[\rho^2+\sigma_\varepsilon^2+(2+\beta)\sigma_\varepsilon\sigma_\mu+(2+\beta)\sigma_\mu^2+\rho(\beta\sigma_\varepsilon+(2+\beta)\sigma_\mu)]}{2(\sigma_\varepsilon+\sigma_\mu)[\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu)]^2} - p_N^H.$$

Comparing these profits, $\pi_S^{III} = \pi_S^{IV}$; and if $\beta \le \min\{\beta_{RC}^{N1}, \beta_{RC}^{N2}\}$, $\pi_S^{II} < \pi_S^I$ and $\pi_S^{III} < \pi_S^I$; if $\beta_{RC}^{N2} < \beta \le \beta_{RC}^{N3}$, $\pi_S^I < \pi_S^{II}$ and $\pi_S^{III} < \pi_S^{II}$; if $\beta > \max\{\beta_{RC}^{N1}, \beta_{RC}^{N3}\}$, $\pi_S^I < \pi_S^{III}$ and $\pi_S^{II} < \pi_S^{III}$. $\beta_{RC}^{N3} \le \beta_{RC}^{N1} \le \beta_{RC}^{N2}$ if $\sigma_\varepsilon > \sigma_\mu$ & $\rho \le \frac{\sigma_\varepsilon - \sigma_\mu}{2}$, $\beta_{RC}^{N3} \ge \beta_{RC}^{N1} \ge \beta_{RC}^{N2}$ otherwise. Hence the claim. □

Proof of Lemma 3.

Similar to Figure 4, we characterize the decisions by the retailers to disclose signals in system RC when they exchange signals, as illustrated in Figure A-1.



Note. $p_S^L = \frac{3\sigma_u^2(\sigma_\varepsilon-\rho)}{4(2+\beta)^2(\sigma_\varepsilon+\sigma_\mu)(\rho+\sigma_\varepsilon+2\sigma_\mu)}$, $p_S^H = \frac{3\sigma_u^2}{4(2+\beta)^2(\sigma_\varepsilon+\sigma_\mu)}$

Figure A-1. Information acquisition in system RC with information sharing

By Figure A-1 and Table 5, the supplier's maximum ex-ante profits in areas I – IV are:

$$\pi_S^I = \pi_S^{(0,0)}, \pi_S^{II} = \pi_S^{(0,0)} + \frac{\sigma_\mu^2}{(2+\beta)(\rho+\sigma_\varepsilon+2\sigma_\mu)} - p_S^L - p_S^H, \pi_S^{III} = \pi_S^{IV} = \pi_S^{(0,0)} + \frac{\sigma_\mu^2}{2(2+\beta)(\sigma_\varepsilon+\sigma_\mu)} - p_S^H.$$

$\pi_S^I < \pi_S^{II}$ and $\pi_S^{III} = \pi_S^{IV} < \pi_S^{II}$. The comparative and sensitive results follow from the expressions for p_S^H and p_S^L. □

Proof of Proposition 2.

We assume $(m_1^*, m_2^*) = (p_\cdot^H, p_\cdot^L)$ under bilateral information acquisition, and $m_1^* = p_\cdot^H$ under unilateral information acquisition. By Lemma 3, bilateral information acquisition will always occur if the retailers share information. By Lemma 2, if the retailers forfeit information sharing, when $\sigma_\varepsilon > \sigma_\mu$ & $\rho \le \frac{\sigma_\varepsilon - \sigma_\mu}{2}$, no information acquisition will occur if $\beta \le \beta_{RC}^{N1}$, while unilateral information acquisition will occur if $\beta > \beta_{RC}^{N1}$. We analyze the retailers' decision to share information by comparing their ex-ante expected profits without and with information sharing, taking into consideration the supplier's signal acquisition. The retailers will share information iff both of them are better off. We use subscript N or S on the relevant profits, with N for "no information sharing" and S for "with information sharing".

When $0 \leq \rho < \frac{(\sigma_\varepsilon - \sigma_\mu)^+}{2}$ and $\beta \leq \beta_{RC}^{N1}$, we compare $(\pi_{Ri}^{(0,0)})_N = \pi_{Ri}^0 + \frac{\sigma_u^2(\sigma_\varepsilon + \sigma_\mu)}{[\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu)]^2}$ in Table 3 with $(\pi_{Ri}^{(1,1)})_S = \pi_{Ri}^0 + \frac{\sigma_\mu^2}{2(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)} + (m_i)_S$ in Table 5, where $(m_{1(2)})_S = p_S^H(p_S^L)$. When $\sigma_\varepsilon > 6\sigma_\mu$, $\rho < \frac{\sigma_\varepsilon - 6\sigma_\mu}{7}$ and $\beta \leq \beta_{RC}^{S1}$, where $\beta_{RC}^{S1} = \frac{\alpha+\gamma}{\omega}$, with $\omega = 3\rho^3 - 5\rho^2\sigma_\varepsilon + 4\rho\sigma_\varepsilon^2 + 4\sigma_\varepsilon^3 + 4\rho^2\sigma_\mu - 2\rho\sigma_\varepsilon\sigma_\mu + 16\sigma_\varepsilon^2\sigma_\mu + 3\rho\sigma_\mu^2 + 15\sigma_\varepsilon\sigma_\mu^2 + 6\sigma_\mu^3$, $\alpha = -2(3\rho^2\sigma_\varepsilon - \rho\sigma_\varepsilon^2 + 4\sigma_\varepsilon^3 + 3\rho^2\sigma_\mu + 4\rho\sigma_\varepsilon\sigma_\mu + 11\sigma_\varepsilon^2\sigma_\mu + 5\rho\sigma_\mu^2 + 13\sigma_\varepsilon\sigma_\mu^2 + 6\sigma_\mu^3)$,

$$\gamma = 4\sqrt{\begin{array}{c}-3\rho^4\sigma_\varepsilon^2 + 8\rho^3\sigma_\varepsilon^3 - 2\rho^2\sigma_\varepsilon^4 - 8\rho\sigma_\varepsilon^5 + 5\sigma_\varepsilon^6 - 6\rho^4\sigma_\varepsilon\sigma_\mu + 12\rho^3\sigma_\varepsilon^2\sigma_\mu + 16\rho^2\sigma_\varepsilon^3\sigma_\mu \\ -44\rho\sigma_\varepsilon^4\sigma_\mu + 22\sigma_\varepsilon^5\sigma_\mu - 3\rho^4\sigma_\mu^2 + 42\rho^2\sigma_\varepsilon^2\sigma_\mu^2 - 72\rho\sigma_\varepsilon^3\sigma_\mu^2 + 33\sigma_\varepsilon^4\sigma_\mu^2 - 4\rho^3\sigma_\mu^2 \\ +28\rho^2\sigma_\varepsilon\sigma_\mu^3 - 44\rho\sigma_\varepsilon^2\sigma_\mu^3 + 20\sigma_\varepsilon{}^3\sigma_\mu^3 + 4\rho^2\sigma_\mu^4 - 8\rho\sigma_\varepsilon\sigma_\mu^4 + 4\sigma_\varepsilon^2\sigma_\mu^4\end{array}}, \beta \leq \beta_{RC}^{N1} \text{ and}$$

$\min\{(\pi_{Ri}^{(1,1)})_S - (\pi_{Ri}^{(0,0)})_N, i \in \{1,2\}\} \geq 0$. Then, under such conditions, the retailers will share signals and the supplier will acquire signals from them both. Otherwise, neither information sharing nor information acquisition will occur.

When $0 \leq \rho < \frac{(\sigma_\varepsilon - \sigma_\mu)^+}{2}$ and $\beta > \beta_{RC}^{N1}$, we compare $(\pi_{Ri}^{(1,0)})_N$ with $(\pi_{Ri}^{(1,1)})_S$. The requirements of $\beta > \beta_{RC}^{N1}$ and $\min\{(\pi_{Ri}^{(1,0)})_S - (\pi_{Ri}^{(1,0)})_N, i \in \{1,2\}\} > 0$ cannot hold simultaneously. Thus, the retailers will not share signals and the supplier will acquire signal from one retailer at price p_N^H.

Similarly, when $\rho \geq \frac{(\sigma_\varepsilon - \sigma_\mu)^+}{2}$, we consider the ex-ante profits of the retailers without information sharing in the three cases of part 1) in Lemma 2, and follow a similar procedure as above to establish the outcomes. Particularly, $\beta_{RC}^{S2} = Root[f(\beta),3]$, where $f(\beta) = -2\rho^2 + 4\rho\sigma_\varepsilon - 2\sigma_\varepsilon^2 + (2\rho^2 + 2\rho\sigma_\varepsilon + 8\sigma_\varepsilon^2 + 6\rho\sigma_\mu + 18\sigma_\varepsilon\sigma_\mu + 12\sigma_u^2)\beta + (2\rho^2 + 6\rho\sigma_\varepsilon + 2\sigma_\varepsilon^2 + 10\rho\sigma_\mu + 10\sigma_\varepsilon\sigma_\mu + 10\sigma_u^2)\beta^2 + (\rho^2 + \rho\sigma_\varepsilon + 3\rho\sigma_\mu + \sigma_\varepsilon\sigma_\mu + 2\sigma_u^2)\beta^3$, and $Root[f(\beta),1] < Root[f(\beta),2] < 0$, $Root[f,k]$ represents the kth root of f. $Root[f,k]$ represents the kth root of f. □

Proof of Proposition 3.

We use $(\pi_{\cdot}^{(0,0)})_N$ in Table 4 as the reference. The subscript T stands for the system profit, i.e., the total profit of the two retailers and the supplier. The results can be proved by comparing the equilibrium profits in Area II, III, and IV of Figure 5. For instance, no information sharing occurs and bilateral information acquisition occurs in Area III. With the condition $\max\{\beta_{RC}^{N2}, \beta_{RC}^{S2}\} < \beta < \beta_{RC}^{N3}$, we can show $(\pi_{Ri}^{(1,1)})_N < (\pi_{Ri}^{(0,0)})_N, i = 1,2$, $(\pi_S^{(1,1)})_N > (\pi_S^{(0,0)})_N$, and $(\pi_T^{(1,1)})_N < (\pi_T^{(0,0)})_N$ with $(m_1^*, m_2^*) = (p_N^H, p_N^L)$. The consumer welfare is defined as $CS = \sum_{i=1}^{2} E[U(q_i) - p_i q_i]$, where $U(q_i) = (a+\mu)q_i - \frac{{q_i}^2}{2}$. It can be verified that $(CS^{(1,1)})_N < (CS^{(0,0)})_N$. Similar comparative analyses can be conducted for the other areas. Most of the comparison results are the same as those in Area III, except in Area IV. In Area IV-a, we have $(\pi_{Ri}^{(1,1)})_S > (\pi_{Ri}^{(0,0)})_N, i = 1,2$, and $(\pi_T^{(1,1)})_S > (\pi_T^{(0,0)})_N$ with $(m_1^*, m_2^*) = (p_S^H, p_S^L)$. In Area IV-b, we have $(\pi_{R1}^{(1,1)})_S > (\pi_{R1}^{(0,0)})_N$ with $m_1^* = p_S^H$. □

Derivation of the operations subgame equilibrium in system SRC, for given (n_1, n_2)

Without information sharing, similar to that in system RC, $q_i(w_1, w_2|x_i) = \frac{2(a_i - w_i) - \beta(a_{3-i} - w_{3-i})}{4-\beta^2} + Bx_i$. If $n = (0,0)$, $\pi_{Si}^{(0,0)} = \mathrm{E}[(w_i - c_i)q_i]$, $i = 1,2$. With $q_i(w_1, w_2|x_i)$ and the first-order condition of $\pi_{Si}^{(0,0)}$, we have $w_i^{(0,0)} = w_i^0 = \frac{(8-\beta^2)a_i - 2\beta a_{3-i} + 8c_i + 2\beta c_{3-i}}{16-\beta^2}$.

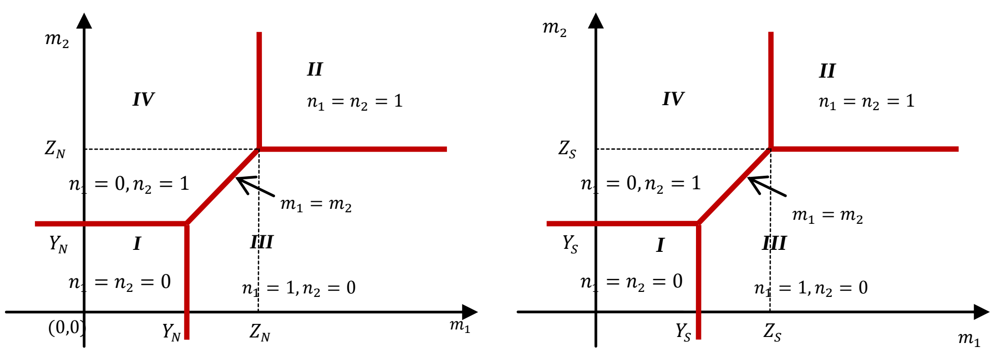
Thus, $q_i^{(0,0)} = q_i^0 + Bx_i = \frac{2((8-\beta^2)(a_i - c_i) - 2\beta(a_{3-i} - c_{3-i}))}{64 - 20\beta^2 + \beta^4} + \frac{\sigma_\mu}{(2+\beta)\sigma_\mu + 2\sigma_\varepsilon + \beta\rho} x_i$.

We can follow a similar procedure as that for $n = (0,0)$ to establish the equilibrium outcomes for the other cases. We consider $w_i(x_i)$ in the form of $w_i(x_i) = w_{i0} + w_{i1}x_i$ for $n = (1,1)$.

When the retailers share information, we can follow a similar procedure as above. The difference is that we now consider $q_i(x_1, x_2)$ in the form of $q_i(x_1, x_2) = q_{i0} + q_{i1}x_1 + q_{i2}x_2, i = 1,2$, and $w_i(x_1, x_2)$ in the form of $w_i(x_1, x_2) = w_{i0} + w_{i1}x_1 + w_{i2}x_2, i = 1,2$ for $n = (1,1)$. □

Proof of Lemma 4.

We analyze the decisions by the retailers to disclose signals in system SRC without (with) information sharing by Table 6(7), and illustrate them in Figure A-2. By Figures A-2, we can derive the suppliers' signal prices in the equilibrium. Their expected ex-ante profits are as shown in Table 6 and Table 7, for the cases without and with information sharing respectively. The equilibria are as illustrated in Table A-2 and Table A-3.



a) With no information sharing b) With information sharing

Figure A-2. Information acquisition in system SRC,

Note. $Y_N = \frac{3\sigma_\mu^2(\sigma_\varepsilon + \sigma_\mu)}{4(\beta\rho + 2\sigma_\varepsilon + (2+\beta)\sigma_\mu)^2}$, $Z_N = \frac{\sigma_\mu^2(4\beta^4\rho^3 + 192\sigma_\varepsilon^3 - \beta^3(8-9\beta)\rho^2\sigma_\mu - 2\beta^2(16+4\beta-3\beta^2)\rho\sigma_\mu^2 + (192-32\beta^2+\beta^4)\sigma_\mu^3 - 8\sigma_\varepsilon^2((4-\beta)\beta^2\rho - (72-4\beta^2+\beta^3)\sigma_\mu) - \sigma_\varepsilon(\beta^3(8+3\beta)\rho^2 + 2\beta^2(32+3\beta^2)\rho\sigma_\mu - (576-64\beta^2+8\beta^3-3\beta^4)\sigma_\mu^2))}{16(\beta\rho - 4\sigma_\varepsilon + (-4+\beta)\sigma_\mu)^2(\beta\rho + 2\sigma_\varepsilon + (2+\beta)\sigma_\mu)^2}$, $Y_S = \frac{3\sigma_\mu^2}{2(2+\beta)^2(\rho + \sigma_\varepsilon + 2\sigma_\mu)}$, $Z_S = \frac{(192-32\beta^2+\beta^4)\sigma_\mu^2}{8(4-\beta)^2(2+\beta)^2(\rho + \sigma_\varepsilon + 2\sigma_\mu)}$.

	$m_2 < Y_N$	$Y_N \le m_2 < Z_N$	$m_2 \ge Z_N$
$m_1 < Y_N$	$\underline{\pi_{S1}^{(0,0)}}$, $\underline{\pi_{S2}^{(0,0)}}$	$\underline{\pi_{S1}^{(0,1)}}$, $\pi_{S2}^{(0,1)}$	$\underline{\pi_{S1}^{(0,1)}}$, $\pi_{S2}^{(0,1)}$
$Y_N \le m_1 < Z_N$	$\pi_{S1}^{(1,0)}$, $\underline{\pi_{S2}^{(1,0)}}$	$\pi_{S1}^{(1,0)}$, $\pi_{S2}^{(1,0)}$ $(m_1 > m_2)$ $\pi_{S1}^{(0,1)}$, $\pi_{S2}^{(0,1)}$ $(m_1 < m_2)$	$\pi_{S1}^{(0,1)}$, $\pi_{S2}^{(0,1)}$
$m_1 \ge Z_N$	$\pi_{S1}^{(1,0)}$, $\underline{\pi_{S2}^{(1,0)}}$	$\pi_{S1}^{(1,0)}$, $\pi_{S2}^{(1,0)}$	$\pi_{S1}^{(1,1)}$, $\pi_{S2}^{(1,1)}$

Table A-2. Derivation of NE for system SRC, without information sharing

	$m_2 < Y_S$	$Y_S \le m_2 < Z_S$	$m_2 \ge Z_S$
$m_1 < Y_S$	~~$\pi_{S1}^{(0,0)}$~~, ~~$\pi_{S2}^{(0,0)}$~~	~~$\pi_{S1}^{(0,1)}$~~, $\pi_{S2}^{(0,1)}$	~~$\pi_{S1}^{(0,1)}$~~, $\pi_{S2}^{(0,1)}$
$Y_S \le m_1 < Z_S$	$\pi_{S1}^{(1,0)}$, ~~$\pi_{S2}^{(1,0)}$~~	$\pi_{S1}^{(1,0)}, \pi_{S2}^{(1,0)}$ $(m_1 > m_2)$ $\pi_{S1}^{(0,1)}, \pi_{S2}^{(0,1)}$ $(m_1 < m_2)$	$\pi_{S1}^{(0,1)}, \pi_{S2}^{(0,1)}$
$m_1 \ge Z_S$	$\pi_{S1}^{(1,0)}$, ~~$\pi_{S2}^{(1,0)}$~~	$\pi_{S1}^{(1,0)}, \pi_{S2}^{(1,0)}$	$\pi_{S1}^{(1,1)}, \pi_{S2}^{(1,1)}$

Table A-3. Derivation of NE for system SRC, with information sharing

Table A-2 and Table A-3 show that there is a unique pure strategy NE in system SRC with or without information sharing, in which, $m_i < Y_N(Y_S)$ and $n_i = 0$, $i = 1,2$. Hence the claim. □

Proof of Proposition 4.

By Lemma 4, regardless of the status of information sharing, information acquisition agreement will be $(n_1, n_2) = (0,0)$ in system SRC. The retailers' decision on information sharing is based on their ex-ante profits with and without information sharing. We still use subscript $N(S)$ on the relevant ex-ante profits, with N for "no information sharing" and S for "with information sharing".

$(\pi_{Ri}^{(0,0)})_N = \pi_{Ri}^0 + \frac{\sigma_\mu^2(\sigma_\epsilon+\sigma_\mu)}{(\beta\rho+2\sigma_\epsilon+(2+\beta)\sigma_\mu)^2}$ in Table 6 and $(\pi_{Ri}^{(0,0)})_S = \pi_{Ri}^0 + \frac{2\sigma_\mu^2}{(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}$ in Table 7.

If $\beta < \beta_{SRC} = \frac{2[\sqrt{2(\sigma_\mu+\sigma_\varepsilon)(2\sigma_\mu+\sigma_\varepsilon+\rho)}-(\sigma_\mu+\sigma_\varepsilon)]}{3\sigma_\mu+\sigma_\varepsilon+2\rho}$, then $(\pi_{Ri}^{(0,0)})_S - (\pi_{Ri}^{(0,0)})_N = \frac{2\sigma_\mu^2}{(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)} - \frac{\sigma_\mu^2(\sigma_\epsilon+\sigma_\mu)}{(\beta\rho+2\sigma_\epsilon+(2+\beta)\sigma_\mu)^2} > 0$, and both retailers will be better off with signal exchange. □

Proof of Proposition 5.

By $w_i(\hat{s})$ and $q_i(s,\hat{s})$, we can rewrite (10) as follows:

$$\pi_R(n|m) = \frac{(a_1-c_1)^2+(a_2-c_2)^2-2\beta(a_1-c_1)(a_2-c_2)}{16(1-\beta^2)} + \frac{E(\mu M)}{1+\beta} - \frac{E(\mu\widehat{M}(n))}{2(1+\beta)} - \frac{E(M^2)}{2(1+\beta)} + \frac{E(\widehat{M}(n)^2)}{8(1+\beta)} + mn \quad \text{(A-1)}$$

$$\frac{\partial\pi_R(n|m)}{\partial n} = \frac{-\sigma_\mu(\sigma_\mu+\sigma_\varepsilon-h-\rho)[\sigma_\mu+\sigma_\varepsilon+(N-1)(h+\rho)][3\sigma_\mu(\sigma_\mu+\sigma_\varepsilon)+(N-1)h(3\sigma_\mu+4\sigma_\varepsilon)-(N-1)\sigma_\mu\rho]}{8(1+\beta)N^2(\sigma_\mu+\sigma_\varepsilon)^2[\sigma_\mu+\sigma_\varepsilon+(n-1)(h+\rho)]^2} + m.$$

$\pi_R(n|m)$ decreases in n for $n \le \bar{n}$ and increases in n for $n > \bar{n}$, where:

$$\bar{n} = \sqrt{\frac{\sigma_\mu(\sigma_\mu+\sigma_\varepsilon-h-\rho)[\sigma_\mu+\sigma_\varepsilon+(N-1)(h+\rho)][3\sigma_\mu(\sigma_\mu+\sigma_\varepsilon)+(N-1)h(3\sigma_\mu+4\sigma_\varepsilon)-(N-1)\sigma_\mu\rho]}{8(1+\beta)N^2(\sigma_\mu+\sigma_\varepsilon)^2(h+\rho)^2 m}} - \frac{\sigma_\mu+\sigma_\varepsilon-h-\rho}{h+\rho}.$$

Note that $n \in [0, N]$. It can be verified that the optimal n can only be 0 or N when $\bar{n} \le 0$, $0 < \bar{n} \le N$, or $\bar{n} > N$. $\pi_R(N|m) - \pi_R(0|m) = -\frac{3\sigma_u^2(\sigma_\mu+\sigma_\varepsilon)+\sigma_\mu(3\sigma_\mu+4\sigma_\varepsilon)(N-1)h-\sigma_u^2(N-1)\rho}{8(1+\beta)N(\sigma_\mu+\sigma_\varepsilon)^2} + mN$.

If $m \ge m^S$, where $m^S = \frac{3\sigma_u^2(\sigma_u+\sigma_\varepsilon)+(N-1)(3\sigma_u^2 h+4\sigma_\mu\sigma_\varepsilon h-\sigma_u^2\rho)}{8(1+\beta)N^2(\sigma_u+\sigma_\varepsilon)^2}$, $n^* = N$; otherwise, $n^* = 0$.

(11) can be rewritten as:

$$\pi_S(m) = \frac{(a_1-c_1)^2+(a_2-c_2)^2-2\beta(a_1-c_1)(a_2-c_2)}{8(1-\beta^2)} + \frac{E(M\widehat{M}(n))}{2(1+\beta)} - \frac{E(\widehat{M}(n)^2)}{4(1+\beta)} - mn. \quad \text{(A-2)}$$

$\pi_S(m)$ decreases in m. So, $m^* = m^S$ if the supplier has an incentive to acquire signals.
Therefore if $\pi_S(m^*)_{(n=N)} - \pi_S(m)_{(n=0)} > 0$ when $m^* = m^S$, information acquisition will occur.

$$\pi_S(m^*)_{(n=N)} - \pi_S(m)_{(n=0)} = \frac{\sigma_\mu[3(N-1)\sigma_\mu\rho-(N-1)\sigma_\mu h-4(N-1)\sigma_\varepsilon h-\sigma_\mu(\sigma_\mu+\sigma_\varepsilon)]}{8(1+\beta)N(\sigma_\mu+\sigma_\varepsilon)^2}.$$

Then it is easy to verify that under the transaction condition specified in the text, the supplier will offer payment to acquire all signals. □

Proof of Proposition 6.

It can be verified that if the retailer has two signals that satisfy signal structure [A2] in system B and system SC, no information acquisition will occur. To maintain consistency, in comparing SC(B) and SRC(RC), we assume that the retailer in SC(B) has two signals that satisfy signal structure [A2]; and in comparing B and SC, we assume that the retailer has only one signal that satisfies signal structure [A1].

$\Delta\pi_S^B = \sum_i \Delta\pi_{Si}^{SC} = \sum_i \Delta\pi_{Si}^{SRC} = 0$, and,

$$\Delta\pi_R^B = \Delta\pi_R^{SC} = \Delta\pi_T^B = \Delta\pi_T^{SC} = \begin{cases} \frac{\sigma_u^2}{2(1+\beta)(\sigma_\mu+\sigma_\varepsilon)}, & \textit{the retailer has one signal in } B(SC) \\ \frac{\sigma_u^2}{(1+\beta)(2\sigma_\mu+\sigma_\varepsilon+\rho)}, & \textit{the retailer has two signals in } B(SC) \end{cases},$$

$$\sum_i \Delta\pi_{Ri}^{SRC} = \Delta\pi_T^{SRC} = \begin{cases} \frac{4\sigma_\mu^2}{(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{if } \beta < \beta_{SRC} \\ \frac{2\sigma_\mu^2(\sigma_\varepsilon+\sigma_\mu)}{(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{else} \end{cases},$$

$$\Delta\pi_S^{RC} = \begin{cases} 0, & \textit{Area I in Figure } 5 \\ \frac{\sigma_u^2\left(2\rho^2-\sigma_\varepsilon^2-2(1-\beta)\sigma_\varepsilon\sigma_\mu+(1+2\beta)\sigma_u^2+2\rho\left(2\sigma_\mu+\beta\left(\sigma_\varepsilon+\sigma_\mu\right)\right)\right)}{4\left(\sigma_\varepsilon+\sigma_\mu\right)\left(\beta\left(\rho+\sigma_\mu\right)+2\left(\sigma_\varepsilon+\sigma_\mu\right)\right)^2}, & \textit{Area II in Figure } 5 \\ \frac{\sigma_u^2((2\beta-1)\rho^2-2\sigma_\varepsilon^2+(1+2\beta)\sigma_u^2+2\rho(2\sigma_\varepsilon+\sigma_\mu+2\beta\sigma_\mu))}{4(\sigma_\varepsilon+\sigma_\mu)(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{Area III in Figure } 5 \\ \frac{(1+2\beta)\sigma_u^2}{2(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{Area IV in Figure } 5 \end{cases},$$

$$\sum_i \Delta\pi_{Ri}^{RC} = \begin{cases} \frac{2\sigma_u^2(\sigma_\varepsilon+\sigma_\mu)}{(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{Area I in Figure } 5 \\ \frac{\sigma_\mu^2\left((1-2\beta)\rho^2+8\sigma_\varepsilon^2-2(1+\beta)\rho\sigma_\mu+5\sigma_\mu^2+2\sigma_\varepsilon\left((\beta-2)\rho+(6+\beta)\sigma_\mu\right)\right)}{4\left(\sigma_\varepsilon+\sigma_\mu\right)\left(\beta\rho+2\sigma_\varepsilon+(2+\beta)\sigma_\mu\right)^2}, & \textit{Areas II and III in Figure } 5 \\ \frac{5\sigma_\mu^2}{2(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{Area IV in Figure } 5 \end{cases},$$

$$\Delta\pi_T^{RC} = \begin{cases} \frac{2\sigma_u^2(\sigma_\varepsilon+\sigma_\mu)}{(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{Area I in Figure } 5 \\ \frac{\sigma_u^2((3-2\beta)\rho^2+7{\sigma_\varepsilon}^2+2(5+2\beta)\sigma_\varepsilon\sigma_\mu+2(3+\beta)\sigma_u^2+2\rho(2(-1+\beta)\sigma_\varepsilon+\sigma_\mu))}{4(\sigma_\varepsilon+\sigma_\mu)(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{Area II in Figure } 5 \\ \frac{\sigma_u^2(3(\sigma_\varepsilon+\sigma_\mu)+\beta(\rho+\sigma_\mu))}{2(\beta(\rho+\sigma_\mu)+2(\sigma_\varepsilon+\sigma_\mu))^2}, & \textit{Area III in Figure } 5 \\ \frac{(3+\beta)\sigma_\mu^2}{(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{Area IV in Figure } 5 \end{cases},$$

$$\Delta CS^B = \Delta CS^{SC} = \begin{cases} \frac{(1+2\beta)\sigma_u^2}{4(1+\beta)^2(\sigma_\varepsilon+\sigma_\mu)}, & \textit{the retailer has one signal in system } B(SC) \\ \frac{(1+2\beta)\sigma_\mu^2}{2(1+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{the retailer has two signals in system } B(SC) \end{cases},$$

$$\Delta CS^{SRC} = \begin{cases} \frac{2(1+2\beta)\sigma_\mu^2}{(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & \textit{if } \beta < \beta_{SRC} \\ \frac{\sigma_\mu^2(2\beta\rho+\sigma_\varepsilon+(1+2\beta)\sigma_\mu)}{(\beta\rho+2\sigma_\varepsilon+(2+\beta)\sigma_\mu)^2}, & \textit{else} \end{cases},$$

$$\Delta CS^{RC}=\begin{cases}\frac{\sigma_\mu^2(2\beta\rho+\sigma_\varepsilon+(1+2\beta)\sigma_\mu)}{(\beta\rho+2\sigma_\varepsilon+(2+\beta)\sigma_\mu)^2}, & Area\ I\ in\ Figure\ 5\\ \frac{\sigma_\mu^2(-3\rho^2+5\sigma_\varepsilon^2-2(3-2\beta)\rho\sigma_\mu+(2+4\beta)\sigma_\mu^2+2\sigma_\varepsilon(2\beta\rho+(5+2\beta)\sigma_\mu))}{8(\sigma_\varepsilon+\sigma_\mu)(\beta\rho+2\sigma_\varepsilon+(2+\beta)\sigma_\mu)^2}, & Area\ II\ in\ Figure\ 5\\ \frac{\sigma_\mu^2(2\beta\rho+\sigma_\varepsilon+(1+2\beta)\sigma_\mu)}{4(\beta\rho+2\sigma_\varepsilon+(2+\beta)\sigma_\mu)^2}, & Area\ III\ in\ Figure\ 5\\ \frac{(1+2\beta)\sigma_\mu^2}{2(2+\beta)^2(\rho+\sigma_\varepsilon+2\sigma_\mu)}, & Area\ IV\ in\ Figure\ 5\end{cases}.$$

Proposition 6 can be established by comparing the relevant profit gains and consumer welfare across various systems. □

Derivation of conditional expectations under signal structure I in a bilateral monopoly

The posterior distribution of μ given s is $E(\mu|s)=\frac{\sigma_\mu}{N(\sigma_\mu+\sigma_\varepsilon)}\sum_{i=1}^{N}x_i=M$, as $E(\mu|s)=\frac{1}{N}\sum_{i=1}^{N}E(\mu_i|x_i)$, and $E(\mu_i|x_i)=\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}x_i$.

For $i=n+1,n+2,\dots,N$, the prior distribution of $(x_i,x_1,x_2,\dots,x_n)$ is multivariate normal with mean zero and covariance matrix as follows:

$$\Sigma_{(x_i,x_1,x_2,\dots,x_n)}=\begin{bmatrix}\sigma_\mu+\sigma_\varepsilon & h+\rho & h+\rho & \cdots & h+\rho\\ h+\rho & \sigma_\mu+\sigma_\varepsilon & h+\rho & \cdots & h+\rho\\ h+\rho & h+\rho & \sigma_\mu+\sigma_\varepsilon & \cdots & h+\rho\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ h+\rho & h+\rho & h+\rho & \cdots & \sigma_\mu+\sigma_\varepsilon\end{bmatrix}=\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}$$

Given $\hat{s}=(x_1,x_2,\dots,x_n)$, the posterior distribution of x_i($i=n+1,n+2,\dots,N$) is $E(x_i|\hat{s})=\Sigma_{12}\Sigma_{22}^{-1}\hat{s}^T=\frac{h+\rho}{\sigma_\mu+\sigma_\varepsilon+(n-1)(h+\rho)}\sum_{i=1}^{n}x_i$.

Hence $E(E(\mu|s)|\hat{s})=\frac{1}{N}\sum_{i=1}^{N}E(E(\mu_i|x_i)|\hat{s})=\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}\left[1+\frac{(N-n)(h+\rho)}{\sigma_\mu+\sigma_\varepsilon+(n-1)(h+\rho)}\right]\frac{\sum_{i=1}^{n}x_i}{N}=\widehat{M}(n)$.

We can derive $E(\mu M)$, $E(\mu\widehat{M}(n))$,$E(M^2)$, $E(\widehat{M}(n)^2)$, and $E\big(M\widehat{M}(n)\big)$ as follows:

$E(\mu M)=\frac{\sigma_\mu}{N^2(\sigma_\mu+\sigma_\varepsilon)}E\left[\left(\sum_{i=1}^{N}\mu_i\right)\left(\sum_{i=1}^{N}x_i\right)\right]=\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}\frac{\sigma_\mu+(N-1)h}{N}$, since $E\left[\mu_j\left(\sum_{i=1}^{N}x_i\right)\right]=E\left(\mu_jx_j\right)+E\left[\mu_j\left(\sum_{i=1,i\neq j}^{N}x_i\right)\right]$, $j=1,2,\dots,N$,

$E(\mu_jx_j)=Cov(\mu_j,x_j)+E(\mu_j)E(x_j)=Cov(\mu_j,\mu_j+\varepsilon_j)=Var(\mu_j)+Cov(\mu_j,\varepsilon_j)=\sigma_\mu$,

$E\left[\mu_j\left(\sum_{i=1,i\neq j}^{N}x_i\right)\right]=(N-1)E(\mu_jx_i), i\neq j$,

$E(\mu_jx_i)=Cov(\mu_j,x_i)+E(\mu_j)E(x_i)=Cov(\mu_j,\mu_i+\varepsilon_i)=Cov(\mu_j,\mu_i)+Cov(\mu_j,\varepsilon_i)=h$,

We can follow similar procedures to show:

$E(\mu M)=\frac{\sigma_\mu}{\sigma_\mu+\sigma_\varepsilon}\frac{\sigma_\mu+(N-1)h}{N}$,$E\big(\mu\widehat{M}(n)\big)=\frac{n\sigma_\mu[\sigma_\mu+\sigma_\varepsilon+(N-1)(h+\rho)][\sigma_\mu+(N-1)h]}{N^2(\sigma_\mu+\sigma_\varepsilon)[\sigma_\mu+\sigma_\varepsilon+(n-1)(h+\rho)]}$,

$E(M^2)=\frac{\sigma_u^2}{(\sigma_\mu+\sigma_\varepsilon)^2}\frac{\sigma_\mu+\sigma_\varepsilon+(N-1)(h+\rho)}{N}$, $E\big(M\widehat{M}(n)\big)=E\big(\widehat{M}(n)^2\big)=\frac{n\sigma_u^2[\sigma_\mu+\sigma_\varepsilon+(N-1)(h+\rho)]^2}{N^2(\sigma_\mu+\sigma_\varepsilon)^2[\sigma_\mu+\sigma_\varepsilon+(n-1)(h+\rho)]}$. □