Online Supplement for "Leader-Based Collective Bargaining: Cooperation Mechanism and Incentive Analysis"

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Online Appendix I: Proofs for Main Results

Proof of Lemma 1. The results can be directly derived from the Nash bargaining solution.

Proof of Proposition 1. The two buyers solve $q_i^S = \arg\max_{q_i} \alpha_{is}(a - q_i - b\widetilde{q}_j - c)q_i$, for $i, j = 1, 2, j \neq i$, simultaneously. Moreover, in equilibrium, we must have $q_i^S = \widetilde{q}_i$, $i = 1, 2$. Given the specific concavity of the objective functions, we can find a fixed point. In particular, $q_i^S = \frac{a-c}{2+b}$, and accordingly, $w_i^S = c + (1 - \alpha_{is})\frac{a-c}{2+b}$, $\pi_i^S = \alpha_{is}\left(\frac{a-c}{2+b}\right)^2$, $i = 1, 2$, and $\pi_s^S = (2 - \alpha_{1s} - \alpha_{2s})\left(\frac{a-c}{2+b}\right)^2$.

Proof of Lemma 2. This lemma follows directly from the arguments preceding the lemma.

Proof of Proposition 2. Taking the first order conditions of $\pi_i(q_i, q_j)$ and $\pi_j(q_i, q_j)$ with respect to their procurement quantities, we obtain $q_i = \frac{a-bq_j-c}{2}$ and $q_j = \frac{\alpha_{is}(a-c)+(1-b-\alpha_{is})q_i}{2(1-(1-\alpha_{is})b)}$. Solving them simultaneously we obtain the equilibrium quantities

$$q_i^L = \frac{(2(1-b) + b\alpha_{is})(a-c)}{(4+b)(1-b) + 3b\alpha_{is}}; \quad q_j^L = \frac{(1-b+\alpha_{is})(a-c)}{(4+b)(1-b) + 3b\alpha_{is}}.$$

The outcomes in (a) can be derived accordingly.

For part (b): first, we have

$$\frac{q_i^L}{q_i^S} = \frac{(2+b)[2(1-b) + b\alpha_{is}]}{[(4+b)(1-b) + 3b\alpha_{is}]}.$$

Note that $(2+b)[2(1-b)+b\alpha_{is}]-[(4+b)(1-b)+3b\alpha_{is}]=b(1-b)(1-\alpha_{is})\geq 0$. Thus, $q_i^L \geq q_i^S$. Second, we have

$$\frac{q_j^L}{q_j^S} = \frac{(2+b)[1-b+\alpha_{is}]}{[(4+b)(1-b)+3b\alpha_{is}]}.$$

Note that $(2+b)[1-b+\alpha_{is}]-[(4+b)(1-b)+3b\alpha_{is}]=-2(1-b)(1-\alpha_{is})\leq 0$. Thus, $q_j^L \leq q_j^S$. Moreover,

$$\begin{aligned} q_i^L - q_i^S + q_j^L - q_j^S &= \frac{(2+b)[3(1-b)+(1+b)\alpha_{is}]-2[(4+b)(1-b)+3b\alpha_{is}]}{[(4+b)(1-b)+3b\alpha_{is}]}(a-c) \\ &= \frac{-(1-b)(2-b)(1-\alpha_{is})}{[(4+b)(1-b)+3b\alpha_{is}]}(a-c) \\ &\leq 0, \end{aligned}$$

which shows $q_i^L + q_j^L \leq q_i^S + q_j^S$.

Proof of Proposition 3. The result holds because $\pi_i^L/\pi_i^S = (q_i^L/q_i^S)^2$ and $q_i^L \geq q_i^S$ which has been shown in the above.

Proof of Proposition 4. We first prove (a). Define $\underline{\alpha}_{js} = \frac{(2+b)^2(1-b+\alpha_{is})[(1-b)^2+\alpha_{is}(1-b^2+b\alpha_{is})]}{[(4+b)(1-b)+3b\alpha_{is}]^2}$, then $\pi_j^L \geq \pi_j^S$ iff $\alpha_{js} \leq \underline{\alpha}_{js}$. We can verify that

$$\begin{aligned} 1-\underline{\alpha}_{js} &= \frac{(1-\alpha_{is})[12-16b+b^3+2b^4+b^5+(4+12b-8b^2-6b^3-2b^4)\alpha_{is}+(4b+4b^2+b^3)(\alpha_{is})^2]}{[(4+b)(1-b)+3b\alpha_{is}]^2} \\ &\geq 0, \end{aligned}$$

thus $\underline{\alpha}_{js} \in [0,1]$. This completes the proof for Proposition 4(a).

We next prove (b). Note that $\frac{\pi_j^L}{\pi_j^S} = \frac{(2+b)^2(1-b+\alpha_{is})[(1-b)^2+\alpha_{is}(1-b^2+b\alpha_{is})]}{\alpha_{js}[(4+b)(1-b)+3b\alpha_{is}]^2}$. Define

$$\begin{aligned} A &= (2+b)^2(1-b+\alpha_{is})[(1-b)^2+\alpha_{is}(1-b^2+b\alpha_{is})]-\alpha_{js}[(4+b)(1-b)+3b\alpha_{is}]^2 \\ &= b(2+b)^2\alpha_{is}^3+(4+8b-3b^2-7b^3-2b^4-9b^2\alpha_{js})\alpha_{is}^2 \\ &\quad +(8-4b-10b^2+b^3+4b^4+b^5-b(24-18b-6b^2)\alpha_{js})\alpha_{is} \\ &\quad +4-8b+b^2+5b^3-b^4-b^5-(16-24b+b^2+6b^3+b^4)\alpha_{js}. \end{aligned}$$

Then $\pi_j^L \geq \pi_j^S$ iff $A \geq 0$.

Rewriting A in terms of b we have

$$\begin{aligned} A &= -(1-\alpha_{is})b^5 - (1 - 4\alpha_{is} + 2\alpha_{is}^2 + \alpha_{js})b^4 + (5 + \alpha_{is} - 7\alpha_{is}^2 + \alpha_{is}^3 - 6\alpha_{js} + 6\alpha_{is}\alpha_{js})b^3 \\ &\quad +(1 - 10\alpha_{is} - 3\alpha_{is}^2 + 4\alpha_{is}^3 - \alpha_{js} + 18\alpha_{is}\alpha_{js} - 9\alpha_{is}^2\alpha_{js})b^2 \\ &\quad -(8 + 4\alpha_{is} - 8\alpha_{is}^2 - 4\alpha_{is}^3 - 24\alpha_{js} + 24\alpha_{is}\alpha_{js})b + 4 + 8\alpha_{is} + 4\alpha_{is}^2 - 16\alpha_{js}. \end{aligned} \tag{1}$$

A is a five-degree function of b, so there may exist at most 5 real roots for $A(b) = 0$. It can be shown that $A(0) = 4(1+\alpha_{is})^2 - 16\alpha_{js} > 4(1+\alpha_{js})^2 - 16\alpha_{js} = 4(1-\alpha_{js})^2 > 0$. It can also be shown that $A(1) = 9(\alpha_{is} - \alpha_{js})\alpha_{is}^2 > 0$. Therefore, only three cases may exist when $b \in [0, 1]$: There are zero real roots, 2 real roots, or 4 real roots. The first two cases are similar since the first is a special case of the second. Next we show that it is impossible for the third case to arise.

First, note that $A(1) > 0$ and $A(+\infty) < 0$, so there at least exists one real root when $b \in (1, +\infty)$. Suppose there exist 4 real roots when $b \in [0, 1]$, then there must exist one real root, labeled as b_1, in the interval $b \in (1, +\infty)$. Label the 4 real roots in $b \in [0, 1]$ as b_2, b_3, b_4 and b_5, respectively. Thus $b_i, i = \{1, 2, 3, 4, 5\}$ is positive. Second, rewriting $A(b)$ in terms of the real roots yields $A(b) = -(1-\alpha_{is})(b-b_1)(b-b_2)(b-b_3)(b-b_4)(b-b_5)$, which can be rewritten as

$$\begin{aligned} A(b) &= -(1-\alpha_{is})[b^5 - (b_1 + b_2 + b_3 + b_4 + b_5)b^4] \\ &\quad -(1-\alpha_{is})(b_1b_2 + b_1b_3 + b_2b_3 + b_4(b_1 + b_2 + b_3) + b_5(b_1 + b_2 + b_3 + b_4))b^3 \\ &\quad +(1-\alpha_{is})(b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4 + b_5(b_1b_2 + b_1b_3 + b_2b_3 + b_4(b_1 + b_2 + b_3)))b^2 \\ &\quad -(1-\alpha_{is})(b_1b_2b_3b_4 + b_5(b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4))b \\ &\quad +(1-\alpha_{is})b_1b_2b_3b_4b_5. \end{aligned}$$

Corresponding to function (1), we find that b^3's coefficient $(b_1b_2+b_1b_3+b_2b_3+b_4(b_1+b_2+b_3)+b_5(b_1+b_2+b_3+b_4))$ should be positive, equal to $-\frac{5+\alpha_{is}-7\alpha_{is}^2+\alpha_{is}^3-6\alpha_{js}+6\alpha_{is}\alpha_{js}}{1-\alpha_{is}}$. However, it can be shown that $-\frac{5+\alpha_{is}-7\alpha_{is}^2+\alpha_{is}^3-6\alpha_{js}+6\alpha_{is}\alpha_{js}}{1-\alpha_{is}} = -[5 - \alpha_{is}^2 + 6(\alpha_{is} - \alpha_{js})] < 0$, which leads to contradiction, and hence it is impossible for 4 real roots to exist in the interval $b \in [0, 1]$. Thus, we can conclude that in the interval $b \in [0, 1]$, there may exist two threshold values $\underline{b}$ and $\bar{b}$, where $\underline{b} < \bar{b}$. LCB hurts the follower j when $b \in [\underline{b}, \bar{b}]$ and benefits him otherwise. This completes the proof for Proposition 4(b).

Proof of Proposition 5. Under LCB, the supply chain's profit is

$$\Pi^L = \pi_i^L + \pi_j^L + \pi_s^L = \frac{[7 - 15b + 9b^2 - b^3 + (2 + 4b - 7b^2 + b^3)\alpha_{is} - (1 - 3b)\alpha_{is}^2](a - c)^2}{[(4 + b)(1 - b) + 3b\alpha_{is}]^2}.$$

Under separate procurement, its profit is $\Pi^S = \pi_i^S + \pi_j^S + \pi_s^S = \frac{2(a-c)^2}{(2+b)^2}$. We find that

$$\frac{\Pi^L}{\Pi^S} = \frac{(2 + b)^2[7 - 15b + 9b^2 - b^3 + (2 + 4b - 7b^2 + b^3)\alpha_{is} - (1 - 3b)\alpha_{is}^2]}{2[(4 + b)(1 - b) + 3b\alpha_{is}]^2}.$$

Define

$$C = (2 + b)^2[7 - 15b + 9b^2 - b^3 + (2 + 4b - 7b^2 + b^3)\alpha_{is} - (1 - 3b)\alpha_{is}^2] - 2[(4 + b)(1 - b) + 3b\alpha_{is}]^2,$$

thus $\Pi^L \geq \Pi^S$ if $C \geq 0$. It can be shown that $C = (1 - b)(1 - \alpha_{is})C_1(b)$, where $C_1(b) = b^4 - 2b^3 - (7 - 3\alpha_{is})b^2 + (12 - 4\alpha_{is})b - 4 + 4\alpha_{is}$. Because $C_1''(b) = 12b^2 - 12b - 2(7 - 3\alpha_{is}) < 0$ for $b \in [0, 1]$, $C_1(b)$ changes sign at most once over $b \in [0, 1]$. This, together with the facts that $C_1(0) = -4(1 - \alpha_{is}) \leq 0$ and $C_1(1) = 3\alpha_{is} \geq 0$, proves the proposition.

Proof of Lemma 3. This lemma follows from the arguments preceding the lemma.

Proof of Lemma 4. It is straightforward to verify that the Nash product is log-concave. Thus, by the first-order condition, we have that the transfer price satisfies

$$\alpha_{is} t q_j = \alpha_{is}\alpha_{ij}((a - q_j - bq_i)q_j - \pi_j^S) - \alpha_{is}(1 - \alpha_{ij})((a - q_i - bq_j)q_i - c(q_i + q_j)) + (1 - \alpha_{ij})\pi_i^S.$$

Then, buyer i's profit function can be written as

$$\pi_i(q_i, q_j) = \alpha_{is}\alpha_{ij}((a - q_i - bq_j - c)q_i + (a - q_j - bq_i - c)q_j - \pi_j^S) + (1 - \alpha_{ij})\pi_i^S,$$

and buyer j's profit function can be written as

$$\pi_j(q_i, q_j) = \frac{\alpha_{is}(1 - \alpha_{ij})((a - q_i - bq_j - c)q_i + (a - q_j - bq_i - c)q_j) + \alpha_{is}\alpha_{ij}\pi_j^S - (1 - \alpha_{ij})\pi_i^S}{\alpha_{is}}.$$

Solving these two functions simultaneously we have the optimal production quantities which achieve the first-best solution:

$$q_i^L = q_j^L = \frac{a - c}{2(1 + b)}.$$

Proof of Proposition 6. The proposition follows directly from the closed-form expressions of the profits of the leader and the follower in Lemma 4.

Proof of Proposition 7.

$$\begin{aligned}
\pi_i^L - \pi_i^S &= (1-\alpha_{ij})\pi_i^S + \alpha_{ij}\alpha_{is}\left(\frac{(a-c)^2}{2(1+b)} - \pi_j^S\right) - \pi_i^S \\
&= \alpha_{ij}\alpha_{is}\left(\frac{1}{4(1+b)} - \frac{1}{(2+b)^2} + \frac{1}{4(1+b)} - \frac{\alpha_{js}}{(2+b)^2}\right)(a-c)^2 \\
&\geq 0.
\end{aligned}$$

$$\begin{aligned}
\pi_j^L - \pi_j^S &= \alpha_{ij}\pi_j^S + (1-\alpha_{ij})\left(\frac{(a-c)^2}{2(1+b)} - \frac{\pi_i^S}{\alpha_{is}}\right) - \pi_j^S \\
&= (1-\alpha_{ij})\left(\frac{(a-c)^2}{2(1+b)} - \frac{\pi_i^S}{\alpha_{is}} - \pi_j^S\right) \\
&= (1-\alpha_{ij})\left(\frac{1}{4(1+b)} - \frac{1}{(2+b)^2} + \frac{1}{4(1+b)} - \frac{\alpha_{js}}{(2+b)^2}\right)(a-c)^2 \\
&\geq 0.
\end{aligned}$$

$$(\pi_i^L + \pi_j^L + \pi_s^L) - (\pi_i^S + \pi_j^S + \pi_s^S) = \frac{(a-c)^2}{2(1+b)} - \frac{2(a-c)^2}{(2+b)^2} = \frac{b^2(a-c)^2}{2(1+b)(2+b)^2} \geq 0.$$

Proof of Proposition 8. The supplier's profit gain under the fixed price LCB is

$$\pi_s^L - \pi_s^S = B(b) = \frac{\alpha_{ij}(1-\alpha_{is})}{\alpha_{is}}\left(\alpha_{is}\frac{(a-c)^2}{2(1+b)} - \alpha_{is}\pi_j^S - \pi_i^S\right) - \frac{1-\alpha_{js}}{\alpha_{js}}\pi_j^S.$$

Thus, the supplier will benefit from the fixed price LCB iff $B(b) \geq 0$. $B(b)$ can be simplified as

$$B(b) = \frac{\alpha_{ij}(1-\alpha_{is})(a-c)^2}{2(1+b)(2+b)^2}\left(b^2 - \frac{2(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})}b - \frac{2(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})}\right),$$

Define $B_1(b) = b^2 - \frac{2(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})}b - \frac{2(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})}$. Then $B_1(b)$ is a convex function of b. If $b=0$, then $B_1(0) = -\frac{2(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})} \leq 0$. If $b=1$, then $B_1(1) = \frac{\alpha_{ij}(1-\alpha_{is}) - 4(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))}{\alpha_{ij}(1-\alpha_{is})}$. $B_1(1)$ will be positive when $\alpha_{ij}(1-\alpha_{is}) \geq 4(1-\alpha_{js})(1-\alpha_{ij}(1-\alpha_{is}))$. If so, then there exists a unique threshold value $\tilde{b}$, larger than which $B(b) \geq 0$. This completes the proof.

Proof of Lemma 5. Given q_i and q_j, the Nash product is log-concave in w, which yields

$$w(q_i, q_j) = c + (1-\alpha_{is})(a - q_i - bq_j - c) + \frac{\alpha_{is}\pi_s^S}{q_i + q_j} - \frac{(1-\alpha_{is})\pi_i^S}{q_i},$$

Substituting $w(q_i, q_j)$ into the profit functions of i and j yields Lemma 5.

Proof of Lemma 6. Given q_i, q_j and t, the Nash product between buyer i and the supplier is log-concave. Substituting $w(q_i, q_j)$ into the profit functions of buyers i and j yields Lemma 6.

Proof of Lemma 7. The results are immediate given that the objective function is log-concave.

Proof of Proposition 9. Maximizing the buyers' profit functions shown in Lemma 7 we have two best-response functions: (1) $q_i(q_j) = (a - c - 2bq_j)/2$; (2) $q_j(q_i) = (a - c - 2bq_i)/2$. Solving them simultaneously yields the equilibrium quantities $q_i^L = q_j^L = \frac{a-c}{2(1+b)}$, based on which the results of Proposition 9 can be obtained.

Online Appendix II: Extra Extensions

In this section, we present four extra extensions of our main model and the associated proofs.

A. Alternative Benchmark: Sequential Price Negotiation

In this subsection, we analyze an alternative benchmark where the two buyers negotiate with the supplier sequentially under separate procurement. Without loss of generality, we assume that buyer i negotiates with the supplier first. If the negotiation fails, then buyer i will earn zero profit, while buyer j will become the monopoly buyer and his quantity will be $q_j = q_j^m = (a-c)/2$. Thus, the supplier's reservation profit for the first negotiation is $\pi_s^m = (1-\alpha_{js})(a-c)^2/4$. Solving this problem, we can obtain the following proposition.

Proposition 10. *Suppose buyer i negotiates with the supplier before buyer j. In equilibrium, the procurement quantities, the wholesale prices and the profits of the supply chain parties follow:*
$q_i^S = \frac{(2-2b+\alpha_{js}b)(a-c)}{4-3b^2+\alpha_{js}b^2}$, $q_j^S = \frac{(4-2b-b^2)(a-c)}{2(4-3b^2+\alpha_{js}b^2)}$,
$w_i^S = c + \frac{[2(4-4b^2+b^3-2\alpha_{js}b(1-b))-\alpha_{is}(8-4(2-\alpha_{js})b-2(3-\alpha_{js})b^2+(2-\alpha_{js})(3-\alpha_{js})b^3)](a-c)}{4(4-3b^2+\alpha_{js}b^2)}$, $w_j^S = c + \frac{(1-\alpha_{js})(4-2b-b^2)(a-c)}{2(4-3b^2+\alpha_{js}b^2)}$,
$\pi_i^S = \alpha_{is}\frac{(2-2b+a_{js}b)^2(a-c)^2}{4(4-3b^2+a_{js}b^2)}$, $\pi_j^S = \alpha_{js}\left(\frac{(4-2b-b^2)(a-c)}{2(4-3b^2+\alpha_{js}b^2)}\right)^2$, *and*
$\pi_s^S = \frac{[8-8b+b^2-4\alpha_{js}(1-b)-\alpha_{is}(2-2b+\alpha_{js}b)^2](a-c)^2}{4(4-3b^2+\alpha_{js}b^2)}$.

From the above results, we can find that if the supplier can decide whom to negotiate with first, he will always choose to first negotiate with the buyer with greater bargaining power. This is because to negotiate with a weaker buyer later will give the supplier a larger reservation profit in the first negotiation, which benefits the supplier.

Performance Comparison with Equal Price LCB. It is analytically challenging to compare the buyers' profits under this new benchmark with those under the equal price LCB. Thus, we rely on a numerical analysis. The parameters are listed in Table 2, which gives us total 1923 feasible combinations. Figure 1 provides an illustration. In general, we find that the equal price LCB benefits the leader, while it may hurt the follower when the competition intensity b is in a middle range. This is similar to our findings in the main model.

Table 2: Summary of Parameters

Buyers' Bargaining Powers:	$\alpha_{is} \in [0.1, 0.9]$, step length=0.05 $\alpha_{js} = 0.5\alpha_{is}$
Competition Intensity:	$b \in [0, 1]$, step length=0.01

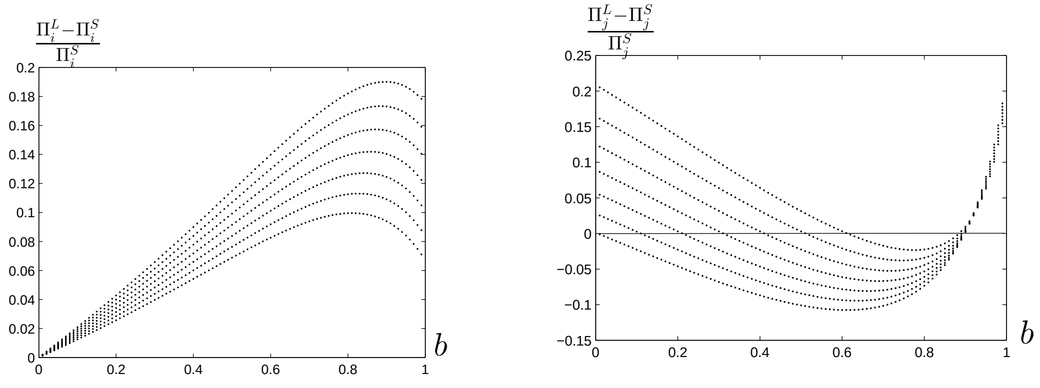

Figure 1: Leader and Follower's Preferences for Equal Price LCB and Separate Procurement

Performance Comparison with Fixed Price LCB. It is straightforward to show that all our analytical results in our main model remain to hold, given that the buyers' profits under separate procurement serve as their reservation profits when they form the fixed price LCB. It is still possible for the supplier to be better off due to the enlargement of the total supply chain surplus under the fixed price LCB.

B. The Existence of Spot Market

In this section, we assume that there is a spot market from which the buyers can procure the component if they fail to negotiate with the supplier. Suppose the spot price c_p satisfies $c < w_i < c_p$.

Separate Procurement. We again assume the buyers negotiate with the supplier in parallel simultaneously and use the Nash-Nash solution concept to solve the problem. Suppose buyer i's negotiation with the supplier fails while buyer j's negotiation with the supplier succeeds. Then, buyer i would purchase from the spot market at c_p for a quantity $q_i^p = (a - bq_j - c_p)/2$ and his profit would be $(a - bq_j - c_p)^2/4$. This serves as buyer i's reservation profit. As before, the supplier's reservation profit for the negotiation with buyer i is: $(w_j - c)q_j$. We can then solve the two parallel negotiations by backward induction. The results are presented in the following proposition.

Proposition 11. *If the buyers can purchase from the spot market, then under separate procurement, their reservation profits are* $\pi^p = \frac{(2a+bc-2c_p-bc_p)^2}{4(2+b)^2}$, *and their equilibrium procurement quantities, the wholesale prices and the profits of the supply chain parties follow:* $q_i^S = \frac{a-c}{2+b}$, $w_i^S = c + (1-\alpha_{is})\frac{(c_p-c)[4a-(2-b)c-(2+b)c_p]}{4(a-c)}$, $\pi_i^S = \pi^p + \alpha_{is}\left[\left(\frac{a-c}{2+b}\right)^2 - \pi^p\right]$, $i = 1, 2$, *and* $\pi_s^S = (2-\alpha_{1s}-\alpha_{2s})\left[\left(\frac{a-c}{2+b}\right)^2 - \pi^p\right]$, *respectively.*

The existence of the spot market increases the buyers' reservation profits and hence their equilibrium profits. The wholesale prices are lower than before, and the supplier obtains less profit.

The Equal Price LCB. In the presence of the spot market, both buyers would source from there if the leader's negotiation with the supplier fails. We can thus obtain the buyers' reservation profits: $\pi_p = \left(\frac{a-c_p}{2+b}\right)^2$. Then, the GNB between the leader i and the supplier can be formulated as:

$$\max_{w} \quad [(a - q_i - bq_j - w)q_i - \pi_p]^{\alpha_{is}}[(w - c)(q_i + q_j)]^{1-\alpha_{is}}. \tag{2}$$

Solving the game backward, we can obtain the following results.

Lemma 8. *Given* q_i *and* q_j*, the bargaining outcome of the wholesale price between the leader and the supplier follows:*

$$w(q_i, q_j) = \alpha_{is}c + (1 - \alpha_{is})(a - q_i - bq_j) - (1 - \alpha_{is})\pi_p/q_i,$$

under which the leader's profit is $\pi_i(q_i, q_j) = \alpha_{is}(a - q_i - bq_j - c)q_i + (1 - \alpha_{is})\pi_p$ *and the follower's profit is* $\pi_j(q_i, q_j) = [\alpha_{is}(a - c) - (1 - b + \alpha_{is}b)q_j + (1 - b - \alpha_{is})q_i + (1 - \alpha_{is})\pi_p/q_i]q_j$.

With the above lemma, we can derive the equilibrium results.

Proposition 12. *If the buyers can purchase from the spot market, then under the equal price LCB, the equilibrium quantities of the leader and the follower are:*

$$q_i^L = \frac{(2(1-b) + b\alpha_{is})(a-c) + \sqrt{(2(1-b) + b\alpha_{is})^2(a-c)^2 + 4b((4+b)(1-b) + 3b\alpha_{is})(1-\alpha_{is})\pi_p}}{2((4+b)(1-b) + 3b\alpha_{is})}$$

and $q_j^L = \frac{\alpha_{is}(a-c)+(1-b-\alpha_{is})q_i^L+\frac{(1-\alpha_{is})\pi_p}{q_i^L}}{2(1-b+\alpha_{is}b)}$*, respectively, and the wholesale price is:* $w^L = c + (1 - \alpha_{is})q_i^L - (1-\alpha_{is})\pi_p/q_i^L$*, under which the supply chain parties' profits are:* $\pi_i^L = \pi_p + \alpha_{is}((q_i^L)^2 - \pi_p)$, $\pi_j^L = (1 - b + \alpha_{is}b)(q_j^L)^2$*, and* $\pi_s^L = (1 - \alpha_{is})((q_i^L)^2 - \pi_p) + (1 - \alpha_{is})\left((q_j^L)^2 - \left(\frac{q_j^L}{q_i^L}\right)^2 \pi_p\right)\frac{q_i^L}{q_j^L}$.

From the above proposition, we can find that it is similar to the case in the main model without spot market that the follower's procurement quantity is smaller than that of the leader because of the competition effect, and we can also establish the following result.

Proposition 13. *The leader is always better off under the equal price LCB; i.e.,* $\pi_i^L \geq \pi_i^S$.

For the follower, it is challenging to analytically compare his profit under the equal price LCB with that under separate procurement. Our numerical analysis (the parameters are presented in

Table 3: Summary of Parameters

Market Potential:	$a = 1$
Buyers' Bargaining Powers:	$\alpha_{is} \in [0, 1]$, step length=0.01 $\alpha_{js} \in [0, 1]$, step length=0.01
Competition Intensity:	$b \in [0, 1]$, step length=0.01
Cost:	$c = 0$
Spot Price:	$c_p \in [0.3, 1]$, step length=0.1

Table 3 and the results are illustrated in Figure 2) again shows that there might exist two threshold values of b between which the follower's profit is reduced by the equal price LCB, which is consistent with our previous finding without spot market. We also observe that the benefit of the equal price LCB relative to separate procurement for the follower is greater when the spot price is higher. This is mostly because when the spot price is small, the buyers can already obtain large profits from separate procurement with their large reservation profits.

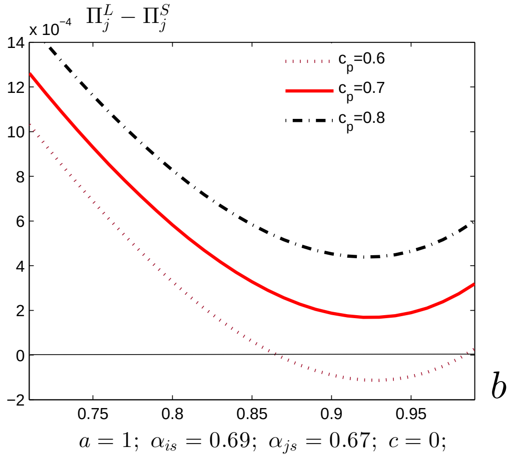

Figure 2: Benefit of equal price LCB for the follower.

The Fixed Price LCB. Similarly to the equal price LCB case, the leader i's reservation profit is π_p, and the GNB can be formulated as:

$$\max_{w} \qquad [(a - q_i - bq_j - w)q_i + (t - w)q_j - \pi_p]^{\alpha_{is}}[(w - c)(q_i + q_j)]^{1-\alpha_{is}}.$$

Lemma 9. *Given q_i and q_j, the bargaining outcome of the wholesale price between the leader and* *the supplier follows:*

$$w(q_i, q_j) = \alpha_{is} c + (1 - \alpha_{is}) \frac{(a - q_i - bq_j)q_i + tq_j - \pi_p}{q_i + q_j},$$

under which the leader's profit is $\pi_i(q_i, q_j) = \alpha_{is}[(a - q_i - bq_j - c)q_i + (t - c)q_j] + (1 - \alpha_{is})\pi_p$.

In the first stage, the buyers negotiate the transfer price t with their reservation profits π_i^S and π_j^S equal to the profits under separate procurement.

$$\max_t \quad [\pi_i(q_i, q_j) - \pi_i^S]^{\alpha_{ij}} [\pi_j(q_i, q_j) - \pi_j^S]^{1-\alpha_{ij}}. \tag{3}$$

It can be shown that t is unique and follows:

$$t(q_i, q_j) = \frac{\alpha_{ij}((a - q_j - bq_i)q_j - \pi_j^S) + (1 - \alpha_{ij})(\frac{\pi_i^S - (1-\alpha_{is})\pi_p}{\alpha_{is}} - ((a - q_i - bq_j)q_i - c(q_i + q_j)))}{q_j}. \tag{4}$$

We can find that this transfer price is lowered with the spot market than without. Substituting $t(q_i, q_j)$ into the buyers' profit functions we can derive the equilibrium.

Proposition 14. *Under the fixed price LCB with the existence of spot market, the buyers' equilibrium procurement quantities are:* $q_i^L = q_j^L = \frac{a-c}{2(1+b)}$*, and the profits of the supply chain parties follow:* $\pi_i^L = (1 - \alpha_{ij})\pi_i^S + \alpha_{ij}(1 - \alpha_{is})\pi_p + \alpha_{ij}\alpha_{is}(\frac{(a-c)^2}{2(1+b)} - \pi_j^S)$, $\pi_j^L = \alpha_{ij}\pi_j^S + \frac{(1-\alpha_{ij})(1-\alpha_{is})\pi_p}{\alpha_{is}} + (1 - \alpha_{ij})(\frac{(a-c)^2}{2(1+b)} - \frac{\pi_i^S}{\alpha_{is}})$*, and* $\pi_s^L = \frac{(a-c)^2}{2(1+b)} - \pi_i^L - \pi_j^L$.

We can see that the procurement quantities remain the same with or without the spot market, and also the buyers' profit functions follow the same form except for the addition of their reservation profits with the spot market compared to those without the spot market. Similar results about the benefits of the fixed price LCB can be obtained. Hence, the existence of the spot market does not change any of our insights qualitatively.

Proposition 15. $\pi_i^L \geq \pi_i^S$, $\pi_j^L \geq \pi_j^S$, $(\pi_i^L + \pi_j^L) \geq (\pi_i^S + \pi_j^S)$*, and* $(\pi_i^L + \pi_j^L + \pi_s^L) \geq (\pi_i^S + \pi_j^S + \pi_s^S)$*; i.e., the fixed price LCB benefits the buyer alliance, as well as the whole supply chain.*

C. Representative-based Collective Bargaining

As we discussed in the introduction, buyers may also form a representative-based alliance to delegate the negotiation with the supplier to an independent agent firm; i.e., the so-called representative-based collective bargaining (RCB). Notice that for RCB, the representative firm often charges a commission for the trade (see Ahn et al. 2011 for a recent A.T. Kearney report on procurement outsourcing). In Li & Fung's case (Li & Fung is a Hong-Kong-based buying agency to negotiate with material/component supplier for its customers), the buying commission is 5% of the F.O.B. Country of Origin Price.[9]

We use r to label the sourcing agent and γ to denote the commission rate. The buyers first decide their procurement quantities independently. Then, the agent will negotiate with the supplier for the wholesale price w. Thus, we can formulate buyer i's profit as $\pi_i = [a - q_i - bq_j - (1+\gamma)w]q_i$, the supplier's profit as $\pi_s = (w - c)(q_i + q_j)$, and the agent's profit as $\pi_r = \gamma w(q_i + q_j)$. Let the agent's bargaining power over the supplier be α, and he will negotiate with the supplier on behalf of the buyers to maximize the total RCB profits $(a - q_i - bq_j - w)q_i + (a - q_j - bq_i - w)q_j$. Hence, we can formulate the GNB problem as:

$$\max_{w} \qquad [(a - q_i - bq_j - w)q_i + (a - q_j - bq_i - w)q_j]^{\alpha}[(w - c)(q_i + q_j)]^{1-\alpha}.$$

We can derive the wholesale price $w(q_i, q_j) = \alpha c + (1-\alpha)[(a - q_i - bq_j)q_i + (a - q_j - bq_i)q_j]/(q_i + q_j)$. Then, buyer i's profit function can be rewritten as:

$$\pi_i(q_i, q_j) = \frac{q_i}{q_i + q_j}[\alpha(a - q_i - bq_j)q_i + (a - q_i - bq_j - (1-\alpha)(a - q_j - bq_i))q_j - (1+\gamma)\alpha c(q_i + q_j)].$$

However, it is challenging to solve the equilibrium procurement quantities analytically from the above equations. Thus, we resort to a numerical analysis. We set $a = 200, c = 1, \gamma = 0.05, \alpha/\alpha_{is} = 1, 1.1, 1.2$, and let b vary in $[0, 1]$ with 0.01 per step. We compare the buyers' profits under RCB with those under the equal price LCB. Figure 3 provides an illustration.

Clearly, if the agent's bargaining power is no greater than the buyers', RCB makes the buyers worse off compared to the equal price LCB because the agent charges a commission. Hence, RCB will be used only if the agent has a sufficiently larger bargaining power. Moreover, we observe that the comparison also depends on the market competition intensity b. In particular, RCB tends to benefit the buyers when the downstream competition is intense (i.e., b is large). This finding is in line with the anecdotal evidence that strong competitors tend to cooperate less in LCB.

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[9] see the buying agency agreement by Designs Apparel, Inc. and Li & Fung via http://www.sec.gov/Archives/edgar/data/813298/000119312506070549/dex1047.htm

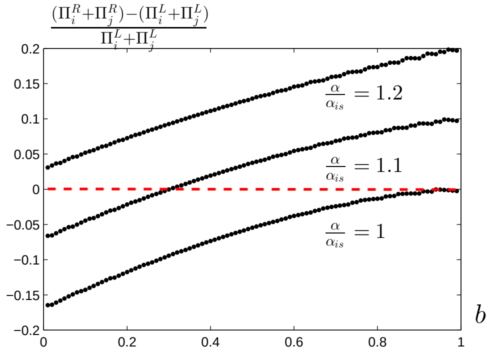

Figure 3: Comparison of the buyers' profits between RCB and equal price LCB.

D. Quantity Decision after Price Negotiation

In our study, we have assumed that the buyers make their quantity decisions before negotiating the prices with the supplier. This is in line with the business examples and anecdotal evidence we find (such as the Mai Wiru and Nuss LCB cases discussed in the introduction section). However, some prior studies have also assumed that buyers make their quantity decisions after the price negotiations (e.g., Feng and Lu 2013a,b). In the following, we provide an analysis for this alternative sequence of events in our context. We shall note that this sequence can induce double marginalization which may make the supply chain less efficient.

Separate Procurement. We use backward induction to first determine the procurement quantities: $q_i(w_i, w_j) = \frac{(2-b)a-2w_i+bw_j}{4-b^2}$ and $q_j(w_i, w_j) = \frac{(2-b)a-2w_j+bw_i}{4-b^2}$, if both negotiations succeed. To solve the wholesale prices in equilibrium, we need to specify the disagreement points in each negotiation. Clearly, without an outside option, the buyer's reservation profit is still zero. For the supplier's reservation profit, following Feng and Lu (2013a,b), it will be the profit he can obtain from the other negotiation, assuming it succeeds and the buyer becomes the monopoly buyer; that is, $(w_j - c)q_j^m$ where $q_j^m(w_j) = \frac{a-w_j}{2}$. Then, we can formulate the GNB problem over the wholesale prices as follows:

$$\max_{w_i} \quad \left[\left(\frac{(2-b)a - 2w_i + bw_j}{4-b^2}\right)^2\right]^{\alpha_{is}} \left(\frac{((2-b)a - 2w_i + bw_j)(2w_i - bw_j - (2-b)c)}{2(4-b^2)}\right)^{1-\alpha_{is}}.$$

Proposition 16. *Under separate procurement, if the quantity decisions are made after the price negotiations, then the equilibrium follows:* $q_i^S = \frac{(1+\alpha_{is})(a-c)}{2(2+b)}$, $w_i^S = c + \frac{[2(1-\alpha_{is})+b(1-\alpha_{js})](a-c)}{2(2+b)}$, $\pi_i^S = \left(\frac{(1+\alpha_{is})(a-c)}{2(2+b)}\right)^2$, $i = 1, 2$, *and* $\pi_s^S = \frac{[2-\alpha_{is}^2-\alpha_{js}^2+b(1-\alpha_{is}\alpha_{js})](a-c)^2}{2(2+b)^2}$, *respectively.*

The Equal Price LCB. We again solve the problem by backward induction. Since the wholesale price is the same for the two buyers, it's easy to show that $q_i(w) = q_j(w) = \frac{a-w}{2+b}$. Thus, we can obtain $\pi_i(w) = \left(\frac{a-w}{2+b}\right)^2$ and $\pi_s(w) = \frac{2(w-c)(a-w)}{2+b}$ for the leader and the supplier if the negotiation succeeds; if the negotiation fails, they obtain zero profit. Hence, we can formulate the GNB problem as:

$$\max_{w} \quad \left[\left(\frac{a-w}{2+b}\right)^2\right]^{\alpha_{is}} \left(\frac{2(w-c)(a-w)}{2+b}\right)^{1-\alpha_{is}}.$$

Proposition 17. *Under the equal price LCB, if the quantity decisions are made after the price negotiation, then the equilibrium follows:* $q_i^L = q_j^L = \frac{(1+\alpha_{is})(a-c)}{2(2+b)}$, $w^L = c + \frac{(1-\alpha_{is})(a-c)}{2}$, $\pi_i^L = \pi_j^L = \left(\frac{(1+\alpha_{is})(a-c)}{2(2+b)}\right)^2$, *and* $\pi_s^L = \frac{(1-\alpha_{is}^2)(a-c)^2}{2(2+b)}$.

Performance Comparison with Separate Procurement. We compare the supply chain parties' profits under these two cases.

Proposition 18. *Suppose* $\alpha_{is} > \alpha_{js}$. *The leader (i.e., buyer i) is indifferent between separate procurement and the equal price LCB, the follower is always better off, while the supplier is always worse off by the equal price LCB.*

Under the equal price LCB, the follower receives the same wholesale price as does the leader. Hence, compared to separate procurement, the follower's wholesale price gets lower and his procurement quantity increases, which increases his profit. For the leader, on the one hand, the equal price LCB leads to a larger procurement quantity of the follower which hurts the leader; on the other hand, the equal price LCB helps lower the wholesale price. Interestingly, the leader's equilibrium profit remains the same in these two cases and thus he is indifferent. These results imply that under this alternative sequence of events, the equal price LCB will always be feasible since neither buyer is worse off. In contrast, the supplier will always be worse off by the equal price LCB, because he negotiates a single wholesale price with a stronger buyer, which determines the buyers' subsequent procurement quantities.

The Fixed Price LCB. For the fixed price LCB, we adopt the following sequence of events. First, the two buyers negotiate a transfer price t and form the alliance. Second, assume buyer i is the leader who negotiates with the supplier for the wholesale price w. Third, given t and w, the two buyers decide their procurement quantities simultaneously. With this sequence, we can derive the two buyers' profit functions as:

$$\pi_i(q_i, q_j) = (a - q_i - bq_j - w)q_i + (t - w)q_j \quad \text{and} \quad \pi_j(q_i, q_j) = (a - q_j - bq_i - t)q_j,$$

and the supplier's profit function as:

$$\pi_s(q_i, q_j) = (w - c)(q_i + q_j).$$

We solve the problem by backward induction. First, we determine the buyers' procurement quantities: $q_i(t, w) = \frac{(2-b)a-2w+bt}{4-b^2}$ and $q_j(t, w) = \frac{(2-b)a-2t+bw}{4-b^2}$. Then, the leader i's profit function can be rewritten as $\pi_i(t, w) = \left(\frac{(2-b)a-2w+bt}{4-b^2}\right)^2 + \frac{(t-w)((2-b)a-2t+bw)}{4-b^2}$, and the supplier's profit function follows $\pi_s(t, w) = \frac{(w-c)(a-w-t)}{2+b}$, if their negotiation succeeds; otherwise, they obtain zero reservation profit. Hence, we can formulate the GNB problem as:

$$\max_{w} \qquad [\pi_i(t, w)]^{\alpha_{is}} [\pi_s(t, w)]^{1-\alpha_{is}}.$$

From the above negotiation, w can be determined as a function of t. Then, in the first stage, the two buyers negotiate to determine the transfer price t by solving the GNB problem:

$$\max_{t} \qquad [\pi_i(t) - \pi_i^S]^{\alpha_{ij}} [\pi_j(t) - \pi_j^S]^{1-\alpha_{ij}},$$

where the reservation profits π_i^S and π_j^S are the profits they can obtain from separate procurement. Clearly, given this feature, the two buyers will always be better off under the fixed price LCB. To assess the supplier's profit, we would need to solve the whole problem, which is technically challenging. From our numerical analysis (with the parameters presented in Table 1), we observe that the supplier now is always worse off under the fixed price LCB (recall from Proposition 8 that in our main model, the supplier might also benefit from the fixed price LCB).

E. Proofs for Extension Results

Proof of Proposition 10. We solve the game backwards. Step 1, the supplier negotiates with buyer j and obtains $w_j(q_i, q_j, w_i) = \alpha_{js}c + (1 - \alpha_{is})(a - q_j - bq_i)$. Step 2, buyer j determines his production quantity $q_j(q_i, w_i) = \frac{a-bq_i-c}{2}$. Step 3, the supplier negotiates with buyer i to determine w_i. It can be shown that $\pi_i - Di = \left(\frac{(2-b)a-(2-b^2)q_i+bc-2w_i}{2}\right) q_i$, and $\pi_s - d_i = \frac{4(w_i-c)q_i+(1-\alpha_{js})(a-bq_i-c)^2-4\pi_s^m}{4}$. Solving the negotiation problem between the supplier and buyer i, we have
$w_i(q_i) = \frac{4\alpha_{is}cq_i+2(1-\alpha_{is})[(2-b)aq_i-(2-b^2)q_i^2+bcq_i]-\alpha_{is}(1-\alpha_{js})(a-bq_i-c)^2+4\alpha_{is}\pi_s^m}{4q_i}$. Therefore, buyer i's profit can be written as $\pi_i(q_i) = \frac{2\alpha_{is}[(2-b)aq_i-(2-b^2)q_i^2+bcq_i]-4\alpha_{is}cq_i+\alpha_{is}(1-\alpha_{js})(a-bq_i-c)^2-4\alpha_{is}\pi_s^m}{4}$, which results in $q_i^S = \frac{(2-2b+\alpha_{js}b)(a-c)}{4-3b^2+\alpha_{js}b^2}$ and then the results of Proposition 10.

We now show that the supplier will prefer to negotiate with the buyer with greater bargaining power first. If the supplier negotiates with buyer i first, his equilibrium profit is

$$\pi_s^S = \frac{[8-8b+b^2-4\alpha_{js}(1-b)-\alpha_{is}(2-2b+\alpha_{js}b)^2](a-c)^2}{4(4-3b^2+\alpha_{js}b^2)}.$$

If the supplier negotiates with buyer j first, his equilibrium profit is

$$\pi_s^{S'} = \frac{[8-8b+b^2-4\alpha_{is}(1-b)-\alpha_{js}(2-2b+\alpha_{is}b)^2](a-c)^2}{4(4-3b^2+\alpha_{is}b^2)}.$$

Comparing the supplier's profits in these two cases, we find that the difference

$$\pi_s^S - \pi_s^{S'} = \frac{(a-c)^2(\alpha_{is}-\alpha_{js})bD(b)}{4(4-3b^2+\alpha_{is}b^2)(4-3b^2+\alpha_{js}b^2)},$$

where $D(b) = 16-8b-4\alpha_{is}b-4\alpha_{js}b+4\alpha_{is}\alpha_{js}b-20b^2+8\alpha_{is}b^2+8\alpha_{js}b^2-4\alpha_{is}\alpha_{js}b^2+13b^3-4\alpha_{is}b^3-4\alpha_{js}b^3+\alpha_{is}\alpha_{js}b^3$.

We further find that

$$\begin{aligned}
D(b)' &= -8-4\alpha_{is}-4\alpha_{js}+4\alpha_{is}\alpha_{js}-40b+16\alpha_{is}b+16\alpha_{js}b-8\alpha_{is}\alpha_{js}b+39b^2-12\alpha_{is}b^2-12\alpha_{js}b^2+3\alpha_{is}\alpha_{js}b^2;\\
D(b)'' &= -40+16\alpha_{is}+16\alpha_{js}-8\alpha_{is}\alpha_{js}+78b-24\alpha_{is}b-24\alpha_{js}b+6\alpha_{is}\alpha_{js}b;\\
D(b)''' &= 78-24\alpha_{is}-24\alpha_{js}+6\alpha_{is}\alpha_{js}\\
&> 0.
\end{aligned}$$

Therefore, $D(b)''$ is increasing in b with the minimizer $D''(b=0) = -40+16\alpha_{is}+16\alpha_{js}-8\alpha_{is}\alpha_{js} < 0$ and the maximizer $D''(b=1) = 38-8\alpha_{is}b-8\alpha_{js}b-2\alpha_{is}\alpha_{js} > 0$. This indicates that $D(b)'$ is convex in b its the domain. Then the maximizer of $D(b)'$ is either $D'(b=0)$ or $D'(b=1)$. Note that $D'(b=0) = -8-4\alpha_{is}-4\alpha_{js}+4\alpha_{is}\alpha_{js} < 0$ and $D'(b=1) = -9-\alpha_{is}\alpha_{js} < 0$, we can show that $D(b)$ is decreasing in b. Therefore, we have $D(b) \geq D(b=1) = 1+\alpha_{is}\alpha_{js} > 0$. Given this finding, we show that $\pi_s^S \geq \pi_s^{S'}$ iff $\alpha_{is} \geq \alpha_{js}$.

Proof of Proposition 11. For separate procurement, when the negotiation with supplier fails, a buyer can purchase components from the spot market at unit price c_p. So the Nash-Nash framework for the buyers' reservation profit is: the failed buyer buys from the spot market, assuming the other negotiation succeeds and that buyer buys from the supplier for the predetermined quantity, then the two buyers compete in the downstream market.

If buyer i fails to negotiate with the supplier, then the buyers' profit functions are

$$\pi_i = (a - q_i - bq_j - c_p)q_i; \quad \pi_j = (a - q_j - bq_i - w_j)q_j.$$

Then, given q_j, we have $q_i^p = \frac{a-bq_j-c_p}{2}$, and $\pi_i^p = \left(\frac{a-bq_j-c_p}{2}\right)^2$. Similarly, we $q_j^p = \frac{a-bq_i-c_p}{2}$, and $\pi_j^p = \left(\frac{a-bq_i-c_p}{2}\right)^2$. For the negotiation between buyer j and the supplier, they need to solve the following problem

$$\max_{w_j} \qquad [\pi_j(q_i, q_j) - \pi_j^p]^{\alpha_{js}}[\pi_s(q_i, q_j) - (w_i - c)q_i]^{1-\alpha_{js}},$$

which yields $w_j = \frac{\alpha_{js}cq_j+(1-\alpha_{js})(a-q_j-bq_i)q_j-(1-\alpha_{js}\pi_j^p)}{q_j}$. Substituting w_j into buyer j's profit function we have $\pi_j = \alpha_{js}(a - q_j - bq_i - c)q_j + (1 - \alpha_{js})\pi_j^p$. Similarly, we can derive $\pi_i = \alpha_{is}(a - q_i - bq_j - c)q_i + (1 - \alpha_{is})\pi_i^p$. We finally solve the buyers' problems with respect to q_i and q_j and find that $q_i^S = q_j^S = \frac{a-c}{2+b}$. Then $\pi_i^p = \pi_j^p = \left(\frac{(2a+bc-2c_p-bc_p)}{2(2+b)}\right)^2 = \pi^p$. The results summarized in Proposition 11 become straightforward. Note that we need guarantee $q_i^p \geq 0$, which requires $q_i^p = \frac{a-bq_j-c_p}{2} = \frac{(2a+bc-2c_p-bc_p)}{2(2+b)} \geq 0$.

Proof of Lemma 8. The proof is similar to that for Lemma 2.

Proof of Proposition 12. In the buyers' quantity determination problems, the best response functions are: $q_i(q_j) = \frac{a-bq_j-c}{2}, q_j(q_i) = \frac{\alpha_{is}(a-c)+(1-b-\alpha_{is})q_i+\frac{(1-\alpha_{is})\pi_p}{q_i}}{2(1-b+\alpha_{is}b)}$. Solving them simultaneously we find that $q_i = \frac{(2(1-b)+b\alpha_{is})(a-c)\pm\sqrt{(2(1-b)+b\alpha_{is})^2(a-c)^2+4b((4+b)(1-b)+3b\alpha_{is})(1-\alpha_{is})\pi_p}}{2((4+b)(1-b)+3b\alpha_{is})}$. Note that $\pi_i = \alpha_{is}q_i^2 + (1 - \alpha_{is})\pi_p$, which is increasing in q_i, we have the optimal production quantity $q_i^L = \frac{(2(1-b)+b\alpha_{is})(a-c)+\sqrt{(2(1-b)+b\alpha_{is})^2(a-c)^2+4b((4+b)(1-b)+3b\alpha_{is})(1-\alpha_{is})\pi_p}}{2((4+b)(1-b)+3b\alpha_{is})}$. Then the follower's production quantity is $q_j^L = \frac{\alpha_{is}(a-c)+(1-b-\alpha_{is})q_i^L+\frac{(1-\alpha_{is})\pi_p}{q_i^L}}{2(1-b+\alpha_{is}b)}$, and the follower's profit can be written as $\pi_j^L = (1 - b + \alpha_{is}b)q_j^L$.

Clearly, $q_i^L > \frac{(2(1-b)+b\alpha_{is})(a-c)}{(4+b)(1-b)+3b\alpha_{is}}$, and we have shown that $\frac{(2(1-b)+b\alpha_{is})(a-c)}{(4+b)(1-b)+3b\alpha_{is}} > q_i^S$ in Proposition 2. This results in $q_i^L > q_i^S$.

Regarding q_j^L, we find that $q_j^L = \frac{\alpha_{is}(a-c)q_i^L+(1-b-\alpha_{is})(q_i^L)^2+(1-\alpha_{is})\pi_p}{2(1-b+\alpha_{is}b)q_i^L}$. Therefore,

$$
\begin{aligned}
q_j^L - q_i^L \quad &= \frac{-(1-b+\alpha_{is}(1+2b))(q_i^L)^2+\alpha_{is}(a-c)q_i^L+(1-\alpha_{is})\pi_p}{2(1-b+\alpha_{is}b)q_i^L} \\
&= \frac{(1-\alpha_{is})(1-b)[(-1+(1-\alpha_{is})b)(a-c)q_i^L+(2+\alpha_{is}b)\pi_p]}{(1-b+\alpha_{is}b)((4+b)(1-b)+3b\alpha_{is})q_i^L} \\
&= \frac{(1-\alpha_{is})(1-b)}{(4+b)(1-b)+3b\alpha_{is}}\left[-(a-c)+\frac{(2+\alpha_{is}b)\pi_p}{(1-b+\alpha_{is}b)q_i^L}\right] \\
&< \frac{(1-\alpha_{is})(1-b)}{(4+b)(1-b)+3b\alpha_{is}}\left[-(a-c)+\frac{(2+\alpha_{is}b)(a-c)^2}{(1-b+\alpha_{is}b)(2+b)^2 q_i^L}\right] \\
&< \frac{(1-\alpha_{is})(1-b)}{(4+b)(1-b)+3b\alpha_{is}}\left[-(a-c)+\frac{(2+\alpha_{is}b)((4+b)(1-b)+3b\alpha_{is})(a-c)}{(1-b+\alpha_{is}b)(2(1-b)+b\alpha_{is})(2+b)^2}\right] \\
&< 0.
\end{aligned}
$$

The first inequality is due to $\pi_p < \frac{(a-c)^2}{(2+b)^2}$. The second inequality is due to $q_i^L > \frac{(2(1-b)+b\alpha_{is})(a-c)}{(4+b)(1-b)+3b\alpha_{is}}$. The last inequality is due to

$$
\begin{aligned}
&(2+\alpha_{is}b)((4+b)(1-b)+3b\alpha_{is})-(1-b+\alpha_{is}b)(2(1-b)+b\alpha_{is})(2+b)^2 \\
&= -(1-\alpha_{is})(2+4b+\alpha_{is}b-4b^2+4\alpha_{is}b^2-2b^3+\alpha_{is}b^3)b \\
&< 0.
\end{aligned}
$$

Proof of Proposition 13. We first show that a buyer's reservation profit under separate procurement (i.e., π^p) is less than that under equal price LCB (i.e., π_p).

$$
\begin{aligned}
\pi_p - \pi^p \quad &= \left(\frac{a-c_p}{2+b}\right)^2 - \left(\frac{(2a+bc-2c_p-bc_p)}{2(2+b)}\right)^2 \\
&= \frac{b(c_p-c)(4a+bc-4c_p-bc_p)}{4(2+b)^2} \\
&> 0.
\end{aligned}
$$

Clearly, $c_p > c$. Thus, the last inequality holds due to $4a+bc-4c_p-bc_p > 2(2a+bc-2c_p-bc_p) = 4(2+b)q_i^p \geq 0$. Then, note that $q_i^L > q_i^S$, and hence the result, $\pi_i^L > \pi_i^S$, becomes immediate.

Proof of Lemma 9. The proof is similar to that for Lemma 2.

Proof of Proposition 14. Substituting (4) into buyer i's profit function yields $\pi_i(q_i,q_j) = \alpha_{is}\alpha_{ij}[(a-q_i-bq_j-c)q_i+(a-q_j-bq_i-c)q_j-\pi_j^S]+(1-\alpha_{ij})\pi_i^S+\alpha_{ij}(1-\alpha_{is})\pi_p$, and $\pi_j(q_i,q_j) = \alpha_{ij}\pi_j^S+\frac{(1-\alpha_{ij})(1-\alpha_{is})\pi_p}{\alpha_{is}}+(1-\alpha_{ij})[(a-q_i-bq_j-c)q_i+(a-q_j-bq_i-c)q_j-\frac{\pi_i^S}{\alpha_{is}}]$. We solve buyers' problems simultaneously and find that $q_i^L = q_j^L = \frac{a-c}{2(1+b)}$. Then Proposition 14 is immediate.

Proof of Proposition 15. The proof is similar to that for Proposition 7.

Proof of Proposition 16. We solve the game by backward induction. The first step is the determination of q_i and q_j: $q_i(w_i, w_j) = \frac{(2-b)a-2w_i+bw_j}{4-b^2}$, and $q_j(w_i, w_j) = \frac{(2-b)a-2w_j+bw_i}{4-b^2}$. The second step is the price negotiation. Take buyer i as the example. In the negotiation, buyer i's reservation profit is 0, while the supplier's reservation profit is his profit from the negotiation with buyer j as the monopoly buyer. That is, $d_i = (w_j - c)q_j^m$, where $q_j^m(w_j) = \frac{a-w_j}{2}$, because the buyers can determine the order quantities after they observe the negotiation outcomes. Thus, we have $\pi_i(w_i, w_j) = (a - q_i - bq_j - w_i)q_i = \left(\frac{(2-b)a-2w_i+bw_j}{4-b^2}\right)^2$, and $\pi_s(w_i, w_j) - d_i = (w_i - c)q_i(w_i, w_j) + (w_j - c)q_j(w_i, w_j) - (w_j - c)q_j^m(w_j) = \frac{((2-b)a-2w_i+bw_j)(2w_i-bw_j-(2-b)c)}{2(4-b^2)}$. They solve the following maximization problem.

$$
\begin{aligned}
&\max_{w_i} \quad \left(\frac{(2-b)a-2w_i+bw_j}{4-b^2}\right)^{2\alpha_{is}} \left(\frac{((2-b)a-2w_i+bw_j)(2w_i-bw_j-(2-b)c)}{2(4-b^2)}\right)^{1-\alpha_{is}} \\
=&\max_{w_i} \quad \left(\frac{(2-b)a-2w_i+bw_j}{4-b^2}\right)^{1+\alpha_{is}} \left(\frac{2w_i-bw_j-(2-b)c}{2}\right)^{1-\alpha_{is}}.
\end{aligned}
$$

We have $w_i(w_j) = \frac{(2-b)((1-\alpha_{is})a+(1+\alpha_{is})c)+2bw_j}{4}$. Symmetrically, we have $w_j(w_i) = \frac{(2-b)((1-\alpha_{js})a+(1+\alpha_{js})c)+2bw_i}{4}$. These two best response functions yield: $w_i^S = \frac{[2(1-\alpha_{is})+b(1-\alpha_{js})]a+[2(1+\alpha_{is})+b(1+\alpha_{js})]c}{2(2+b)}, i, j \in \{1, 2\}, i \neq j$. The other results can be obtained from w_i^S.

Proof of Proposition 17. This proposition follows from the arguments preceding the proposition.

Proof of Proposition 18. It can be shown that $\pi_i^L = \pi_i^S$, and $\pi_j^L \geq \pi_j^S$ iff $\alpha_{is} \geq \alpha_{js}$. Regarding the supplier's performance, we have $\pi_s^S - \pi_s^L = \frac{(\alpha_{is}-\alpha_{js})(\alpha_{is}+\alpha_{js}+\alpha_{is}b)}{2(2+b)^2}$.