

Supplementary Appendix I: Proofs

In this appendix, we present all the proofs of the propositions in the paper.

Proof of Proposition 1: Let's look for participatory RE equilibrium, where $\phi = 1$. All we need to do is to check the four conditions specified in Definition 1.

First, we look at the retailer's problem: Note the profit function can be expressed as $\pi = (p - w + r\frac{1-\theta}{\theta})E \min(\hat{\phi}\theta D, q) - cq + (w\theta - r(1-\theta))ED$, which is a typical newsvendor problem (plus a constant $(w\theta + r(1-\theta))ED$), and therefore the optimal order quantity q° is given by $\bar{F}\left(\frac{q^\circ}{\hat{\phi}\theta}\right) = \frac{c}{p-w+r\frac{1-\theta}{\theta}}$.

Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\phi} = \phi = 1$. Thus, $q^\circ = \theta\bar{F}^{-1}\left(\frac{c}{p-w+r\frac{1-\theta}{\theta}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q^\circ) = E \min(\phi\theta D, q^\circ)/E(\phi\theta D) = E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r\frac{1-\theta}{\theta}}))/ED$.

Finally, we need to check, given $\hat{\xi} = E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r\frac{1-\theta}{\theta}}))/ED$, consumers are indeed willing to visit store, i.e., $u_s > u_o$, which gives us the condition

$$h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r\frac{1-\theta}{\theta}}))}{ED} [h_o + (1-\theta)h_r]. \quad (\text{SA.1})$$

Note the right hand side of condition (SA.1) is decreasing in θ and is also boundless for $\theta \in \mathbb{R}$.

Therefore, there exists a unique $\theta^\circ \in \mathbb{R}$ such that $h_s = \frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r\frac{1-\theta^\circ}{\theta^\circ}}))}{ED} [h_o + (1-\theta^\circ)h_r]$ and condition (SA.1) is equivalent to $\theta < \theta^\circ$. Since $\theta \in (0, 1)$ in our model, we truncate the cutoff point and set $\psi^\circ = \min(\max(\theta^\circ, 0), 1)$.

Based on the analysis above, we find the condition for a participatory equilibrium is $\theta < \psi^\circ$. In such case, the equilibrium store inventory level is $q^\circ = \theta\bar{F}^{-1}\left(\frac{c}{p-w+r\frac{1-\theta}{\theta}}\right)$, which is strictly positive, since condition (SA.1) will not hold otherwise. \square

Proof of Proposition 2: Let's look for participatory RE equilibrium, where $\phi = 1$.

First, we look at the retailer's problem: Note the profit function can be expressed as $\pi = (p - w)E \min(\hat{\phi}\theta D, q) - cq + (w\theta - r(1-\hat{\phi})(1-\theta))ED$, which is a typical newsvendor problem (plus a constant $(w\theta - r(1-\hat{\phi})(1-\theta))ED$), and therefore the optimal order quantity q^p is given by $\bar{F}\left(\frac{q^p}{\hat{\phi}\theta}\right) = \frac{c}{p-w}$.

Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\phi} = \phi = 1$. Thus, $q^p = \theta\bar{F}^{-1}\left(\frac{c}{p-w}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = A(q^p) = E \min(\phi\theta D, q^p)/E(\phi\theta D) = E \min(D, \bar{F}^{-1}(\frac{c}{p-w}))/ED$.

Finally, we need to check given $\hat{\xi} = E \min(D, \bar{F}^{-1}(\frac{c}{p-w}))/ED$, consumers are indeed willing to

visit store, i.e., $u_s > u_o$, which gives us the condition

$$h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w}))}{ED} \theta h_o + (1 - \theta)(h_o + h_r). \quad (\text{SA.2})$$

Note the left hand side of condition (SA.2) is decreasing in θ and is also boundless for $\theta \in \mathbb{R}$. Therefore, there exists a unique $\theta^p \in \mathbb{R}$ such that $h_s = \frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w}))}{ED} \theta^p h_o + (1 - \theta^p)(h_o + h_r)$ and condition (SA.2) is equivalent to $\theta < \theta^p$. Since $\theta \in (0, 1)$ in our model, we truncate the cutoff point and set $\psi^p = \min(\max(\theta^p, 0), 1)$ and thus we get the equilibrium result.

Note $\frac{\partial \psi^o}{\partial r} \geq 0$ and $\frac{\partial \psi^p}{\partial r} = 0$. Also, when $r = 0$, condition (SA.1) implies condition (SA.2), and thus $\theta^p \geq \theta^o$ or $\psi^p \geq \psi^o$. Therefore, $\exists \bar{r} \geq 0$ such that $\psi^p > \psi^o$ if and only if $r < \bar{r}$. \square

Proof of Proposition 3: If $\theta < \psi^p$, then we have

$$\begin{aligned} \pi^o &= pE \min(\theta D, q^o) - cq^o + wE(\theta D - q^o)^+ - rE \frac{1 - \theta}{\theta} (\theta D - q^o)^+ \\ &< pE \min(\theta D, q^o) - cq^o + wE(\theta D - q^o)^+ \\ &< pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ = \pi^p \end{aligned}$$

where the second inequality is because q^p rather than q^o is the optimal solution that maximizes the newsvendor profit function π^p .

If $\psi^p < \psi^o$ and $\theta \in [\psi^p, \psi^o]$, then we have

$$\begin{aligned} \pi^o &= pE \min(\theta D, q^o) - cq^o + wE(\theta D - q^o)^+ - rE \frac{1 - \theta}{\theta} (\theta D - q^o)^+ \\ &= (p - w - r \frac{1 - \theta}{\theta}) E \min(\theta D, q^o) - cq^o + w\theta ED - rE(1 - \theta)D \\ &> wE\theta D - rE(1 - \theta)D = \pi^p \end{aligned}$$

If $\theta \geq \max(\psi^p, \psi^o)$, then $\pi^o = wE\theta D - rE(1 - \theta)D = \pi^p$. \square

Proof of Proposition 4: Since this is just the base model with a new set of parameter θ' and D' , similar to the proof of Proposition 1, we can show that the participatory equilibrium exists if and only if

$$h_s < \frac{E \min(D', \bar{F}'^{-1}(\frac{c}{p-w+r \frac{1-\theta'}{\theta'}}))}{ED'} [h_o + (1 - \theta')h_r]. \quad (\text{SA.3})$$

and in the participatory equilibrium, we have $\phi^v = 1$ and $q^v = \theta' \bar{F}'(\frac{c}{p-w+r \frac{1-\theta'}{\theta'}}) > 0$. Since $D' = [1 -$

$\alpha(1 - \theta)]D$ and $F'(x) = F(\frac{x}{1 - \alpha(1 - \theta)})$, the right hand side of condition (SA.3) equals to $\frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r \frac{1-\theta'}{\theta'}}))}{ED} [h_o + (1 - \theta')h_r]$, which is decreasing in θ and is also boundless for $\theta \in \mathbb{R}$. Therefore, there exists a unique $\theta^v \in \mathbb{R}$ such that $h_s = \frac{E \min(D, \bar{F}^{-1}(\frac{c}{p-w+r \frac{1-\theta'}{\theta'}}))}{ED} [h_o + (1 - \theta'(\theta^v))h_r]$ and condition (SA.3) is equivalent to $\theta < \theta^v$, where $\theta'(\theta^v) = \frac{\theta^v}{1 - \alpha(1 - \theta^v)}$, i.e., the posterior fraction of high-type customers given prior θ^v . Since $\theta \in (0, 1)$ in our model, we truncate the cutoff point and set $\psi^v = \min(\max(\theta^v, 0), 1)$ and thus we get the equilibrium result.

Since $\theta' > \theta$, it is easy to show that condition (SA.3) implies condition (SA.1), and thus $\theta^\circ \geq \theta^v$. Therefore, their truncated counterparts $\psi^\circ \geq \psi^v$. \square

Proof of Proposition 5: If $\theta < \psi^v$, then we have

$$\begin{aligned}\pi^\circ &= pE \min(\theta D, q^\circ) - cq^\circ + wE(\theta D - q^\circ)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^\circ)^+ \\ &< pE \min(\theta D, q^\circ) - cq^\circ + wE(\theta D - q^\circ)^+ - rE \frac{1-\theta'}{\theta'} (\theta D - q^\circ)^+ \\ &= pE \min(\theta' D', q^\circ) - cq^\circ + wE(\theta' D' - q^\circ)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^\circ)^+ \\ &< pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^v)^+ = \pi^v\end{aligned}$$

where the first inequality is because of $\theta' > \theta$, the second inequality is because q^v rather than q° is the optimal solution that maximizes the newsvendor profit function π^v .

If $\theta \geq \psi^\circ$, then we have $\pi^\circ = wE\theta D - rE(1-\theta)D < wE\theta D - rE(1-\alpha)(1-\theta)D = wE\theta' D' - r(1-\theta')D' = \pi^v$.

If $\theta \in [\psi^v, \psi^\circ)$, then $\pi^\circ = (p-w+r\frac{1-\theta}{\theta})E \min(\theta D, q^\circ) - cq^\circ + (w\theta - r(1-\theta))ED$, while $\pi^v = wE\theta' D' - rE(1-\theta')D' = wE\theta D - rE(1-\alpha)(1-\theta)D$. Then, by Envelop Theorem, $\frac{\partial(\pi^\circ - \pi^v)}{\partial w} = \frac{\partial \pi^\circ}{\partial w} - \frac{\partial \pi^v}{\partial w} = -E \min(\theta D, q^\circ) < 0$. To conclude the result, we just need to note that it is indeed possible to have $\pi^\circ > \pi^v$, e.g., when α is very close to 0. \square

Proof of Proposition 6: Comparing $u_{s,in}$ and u_o , we have

- $u_{s,in} > u_o$ if the following condition holds

$$h_s < h_o + (1-\theta)h_r \tag{SA.4}$$

Then, consumers visit store if store is in stock. Thus, retailer profit function is $\pi = pE \min(\theta D, q) - cq + wE(\theta D - q)^+ - rE \frac{1-\theta}{\theta} (\theta D - q)^+ = (p-w+r\frac{1-\theta}{\theta})E \min(\theta D, q) - cq + (w\theta - r(1-\theta))ED$, which is a typical newsvendor problem (plus a constant $(w\theta + r(1-\theta))ED$), and therefore the optimal order quantity $q^a = \theta \bar{F}^{-1} \left(\frac{c}{p-w+r\frac{1-\theta}{\theta}} \right)$. Note condition (SA.4) is equivalent to $\theta < \tilde{\psi}^a$, where $\tilde{\psi}^a = \min(\max(\frac{h_o+h_r-h_s}{h_r}, 0), 1)$, given $\theta \in (0, 1)$. Moreover, note if the critical fractile $\frac{c}{p-w+r\frac{1-\theta}{\theta}} \geq 1$ ($\Leftrightarrow \theta \geq \frac{r}{(w+r-p+c)^+}$, given $\theta \in (0, 1)$), then $q^a = 0$, i.e., store is never in stock and thus consumers actually always buy online as a result.

- If condition (SA.4) does not hold, i.e., $\theta \geq \tilde{\psi}^a$, then $u_{s,in} \leq u_o$ and consumers buy online directly. Then, it is easy to see that the optimal store inventory level is $q^a = 0$.

By setting $\psi^a = \min(\frac{r}{(w+r-p+c)^+}, \tilde{\psi}^a)$, we get the market outcome.

Finally, to prove $\psi^a \geq \psi^\circ$, we only need to note the following two facts: First, since condition (SA.1) implies condition (SA.4), we have $\tilde{\psi}^a \geq \psi^\circ$; second, since $\frac{c}{p-w+r\frac{1-\theta}{\theta}} \geq 1$ (or $\theta \geq \frac{r}{(w+r-p+c)^+}$) implies that $q^\circ = 0$, we have $\frac{r}{(w+r-p+c)^+} \geq \psi^\circ$. \square

Proof of Proposition 7: If $\theta < \psi^\circ$, then we have

$$\begin{aligned}\pi^\circ &= pE \min(\theta D, q^\circ) - cq^\circ + wE(\theta D - q^\circ)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^\circ)^+ \\ &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

If $\theta \geq \psi^a$, then we have $\pi^\circ = wE\theta D - rE(1-\theta)D = \pi^a$.

If $\theta \in [\psi^\circ, \psi^a)$, then we have

$$\begin{aligned}\pi^\circ &= wE\theta D - rE(1-\theta)D \\ &< (p-w+r\frac{1-\theta}{\theta})E \min(\theta D, q^a) - cq^a + wE\theta D - rE(1-\theta)D \\ &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

□

Proof of Proposition 8: First, note condition (10) in the paper implies condition (SA.2), thus $\underline{\psi}^{pa} \leq \psi^p$. Moreover, it is easy to check that $\overline{\psi}^{pa} = \psi^a$. Since $\underline{\psi}^{pa} \leq \overline{\psi}^{pa}$, we have $\underline{\psi}^{pa} \leq \psi^a$.

1. If $\theta < \underline{\psi}^{pa}$, then $\pi^p = pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ = pE \min(\theta D, q^{pa}) - cq^{pa} + wE(\theta D - q^{pa})^+ = \pi^{pa}$ and

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

where the first inequality is because q^p is the unique maximal of π^p . Thus, we have $\pi^p = \pi^{pa} > \pi^a$.

2. If $\theta \in [\underline{\psi}^{pa}, \min(\psi^p, \psi^a))$, then

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

and

$$\begin{aligned}\pi^a &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ \\ &= pE \min(\theta D, q^{pa}) - cq^{pa} + wE(\theta D - q^{pa})^+ - rE \frac{1-\theta}{\theta} (\theta D - q^{pa})^+ = \pi^{pa}\end{aligned}$$

Thus, we have $\pi^p > \pi^a = \pi^{pa}$

3. If $\theta \in [\min(\psi^p, \psi^a), \max(\psi^p, \psi^a))$,

- if $\psi^p < \psi^a$, then

$$\begin{aligned}\pi^a &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ \\ &= pE \min(\theta D, q^{pa}) - cq^{pa} + wE(\theta D - q^{pa})^+ - rE \frac{1-\theta}{\theta} (\theta D - q^{pa})^+ = \pi^{pa}\end{aligned}$$

and

$$\begin{aligned}\pi^a &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ \\ &= \left(p - w + r \frac{1-\theta}{\theta} \right) E \min(\theta D, q^a) - cq^a + wE\theta D - rE(1-\theta)D \\ &> wE\theta D - rE(1-\theta)D = \pi^p\end{aligned}$$

Thus, we have $\pi^a = \pi^{pa} > \pi^p$;

- if $\psi^p > \psi^a$, then

$$\begin{aligned}\pi^a &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ \\ &= pE \min(\theta D, q^{pa}) - cq^{pa} + wE(\theta D - q^{pa})^+ - rE \frac{1-\theta}{\theta} (\theta D - q^{pa})^+ = \pi^{pa}\end{aligned}$$

and

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

Thus, we have $\pi^p > \pi^a = \pi^{pa}$.

4. If $\theta \geq \max(\psi^p, \psi^a)$, then $\pi^p = \pi^a = \pi^{pa} = wE\theta D - rE(1-\theta)D$.

□

Proof of Proposition 9: The addition of virtual showrooms only creates a new customer pool with D' and θ' . Then since ψ^p is determined by condition (SA.2), ψ^{pv} should be determined by condition $h_s < \frac{E \min(D', E'^{-1}(\frac{c}{p-w}))}{ED'} \theta h_o + (1-\theta')(h_o + h_r)$, which is implied by condition (SA.2). Thus, we have $\psi^{pv} \leq \psi^p$.

If $\theta < \psi^{pv}$, then $\pi^p = pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ = pE \min(\theta' D', q^{pv}) - cq^{pv} + wE(\theta' D' - q^{pv})^+ = \pi^{pv}$ and

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> \max(pE \min(\theta D, q^v) - cq^v + wE(\theta D - q^v)^+, wE\theta D) \\ &> \max\left(pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^v)^+, wE\theta' D' - rE(1-\theta')D'\right) = \pi^v\end{aligned}$$

If $\theta \in [\psi^{pv}, \psi^p]$, then $\pi^p = pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ = (p-w)E \min(\theta D, q^p) -$

$cq^p + wE\theta D > wE\theta' D' - rE(1 - \theta) D = \pi^{pv}$ and

$$\begin{aligned} \pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> \max(pE \min(\theta D, q^v) - cq^v + wE(\theta D - q^v)^+, wE\theta D) \\ &> \max\left(pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1 - \theta'}{\theta'} (\theta' D' - q^v)^+, wE\theta' D' - rE(1 - \theta') D'\right) = \pi^v \end{aligned}$$

If $\theta \in [\psi^p, \max(\psi^p, \psi^v))$, then $\pi^p = wE\theta D - rE(1 - \theta) D$ and $\pi^{pv} = wE\theta' D' - r(1 - \theta') D'$, and

$$\begin{aligned} \pi^v &= pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1 - \theta'}{\theta'} (\theta' D' - q^v)^+ \\ &= \left(p - w + r \frac{1 - \theta'}{\theta'}\right) E \min(\theta' D', q^v) - cq^v + wE\theta' D' - rE(1 - \theta') D' \\ &> wE\theta' D' - rE(1 - \theta') D' \\ &= wE\theta D - r(1 - \alpha) E(1 - \theta) D \\ &\geq \max(\pi^{pv}, \pi^p) \end{aligned}$$

If $\theta \geq \max(\psi^p, \psi^v)$, $\pi^v = \pi^{pv} = wE\theta' D' - r(1 - \theta') D' = wE\theta D - r(1 - \alpha) E(1 - \theta) D > wE\theta D - rE(1 - \theta) D = \pi^p$.

□

Proof of Proposition 10: Let's first prove the part (i):

Think of the case with virtual showroom as a new base model with a new customer pool. Then, since $\psi^a \geq \psi^o$, we should also have $\psi^{va} \geq \psi^v$.

If $\theta \in [\psi^v, \psi^{va})$, then

$$\begin{aligned} \pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1 - \theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &= \left(p - w + r \frac{1 - \theta'}{\theta'}\right) E \min(\theta' D', q^{va}) - cq^{va} + wE\theta' D' - rE(1 - \theta') D' \\ &> wE\theta' D' - rE(1 - \theta') D' = \pi^v \end{aligned}$$

and

$$\begin{aligned} \pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1 - \theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &> pE \min(\theta' D', q^a) - cq^a + wE(\theta' D' - q^a)^+ - rE \frac{1 - \theta'}{\theta'} (\theta' D' - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1 - \theta}{\theta} (\theta D - q^a)^+ = \pi^a \end{aligned}$$

Thus, we get part (i).

Next, let's look at part (ii):

Since we can think of the case with virtual showroom as a new base model with new customer pool (θ' and D'), the proof should be similar to the proof of Proposition 8 and thus omitted.

□

Proof of Proposition 11: Let's first look at the case when $\theta < \psi^p$. In Proposition 3, we have shown that $\pi^p > \pi^\circ$. Thus, all we need to show is that physical showroom is strictly better than the other three options as well.

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> \max(pE \min(\theta D, q^v) - cq^v + wE(\theta D - q^v)^+, wE\theta D) \\ &> \max\left(pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^v)^+, wE\theta' D' - rE(1-\theta') D'\right) = \pi^v\end{aligned}$$

A similar argument could be used to prove $\pi^p > \pi^{va}$, by simply replacing the superscript $.^v$ with $.^{va}$ in the equation above. Finally, compared with the case where there is only availability information

$$\begin{aligned}\pi^p &= pE \min(\theta D, q^p) - cq^p + wE(\theta D - q^p)^+ \\ &> \max(pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+, wE\theta D) \\ &> \max\left(pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+, wE\theta D - rE(1-\theta) D\right) = \pi^a\end{aligned}$$

So, we can conclude $\pi^* = \pi^p$.

Next, let's look at the case where $\theta \geq \psi^p$. Let's first show the suboptimality of the option of offering only physical showroom:

- if $\theta < \max(\psi^p, \psi^\circ)$, then

$$\begin{aligned}\pi^\circ &= pE \min(\theta D, q^\circ) - cq^\circ + wE(\theta D - q^\circ)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^\circ)^+ \\ &= \left(p - w + r \frac{1-\theta}{\theta}\right) E \min(\theta D, q^\circ) - cq^\circ + wE\theta D - rE(1-\theta) D \\ &> wE\theta D - rE(1-\theta) D = \pi^p\end{aligned}$$

- if $\theta > \max(\psi^p, \psi^\circ)$, then $\pi^\circ = wE\theta D - rE(1-\theta) D = \pi^p$

So in the following analysis, we only need to look at the other four types of information.

- Case 1: $\theta < \psi^\circ$. Note that if $\psi^p \geq \psi^\circ$, we don't even have this case. So in the following proof, we only focus on the case when $\psi^p < \psi^\circ$. Note when $\alpha = 0$, we have $\psi^v = \psi^\circ$ and $\psi^{va} = \psi^a$. Also, it is easy to check that $\frac{\partial \psi^v}{\partial \alpha} \leq 0$ and $\frac{\partial \psi^{va}}{\partial \alpha} \leq 0$. Then, let's set $\alpha_1 = \arg \min_{\alpha \in [0,1]} |\theta - \psi^v(\alpha)|$

and $\alpha_2 = \arg \min_{\alpha \in [0,1]} |\theta - \psi^{va}(\alpha)|$. Since $\forall \alpha \psi^v \leq \psi^{va}$, we have $\alpha_1 \leq \alpha_2$.

- If $\alpha < \alpha_1$, then $\theta < \psi^v \leq \min(\psi^\circ, \psi^{va}) \leq \psi^a$. Therefore, $\pi^v = \pi^{va}$ and $\pi^a = \pi^\circ$. Moreover,

$$\begin{aligned}\pi^v &= pE \min(\theta' D', q^v) - cq^v + wE(\theta' D' - q^v)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^v)^+ \\ &> pE \min(\theta' D', q^a) - cq^a + wE(\theta' D' - q^a)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

Thus, $\pi^* = \pi^v$.

– If $\alpha \in (\alpha_1, \alpha_2)$, then $\psi^v < \theta < \min(\psi^\circ, \psi^{va}) \leq \pi^a$. Therefore, $\pi^a = \pi^\circ$. Also,

$$\begin{aligned}\pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &> pE \min(\theta' D', q^a) - cq^a + wE(\theta' D' - q^a)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

and

$$\begin{aligned}\pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &= \left(p - w + r \frac{1-\theta'}{\theta'} \right) E \min(\theta' D', q^{va}) - cq^{va} + wE\theta' D' - rE(1-\theta') D' \\ &> wE\theta' D' - rE(1-\theta') D' = \pi^v\end{aligned}$$

Thus, we can conclude $\pi^* = \pi^{va}$.

– If $\alpha > \alpha_2$, then $\psi^v \leq \psi^{va} < \theta < \psi^\circ \leq \psi^a$. Therefore, $\pi^a = \pi^\circ$ and $\pi^v = \pi^{va}$. Thus, we only need to compare π° and π^v . Because $\frac{\partial(\pi^v - \pi^\circ)}{\partial \alpha} = r(1-\theta)ED > 0$, there exists $\alpha_3 \geq \alpha_2$ such that $\pi^\circ > \pi^v$ if and only if $\alpha \in (\alpha_2, \alpha_3)$. Thus, $\pi^* = \pi^\circ$ if $\alpha \in (\alpha_2, \alpha_3)$; $\pi^* = \pi^v$ if $\alpha > \alpha_3$.

• Case 2: $\theta > \psi^\circ$.

– If $\theta \in [\max(\psi^\circ, \psi^p), \max(\psi^a, \psi^p)]$, then let's set $\alpha'_1 = \arg \min_{\alpha \in [0,1]} |\theta - \psi^{va}(\alpha)|$.

* If $\alpha < \alpha'_1$, then $\psi^v \leq \psi^\circ < \theta < \psi^{va} \leq \psi^a$. Therefore, $\pi^v = wE\theta' D' - rE(1-\theta') D' > wE\theta D - r(1-\theta) D = \pi^\circ$. Also,

$$\begin{aligned}\pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &> pE \min(\theta' D', q^a) - cq^a + wE(\theta' D' - q^a)^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^a)^+ \\ &> pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ = \pi^a\end{aligned}$$

and

$$\begin{aligned}\pi^{va} &= pE \min(\theta' D', q^{va}) - cq^{va} + wE(\theta' D' - q^{va})^+ - rE \frac{1-\theta'}{\theta'} (\theta' D' - q^{va})^+ \\ &= \left(p - w + r \frac{1-\theta'}{\theta'} \right) E \min(\theta' D', q^{va}) - cq^{va} + wE\theta' D' - rE(1-\theta') D' \\ &> wE\theta' D' - rE(1-\theta') D' = \pi^v\end{aligned}$$

Thus, we can conclude $\pi^* = \pi^{va}$.

* If $\alpha > \alpha'$, then $\psi^v \leq \min(\psi^\circ, \pi^{va}) \leq \max(\psi^\circ, \psi^{va}) < \theta < \psi^a$. Therefore, $\pi^{va} = \pi^v$ and

$$\begin{aligned}\pi^a &= pE \min(\theta D, q^a) - cq^a + wE(\theta D - q^a)^+ - rE \frac{1-\theta}{\theta} (\theta D - q^a)^+ \\ &= \left(p - w + r \frac{1-\theta}{\theta} \right) E \min(\theta D, q^a) - cq^a + wE\theta D - rE(1-\theta)D \\ &> wE\theta D - rE(1-\theta)D = \pi^\circ\end{aligned}$$

Thus, we only need to compare π^v and π^a . Because $\frac{\partial(\pi^v - \pi^a)}{\partial \alpha} = r(1-\theta)ED > 0$, there exists $\alpha'_2 \geq \alpha'_1$ such that $\pi^a > \pi^v$ if and only if $\alpha \in (\alpha'_1, \alpha'_2)$. Thus $\pi^* = \pi^a$ if $\alpha \in (\alpha'_1, \alpha'_2)$; $\pi^* = \pi^v$ if $\alpha > \alpha'_2$.

– If $\theta > \max(\psi^a, \psi^p)$, then since $\psi^v \leq \psi^\circ$ and $\psi^a \geq \psi^\circ$, we have $\psi^a \geq \psi^v$. Also, note ψ^a is determined by condition (SA.4), which is implied by condition $h_s < h_o + (1-\theta')h_r$, which could be shown to determine the cutoff ψ^{va} . Thus, we also have $\psi^{va} \leq \psi^a$. Then, if $\theta \geq \max(\psi^p, \psi^a)$, we have $\pi^v = \pi^{va} = wE\theta'D' - rE(1-\theta')D' > wE\theta D - rE(1-\theta)D = \pi^a = \pi^p = \pi^\circ$. So we can conclude $\pi^* = \pi^v$. In this case, we can simply set $\alpha'_1 = \alpha'_2 = 0$.

□

Supplementary Appendix II: Extensions

In the paper, we have the following main findings:

1. Adding physical showrooms may reduce profits.
2. Adding virtual showrooms may reduce profits.
3. Providing only availability information never reduces profits.
4. There is no complementary effect between physical showrooms and availability information.
5. There is no complementary effect between physical showrooms and virtual showrooms.

In this appendix, we extend our basic model in three different ways. The goal is to show the robustness of the results listed above.

III Endogenous Online Channel

In this section, we relax the assumption in the paper that the online channel is exogenous. Suppose both online and offline channels follow the standard newsvendor setup. The retail price is p , which is the same across both channels. Unit inventory costs are c_s and c_o in the store and online channel, both of which are smaller than p . For each unit sold online, if it is not returned, the retailer can get the revenue p , i.e., the price of the product; if it is returned, the retailer incurs net cost k (i.e., the retailer cannot make money from dealing with returns). The retailer decides the inventory levels in both channels, q_s and q_o , in the beginning.

Consumers setup is the same as before. We assume when the online channel is out of stock, those who are willing to buy online will leave for other websites to buy the product at the same price and obtain the same utility u_o . Note, when consumers encounter a stockout in store, they will buy from the retailer's online channel first. In many companies (e.g., Bonobos and Warby Parker), store employees are trained and equipped with digital devices to help store customers order online. However, when customers are shopping online at home, it is hard for a firm to persuade customers to come to store when online is out of stock. Our model setup is to capture this difference. Gao and Su (2016) use a similar assumption to study a specific omnichannel fulfillment strategy, i.e., buy-online-and-pickup-in-store.

With this model setup, we can find that consumer's utility functions, and thus their channel choices, remain unchanged.

III.1 Base Model

Given belief $\hat{\phi}$, the retailer's total profit is

- If $\hat{\phi} = 1$, then

$$\pi(q_s, q_o) = pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1 - \theta)]E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o$$

- If $\hat{\phi} = 0$, then

$$\pi(q_s, q_o) = [p\theta - k(1 - \theta)]E \min(D, q_o) - c_o q_o$$

Definition III.1. A RE equilibrium $(q_s, q_o, \phi, \hat{\phi}, \hat{\xi})$ satisfies the following:

- i. Given $\hat{\xi}$, if $u_s > u_o$, then $\phi = 1$; otherwise $\phi = 0$;
- ii. Given $\hat{\phi}$, $(q_s, q_o) = \arg \max \pi(q_s, q_o)$;
- iii. $\hat{\xi} = A(q_s)$;
- iv. $\hat{\phi} = \phi$.

Proposition III.1. In the base model, the RE equilibrium is given as follows:

- If $h_s < \xi_1^\circ [h_o + (1 - \theta)h_r]$ and $c_o \geq [p\theta - k(1 - \theta)]\frac{c_s}{p}$, then consumers visit store (i.e., $\phi^\circ = 1$) and $q_s^\circ = \theta \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^\circ = 0$;
- if $h_s < \xi_2^\circ [h_o + (1 - \theta)h_r]$ and $c_o < [p\theta - k(1 - \theta)]\frac{c_s}{p}$ and $c_s \leq k\frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then consumers visit store (i.e., $\phi^\circ = 1$) and $q_s^\circ = \theta \bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$, $q_o^\circ = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right) - \bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$;
- otherwise, no one comes to store (i.e., $\phi^\circ = 0$) and $q_s^\circ = 0$, $q_o^\circ = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right)$,

where $\xi_1^\circ = \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED}$, $\xi_2^\circ = \frac{E \min(D, \bar{F}^{-1}(\frac{\theta c_s - c_o}{k(1-\theta)}))}{ED}$.

III.2 Physical Showrooms

Suppose there is a physical showroom in the store. Given belief $\hat{\phi}$, the retailer's total profit is

- If $\hat{\phi} = 1$, then

$$\pi = pE \min(\theta D, q_s) - c_s q_s + pE \min((\theta D - q_s)^+, q_o) - c_o q_o$$

- If $\hat{\phi} = 0$, then

$$\pi = [p\theta - k(1 - \theta)]E \min(D, q_o) - c_o q_o$$

Proposition III.2. With physical showrooms, the RE equilibrium is given as follows:

- If $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} \theta h_o + (1 - \theta)(h_o + h_r)$ and $c_s < c_o$, then consumers come to store (i.e., $\phi^p = 1$) and $q_s^p = \theta \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^p = 0$;
- if $h_s < (1 - \theta)(h_o + h_r)$ and $c_s \geq c_o$, then consumers come to store (i.e., $\phi^p = 1$) and $q_s^p = 0$, $q_o^p = \theta \bar{F}^{-1}(\frac{c_o}{p})$;
- otherwise, no one comes to store (i.e., $\phi^p = 0$) and $q_s^p = 0$, $q_o^p = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right)$.

Proposition III.3. *If $(1 - \theta)(h_o + h_r) < h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta)h_r]$ and $[p\theta - k(1 - \theta)]\frac{c_s}{p} < c_o < c_s$, then consumers visit store if there is no physical showroom (i.e., $\phi^\circ = 1$) and they buy online directly if there is physical showroom (i.e., $\phi^\circ = 0$). Also, in this case, physical showroom decreases profit, i.e., $\pi^\circ > \pi^p$.*

This shows that our first finding, i.e., physical showrooms could backfire, is robust.

III.3 Virtual Showroom

Suppose there is a virtual showroom online. This is just a special case of the base model, with D' and θ' .

Proposition III.4. *With virtual showrooms, the RE equilibrium is given as follows:*

- If $h_s < \xi_1^v [h_o + (1 - \theta')h_r]$ and $c_o \geq [p\theta' - k(1 - \theta')]\frac{c_s}{p}$, then consumers visit store (i.e., $\phi^v = 1$) and $q_s^v = \theta' \bar{F}'^{-1}(\frac{c_s}{p})$, $q_o^v = 0$;
- if $h_s < \xi_2^v [h_o + (1 - \theta')h_r]$ and $c_o < [p\theta' - k(1 - \theta')]\frac{c_s}{p}$ and $c_s \leq k\frac{1 - \theta'}{\theta'} + \frac{c_o}{\theta'}$, then consumers visit store (i.e., $\phi^v = 1$) and $q_s^v = \theta' \bar{F}'^{-1}\left(\frac{\theta' c_s - c_o}{k(1 - \theta')}\right)$, $q_o^v = \bar{F}'^{-1}\left(\frac{c_o}{p\theta' - k(1 - \theta')}\right) - \bar{F}'^{-1}\left(\frac{\theta' c_s - c_o}{k(1 - \theta')}\right)$;
- otherwise, no one comes to store (i.e., $\phi^v = 0$) and $q_s^v = 0$, $q_o^v = \bar{F}'^{-1}\left(\frac{c_o}{p\theta' - k(1 - \theta')}\right)$,

where $\xi_1^v = \frac{E \min(D', \bar{F}'^{-1}(\frac{c_s}{p}))}{ED'}$, $\xi_2^v = \frac{E \min(D', \bar{F}'^{-1}(\frac{\theta' c_s - c_o}{k(1 - \theta')}))}{ED'}$.

Proposition III.5. *If $\frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta)h_r] < h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta)h_r]$ and $[p\theta' - k(1 - \theta')]\frac{c_s}{p} < c_o$, then consumers visit store if there is no virtual showroom (i.e., $\phi^\circ = 1$) and they buy online directly if there is virtual showroom (i.e., $\phi^\circ = 0$). Also, in this case, there exists \bar{c}_o such that virtual showroom decreases profit (i.e., $\pi^\circ > \pi^v$) if $c_o > \bar{c}_o$.*

This shows that our second finding, i.e., virtual showrooms could backfire, is robust.

III.4 Availability Information

Suppose there is availability information.

- If $h_s < h_o + (1 - \theta)h_r$, then consumers will come to store only if it is in stock. Then, the retailer's profit is

$$\pi = pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1 - \theta)]E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o$$

- If $h_s \geq h_o + (1 - \theta)h_r$, then consumers never come to store. Then, the retailer's profit is $\pi = [p\theta - k(1 - \theta)]E \min(D, q_o) - c_o q_o$.

Proposition III.6. *With availability information, the market outcome is given as follows:*

- If $h_s < h_o + (1 - \theta)h_r$ and $c_o \geq [p\theta - k(1 - \theta)]\frac{c_s}{p}$, then consumers visit store if store is in stock and $q_s^a = \theta\bar{F}^{-1}\left(\frac{c_s}{p}\right)$, $q_o^a = 0$;
- if $h_s < h_o + (1 - \theta)h_r$ and $c_o < [p\theta - k(1 - \theta)]\frac{c_s}{p}$ and $c_s \leq k\frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then consumers visit store if store is in stock and $q_s^a = \theta\bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$, $q_o^a = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right) - \bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$;
- otherwise, no one ever comes to store and $q_s^a = 0$, $q_o^a = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right)$,

Proposition III.7. *Availability info never reduces profit, i.e., $\pi^a \geq \pi^\circ$.*

This shows that our third finding, i.e., availability information never reduces profit, is robust.

III.5 Joint Implementation

Here, we only look at the two pairs, i.e., physical showrooms and availability information, and physical showrooms and virtual showrooms. Our goal is to check if there is any complementary effect between them.

III.5.1 Physical showrooms and availability information

Suppose there are both physical showrooms and availability information.

- If $h_s < (1 - \theta)(h_o + h_r)$, then consumers would always visit store. Thus,

$$\pi = pE \min(\theta D, q_s) - c_s q_s + pE \min((\theta D - q_s)^+, q_o) - c_o q_o$$

- if $(1 - \theta)(h_o + h_r) \leq h_s < h_o + (1 - \theta)h_r$, then consumers come to store only if it is in stock. Thus,

$$\pi = pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1 - \theta)] E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o$$

- if $h_s \geq h_o + (1 - \theta)h_r$, then no one ever comes to store. Thus,

$$\pi = [p\theta - k(1 - \theta)] E \min(D, q_o) - c_o q_o$$

Proposition III.8. $\pi^{pa} \leq \max(\pi^p, \pi^a)$

This shows that there is no complementary effect between physical showroom and availability information.

III.5.2 Physical showrooms and virtual showrooms

Suppose there are both physical and virtual showrooms. This is just the same as the physical showroom only scenario with a different customer pool D' and θ' .

Proposition III.9. $\pi^{pv} \leq \max(\pi^p, \pi^v)$

This shows that there is no complementary effect between these two types of showrooms.

II2 Continuous Valuation

Suppose consumer valuation V is continuously distributed on $[0, +\infty)$. Let $G(v)$ be the proportion of consumers with a valuation v or lower. Ex ante, consumers know the distribution G but not their valuations, so they are homogeneous. Ex post, consumers will learn their valuations after purchase or by checking the product in store.

All other model elements remain the same as before.

II2.1 Base Model

If consumers buy online directly, after they get the delivery, they can realize their valuation. For a consumer with V , if she keeps it, she gets payoff $V - p - h_o$; if she returns it, her payoff is $-h_o - h_r$. Then, only those with $V < p - h_r$ will return the product. The ex ante expected payoff of buying online directly is

$$u_o = E_V \max(V - p - h_o, -h_o - h_r)$$

Note those with $V \in (p - h_r, p)$ will not like the product (since $V < p$) but they don't return it because the return cost is too high (since $V - p > -h_r$).

In the store, if there is stock, then consumers can realize their valuation, and only those with $V \geq p$ will make a purchase on the spot, and the others will leave. If store is out of stock, they can buy online. So the expected payoff is

$$u_s(\hat{\xi}) = -h_s + \hat{\xi} E_V \max(V - p, 0) + (1 - \hat{\xi}) u_o$$

Here, as what we did in the paper, we assume consumers would consider both channels, i.e., $u_o \geq 0$ and $u_s(1) \geq 0$.

Denote the fraction of customers who visit store as ϕ . In an equilibrium, this could be either 0 or 1, since consumers are ex ante homogeneous.

Given belief $\hat{\phi}$, the retailer's profit function is

$$\pi(q) = p\bar{G}(p) E_D D_{in}(q) - cq \tag{II2.1}$$

$$+ [w\bar{G}(p - h_r) - rG(p - h_r)] E_D D_{out}(q) \tag{II2.2}$$

$$+ [w\bar{G}(p - h_r) - rG(p - h_r)] (1 - \hat{\phi}) E_D D \tag{II2.3}$$

where $D_{in}(q) = \min(\hat{\phi}D, \frac{q}{\bar{G}(p)})$, and $D_{out}(q) = (\hat{\phi}D - \frac{q}{\bar{G}(p)})^+$.

Definition II2.1. A RE equilibrium $(q, \phi, \hat{\phi}, \hat{\xi})$ satisfies the following:

- i. Given $\hat{\xi}$, if $u_s > u_o$, then $\phi = 1$; otherwise $\phi = 0$;
- ii. Given $\hat{\phi}$, $q = \arg \max \pi(q)$;
- iii. $\hat{\xi} = A(q)$, where $A(q) = \frac{\min(\phi D, q/\bar{G}(p))}{E\phi D}$;
- iv. $\hat{\phi} = \phi$.

Proposition II2.1. *If $h_s < h_s^\circ$, then consumers visit store (i.e., $\phi^\circ = 1$), and $q^\circ = \bar{G}(p) \bar{F}^{-1} \left(\frac{c}{p-w \frac{G(p-h_r)}{G(p)} + r \frac{G(p-h_r)}{G(p)}} \right)$ where $h_s^\circ = \xi^\circ (E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r))$ and $\xi^\circ = \frac{E_D \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w \frac{G(p-h_r)}{G(p)} + r \frac{G(p-h_r)}{G(p)}} \right) \right)}{E_D D}$; otherwise, no one comes to store (i.e., $\phi^\circ = 0$), and $q^\circ = 0$.*

II2.2 Physical Showrooms

With physical showrooms, when store is out of stock, consumers are still able to realize their valuation. In such case, only those store customers with $V-p-h_o \geq 0$ will keep on buying online. So

$$u_s(\hat{\xi}) = -h_s + \hat{\xi} E_V \max(V-p, 0) + (1-\hat{\xi}) E_V \max(V-p-h_o, 0)$$

Then, the retailer's profit is

$$\pi = p \bar{G}(p) E_D D_{in}(q) - cq \tag{II2.4}$$

$$+ w \bar{G}(p+h_o) E_D D_{out}(q) \tag{II2.5}$$

$$+ [w \bar{G}(p-h_r) - r G(p-h_r)] (1-\hat{\phi}) E_D D \tag{II2.6}$$

where $D_{in}(q) = \min(\hat{\phi} D, \frac{q}{\bar{G}(p)})$, and $D_{out}(q) = (\hat{\phi} D - \frac{q}{\bar{G}(p)})^+$.

Proposition II2.2. *With physical showrooms, if $h_s < h_s^p$, then consumers visit store (i.e., $\phi^p = 1$), and $q^p = \bar{G}(p) \bar{F}^{-1} \left(\frac{c}{p-w \frac{G(p+h_o)}{G(p)}} \right)$, where $h_s^p = \xi^p E_v \max(V-p, 0) + (1-\xi^p) E_V \max(V-p-h_o, 0) + h_o - E_V \max(V-p, -h_r)$ and $\xi^p = \frac{E_D \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w \frac{G(p+h_o)}{G(p)}} \right) \right)}{E_D D}$; otherwise, no one comes to store (i.e., $\phi^p = 0$), and $q^p = 0$. Moreover, $h_s^p > h_s^\circ$ if and only if r is small enough.*

Proposition II2.3. *If $h_s^p < h_s^\circ$ and $h_s \in [h_s^p, h_s^\circ)$, then consumers come to store in the base case but buy online directly if there is physical showroom. As a result, providing physical showrooms reduces total profit (i.e., $\pi^p < \pi^\circ$).*

This shows that our first finding, i.e., physical showrooms could backfire, is robust.

II2.3 Virtual Showrooms

Similar to what we did in the paper, we assume virtual showrooms help to screen out some “low-type” customers. Specifically, the signal consumers receive after checking with the virtual showroom is still binary: those whose valuation $V < \bar{v}$ will realize their valuation is low and therefore leave the market without any purchase, while those whose valuation $V \geq \bar{v}$ will know their valuation is greater than \bar{v} and update their belief about the valuation distribution to G' such that $\forall v \geq \bar{v}$, $G'(v) = \frac{G(v)-G(\bar{v})}{1-G(\bar{v})}$. Here, we assume threshold $\bar{v} \leq p-h_r$, i.e., virtual showrooms would screen out only those who really don't like the product and are bound to return the product if they buy it online. The total number of customers left in the market is $D' = [1-G(\bar{v})]D$.

Thus, the model is the same as base model except for a new customer pool, D' and G' . We denote E'_V as the expectation over V given G' .

Proposition II2.4. *With virtual showrooms, if $h_s < h_s^v$, then consumers visit store (i.e., $\phi^v = 1$), and $q^v = \bar{G}(p) \bar{F}^{-1} \left(\frac{c}{p-w \frac{\bar{G}'(p-h_r)}{\bar{G}'(p)} + r \frac{G'(p-h_r)}{\bar{G}'(p)}} \right)$, where $h_s^v = \xi^v (E'_V \max(V-p, 0) + h_o - E'_V \max(V-p, -h_r))$ and $\xi^v = \frac{E_D \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w \frac{G'(p-h_r)}{\bar{G}'(p)} + r \frac{G'(p-h_r)}{\bar{G}'(p)}} \right) \right)}{E_D D}$; otherwise, no one comes to store (i.e., $\phi^v = 0$), and $q^v = 0$. Moreover, $h_s^v < h_s^\circ$.*

Proposition II2.5. *If $h_s \in [h_s^v, h_s^\circ)$, then consumers come to store in the base case but buy online directly if there is virtual showroom. As a result, there exists \bar{w} such that providing virtual showrooms reduces total profit (i.e., $\pi^v < \pi^\circ$) if $w < \bar{w}$.*

This shows that our second finding, i.e., virtual showrooms could backfire, is robust.

II2.4 Availability Information

With availability information, if it is shown that store is in stock, then $u_{s,in} = -h_s + E_V \max(V-p, 0)$. Apparently, consumer will not go to store if store is out of stock.

Denote ϕ_{in} as the fraction of customers visiting store when it is in stock. Then, the retailer's profit function is

$$\pi(q) = p\bar{G}(p)\phi_{in}ED_{in}(q) - cq \quad (\text{II2.7})$$

$$+ [w\bar{G}(p-h_r) - rG(p-h_r)](1-\phi_{in})ED_{in}(q) \quad (\text{II2.8})$$

$$+ [w\bar{G}(p-h_r) - rG(p-h_r)]ED_{out}(q) \quad (\text{II2.9})$$

where $D_{in}(q) = \min(D, \frac{q}{\theta\phi_{in}})$ and $D_{out}(q) = (D - \frac{q}{\theta\phi_{in}})^+$.

Proposition II2.6. *With availability information, if $h_s \leq h_s^a$, then consumers come to store if store is in stock and buy online if store is out of stock, where $h_s^a = E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)$ and $q^a = \bar{G}(p) \bar{F}^{-1} \left(\frac{c}{p-w \frac{\bar{G}'(p-h_r)}{\bar{G}'(p)} + r \frac{G'(p-h_r)}{\bar{G}'(p)}} \right)$; otherwise, no one comes to store, and $q^a = 0$.*

Proposition II2.7. *Compared to the base case, providing availability information never decreases profit, i.e., $\pi^a \geq \pi^\circ$.*

This shows that our third finding, i.e., availability never reduces profit, is robust.

II2.5 Joint Implementation

Here, we only look at the two pairs, i.e., physical showrooms and availability information, and physical showrooms and virtual showrooms. Our goal is to check if there is any complementary effect between them.

II2.5.1 Physical showrooms and availability information

Suppose there are both physical showrooms and availability information. Now even if store is out of stock, consumers may still want to visit store, $u_{s,out} = -h_s + E_V \max(V - p - h_o, 0)$; if store is in stock, $u_{s,in} = -h_s + E_V \max(V - p, 0)$.

- If $h_s \leq E_V \max(V - p - h_o, 0) + h_o - E_V \max(V - p, -h_r)$, then consumers come to store even if store is out of stock. Thus,

$$\pi = pE_D \min(\bar{G}(p) D, q) - cq + w\bar{G}(p + h_o) E_D \left(D - \frac{q}{\bar{G}(p)} \right)^+$$

- If $E_V \max(V - p - h_o, 0) + h_o - E_V \max(V - p, -h_r) < h_s \leq E_V \max(V - p, 0) + h_o - E_V \max(V - p, -h_r)$, then consumers come to store only if it is in stock. Thus,

$$\pi = pE_D \min(\bar{G}(p) D, q) - cq + [w\bar{G}(p - h_r) - rG(p - h_r)] E_D \left(D - \frac{q}{\bar{G}(p)} \right)^+$$

- If $h_s > E_V \max(V - p, 0) + h_o - E_V \max(V - p, -h_r)$, then no one comes to store. Thus,

$$\pi = [w\bar{G}(p - h_r) - rG(p - h_r)] E_D D$$

Proposition II2.8. $\pi^{pa} \leq \max(\pi^p, \pi^a)$

This shows that there is no complementary effect between physical showrooms and availability information.

II2.5.2 Physical showrooms and virtual showroom

Suppose there are both physical and virtual showrooms. This is just the same as the physical showrooms only scenario with a different customer pool D' and G' .

Proposition II2.9. $\pi^{pv} \leq \max(\pi^p, \pi^v)$.

This shows that there is no complementary effect between these two types of showroom.

II3 Informed and Uninformed Customers

Suppose that there are two groups of customers: a fraction λ of them are informed and a fraction $1 - \lambda$ of them are uninformed. Informed customers know they are high type (v). But uninformed customers don't know their type ex ante; they just know θ of them are high type (v) while the others are low type (0).

Everything else remains the same as the base model. Note the uninformed customers' behavior remains the same as what we described in the simple model. So, here we only describe informed customers' behavior. In this section, we use subscripts \cdot_u and \cdot_i to denote the parameters regarding uninformed and informed customers, respectively.

II3.1 Base Model

For uninformed customers, their utility functions are given in the paper and are also presented here:

- $u_{o,u} = \theta(v - p - h_o) - (1 - \theta)(h_o + h_r)$
- $u_{s,u}(\hat{\xi}) = -h_s + \hat{\xi}\theta(v - p) + (1 - \hat{\xi})u_{o,u}$

For informed customers, they don't have valuation uncertainty, but still they need to face availability uncertainty. Specifically, their utility functions are

- $u_{o,i} = v - p - h_o$
- $u_{s,i}(\hat{\xi}) = -h_s + \hat{\xi}(v - p) + (1 - \hat{\xi})u_{o,i}$

Denote the fraction of informed customers who visit store as ϕ_i , and the fraction of uninformed customers who visit store as ϕ_u . We only consider the cases where people from the same group choose the same channel. Therefore, we have four possible equilibrium outcomes:

1. $\phi_i = 0, \phi_u = 0$
2. $\phi_i = 1, \phi_u = 0$
3. $\phi_i = 0, \phi_u = 1$
4. $\phi_i = 1, \phi_u = 1$

Note if $u_{s,i}(\hat{\xi}) > u_{o,i}$, then we must have $u_{s,u}(\hat{\xi}) > u_{o,u}$. Thus, $\phi_i = 1, \phi_u = 0$ cannot be an equilibrium. So we have 3 left.

Given beliefs $\hat{\phi}_i$ and $\hat{\phi}_u$, the retailer's profit function is

- if $\hat{\phi}_i = 1$ and $\hat{\phi}_u = 1$, then

$$\begin{aligned} \pi(q) = & pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq \\ & + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \\ & + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \end{aligned}$$

- if $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 1$, then

$$\begin{aligned} \pi(q) = & pE \min((1 - \lambda)\theta D, q) - cq \\ & + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ \\ & + wE\lambda D \end{aligned}$$

- if $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 0$, then

$$\pi(q) = [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D$$

Definition II3.1. A RE equilibrium $(\phi_u, \phi_i, q, \hat{\xi}, \hat{\phi}_u, \hat{\phi}_i)$ satisfies the following:

i. Given $\hat{\xi}$,

$$\phi_u = \begin{cases} 1 & \text{if } u_{s,u} > u_{o,u} \\ 0 & \text{if } u_{s,u} \leq u_{o,u} \end{cases} \quad \text{and} \quad \phi_i = \begin{cases} 1 & \text{if } u_{s,i} > u_{o,i} \\ 0 & \text{if } u_{s,i} \leq u_{o,i} \end{cases}$$

ii. Given $\hat{\phi}_u, \hat{\phi}_i$, $q = \arg \max_q \pi(q)$;

iii. $\hat{\xi} = A(q)$, where $A(q) = \frac{E \min((\lambda \phi_i + (1-\lambda) \phi_u \theta) D, q)}{E(\lambda \phi_i + (1-\lambda) \phi_u \theta) D}$;

iv. $\hat{\phi}_u = \phi_u, \hat{\phi}_i = \phi_i$.

Proposition II3.1. In the base model, the RE equilibrium is as follows:

- if $h_s < \xi_1^\circ h_o$, then all customers come to store, i.e., $\phi_i^\circ = 1, \phi_u^\circ = 1$
- if $\xi_2^\circ h_o \leq h_s < \xi_2^\circ [h_o + (1-\theta)h_r]$, then only uninformed customers come to store, i.e., $\phi_i^\circ = 0, \phi_u^\circ = 1$;
- otherwise, no one comes to store, i.e., $\phi_i^\circ = 0, \phi_u^\circ = 0$,

$$\text{where } \xi_1^\circ = \frac{E \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w+r \frac{(1-\theta)(1-\lambda)}}{\lambda+(1-\lambda)\theta} \right) \right)}{ED} \quad \text{and} \quad \xi_2^\circ = \frac{E \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w+r \frac{1-\theta}}{\theta} \right) \right)}{ED}.$$

II3.2 Physical Showrooms

Suppose there is a physical showroom. The informed customers' utility functions remain the same as in the base model, since showrooms do not have any effect on them.

Given beliefs $\hat{\phi}_i$ and $\hat{\phi}_u$, the retailer's profit function is

- if $\hat{\phi}_i = 1$ and $\hat{\phi}_u = 1$, then

$$\begin{aligned} \pi = & pE \min((\lambda + (1-\lambda)\theta)D, q) - cq \\ & + wE[(\lambda + (1-\lambda)\theta)D - q]^+ \end{aligned}$$

- if $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 1$, then

$$\begin{aligned} \pi = & pE \min((1-\lambda)\theta D, q) - cq \\ & + w\theta E \left[(1-\lambda)D - \frac{q}{\theta} \right]^+ \\ & + wE\lambda D \end{aligned}$$

- if $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 0$, then

$$\pi = [w\theta - r(1-\theta)]E(1-\lambda)D + wE\lambda D$$

Proposition II3.2. With physical showrooms, the RE equilibrium is as follows:

- if $h_s < \xi^p h_o$, then all customers come to store, i.e., $\phi_i^p = 1, \phi_u^p = 1$
- if $\xi^p h_o \leq h_s < \xi^p \theta h_o + (1 - \theta)(h_o + h_r)$, then only uninformed customers come to store, i.e., $\phi_i^p = 0, \phi_u^p = 1$;
- otherwise, no one comes to store, i.e., $\phi_i^p = 0, \phi_u^p = 0$,

where $\xi^p = \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}$.

Proposition II3.3. *If $\max\{\xi_2^\circ h_o, \xi^p \theta h_o + (1 - \theta)(h_o + h_r)\} < h_s < \xi_2^\circ [h_o + (1 - \theta)h_r]$, then uninformed customers come to store in the base case and buy online if there is physical showroom while informed customers always buy online. As a result, physical showroom decreases total profit, i.e., $\pi^\circ > \pi^p$.*

This shows that our first finding, i.e., physical showrooms could backfire, is robust.

II3.3 Virtual Showrooms

Suppose there is a virtual showroom online. Note virtual showrooms do not affect informed customers. It simply screens out α of low-type uninformed customers. Then $D' = [1 - \alpha(1 - \theta)(1 - \lambda)]D$, $\theta' = \frac{\theta}{1 - \alpha(1 - \theta)}$ and $\lambda' = \frac{\lambda}{1 - \alpha(1 - \theta)(1 - \lambda)}$.

Here, we refer to those remaining in the market as the customers.

Proposition II3.4. *With virtual showroom, the RE equilibrium is as follows:*

- if $h_s < \xi_1^v h_o$, then all customers come to store, i.e., $\phi_i^v = 1, \phi_u^v = 1$
- if $\xi_2^v h_o \leq h_s < \xi_2^v [h_o + (1 - \theta)h_r]$, then only uninformed customers come to store, i.e., $\phi_i^v = 0, \phi_u^v = 1$;
- otherwise, no one comes to store, i.e., $\phi_i^v = 0, \phi_u^v = 0$,

where $\xi_1^v = \frac{E \min\left(D', \bar{F}'^{-1}\left(\frac{c}{p-w+r \frac{(1-\theta')(1-\lambda')}{\lambda'+(1-\lambda')\theta'}}\right)\right)}{ED'}$ and $\xi_2^v = \frac{E \min\left(D', \bar{F}'^{-1}\left(\frac{c}{p-w+r \frac{1-\theta'}{\theta'}}\right)\right)}{ED'}$.

Proposition II3.5. *If $\max\{\xi_2^\circ h_o, \xi_2^v [h_o + (1 - \theta')h_r]\} < h_s \leq \xi_2^\circ [h_o + (1 - \theta)h_r]$, then uninformed customers come to store in the base case and buy online if there is virtual showroom while informed customers always buy online. As a result, there exists \bar{w} such that providing virtual showroom decreases total profit (i.e., $\pi^\circ > \pi^v$) if $w < \bar{w}$.*

This shows that our second finding, i.e., virtual showrooms could backfire, is robust.

II3.4 Availability Information

With availability info, customers (including informed) don't need to form beliefs.

- If $h_s < h_o$, then both informed and uninformed customers go to store if in stock. Thus, retailer's profit

$$\begin{aligned}\pi = & pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq \\ & + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \\ & + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+\end{aligned}$$

- if $h_o \leq h_s < h_o + (1 - \theta)h_r$, then only uninformed customer will go to store if in stock. Thus,

$$\begin{aligned}\pi = & pE \min((1 - \lambda)\theta D, q) - cq \\ & + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ \\ & + wE\lambda D\end{aligned}$$

- if $h_s \geq h_o + (1 - \theta)h_r$, then no one ever comes to store. Thus,

$$\pi = [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D$$

Proposition II3.6. *With availability information,*

- If $h_s < h_o$, then both informed and uninformed customers go to store if store is in stock, and $q^a = (\lambda + (1 - \lambda)\theta) \bar{F}^{-1} \left(\frac{c}{p - w + r \frac{(1 - \lambda)(1 - \theta)}{\lambda + (1 - \lambda)\theta}} \right)$;
- If $h_o \leq h_s < h_o + (1 - \theta)h_r$, then only uninformed customer will go to store if in stock, and $q^a = (1 - \lambda)\theta \bar{F}^{-1} \left(\frac{c}{p - w + r \frac{1 - \theta}{\theta}} \right)$;
- If $h_s \geq h_o + (1 - \theta)h_r$, then no one ever comes to store, and $q^a = 0$.

Proposition II3.7. *Compared to the base case, providing availability information never decreases profit, i.e., $\pi^a \geq \pi^\circ$.*

This shows that our third finding, i.e., availability information never reduces profit, is robust.

II3.5 Joint Implementation

Here, we only look at the two pairs, i.e., physical showrooms and availability information, and physical showrooms and virtual showrooms. Our goal is to check if there is any complementary effect between them.

II3.5.1 Physical showrooms and availability information

Suppose there are both physical showrooms and availability information.

- If $h_s < \min(h_o, (1 - \theta)(h_o + h_r))$, then informed customers go to store if in-stock and buy online if stockouts; uninformed customers always go to store. Thus,

$$\pi = pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+$$

- If $h_o < (1 - \theta)(h_o + h_r)$ and $h_o \leq h_s < (1 - \theta)(h_o + h_r)$, then informed customers always buy online; uninformed customers always go to store. Thus,

$$\pi = pE \min((1 - \lambda)\theta D, q) - cq + wE((1 - \lambda)\theta D - q)^+ + wE\lambda D$$

- If $h_o > (1 - \theta)(h_o + h_r)$ and $(1 - \theta)(h_o + h_r) \leq h_s < h_o$, then informed customers go to store if in stock and buy online if stockouts; uninformed customers go to stock if in stock and buy online if stockouts. Thus,

$$\begin{aligned} \pi = & pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq \\ & + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \\ & + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \end{aligned}$$

- If $\max(h_o, (1 - \theta)(h_o + h_r)) \leq h_s < h_o + (1 - \theta)h_r$, then informed customers always buy online; uninformed customers go to store if in stock and buy online if out of stock. Thus,

$$\begin{aligned} \pi = & pE \min((1 - \lambda)\theta D, q) - cq \\ & + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ \\ & + wE\lambda D \end{aligned}$$

- If $h_s \geq h_o + (1 - \theta)h_r$, then no one ever comes to store. Thus,

$$\pi = [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D$$

Proposition II3.8. $\pi^{pa} > \max(\pi^p, \pi^a)$ if and only if $\xi^p h_o < h_s < \min(h_o, (1 - \theta)(h_o + h_r))$.

Let us compare consumer equilibrium behavior in the three scenarios when $\xi^p h_o < h_s < \min(h_o, (1 - \theta)(h_o + h_r))$:

i. Physical showrooms only:

- informed customer: buy online
- uninformed customer: go to store

ii. Availability information only:

- informed customer: go to store if in stock, buy online if stockouts
- uninformed customer: go to store if in stock, buy online if stockouts

iii. Physical showrooms and availability information:

- informed customer: go to store if in stock, buy online if stockouts

– uninformed customer: always go to store

Proposition II3.8 shows that when different types of customers require different types of information, we may need to offer both physical showroom and availability info. Specifically, we need physical showrooms to attract uninformed customer, and availability information to attract informed customers to the store.

II3.5.2 Physical showrooms and virtual showrooms

Suppose there are both physical and virtual showrooms. This is similar to the physical showrooms only scenario with D' , θ' and λ' .

Proposition II3.9. $\pi^{pv} \leq \max(\pi^p, \pi^v)$.

This shows that there is no complementary effect between these two types of showrooms.

II4 Proofs

Proof of Proposition III.1: Note nonparticipatory equilibrium, $(0, \bar{F}^{-1}(\frac{c_o}{p\theta - k(1-\theta)}), 0, 0, 0)$, always exists. Same as what we did in the paper, we are going to look for participatory equilibrium, where $\phi^\circ = 1$ and $q_s^\circ > 0$. All we need to do is to check the four conditions specified in Definition III.1.

First, we look at retailer's problem: In the participatory equilibrium, retailer's belief is consistent, i.e., $\hat{\phi} = \phi^\circ = 1$. Then, the retailer's profit is

$$\pi = pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1-\theta)] E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o.$$

It is easy to verify the profit function is jointly concave in (q_s, q_o) . Note the first order derivatives are given as follows: $\frac{\partial \pi}{\partial q_s} = k \frac{1-\theta}{\theta} \bar{F}\left(\frac{q_s}{\theta}\right) + (p - k \frac{1-\theta}{\theta}) \bar{F}\left(\frac{q_s}{\theta} + q_o\right) - c_s$ and $\frac{\partial \pi}{\partial q_o} = [p\theta - k(1-\theta)] \bar{F}\left(\frac{q_s}{\theta} + q_o\right) - c_o$. Given that $q_s \geq 0$ and $q_o \geq 0$, it is easy to find the optimal solution is given as follows:

- if $c_o \geq [p\theta - k(1-\theta)] \frac{c_s}{p}$, then $\bar{F}\left(\frac{q_s}{\theta}\right) = \frac{c_s}{p}$ and $q_o = 0$
- if $c_o < [p\theta - k(1-\theta)] \frac{c_s}{p}$ and $c_s \leq k \frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then $\bar{F}\left(\frac{q_s}{\theta}\right) = \frac{\theta c_s - c_o}{k(1-\theta)}$ and $\bar{F}\left(\frac{q_s}{\theta} + q_o\right) = \frac{c_o}{p\theta - k(1-\theta)}$
- if $c_o < [p\theta - k(1-\theta)] \frac{c_s}{p}$ and $c_s > k \frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then $q_s = 0$ and $\bar{F}(q_o) = \frac{c_o}{p\theta - k(1-\theta)}$

Thus, to ensure participatory equilibrium, we need (i) $c_o \geq [p\theta - k(1-\theta)] \frac{c_s}{p}$ or (ii) $c_o < [p\theta - k(1-\theta)] \frac{c_s}{p}$ and $c_s \leq k \frac{1-\theta}{\theta} + \frac{c_o}{\theta}$

Let's first look at case (i): In equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q_s^\circ) = E \min(\phi^\circ D, q_s^\circ/\theta)/E(\phi^\circ D) = E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))/ED$. Then, we go back to consumer's decision. To ensure $\phi^\circ = 1$, we need $u_s > u_o$, i.e., $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1-\theta)h_r]$.

Next, let's look at case (ii): In equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q_s^\circ) = E \min(\phi^\circ D, q_s^\circ/\theta)/E(\phi^\circ D) = E \min(D, \bar{F}^{-1}(\frac{\theta c_s - c_o}{k(1-\theta)}))/ED$. Then, we go back to consumer's decision. To ensure $\phi^\circ = 1$, we need $u_s > u_o$, i.e., $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{\theta c_s - c_o}{k(1-\theta)}))}{ED} [h_o + (1-\theta)h_r]$.

Based on the analysis above, we find the conditions for a participatory equilibrium are

- $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1-\theta)h_r]$ and $c_o \geq [p\theta - k(1-\theta)] \frac{c_s}{p}$, in which the equilibrium outcome is $\phi^\circ = 1$, $q_s^\circ = \theta \bar{F}^{-1}\left(\frac{c_s}{p}\right)$ and $q_o^\circ = 0$.
- $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{\theta c_s - c_o}{k(1-\theta)}))}{ED} [h_o + (1-\theta)h_r]$ and $c_o < [p\theta - k(1-\theta)] \frac{c_s}{p}$ and $c_s \leq k \frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, in which the equilibrium outcome is $\phi^\circ = 1$, $q_s^\circ = \theta \bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$, $q_o^\circ = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right) - \bar{F}^{-1}\left(\frac{\theta c_s - c_o}{k(1-\theta)}\right)$.

□

Proof of Proposition III.2: Note nonparticipatory equilibrium, $(0, \bar{F}^{-1}(\frac{c_o}{p\theta - k(1-\theta)}), 0, 0, 0)$, always exists. Same as what we did in the paper, we are going to look for participatory equilibrium, where $\phi^p = 1$.

First, we look at retailer's problem: In the participatory equilibrium, retailer's belief is consistent, i.e., $\hat{\phi} = \phi^p = 1$. Then, retailer's profit is $\pi = pE \min(\theta D, q_s) - c_s q_s + pE \min((\theta D - q_s)^+, q_o) - c_o q_o = pE \min(\theta D, q_s + q_o) - c_s q_s - c_o q_o$. Then, it is easy to find that the optimal solution is as follows

- if $c_s < c_o$, then $q_s^p = \theta \bar{F}^{-1}(\frac{c_s}{p})$ and $q_o^p = 0$
- if $c_s \geq c_o$, then $q_s^p = 0$ and $q_o^p = \theta \bar{F}^{-1}(\frac{c_o}{p})$.

Let's first look at the case when $c_s < c_o$. In equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = A(q_s^p) = E \min(\phi^p D, q_s^p / \theta) / E(\phi^p D) = E \min(D, \bar{F}^{-1}(\frac{c_s}{p})) / ED$. Then, we go back to consumer's decision. To ensure $\phi^p = 1$, we need $u_s > u_o$, i.e., $h_s < \hat{\xi}^p \theta h_o + (1 - \theta)(h_o + h_r)$. Therefore, we need $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} \theta h_o + (1 - \theta)(h_o + h_r)$.

Next let's look at the case when $c_s \geq c_o$. In equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = A(q_s^p) = 0$. Then, we go back to consumer's decision. To ensure $\phi^p = 1$, we need $u_s > u_o$, i.e., $h_s < \hat{\xi}^p \theta h_o + (1 - \theta)(h_o + h_r)$. Therefore, we need $h_s < (1 - \theta)(h_o + h_r)$.

Based on the analysis above, we find the conditions for a participatory equilibrium are

- $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} \theta h_o + (1 - \theta)(h_o + h_r)$ and $c_s < c_o$, in which the equilibrium outcome is $\phi^p = 1$, $q_s^p = \theta \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^p = 0$;
- $h_s < (1 - \theta)(h_o + h_r)$ and $c_s \geq c_o$, in which the equilibrium outcome is $\phi^p = 1$, $q_s^p = 0$, $q_o^p = \theta \bar{F}^{-1}(\frac{c_o}{p})$;

□

Proof of Proposition III.3: If $(1 - \theta)(h_o + h_r) < h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta)h_r]$ and $[p\theta - k(1 - \theta)]\frac{c_s}{p} < c_o < c_s$, according to Propositions III.1 and III.2, we can easily find that $\phi^o = 1$, $q_s^o = \theta \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^o = 0$ and $\phi^p = 0$, $q_s^p = 0$, $q_o^p = \bar{F}^{-1}(\frac{c_o}{p\theta - k(1 - \theta)})$. Then,

$$\begin{aligned} \pi^o &= pE \min(\theta D, q_s^o) - c_s q_s^o \\ &> [p\theta - k(1 - \theta)] E \min(D, q_o^p) - c_o q_o^p = \pi^p \end{aligned}$$

where the inequality is because $q_s = 0$ and $q_o = q_o^p$ is a feasible but not the optimal solution to the case where there is no physical showroom. □

Proof of Proposition III.4: The proof is similar to the proof of Proposition III.1. We only need to replace D and θ with D' and θ' . Details are omitted. □

Proof of Proposition III.5: If $\frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta')h_r] < h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} [h_o + (1 - \theta)h_r]$ and $[p\theta' - k(1 - \theta')]\frac{c_s}{p} < c_o$, then $\phi^o = 1$, $q_s^o = \theta \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^o = 0$ and $\phi^v = 0$, $q_s^v = 0$, $q_o^v = \bar{F}^{-1}(\frac{c_o}{p\theta' - k(1 - \theta')})$. Then, $\pi^o = pE \min(\theta D, q_s^o) - c_s q_s^o$ and $\pi^v = [p\theta' - k(1 - \theta')] E \min(D, q_o^v) - c_o q_o^v$. Note by Envelop Theorem, we have $\frac{\partial(\pi^o - \pi^v)}{\partial c_o} = q_s^v > 0$. Also, note when $c_o = p\theta' - k(1 - \theta')$, $\pi^v = 0 < \pi^o$. Thus, there exists \bar{c}_o such that $\pi^o > \pi^v$ if $c_o > \bar{c}_o$. □

Proof of Proposition III.6: If $h_s < h_o + (1 - \theta)h_r$, then consumers come to store only if it is in stock. For the profit function

$$\pi = pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1 - \theta)]E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o$$

it is easy to check it is jointly concave in (q_s, q_o) . Note the first order derivatives are $\frac{\partial \pi}{\partial q_s} = k\frac{1-\theta}{\theta}\bar{F}\left(\frac{q_s}{\theta}\right) + (p - k\frac{1-\theta}{\theta})\bar{F}\left(\frac{q_s}{\theta} + q_o\right) - c_s$ and $\frac{\partial \pi}{\partial q_o} = [p\theta - k(1 - \theta)]\bar{F}\left(\frac{q_s}{\theta} + q_o\right) - c_o$. Given that $q_s \geq 0$ and $q_o \geq 0$, it is easy to find the optimal solution is given as follows:

- if $c_o \geq [p\theta - k(1 - \theta)]\frac{c_s}{p}$, then $\bar{F}\left(\frac{q_s^a}{\theta}\right) = \frac{c_s}{p}$ and $q_o^a = 0$
- if $c_o < [p\theta - k(1 - \theta)]\frac{c_s}{p}$ and $c_s \leq k\frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then $\bar{F}\left(\frac{q_s^a}{\theta} + q_o^a\right) = \frac{c_o}{p\theta - k(1-\theta)}$ and $\bar{F}\left(\frac{q_s^a}{\theta}\right) = \frac{\theta c_s - c_o}{k(1-\theta)}$
- if $c_o < [p\theta - k(1 - \theta)]\frac{c_s}{p}$ and $c_s > k\frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then $q_s^a = 0$ and $\bar{F}(q_o^a) = \frac{c_o}{p\theta - k(1-\theta)}$

If $h_s \geq h_o + (1 - \theta)h_r$, then all customers always buy online. Then, profit function is $\pi = [p\theta - k(1 - \theta)]E \min(D, q_o) - c_o q_o$. Thus, $q_s^a = 0$, $q_o^a = \bar{F}^{-1}\left(\frac{c_o}{p\theta - k(1-\theta)}\right)$. \square

Proof of Proposition III.7:

- If $h_s < h_o + (1 - \theta)h_r$ and $c_o \geq [p\theta - k(1 - \theta)]\frac{c_s}{p}$, then $\pi^a = pE \min(\theta D, q_s^a) - c_s q_s^a$.
 - If $h_s < \xi_1^\circ[h_o + (1 - \theta)h_r]$, then $\pi^\circ = pE \min(\theta D, q_s^\circ) - c_s q_s^\circ = \pi^a$;
 - If $h_s \geq \xi_1^\circ[h_o + (1 - \theta)h_r]$, then $\pi^\circ = [p\theta + k(1 - \theta)]E \min(D, q_o^\circ) - c_o q_o^\circ < \pi^a$, where the inequality is because $q_s = 0$ and $q_o = q_o^\circ$ is also a feasible but not the optimal solution to the case where there is availability information;
- If $h_s < h_o + (1 - \theta)h_r$ and $c_o < [p\theta - k(1 - \theta)]\frac{c_s}{p}$ and $c_s \leq k\frac{1-\theta}{\theta} + \frac{c_o}{\theta}$, then $\pi^a = pE \min(\theta D, q_s^a) - c_s q_s^a + [p\theta + k(1 - \theta)]E \min\left(\left(D - \frac{q_s^a}{\theta}\right)^+, q_o^a\right) - c_o q_o^a$
 - If $h_s \leq \xi_2^\circ[h_o + (1 - \theta)h_r]$, then $\pi^\circ = pE \min(\theta D, q_s^\circ) - c_s q_s^\circ + [p\theta - k(1 - \theta)]E \min\left(\left(D - \frac{q_s^\circ}{\theta}\right)^+, q_o^\circ\right) - c_o q_o^\circ = \pi^a$;
 - $h_s > \xi_2^\circ[h_o + (1 - \theta)h_r]$, then $\pi^\circ = [p\theta - k(1 - \theta)]E \min(D, q_o^\circ) - c_o q_o^\circ < \pi^a$, where the inequality is because $q_s = 0$ and $q_o = q_o^\circ$ is also a feasible but not the optimal solution to the case where there is availability information;
- Otherwise, then $\pi^a = [p\theta - k(1 - \theta)]E \min(D, q_o^a) - c_o q_o^a = \pi^\circ$.

\square

Proof of Proposition III.8. • If $h_s \leq (1 - \theta)(h_o + h_r)$, then

$$\begin{aligned} \pi^{pa} &= \max_{q_s, q_o \geq 0} \{pE \min(\theta D, q_s) - c_s q_s + pE \min((\theta D - q_s)^+, q_o) - c_o q_o\} \\ &= pE \min(\theta D, q_s^p) - c_s q_s^p + pE \min((\theta D - q_s^p)^+, q_o^p) - c_o q_o^p = \pi^p \end{aligned}$$

- If $(1 - \theta)(h_o + h_r) \leq h_s < h_o + (1 - \theta)h_r$, then

$$\begin{aligned}\pi^{pa} &= \max_{q_s, q_o \geq 0} \left\{ pE \min(\theta D, q_s) - c_s q_s + [p\theta - k(1 - \theta)] E \min\left(\left(D - \frac{q_s}{\theta}\right)^+, q_o\right) - c_o q_o \right\} \\ &= pE \min(\theta D, q_s^a) - c_s q_s^a + [p\theta - k(1 - \theta)] E \min\left(\left(D - \frac{q_s^a}{\theta}\right)^+, q_o^a\right) - c_o q_o^a = \pi^a\end{aligned}$$

- If $h_s > h_o + (1 - \theta)h_r$, then

$$\begin{aligned}\pi^{pa} &= \max_{q_o \geq 0} \{ [p\theta - k(1 - \theta)] E \min(D, q_o) - c_o q_o \} \\ &= [p\theta + k(1 - \theta)] E \min(D, q_o^a) - c_o q_o^a = \pi^a\end{aligned}$$

□

Proof of Proposition III.9: First, similar to the proof of Proposition III.2, we can find the RE equilibrium for the case when there are both physical and virtual showrooms. We simply replace D and θ with D' and θ' . The results are given as follows:

- If $h_s < \frac{E \min(D', \bar{F}'^{-1}(\frac{c_s}{p}))}{ED'} \theta' h_o + (1 - \theta')(h_o + h_r)$ and $c_s < c_o$, then consumers come to store (i.e., $\phi^{pv} = 1$) and $q_s^{pv} = \theta' \bar{F}'^{-1}(\frac{c_s}{p})$, $q_o^{pv} = 0$;
- if $h_s < (1 - \theta')(h_o + h_r)$ and $c_s \geq c_o$, then consumers come to store (i.e., $\phi^{pv} = 1$) and $q_s^{pv} = 0$, $q_o^{pv} = \theta' \bar{F}'^{-1}(\frac{c_o}{p})$;
- otherwise, no one comes to store (i.e., $\phi^{pv} = 0$) and $q_s^{pv} = 0$, $q_o^{pv} = \bar{F}'^{-1}\left(\frac{c_o}{p\theta' - k(1 - \theta')}\right)$.

Then, let's compare profits.

- If $h_s < \frac{E \min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} \theta' h_o + (1 - \theta')(h_o + h_r)$ and $c_s < c_o$, then

$$\begin{aligned}\pi^{pv} &= pE \min(\theta' D', q_s^{pv}) - c_s q_s^{pv} + pE \min\left((\theta' D' - q_s^{pv})^+, q_o^{pv}\right) - c_o q_o^{pv} \\ &= pE \min(\theta' D', q_s^p) - c_s q_s^p + pE \min\left((\theta' D' - q_s^p)^+, q_o^p\right) - c_o q_o^p = \pi^p\end{aligned}$$

- If $h_s < (1 - \theta')(h_o + h_r)$ and $c_s > c_o$, then

$$\begin{aligned}\pi^{pv} &= pE \min(\theta' D', q_o^{pv}) - c_o q_o^{pv} \\ &= pE \min(\theta' D', q_o^p) - c_o q_o^p = \pi^p\end{aligned}$$

- Otherwise,

$$\begin{aligned}\pi^{pv} &= [p\theta' - k(1 - \theta')] E \min(D', q_o^{pv}) - c_o q_o^{pv} \\ &\leq pE \min(\theta' D', q_s^v) - c_s q_s^v + [p\theta' - k(1 - \theta')] E \min\left(\left(D' - \frac{q_s^v}{\theta'}\right)^+, q_o^v\right) - c_o q_o^v = \pi^v\end{aligned}$$

where the inequality is because $q_s = 0, q_o = q_o^{pv}$ is also a feasible solution to the case where there is only virtual showroom.

□

Proof of Proposition II2.1: Note nonparticipatory equilibrium, $(0, 0, 0, 0)$, always exists. Same as what we did in the simple model, we are going to look for participatory equilibrium, where $\phi^\circ = 1$ and $q^\circ > 0$. All we need to do is to check the four conditions specified in Definition II2.1.

First, we look at retailer's problem: Given belief $\hat{\phi} = 1$, the retailer maximizes total profit $\pi = pE \min(\bar{G}(p)D, q) - cq + [w\bar{G}(p - h_r) - rG(p - h_r)]E(D - \frac{q}{\bar{G}(p)})^+$
 $= (p - w\frac{\bar{G}(p-h_r)}{\bar{G}(p)} + r\frac{G(p-h_r)}{\bar{G}(p)})E \min(\bar{G}(p)D, q) - cq + (w\bar{G}(p - h_r) - rG(p - h_r))ED$, which is a typical newsvendor problem (plus a constant $(w\bar{G}(p - h_r) - rG(p - h_r))ED$), and therefore the optimal order quantity $q^\circ = \bar{G}(p)\bar{F}^{-1}\left(\frac{c}{p-w\frac{\bar{G}(p-h_r)}{\bar{G}(p)}+r\frac{G(p-h_r)}{\bar{G}(p)}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q^\circ) = E \min(\phi D, q^\circ/\bar{G}(p))/E(\phi D) = E \min(D, \bar{F}^{-1}(\frac{c}{p-w\frac{\bar{G}(p-h_r)}{\bar{G}(p)}+r\frac{G(p-h_r)}{\bar{G}(p)}}))/ED$.

Finally, we go back to consumer's decision. To ensure $\phi = 1$, we need $u_s > u_o$, which gives us the condition $h_s < h_s^\circ$. □

Proof of Proposition II2.2: Let's look for participatory RE equilibrium, where $\phi = 1$.

First, we look at retailer's problem: Given belief $\hat{\phi} = 1$, the retailer maximizes total profit $\pi = pE \min(\bar{G}(p)D, q) - cq + w\bar{G}(p + h_o)E(D - \frac{q}{\bar{G}(p)})^+ = (p - w\frac{\bar{G}(p+h_o)}{\bar{G}(p)})E \min(\bar{G}(p)D, q) - cq + (w\bar{G}(p + h_o))ED$, which is a typical newsvendor problem (plus a constant $((w\bar{G}(p + h_o))ED)$), and therefore the optimal order quantity $q^p = \bar{G}(p)\bar{F}^{-1}\left(\frac{c}{p-w\frac{\bar{G}(p+h_o)}{\bar{G}(p)}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = A(q^p) = E \min(\phi D, q^p/\bar{G}(p))/E(\phi D) = E \min(D, \bar{F}^{-1}(\frac{c}{p-w\frac{\bar{G}(p+h_o)}{\bar{G}(p)}}))/ED$.

Finally, we go back to consumer's decision. To ensure $\phi = 1$, we need $u_s > u_o$, which gives us the condition $h_s < h_s^p$.

Note $\frac{\partial h_s^\circ}{\partial r} \geq 0$ and $\frac{\partial h_s^p}{\partial r} = 0$. Also, when $r = 0$, we have $h_s^\circ < h_s^p$. Therefore, $\exists \bar{r} \geq 0$ such that $h_s^p > h_s^\circ$ if and only if $r < \bar{r}$. □

Proof of Proposition II2.3: If $h_s^p < h_s^\circ$ and $h_s \in [h_s^p, h_s^\circ]$, then

$$\begin{aligned} \pi^\circ &= pE \min(\bar{G}(p)D, q^\circ) - cq^\circ + [w\bar{G}(p - h_r) - rG(p - h_r)]E\left(D - \frac{q^\circ}{\bar{G}(p)}\right)^+ \\ &> [w\bar{G}(p - h_r) - rG(p - h_r)]ED = \pi^p \end{aligned}$$

where the inequality is because $q = 0$ is a feasible but not the optimal solution to the case where there is no physical showroom. □

Proof of Proposition II2.4: Since this is just the base model with a new set of parameters G' and D' , similar to the proof of Proposition II2.1, we can show that the participatory equilibrium exists if and only if $h_s < h_s^v$.

Next, let's show that $h_s^v < h_s^\circ$: Note that

$$\begin{aligned}\xi^v &= \frac{E_{D'} \min \left(D', \bar{F}'^{-1} \left(\frac{c}{p-w \frac{\bar{G}'(p-h_r)}{\bar{G}'(p)} + r \frac{\bar{G}'(p-h_r)}{\bar{G}'(p)}} \right) \right)}{E_{D'} D'} \\ &= \frac{E_D \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w \frac{\bar{G}(p-h_r)}{\bar{G}(p)} + r \frac{\bar{G}(p-h_r) - \bar{G}(\bar{v})}{\bar{G}(p)}} \right) \right)}{E_D D} \\ &< \frac{E_D \min \left(D, \bar{F}^{-1} \left(\frac{c}{p-w \frac{\bar{G}(p-h_r)}{\bar{G}(p)} + r \frac{\bar{G}(p-h_r)}{\bar{G}(p)}} \right) \right)}{E_D D} = \xi^\circ\end{aligned}$$

and then,

$$\begin{aligned}h_s^v &= \xi^v (E'_V \max(V-p, 0) + h_o - E'_V \max(V-p, -h_r)) \\ &= \xi^v \left(\frac{E_V \max(V-p, 0) - E_V \max(V-p+h_r, 0)}{1-G(\bar{v})} + h_o + h_r \right) \\ &< \xi^\circ \left(\frac{E_V \max(V-p, 0) - E_V \max(V-p+h_r, 0)}{1-G(\bar{v})} + h_o + h_r \right) \\ &< \xi^\circ (E_V \max(V-p, 0) - E_V \max(V-p+h_r, 0) + h_o + h_r) = h_s^\circ\end{aligned}$$

□

Proof of Proposition II2.5: If $h_s \in [h_s^v, h_s^\circ)$, then

$$\begin{aligned}\pi^\circ - \pi^v &= pE \min(\bar{G}(p)D, q^\circ) - cq^\circ + [w\bar{G}(p-h_r) - rG(p-h_r)] E \left(D - \frac{q^\circ}{\bar{G}(p)} \right)^+ \\ &\quad - [w\bar{G}(p-h_r) - r(G(p-h_r) - G(\bar{v}))] ED \\ &= \left[p - w \frac{\bar{G}(p-h_r)}{\bar{G}(p)} + r \frac{G(p-h_r)}{\bar{G}(p)} \right] E \min(\bar{G}(p)D, q^\circ) - cq^\circ - rG(\bar{v}) ED\end{aligned}$$

By Envelop Theorem, we have $\frac{\partial(\pi^\circ - \pi^v)}{\partial w} = -\frac{\bar{G}(p-h_r)}{\bar{G}(p)} E \min(\bar{G}(p)D, q^\circ) < 0$. To conclude the result, we just need to note that it is indeed possible to have $\pi^\circ > \pi^v$, e.g., when α is very close to 0. □

Proof of Proposition II2.6: Solving for $u_{s,in} > u_o$, we have the condition $h_s < h_s^a$. In this case, the retailer's profit is

$$\pi = pE \min(\bar{G}(p)D, q) - cq + [w\bar{G}(p-h_r) - rG(p-h_r)] E \left(D - \frac{q}{\bar{G}(p)} \right)^+$$

Thus, $q^a = \bar{G}(p) \bar{F}^{-1} \left(\frac{c}{p-w \frac{\bar{G}(p-h_r)}{\bar{G}(p)} + r \frac{G(p-h_r)}{\bar{G}(p)}} \right)$.

If $u_{s,in} \leq u_o$, i.e., $h_s \geq h_s^a$, then no one comes to store. So clearly, $q^a = 0$. □

Proof of Proposition II2.7:

- If $h_s < \xi^\circ(E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r))$,

$$\begin{aligned}\pi^a &= pE \min(\bar{G}(p)D, q^a) - cq^a + [w\bar{G}(p-h_r) - rG(p-h_r)] E \left(D - \frac{q^a}{\bar{G}(p)} \right)^+ \\ &= pE \min(\bar{G}(p)D, q^\circ) - cq^\circ + [w\bar{G}(p-h_r) - rG(p-h_r)] E \left(D - \frac{q^\circ}{\bar{G}(p)} \right)^+ = \pi^\circ\end{aligned}$$

- If $\xi^\circ(E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)) \leq h_s < E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)$, then

$$\begin{aligned}\pi^a &= pE \min(\bar{G}(p)D, q^a) - cq^a + [w\bar{G}(p-h_r) - rG(p-h_r)] E \left(D - \frac{q^a}{\bar{G}(p)} \right)^+ \\ &> [w\bar{G}(p-h_r) - rG(p-h_r)] ED = \pi^\circ\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is availability information.

- If $h_s \geq E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)$, then $\pi^a = [w\bar{G}(p-h_r) - rG(p-h_r)] ED = \pi^\circ$.

□

Proof of Proposition II2.8: • If $h_s \leq E_V \max(V-p-h_o, 0) + h_o - E_V \max(V-p, -h_r)$, then

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \left\{ pE_D \min(\bar{G}(p)D, q) - cq + w\bar{G}(p+h_o) E_D \left(D - \frac{q}{\bar{G}(p)} \right)^+ \right\} \\ &= pE_D \min(\bar{G}(p)D, q^p) - cq^p + w\bar{G}(p+h_o) E_D \left(D - \frac{q^p}{\bar{G}(p)} \right)^+ = \pi^p\end{aligned}$$

- If $E_V \max(V-p-h_o, 0) + h_o - E_V \max(V-p, -h_r) < h_s \leq E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)$, then,

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \left\{ pE_D \min(\bar{G}(p)D, q) - cq + [w\bar{G}(p-h_r) - rG(p-h_r)] E_D \left(D - \frac{q}{\bar{G}(p)} \right)^+ \right\} \\ &= pE_D \min(\bar{G}(p)D, q^a) - cq^a + [w\bar{G}(p-h_r) - rG(p-h_r)] E_D \left(D - \frac{q^a}{\bar{G}(p)} \right)^+ = \pi^a\end{aligned}$$

- If $h_s > E_V \max(V-p, 0) + h_o - E_V \max(V-p, -h_r)$, then $\pi = [w\bar{G}(p-h_r) - rG(p-h_r)] E_D D = \pi^a$.

□

Proof of Proposition II2.9: Similar to the proof of Proposition II2.2, we can get the RE equilibrium of the case when there are both physical and virtual showrooms. We just need to replace D and G with D' and G' . Thus, the RE equilibrium is given as follows:

With physical and virtual showrooms, if $h_s \leq h_s^{pv}$, then consumers visit store (i.e., $\phi^{pv} = 1$), and

$q^{pv} = \bar{G}'(p) \bar{F}'^{-1} \left(\frac{c}{p-w \frac{\bar{G}'(p+h_o)}{\bar{G}'(p)}} \right)$, where $h_s^{pv} = \xi^{pv} E'_V \max(V-p, 0) + (1-\xi^{pv}) E'_V \max(V-p-h_o, 0) + h_o - E'_V \max(V-p, -h_r)$ and $\xi^{pv} = \frac{E_{D'} \min \left(D', \bar{F}'^{-1} \left(\frac{c}{p-w \frac{\bar{G}'(p+h_o)}{\bar{G}'(p)}} \right) \right)}{E_{D'} D'}$; otherwise, no one comes to store (i.e., $\phi^{pv} = 0$), and $q^{pv} = 0$.

Note that $\frac{\bar{G}'(p+h_o)}{\bar{G}'(p)} = \frac{\bar{G}(p+h_o)}{\bar{G}(p)}$. Then, we can find $\xi^{pv} = \xi^p$.

$$\begin{aligned} h_s^{pv} &= \xi^{pv} E'_V \max(V-p, 0) + (1-\xi^{pv}) E'_V \max(V-p-h_o, 0) + h_o - E'_V \max(V-p, -h_r) \\ &= \frac{\xi^{pv} E_V \max(V-p, 0) + (1-\xi^{pv}) E_V \max(V-p-h_o, 0) - E_V \max(V-p+h_r, 0)}{1-G(\bar{v})} + h_o + h_r \\ &< \xi^{pv} E_V \max(V-p, 0) + (1-\xi^{pv}) E_V \max(V-p-h_o, 0) - E_V \max(V-p+h_r, 0) + h_o + h_r \\ &= \xi^p E_V \max(V-p, 0) + (1-\xi^p) E_V \max(V-p-h_o, 0) - E_V \max(V-p+h_r, 0) + h_o + h_r = h_s^p \end{aligned}$$

- If $h_s \leq h_s^{pv}$, then

$$\begin{aligned} \pi^{pv} &= pE \min \left(\bar{G}'(p) D', q^{pv} \right) - cq^{pv} + w \bar{G}'(p+h_o) E \left(D' - \frac{q^{pv}}{\bar{G}'(p)} \right)^+ \\ &= pE \min \left(\bar{G}(p) D, q^{pv} \right) - cq^{pv} + w \bar{G}(p+h_o) E \left(D - \frac{q^{pv}}{\bar{G}(p)} \right)^+ \\ &= pE \min \left(\bar{G}(p) D, q^p \right) - cq^p + w \bar{G}(p+h_o) E \left(D - \frac{q^p}{\bar{G}(p)} \right)^+ = \pi^p \end{aligned}$$

- If $h_s \in (h_s^{pv}, \max(h_s^{pv}, h_s^v)]$, then

$$\begin{aligned} \pi^{pv} &= [w \bar{G}'(p-h_r) - r G'(p-h_r)] E_{D'} D' \\ &< pE \min \left(\bar{G}'(p) D', q^v \right) - cq^v + [w \bar{G}'(p-h_r) - r G'(p-h_r)] E_{D'} \left(D' - \frac{q^v}{\bar{G}'(p)} \right)^+ = \pi^v \end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case when there is only virtual showroom.

- If $h_s > \max(h_s^{pv}, h_s^v)$, then $\pi^{pv} = [w \bar{G}'(p-h_r) - r G'(p-h_r)] E_{D'} D' = \pi^v$.

□

Proof of Proposition II3.1: Note nonparticipatory equilibrium, $(0, 0, 0, 0, 0)$, always exists. Same as what we did in the paper, we are going to look for participatory equilibrium, where at least one of ϕ_i and ϕ_u is 1.

- Let's first look for conditions where $\phi_u = \phi_i = 1$ is an equilibrium.

Given $\hat{\phi}_u = \hat{\phi}_i = 1$, then the retailer's profit function is

$$\begin{aligned}\pi(q) &= pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq \\ &\quad + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \\ &= \left[p - w + r \frac{(1 - \theta)(1 - \lambda)}{\lambda + (1 - \lambda)\theta} \right] E \min((\lambda + (1 - \lambda)\theta)D, q) - cq \\ &\quad + [w(\theta(1 - \lambda) + \lambda) - r(1 - \theta)(1 - \lambda)]ED\end{aligned}$$

which is a typical newsvendor problem (plus a constant $[w(\theta(1 - \lambda) + \lambda) - r(1 - \theta)(1 - \lambda)]ED$), and therefore the optimal order quantity $q^\circ = (\lambda + (1 - \lambda)\theta)\bar{F}^{-1}\left(\frac{c}{p - w + r\frac{(1 - \theta)(1 - \lambda)}{\lambda + (1 - \lambda)\theta}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e.,

$$\hat{\xi}^\circ = \frac{E \min((\lambda + (1 - \lambda)\theta)D, q^\circ)}{E(\lambda + (1 - \lambda)\theta)D} = \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c}{p - w + r\frac{(1 - \theta)(1 - \lambda)}{\lambda + (1 - \lambda)\theta}}\right)\right)}{ED} = \xi_1^\circ.$$

Finally, we go back to consumer's decision. To ensure $\phi_u = \phi_i = 1$, we need $u_{s,u} \geq u_{o,u}$ and $u_{s,i} > u_{o,i}$, which gives us the condition $h_s < \xi_1^\circ h_o$.

- Next, let's look for the condition where $\phi_i = 0$ and $\phi_u = 1$.

Given $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 1$, the retailer's profit is

$$\begin{aligned}\pi(q) &= pE \min((1 - \lambda)\theta D, q) - cq + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ + wE\lambda D \\ &= \left[p - w + r \frac{1 - \theta}{\theta} \right] E \min((1 - \lambda)\theta D, q) - cq + [(w\theta - r(1 - \theta))(1 - \lambda) + w\lambda]ED\end{aligned}$$

which is a typical newsvendor problem (plus a constant $[(w\theta - r(1 - \theta))(1 - \lambda) + w\lambda]ED$), and therefore the optimal order quantity $q^\circ = (1 - \lambda)\theta\bar{F}^{-1}\left(\frac{c}{p - w + r\frac{1 - \theta}{\theta}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e.,

$$\hat{\xi}^\circ = \frac{E \min((1 - \lambda)\theta D, q^\circ)}{E(1 - \lambda)\theta D} = \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c}{p - w + r\frac{1 - \theta}{\theta}}\right)\right)}{ED} = \xi_2^\circ.$$

Finally, we go back to consumer's decision. To sure $\phi_u = 1$ and $\phi_i = 0$, we need $u_{s,u} > u_{o,u}$ and $u_{s,i} \leq u_{o,i}$, which gives us the condition $\xi_2^\circ \leq h_s < \xi_2^\circ[h_o + (1 - \theta)h_r]$.

In the end, we note that $\xi_2^\circ \geq \xi_1^\circ$. Thus, the conditions for the two types of participatory RE equilibrium is disjoint. Then, the participatory RE equilibrium is unique. Also, for all the other cases, we only have nonparticipatory equilibrium. \square

Proof of Proposition II3.2: Let's look for participatory RE equilibrium,, where at least one of ϕ_i and ϕ_u is 1.

- Let's first look for conditions where $\phi_u = \phi_i = 1$ is an equilibrium.

Given $\hat{\phi}_u = \hat{\phi}_i = 1$, the retailer's profit is

$$\begin{aligned}\pi &= pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+ \\ &= (p - w)E \min((\lambda + (1 - \lambda)\theta)D, q) - cq + w(\lambda + (1 - \lambda)\theta)ED\end{aligned}$$

which is a typical newsvendor problem (plus a constant $w(\lambda + (1 - \lambda)\theta)ED$), and therefore the optimal order quantity $q^p = (\lambda + (1 - \lambda)\theta)\bar{F}^{-1}\left(\frac{c}{p-w}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = \frac{E \min((\lambda + (1 - \lambda)\theta)D, q^p)}{E(\lambda + (1 - \lambda)\theta)D} = \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}$.

Finally, we go back to consumer's decision. To ensure $\phi_u = \phi_i = 1$, we need $u_{s,u} \geq u_{o,u}$ and $u_{s,i} > u_{o,i}$, which gives us the condition $h_s < \xi^p h_o$.

- Next, let's look for the condition where $\phi_i = 0$ and $\phi_u = 1$ is an equilibrium.

Given $\hat{\phi}_i = 0$ and $\hat{\phi}_u = 1$, the retailer's profit is

$$\begin{aligned}\pi(q) &= pE \min((1 - \lambda)\theta D, q) - cq + wE[(1 - \lambda)D - q]^+ + wE\lambda D \\ &= (p - w)E \min((1 - \lambda)\theta D, q) - cq + [w(1 - \lambda)\theta + w\lambda]ED\end{aligned}$$

which is a typical newsvendor problem (plus a constant $[w(1 - \lambda)\theta + w\lambda]ED$), and therefore the optimal order quantity $q^p = (1 - \lambda)\theta\bar{F}^{-1}\left(\frac{c}{p-w}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^p = \frac{E \min((1 - \lambda)\theta D, q^p)}{E(1 - \lambda)\theta D} = \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}$.

Finally, we go back to consumer's decision. To sure $\phi_u = 1$ and $\phi_i = 0$, we need $u_{s,u} > u_{o,u}$ and $u_{s,i} \leq u_{o,i}$, which gives us the condition $\xi^p \leq h_s < \xi^p \theta h_o + (1 - \theta)(h_o + h_r)$.

□

Proof of Proposition II3.3: If $\max\{\xi_2^\circ h_o, \xi^p \theta h_o + (1 - \theta)(h_o + h_r)\} < h_s < \xi_2^\circ [h_o + (1 - \theta)h_r]$, then

$$\begin{aligned}\pi^p &= [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D \\ &< pE \min((1 - \lambda)\theta D, q^\circ) - cq^\circ + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q^\circ}{\theta} \right]^+ + wE\lambda D = \pi^\circ\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is no physical showroom. □

Proof of Proposition II3.4: Since this is just the base model with a new set of parameters D', θ', λ' , the proof is similar to the proof of Proposition II3.1, and thus omitted. □

Proof of Proposition II3.5: If $\max\{\xi_2^\circ h_o, \xi_2^v [h_o + (1 - \theta')h_r]\} \leq h_s < \xi_2^\circ [h_o + (1 - \theta)h_r]$, then

$$\begin{aligned}\pi^\circ - \pi^v &= \left(p - w + r\frac{1 - \theta}{\theta}\right)E \min((1 - \lambda)\theta D, q^\circ) - cq^\circ \\ &\quad + ((w\theta - r(1 - \theta))(1 - \lambda) + w\lambda)ED - ((w\theta' - r(1 - \theta'))(1 - \lambda') + w\lambda')ED' \\ &= \left(p - w + r\frac{1 - \theta}{\theta}\right)E \min((1 - \lambda)\theta D, q^\circ) - cq^\circ - r\alpha(1 - \theta)(1 - \lambda)ED\end{aligned}$$

By Envelop Theorem, we have $\frac{\partial(\pi^\circ - \pi^v)}{\partial w} = -E \min((1 - \lambda)\theta D, q^\circ) < 0$. To conclude the result, we just need to note that it is indeed possible to have $\pi^\circ > \pi^v$, e.g., when α is very close to 0. □

Proof of Proposition II3.6:

- If $h_s < h_o$, then the retailer's profit can be expressed as

$$\pi = \left[p - w + r \frac{(1-\theta)(1-\lambda)}{\lambda + (1-\lambda)\theta} \right] E \min((\lambda + (1-\lambda)\theta)D, q) - cq + [w(\theta(1-\lambda) + \lambda) - r(1-\theta)(1-\lambda)] ED,$$

which is a newsvendor profit (plus a constant $[w(\theta(1-\lambda) + \lambda) - r(1-\theta)(1-\lambda)] ED$). Thus, the optimal inventory level is $q^a = (\lambda + (1-\lambda)\theta) \bar{F}^{-1} \left(\frac{c}{p-w+r \frac{(1-\lambda)(1-\theta)}{\lambda+(1-\lambda)\theta}} \right)$;

- If $h_o \leq h_s < h_o + (1-\theta)h_r$, then the retailer's profit can be expressed as

$$\pi = \left[p - w + r \frac{1-\theta}{\theta} \right] E \min((1-\lambda)\theta D, q) - cq + [w(\theta(1-\lambda) + \lambda) - r(1-\theta)(1-\lambda)] ED,$$

which is a newsvendor profit (plus a constant $[w(\theta(1-\lambda) + \lambda) - r(1-\theta)(1-\lambda)] ED$). Thus, the optimal inventory level is $q^a = (1-\lambda)\theta \bar{F}^{-1} \left(\frac{c}{p-w+r \frac{1-\theta}{\theta}} \right)$;

- If $h_s \geq h_o + (1-\theta)h_r$, then no one ever comes to store, and thus $q^a = 0$.

□

Proof of Proposition II3.7:

- If $h_s < \xi_1^\circ h_o$, then

$$\begin{aligned} \pi^a &= \max_{q \geq 0} \left\{ pE \min((\lambda + (1-\lambda)\theta)D, q) - cq + [w\theta - r(1-\theta)]E(1-\lambda) \left[D - \frac{q}{\lambda + (1-\lambda)\theta} \right]^+ \right. \\ &\quad \left. + wE\lambda \left(D - \frac{q}{\lambda + (1-\lambda)\theta} \right)^+ \right\} \\ &= pE \min((\lambda + (1-\lambda)\theta)D, q^\circ) - cq^\circ + [w\theta - r(1-\theta)]E(1-\lambda) \left[D - \frac{q^\circ}{\lambda + (1-\lambda)\theta} \right]^+ \\ &\quad + wE\lambda \left(D - \frac{q^\circ}{\lambda + (1-\lambda)\theta} \right)^+ \\ &= \pi^\circ \end{aligned}$$

- If $\xi_1^\circ h_o < h_s < h_o$,

– if $\xi_2^\circ < h_s \leq \xi_2^\circ[h_o + (1 - \theta)h_r]$, then

$$\begin{aligned}
\pi^a &= \max_{q \geq 0} \left\{ pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \right. \\
&\quad \left. + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \right\} \\
&\geq pE \min((\lambda + (1 - \lambda)\theta)D, q^\circ) - cq^\circ + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q^\circ}{\lambda + (1 - \lambda)\theta} \right]^+ \\
&\quad + wE\lambda \left(D - \frac{q^\circ}{\lambda + (1 - \lambda)\theta} \right)^+ \\
&= \left[p - w + r(1 - \theta) \frac{1 - \lambda}{\lambda + (1 - \lambda)\theta} \right] E \min((\lambda + (1 - \lambda)\theta)D, q^\circ) - cq^\circ \\
&\quad + [(w\theta - r(1 - \theta))(1 - \lambda) + w\lambda] ED \\
&\geq \left[p - w + r(1 - \theta) \frac{1}{\theta} \right] E \min((1 - \lambda)\theta D, q^\circ) - cq^\circ + [(w\theta - r(1 - \theta))(1 - \lambda) + w\lambda] ED \\
&= \pi^\circ
\end{aligned}$$

– otherwise, then

$$\begin{aligned}
\pi^a &= \max_{q \geq 0} \left\{ pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \right. \\
&\quad \left. + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \right\} \\
&\geq [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D \\
&= \pi^\circ
\end{aligned}$$

where the inequality is because $q = 0$ is a feasible but not the optimal solution to the case where there is availability information.

• If $h_o < h_s < h_o + (1 - \theta)h_r$, then

– if $\xi_2^\circ < h_s \leq \xi_2^\circ[h_o + (1 - \theta)h_r]$, then

$$\begin{aligned}
\pi^a &= \max_{q \geq 0} \left\{ pE \min((1 - \lambda)\theta D, q) - cq + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ + wE\lambda D \right\} \\
&= pE \min((1 - \lambda)\theta D, q^\circ) - cq^\circ + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q^\circ}{\theta} \right]^+ + wE\lambda D \\
&= \pi^\circ
\end{aligned}$$

– otherwise, then

$$\begin{aligned}
\pi^a &= \max_{q \geq 0} \left\{ pE \min((1 - \lambda)\theta D, q) - cq + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ + wE\lambda D \right\} \\
&\geq [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D \\
&= \pi^\circ
\end{aligned}$$

where the inequality is because $q = 0$ is a feasible but not the optimal solution to the case

where there is availability information.

- If $h_s > h_o + (1 - \theta)h_r$, then $\pi^a = [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D = \pi^\circ$.

□

Proof of Proposition II3.8:

- If $h_s < \min(\xi^p h_o, h_o, (1 - \theta)(h_o + h_r))$, then

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \{pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+\} \\ &= pE \min((\lambda + (1 - \lambda)\theta)D, q^p) - cq^p + wE((\lambda + (1 - \lambda)\theta)D - q^p)^+ \\ &= \pi^p\end{aligned}$$

- If $\xi^p h_o < h_s < \min(h_o, (1 - \theta)(h_o + h_r))$, then

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \{pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+\} \\ &\geq pE \min((\lambda + (1 - \lambda)\theta)D, q^p) - cq^p + wE((\lambda + (1 - \lambda)\theta)D - q^p)^+ \\ &= (p - w)E \min((\lambda + (1 - \lambda)\theta)D, q^p) - cq^p + w(\lambda + (1 - \lambda)\theta)ED \\ &> (p - w)E \min((1 - \lambda)\theta D, q^p) - cq^p + w(\lambda + (1 - \lambda)\theta)ED \\ &= \pi^p\end{aligned}$$

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \{pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+\} \\ &\geq pE \min((\lambda + (1 - \lambda)\theta)D, q^a) - cq^a + wE((\lambda + (1 - \lambda)\theta)D - q^a)^+ \\ &= (p - w)E \min((\lambda + (1 - \lambda)\theta)D, q^a) - cq^a + w(\lambda + (1 - \lambda)\theta)ED \\ &> (p - w)E \min((\lambda + (1 - \lambda)\theta)D, q^a) - cq^a + w(\lambda + (1 - \lambda)\theta)ED \\ &\quad - r(1 - \theta)(1 - \lambda)[ED - E \min(D, \frac{q^a}{\lambda + (1 - \lambda)\theta})] \\ &= \pi^a\end{aligned}$$

- If $h_o < (1 - \theta)(h_o + h_r)$ and $h_o < h_s < (1 - \theta)(h_o + h_r)$, then

$$\begin{aligned}\pi^{pa} &= \max_{q \geq 0} \{pE \min((1 - \lambda)\theta D, q) - cq + wE((1 - \lambda)\theta D - q)^+ + wE\lambda D\} \\ &= pE \min((1 - \lambda)\theta D, q^p) - cq^p + wE((1 - \lambda)\theta D - q^p)^+ + wE\lambda D \\ &= \pi^p\end{aligned}$$

- If $h_o > (1 - \theta)(h_o + h_r)$ and $(1 - \theta)(h_o + h_r) < h_s < h_o$, then

$$\begin{aligned}
\pi^{pa} &= \max_{q \geq 0} \left\{ pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q}{\lambda + (1 - \lambda)\theta} \right]^+ \right. \\
&\quad \left. + wE\lambda \left(D - \frac{q}{\lambda + (1 - \lambda)\theta} \right)^+ \right\} \\
&= pE \min((\lambda + (1 - \lambda)\theta)D, q^a) - cq^a + [w\theta - r(1 - \theta)]E(1 - \lambda) \left[D - \frac{q^a}{\lambda + (1 - \lambda)\theta} \right]^+ \\
&\quad + wE\lambda \left(D - \frac{q^a}{\lambda + (1 - \lambda)\theta} \right)^+ \\
&= \pi^a
\end{aligned}$$

- If $\max(h_o, (1 - \theta)(h_o + h_r)) < h_s < h_o + (1 - \theta)h_r$, then

$$\begin{aligned}
\pi &= \max_{q \geq 0} \left\{ pE \min((1 - \lambda)\theta D, q) - cq + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q}{\theta} \right]^+ + wE\lambda D \right\} \\
&= pE \min((1 - \lambda)\theta D, q^a) - cq^a + [w\theta - r(1 - \theta)]E \left[(1 - \lambda)D - \frac{q^a}{\theta} \right]^+ + wE\lambda D \\
&= \pi^a
\end{aligned}$$

- If $h_s > h_o + (1 - \theta)h_r$, then $\pi^{pa} = [w\theta - r(1 - \theta)]E(1 - \lambda)D + wE\lambda D = \pi^a$.

□

Proof of Proposition II3.9: Similar to the proof of Proposition II3.2, we can get the RE equilibrium of the case when there are both physical and virtual showrooms. We just need to replace D, θ, λ with D', θ', λ' . Thus, the RE equilibrium is given as follows:

- if $h_s \leq \xi^v h_o$, then all customers come to store, i.e., $\phi_i^v = 1, \phi_u^v = 1$
- if $\xi^v h_o < h_s \leq \xi^v \theta' h_o + (1 - \theta')(h_o + h_r)$, then only uninformed customers come to store, i.e., $\phi_i^v = 0, \phi_u^v = 1$;
- otherwise, no one comes to store, i.e., $\phi_i^v = 0, \phi_u^v = 0$,

where $\xi^v = \frac{E \min\left(D', \bar{F}'^{-1}\left(\frac{c}{p-w}\right)\right)}{ED'}$.

□

It is easy to find that $\xi^v = \xi^p$.

- If $h_s < \xi^p h_o$, then

$$\begin{aligned}
\pi^{pv} &= \max_{q \geq 0} \left\{ pE \min((\lambda' + (1 - \lambda')\theta')D', q) - cq + wE((\lambda' + (1 - \lambda')\theta')D' - q)^+ \right\} \\
&= \max_{q \geq 0} \left\{ pE \min((\lambda + (1 - \lambda)\theta)D, q) - cq + wE((\lambda + (1 - \lambda)\theta)D - q)^+ \right\} \\
&= pE \min((\lambda + (1 - \lambda)\theta)D, q^p) - cq^p + wE((\lambda + (1 - \lambda)\theta)D - q^p)^+ \\
&= \pi^p
\end{aligned}$$

- If $\xi^p h_o < h_s < \xi^p \theta' h_o + (1 - \theta')(h_o + h_r)$, then

$$\begin{aligned}
\pi^{pv} &= \max_{q \geq 0} \left\{ pE \min \left((1 - \lambda') \theta' D', q \right) - cq + w\theta' E \left((1 - \lambda') D - \frac{q}{\theta'} \right)^+ + wE\lambda' D' \right\} \\
&= \max_{q \geq 0} \left\{ pE \min \left((1 - \lambda) \theta D, q \right) - cq + w\theta E \left((1 - \lambda) D - \frac{q}{\theta} \right)^+ + wE\lambda D \right\} \\
&= pE \min \left((1 - \lambda) \theta D, q^p \right) - cq^p + w\theta E \left((1 - \lambda) D - \frac{q^p}{\theta} \right)^+ + wE\lambda D \\
&= \pi^p
\end{aligned}$$

- If $\xi^p \theta' h_o + (1 - \theta')(h_o + h_r) < h_s < \xi^p \theta h_o + (1 - \theta)(h_o + h_r)$, then

$$\begin{aligned}
\pi^{pv} &= [w\theta' - r(1 - \theta')] E(1 - \lambda') D + wE\lambda' D' \\
&\leq w\theta' E(1 - \lambda') D' + wE\lambda' D' \\
&= w\theta E(1 - \lambda) D + wE\lambda D \\
&\leq pE \min \left((1 - \lambda) \theta D, q^p \right) - cq^p + w\theta E \left((1 - \lambda) D - \frac{q^p}{\theta} \right)^+ + wE\lambda D \\
&= \pi^p
\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is only physical showroom.

- If $h_s > \xi^p \theta h_o + (1 - \theta)(h_o + h_r)$, then $\pi^{pv} = [w\theta' - r(1 - \theta')] E(1 - \lambda') D' + wE\lambda' D'$. Let's compare π^{pv} with π^v . Note π^v can take only three values:

- if $\phi_i^v = \phi_u^v = 1$, then

$$\begin{aligned}
\pi^v &= pE \min \left((\lambda' + (1 - \lambda') \theta') D', q^v \right) - cq^v \\
&\quad + [w\theta' - r(1 - \theta')] E(1 - \lambda') \left[D' - \frac{q^v}{\lambda' + (1 - \lambda') \theta'} \right]^+ + wE\lambda' \left(D' - \frac{q^v}{\lambda' + (1 - \lambda') \theta'} \right)^+ \\
&\geq [w\theta' - r(1 - \theta')] E(1 - \lambda') D' + wE\lambda' D' = \pi^{pv}
\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is only virtual showroom;

- if $\phi_i^v = 0$ and $\phi_u^v = 1$, then

$$\begin{aligned}
\pi^v &= pE \min \left((1 - \lambda') \theta' D', q^v \right) - cq^v + [w\theta' - r(1 - \theta')] E \left[(1 - \lambda') D' - \frac{q^v}{\theta'} \right]^+ + wE\lambda' D' \\
&\geq [w\theta' - r(1 - \theta')] E(1 - \lambda') D' + wE\lambda' D' = \pi^{pv}
\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is only virtual showroom;

- if $\phi_i^v = \phi_u^v = 0$, then $\pi^v = [w\theta' - r(1 - \theta')] E(1 - \lambda') D' + wE\lambda' D' = \pi^{pv}$.

References

Gao, F. and X. Su (2016). Omnichannel retail operations with buy-online-and-pickup-in-store. *Management Science*, Forthcoming.

Supplementary Appendix III: Numerical Study

The goals of this numerical study are as follows:

1. Investigate the likelihood of $\pi^o > \pi^p$;
2. Investigate the likelihood of $\pi^o > \pi^v$.

Here, we consider the following parameter values:

- $v = 60$
- $p = 30, w = 15$
- $\frac{p-c}{w} = \{0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4\}$
- $r = \{1, 3, 5, 7, 9\}$
- $h_o = \{1, 2, 3, 4, 5\}$
- $h_s = \{1, 2, 3, 4, 5\}$
- $h_r = \{1, 2, 3, 4, 5\}$
- $D \sim N(\mu, \sigma^2)$, where $\mu = 100, \frac{\sigma}{\mu} = \{\frac{1}{4}, \frac{1}{4.5}, \frac{1}{5}, \frac{1}{5.5}, \frac{1}{6}, \frac{1}{6.5}, \frac{1}{7}, \frac{1}{7.5}, \frac{1}{8}\}$
- $\alpha = \{0.1, 0.3, 0.5, 0.7, 0.9\}$
- $\theta = \{0.1, 0.3, 0.5, 0.7, 0.9\}$

There are 1,265,625 cases in total. After checking with the assumptions we made in the paper, we end up having 914,895 cases. For each case, we calculate the equilibrium profit π^o , π^p and π^v based on the equilibrium outcomes described in the paper.

We are interested in how frequent $\pi^o > \pi^p$ and $\pi^o > \pi^v$ happen.

Among all the 914,895 cases, we find that 4.34% of them have $\pi^o > \pi^p$, and 4.58% of them have $\pi^o > \pi^v$.

Figures 1 and 2 show how the results change given different profit margin ratios. Specifically, given $\frac{p-c}{w}$, we calculate the fraction of cases where $\pi^o > \pi^p$ (see Figure 1) and $\pi^o > \pi^v$ (see Figure 2). From these two figures, we find a general pattern: physical showrooms are more likely to reduce profits when the store channel is less profitable, while virtual showrooms are more likely to reduce profits when the online channel is less profitable. An implication to retailers: showrooms should be implemented in their strong channels.

Figure 1: Proportion of instances that physical showrooms backfire (i.e., $\pi^o > \pi^p$)

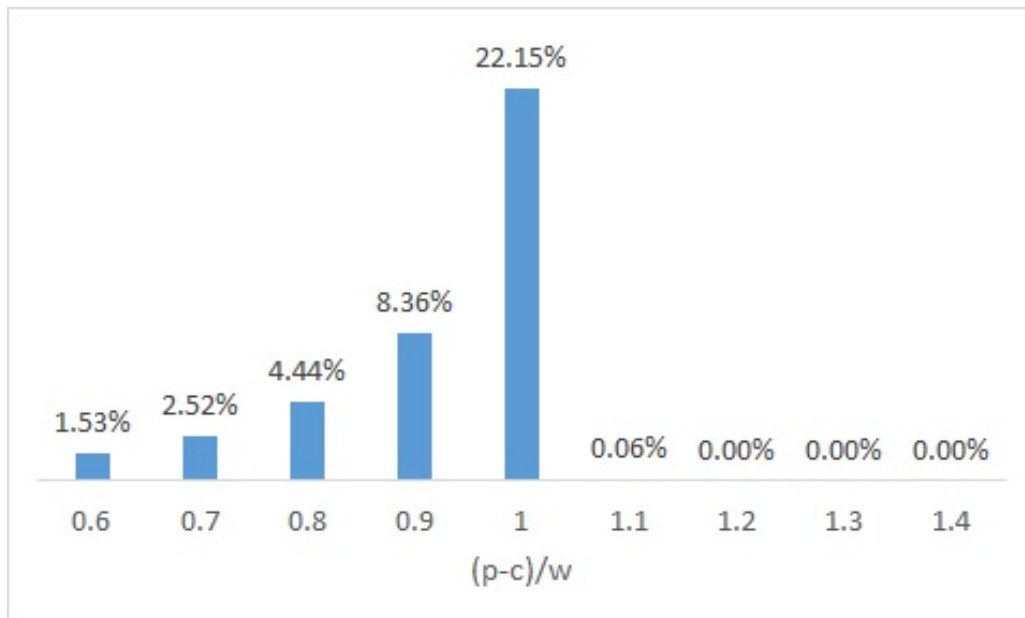


Figure 2: Proportion of instances that virtual showrooms backfire (i.e., $\pi^o > \pi^v$)

