# Would You Like to Upgrade to a Premium Room? Evaluating the Benefit of Offering Standby Upgrades

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## Online Appendix

Proof of Lemma 1:

In this proof, we derive the optimal prices and expected revenues in the myopic case for a given $X_U$.

For each $X_U$ value, we have three candidate pricing strategies for the optimal price set. These strategies have different expected revenue functions and constraints. We first analyze Strategy 1 and Strategy 3 and show the optimal price sets under these, and then use them to construct a solution for Strategy 2.

- For Strategy 1, the expected revenue function can be written as:

$$\Pi_{MY1} = \frac{X_U}{2}[p_S p - p_S^2 + p - p^2] \text{ where } X_U(1-p_S) \leq 1$$

We solve the unconstrained problem first. First order conditions (FOC) give us (the constant is ignored):

$$\frac{\partial \Pi_{MY1}}{\partial p} = p_S + 1 - 2p = 0 \quad \text{and} \quad \frac{\partial \Pi_{MY1}}{\partial p_S} = p - 2p_S = 0$$

The optimal price set is $(p_S, p) = (\frac{1}{3}, \frac{2}{3})$ for the unconstrained problem (the objective function is concave as its Hessian is negative definite). We have two possibilities:

⇒ The constraint is satisfied when $X_U \leq \frac{3}{2}$; therefore, the optimal price set for the unconstrained problem is the optimal price set for the constrained problem in this range.

⇒ The constraint is violated when $X_U > \frac{3}{2}$. Because of the concavity of the expected revenue function, $X_U(1-p_S) = 1$ must be binding at optimality. Since the constraint is not a function of $p$, we can show (through the Lagrange method) that $p_S + 1 - 2p = 0$ must be satisfied. Therefore, $p_S = \frac{X_U - 1}{X_U}$ and $p = \frac{2X_U - 1}{2X_U}$.

For Strategy 1, the optimal price set and resulting expected revenue for different $X_U$ values are as follows:

$$(p_s, p) = \begin{cases} (\frac{1}{3}, \frac{2}{3}) & \text{if } X_U \leq \frac{3}{2} \\ (\frac{X_U - 1}{X_U}, \frac{2X_U - 1}{2X_U}) & \text{if } X_U \geq \frac{3}{2} \end{cases} \qquad \Pi_{MY1} = \begin{cases} \frac{X_U}{6} & \text{if } X_U \leq \frac{3}{2} \\ \frac{4X_U - 3}{8X_U} & \text{if } X_U \geq \frac{3}{2} \end{cases}$$

- For Strategy 3, the expected revenue function can be written as:

$$\Pi_{MY3} = \frac{1}{X_U}\left[pX_U - \frac{p-p_S}{2-2p} - \frac{p_S}{2-2p_S}\right] \text{ where } X_U(1-p) \geq 1$$

We solve the unconstrained problem first. FOC give us (the constant is ignored):

$$\frac{\partial \Pi_{MY3}}{\partial p} = X_U - \frac{1-p_S}{2(1-p)^2} = 0 \quad \text{and} \quad \frac{\partial \Pi_{MY3}}{\partial p_S} = \frac{1}{2-2p} - \frac{1}{2(1-p_S)^2} = 0$$

The optimal price set is $(p_S, p) = \left(1 - \frac{1}{\sqrt[3]{2X_U}}, 1 - \frac{1}{\sqrt[3]{(2X_U)^2}}\right)$ for the unconstrained problem (the objective function is concave as its Hessian is negative definite). We have two possibilities:

⇒ The constraint is satisfied when $X_U \geq 4$; therefore, the optimal price set for the unconstrained problem is the optimal price set for the constrained problem in this range.

⇒ The constraint is violated when $X_U < 4$. Because of the concavity of the expected revenue function, $X_U(1-p) = 1$ must be binding. Since the constraint is not a function of $p_S$, we can show (through Lagrange method) that $\frac{1}{2-2p} - \frac{1}{2(1-p_S)^2} = 0$ must be satisfied. Therefore, $p_S = \frac{\sqrt{X_U}-1}{\sqrt{X_U}}$ and $p = \frac{X_U - 1}{X_U}$.

For Strategy 3, the optimal price set and resulting expected revenue for different $X_U$ values are as follows:

$$(p_S, p) = \begin{cases} \left(\frac{\sqrt{X_U}-1}{\sqrt{X_U}}, \frac{X_U-1}{X_U}\right) & \text{if } X_U \leq 4 \\ \left(1 - \frac{1}{\sqrt[3]{2X_U}}, 1 - \frac{1}{\sqrt[3]{(2X_U)^2}}\right) & \text{if } X_U \geq 4 \end{cases} \qquad \Pi_{MY3} = \begin{cases} \frac{\sqrt{X_U}-1}{\sqrt{X_U}} & \text{if } X_U \leq 4 \\ 1 - \frac{3}{\sqrt[3]{(2X_U)^2}} + \frac{1}{X_U} & \text{if } X_U \geq 4 \end{cases}$$

- We have found two boundary points so far: $X_U = \frac{3}{2}$ and $X_U = 4$. Our expected revenue function for Strategy 2 was as follows:

$$\Pi_{MY2} = \int_0^{1/(1-p_S)} \frac{R_1}{X_U} dx + \int_{1/(1-p_S)}^{X_U} \frac{R_2}{X_U} dx \quad \text{where } X_U(1-p_S) \geq 1 \text{ and } X_U(1-p) \leq 1$$

1) When $X_U \geq \frac{3}{2}$, the optimal price set of Strategy 1 is an asymptotically feasible price set for Strategy 2 since the constraint $X_U(1-p_S) = 1$ is binding.

2) When $X_U \leq 4$, the optimal price set of Strategy 3 is an asymptotically feasible price set for Strategy 2 since the constraint $X_U(1-p) = 1$ is binding.

By definition, the optimal price set under Strategy 2 is the one with the maximum expected revenue of all feasible price sets. Thus, Strategy 2 cannot be worse than Strategies 1 and 3 in the range of $1.5 \leq X_U \leq 4$.

After noting these, the expected revenue function can be written as:

$$\Pi_{MY2} = \frac{1}{X_U}\left[p_S X_U + \frac{X_U^2}{2}(p_S p - p_S + p - p^2) - \frac{p_S}{2-2p_S}\right]$$

We solve the unconstrained problem first. FOC give us:

$$\frac{\partial \Pi_{MY2}}{\partial p} = X_U(p_S + 1 - 2p) = 0 \quad \text{and} \quad \frac{\partial \Pi_{MY2}}{\partial p_S} = 1 - \frac{X_U(1-p)}{2} - \frac{1}{2X_U(1-p_S)^2} = 0$$

The optimal price set must satisfy:

$$p = \frac{p_s + 1}{2} \quad \text{and} \quad p_S = 1 - \frac{4}{X_U} + \frac{2}{X_U^2(1-p_S)^2}$$

We discard the solutions violating $0 \leq p_S \leq p \leq 1$. Our analysis shows that the optimal price set for the unconstrained problem violates $X_U(1-p_S) \geq 1$ and $X_U(1-p) \leq 1$ when $X_U < \frac{3}{2}$ and $X_U > 4$. Moreover,

$X_U(1-p_S)=1$ must be binding for $X_U<\frac{3}{2}$; therefore, $p_S=\frac{X_U-1}{X_U}$. Through Lagrange method, we know that $\frac{\partial \Pi_{MY2}}{\partial p}=0$ must be satisfied in this scenario. Therefore $p=\frac{2X_U-1}{2X_U}$. With the same approach, $X_U(1-p)=1$ must be binding for $X_U>4$; therefore, $p=\frac{X_U-1}{X_U}$. Through Lagrange method, we know that $\frac{\partial \Pi_{MY2}}{\partial p_S}=0$ must be satisfied in this scenario. Therefore, $p_S=\frac{\sqrt{X_U}-1}{\sqrt{X_U}}$.

For Strategy 2, the optimal price set and resulting expected revenue for different $X_U$ values are as follows:

$$(p_S,p)=\begin{cases}\left(\frac{X_U-1}{X_U},\frac{2X_U-1}{2X_U}\right) & \text{if } X_U\leq\frac{3}{2}\\ \left(\frac{3X_U-4}{3X_U}-\frac{8(1-i\sqrt{3})}{3\kappa}-\frac{\kappa(1+i\sqrt{3})}{6X_U^2},\frac{3X_U-2}{3X_U}-\frac{4(1-i\sqrt{3})}{3\kappa}-\frac{\kappa(1+i\sqrt{3})}{12X_U^2}\right) & \text{if } \frac{3}{2}<X_U<4\\ \left(\frac{\sqrt{X_U}-1}{\sqrt{X_U}},\frac{X_U-1}{X_U}\right) & \text{if } X_U\geq 4\end{cases}$$

where $\kappa=\sqrt[3]{27X_U^4-64X_U^3+3\sqrt{3}\sqrt{27X_U^8-128X_U^7}}$.

Let $\kappa=a+ib$, and its complex conjugate $\bar{\kappa}=a-ib$. Our analysis shows that $\kappa\bar{\kappa}=16X_U^2$ when $\frac{3}{2}\leq X_U\leq 4$. Multiplying the second term of $p_S$ in the second row with $\bar{\kappa}$ gives $p_S=\frac{3X_U-4}{3X_U}-\frac{\bar{\kappa}(1-i\sqrt{3})}{6X_U^2}-\frac{\kappa(1+i\sqrt{3})}{6X_U^2}$. The complex parts of the second and third terms cancel out and we have $p_S=\frac{3X_U-4}{3X_U}-\frac{2b\sqrt{3}-2a}{6X_U^2}$. Using the same iterations for $p$, we can show that $p=\frac{3X_U-2}{3X_U}-\frac{b\sqrt{3}-a}{6X_U^2}$. In order to make it easier to follow, we use $\Upsilon=\frac{b\sqrt{3}-a}{6X_U^2}$.

Based on the prices given above, the resulting expected revenue for different $X_U$ values are as follows:

$$\Pi_{MY2}=\begin{cases}\frac{4X_U-3}{8X_U} & \text{if } X_U\leq\frac{3}{2}\\ \frac{9X_U(6\Upsilon(X_U(\Upsilon(X_U\Upsilon-2)+2)-3)+5)-44}{36X_U(3X_U\Upsilon+2)} & \text{if } \frac{3}{2}\leq X_U\leq 4\\ \frac{\sqrt{X_U}-1}{\sqrt{X_U}} & \text{if } X_U\geq 4\end{cases}$$

We compare $\Pi_{MY1}$, $\Pi_{MY2}$ and $\Pi_{MY3}$ for a given $X_U$, which leads to Eq (3) and Eq (4). □

Derivation of Eq.(5):

In this proof, we derive the optimal price and expected revenues for the no-standby case for a given $X_U$. Two possible strategies exist based on $X_U$ values: $X_U(1-p)\leq 1$ and $X_U(1-p)\geq 1$

- For Strategy 1, the expected revenue function can be written as:

$$\Pi_{NS1}=\int_0^{X_U}\frac{x(1-p)p}{X_U}\,dx=\frac{X_U}{2}p(1-p)\ \text{ where }\ X_U(1-p)\leq 1$$

We solve the unconstrained problem first. FOC give us (the constant is ignored) $\frac{\partial \Pi_{NS1}}{\partial p}=1-2p=0$.

Using a similar approach as in Strategy 1 of the myopic case gives:

$$p=\begin{cases}\frac{1}{2} & \text{if } X_U\leq 2\\ \frac{X_U-1}{X_U} & \text{if } X_U\geq 2\end{cases}\qquad \Pi_{NS1}=\begin{cases}\frac{X_U}{8} & \text{if } X_U\leq 2\\ \frac{X_U-1}{2X_U} & \text{if } X_U\geq 2\end{cases}$$

- For Strategy 2, the expected revenue function can be written as:

$$\Pi_{NS2}=\int_0^{1/(1-p)}\frac{x(1-p)p}{X_U}\,dx+\int_{1/(1-p)}^{X_U}\frac{p}{X_U}\,dx=\frac{1}{X_U}\left(pX_U-\frac{p}{2-2p}\right)\ \text{ where }\ X_U(1-p)\geq 1$$

We solve the unconstrained problem first. FOC give us $\frac{\partial \Pi_{NS2}}{\partial p}=X_U-\frac{1}{2(1-p)^2}=0$.

Using a similar approach as in Strategy 3 of the base model gives:

$$p=\begin{cases}\frac{X_U-1}{X_U} & \text{if } X_U\leq 2\\ \frac{\sqrt{2X_U}-1}{\sqrt{2X_U}} & \text{if } X_U\geq 2\end{cases}\qquad \Pi_{NS2}=\begin{cases}\frac{X_U-1}{2X_U} & \text{if } X_U\leq 2\\ 1-\frac{2}{\sqrt{2X_U}}+\frac{1}{2X_U} & \text{if } X_U\geq 2\end{cases}$$

We compare $\Pi_{NS1}$ and $\Pi_{NS2}$ for a given $X_U$ and choose the optimal price, leading us to Eq (5). □

Proof of Proposition 1:
Comparison of the expected revenue functions and optimal price sets in Eq (3), Eq (4) and Eq (5) leads to $\Pi_{MY} > \Pi_{NS}$ and $p_S^{MY} < p^{NS} < p^{MY}$. □

Derivation of the Model Extension for Overbooking (§3.4):
With the new booking constraint on the premium rooms, there are four different sets of possible outcomes (instead of the three in the base model) for a given price set $(p_S, p)$ and a realized market size $x$: In $O_1^W$, the hotel can fully satisfy the premium room and standby upgrade demand ($D_P \le 1-w$ and $D_S + D_P \le 1$). In $O_2^W$, the hotel can fully satisfy the premium room demand ($D_P \le 1-w$), but can only partially satisfy the standby upgrade demand ($D_S + D_P > 1$). In $O_3^W$, the premium room demand exceeds the new booking constraint ($D_P > 1-w$), but the standby upgrade demand is fully satisfied ($D_S \le w$). In $O_4^W$, the premium room demand exceeds the booking constraint and the hotel can only partially satisfy standby upgrade demand ($D_S > w$).[1] Using a similar approach as in the base model, we can write the conditions for each outcome, the premium room capacity allocated to premium room bookings and standby upgrades, and resulting revenue as follows:

| $x$ | Outcome | $(C_P, C_S)$ | Revenue |
|---|---|---|---|
| $0 \le x \le \frac{1-w}{1-p}$ and $0 \le x \le \frac{1}{1-p_S}$ | $O_1^W$ | $(D_P, D_S)$ | $R_1^W = (x(1-p))p + (x(p-p_S))p_S$ |
| $\frac{1}{1-p_S} < x \le \frac{1-w}{1-p}$ | $O_2^W$ | $(D_P, 1-D_P)$ | $R_2^W = (x(1-p))p + (1-x(1-p))p_S$ |
| $\frac{1-w}{1-p} < x \le \frac{w}{p-p_S}$ | $O_3^W$ | $(1-w, D_S)$ | $R_3^W = (1-w)p + (x(p-p_S))p_S$ |
| $\frac{1-w}{1-p} < x$ and $\frac{w}{p-p_S} < x$ | $O_4^W$ | $(1-w, w)$ | $R_4^W = (1-w)p + wp_S$ |

Note that no strategy can lead to both $O_2^W$ and $O_3^W$ for the given conditions. Therefore, we can show that five possible strategies exist:

**Strategy 1**: Choosing a price set $(p_S, p)$ leading to an outcome set $\{O_1^W\}$ where the hotel's expected revenue is $\Pi_{W1} = \int_0^{X_U} \frac{R_1^W}{X_U} dx$.

**Strategy 2**: Choosing a price set $(p_S, p)$ leading to an outcome set $\{O_1^W, O_2^W\}$ where the hotel's expected revenue is $\Pi_{W2} = \int_0^{1/(1-p_S)} \frac{R_1^W}{X_U} dx + \int_{1/(1-p_S)}^{X_U} \frac{R_2^W}{X_U} dx$.

**Strategy 3**: Choosing a price set $(p_S, p)$ leading to an outcome set $\{O_1^W, O_3^W\}$ where the hotel's expected revenue is $\Pi_{W3} = \int_0^{(1-w)/(1-p)} \frac{R_1^W}{X_U} dx + \int_{(1-w)/(1-p)}^{X_U} \frac{R_3^W}{X_U} dx$.

**Strategy 4**: Choosing a price set $(p_S, p)$ leading to an outcome set $\{O_1^W, O_3^W, O_4^W\}$ where the hotel's expected revenue is $\Pi_{W4} = \int_0^{(1-w)/(1-p)} \frac{R_1^W}{X_U} dx + \int_{(1-w)/(1-p)}^{w/(p-p_S)} \frac{R_3^W}{X_U} dx + \int_{w/(w-p_S)}^{X_U} \frac{R_4^W}{X_U} dx$.

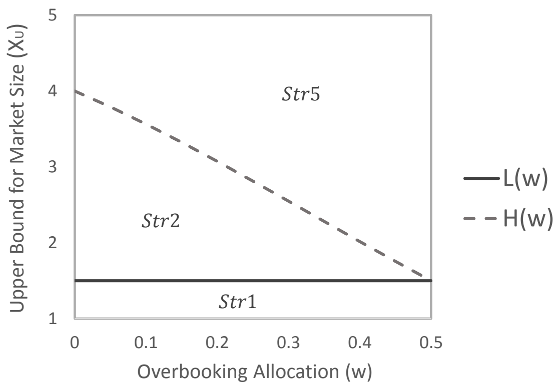
**Strategy 5**: Choosing a price set $(p_S, p)$ leading to an outcome set $\{O_1^W, O_2^W, O_4^W\}$ where the hotel's expected revenue is $\Pi_{W5} = \int_0^{1/(1-p_S)} \frac{R_1^W}{X_U} dx + \int_{1/(1-p_S)}^{(1-w)/(1-p)} \frac{R_2^W}{X_U} dx + \int_{(1-w)/(1-p)}^{X_U} \frac{R_4^W}{X_U} dx$.

The following lemma presents the optimal strategies for a given set of $X_U$ and $w$.

Lemma 1. *When the hotel allocates $w \in [0, 0.5]$ of its premium room capacity to satisfy the excess standard room demand and utilizes standby upgrades, the hotel should follow Strategy 1 when $X_U \le L(w) = \frac{3}{2}$, Strategy 5 for $X_U \ge H(w) = 4(1+w)(1-w)^2$, and Strategy 2 otherwise. (see Figure 1 for $L(w)$ and $H(w)$).*

[1] Note that as $w \to 0$, $O_4^W$ converges to $O_3$ in our base model with $(C_P, C_S) = (1, 0)$.

**Figure 1 Bounds of the Optimal Strategies for Different $w$ and $X_U$ values**




Proof of Lemma 4:

For Strategy 1, the expected revenue function can be written as:

$$\Pi_{W1} = \frac{X_U}{2}[p_S p - p_S^2 + p - p^2] \text{ where } X_U(1-p_S) \leq 1 \text{ and } X_U(1-p) \leq 1-w$$

Note that this function is identical to $\Pi_{MY1}$ in Lemma 1. Therefore, the optimal price set is $(p_S, p) = (\frac{1}{3}, \frac{2}{3})$ for the unconstrained problem. The constraints can be rewritten as $X_U \leq 3(1-w)$ and $X_U \leq \frac{3}{2}$. Since we only consider $w \in [0, 0.5]$, the optimal price set for the unconstrained problem is the optimal price set for the constrained problem only if $X_U \leq \frac{3}{2}$. Parallel to Lemma 1, Strategy 1 is the optimal strategy for the maximization problem only when $X_U \leq \frac{3}{2}$. Let $L(w) = \frac{3}{2}$ denote the upper bound of $X_U$ for Strategy 1.

For Strategy 5, the expected revenue function can be written as:

$$\Pi_{W5} = \frac{-2p^2(1-p_S-1)(1-w)X_U + p\left(2p_S^2 wX_U + p_S((w-2)w - 2X_U + 2) - (w-1)(w+2X_U-1)\right)}{2(1-p)(1-p_S)X_U}$$
$$-\frac{p_S\left(p_S\left(2wX_U + (1-w)^2\right) + w(w+2X_U-2)\right)}{2(1-p)(1-p_S)X_U} \text{ where } X_U(1-p) \geq 1-w \text{ and } X_U(p-p_S) \geq w$$

FOC give us (the constant is ignored):

$$\frac{\partial \Pi_{W5}}{\partial p} = \frac{(p_S-1)(1-w)^2}{2(1-p)^2 X_U} - w + 1 = 0 \quad \text{and}$$

$$\frac{\partial \Pi_{W5}}{\partial p_S} = \frac{p\left(2(1-p_S)^2 wX_U - 1\right) + p_S^2\left(-\left(2wX_U + (1-w)^2\right)\right) + 2p_S\left(2wX_U + (1-w)^2\right) - w(w+2X_U-2)}{2(p-1)(1-p_S)^2 X_U} = 0$$

Note that Strategy 3 in Lemma 1 is a special case of Strategy 5 when $w = 0$. Using a similar approach as in Lemma 1, the constraints are satisfied when $X_U$ is greater than some lower bound $H(w)$ for the optimal price set for the unconstrained problem. Similarly, the constraint $X_U(1-p) = 1-w$ must be binding for $X_U \leq H(w)$. Solving the binding constraint and FOC gives us $H(w) = 4(1+w)(1-w)^2$. Parallel to Lemma 1, Strategy 5 is the optimal strategy for the maximization problem only when $X_U \geq 4(1+w)(1-w)^2$. The proof on the suboptimality of Strategy 3 and Strategy 4 when $w \leq 0.5$ is available from authors upon request. □

Proof of Proposition 2:

For the first part of the proposition, it is easy to see $\Pi_{W1} = \Pi_{MY1}$ and $\Pi_{W2} = \Pi_{MY2}$. The optimal price sets are identical for the two unconstrained problems with the same objective function. Following the lemma above, $p(w) = p^{MY}$ and $p_S(w) = p_S^{MY}$ when $X_U \leq H(w)$ (when Strategy 1 or 2 is optimal). Given $X_U(1-p) \leq 1-w$ is not violated, $C_P$ and $C_S$ are the same for both problems resulting in no change in the expected revenues.

The proof of the price comparisons in the second part of the proposition is straightforward but lengthy, therefore available from the authors upon request. For the expected revenue comparisons, let $\Pi(p_S,p,w)$ denote the expected revenue of a price set $(p_S,p)$ for a given $w$. Consider a hotel using a price set $\big(p_S^*(w),p^*(w)\big)$ which is optimal for a given $X_U$ and $w$ when Strategy 5 is used. When $O_1$ or $O_2$ is observed, $C_P$ and $C_S$ do not change in $w$. On the other hand, recall that $(C_P,C_S)=(1-w,w)$ when $O_4$ is observed. In this outcome, $C_P$ decreases in $w$ while $C_S$ increases. Given $p>p_S$ and $R_4^W=(1-w)p+wp_S$, $\Pi\big(p_S^*(w),p^*(w),w\big)<\Pi\big(p_S^*(w),p^*(w),w-\Delta\big)$ where $\Delta$ is a very small positive number. Since we have $\Pi\big(p_S^*(w),p^*(w),w-\Delta\big)\leq\Pi\big(p_S^*(w-\Delta),p^*(w-\Delta),w-\Delta\big)$ given that the latter is the optimal expected revenue for $\theta+\Delta$, we can deduce $\Pi\big(p_S^*(w),p^*(w),w\big)<\Pi\big(p_S^*(w-\Delta),p^*(w-\Delta),w-\Delta\big)$. Thus, we conclude that $\Pi(w)$ decreases in $w$. □

Proof of Lemma 3:

In this proof, we show the optimal strategy for strategic guests for a given $X_U$.

For each $X_U$ value, we have three candidate pricing strategies for the optimal price set. These strategies have different expected revenue functions and constraints.

- For Strategy 1 where $X_U(1-p_S)\leq 1$, the expected revenue function is $\Pi_{ST1}=\int_0^{X_U}\frac{R_1}{X_U}dx$.

Since premium room bookings and standby upgrade demand are fully satisfied in this strategy, $r'=1$ ($v^*=\infty$) and $z=1$. Therefore, $R_1$ simplifies to $(x(1-p_S))p_S$, and $\Pi_{ST1}=\Pi_{NS1}$.

- For Strategy 2 where $X_U(1-p_S)>1$ and $X_U(1-z)\leq 1$, the expected revenue function is:

$$\Pi_{ST2}=\int_0^{1/(1-p_S)}\frac{R_1}{X_U}dx+\int_{1/(1-p_S)}^{X_U}\frac{R_2}{X_U}dx$$

There are two cases:

— $r'\geq\frac{1-p}{1-p_S}$. This results in $z=1$ and the following constraint holds:

$$(1-p)\leq(1-p_S)\left[\int_0^{1/(1-p_S)}\frac{1}{X_U}dx+\int_{1/(1-p_S)}^{X_U}\frac{1}{x(1-p_S)}\frac{1}{X_U}dx)\right]$$

In this case, $R_1$ simplifies to $(x(1-p_S))p_S$ and $R_2$ simplifies to $p_S$, therefore $\Pi_{ST2a}=\Pi_{NS2}$.

— $r'<\frac{1-p}{1-p_S}$. This results in $z<1$ and the following constraint holds:

$$(z-p)=(z-p_S)\left[\int_0^{1/(1-p_S)}\frac{1}{X_U}dx+\int_{1/(1-p_S)}^{X_U}\frac{1-x(1-z)}{x(z-p_S)}\frac{1}{X_U}dx\right]$$

Note that the constraint can be simplified to $X_U(1-p)=1+log(X_U(1-p_S))$. Through this constraint and the condition $X_U(1-p_S)>1$ , we get $p<\frac{X_U-1}{X_U}$ and $p_S<\frac{X_U-1}{X_U}$. Since $z$ disappears from the constraint, the equilibrium is not unique. However, the smallest possible $z$ results in the highest expected revenue, as it implies that more people will directly book the premium rooms. Given $p_S<\frac{X_U-1}{X_U}$, the expected revenue for a given price set $(p_S,p)$ reaches its upper bound at $z=\frac{X_U-1}{X_U}$ (i.e. the $X_U(1-z)\leq 1$ constraint is binding).

- For Strategy 3 where $X_U(1-z) > 1$, the expected revenue function is:

$$\Pi_{ST3} = \int_{0}^{1/(1-p_S)} \frac{R_1}{X_U} dx + \int_{1/(1-p_S)}^{1/(1-z)} \frac{R_2}{X_U} dx + \int_{1/(1-z)}^{X_U} \frac{R_3}{X_U} dx$$

The following constraint holds:

$$(z-p) = (z-p_S))\left[\int_{0}^{1/(1-p_S)} \frac{1}{X_U} dx + \int_{1/(1-p_S)}^{1/(1-z)} \frac{1-x(1-z)}{x(z-p_S)} \frac{1}{X_U} dx)\right]$$

which can be simplified to $X_U(z-p) = log\left(\frac{1-p_S}{1-z}\right)$.

Let $\Pi_{ST3}(p_S,p,z)$ and $\Pi_{ST2b}(p_S,p,z)$ denote the expected revenues of a price set $(p_S,p)$ and a $z$ value satisfying the constraints of Strategy 3 and the second case of Strategy 2, respectively. For a given $X_U$, the upper bound for $\Pi_{ST2b}(p_S,p,z)$ is $\Pi_{ST2b}\left(p_S^*,p^*,\frac{X_U-1}{X_U}\right)$ where $(p_S^*,p^*)$ is the optimal price set of the second scenario of Strategy 2. The set $(p_S^*,p^*,z)$ asymptotically satisfies the constraints of Strategy 3, therefore $\Pi_{ST3}\left(p_S^*,p^*,\frac{X_U-1}{X_U}\right) = \Pi_{ST2b}\left(p_S^*,p^*,\frac{X_U-1}{X_U}\right)$. Given $\Pi_{ST3}\left(p_S^*,p^*,\frac{X_U-1}{X_U}\right) \leq \Pi_{ST3}(p_S^{**},p^{**},z^{**})$ where the left-hand side is the expected revenue of a feasible price set under Strategy 3 while the right-hand-side is the optimal expected revenue under Strategy 3, $\Pi_{ST2b} \leq \Pi_{ST3}$. Thus, Strategy 2 is weakly dominated by Strategy 3, and we ignore Strategy 2 from further consideration.

Now, we can compare Strategy 1, the first scenario of Strategy 2 and Strategy 3 under two cases of $X_U$:

- $X_U \leq 2$: In this range, $\Pi_{NS2} \leq \Pi_{NS1}$; therefore, Strategy 1 (with the optimal $p_S = \frac{X_U-1}{X_U}$) dominates the first scenario of Strategy 2. Consider a hotel trying to implement Strategy 3. Given a low $X_U$ and its resulting high $r'$ values in this range, the asymptotic upper bound of the expected revenue under Strategy 3 is the expected revenue under $(p,p_S,z) = \left(\frac{X_U-1}{X_U},\frac{X_U-1}{X_U},\frac{X_U-1}{X_U}\right)$, which is also the optimal expected revenue under Strategy 2. Therefore, Strategy 1 is the optimal strategy in this range.
- $X_U > 2$: In this range, $\Pi_{NS2} > \Pi_{NS1}$; therefore, the first scenario of Strategy 2 (with an optimal $p_S = \frac{\sqrt{2X_U}-1}{\sqrt{2X_U}}$) dominates Strategy 1. Consider a hotel that chooses a price set $\left(\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}},\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}}+\Delta\right)$, where $\Delta$ is a very small positive number. In this case, we can have $z<1$, i.e., some of the guests with $v > \frac{\sqrt{2X_U}-1}{\sqrt{2X_U}}+\Delta$ book a premium room, therefore $\Pi_{NS2} < \Pi_{ST3}\left(\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}},\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}}+\Delta,z\right)$. Given $\Pi_{ST3}\left(\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}},\frac{\sqrt{2X_U}-1}{\sqrt{2X_U}}+\Delta,z\right) \leq \Pi_{ST3}(p_S^{**},p^{**},z^{**})$ where the left-hand side is the expected revenue of a feasible price set under Strategy 3 while the right-hand-side is the optimal expected revenue under Strategy 3, $\Pi_{NS2} < \Pi_{ST3}(p_S^{**},p^{**},z^{**})$. Thus, Strategy 3 is the optimal strategy in this range. □

Proof of Proposition 3:

In this proof, we compare the expected revenues of the strategic case with the no-standby benchmark and the myopic case for a given $X_U$.

Let $\widetilde{\Pi}(p_S,p)$ denote the expected revenue of the price set $(p_S,p)$ when guests are myopic and $\widehat{\Pi}(p_S,p)$ denote the expected revenue of the price set $(p_S,p)$ when guests are strategic.

- $X_U \leq 2$: In this case, we know from Lemma 3 that a hotel facing strategic guests should use Strategy 1, i.e., a price set where all guests choose standby upgrades, and $\Pi_{ST1} = \Pi_{NS1}$. Given $\Pi_{NS} < \widetilde{\Pi}(p_S^{MY},p^{MY})$, we deduce $\Pi_{ST1} < \widetilde{\Pi}(p_S^{MY},p^{MY})$.

- $X_U > 2$: In this case, we know from Lemma 3 that the hotel's optimal strategy is to choose a price set that leads to $z < 1$. We make two comparisons for this range of $X_U$ values:

⇒ Comparison of $\Pi_{ST}$ and $\Pi_{NS}$: Given $z < 1$, standby upgrade program price discriminates. Consider a hotel facing strategic guests that uses a standby upgrade program and chooses a price set $(p^{NS}, p^{NS} + \Delta)$ where $\Delta$ is a very small positive number. $\widehat{\Pi}(p^{NS}, p^{NS} + \Delta) > \Pi_{NS}$, since some of the guests with $v > p^{NS} + \Delta$ book a premium room. Given $\widehat{\Pi}(p^{NS}, p^{NS} + \Delta) \leq \widehat{\Pi}(p_S^{ST}, p^{ST})$, we find $\widehat{\Pi}(p_S^{ST}, p^{ST}) > \Pi_{NS}$.

⇒ Comparison of $\Pi_{ST}$ and $\Pi_{MY}$: Consider a hotel using the optimal price set for the strategic case, i.e., $(p_S^{ST}, p^{ST})$. Given $1 - z < 1 - p^{ST}$, the number of premium room bookings when guests are myopic is greater than the number of premium room bookings when guests are strategic. Therefore, $\widehat{\Pi}(p_S^{ST}, p^{ST}) < \widetilde{\Pi}(p_S^{ST}, p^{ST})$. Since $\widetilde{\Pi}(p_S^{ST}, p^{ST}) \leq \widetilde{\Pi}(p_S^{MY}, p^{MY})$ as the latter is the optimal profit for the strategic customer case, we find $\widetilde{\Pi}(p_S^{MY}, p^{MY}) > \widehat{\Pi}(p_S^{ST}, p^{ST})$. □

Proof of Proposition 4:
In this proof, we show that strategic guests never book premium rooms if the hotel uses the optimal price set in the myopic case for a given $X_U$. Let us evaluate $X_U \leq 1.5$ and $X_U > 1.5$ separately:

- $X_U \leq 1.5$: In this case, the hotel uses Strategy 1, i.e., fully satisfies the premium room and standby upgrade demand. Therefore, $r' = r = 1$ and all strategic guests choose standby upgrades.
- $X_U > 1.5$: In this case, the hotel uses Strategy 2 or Strategy 3. For a given $X_U$, we need to show that the price set $(p_S^{MY}, p^{MY})$ always leads to $z = 1$. In order to show this, we test whether the utility of choosing a standby upgrade dominates booking a premium room for all guests. For a given $(p_S^{MY}, p^{MY}, X_U)$ set, the following inequality always holds:

$$(1 - p^{MY}) \leq (1 - p_S^{MY}) \left[ \int_0^{1/(1-p_S^{MY})} \frac{1}{X_U} dx + \int_{1/(1-p_S^{MY})}^{X_U} \frac{1}{X_U(1 - p_S^{MY})x} dx \right]$$

where the left-hand side of the inequality is the utility of booking a premium room for a guest with $v = 1$ and the right hand side of the inequality is the utility of choosing a standby upgrade for a guest with $v = 1$ times her expectation on the fill rate. Given that this inequality always holds, even a guest with $v = 1$ would choose the standby upgrade offer. Therefore, standby upgrades fully cannibalize the premium room bookings. □