Online Supplement to "Is reshoring better than offshoring? The effect of offshore supply dependence"

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Proof of Proposition 1. The general result with $\delta \geq 0$ follows from Propositions A3 with $e = 0$ (no OSD is equivalent to having free expedited shipping). The special result with $\delta = 0$ follows from the argument that a reshoring manufacturer can replicate any offshoring production quantity at no higher costs. □

Proof of Proposition 2. Case (a) follows from Proposition A3. Cases (b) and (c) follow from Proposition A4. □

Proof of Lemma A1. By (A3), it is straightforward to verify that $\gamma_U^0(\Delta)$ is increasing in Δ, $\gamma_D^0(\Delta)$ is decreasing in Δ, $\gamma_U^0 = 0$ for $\Delta \leq z^0 - z_U^0$, and $\gamma_D^0 = 1$ for $\Delta \leq z_D^0 - z^0$. Therefore, $\gamma_U^0(\Delta)$ and $\gamma_D^0(\Delta)$ intersect at most once. Since when $\Delta^0 = z_D^0 - z_U^0$, $\gamma_U^0(\Delta^0) = \gamma_D^0(\Delta^0) = \gamma^0 = \frac{c+m_0}{c+m_0+(1+t_g)r}$, (Δ^0, γ^0) characterizes the intersection.

Recall that $\Delta \leq \Delta^0 = z_D^0 - z_U^0$ implies $\mu_H + \sigma z_U^0 \leq \mu_L + \sigma z_D^0$. From the analysis preceding the lemma, when $x_m^0 \in [\mu_H + \sigma z_U^0, \mu_L + \sigma z_D^0]$, the manufacturer would do nothing after learning the demand type. The problem thus becomes $\max_{x_m^0 \geq 0} \{-((1+t_g)(c+m_0)+s_g)x_m^0 + p\mathbb{E}[\min\{\Psi, x_m^0\}]\}$. It is straightforward to show that the optimal solution solves the FOC: $\gamma\Phi((x-\mu_H)/\sigma) + (1-\gamma)\Phi((x-\mu_L)/\sigma) = \Phi(z^0)$. Note that the left-hand-side of the FOC is increasing in x and decreasing in γ. Therefore, $x \geq \mu_H + \sigma z_U^0$ if and only if $\gamma \leq \gamma'$, where γ' is determined by

$$\gamma'\Phi((\mu_H + \sigma z_U^0 - \mu_H)/\sigma) + (1-\gamma')\Phi((\mu_H + \sigma z_U^0 - \mu_L)/\sigma) = \Phi(z^0) \Rightarrow \gamma' = \frac{\Phi(z_U^0 + \Delta) - \Phi(z^0)}{\Phi(z_U^0 + \Delta) - \Phi(z_U^0)}$$

which is the same as the threshold $\gamma_U^0(\Delta)$ defined in (A3).

Symmetrically, $x \leq \mu_L + \sigma z_D^0$ if and only if $\gamma \leq \overline{\gamma}$, where $\overline{\gamma}$ is determined by

$$\overline{\gamma}\Phi((\mu_L + \sigma z_D^0 - \mu_H)/\sigma) + (1-\overline{\gamma})\Phi((\mu_L + \sigma z_D^0 - \mu_L)/\sigma) = \Phi(z^0) \Rightarrow \overline{\gamma} = \frac{\Phi(z_D^0) - \Phi(z^0)}{\Phi(z_D^0) - \Phi(z_D^0 - \Delta)}$$

which is the same as the threshold $\gamma_D^0(\Delta)$ defined in (A3). Therefore, we conclude that $\mu_H + \sigma z_U^0 \leq x_m^{0*} \leq \mu_L + \sigma z_D^0$ if and only if $\gamma_U^0(\Delta) \leq \gamma \leq \gamma_D^0(\Delta)$. □

Proof of Proposition A1. We first introduce a relation that follows straightforward integration by parts. Recall that Φ and ϕ denote the standard normal cumulative distribution and probability density functions, respectively. Suppose Ψ follows a normal distribution with mean μ and standard deviation σ. Then

$$\int_{-\infty}^{x} \frac{\psi}{\sqrt{2\pi}\sigma} \mathrm{e}^{-\frac{(\psi-\mu)^2}{2\sigma^2}} \,\mathrm{d}\psi = \mu\Phi\left(\frac{x-\mu}{\sigma}\right) - \sigma\phi\left(\frac{x-\mu}{\sigma}\right). \tag{S1}$$

Case I: $\Delta \le \Delta^0$ and $\gamma_U^0 \le \gamma \le \gamma_D^0$. Due to Lemma A1, we know that the manufacturer would do nothing after learning the demand type. Therefore, the optimal solution is $x_m^{0*} = x_{sH}^{0*} = x_{sL}^{0*} = x$, where x uniquely solves $\gamma\Phi((x-\mu_H)/\sigma) + (1-\gamma)\Phi((x-\mu_L)/\sigma) = \Phi(z^0)$. The optimal profit is $-((1+t_g)(c+m_0)+s_g)x_m^{0*} + p\mathbb{E}[\min\{\Psi, x_m^{0*}\}]$.

Case II: $\Delta \le \Delta^0$ and $\gamma < \gamma_U^0$, or $\Delta > \Delta^0$ and $\gamma \le \gamma^0$. First consider the subcase of $\Delta \le \Delta^0$ and $\gamma < \gamma_U^0$. Due to Lemma A1, we know that $x_m^{0*} < \mu_H + \sigma z_U^0 \le \mu_L + \sigma z_D^0$, hence it is optimal for the manufacturer to adjust the final inventory level upward to $\mu_H + \sigma z_U^0$ if the demand type turns out to be high and do nothing if it turns out to be low. The problem becomes the following:

$$\Pi_m^0 \doteq \max_{x_m^0 \ge 0}\{-(c+m_0)x_m^0 + \gamma\Pi_s^0(\mu_H, x_m^0) + (1-\gamma)\Pi_s^0(\mu_L, x_m^0)\}, \tag{S2}$$

where $\Pi_s^0(\mu_H, x_m^0) = -(1+t_g)(c+m_0+r)(x_{sH}^{0*} - x_m^0) - t_g(c+m_0)x_m^0 - s_g x_{sH}^{0*} + p\mathbb{E}[\min\{\Psi, x_{sH}^{0*}\}|\mu_H]$ with $x_{sH}^{0*} = \mu_H + \sigma z_U^0$, $\Pi_s^0(\mu_L, x_m^0) = -(t_g(c+m_0)+s_g)x_m^0 + p\mathbb{E}[\min\{\Psi, x_m^0\}|\mu_L]$. One can show that the optimal x_m^{0*} solves the following FOC: $\Phi((x_m^0 - \mu_L)/\sigma)) = [\gamma(1+t_g)r + (1-\gamma)(p-(1+t_g)(c+m_0)-s_g)]/[(1-\gamma)p]$. Therefore, $x_m^{0*} = \mu_L + \sigma z_{mU}^0$. Plugging x_m^{0*} into Π_m^0 and utilizing (S1), we obtain the optimal profit

$$\Pi_m^0 = p\sigma\gamma\Phi(z_U^0)\Delta - p\sigma[\gamma\phi(z_U^0) + (1-\gamma)\phi(z_{mU}^0)] + p\Phi(z^0)\mu_L. \tag{S3}$$

Now consider the subcase of $\Delta > \Delta^0$ and $\gamma \le \gamma^0$. Recall that $\Delta > \Delta^0$ implies $\mu_L + \sigma z_D^0 < \mu_H + \sigma z_U^0$, hence if $x_m^0 \le \mu_L + \sigma z_D^0$, then the manufacturer would adjust the final inventory level upward if the demand type turns out to be high and do nothing if it turns out to be low. The problem becomes the same as (S2), and it follows that $x_m^{0*} = \mu_L + \sigma z_{mU}^0$. It is straightforward to verify that z_{mU}^0 increases from z to z_D^0 as γ increases from 0 to $\gamma^0 = \frac{c+m_0}{c+m_0+(1+t_g)r}$. Therefore, when $\Delta > \Delta^0$ and $\gamma \le \gamma^0$, the optimal solution is $x_m^{0*} = \mu_L + \sigma z_{mU}^0$ and the optimal profit is given by (S3).

Case III: $\Delta \le \Delta^0$ and $\gamma > \gamma_D^0$, or $\Delta > \Delta^0$ and $\gamma > \gamma^0$. Symmetric to Case II, we can show that it is optimal for the manufacturer to adjust the final inventory level downward to $\mu_L + \sigma z_D^0$ if the demand type turns out to be low and do nothing if it turns out to be high. The problem becomes the following: $\Pi_m^0 \doteq \max_{x_m^0 \ge 0}\{-(c+m_0)x_m^0 + \gamma\Pi_s^0(\mu_H, x_m^0) + (1-\gamma)\Pi_s^0(\mu_L, x_m^0)\}$, where $\Pi_s^0(\mu_H, x_m^0) = -((1+t_g)(c+m_0)+s_g)x_\psi^{0*} + p\mathbb{E}[\min\{\Psi, x_\psi^{0*}\}|\mu_H]$, and $\Pi_s^0(\mu_L, x_m^0) = -((1+t_g)(c+m_0)+s_g)x_{sL}^{0*} + p\mathbb{E}[\min\{\Psi, x_{sL}^{0*}\}|\mu_L]$ with $x_{sL}^{0*} = \mu_L + \sigma z_D^0$. Similar to Case II, the optimal x_m^{0*} solves the following FOC: $\Phi((x_m^0 - \mu_H)/\sigma)) = [\gamma(p-s_g) - (1+\gamma t_g)(c+m_0)]/(\gamma p)$. Therefore, the optimal solution is $x_m^{0*} = x_{sH}^{0*} = \mu_H + \sigma z_{mD}^0$, $x_{sL}^{0*} = \mu_L + \sigma z_D^0$. Plugging x_m^{0*} into Π_m^0 and utilizing (S1), we obtain the optimal profit $\Pi_m^0 = p\sigma\gamma\Phi(z_{mD}^0)\Delta - p\sigma[\gamma\phi(z_{mD}^0) + (1-\gamma)\phi(z_D^0)] + p\Phi(z^0)\mu_L$. $\square$

Proof of Proposition A3. We show the proof in four parts. First we show that the manufacturer prefers to remain offshoring for sufficiently small Δ. Second we show the manufacturer's limit preference for sufficiently large Δ. Third we show that if the manufacturer's preference for $\Delta \to 0$ and $\Delta \to \infty$ differs, then as Δ increases, the manufacturer's preference changes exactly once. Finally we show that if the manufacturer's preference for $\Delta \to 0$ and $\Delta \to \infty$ is the same, then as Δ increases, the manufacturer's preference never changes.

We use short-hand notations $\Pi^i_X(\Delta,\gamma)$ to denote the offshoring ($i=0$) or reshoring ($i=1$) manufacturer's profit expressions in the three cases ($X=N,U,D$) presented in Propositions A1 and A2. For example, $\Pi^0_N(\Delta,\gamma) = -b_0 x_m^{0*} + p\mathbb{E}[\min\{\Psi, x_m^{0*}\}]$ and $\Pi^1_N(\Delta,\gamma) = -b_1 x_c^{1*} + p\mathbb{E}[\min\{\Psi, x_c^{1*}\}]$. We also denote the inverse functions of $\gamma^i_X(\Delta)$ by $\Delta^i_X(\gamma)$.

I. When $\Delta < \min(\Delta^0_U(\gamma), \Delta^1_U(\gamma), \Delta^0_D(\gamma), \Delta^0_D(\gamma))$, both offshoring and reshoring are in Case N, namely the manufacturer uses neither flexibility regardless of the demand type. Since $b_0 \le b_1$, in this case an offshoring manufacturer could mimic the reshoring strategy at no higher costs and receive the same revenue, thus the manufacturer prefers to remain offshoring.

II. When $\Delta \to \infty$, each of offshoring and reshoring will be either in Case U or Case D, and the Π terms' derivatives with respect to Δ determine the limit profit comparisons. The conditions in the proposition follow straightforwardly from Lemma A3.

III. We make an important observation that for any γ, the manufacturer's profit is concave in Δ. We already know in Cases U and D the profits are linear in Δ. For Case N, take the example of offshoring and $\gamma < \gamma^0$ so as Δ increases the solution case shifts from N to U (the proof for the other cases are similar). Consider any $\Delta < \Delta^0_U(\gamma)$, and we know $\Pi^0_U(\Delta,\gamma) < \Pi^0_N(\Delta,\gamma)$. Since Π^0_U is decreasing in u_0, there must exist a $u_0' < u_0$ such that $\Pi^0_U(\Delta,\gamma,u_0') = \Pi^0_N(\Delta,\gamma,u_0)$. Therefore, we know that $\mathrm{d}\Pi^0_N(u_0)/\,\mathrm{d}\Delta = \mathrm{d}\Pi^0_U(u_0')/\,\mathrm{d}\Delta = p\sigma\gamma(p - b_0 - u_0')$. Such u_0' must increase in Δ. As a result, we know that Π^0_N is concave in Δ. Finally note that $\Pi^0_U(\Delta,\gamma) < \Pi^0_N(\Delta,\gamma) \Leftrightarrow \Delta < \Delta^0_U(\gamma)$, which guarantees that the left- and right-derivatives at $\Delta^0_U(\gamma)$ are equal, and thus Π^0 is first concave then linear, and is generally concave. The proof for the other cases are similar.

Using this property, we show that if the manufacturer's preference for $\Delta \to 0$ and $\Delta \to \infty$ differs, then as Δ increases, the manufacturer's preference changes exactly once. When both offshoring and reshoring are in Case N, we know that the manufacturer's preference does not change in Δ. When one production mode is out of Case N, for example offshoring is in Case U, then $\Pi^0_U(\Delta)$ is linear and $\Pi^1(\Delta)$ is generally concave and increasing. When $\Pi^0_U(0) < \Pi^1(0)$ and $\Pi^0_U(\infty) > \Pi^1(\infty)$, or vice versa, it follows that they intersect exactly once.

IV. Take the example of a small γ such that as Δ increases, both offshoring and reshoring shift from Case N to Case U (the proof for the other cases are similar). We need to show that if $\Pi_N^0(0) > \Pi_N^1(0)$ and $\Pi_U^0(\infty) > \Pi_N^1(\infty)$, then $\Pi_{N/U}^0(\Delta)$ and $\Pi_{N/U}^1(\Delta)$ do not intersect.

We already know from Bullet I that for $\Delta \leq \min(\Delta_U^0(\gamma), \Delta_U^1(\gamma))$, $\Pi_N^0(\Delta)$ and $\Pi_N^1(\Delta)$ do not intersect, and only need to prove they do not intersect for $\Delta > \min(\Delta_U^0(\gamma), \Delta_U^1(\gamma))$. First consider $\Delta_U^1(\gamma) < \Delta_U^0(\gamma)$. In this case, we know for $\Delta > \Delta_U^1(\gamma)$, reshoring is always in Case U, and offshoring shifts from Case N to Case U at $\Delta_U^0(\gamma)$. We also know from Bullet III that in this region $\Pi_U^1(\Delta)$ is linear and $\Pi_{N/U}^0(\Delta)$ is concave. Recall that $\Pi_N^0(\Delta_U^1(\gamma)) > \Pi_U^0(\Delta_U^1(\gamma))$, and $\Pi_U^0(\infty) > \Pi_U^1(\infty)$. One can easily see graphically that they do not intersect in this region.

Next consider $\Delta_U^0(\gamma) < \Delta_U^1(\gamma)$. In this case, we know for $\Delta > \Delta_U^0(\gamma)$, offshoring is always in Case U, and reshoring shifts from Case N to Case U at $\Delta_U^1(\gamma)$. For $\Delta \in [\Delta_U^0(\gamma), \Delta_U^1(\gamma)]$, we know that $\Pi_U^0(\Delta) > \Pi_N^0(\Delta) \geq \Pi_N^1(\Delta)$ (the latter due to $b_0 \leq b_1$). Therefore, $\Pi_U^0(\Delta)$ and $\Pi_N^1(\Delta)$ do not intersect in this region. For $\Delta > \Delta_U^1(\gamma)$, note that $\Pi_U^0(\Delta_U^1(\gamma)) > \Pi_U^1(\Delta_U^1(\gamma))$, and $\Pi_U^0(\Delta)' > \Pi_U^1(\Delta)'$. Therefore, $\Pi_U^0(\Delta)$ and $\Pi_U^1(\Delta)$ do not intersect in this region. This concludes the proof that in generally, $\Pi_{N/U}^0(\Delta)$ and $\Pi_{N/U}^1(\Delta)$ do not intersect.

Finally, $\overline{\gamma} \leq \overline{\gamma}_B$ and $\Delta^*(\gamma) \geq \Delta_B^*(\gamma)$ are due to the following argument: a reshoring manufacturer without OSD can always replicate the optimal decisions of a reshoring manufacturer under OSD at no higher costs, and that an offshoring manufacturer's profit is not affected by OSD, thus accounting for OSD reduces the reshoring region. □

Proof of Proposition A4. For the case of $d_0 \geq d_1$, the proof is similar to that of Proposition A3 and omitted. For the case of $d_0 < d_1$, we provide a sample-path proof. Note that under either offshoring or reshoring, the manufacturer's strategy can be described by three decisions: the inventory obtained regularly; the inventory obtained with expedition; and the components/finished goods discarded before the selling season (not including inventory discarded after the selling season, which is out of the manufacturer's control). The costs associated with the three decisions are b_i, $b_i + u_i$, and b_i, where $i = 0, 1$ respectively represent offshoring and reshoring. Since $b_0 \leq b_1$, $b_0 + u_0 < b_1 + u_1$, and $d_0 < d_1$, an offshoring manufacturer could mimic any reshoring strategy at no higher costs and receive the same revenue, thus the manufacturer prefers to remain offshoring. Finally, $\overline{\gamma} \leq \overline{\gamma}_B$ and $\Delta^*(\gamma) \geq \Delta_B^*(\gamma)$ are due to the following argument: a reshoring manufacturer without OSD can always replicate the optimal decisions of a reshoring manufacturer under OSD at no higher costs, and that an offshoring manufacturer's profit is not affected by OSD, thus accounting for OSD reduces the reshoring region. □

Proof of Proposition A5. Cases (a) follows from the fact that $m \le s$ and $r \le e$ respectively ensure that reshoring has higher downward and upward inventory adjustment costs than offshoring. Cases (b) follows from the fact that $m > s$ and $r > e$ respectively ensure that reshoring has lower downward and upward inventory adjustment costs than offshoring. In Case (c) we have $m > s$ and $r \le e$. Due to Lemmas A1 and A2, we know that $\gamma_U^0(\Delta)$ is increasing in Δ and $\gamma_D^1(\Delta)$ is decreasing in Δ. It is easy to verify that $\gamma_U^0(\Delta)$ and $\gamma_D^1(\Delta)$ intersect at (Δ^*, γ^*), where $\Delta^* = z_D^1 - z_U^0$ and $\gamma^* = \frac{c+s}{c+s+r}$. It is also easy to verify that $\gamma_U^0(\Delta) > \gamma_U^1(\Delta)$, $\gamma_D^0(\Delta) > \gamma_D^1(\Delta)$, $\Delta^* < \min\{\Delta^0, \Delta^1\}$, and $\gamma^1 < \gamma^* < \gamma^0$, hence when $\Delta \le \Delta^*$, $\mu_H + \sigma z_U^1 \le \mu_H + \sigma z_U^0 \le \mu_L + \sigma z_D^1 \le \mu_L + \sigma z_D^0$.

We first consider Case 1: $\Delta \le \Delta^*$ and $\gamma_U^0(\Delta) \le \gamma \le \gamma_D^1(\Delta)$. Due to Lemmas A1 and A2, we know that $\mu_H + \sigma z_U^0 \le x_m^{0*} \le \mu_L + \sigma z_D^1$ and $\mu_H + \sigma z_U^0 \le x_c^{1*} \le \mu_L + \sigma z_D^1$. In this case, the manufacturer would do nothing after learning the demand type, and the offshoring and reshoring problems become identical, hence $x_m^{0*} = x_c^{1*}$ and the manufacturer is indifferent toward reshoring.

We next consider Case 2: $\Delta \le \Delta^*$ and $\gamma < \gamma_U^0(\Delta)$, or $\Delta > \Delta^*$ and $\gamma \le \gamma^*$. The first subcase is $\Delta \le \Delta^*$ and $\gamma < \gamma_U^0(\Delta)$, which has two possible scenarios: 1) $\gamma_U^1(\Delta) < \gamma < \gamma_U^0(\Delta)$, and 2) $\gamma \le \gamma_U^1(\Delta)$. In Scenario 1), due to Propositions A1 and A2, we know that if the demand type turns out to be high, an offshoring manufacturer would adjust the final inventory level upward, but a reshoring manufacturer would do nothing. Because an offshoring manufacturer always has the option of doing nothing, offshoring must yield higher profits than reshoring. In Scenario 2), due to Propositions A1 and A2, we know that if the demand type turns out to be high, both an offshoring and a reshoring manufacturer would adjust the final inventory level upward, which incurs r per unit under offshoring and e per unit under reshoring. Since $r \le e$, offshoring yields higher profits than reshoring. Therefore, the manufacturer prefers to remain offshoring in this subcase.

The second subcase is $\Delta > \Delta^*$ and $\gamma \le \gamma^*$. Recall that $\gamma^1 < \gamma^* < \gamma^0$. Due to Proposition A1, we know that an offshoring manufacturer would adjust the final inventory level upward if the demand type turns out to be high. On the other hand, due to Proposition A2, a reshoring manufacturer may do nothing, adjust the final inventory level upward if the demand type turns out to be high, or adjust the final inventory level downward if it turns out to be low. When a reshoring manufacturer does nothing or adjusts the final inventory level upward if the demand type turns out to be high, offshoring yields higher profits than reshoring following similar arguments as the first subcase.

It remains to compare offshoring and reshoring profits with $\Delta > \Delta^*$ and $\max\{\gamma_D^1(\Delta), \gamma^1\} < \gamma < \gamma^*$, when an offshoring manufacturer adjusts production upward if the demand type turns out to be high, and a reshoring manufacturer adjusts the component inventory level downward if the demand type turns out to be low. Due to Propositions A1 and A2, the optimal offshoring profit is given by

$\Pi_m^0 = p\sigma\gamma\Phi(z_U^0)\Delta - p\sigma[\gamma\phi(z_U^0) + (1-\gamma)\phi(z_{mu}^0)] + p\Phi(z)\mu_L = (p-c-m-s-r)\gamma\sigma\Delta - p\sigma[\gamma\phi(z_U^0) + (1-\gamma)\phi(z_{mu}^0)] + p\Phi(z)\mu_L$, whereas the optimal reshoring profit is given by $\Pi_c^1 = p\sigma\gamma\Phi(z_{cd}^1)\Delta - p\sigma[\gamma\phi(z_{cd}^1) + (1-\gamma)\phi(z_D^1)] + p\Phi(z)\mu_L = [(p-m)\gamma - (c+s)]\sigma\Delta - p\sigma[\gamma\phi(z_{cd}^1) + (1-\gamma)\phi(z_D^1)] + p\Phi(z)\mu_L$.. The difference between the two expressions is $\Pi_m^0 - \Pi_c^1 = \sigma[(c+s) - \gamma(c+s+r)]\Delta - p\sigma[\gamma\phi(z_U^0) + (1-\gamma)\phi(z_{mu}^0) - \gamma\phi(z_{cd}^1) - (1-\gamma)\phi(z_D^1)]$. Note that for any $\gamma' \in (\max\{\gamma_D^1(\Delta), \gamma^1\}, \gamma^*)$ where $\gamma^* = (c+s)/(c+s+r)$, the above expression is strictly increasing in Δ. Define $\tilde{\Delta}$ as the solution to $\gamma_D^1(\tilde{\Delta}) = \gamma'$. We know that at the point $(\tilde{\Delta}, \gamma')$, the reshoring manufacturer would do nothing regardless of the revealed demand type. Therefore, offshoring yields higher profits than reshoring. It then follows that offshoring yields strictly higher profits than reshoring for all $\Delta > \tilde{\Delta}$ and $\gamma' \in (\max\{\gamma_D^1(\Delta), \gamma^1\}, \gamma^*)$. Combining the above cases, we conclude that the manufacturer prefers to remain offshoring in Case 2.

This leaves Case 3: $\Delta \leq \Delta^*$ and $\gamma > \gamma_D^1(\Delta)$, or $\Delta > \Delta^*$ and $\gamma > \gamma^*$. The proof is similar to Case 2 and omitted.

The proof of Case (d) mirrors that of Case (c) and is omitted. $\square$